

PREFACE

Considerable interest has been shown in the past for solving the non-linear problems of plates of different shapes. It is evident that the interest has been ingenerated by the demands placed on the designers of the sophisticated structure and space vessels of the present day high technology industries.

It is well known that the deflections 'W' of thin plates are small in comparison with its thickness. A very satisfactory approximate theory of bending of the plate by lateral loads can be developed by making the following assumptions.

- 1) There is no deformation in the middle plane of the plate and this plane remains neutral during bending.
- 2) Points initially lying on a normal to the middle plane of the plate remain on the normal to the middle surface of the plate after bending and
- 3) finally the normal stresses in the direction transverse to this plate can be disregarded.

The above assumptions constitute the simplest and most widely used classical theory developed by Lagrange [1811].

In continuation to this linear theory different investigations on the bending of the thin plates have been carried out by different investigators and the bibliography of these works has been incorporated in a well organized manner, in the book of Theory of Plates and Shells by Timoshenko .

The different approaches have been adopted in these works of which the method of 'Constant Deflection Contour Lines' by Mazumdar J.[1970] is a handy and powerful one. 'Constant Deflection Contour Lines' i.e. the lines of equal deflections are obtained by intersecting the bent plates by planes to the original plane of the plate. Mazumdar J. [1970] proposed a method of solving problems of elastic plates and used the

concept of 'Constant Deflection Contour Lines' or lines of equal deflection.

In line with the concept of lines of equal deflection the important works of different research workers like Mazumdar J.[1970,1971,1973,1974] ,Hewitt J.S. & Mazumdar J.[1974] Jones R. & Mazumdar J. ,Fu-Pen –Chiang [1975], Bucco D., Mazumdar J. and Sved G.[1979] may be referred. All these works are based on linear theory of thin plates . In a series of papers from 1970 to 1974 Mazumdar J. quite elegantly analysed the mentioned problems using the method of 'Constant Deflection Contour Lines' .

1)Bending problems in which the plates are subjected to lateral loading only[1970].

2) Determination of fundamental frequency of Transverse vibration of thick plates of arbitrary shape. As an illustration of the method an elliptical plate for both clamped and simply supported edge condition is discussed [1971].

3(a) Buckling problems in which the plates are subjected to edge loading only.]

(b) Combined loading problems in which the plates are subjected to lateral loading and edge loading simultaneously [1971]

4. Approximate computation of the fundamental frequency of membrane of arbitrary shape vibrating harmonically has been investigated. As an illustration of the investigation, the calculation of gravest mode of an annular elliptic and parabolic membrane has been discussed [1973].

Hewitt J.S. and Mazumdar J. [1974] developed a method for determining the behaviors of an arbitrary shape plate composed of visco elastic material under transverse load which vibrates in normal mode. The method is based upon the method of lines of equal deflection on the surface of the plate. It was pointed out during investigation that visco-elastic plate will vibrate with same normal mode as an elastic plate of same shape and with the same boundary condition, and that the time

behaviours can be found by using the frequency of free vibration of the associated elastic plate. As an illustration of the developed technology the unsolved problems of clamped vibrating elliptic plate of Kelvin material was found and some numerical results were given. It was observed that the technique is applicable to any panel of visco-elastic material of arbitrary shape with clamped or simply supported boundary conditions.

Jones R , Mazumdar J. and Fu Pen Chiang [1975] undertook a more general study of the elastic plate problems by the method of 'Constant Deflection Contour Lines' through considerations of determining the differential equations of lines of equal deflection under various boundary and load conditions and show the simultaneously of this method for plates of any geometry.

Bucco D., Mazumdar J. and sved G. [1979] derived the fundamental frequency of plates of arbitrary shape using the method of 'Constant Deflection Contour Lines'.

When the deflections are no longer small in comparison with the plate thickness, the supplementary stresses in the middle plane of the plate must be taken into account in deriving the differential equations governing the deflections of plates. Thus non-linear differential equations are obtained and the solutions of the problem become much more complicated.

The coupled non-linear differential equations for large amplitude axisymmetric deformations were initially derived by Von Karman[1910]. These equations in the coupled form were difficult to solve, and different numerical methods had been employed by different authors for investigations of large deflection of plates.

The out standing research workers in this field are Chu H.N. and Herman G. [1956] , Weil N.A. and Newmark N.M.[1956], Nash⁵⁸ W.A. and Cooley I.D.[1959], Nowinski J.L. [1963], Nowinski J.L. and Ismail I.A. [1965], Schmidt R.⁵⁹ [1968] , Bauer⁶⁶ H.F. [1968] ,Bolton

R.[1972] , Banerjee B. and Dutta S. [1981] who employed Von Karman equations to analyse the non-linear behaviour of thin plates.

Chu H.N. and Herman G. quite elegantly analysed the influence of large amplitude on free flexural vibrations of rectangular elastic plate. The authors' investigations are based on Von Karmann's equation which have been solved by numerical method.

Weil and Newmark investigated the large deflection of elliptic plates .The authors solved Von Karman equation and the method of solution is laborious.

Nash and Cooley investigated large deflection problems of elliptic plates using the method of perturbation .Nowinski quite elegantly analysed the nonlinear vibration of elastic circular plates exhibiting the rectilinear orthotropy. In the solution of differential equation for stress function the author used a trial function involving fourteen unknown constants.

Nowinski and Ismail treated the non linear dynamic behavior of triangular plate by semi inverse method. Schmidt 's analysis on the large deflection of a clamped circular plate under a concentrated load at the center by iterative method is noteworthy.

Bauer investigated the dynamic response of circular as well as rectangular plates due to pulse excitement. The author solved the corresponding differential equation for stress functions completely.

Bolton used error minimizing technique showing practical interest in modern design. The other useful workers who treated Von Karman equation to analyse non linear behaviors of different elastic plates are Herman G. ⁶¹ [1955] , Sayed M.A. & Schmidt R. ⁵⁹ [1977], Satyamoorthy M. [1979] , Banerjee B. &Dutta S. [1980] and Banarjee B. [1982] etc.

Berger H.M. [1955] gives us an approximate method for solving large deflection of thin plates . The author neglects second second strain invariant of the middle surface strains in the expressions corresponding to the total potential energy of the system. Following this approximate

method many investigators like Nowinski J.L. [1957] , Nash W.A. & Modeer J.R. [1959] ,Basuli S. [1961], Sinha S.N. [1963] , Banerjee B. [1967] etc.

It is to be noted that Berger's method is still an intriguing subject.

Iwinski and Nowinski analysed orthotropic circular and rectangular plates using Berger's method to obtain satisfactory results.

Nash & Modeer investigated non-linear dynamic behaviors of elastic plates whereas Basuli employed this Berger's equation for clamped circular plates under concentrated load by a quite interesting method.

From the engineering point of view results of Sinha were very interesting for the solution of large deflection of circular and rectangular plates based on elastic foundation.

Banerjee B. discussed the large amplitude free vibration of elliptic plates based on Berger's approximation. Following Berger's line of thought , Mazumdar J. and Jones R.[1974] quite elegantly analysed the large deflections of thin elastic plates of arbitrary shape using concept of 'Constant Deflection Contour Lines'. The authors also analysed the large amplitude transverse vibration of plates by using Berger's method with the help of 'Constant Deflection Contour Lines'. In 1982 Banerjee B. analysed the problem of large deflection of an elliptic plate under concentrated load at the center by using Berger's method and Constant Deflection Contour Lines.

All these investigations have certain limitations, namely deflections been obtained for immovable edges only.

Banerjee B. and Datta S. [1981] introduced a modified strain energy expression to investigate the large deflection of different elastic plates, for movable as well as immovable edge conditions. The authors were able to decouple the differential equations and solved in simplified manner. Later Banerjee B. utilized his idea in a series of papers [1983,1984,1988,1990,1993,1995] to investigate non-linear dynamic,

thermal behaviors of different elastic plates and achieved satisfactory results.

The survey of different literature on the non linear analysis of different elastic plates reveals the fact that the concept of ‘Constant Deflection Contour Lines’ is a powerful tool for analyzing the nonlinear behaviors of thin elastic plates. All these works are based on Berger’s approximation which yields satisfactory results only for immovable edge condition.

The aim of present thesis is to offer a novel computerized approach for non-linear analysis of plates under different loading using boundary value theorem along with simulation technique . The present investigation seems to be more advantageous than those obtained from other investigations because

- 1)single differential equation gives the behaviors of plates of any shape just changing ‘u’.
- 2) computation labor is minimum which is urgently needed to the present age.
- 3)investigations is valid for movable as well as immovable edge conditions.
- 4) the software developed gives directly the form of the deflection form.

The present investigation has been divided into several chapters. The first chapter deals with the generalization of the form of deflection ‘W’ by a computerized approach for the solution in case of an elliptic plate under mechanical loading using shooting method with the help of simulation technique .The analysis is done by BASIC language. This is an attempt to choose the form of ‘W’ to avoid the method of induction.

The second chapter gives the solution methodology using boundary value theorem of finite difference technique.

The third chapter gives the pattern of finite difference equation for the partial differential equations from which the program is developed using MATLAB.

In the fourth chapter we deal with analysis of orthotropic laminated plates using the program based on Matlab. To test the accuracy of the developed program we consider the several interesting problems.

Berger's approximation method has been employed in the investigation. Central Deflections have been obtained for various load functions. The corresponding linear problem is due to Timoshenko.

The fifth chapter describes nonlinear analysis of polygonal sandwich plates under thermal load conditions to have closed solution. Results are verified with the developed program.

Large amplitude free vibration of thick polygonal plates placed on Winkler type foundation is also analysed and presented in the sixth chapter .

It is observed that the present results are in excellent agreement with those found by other method.

It is believed that results obtained in the present study showing application of Berger's method under special type of edge conditions are completely new.