CHAPTER - I

DEVELOPMENT IN THE THEORETICAL CALCULATION OF ATOMIC PAIR PRODUCTION CROSS SECTION

1.1 INTRODUCTION

Pair production is due to the interaction of a photon in the nuclear Coulomb field or the field of an electron whereby a photon completely disappears and in its place appears a positron electron pair whose total energy is just equal to the energy of the photon.

$h\nu = (E_{+} + m_0c^2) + (E_{-} + m_0c^2)$ where E_+ and E_- are the kinetic energies of the leptons, $+$ being for the positron. For the interaction to take place in the nuclear field the energy of the incident photon must exceed 1.02 Mev. Threshold for pair production in the field of an electron is 4 mc^2. Presence of an electron or the nucleus is essential to conserve momentum.

As early as 1930 (Ch-30, Ta-30,32) it was apparent from the absorption measurements of 2.62 Mev photons from ThC" that the known photon matter interaction processes i.e. Compton and photo electric process were not sufficient to explain the experimental results. It was, therefore, evident from the anomalous experimental results that some sort of hitherto unknown type of interaction must be involved. Anderson and Neddermeyer (An-33) showed that 2.62 Mev photons from ThC" eject electron positron pairs when incident on lead foils. Combining this and other related observations Oppenheimer and Plessett (O-33) first offered an explanation of the observed anomaly of the experimental results on the basis of Dirac's electron theory, according to which negative energy states of the electron were assumed to be completely filled up. To lift an electron from negative energy states to that of a positive one 2 m_oc^2 energy is required, and removal of an electron from negative energy state creates a vacancy there and thus become observable but with positive charge, known as positron and the lifted particle is simply the electron. Thus a pair of negetron and positron can be created out of 2 m_oc^2 i.e. 1.02 Mev in the nuclear field. The phenomena of pair production is closely related to bremsstrahlung where an electron undergoes a transition between two states of positive energy and a photon is created instead of being absorbed.

Theoretical calculations pertaining to the pair production process in the field of a nucleus was offered shortly after Andersons discovery by Nishina and Tomanga (Ni-33), Oppenheimer and Plessett (O-33) and Heitter and Sauter (He-33). Later relativistic calculations for the production of electron positron pair by photons and for the related process of Bremsstrahlung were carried out by Bethe and Heitler (Be-34) which inspite of its plane wave approximation has been proved to be remarkably successful in predicting the important features of these process.

1.2 Bethe and Heitler

Quantum mechanical solution to the phenomena of pair production has been obtained by Bethe and Heitler (Be-34) using plane waves for both the electrons. When the interaction takes place in the field of the nucleus it is assumed that electrons surrounding the nucleus play no part in the kinetics of the reaction, their only effect being to screen the field of the nucleus if the reaction takes place at a distance comparable with the radius of the electron orbits. Such pair creation is sometimes called coherent or elastic pair production. Inelastic or incoherent pair production results when some of the energy transferred in the interaction is taken by an atomic electron for exitation or ionization. If the atom is ionized the interaction is called triplet production and can be approximated by considering the pair formation to take place in the field of a free electron. Although elastic and inelastic pair production have different final states they are independent of each other and cross section of the two processes are additive. Therefore any given form of the pair cross section such as σ or $\frac{d\sigma}{dE_+}$ is equal to the sum σ (elastic) + σ (inelastic).

The major contribution however is from elastic component except for very low Z- atoms. Bethe Heitler's treatment neglects screening effects of other electrons and interaction between the electrons and the nucleus is considered as only a small perturbation assuming plane waves for the electrons, i.e. Born approximation.

They find the differential cross section for the creation of a positron with total energy between E and E + dE and an electron with total energy between E and E -dE by a photon of energy $h\nu$ in the Coulomb field of a nucleus of charge Ze to be

$$\frac{d\sigma}{dE_+} = \Phi \frac{p_+ p_-}{(h\nu)^3}\Bigg\{-\frac{4}{3} - \frac{2E_+E_-(p_+^2+p_-^2)}{p_+^2 p_-^2} + m^2c^4\left(\frac{E_+L_-}{p_-^3} + \frac{E_-L_+}{p_+^3} - \frac{L_+L_-}{p_+p_-}\right) + L\Bigg[\frac{(h\nu)^2}{p_+^3p_-^3}\left(E_+^2E_-^2 + p_+^2p_-^2\right) - \frac{8}{3}\frac{E_+E_-}{p_+p_-} - \frac{h\nu\, m^2c^4}{2p_+p_-}\left(\frac{E_+E_- - p_-^2}{p_-^3}L_- + \frac{E_+E_- - p_+^2}{p_+^3}L_+ + \frac{2h\nu E_+E_-}{p_+^2p_-^2}\right)\Bigg]\Bigg\} \quad \ldots (1.1)$$

where
$$L_+ = 2\ln\frac{(E_+ + p_+)}{mc^2}, \quad L_- = 2\ln\frac{(E_- + p_-)}{mc^2}$$
$$L = 2\ln\left[(E_+E_- + p_+p_- + 1)/mc^2h\nu\right]$$
$$E_- = h\nu - E_+$$

$\Phi = Z^2 r_o^2/137$, p_+ and p_- are the momenta of the electrons in energy units, Z is the atomic number, ro the classical electron radius, and $\frac{1}{137}$ is the fine structure constant. Equation (1.1) is limited by the condition for the validity of Born approximation which is

$$Ze^2/\hbar\nu \ll 1 \qquad \ldots (1.2)$$

It is therefore not valid for high Z or for small electron and positron velocities. It is also limited to energies where screening of the electrons is unimportant i.e. energies such that most probable distance for pair production to take place is considerably less than the atomic radius. This consideration leads to the conclusion that equation (1.1) is valid only if

$$\frac{137\, mc^2 h\nu}{2E_+E_-Z^{1/3}} \gg 1 \qquad \ldots (1.3)$$

For high Z elements $Ze^2/\hbar v$ will not be negligible even for the fastest component of the electron pair. For example minimum value of $Ze^2/\hbar v$ for Pb is $0{\cdot}6$. Actually the **ratio** of theoretical and experimental values for pair production of 88 Mev photon varies linearly with $(Z/137)^2$ as would be expected from inadequacy of Born's approximation. For lead the theoretical cross section is 12% higher than experimental value at this energy (L-49). Even at 17.6 Mev the discrepancy appears to be about 10% in the case of lead (W-49). Again for high energy photons an appreciable contribution to the pair production cross section may come from a distance r of the nucleus which is greater than the radius of the K-shell. Therefore effective nuclear charge is reduced because of screening of the atomic electrons. In Bethe Heitler's theory of pair production the effect of atomic electron is approximated by using Thomas-Fermi statistical model for the atom. In place of equation (1.1) they obtain

$$\frac{d\sigma}{dE_+} = \frac{\Phi}{(h\nu)^3}\Big\{(E_+^2 + E_-^2)\left[\phi_1(\gamma) - \tfrac{4}{3}\ln Z\right] + \tfrac{2}{3}E_+E\left[\phi_2(\gamma) - \tfrac{4}{3}\ln Z\right]\Big\} \quad \ldots (1.4)$$

where $\gamma = \frac{100\, mc^2 h\nu}{E_+ E_- Z^{1/3}}$ and $\phi_1(\gamma)$ and $\phi_2(\gamma)$ are functions of γ whose values were obtained by numerical integration, for γ between 0 and 2.

For smaller energies ($2 < \gamma < 15$) a more convenient formula is (Be-34)

$$\frac{d\sigma}{dE_+} = \frac{4\alpha Z r_0^2}{(h\nu)^3}\left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right)\left[\ln\frac{2E_+E_-}{h\nu\, mc^2} - \frac{1}{2} - C(\gamma)\right] \quad \ldots (1.5)$$

$\phi_1(\gamma)$ and $\phi_2(\gamma)$ and $C(\gamma)$ are given by Bethe and Heitler (Be-34) and Bethe and Ashkin (Be-53). At very high energy ($\gamma \simeq 0$) screening is complete and cross section becomes

$$\frac{d\sigma}{dE_+} = \frac{4\alpha Z^2 r_0^2}{(h\nu)^3}\left[\left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right)\ln\left(183 Z^{-1/3}\right) + \frac{1}{9}E_+E_-\right] \quad \ldots (1.6)$$

Hough has given a formula for the energy distribution of pair electrons without screening which approximates equation (1.1) is valid for $h\nu < 20\ mc^2$ and overlaps equation (1.6) between 10 mc^2 and 20 mc^2. It can be written as

$$\frac{d\sigma}{dE_+} = \bar{\Phi}_m \zeta \left\{ 1 + \cdot 135 \left(\bar{\Phi}_m / \bar{\Phi} - \cdot 52 \right) \zeta (1 - \zeta) \right\} \quad \ldots (1.7)$$

where $\zeta = 2\sqrt{x(1-x)}$, $x = \frac{E_+ - mc^2}{h\nu - 2mc^2}$ and $\bar{\Phi}_m$ is the differential cross section for the equal partition of energy i.e. value of $\frac{d\sigma}{dE_+}$ (E) from equation (1.1) when $E_+ = E_- = \frac{1}{2}h\nu$. Second term in the curly bracket of equation (1.7) should be dropped when it is negative. Values from equation (1.7) agree with those from the Bethe-Heitler equation within a fraction of 1% for $.05 < x < .95$ and $K < 10\ mc^2$.

For most cases the total pair production cross section may be obtained by integrating equation (1.1), (1.2) and (1.3). An analytic integration can be obtained for very high energy where for no screening integration of equation (1.1) gives (Heitler-Sauter)

$$\sigma = \bar{\Phi} \left(\frac{28}{9} \ln \frac{2h\nu}{mc^2} - \frac{218}{17} \right) \quad \ldots (1.8)$$

and for complete screening integration of (1.6) gives (Bethe-Heilter)

$$\sigma = \Phi\left[\frac{28}{9}\ln\left(183 Z^{-\frac{1}{3}}\right) - \frac{2}{27}\right] \quad \ldots (1.9)$$

Integrated cross sections equivalent to the integration of equation (1.1) have been derived by Racah (Ra-36) and by Jost et al (Jo-50) but not in terms of tabulated functions.

Racah formula has been reduced by Maximon (Ma-68) to a simple analytical expression, containing rapidly convergent series expansions.

For energy near threshold such that K-2 1

$$\sigma = \alpha Z r_0^2 \frac{2\pi}{3}\left(\frac{K-2}{K}\right)^3\left[1 + \frac{1}{2}\epsilon + \frac{23}{40}\epsilon^2 \ldots\right] \quad \ldots (1.10)$$

$$\epsilon = K-2/K+2$$

An equation for the momentum distribution of the recoil nucleus and its angular distribution have been obtained by Jost et al (Jo-50) in terms of atomic form factor F(q,Z) which can be integrated numerically to obtain total pair production cross section.

Borsellino (Bo-53) working with Bethe-Heitler cross section obtained an equivalent equation for the nuclear recoil momentum distribution. In addition Borsellino obtained an equation for the distribution of total momentum of the pair electrons as well as one for the distribution of the energy of the pair in the center of mass of the electron and positron.

The above results all make use of Born approximation and take no account of any Coulomb correction. That is they show symmetrical energy distribution for both the lepton pairs. This is because they neglect the interaction between the electrons and nucleus which causes the energy distribution of the two to be different because of repulsion of the positron and attraction of the electron. If a pair is created at a distance r from the nucleus of charge Ze then potential energy of each member of the pair is Ze^2/r. Nuclear repulsion on the positron and attraction on the electron will increase the original difference in their kinetic energy by $2Ze^2/r$. Principal contribution to the matrix element for the pair production comes from a region of the nuclear field for which r lies between $\hbar/m_0c$ and $\frac{\hbar}{m_0c}\frac{h\nu}{2m_0c^2}$ (Be-34). Therefore on the average, positron should receive a maximum of $2Ze^2/\frac{\hbar}{m_0c}$ .0075 Z Mev more kinetic energy than the electron for

* the same value of $h\nu$. For high energy assymmetry should be less. For $h\nu$ = 2.62 Mev in Pb calculations using exact wave functions (Ja-56) indicate that average positron energy should be expected to be .28 Mev greater than the average electron. Some assymmetry appears to have been observed experimentally (Ali-38). The most probable positron energy being 1.1 Mev instead of $\frac{2.62 - 1.02}{2} = .8$ Mev for 2.62 Mev γ -ray photon.

1.2 Jaeger and Hulme (Ja-36)

Attempts for making theoretical calculations of pair production cross section using exact coulomb field wave functions are as old as Born approximation calculations. For energy between 2 mc^2 and 12 mc^2 Zerby and Moran (Ze-56) suggest the empirical equation

$$\sigma = \sigma_B \left[1 + (7.824 \times 10^{-4}) Z e^{-.612K} \right] \quad \ldots (1.11)$$

where K is the photon energy in mc^2 unit and σ_B is obtained by integration of (1.7). For high energy region Bethe et al (Be -54) and Davies Bethe Maximon (Da-54) obtained a Coulomb correction by the use of Furry Sommerfield Maue wave functions. Their results yield a Coulomb correction to be substracted from the Born approximation equations with or without screening. The unscreened pair production cross section in the point Coulomb nuclear field for relativistic energy was found to be

$$\sigma = \alpha Z r_0^2 \left[\frac{28}{9} \ln 2k - \frac{28}{9} f(Z) - \frac{218}{17} \right] \quad \ldots (1.12)$$

where f(Z) is the Coulomb correction factor which can be written as

$$f(Z) = a^2 \left[(1+a^2)^{-1} + {\cdot}2026 - {\cdot}036a^2 + {\cdot}00836a^4 - {\cdot}002a^6 \right] \quad \ldots (1.13)$$

where $$a^2 = \left(Z/137 \right)^2$$

This correction is valid only for energies greater than about 50 Mev.

For the case of complete screening Davies Bethe Maximon gives the following expression for the pair production cross section.

$$\sigma = \alpha Z r_0^2 \left[\frac{28}{9} \ln\left(183 Z^{-\frac{1}{3}}\right) - \frac{28}{9} f(Z) - \frac{2}{27} \right] \quad \ldots (1.14)$$

equation (1.14) was evaluated by Sørenssen (Sø - 65,66) for various elements with Thomas-Fermi Moliere form factor and the more accurate Hartee-Fock-Slater atomic form factor.

For the lower energy region below 20-30 Mev the Davies Bethe Maximon high energy approximation breaks down, besides Furry Sommerfield Maue wave functions themselves are not good for high Z elements and small energies. There is however the possibility of using exact coulomb field wave functions. Since Dirac equation is separable only in polar co-ordinates, the wave function and hence the cross sections must be written as partial wave expansions. As the necessary number of terms in such expansion increase very rapidly with energy the method is best suited for low energy region. Such a method was used by Jaeger and Hulme (Ja-35), Nishina, Tomanga and Sakata (Ni-34) for the calculation on internal conversion and pair production cross section. Because of great numerical task these authors could calculate only a limited number of terms still Jaeger Hulme calculations showed that the exact cross section deviates by a factor of the order 2 from the Born approximation results for a photon energy 3 mc^2 in Pb which is not surprising however since Bethe Heitler's formula is a crude approximation for high energy and worst for the low energy region. Calculated results of Jaeger and Hulme along with Born approximation results are given in table 1.1.

Table 1.1

Calculated results of Jaeger and Hulme along with Born approximation results.

$h\nu = 3\ mc^2$				$h\nu = 5\ mc^2$
Z	50	65	82	82
$\sigma \times 10^{-24} cm^2/atom$	.17	.34	.67	3.1
$\sigma_B \times 10^{-24} cm^2/atom$	.13	.21	.34	2.5

It is evident from the table that Born approximation gives values which are too low by as much a factor 2 from the exact results, but the discrepancy decreases rapidly with increasing energy and decreasing atomic number.

Cross section formula taking screening and atomic excitation into account has been given by Wheeler and Lamb (Wh-56). This expression is in the form

$$\sigma = \alpha Z r_0^2 (22.6 - 2.08 \ln Z) \qquad \ldots(1.15)$$

More accurate and detailed calculations of the pair production cross section were presented by Øverbo et al (Øv-68). Semiempirical formula for Pb in the energy range $6 \leq K \leq 30$ Mev as was given by Øverbo et al is

$$\sigma = \sigma_B - 4.02 + \frac{16.8}{K} \ln(K - .75) \quad \ldots (1.16)$$

Inspite of the existence of different cross section formula for the evaluation of pair production cross section which are valid in different energy range accurate numerically calculated values of pair production cross section (Ø0-67, Ts-72, Ø0-79) are now available. These calculations have made it possible to find the pair production cross section of any element in the energy range of interest.

Numerical results of angular cross section of electron positron pair production in a point coulomb potential have been obtained by Dugne and J.Proriol (Du-76). Analytic method used by them is exactly similar to that used by Øverbo et al (Øo-67) and when the cross sections are integrated over all emmission angles the results of Øverbo et al are recovered.

Su(79)
Also Sud et al[7] have developed a method for calculation of electron pair production in a point coulomb potential at high energy ($E > 30$ Mev).

1.3 Øverbo (70) ($K \leq 10\ mc^2$)

With the development of computor the method of partial wave for calculation of atomic pair production cross section has become much more powerful.

The main difficulty in the calculation is the evaluation of radial parts of the matrix element. The methods used to evaluate this may be divided into two categories of which first is to integrate the radial integrals analytically. The integral may then be expressed in terms of generalised hypergeometric functions which are more or less difficult to evaluate numerically. This is the analytic method. The other is to perform the integrals numerically using numerical solutions to the wave equation. While the analytic method applies only to coulomb wave functions the numerical method has the additional advantage of being applicable to screened potentials.

Øverbo (Øo -67() used analytic method for the calculation of pair production cross section using a point Coulomb wave function. Nucleus was considered to be infinitely heavy point mass, thus no account of nuclear recoil and finite nuclear size was taken. Radiative correction was also neglected. He has calculated the pair production cross section for different elements in the energy range 0-10 mc^2. Calculated results of Øverbo is given for two different energies along with JH calculations and Born approximation results for comparison in Table 1.2.

Table - 1.2

PAIR PRODUCTION CROSS SECTION ACCORDING TO ØVERBO, JH AND BORN APPROXIMATION CALCULATIONS

$K(mc^2)$	Z	50	65	82
3.00	$\sigma_{\emptyset} \times 10^{-24} cm^2/atom$	.177	.354	.671
	$\sigma_{JH} \times 10^{-24} cm^2/atom$	.17	.34	.67
	$\sigma_{B} \times 10^{-24} cm^2/atom$	.13	.21	.34
5.00	$\sigma_{\emptyset} \times 10^{-24} cm^2/atom$			3.177
	$\sigma_{JH} \times 10^{-24} cm^2/atom$			3.00
	$\sigma_{B} \times 10^{-24} cm^2/atom$			2.5

The cross section σ_{JH} are smaller than $\sigma_{\emptyset}$ because the large contribution which were obtained by extrapolation by JH were underestimated.

Since σ varies by several orders in magnitude in the energy region 0-10 cm^2 it is convenient to base the representation of the cross section on the ratio σ/σ_B for the purpose of comparison with the calculations of Øverbo. This has also the advantage of being a direct presentation of the coulomb correction. In fig. 1.1 - 1.5 we give σ/σ_B as a function of photon energy for five elements viz. Aluminium, Copper, Iodine, Lead and Uranium. From the figs. it is apparent that Coulomb correction vanishes for photon energy around 10-11 cm^2 the position of crossing however depend to some extent on the atomic number of elements. As the energy decreases from this value the ratio σ/σ_B increases towards a maximum for a given Z and after this the ratio decreases rapidly with decreasing ____ energy. The Z dependence of σ/σ_B is found to satisfy a relation of the form

$$\sigma/\sigma_B = 1 + C_1(K)(\alpha Z)^2 + C_2(K)(\alpha Z)^4 + \cdots \quad \ldots (1.17)$$

coefficients $C_i(K)$ increases strongly with decreasing energy. For large photon energy a few powers of $(\alpha Z)^2$ is sufficient to describe the behaviour even for large Z. With decreasing energy an increasing number of terms

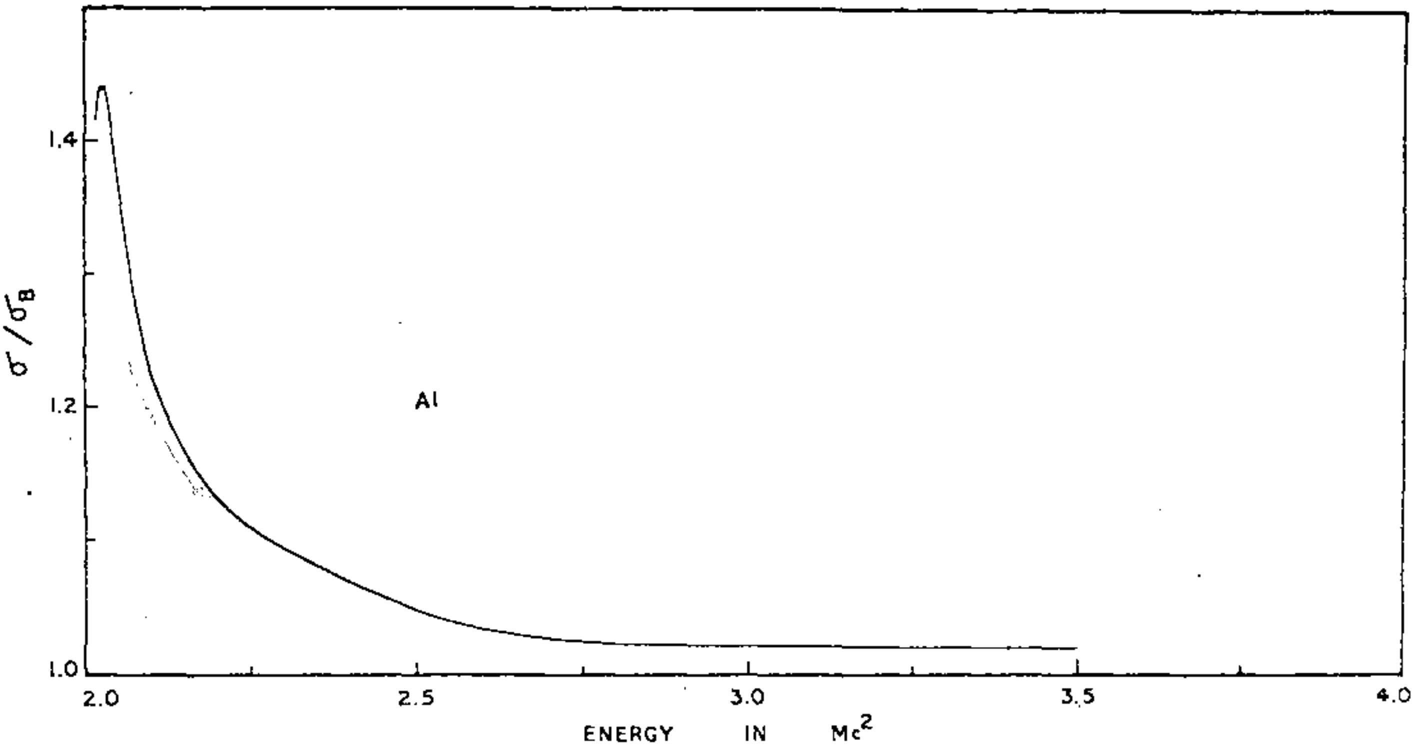

FIG. I.1. SHOWING THE PAIR PRODUCTION CROSS SECTION ACCORDING TO POINT COULOMB CALCULATION OF ØVERBO Ref. (ØO - 67) RELATIVE TO BORN APPROXIMATION RESULTS FOR ALUMINIUM.

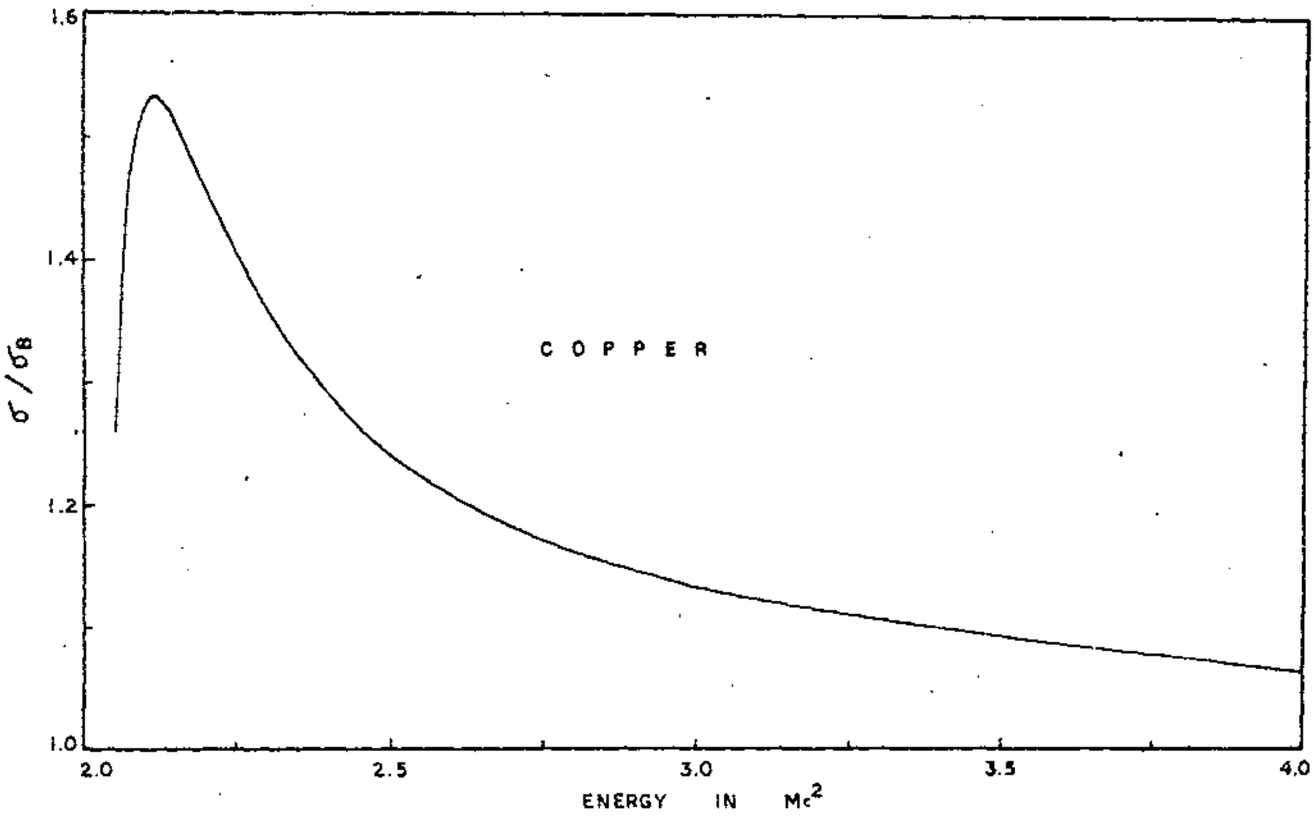

FIG. 1.2. SAME AS IN FIG. 1.1 BUT FOR COPPER.

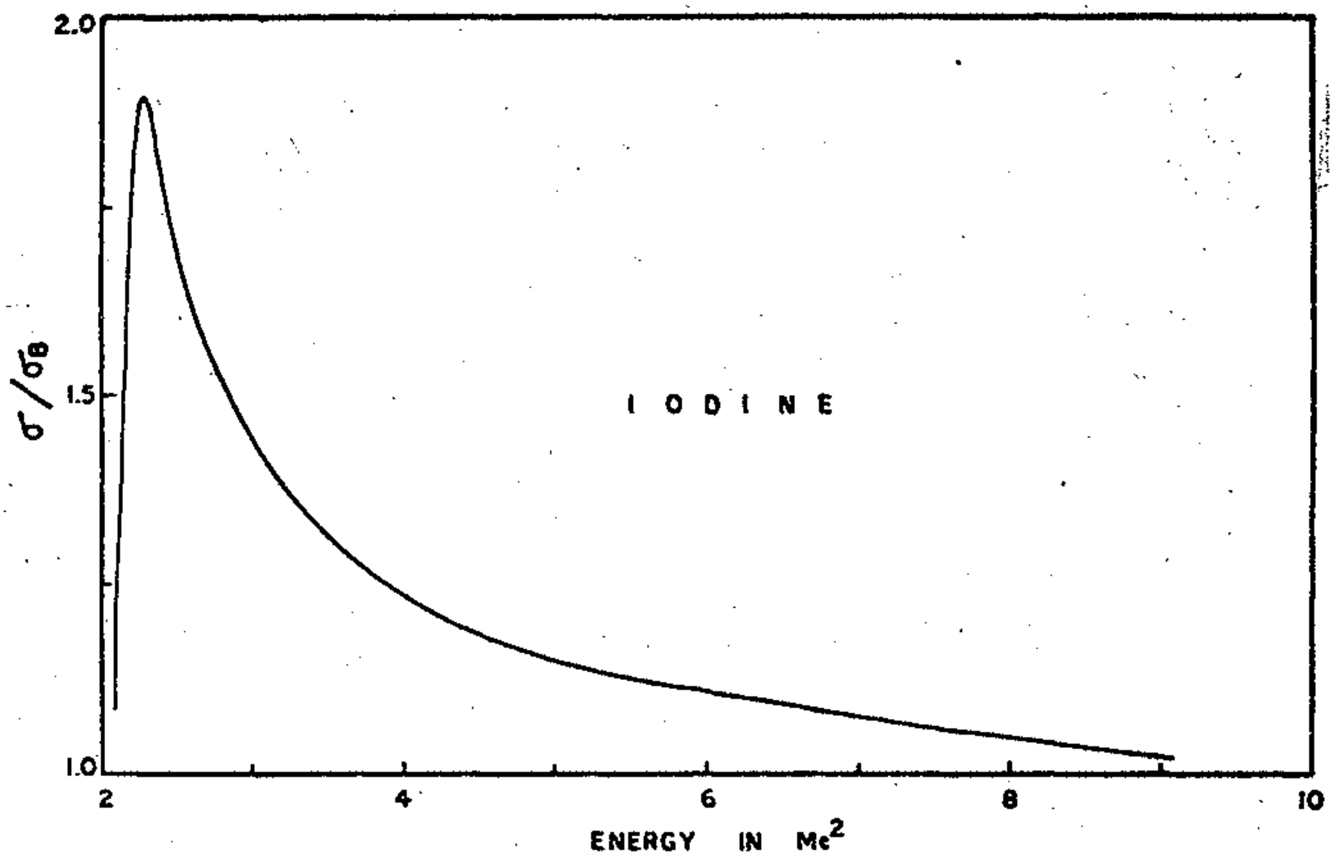

FIG. 1.3. SAME AS IN FIG. 1.1 BUT FOR IODINE.

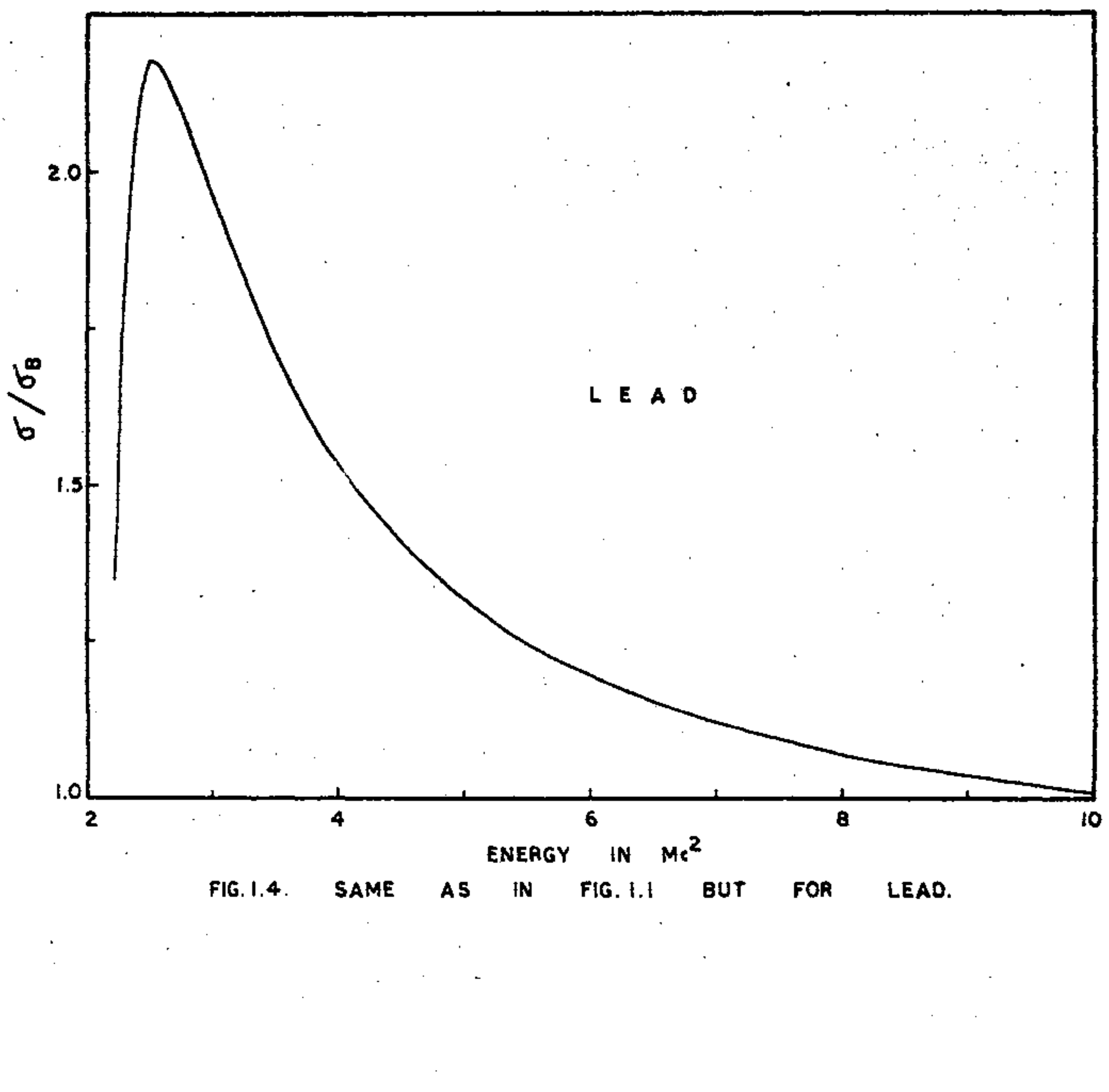

FIG. 1.4. SAME AS IN FIG. 1.1 BUT FOR LEAD.

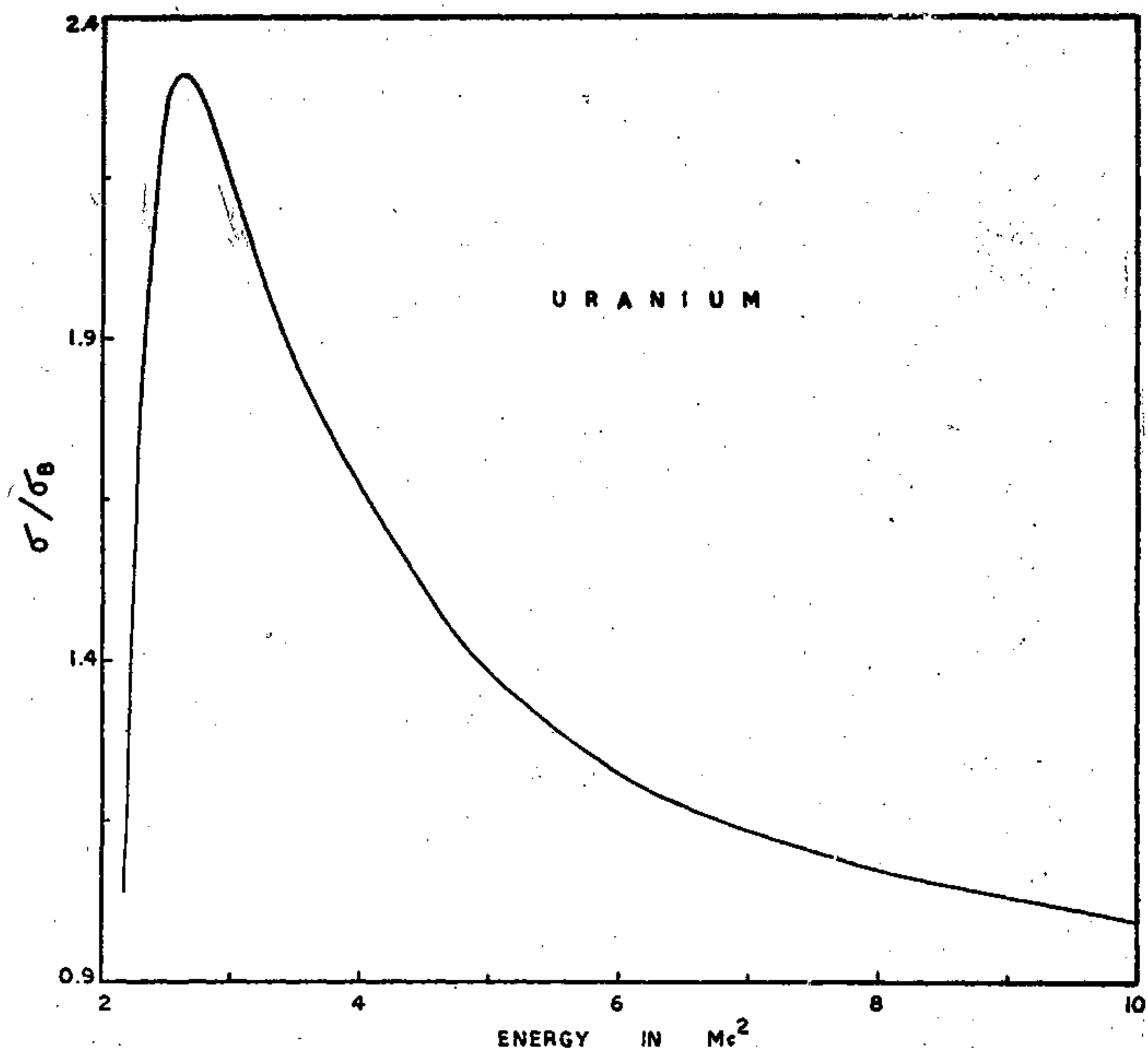

FIG. 1.5. SAME AS IN FIG. 1.1 BUT FOR URANIUM.

contribute to the ratio significantly. According to Øverbo the ratio σ/σ_B can be found out to an accuracy of 1% for any combination of energy and atomic number.

1.4 TSENG AND PRATT (72)

Tseng and Pratt however made calculation of atomic pair production cross section in screened potential using numerical methods and found that near threshold the screening is important and that screening correction results in an increase in the cross section at low energy rather than a decrease which might be argued from some what naive picture involving a simple replacement of the nuclear charge Ze by a smaller effective charge. This can be qualitatively understood by requiring that lepton wave function be normalised such that the probability current for both leptons are the same at large distance from the nucleus as observed. If now one pictures the time reversed process in which it can be seen that probability density of the positron in the vicinity of the nucleus behaves very differently from that of the electron. This is because the amplitude of the incoming positron will be enhanced in the vicinity of the nucleus by the screening electrons which effectively reduce the nuclear charge. At electron Compton wave length distance an electron sees a point Coulomb potential corresponding to the nuclear charge Ze. The electron wave function has a hydrogen like shape, the only effect of the atomic electron screening as described by a central potential V deviating from the point Coulomb form is to modify the normalization. For very low energy continuum wave function this normalization is indeed sensitive to screening. Tseng and Pratt has shown that screened pair production cross section can roughly be obtained from point coulomb cross section by simply using a multiplicative normalization factor. They examined the shapes of the electron wave function near but outside the atomic nucleus from which they could predict the effect of atomic electron screening in the entire range of the point coulomb calculation of Øverbo.

According to Tseng and Pratt the screened electron or positron wave function of shifted energy $\delta E = \pm V_0$ (+sign being for the positron and -ve sign for the electron) at small distance is even closer in shape to a point coulomb wave function of same energy ($\delta E = 0$). Here $\delta E = E - E_c$ subscript C stands for Coulomb potential case $V_c = -\frac{a}{r}$ with $a = Z\alpha$ and they assumed a central potential $V = -(\frac{a}{r} + V_0 + \tilde{V})$ with $\tilde{V}(r=0) = 0$, V_0 is a constant. An analytic calculation shows that deviation from the point Coulomb shape at small distances is approximately $\alpha^2 Z^4 \gamma^2/2l+3$ for $\delta E = 0$ and $\alpha^3 Z^{5/3}\gamma^3/6(l+2)$ for $\delta E_{\pm} = V_0$ for l-th partial wave which has been verified by numerical calculations. Since the energy shifts for electron and positron are equal and opposite a point Coulomb pair production angular distribution with a specified split (E_+, $E-$) of photon energy $K = E_+ + E-$ between pair differs only by a normalization factor from the screened distribution resulting from the same photon energy but a different split ($E_+ + V_0$, $E_- - V_0$) between the positron and electron.

For low K partial waves except at very low energy $\tilde{N}_S = (p_s E_s)^{1/2} N_S$ is equal to $\tilde{N}_C = (p_c E_c)^{1/2} N_c$ for the case with energy shift where Ns and Nc are the normalizations of the wave functions of screened and point Coulomb potentials respectively. At high energy $\tilde{N}_S = \tilde{N}_C$ with or without energy shift.

For atomic pair production low K partial waves dominate the cross section for low photon energies. Consequently they predicted for photon energies above 1.2 Mev the relation $\sigma_S(E_+ + V_0, E_- - V_0) = \sigma_C(E_+ E_-)$ between screened and point coulomb pair production energy distributions. Using the energy shift screening theory they showed that for photon energies above 3 mc^2 screening effect is not important if one is interested only in total pair production cross section, however if one is interested in the pair production energy distribution screening effect is still important in this energy region. At low energy the screening will increase the cross section but with increasing energy an increasing section of the spectrum is decreased by screening. In total cross section due to cancellation between these two sections of the spectrum there is a large energy region for which screening is not important. Cross section values for a number of elements in the energy range $2.10 - 10\ mc^2$ have been calculated by them.

1.5 Øverbo - 79

Recently Øverbo has calculated the pair production cross section taking screening of the atomic electrons into account for photon energy from threshold to 45 Mev. The correction is given as a sum of (Negative Born Screening term) and a contribution which is obtained by simply shifting the point coulomb positron energy spectrum in energy by an amount Vel (0), Vel(r) being the potential caused by the atomic electrons.

This contribution is positive and relatively large in the threshold region but decreases fast with increasing energy. The importance of screening at low energies can be understood from the general behaviour of the positron energy spectrum (Ø0-67, 73). The two main features are (i) An enhancement for high positron energies E_+ and (ii) a suppression of low E_+.

A semiqualitative description of both these effect can be obtained by considering the non relativistic (unscreened) case for which the form of the spectrum is given by symmetric Born approximation spectrum multiplied by the densities of the positron and electron wave at the nucleus

$$\frac{d\sigma}{dE_+} \propto \left(\frac{d\sigma}{dE_+}\right)_B \frac{2\pi y_+}{e^{2\pi y_+}-1} \cdot \frac{2\pi y_-}{1-e^{-2\pi y_-}} \quad \ldots (1.18)$$

where $y = \frac{\alpha Z E}{p}$. The suppression for small E_+ is seen to be due to repulsion of the positron by the nuclear field which makes the cross section approximately proportional to the factor $e^{-2\pi\alpha p/p_+}$ for small p_+ while enhancement for high E_+ or low E_- is connected with attraction of the low energy electron which gives rise to a factor $2\pi\alpha Z/p_-$ for small p^-. As the photon energy K decreases these factors govern an increasing part of the spectrum leading to larger degree of assymetry. The main effect of the atomic electron is to reduce the repulsion of the positron which corresponds to a weakening of the exponential factor. The screening will therefore increase the cross section and relative correction will increase as energy decreases. An estimate of the effect can be obtained by the following argument. The relative smallness of Born screening term indicates that almost all the pairs are produced inside the electron shells. However as the pair particles move from this region to free space their energies will be shifted by a certain amount $\Delta = \Delta(Z)$ due to the electric field from the atomic electrons. One therefore expects the screened differential cross section corresponding to a pair detected with energies E_+ and E_- to be given approximately by

$$d\sigma^s(E_+, E_-) = d\sigma_c(E_+ + \Delta, E_- - \Delta) \qquad \ldots (1.19)$$

where $d\sigma_c$ denotes the pure Coulomb cross section. The energy shift Δ is given by $\Delta = V_{el}(0)$ where $V_{el}(r)$ being the contribution from the atomic electrons to the potential energy of the created electrons. The total cross section excluding the radiative correction has been given by

$$\sigma = \sigma_B(1-R) + \Delta\sigma_c + \Delta\sigma_c^s \quad \text{for } K > 3.5\,mc^2 \qquad \ldots(1.20)$$

where σ_B is the unscreened Born approximation cross section, R = relative Born screening term given by

$$R = \frac{\Delta\sigma_B}{\sigma_B} = (1.61 - 5.62x + 4.93x^2)/100K$$
$$+(.048 - 2.61x + 6.36x^2 - 2.65x^3)\times 10^{-2}$$
$$+(-3.69 + 42.3x - 17.9x^2 + .826x^3)K\times 10^{-}$$
$$+(.119 + 3.58x - 17.2x^2 + 11.7x^3)K^2\times 10^{-5}$$
$$+(.087 - 2.8x + 7.65x^2)K^3\times 10^{-7} \qquad \ldots(1.21)$$

where $x = (\alpha Z)^{2/3}$, $\alpha = \frac{1}{137.036}$ and K is the photon energy in mc^2 unit. The coulomb correction $\Delta\sigma_c$ (Ø0·67·,73) can be obtained with an accuracy of the order of a few tenths of a percent,

$\Delta\sigma_c^s = \alpha^2 Z^3 e^S$ barns where

$$S = -7.904 - 1.737a + 7.465a^2 + (-0.048 \quad .894a - 2.54a^2)\ \ln K$$

$$(-1.216 + 5.158a - 4.83a^2)/K-2 - 0.219/K-2)^2 \qquad \ldots (1.22)$$

with $a = \alpha Z$ and K in mc^2 unit.
This method differs from that of Tseng and Pratt by the added Born term and also the energy shift used here is greater than that of Tseng and Pratt. A comparison of the results of different calculations are made in Table (1.3) for some selected elements in the energy range from 2.10 - 2.80 mc^2. Since the cross sections change by several orders in magnitude it is convenient to base the presentation in terms of Born approximation results. In table 1.4 the cross sections according to different calculations are presented in terms of Born approximation calculations in the same energy range and for ~~A~~aluminium, ~~C~~copper, ~~I~~iodine, ~~L~~lead and ~~U~~uranium and also the cross sections relative to Born approximations results for the above elements have been displayed in figs. 1.6 - 1.10.

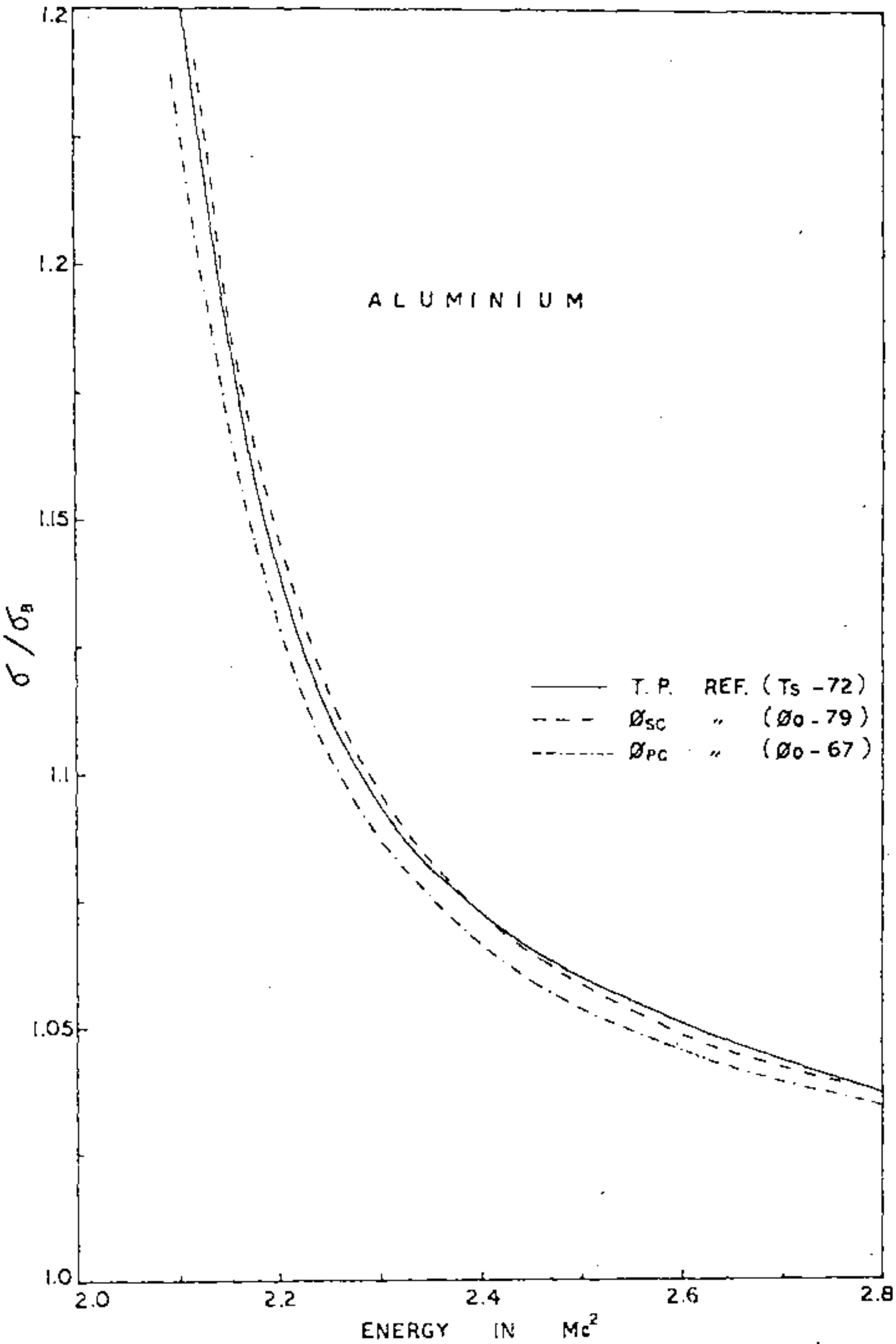

FIG. I.6. COMPARISON BETWEEN THE RATIO σ / σ_B ACCORDING TO DIFFERENT THEORETICAL CALCULATIONS FOR ALUMINIUM. σ_B = UNSCREENED BORN APPROXIMATION PAIR PRODUCTION CROSS SECTION, σ = PAIR PRODUCTION CROSS SECTION.

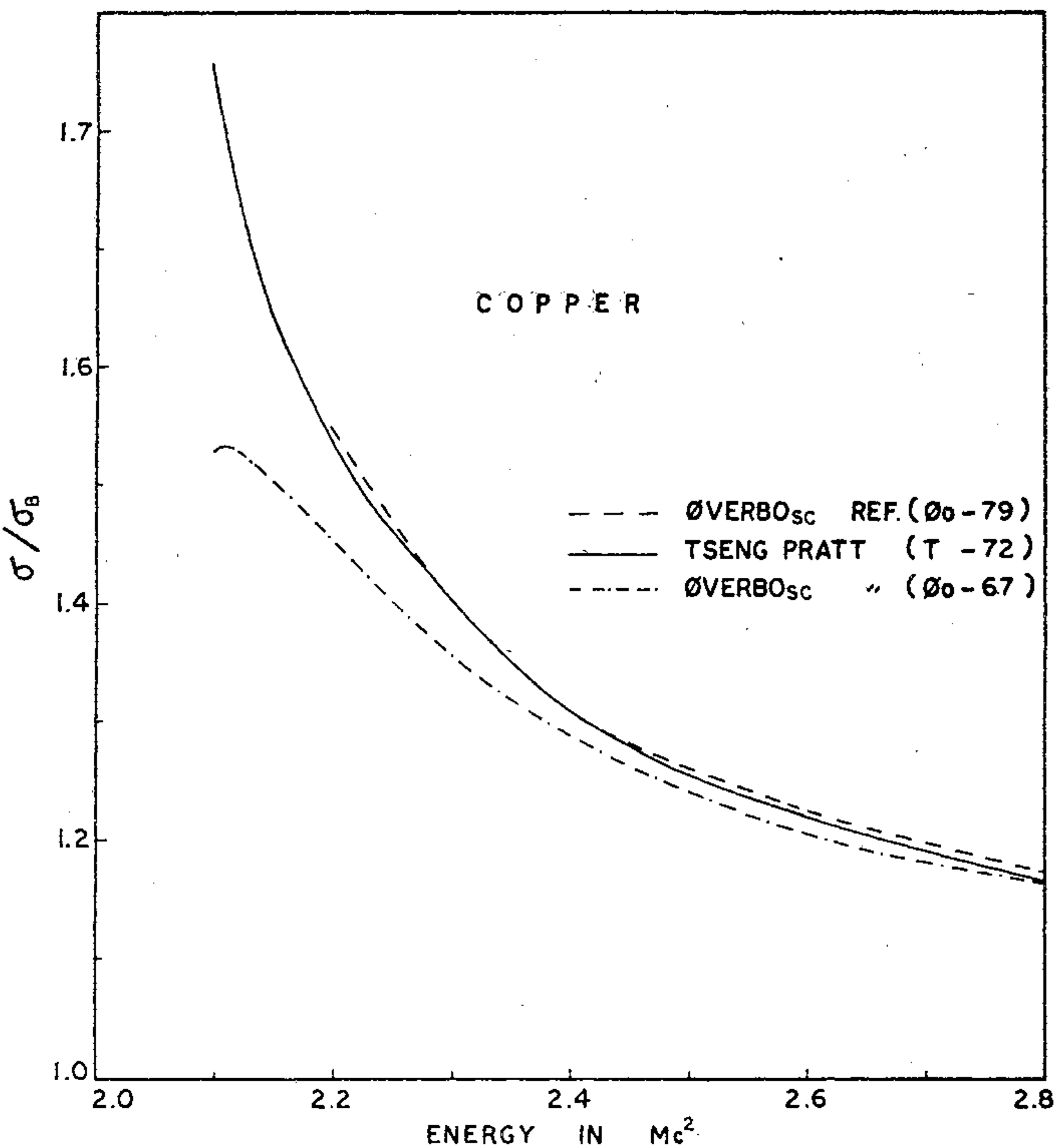

FIG. I.7. SAME AS IN FIG. I.6. BUT FOR COPPER.

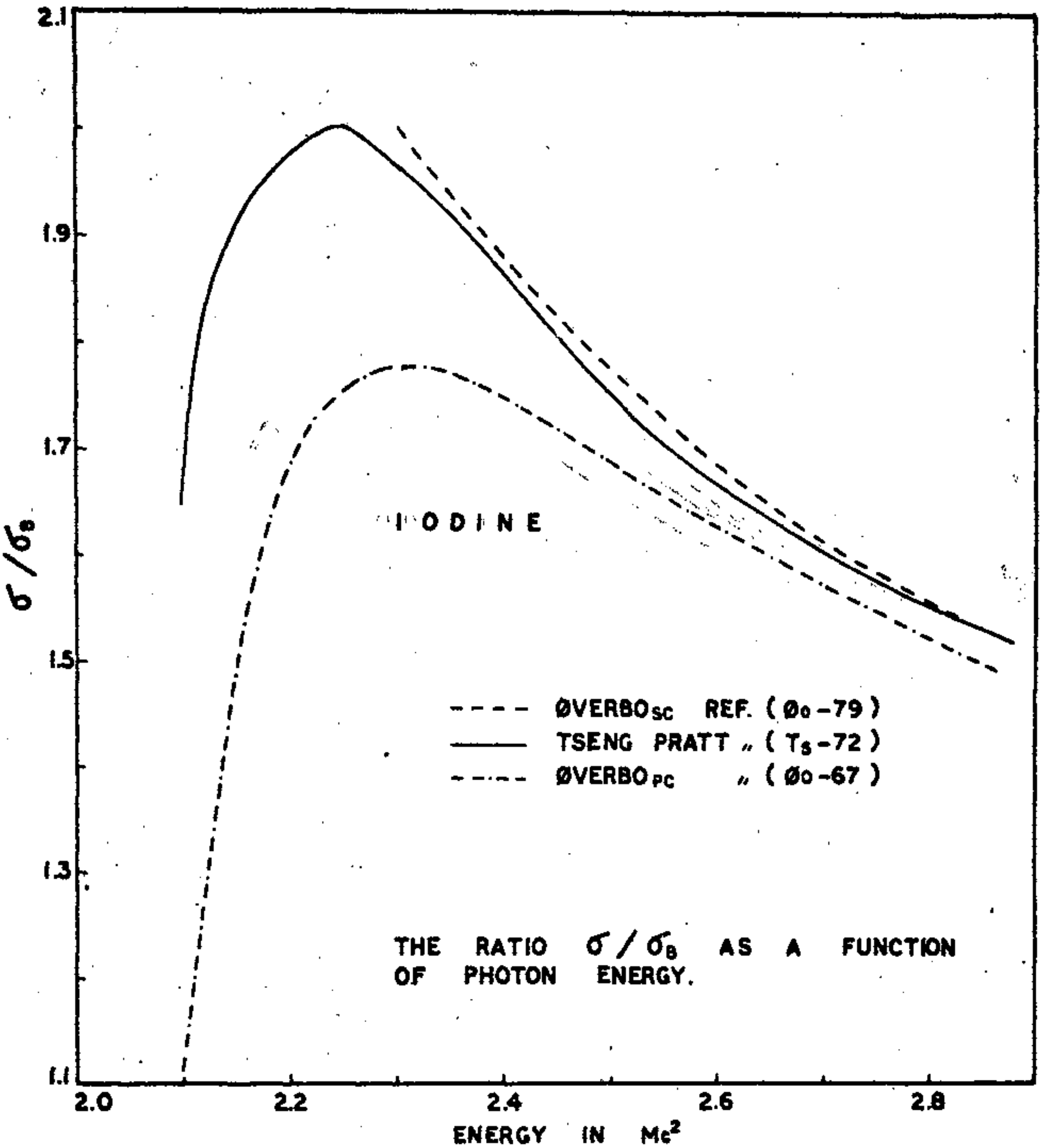

FIG. 1.8. SAME AS FIG. 1.6 BUT FOR IODINE.

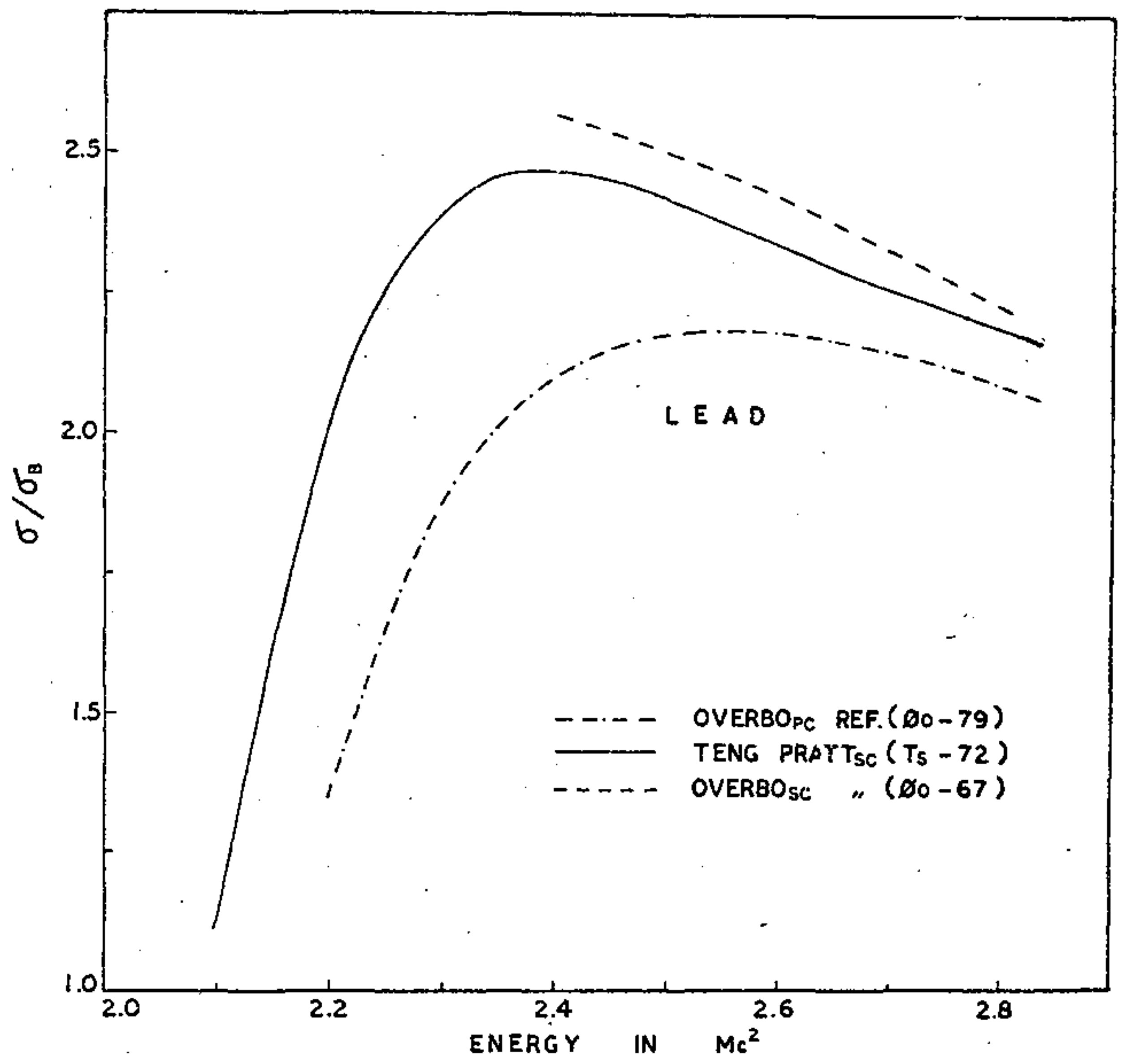

FIG. 1.9. SAME AS FIG. 1.6 BUT FOR LEAD.

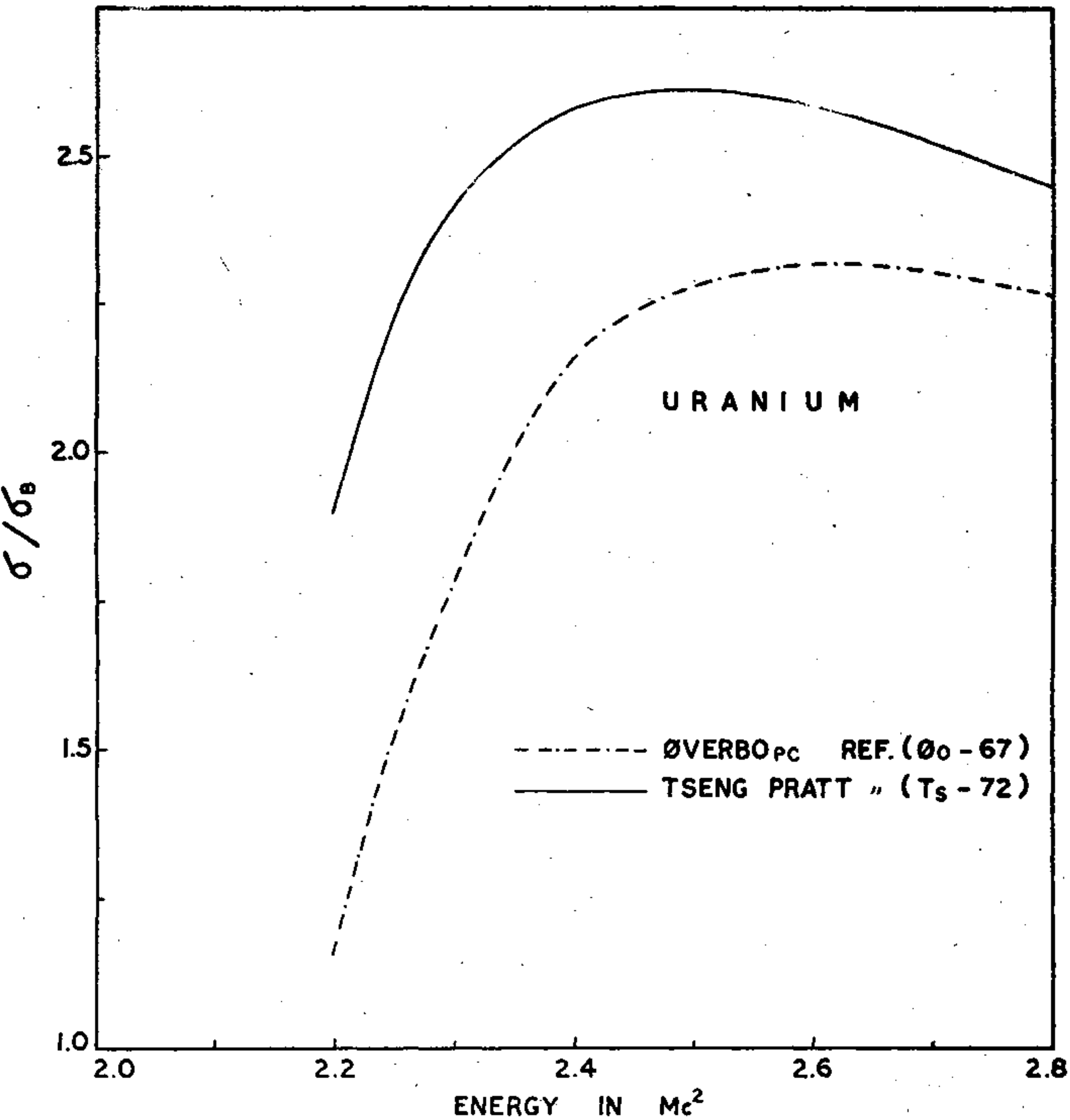

FIG. 1.10. SAME AS FIG. 1.6 BUT FOR URANIUM.

Table - 1.3

COMPARISON OF THE CROSS SECTIONS OBTAINED BY DIFFERENT AUTHORS ALONG WITH BORN APPROXIMATION RESULTS FOR A SELECTED ELEMENTS.

Atomic number of the element	Energy in mc^2 unit	σ_B in barns per atom	σ^{ϕ}_{Pc} in barns per atom	σ^{TP}_{Sc} in barns per atom	σ^{ϕ}_{Sc} in barns per atom
13	2.10	*2.242-5	2.768-5	2.840-5	2.870-5
29		1.115-4	1.705-4	1.940-4	
53		3.726-4	4.134-4	6.250-4	
82		8.921-4	3.719-4	1.010-3	
92		1.123-3	3.062-4	1.030-3	
13	2.20	1.579-4	1.784-4	1.800-4	1.810-4
29		7.860-4	1.144-3	1.210-3	1.220-3
53		2.625-3	4.437-3	5.170-3	
82		6.284-3	8.486-3	1.260-2	
92		7.911-3	9.161-3	1.510-2	
13	2.30	4.723-4	5.143-4	5.170-4	5.180-4
29		2.350-3	3.191-3	3.310-3	3.300-3
53		7.850-3	1.397-2	1.540-2	1.570-2
82		1.879-2	3.530-2	4.490-2	
92		2.365-2	4.241-2	5.700-2	
13	2.40	9.974-4	1.065-3	1.070-3	1.071-3
29		4.963-3	6.403-3	6.530-3	6.550-3
53		1.657-2	2.895-2	3.090-2	3.110-2
82		3.968-2	8.354-2	9.790-2	1.020-1
92		4.995-2	1.062-1	1.290-1	
13	2.50	1.744-3	1.840-3	1.850-3	1.847-3
29		8.679-3	1.079-2	1.090-2	1.097-2
53		2.899-2	4.890-2	5.070-2	5.140-2
82		6.939-2	1.512-1	1.680-1	1.740-1
92		8.735-2	1.984-1	2.280-1	2.400-1

Contd..

Table - 1-3 (Contd..)

Atomic number of the element	Energy in mc^2 unit	σ_B in barns per atom	$\sigma^{\emptyset}_{Pc}$ in barns per atom	σ^{TP}_{Sc} in barns per atom	$\sigma^{\emptyset}_{Sc}$ in barns per atom
13	2.60	2.711-3	2.838-3	2.850-3	2.846-3
29		1.349-2	1.632-2	1.650-2	1.652-2
53		4.506-2	7.328-2	7.510-2	7.610-2
82		1.078-1	2.349-1	2.320-1	2.610-1
92		1.357-1	3.143-1	3.510-1	3.620-1
13	2.80	5.266-3	5.455-3	5.460-3	5.466-3
29		2.620-2	3.053-2	3.070-2	3.077-2
53		8.753-2	1.333-1	1.360-1	1.363-1
82		2.095-1	4.376-1	4.590-1	4.660-1
92		2.637-1	5.971-1	6.440-1	6.520-1

* The form 2.711-3 means 2.711×10^{-3}

Table -1-4

CROSS SECTIONS FOR SOME SELECTED ELEMENTS ACCORDING TO DIFFERENT AUTHORS RELATIVE TO BORN APPROXIMATION RESULTS IN THE ENERGY RANGE 2.10 - 2.80 mc^2

Atomic number	Energy in mc^2 unit	$\sigma^{\emptyset o}_{Pc} / \sigma_B$	$\sigma^{TP}_{Sc} / \sigma_B$	$\sigma^{\emptyset o}_{Sc} / \sigma_B$
13	2.10	1.234	1.266	1.280
29		1.529	1.739	
53		1.109	1.677	
82		.4168	1.132	
92		.2726	.917	

Contd..

Table - 1.4 (Contd..)

Atomic number	Energy in mc^2 unit	$\frac{\sigma_{Pc}^{\emptyset o}}{\sigma_B}$	$\frac{\sigma_{Sc}^{TP}}{\sigma_B}$	$\frac{\sigma_{Sc}^{\emptyset o}}{\sigma_B}$
13	2.20	1.129	1.139	1.146
29		1.455	1.539	1.552
53		1.690	1.969	
82		1.350	2.005	
92		1.158	1.908	
13	2.30	1.088	1.094	1.096
29		1.357	1.408	1.404
53		1.779	1.960	1.999
82		1.878	2.389	
92		1.793	2.410	
13	2.40	1.067	1.072	1.073
29		1.290	1.310	1.319
53		1.747	1.864	1.876
82		2.105	2.467	2.570
92		2.126	2.582	
13	2.50	1.084	1.060	1.059
29		1.243	1.255	1.263
53	1.	1.686	1.748	1.773
82		2.178	2.42	2.507
92		2.271	2.610	2.74
13	2.60	1.046	1.051	1.049
29		1.209	1.223	1.224
53		1.626	1.666	1.688
82		2.179	2.337	2.42
92		2.316	2.586	2.667
13	2.80	1.0358	1.0368	1.0379
29		1.165	1.171	1.174
53		1.519	1.553	1.557
82		2.088	2.19	2.224
92		2.264	2.44	2.472

From the table it is apparent that screening correction is not very important for low Z elements near threshold but with increasing atomic number screening correction becomes increasingly important.