# CHAPTER I

# INTRODUCTION

## I.1 *INTRODUCTION :*

During the last two decades, the developments in the elementary Particle Physics and in Cosmology have become increasingly dependent on the interplay between the two subjects. The elementary particle physics has entered a very interesting stage with the development of unified theories.[1,2] The superstring theory[2] even raised the hope of unifying gravitational interactions along with the other interactions, viz. electro-weak and strong. However, this ultimate unification can occur only at an energy scale $M_P \sim 10^{19}$ GeV. The standard cosmological model suggests that such an energy scale could have occurred only in the very early universe, close to the Big Bang. The early universe, therefore, may be considered as a unique physical Laboratory for testing the new theories of particle interactions. Fortunately, there has also been considerable progress in the understanding of cosmological models in recent years. It has been realised[3] that the standard Big Bang model is fairly successful in explaining the $2.7^{\circ}$ K cosmic microwave background radiation (in short, MBR), the expansion of the universe (Hubble redshifts), primordial nucleosynthesis and the cosmic abundances of the light elements e.g. the weakly-bound deuteron and $^4$He ( about 23% by mass ). Nevertheless, the model could not be considered as an acceptable model of the early universe because it fails to explain some other observed features of the universe.

Some of these problems are :

**i) the horizon problem :** This is related to the fact that although different distant parts of the universe did not have the opportunity of making causal contact with each other in the past in this model, the present universe is highly isotropic and homogeneous, as may be guessed from the observed uniformity of MBR.

**ii) the flatness problem :** This is related to the observational fact that the present energy density of the universe is very close to the critical density ($\rho_c$), which would correspond to a spatially flat universe. Within the framework of the Big Bang cosmology this leads to a naturalness problem. Thus if the energy density of the universe close to the Planck time is slightly greater than $\rho_c$ say even by one part in $10^{-55}$, the universe would be closed, and would have collapsed millions of years ago. On the other hand, if the density is less than $\rho_c$ by one part in $10^{-55}$, the universe would be open and the present energy density of the universe would be very small. This fine-tuning of the initial conditions finds no explanation in the Big Bang cosmology. The solution of this problem possibly involves a mechanism that produces considerable entropy at a later stage of evolution.

**iii) the small scale inhomogeneity :** Although the universe is homogeneous on a large scale it is very inhomogeneous on the galactic scale and at lower scales. An explanation of the observed structures in the universe may be due to small density perturbations occurring in the very early universe. The origin of these perturbations and the nature of their spectrum seem to need a more sophisticated model.

In addition to these problems there are a few other problems which are in conflict with the concepts of theories of particle physics. It is, therefore, felt essential to look for cosmologies which can solve these basic problems. This led to various inflationary models.[4,5] Although it was Guth[4] who first emphasized the merits of an inflation, ideas similar or close to the inflationary models had been suggested by some other cosmologists.[6-10] Gliner[6] was the first to note that when all four eigenvalues of the stress tensor were equal, the cosmology corresponds to matter with the properties of a vacuum. Gliner and Dymnikova[8] later on showed that this vacuum dominated state makes a transition to a radiation dominated era. This process led to an enormous increase in the scale factor, an idea very similar to the present day inflationary model. However, at present it seems that the equation of state of the superdense baryonic matter taken by them is not appropriate. The possibility of an exponential expansion was also pointed out by Starobinsky[9] from a consideration of the conformal anomaly of the stress tensor, occurring in the semi-classical Einstein equation. Kirzhnits and Linde and Linde studied the vacuum phase transition in an abelian Higgs model.[10] Sato[11] studied the effects of a first order phase transition in the early universe with a motivation to build a consistent baryon number symmetric universe in which CP-violation occurs spontaneously in an exponentially expanding domain. It was, however, Guth[4] who explicitly emphasized the role of a phase transition in solving the cosmological problems.

In the inflationary models gravity is described classically while matter is treated in terms of quantum fields. The basic idea of an inflationary scenario is that in the evolution of the universe there is an epoch when the vacuum energy density dominates the energy density of the universe. During this epoch, the vacuum energy behaves as an effective positive cosmological constant allowing the universe to expand exponentially ( or quasi-exponentially ), $a(t) \sim a_o \exp \left[ \int H \, dt \right]$ where $H = \dot{a}/a$ is the Hubble parameter. This expansion in the scale factor allows a small causally coherent region to grow bigger than the observable universe, provided the exponent $\int H \, dt > 65$ is satisfied. This huge expansion in a very short time is called 'inflation'. A less rapid inflation may be obtained in some other models.[12-15] It has been shown that a non-exponential accelerated phase of expansion

$$\ddot{a}(t) > 0 \qquad (1.1)$$

where a dot denotes differentiation with respect to the proper time t, also leads to an inflation characterized by a period in which a(t) grows as $t^A$ with $A > 1$. An example of this type may be obtained by using an exponential type potential $V(\phi) = V_o \exp(-\lambda \phi)$ with $\lambda \gtrsim 0$, which occurs in the Salam-Sezgin model[16] of N = 2 Supergravity coupled to matter and in some Kaluza-Klein theories.[13] The causal structure of the space-time in all these inflationary models differs considerably from what one gets in the Big Bang model. Consequently, the horizon problem is resolved as the observable universe in the inflationary model had a smaller size compared to a causal horizon volume. In this scenario our observable universe today ( $\sim 10^{28}$ cm ) had a physical size of about 10 cm at the GUT era ( $\sim 10^{-35}$ sec. ) which lies well within one of the smooth regions produced by the inflation. The model also helps us to solve the primordial monopole problem which was introduced in cosmology by unified theories. At the end of inflation the density of the primordial monopoles, if any, will be very small. The fresh production of monopoles during reheating will not occur if the reheating temperature is below the temperature of GUT symmetry breaking ( $T < 10^{14}$ GeV ). It may be added that an inflation also provides a mechanism for the origin of the small scale density inhomogeneities. The original fluctuations present in a comoving volume before inflation are expected to be smeared away by the expansion. But the quantum fluctuations generated during inflation may lead to the observed density perturbations.[17,18] The scale invariant Harrison-Zeldovich[19] spectrum emerges naturally within the exponential expansion phase because the de Sitter space is time translation invariant and quantum fluctuations in this space might act as initial seeds for the galaxy formation. However, such a scale invariant spectrum cannot be obtained in power law inflation[12] for a finite exponent ( A ) . The exponential expansion is , therefore, more attractive.

In the original model the idea of an exponential expansion in the early universe was utilized by Guth[4] to resolve the horizon and the flatness problems. However, the model was not successful as it did not permit a universe with sufficient inflation to return gracefully to the usual Friedman-Robertson-Walker universe. There were several other problems, e.g. the domain wall problem, the problem of generation of inhomogeneities which lead to galaxy formation, the problem of large scale inhomogeneities etc.. It was soon realised from the subsequent work of others that the defects in the model[4] (known as old inflation) could not be removed.[20-23] Fortunately, in 1982 Linde[24] and Albrecht and Steinhardt[25] independently suggested a new mechanism to overcome the problems of the old inflation. The model is known as 'new inflation', which will be briefly reviewed in the next section. Subsequently, a more attractive model was advocated by Linde[26] which has apparently eliminated the fine tuning of the potentials needed to get the new inflationary universe scenario. The modified model is known as chaotic inflation. In this scenario, the universe evolves out of a chaotic distribution of initial data. A simple version of this model may be realized by considering a chaotic distribution of the values of the scalar field $\phi$ ( inflaton ) at time $t \sim t_P = M_P^{-1}$ , where $t_P$ is the Planck time and $M_P$ ,the Planck energy. However, Linde proposed the scenario with the assumption of an isotropic inflation of the early universe. The scenario can be realized even in an anisotropic universe as will be shown in chapter II. A scenario may be realized even with a wide variety of interaction potentials. However, the inflationary models in general need an inflaton field with the exception of Starobinsky model.[9] It may be noted that Starobinsky proposed the model long before the advantages of an inflation were fully realized. The motivation for the model was to remove the singularity problem occurring in the standard model. The de Sitter stage in this model is a self-consistent solution of the semi-classical Einstein equation. It is known that the vacuum expectation value of the stress tensor in curved space time develops divergences, which may be removed by adding curvature squared terms to the Einstein action. However, the regularization process[27-31] causes the trace of the stress tensor to develop a non-zero value leading to the well-known conformal anomaly. It is this anomaly which drives inflation in Starobinsky model. But, the original Starobinsky model[9] faces serious problems when confronted with observed limits of the MBR.[32] It was suggested[33,34] later that the inconsistencies in the model may be removed by adding a term $\alpha R^2$ to the Einstein action where $\alpha$ is very large. With this modification, the Starobinsky model reduces to the study of the field equations derived from a Lagrangian

$$L = R + \alpha R^2 \quad . \qquad (1.2)$$

This is the most general Lagrangian including quadratic terms in curvature in four dimensional conformally flat spaces. The cosmological models obtained in this framework have a number of attractive features. A theory with quadratic curvature terms can be made perturbatively renormalizable[35] and asymptotically free[36,37]. The unitarity issue of the theory is, however, yet to be settled. Nevertheless, the higher derivative theory may be considered as an interesting model for testing various ideas related to the quantum properties of gravity.

Considerable work has been done during the last few decades to formulate a quantum theory of gravity.[38] The investigation seems to lead some people to believe that a consistent theory of quantum gravity cannot be obtained within the framework of point-field theories. The advent of the string theory has opened up new and interesting possibilities in this context. String theories[2] may be looked upon as describing the interactions of a few massless and an infinite set of massive states, with masses which are multiples of the Planck mass. The standared low energy physics is now described by the interactions of the massless modes after one integrates over the massive ones. The supergravity theories may be regarded as the low energy regime of superstring theories. The striking discovery[39] that in ten dimensions, a supergravity theory coupled to Yang-Mills fields with a gauge group SO(32) or $E_8 \times E_8'$ is anomaly free had inspired considerable activities in this area. Although the expected breakthrough had not yet come, the world wide hectic activities have served to focus on a number of issues which need further investigation. Since the quantum consistency of the superstring theory is obtained in the critical dimension D = 10, one has to look for a realistic compactification scheme. Candelas *et al*[40] set out to achieve this by requiring that the ten dimensions should compactify to $M^4_* \times K$, where $M^4_*$ is maximally symmetric and K is a compact six dimensional manifold. It was also demanded that the four dimensional theory should have an unbroken N = 1 supersymmetry, so that the hierarchy problem can be tackled. Candelas *et al* obtained a solution with a Minkowskian ($\Lambda$ = 0) space for $M^4_*$ and a Ricci flat Calabi-Yau manifold for K. While the discovery of Candelas *et al* is striking, the success of the compactification scheme partly rests on its ability to be accommodated in a realistic dynamical theory of evolution. However, attempts[41-45] to build cosmological models based on the N = 1 Yang-Mills supergravity action have not been very successful. Weiss[46] has even questioned the consistency of a Ricci flat compact manifold with observed matter-dominated universe. It is not easy to get inflation in this model. The difficulties stem from the Ricci flatness of the internal space and also from the absence of any dimensionless free parameter. In the absence of SUSY breaking, the naive potential has only one minimum at $g^2_{GUT}$ = 0 and the realisation of the cosmological evolution of the compactified theory to the correct vacuum, $g^2_{GUT} \simeq O(1)$, is a non-trivial problem. The available mechanism for SUSY breaking is to invoke gaugino condensation in the hidden $E'_8$ sector of the $E_8 \times E'_8$ theory. Ellis *et al*[41] have considered the tree level potential with some fine-tuning of the condensation temperature and obtained an inflationary phase in the $d = 4$ effective theory. However, the one loop effective potential does not have a minimum and the situation is not very clear. Maeda *et al*[43] have considered another SUSY breaking potential, given by Binetruy and Gaillard[47], which has only asymptotic validity. It is clear that further investigation is called for to understand the detailed dynamics of both the processes, compactification from ten to four dimensions and an inflation in four dimensions.

As the first step to achieve this goal, we intend to look for cosmological models based on a 10-dimensional action choosing the Gauss-Bonnet combination for the quadratic terms in curvature. Keeping in view the possibilities that the conditions necessary for an acceptable compactification may be realised transiently (even approximately), a systematic study of the solution will be undertaken. The calculations of Ellis *et al*[41], Maeda[42] and Maeda *et al*[43] have high-lighted some of the problems in this approach. The major problem is to get an inflationary phase and also a graceful exit from this phase. While the precise mechanism which allows this is not yet understood, it is useful to look afresh at all the possible mechanisms for inflation in 4-dimensions, viz Kaluza-Klein type inflation[48,49] ( $M^4$ expanding, K contracting ), contribution of the higher derivative terms,[9,33,34,50-52] a realistic temperature dependence of the effective potential etc., to see if field equations allow such a solution.

Cosmological models with different higher derivative terms, and, in particular, the Gauss-Bonnet combination have already been studied by various authors.[53-59] But a consistent scenario is yet to emerge. It is clear that cosmological model building in higher dimension is an exercise which may prove useful in our understanding of both cosmology and particle interactions at higher energies. Keeping this objective in view, we have studied a model with Gauss-Bonnet terms in the action, including both matter loop corrections and temperature effects. We have looked for an acceptable scenario of the pre-compactified universe which eventually passes on to a radiation dominated FRW universe. As will be shown in Chapter IV, a mechanism which helps to keep the cosmological constant small emerges naturally in this model.

## I.2 *NEW INFLATION & ITS PROBLEMS :*

In this section we shall discuss briefly the new inflationary model and its shortcomings.[5] Let us recall that in the old inflationary model[4], Guth first made an attempt to resolve the horizon, flatness and monopole problems of the Big Bang model. He obtained the inflationary model by combining the ideas of a phase transition mechanism which the universe might have gone through during the first $10^{-35}$sec, with the effects of general relativity. Inflation in this model ends with a transition to the broken symmetric phase by nucleation and growth of bubbles. However, it was realised soon that the model had a number of problems which lead to unacceptable consequences.[4,20-23] The energy released during the phase transition will thermalize if the bubbles undergo frequent collisions. But this process makes the universe highly anisotropic, contradicting the observed homogeneity of the universe. It is now known that the assumption of a quick phase transition in this model is not tenable. It was also noted that a large expansion factor required for this stage implies that the bubble nucleation rate is slow compared to the expansion rate of the universe.[4] In this scenario the bubbles formed will never percolate. There is no mechanism in this scenario by which the universe may successfully complete the phase transition to the broken symmetric phase and exit gracefully to the usual FRW-universe. To overcome these problems a modified model was suggested by Linde[24] and Albrecht and Steinhardt[25] independently which is known as 'new inflation'.

The basic idea behind this proposal was to construct a model in which inflation occurs after the bubbles of new phase have been formed. The new element introduced in the model is based on a different kind of phase transition commonly referred to as the 'slow rollover' phase transition . Linde and Albrecht and Steinhardt have suggested the model in the framework of the grand unified theories[1] in which the gauge symmetry is broken by the radiative corrections to the effective potential as in the Coleman and Weinberg[60] ( in short, CW ) model. In the abelian Higgs model the effective potential calculated in one loop approximation is given by

$$V_{eff}^{(1)} ( \phi ) = V (0) + B \phi^4 \left[ \ln \left( \frac{\phi}{\phi_o} \right)^2 - \frac{1}{2} \right] \qquad ( 1.4 )$$

where B is a constant determined by the gauge coupling constant and $\phi_o$ corresponds to the global minimum of $V_{eff}^{(1)}$ . In CW-model spontaneous symmetry breaking ( SSB ) is a consequence of quantum corrections to the potential. The effective potential acquires a temperature dependent mass term when finite temperature corrections are taken into account. The effective potential is given by

$$V_{eff}^{(1)} ( \phi, T ) = V_{eff}^{(1)} (\phi) + C\, T^2 \phi^2 \qquad ( 1.5 )$$

where C is a constant $O(g^2)$ and T is the temperature. This potential provides an energy barrier that can trap the universe in a metastable symmetric phase which occurs during its cooling from a high temperature. At a very high temperature, the absolute minimum of $V^{(1)}_{eff}(\phi,T)$ is located at $\phi = 0$. The position of the absolute minimum does not change until T is less than a critical value $T_c = (B/2C)^{1/2}\phi_o$, where $\phi_o$ is the maximal value of the field $\phi$ inside the bubble immediately after its formation.

The general feature of new inflation is as follows : Consider a first order phase transition in the early universe which occurs due to a spontaneous symmetry breaking at an energy scale $M_G$ (GUT scale) . At a temperature $T \gg T_c \approx M_G$, the vacuum state $\phi = 0$ is the global minimum of the effective potential and the thermal part of the energy momentum tensor dominates. Consequently the universe expands like a radiation dominated phase. As the universe expands, the temperature of the universe drops and falls to $T_c$. when $T \to T_c$, a second minimum develops at $\phi = \phi_o$ and there are degenerate vacuum states at $T = T_c$. At this epoch the vacuum energy starts dominating over the energy momentum tensor. For $T < T_c$, the potential barrier which develops between the two minima of the effective potential does not permit the universe to evolve classicaly from the symmetric state $\phi = 0$ to $\phi = \phi_o$ state instantly. It is known that the quantum mechanical tunneling is also not possible because the transition probability is insignificant.[61] However, at a very low temperature ( $T \approx 0$ ) less than the Hawking temperature $T_H \sim \frac{H}{2\pi}$, ( H is the Hubble parameter ) the barrier effectively disappears and the $\phi = 0$ state becomes unstable. It is now possible to drive the universe away from $\phi = 0$ by a small amount which may be triggared by a small quantum fluctuation. Let $\tilde{\phi}$ be the value of $\phi$ when it penetrates the barrier. The subsequent evolution from $\phi = \tilde{\phi}$ to $\phi = \phi_o$ can be described by the equation ($\phi$ is assumed to be spatially homogeneous):

$$\ddot{\phi} + 3 H \dot{\phi} + \Gamma \dot{\phi} + V'( \phi ) = 0 \qquad ( 1.6 )$$

where the prime indicates differentiation with respect to $\phi$ and dot that with respect to t. The third term accounts for particle creation due to the time variation of $\phi$. The quantity $\Gamma$ is determined by the nature and coupling of the particles produced. One defines the life time of the $\phi$ particles by $\tau = \Gamma^{-1}$. For $T \ll T_c$, we may write $V_T(\phi) \simeq V_o(\phi)$, and the square of the Hubble parameter $H^2 = \frac{8\pi G}{3} \rho$, can be determined from the expression for $\rho$

$$\rho = \frac{1}{2} \dot{\phi}^2 + V( \phi ) + \rho_r , \qquad ( 1.7 )$$

where $\rho_r$ is the radiation energy density produced by the time variation of $\phi$ following the equation

$$\dot{\rho}_r + 4 H \rho_r = \Gamma \dot{\phi}^2 , \qquad ( 1.8 )$$

the r.h.s. takes into account the effect of particle creation.

The scenario can now be described. Suppose V is very flat in some region between $\phi = \tilde{\phi}$ and $\phi = \phi_o$; $\phi$ will evolve very slowly and the radiation energy density can be neglected at this stage. Under this circumstances the eq. (1.6) can be rewritten as

$$\dot{\phi} = - \frac{1}{3} H^{-1} V'(\phi) . \qquad ( 1.9 )$$

If $V(\phi)$ is sufficiently flat, the time taken by $\phi$ to traverse the flat region may be sufficiently large compared to the expansion time scale $H^{-1}$. During this period

$$H \approx \left\{ 8\pi G\ V(0)/3 \right\}^{1/2} \approx \text{const.} \qquad (1.10)$$

and

$$a(t) \propto \exp(\ Ht\ ). \qquad (1.11)$$

Thus the scale factor has a large exponential growth during this slow rollover period. This mechanism of transition has been studied in details by Guth and Pi[62], Hawking[63], Brandenberger and Kahn[64] and others[65] considering the evolution of a single inflaton field in the theory. It has been suggested that a single fluctuation region inflates to encompass the entire observable universe. In this scenario the inflation continues to occur even when the universe evolves towards the stable phase. To allow for a natural transition to a FRW universe, the flat portion of $V(\phi)$ is followed by a region where $V(\phi)$ has a sharp drop and the slow rollover transition ceases to occur. The value of $\phi$ changes rapidly in this stage. Around the minimum, it oscillates with a frequency $\omega = (\ V''(\phi_o))^{1/2} \approx M_G^2 \gg H^2$. These oscillations are damped quickly by both particle creation and the expansion of the universe. Two possibilities exist in this case :

i) If $\Gamma^{-1} \ll H^{-1}$, the rapid time variation of $\phi$ leads to the conversion of the coherent field energy $V(\phi) + \frac{1}{2}\dot{\phi}^2$ into radiation by quantum particle creation,[66] thereby reheating the universe. This is a case of efficient reheating.

ii) If $\Gamma^{-1} \gg H^{-1}$, $\phi$ continues to oscillate, however, the coherent field energy is redshifted away very soon by the expansion of the universe as $a^{-3}$. At a later time $t \simeq \Gamma^{-1}$, the universe is reheated by the radiation energy density leading to a poor reheating.

However, in order to keep the reheating temperature below the GUT scale it is necessary to adjust the parameters in the theory. This fine-tuning is required to avoid fresh production of monopoles. Another success of the new inflation is that it suggests an explanation for the galaxy formation problem. However, alongwith these successes, the new inflationary scenario suffers from some obvious drawbacks, which are outlined below :

a) The CW type potential, which has been found suitable for the scenario needs some fine-tuning of the parameters and is, therefore, not a natural choice.

b) The slow roll equation is not true in general for an interacting field theories.[67]

c) The transition from the symmetric SU(5) may not occur in the direction of the global minimum SU(3) X SU(2) X U(1) . It was pointed out by Guth and Weinberg[20] and Breit *et al*[68] that the universe may instead be trapped in the wrong phase SU(4) X U(1) due to symmetry breaking.

d) Although a scale invariant spectrum of density perturbation is obtained in the new inflation,[18,69] SU(5) GUT produces a very large value for $\frac{\delta\rho}{\rho}$ ( $\sim$ 50 ) which is many orders larger than the experimental value ( $\frac{\delta\rho}{\rho} < 10^{-4}$ ).

To overcome the problem discussed in (d), a proposal has been made by Shafi and Vilenkin[70] which requires an extra inflaton field. It was suggested that the overproduction of the density perturbation may be overcome considering a very weakly interacting field ( $\lambda \sim 10^{-12}$ ). Unfortunately, SU(5) GUT theory does not permit such a weak field. Therefore, it was concluded that the SU(5) GUT is cosmologically unacceptable. However, such fields may occur in supersymmetric and supergravity theories. Inflation of the early universe can be incorporated in a natural way in such theories.[71]

Mazenko *et al*[72] have raised another serious objection against the new inflationary models of the early universe . It was pointed out that due to the violent fluctuations of the field at a very high temperature in the early universe the field may not settle into the false vacuum state ( $\phi = 0$ ) as the universe cools. They have claimed that the models of new inflation are based on wrong physics. It was argued by them that many models previously considered as candidates for new inflation do not enter into an inflationary regime. To see this we note the following: If $\phi$ is only gravitationaly coupled to other fields, at a high temperature the mechanism which confines the field at the value $\phi = 0$ will no longer be valid. In the absence of such confining forces we expect large thermal fluctuations. As a result $\phi(x,t)$ will be inhomogeneous. For $\phi_o < M_p$ the spatial fluctuations in $\phi$ may be much larger than the separation between the minima of $V(\phi)$. The energy momentum tensor $T_{\mu\nu}$ is now dominated by radiation, consequently inflation will not occur. They argued that as the universe expands the scalar field will lose energy. At $T = T_c$, the energy content of the field is insufficient to cross the potential barrier at $\phi = 0$ and, therefore, $\phi(x)$ will rapidly relax to a value $-\phi_0$ if initially $\phi_0(\vec{x}) < 0$. Similarly $\phi(\vec{x}, t)$ will relax to $+\phi_0$ if initially $\phi(\vec{x}) > 0$. Therefore, at $T < T_c$ spatial domains with $\phi(x) = \pm\phi_0$ will be formed. It can be shown that in such domains $V(\phi) = 0$. Consequently, the equation of state necessary for inflation cannot be obtained. However, Albrecht and Brandenberger[73] and Albrecht *et al*[74] almost simultaneously showed that the above argument is not correct in general. The discrepancies can be avoided by a new approach to phase transition in the cosmological models which does not use the concept of an effective potential. They have shown that in many models the expansion of the universe leads to a sufficient Hubble damping of the field configuration which gives rise to a long intermediate phase of inflation. For the CW-model as well as in quartic scalar field potential with double well, the new inflation can be realized. Another objection was pointed by Linde.[26] He pointed out that in theories with a large value of the vacuum potential energy $V(\phi)$ and a sufficiently small coupling constant $\lambda$, the inflationary universe scenario based on a temperature dependent phase transition cannot be realized. This observation provided a strong motivation for the chaotic model suggested by him. This will be discussed in the next chapter.

## I.3 *AIM OF THE WORK :*

The aim of the work is to study some cosmological models of the early universe which are consistent with the recent theories of elementary particle interactions at high energies and also with the constraints imposed by various astrophysical observations. We will study cosmological models within the framework of a quantum field theory in curved space-time. Since an inflationary stage seems to be an essential ingradient of modern cosmological models, we will study various types of inflationary models. Different aspects of any of these models need to be studied carefully before it can be considered as an acceptable theory of the early universe. We have undertaken such a study with particular emphasis on the mechanisms that lead to an inflationary stage and also allows a natural exit from it.

The models which we have chosen for this study are : (i) the chaotic inflation, (ii) the higher derivative theories in 4-dimensions and (iii) higher derivative theories in higher dimensions. We have analysed these models critically and also obtained some new results. Our aim has been to study the efficacy of these models in providing a realistic scenario of the early universe.

## I.4 *SUMMARY OF THE WORK :*

The summary of the chapters contained in this thesis is as follows :

Chapter I gives an introduction into the background and nature of the problems studied and also summarizes the results obtained.

Chapter II : In this chapter, the chaotic inflationary scenario,[26] as proposed by Linde is briefly reviewed. The chaotic scenario is then studied in an anisotropic universe model. We have considered, in particular, an anisotropic Kantowski-Sachs (KS) metric.[75] For a weakly coupled inflaton field the Einstein's field equation admits here two types of cosmological solutions. For one of these solutions, the initial anisotropy washes out as the universe expands while for the other there is no such transition to an isotropic universe. In the second case, the measure of anisotropy remains constant. We have shown that the chaotic model can be realized in the KS-metric. This observation justifies the claim of Linde that the ideas of chaotic scenario are fairly general. The concept that the present state of the universe should not depend on any specific choice of the initial data gets realized in our results, since KS-model, though simple describes a rather exceptional type of anisotropy.

Chapter III : In this chapter, the evolution of the early universe in higher derivative theories has been studied. As a special case, we have studied in detail the modified Starobinsky model, which is described by an action quadratic in curvature :

$$S = \frac{M_P^2}{16\pi} \int \left( R + \alpha R^2 \right) \sqrt{-g}\ d^4x \ . \qquad (1.3)$$

Depending on the initial conditions, the early universe in this model may evolve from one of the two different initial stages : (i) de Sitter and (ii) Non-de Sitter (NdS). In the original Starobinsky model[9] the de Sitter phase is an exact solution which is, however, unstable. On the other hand, the de Sitter stage is only an approximate solution in the modified model. The NdS solution noted by us exhibits entirely new features in the initial stages of the evolution. We have emphasized these new features. We also follow the evolution of the universe in different stages, e.g. the initial NdS-phase, quasi-exponential inflation, particle production, reheating etc. . An interesting result obtained[110] in the flat ( K = 0 ) model is that the choice of the initial value of the scalar curvature ( R ) determines whether R should increase or decrease. In both the cases R tends asymptotically to the value $\left( \frac{1}{12\beta^2} \right)^{1/3}$, where $\beta$ is a constant. However, the magnitude of the inflation is almost independent of this undetermined constant ($\beta$), which is undoubtedly a good feature of the model. In the closed model ( K = + 1 ), $\beta$ is found to be time dependent. Here, the scale factor of the universe grows linearly with time. The solution is unstable and eventually the universe enters into a different phase of expansion, as in the flat model. The solution found in open ( K = -1 ) model is also discussed. However, the scalar curvature (R) here is zero and the solution is not very interesting from a physical point of view. The modification in the Starobinsky model[9] brings in a substantial change in the original scenario in the earlier stages and the new results may have interesting applications in the formulation of quantum cosmology.

**Chapter IV :** In this chapter, we have presented a scenario of the evolution of a ten dimensional universe. As theories of particle interactions often require dimensions more than four for their formulation, it is essential to check if consistent cosmological solutions, which can accommodate these theories are also allowed. Higher derivative terms in the gravitational action and, in particular, the Gauss-Bonnet combinations occur in a natural way in many theories. Cosmological solutions for these theories in ten dimensions have been studied. An attempt has been made to construct a scenario which includes a spontaneous compactification, an inflationary epoch followed by a four dimensional radiation dominated stage. It is noted that with an action containing matter loop correction, including the temperature effect, it is possible to have a realistic scenario, provided there is an inflationary stage of the early

10-dimensional universe. A mechanism is suggested which keeps the 4-dimensional cosmological constant small, close to zero. The study reveals a rich structure of the theory due to the presence of Gauss-Bonnet terms, although one gets back the usual Einstein equation in four dimensions at a large time.