MODIFIED THEORIES OF GRAVITY AND DARK ENERGY MODELS OF THE UNIVERSE

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Dedicated to my family

DECLARATION

I declare that the thesis entitled "**MODIFIED THEORIES OF GRAVITY AND DARK ENERGY MODELS OF THE UNIVERSE**" has been prepared by me under the guidance of **Dr. Bikash Chandra Paul**, Associate Professor of **Department of Physics, University of North Bengal**. No part of this thesis has formed the basis for the award of any degree or fellowship previously.

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CERTIFICATE

I certify that Shri Arindam Saha has prepared the thesis entitled "**MODIFIED THEORIES OF GRAVITY AND DARK ENERGY MODELS OF THE UNIVERSE**", for the award of Ph.D. degree of the **University of North Bengal**, under my guidance. He has carried out the work at the **Department of Physics, University of North Bengal**.

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ABSTRACT

In this thesis specific issues relevant for cosmological model building of the universe are investigated. Different aspects of the universe are also studied in the framework of modified theories of gravity with or without considering exotic matter.

Gravitational instantons, which are the Euclidean solution of the field equation of the modified theories of gravity, are employed to estimate the probability of creation of an inflationary universe with a pair of primordial black holes. Gravitational instantons are obtained considering a gravitational action which is a polynomial function in Ricci scalar (R) given by the Lagrangian $L = \Sigma_i \lambda_i R^i$, where Einstein action corresponds to the case $i = 1$ only, and for $i \geq 2$ corresponds to modified gravity Lagrangian. Using semiclassical approximation and Hartle-Hawking 'no boundary proposal', the probability of quantum creation of an inflationary universe with or without PBH is evaluated.

The method is then extended to a higher dimensional universe with a gravitational action which is described by a polynomial function of scalar curvature. A class of new gravitational instanton solutions relevant for cosmological model building are obtained here in the non-linear theory of gravity considering terms upto R^4 order.

We study creation of an inflationary open universe in a higher order theory of gravity making use of the Hawking-Turok instantons. Considering a non-linear γR^4 term to the Einstein gravitational action with a cosmological constant, gravitational instantons are obtained which are relevant for describing the early universe. It is shown that unlike Bousso and Hawking, the theory permits gravitational instantons even in the absence of a cosmological constant. We obtain gravitational instanton in the framework of the higher order theory of gravity making use of conformal transformation also. The higher order theory of gravity may be converted into a theory of self-interacting scalar field minimally coupled to Einstein gravity which is then employed to obtain relevant gravitational instantons. The singular instantons obtained in the conformally equivalent case is taken up to obtain open inflationary scenario. It is noted here that non-singular instanton solution not only permitted with a positive cosmological constant but

also in the absence of a cosmological constant.

To describe dark energy a holographic dark energy model of the universe is considered with modified generalized Chaplygin gas (MCG). We determine the corresponding holographic dark energy field and its potential. The stability of the holographic dark energy is also discussed.

The modified GCG (MCG) is considered as a single fluid model to unify dark matter with dark energy. Thus it is relevant for describing late accelerating universe. MCG is taken up in the framework of Hořava-Lifshitz (HL) theory of gravity to obtain cosmological models. Using the recent cosmological observational data we determine the constraints that are imposed on its equation of state (EoS) parameter.

In standard model cosmological solutions are obtained with non-interacting fluids but recently from cosmological observations it is found important to explore evolution in the presence of interacting fluids. In this direction Holographic dark energy correspondence of interacting generalized Chaplygin gas model in a compact Kaluza-Klein cosmology is taken up. The evolution of the holographic dark energy and the equation of state parameter are determined using recent observational predicted density parameter. A connection between dark Energy and phantom field is also established.

Cosmological models with power-law expansion in a modified $F(R)$ Hořava-Lifshitz gravity are obtained taking into account a mixture of two interacting fluids in which one of them is dark energy. We obtain the constraints imposed on the EoS parameters of the interacting fluids that permit a viable cosmological model admitting late time acceleration.

PREFACE

The research work presented in this thesis is carried out at the University of North Bengal, West Bengal, India. The thesis is the result of my own work, as well as the outcome of scientific collaborations.
The content of this thesis is based on the following published and communicated research papers:

- Chapter 2 is based on the article entitled:
  Probability for Primordial Black Holes in Higher Derivative Theories- B. C. Paul and A. Saha, *International Journal of Modern Physics* **D 11**, 4(2002).

- Chapter 3 is based on the article entitled:
  Probability of Primordial Black Holes in a multidimensional universe with Nonlinear Scalar Curvature Terms- B. C. Paul, A. Saha, S. Ghose, *Phys. Rev.* **D 78**, 084007(2008).

- Chapter 4 is based on the article entitled:
  Singular Instanton in R^4 Gravity for Open Inflation, B. C. Paul and A. Saha, *Class. Quantum Grav.* **27**, 215004(2010).

- Chapter 5 is based on the article entitled:
  Holographic Dark Energy Model with Generalized Chaplygin Gas- B. C. Paul, P. Thakur and A. Saha, *Internal Report* **ICTP IC/ IR/2007/006**

- Chapter 6 is based on the article entitled:
  Modified Chaplygin Gas in Hořava-Lifshitz Gravity and Constraints on Its B Parameter-B. C. Paul, P. Thakur and A. Saha, *Phys. Rev.* **D 85**, 024039(2012)

- Chapter 7 is based on the article entitled:
  Interacting Holographic Generalized Chaplygin Gas in Compact Kaluza-Klein Cosmology- S. Ghose, A. Saha and B. C. Paul, *communicated.*

- Chapter 8 is based on the article entitled:
  Cosmologies with Interacting Fluids in Modified $F(R)$ Hořava-Lifshitz Gravity- A. Saha and B. C. Paul, *to be communicated.*

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Chapter 1

Introduction

The theoretical understanding of our universe has gone through a profound change after 1915 due to Einstein's General Theory of Relativity (in short, GTR). In 1917, Einstein wanted to obtain a static model of the universe, as during that time he was motivated by the then available information from astronomical observation. But he failed to realize a static universe in his theory of Relativity with usual matter. Einstein then modified his field equation introducing a cosmological constant (Λ) which successfully accommodate a static universe. Later Hubble discovered (1927) that galaxies are moving away from each other leading to an expanding universe scenario. The remarkable discovery by Hubble led Einstein to give up Λ term that was considered for obtaining a static universe. It may be interesting to note that in the absence of a cosmological constant (Λ) Friedmann obtained an expanding universe solution in 1922. The dynamical solution obtained during that time remained only an academic interest until the Hubble's discovery. GTR is now widely accepted as a fundamental theory that describes the geometric properties of space-time. The proposal of GTR made it possible to study the structure of space-time and to realize the evolution of the universe in terms of physical laws. In 1946, Gamow and his collaborators obtained cosmological solutions which permits dynamical evolution of a universe from a phase of very hot and dense state. The model is known as the standard hot big-bang model. It predicts the existence of cosmic microwave background radiation (in short, CMBR) originated during nucleosynthesis as one of the remnants of Big Bang. Penzias and Wilson discovered existence of such primordial radiation while mapping the radio signals from the Milky Way. The discovery of CMBR is one of the major confirmations of the Big Bang theory. The standard Big Bang cosmology based on perfect fluid model is successful in obtaining a radiation or a matter dominated phase of the universe. It has been realized that the standard model is fairly successful in explaining *2.7 K* CMBR spectrum, primordial nucleosynthesis and the cosmic abundances of light elements *e.g.*, deuteron and ^4He (about 23 % by mass). Inspite of several successes of Big Bang model it is known that some of the observed facts of the universe cannot be addressed within the framework of perfect fluid assumption. Big Bang model fails to account early universe at time before $t \sim 10^{-2}$s. The major problems that the Big Bang model encountered are known as horizon problem, flatness problem, small scale inhomogeneity problem etc. [1, 2, 3, 4, 5, 6, 7]. To resolve the above issues in cosmology theories of Particle Physics that are relevant at very high energy are taken into account. Consequently the interface of Particle Physics and Cosmology opened up new avenues in both the areas to understand conceptual issues by invoking inflation. It is assumed that in the early epoch there was a phase when the universe expands enormously in a very short time known as inflation. A small causally connected coherent region grow sufficiently big to encompass the present observed universe.

Gliner [8] was the first to obtain inflation in GTR for an appropriate stress-energy tensor corresponding to matter with the properties of a vacuum. Later, Starobinsky [9, 10] obtained inflationary solution using trace anomaly. However, the efficacy of the theory is known only after the seminal work of Guth [11]. The basic mechanism of inflation is based on temperature dependent phase transition mechanism. The scenario proposed by Guth is known as *'old inflation'*. It corresponds to de-Sitter inflation where first order phase transition from false vacuum ($\phi_i = 0$) to a true vacuum is considered. However at the end of inflation, the universe becomes inhomogeneous which led to a serious problem with present observations [11, 12]. In 1982 Linde [1], Albrecht and Steinhardt [13] independently, proposed another version of inflation known as *'new inflation'* which is based on slow-roll inflation. In the new inflation model a rapid expansion in a very short duration is obtained using a homogeneous scalar field, when it rolls down the potential hill instead of tunneling out of a false vacuum state. At the minimum of the potential inflation ends and subsequently the field oscillates and the universe reheats. It was shown that although new inflation does not produce a perfectly symmetric universe, a tiny quantum fluctuation of the scalar field that originates during this phase leads to the observed structure of the universe. Mukhanov and Chibisov [14] shown that the quantum fluctuations of the field originated during inflation might act as the primordial seed for the structure formation of the universe. Due to several shortcomings in new inflation, cosmological models with second order phase transition mechanism have been employed to obtain a viable cosmology. However, the model encountered with fine-tuning problem. In 1983, Linde proposed an inflationary model which apparently needs no specific fine tunning or phase transition mechanism [15]. It is based on slow-roll inflation where the inflaton field can pick up a value ($\phi \sim M_P$, where M_P is the Planck mass) to begin with from a chaotic distribution of fields. Linde's model is known as *chaotic inflation*. The advantage of the model is that it is not essential to restrict to a particular initial configuration for the field ($\phi_i = 0$) as was required in the new inflation model. The homogeneous scalar field instead can take any value permitted by a random initial distribution of the field satisfying the constraint $V(\phi) \leq M_P^4$ imposed by Planck scale. The upper bound on potential is essential to avoid quantum gravity. Thus Chaotic inflationary expansion of the universe may begin close to Planck time. The early inflation satisfactorily solves some of the issues not understood earlier *e.g.* the flatness problem and horizon problems including a nearly flat spectrum of temperature anisotropies observed in Cosmic Microwave Background (CMB) [16]. In chaotic model, matter in the universe might have created at the end of inflation from quantum fluctuations of the inflaton fields responsible for evolution. The matter dominated phase is a decelerating phase of expansion.

Recent observational data predict another phase of accelerating expansion of the universe which might have occurred in the recent past. The late accelerating phase is believed to have started after the matter dominated phase of the universe. The recent astronomical data when interpreted in the context of Big Bang model have provided some interesting information about the composition of the universe. It has been predicted that our universe is dominated by an unknown matter having negative pressure known as **dark energy** (in short, DE). The existence of DE has been confirmed by a number of observations namely, supernovae Ia (SN Ia) [17], Large scale structure (LSS) [18], Baryon Acoustic Oscillations (BAO) [19], Cosmic microwave background (CMB) anisotropic spectrum [16] etc. Einstein's gravity with matter fields permitted by standard model of Particle Physics do not allow such accelerating phase. Thus it is a challenge in theoretical physics to settle the issue.

It is known that Einstein field equation with a cosmological constant permits DE as the equation of state parameter (EoS) which leads to $\omega = -1$, but the problem is with the low value of the Cosmological constant at the present epoch. A number of cosmological models with a modification of the matter sector have also been proposed. In this context exotic matter namely **quintessence field** [20, 21, 22, 23], **phantom field** [24], **K-essence** [25, 26, 27, 28], **tachyon**, [29, 30, 31, 32, 33, 34], **Chaplygin gas** [35, 36, 37, 38, 39, 40], **Chameleon fields** [41] etc. are considered for obtaining late acceleration.

Alternatively modified theory of gravity, where the gravitational sector of the theory is modified by adding a polynomial function of scalar curvature terms or some curvature invariants *e.g.*, Gauss-Bonnet terms etc., have received increased attention recently due to the need of such terms in high-energy physics and in cosmology. Among different classes of modified theories, there is a particular class which involves non-linear generalizations to the Einstein-Hilbert action. Weyl [42] in 1919 and Eddington [43] in 1923 introduced such modifications by introducing higher order invariants in the gravitational action. But the terms used in gravitational theories mainly due to the scientific curiosity without experimental and theoretical motivation. It is known that GTR is non-renormalizable and therefore, could not be conventionally quantized. In fact, Utiyama and deWitt [44] showed that Einstein-Hilbert action needs to be supplemented by higher order curvature terms in order to renormalize the theory at one-loop level. However, the corrections to GTR are considered to be relevant only at the Planck scale or near the black hole singularities. The importance of such terms arises at a very strong gravity regimes and these are expected to be strongly suppressed by small coupling parameters. As mentioned earlier Starobinsky first obtained inflationary universe solution with R^2-term [9]. The contribution of R^2 term arises as vacuum polarization. Subsequently it is shown that R^2-theory is equivalent to Einstein gravity with a scalar field [45]. In a higher derivative theory of gravity both cosmological singularity and black hole singularities may be avoided [46, 47]. It reveals from recent studies that when quantum loop corrections in field theory or higher order corrections in the low energy string dynamics are considered, the effective gravitational action admits higher order curvature invariants [48, 49, 50]. Another motivation for considering modified theories of gravity is that generalized Lagrangian consisting of negative and positive powers of the scalar curvature (R) admit both the early [9] and late-time [51] accelerating phases of the universe. In the early universe quantum fluctuations that generate during inflation lead to large scale structure formation of the universe. During this epoch it is believed that the primordial black holes are also produced copiously.

It is known that black holes may be formed either (i) due to a gravitational collapse of a massive star or (ii) due to quantum fluctuation that generated during inflation. The later black holes are called *primordial black holes* (in short, PBHs). PBHs are topological black holes [52] which are important to understand the physics of the early universe during their formation era. In 1975, the discovery of Hawking radiation ushered in a new era in black hole physics [53]. The lifetimes of black holes formed by gravitational collapse of a massive star are quite long, which are in fact comparable to the age of the universe. However PBHs are tiny and comparatively short lived. Therefore, the hope of confirming Hawking radiation by observation through PBH hunting is challenging. A number of literature appeared in which formation of PBHs are explored from an initial density inhomogeneities [54, 55], non-linear metric perturbations [56, 57, 58], softening of the equation of state [59, 60, 61], collapse of cosmic strings [62, 63, 64]. The particle horizon mass for a PBH during their formation epoch is given by [65]:

$$M_{PBH}(t) \approx \frac{c^3 t}{G} \approx 10^{15} \left(\frac{t}{10^{-23} s} \right) gm, \tag{1.1}$$

where t is cosmic time after the Big Bang. Therefore PBHs that are formed during the early universe have mass less than that formed at late time. PBHs if survived up to the present epoch, may contribute to a specific kind of dark matter. According to Hawking due to evaporation of PBHs thermal spectrum [53] results near a black hole which led him to propose that the small PBHs would evaporate away in a finite time [66]. Page showed [67] that the Hawking evaporation process leads to loss of mass as

$$M_h = M_i \left(1 - \frac{t}{t_h} \right)^{\frac{1}{3}}, \tag{1.2}$$

where the Hawking evaporation time scale t_h is given by $t_h = \frac{G^2 M_i^3}{3\hbar c^4 \alpha}$, α is the spin parameter of the emitting particles. If one chooses t_i as the present age of the universe, then PBH of mass $M_i \approx 10^{12}\ kg.$ should evaporate now and it might have formed at $t \sim 10^{-23} s$. The knowledge of initial mass M_i of PBHs helps us to estimate the Hawking evaporation time scale. Therefore PBHs may play a significant role which contributes to the total density of the universe. The effect of evaporation makes PBHs a source for γ-radiation which might influence on the light element abundance and spectrum of cosmic microwave background (CMB). Hence PBHs are considered to be important cosmological probe for understanding early universe. It is also important to estimate the probability of creation of a universe with such PBH. In the Euclidean gravity Bousso and Hawking (in short, BH) prescribed a mechanism to estimate the probability making use of *gravitational instantons*. The method of BH will be employed in the modified theories of gravity.

Gravitational instantons are the classical solutions obtained by solving the Euclidean gravitational field equations. The action corresponding to the solution is finite. At stationary phase points, they may be taken as the first approximations to the configurations which dominate the path integrals of quantum gravity. It is important to mention that Hawking introduced Gravitational instantons in the late seventies [68], as a building block for Euclidean quantum gravity. Gravitational instantons are also important to study open inflationary universe. Hawking-Turok (in short, HT) mechanism is helpful to obtain open inflationary universe which will be used here in a modified gravity.

To understand recent cosmic acceleration "The Holographic Principle" [69] is employed in cosmology. According to 't Hooft, a three dimensional universe is considered as an image which can be stored on a two dimensional projection similar to a holographic image. The two dimensional description is rich enough to describe the three dimensional phenomena. So the most radical version of this principle asserts that under certain conditions all the information about a physical system is encoded on its boundary and this demands that the entropy of the system should not exceed the area of the boundary in Planck units. It is motivated by the important theorem of black hole theory which states that the total entropy of matter S_{matter} inside a black hole cannot be greater than the entropy, known as 'Bekenstein-Hawking entropy'. Entropy of a black hole is equal to a quarter of the area of the event horizon $S_{matter} \leq S_{BH} = \frac{A}{4}$, where A is the horizon area.

Recently, a new form of dark energy based on holographic notion has been proposed considering the existence of some cosmic horizon. The basic idea of a holographic dark energy in cosmology is that the saturation of the entropy bound may be related to an unknown ultra-violet (UV) scale Λ to some known cosmological scale in order to enable it to find a viable formula for the dark energy which may be quantum gravity in origin and characterized by Λ. It was shown by Cohen et. al. [70] that if ρ_Λ is the quantum zero point energy density of a system of size L (its infra-red (IR) cut-off), the total energy of the system should not exceed the mass of a black hole of same size where $L^3\rho_\Lambda \leq LM_P^2$. By setting the largest L allowed and identifying $\rho_\Lambda \sim \Lambda^4$ as the holographic dark energy density, we can write

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \tag{1.3}$$

where $M_P^{-2} = 8\pi G$. Taking L as the size of the current universe in Hubble scale, where H^{-1} is the Hubble radius, the resulting energy density is comparable to the present dark energy [71, 72]. However, Hsu [73] pointed out that isotropic, homogeneous and spatially flat universe dominated by nonrelativistic matter and dark energy. The above model does not permit acceleration, because the accelerating universe requires $\omega < -\frac{1}{3}$. The most recent data indicate that $\omega < -0.76$ at the 95% confidence level [74].

A different scale other than Hubble scale was proposed by Fischler and Susskind [75] using particle horizon as IR cut-off. Assuming dark energy which dominates the expansion, one can find $H \propto a^{-(1+\frac{1}{3})}$ and consequently the state parameter of dark energy becomes $\omega = -\frac{1}{3} + \frac{2}{3c} > -\frac{1}{3}$, since c is always positive. Hence the theory is not suitable for accommodating late acceleration. Later Li [76] obtained an accelerating universe considering event horizon as the cosmological scale. The model is then found consistent with the cosmological observations. Subsequently Setare [77] obtained holographic model of interacting dark energy with an equation of state in a non-flat (closed) universe enveloped by L as the system's IR cut-off, where L is the radial size of the event horizon measured on the sphere of the horizon. Thus interacting holographic dark energy is an important issue needs to be investigated.

Another interesting class of dark energy models involving a fluid known as a Chaplygin gas (CG) is found useful in cosmology. It may be mentioned here that Chaplygin gas was first introduced in 1904 in aerodynamics which now-a-days considered to be one of the prospective candidate for dark energy. CG can be described by a complex scalar field obtained from generalized Born-Infeld action. The equation of state for CG contains only one free parameter which is given by [35]

$$p = -\frac{A}{\rho}, \tag{1.4}$$

where A is a positive constant. However, the CG is found not suitable in cosmology and hence ruled out by observation. Subsequently a modified form of CG introduced which is given by

$$p = -\frac{A}{\rho^{\alpha}}, \tag{1.5}$$

where $0 \leq \alpha \leq 1$ [36, 37, 38]. The fluid satisfying eq. (1.5) is known as Generalized Chaplygin Gas (in short, GCG). It has two free parameters A and α. At high energy GCG behaves almost like a pressureless dust whereas at low energy regime it behaves like a dark energy its pressure being negative and almost constant. Thus GCG smoothly interpolates between a non-relativistic matter dominated phase in the early universe to a dark energy dominated phase in the late universe. This interesting property of GCG is useful for considering it as a candidate for unified dark matter and dark energy models as ΛCDM model. Recently another form of equation of state for Chaplygin gas [39] is proposed similar to that considered in [40], which is given by

$$p = A\rho - \frac{B}{\rho^{\alpha}} \quad with \;\; 0 \leq \alpha \leq 1, \tag{1.6}$$

where α, A and B are three parameters. It is known as modified GCG (henceforth, MCG). In the early universe when the size of the universe was small, the MCG corresponds to a barotropic fluid (if one considers $A = \frac{1}{3}$ it corresponds to radiation and $A = 0$ it corresponds to matter). So, the MCG at one extreme end behaves as an ordinary fluid and at the other extreme behaves as cosmological constant. So at all other intermediate stages it mimics as a mixture. The MCG is an attempt to find something interesting that is not exactly ΛCDM. It may be pointed out here that MCG is a single fluid model to unify dark matter and dark energy. In a flat Friedmann model using method outlined by Barrow [78], Gorini *et. al.* showed [79] that MCG is equivalent to a homogeneous minimally coupled scalar field ϕ. Thus it is an important candidate in cosmological model building which will be taken up here for further study making use of observational data.

The concept of non-Euclidean geometry proposed by Riemann led to a formalism to describe geometry of arbitrary dimensional manifold. Guided by this idea Clifford, Helmholtz and Hinton in late 1800 speculated a higher dimensional space-time. The concept of extra-dimensions is found important to realize unification of the fundamental forces in nature including quantization of the gravitation. Kaluza and Klein [80, 81] introduced an extra dimension to unify gravity with electromagnetism and the resulting theory of gravity based on a compact space-time is known as *Kaluza-Klein theory (in short, KK)*. The modified versions of the KK theory requires numerous and extremely small scale for the extra dimension. However the KK approach does not work well. The advent of string theory played an important role in understanding unification scheme further. The string theory, which includes only bosons, is consistent if the dimension of spacetime is 26. Adding fermions to this string theory, a consistent theory in 10 dimensions results which is known as *superstring theory* (in short, SST) [82]. SST is considered to be a promising candidate for a consistent theory of quantum gravity. It is therefore important to investigate cosmological issues in spacetime dimensions more than the usual four dimensions.

Chodos and Detweiler [83] first attempted to build a cosmological model in higher dimensions. Freund [84] obtained cosmological models in higher dimensions by introducing suitable matter tensors. These ideas are used to build subsequent models [85, 86, 87] which accommodates compactification. Again in more than five dimensions, this compactification requires matter terms explicitly or modifications to the Einstein equations. In higher dimensional framework Einstein's theory of gravity is generalized adding quadratic [88], cubic [89], quartic terms [90] in the curvature R and some special combinations like Gauss-Bonnet terms [91, 92, 93, 94] etc. Recently Sharif [95, 96] have explored the evolution of interacting modified holographic dark energy model with Hubble horizon and event horizon as an IR cutoff in a flat KK universe and also examined the validity of generalized second law of thermodynamics (GSLT). The above theory will be considered here to reconstruct the holographic dark energy model with GCG in KK cosmology.

At the Planck epoch a quantum theory of gravity is essential but a consistent theory of quantum gravity is yet to emerge. In recent times to understand cosmological evolution at Planck time *Loop quantum gravity* (in short, LQG) [97] is developed, where singularity problem is avoided. However, the LQG lacks proper description of time evolution for the quantum space-time. Several attempts have been made in the recent past to achieve a complete quantum gravitational theory (UV complete theory). In 2009 Hořava formulated a new proposal for realization of a quantum theory of gravity (now known as Hořava-Lifshitz gravity (in short, HL) [98, 99]) based on power counting renormalizablity which is expected to reduce to Einstein's general relativity (GR) at the infrared (IR) limit. The HL gravity is based on the idea that the Lorentz symmetry is restored in IR limit of a given theory whereas it is absent in the high energy limit.

In obtaining gravitational theory, Horava assumed two conditions: detailed balance and projectability. Sotiriou, Visser and Weifurtner(SVW) [100], proposed a general HL theory with projectability but without detailed-balance conditions. In the ultraviolet (UV) limit, HL has a Lifshitz-like anisotropic scaling namely

$t \rightarrow l^z t$ and $x^i \rightarrow l x^i$, between space and time characterized by the dynamical critical exponent $z = 3$ and thus breaks the Lorentz invariance, while in the infrared (IR), it is obtained with $z = 1$. Thus classical general relativistic theory of gravity may be a case of HL gravity in the low energy limit.

A number of literature appeared using HL gravity to study production of gravitational wave [101, 102, 103], perturbation spectrum [104, 105, 106], properties of black-hole [107, 108, 109], dark energy phenomenology [110, 111], observational consistency [112], astrophysical phenomenology [113, 114, 115], thermodynamical properties [116, 117] etc. Recently a modified form of Hořava-Lifshitz $F(R)$ gravity is proposed by Chaichian [111]. Such a modification may be easily related with the traditional modified gravity approach, but it turns out that the theory is rich in terms of cosmologies. The modified $F(R)$ HL gravity will be taken up to study cosmological evolution.

Cosmological models are usually constructed taking into account perfect fluids or a mixture of non-interacting perfect fluids [118] and each of them evolves separately according to energy conservation law. However, there is no evidence confirming that this will be the only scenario, hence it is interesting to study cosmologies with an interacting mixture of fluids. The exchange of energy among these fluids might play an important role in the evolution of the universe. The interaction between dust-like matter and radiation was first considered by Tolman [119] and Davidson [120]. Cosmological models with decay of massive particles into radiation, or with matter creation [121] and models with a mutual exchange of energy between two fluids at rates which are proportional to a linear combination of their individual densities are considered to study the expansion rate of the universe [122]. Recent observations predict that dark matter and dark energy are the major components of the universe. Due to the lack of knowledge on dark matter and dark energy, most of the investigations in the dark sector rely on the assumption that these two unknown components evolve independently. However, it is possible that the coupling in the dark sector might be responsible for late acceleration of the universe. Wettrich [123] introduces model featuring an interaction between matter and dark energy. Billyard and Coley [124] studied spatially flat isotropic cosmological model in the presence of a mixture of scalar field with an exponential potential and a perfect fluid. They included an interaction term, which helps to understand the transfer of energy from the scalar field to matter component. It is found that the interaction significantly affect the qualitative behaviour of the evolution. The late-time behaviour of these models may be of cosmological interest. Assuming an exponential potential and a linear coupling, Amendola [125] studied the effect of a coupled quintessence field. Farrar and Peebles [126] studied physical processes where the dark matter particles are coupled to scalar field by a Yukawa coupling and other processes are explored in Ref.[127]. A mixture of two interacting fluids is also considered here to study cosmological evolution.

1.1 Aim of the Work

The objective of the research work is to study some specific issues relevant for cosmological model building, which incorporates the quantum nature of matter or both matter and gravity in the early universe. We investigate cosmological models taking in to account the predictions from astronomical and cosmological observations. Different aspects of the observed universe will be explored employing the modified theories gravity and/or taking into account exotic kind of matter in the matter sector also.

1.1.1 Summary of the Work

In *chapter 1* a brief review of inflationary scenario of the universe and different aspects of the cosmological models are discussed.

In *chapter 2*, primordial black holes are discussed with detail calculations of the estimation of probability for quantum creation of PBH. Using NBP proposed by HH, in the framework of higher dimensional gravity a saddle point approximation to the Euclidean path integral permits gravitational instantons which are employed to evaluate the probability of pair creation of PBHs in a four dimensional universe. For this two different types of topologies are considered: (i) a universe with space-like sections with S^3- topology and (ii) a universe with space-like section with $S^1 \times S^2$- topology. An inflationary (de Sitter) universe without a pair of PBH permits in the former case, while the later describes a Nariai universe, an inflationary universe with a pair of black holes.

In *chapter 3*, the PBH pair creation probability are estimated in a multidimensional universe with a Lagrangian which is a polynomial in Ricci scalar (R). The objective of the work is to explore the effect of extra dimensions in the creation of gravitational instantons which are important for cosmological model building.

In *chapter 4*, creation of an open inflationary universe in a higher order gravity is discussed making use of the Hawking-Turok (in short, HT) instantons. HT provided a technique for creating open inflationary universe described by a singular instanton obtained in a minisuperspace model with an inflation field. The potential of the inflaton field does not require any special property for open universe. Using a conformal transformation in a modified gravitational action we determine the conditions under which both the singular and non-singular instanton solutions exist. The corresponding potential of a conformally equivalent picture of fields permits enough inflation for a viable cosmological model.

In *chapter 5*, Holographic dark energy model in FRW universe is discussed. We consider modified GCG for the HDE field and the corresponding potential is obtained in a non-flat universe. Also the viability of HDE in the model is studied. It is found that the HDE is suitable for a restricted domain of the values of the density parameter Ω_Λ in a closed model of the universe.

In *chapter 6*, cosmological models with MCG in HL gravitational theory is taken up. The limits of the EoS parameters are determined using the recent observational data. The equation of state parameter of the total cosmic fluid defined by, $w(z) = \frac{p_{tot}}{\rho_{tot}}$, is evaluated at different redshifts. The suitability of the model is then tested using supernovea magnitudes μ vs z data for closed and open universe respectively which then compared with the curve obtained from union compilation data.

In *chapter 7*, Holographic dark energy correspondence of interacting generalized Chaplygin gas in a compact Kaluza-Klein cosmology is discussed. The evolution of the modified holographic dark energy and the equation of state parameter is obtained here. Using the present observational value of density parameter we obtain a stable configuration which accommodates dark energy. In this case a connection between dark energy and phantom field is established. It is noted that the dark energy might have evolved from a phantom state in the past.

In *chapter 8*, cosmological models with an interacting mixture of two fluids in a modified Hořava-Lifshitz $F(R)$-gravity taking into account power-law scale factors of the universe is studied. We determine the range of values of the state parameters for the interacting fluids.

In *chapter 9*, concluding remarks and future work are presented.

Chapter 2

Primordial Black Holes Creation Probability in Higher Derivative Theories Using Gravitational Instantons

2.1 Introduction

Primordial black holes (PBHs) are comparatively short lived topological black holes which might have formed due to quantum fluctuations of matter distribution in the early universe. The existence of a pair of PBH may be important in the context of the recent cosmological observations. To explain the structure formation of the universe PBH are considered to be one of the promising candidates. Creation of universe at Planck time with such PBH pairs may be studied in the framework of Euclidean gravity via Gravitational instanton. Gravitational instantons are the Euclidean solutions of gravitational field equations. PBHs pair creation is possible only on a background which provides a force that pulls the pair apart. In an inflationary universe scenario the pair of PBHs will accelerate away from each other. Hence one can evaluate the probability of pair creation knowing the corresponding the wave function of the universe in quantum cosmology. Hartle-Hawking *'No Boundary Proposal'* (in short, NBP) [128] is useful to calculate the wave function of the universe and subsequently in determining the probability.

2.1.1 The No Boundary Proposal and Wave Function of the Universe

No Boundary Proposal (NBP) proposed by Hartle-Hawking (HH), states that 'the path integral for quantum gravity should be taken over all compact Euclidean matrices'.

Let us consider a spacetime manifold M having an embedded three dimensional manifold Σ with induced three metric h_{ij}. According to NBP, the quantum state of the universe is defined by the path integrals over Euclidean matrices $g_{\mu\nu}$ on the compact manifold M. The probability of finding h_{ij} on Σ is given by a path integral over $g_{\mu\nu}$ on M. Assuming the spacetime manifold M to be simply connected, the surface Σ can be divided into two parts M_+ and M_-. HH factorized the probability for Σ to have metric h_{ij} into a product of two wavefunctions Ψ_+ and Ψ_-. The wavefunctions are given by a path integral over all matrices $g_{\mu\nu}$ on the half-manifolds M_+ and M_- that induce h_{ij} on Σ. Since in most cases Ψ_+ and Ψ_- are equal, HH referred to Ψ as the wave function of the universe by dropping the suffixes. The wave function of the universe satisfies the differential equation, known as Wheeler de-Witt equation which is given by

$$\left(G_{ijkl}\frac{\partial^2}{\partial h_{ij}\partial h_{kl}} - \sqrt{h}\,{}^3R\right)\Psi = 0. \tag{2.1}$$

In the presence of matter fields Φ, the wave function Ψ of the universe depends on the matter field configurations. But Ψ does not depend on the time explicitly. According to NBP the wave function of the universe is given by the path integral over fields on a compact manifold M_+ whose only boundary is Σ and the path integral is taken over all compact Euclidean matrices $g_{\mu\nu}$ and matter fields Φ on M_+ that admits the metric h_{ij} and matter field configurations Φ on Σ. Consequently one obtains the wave function of the universe with the inclusion of matter fields which is given by

$$\Psi[h_{ij}, \Phi_{\partial M}] = \int D(g_{\mu\nu}, \Phi)\ exp[-I(g_{\mu\nu}, \Phi)], \tag{2.2}$$

where (h_{ij}, Φ_{Σ}) are the three-metric and matter fields on a spacelike boundary Σ, the path integral is taken over all compact Euclidean metrics $g_{\mu\nu}$ and $I(g_{\mu\nu}, \Phi)$ is the corresponding action. The gravitational part of the action is given by

$$I_E = -\frac{1}{16\pi}\int d^4\, x\sqrt{g}(R - \Lambda) - \frac{1}{8\pi}\int_{\Sigma} d^3\, x\sqrt{h}K, \tag{2.3}$$

where g is the determinant of the 4-dimensional Euclidean metric, R is the Ricci scalar, Λ is the cosmological constant, and $K = h^{ab}K_{ab}$ is the trace of the second fundamental form of the boundary Σ for the metric $g_{\mu\nu}$, we choose Gravitational constant $G = 1$ and $c = 1$.

2.1.2 Method of Evaluation of Probability Measure

Bousso and Hawking (in short, BH) [129] estimated the wave function of the universe using a saddle-point approximation to the path integral and estimated the probability of the quantum creation of a universe with a pair of PBH. They considered two types of topology: one with a pair of black holes and the other without black holes. In this case gravitational instanton solutions are obtained which are employed to determine the probability. The gravitational instantons are the classical Euclidean solution of the Einstein's field equation, corresponding to each of the universe which can be analytically continued to match a boundary Σ of the appropriate topology. The Euclidean action (I) is then evaluated for the above solutions. The wave function of the universe in the semiclassical approximation is given

$$\Psi[h_{ij}, \Phi_{\partial M}] \approx \sum_n A_n e^{-I_n}, \tag{2.4}$$

where the sum is over the saddle points of the path integral, and I_n denotes the corresponding Euclidean action. The probability measure to each type of universe is given by

$$P[h_{ij}, \Phi_{\partial M}] \sim e^{\left(-2I^{Re}\right)}, \tag{2.5}$$

where the superscript Re denotes the real part of the action corresponding to the dominant saddle point, *i.e.*, the classical solution satisfying the Hartle-Hawking (HH) boundary condition [128]. Following Bousso and Hawking [129] we determine the probability of the quantum creation of a universe with a pair of primordial black holes in a 4 dimensional universe taking into account Euclidean space-times : (i) space-like sections with S^3- topology and (ii) space-like section with $S^1 \times S^2$-topology. The first kind of spatial structure describes an inflationary (de Sitter) universe without a pair of PBH, while the later describes a Nariai universe [130], an inflationary universe with a pair of black holes. BH determine the probability measure of PBH in Einstein gravity in presence of Λ. A modified gravity with Lagrangian density $f(R) = f(R) = \Sigma_{n=0}^{N} \lambda_n R^n$ is considered here to estimate the probability of creation of a pair of PBHs.

It is well known that a theory with higher order Lagrangian is conformally equivalent to Einstein gravity with a matter sector containing a minimally coupled self interacting scalar field. However, the renormalization of higher loop contributions introduces terms into the effective action that are higher than quadratic in R. Consequently it is important to study the effects of these terms in the quantum creation of a universe with a pair of PBH. In the case of modified gravity Lagrangian density containing cubic or higher order terms admit a potential which is extremely complicated. It was shown by Henk van Elst *et. al.* [131] that $N = 2$ contribution is rather special in four dimensions, while R^3-term gives a more generic perturbation. The qualitative behaviour of the potential $V(\phi)$ does not change relative to the R^3-term even if one considers $N > 3$. We, therefore, consider terms upto R^3 in the action and look for the probabilities for the creation of a universe with two types of topologies as considered by BH [129] in order to compare their creation.

2.2 Gravitational Instanton Solutions with or without a Pair of Black Holes

Let us consider a Euclidean action containing higher derivative terms which is given by

$$I_E = -\frac{1}{16\pi}\int d^4x\sqrt{g}f(R) - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}Kf'(R), \tag{2.6}$$

an action different from that in eq. (2.3). The polynomial function chosen here is $f(R) = R+\alpha R^2+\beta R^3-2\Lambda$, R is the Ricci scalar and Λ is the cosmological constant. In the gravitational action, the term, h is the determinant of the boundary metric h_{ab} and $K = h^{ab}K_{ab}$ is the trace of the second fundamental form of the boundary ∂M. The second term is the contribution from $\tau = 0$ back in the action. It vanishes for a universe with S^3 topology, but gives a non-vanishing contribution for $S^1 \times S^2$- topology.

2.2.1 S^3-Topology

In this section we study vacuum solutions of the Euclidean Einstein equation with a cosmological constant in four-dimensions. The four dimensional Euclidean metric ansatz is given by

$$dS^2 = d\tau^2 + a^2(\tau)\left[dx_1^2 + \sin^2 x_1 d\Omega_2^2\right], \tag{2.7}$$

where $a(\tau)$ is the scale factor of a four-dimensional universe and $d\Omega_2^2$ is a line element on the unit two sphere. The scalar curvature is given by

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right), \tag{2.8}$$

where an overdot denotes differentiation with respect to τ. Rewriting the action (2.6), using the constraint through a undetermined Lagrangian multiplier β, one obtains

$$I_E = -\frac{1}{8\pi}\int\left[f(R)a^3 - \beta\left(R + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2-1}{a^2}\right)\right]d\sigma - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}Kf'(R). \tag{2.9}$$

Varying the action *w.r.t.* R, one obtains β which is given by

$$\beta = a^3 f'(R). \tag{2.10}$$

Using β and treating a and R as independent variables in eq. (2.9), we get

$$I_E = -\frac{\pi}{8}\int_{\tau=0}^{\tau=\frac{\pi}{2H_o}}\left[a^3 f(R) - f'(R)\left(a^3R - 6a\dot{a}^2 - 6a\right) + 6a^2\dot{a}\dot{R}f''(R)\right]d\tau - \frac{3\pi}{4}\left[\dot{a}a^2 f'(R)\right]_{\tau=0}, \tag{2.11}$$

In the above $\ddot{a}$ is eliminated by integration by parts. The following field equations are obtained by varying I_E with respect to a and R:

$$f''(R)\left[R + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2-1}{a^2}\right] = 0, \tag{2.12}$$

$$2f'''(R)\dot{R}^2 + 2f''(R)\left[\ddot{R} + 2\frac{\dot{a}}{a}\dot{R}\right] + f'(R)\left[4\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} - \frac{2}{a^2} + R\right] - f(R) = 0. \tag{2.13}$$

In the above $f''(R) \neq 0$ leads to eq. (2.8), therefore, eq. (2.13) gives the dynamics of evolution. Considering $f(R) = R + \alpha R^2 + \beta R^3 - 2\Lambda$ in the above, we obtain a gravitational instanton solution which is given by

$$a(\tau) = \frac{1}{H_o}\sin H_o\tau, \tag{2.14}$$

where $H_o^2[\beta, \Lambda]$ is determined from the constraint equation

$$432\beta H_o^6 - 3H_o^2 + \Lambda = 0$$

obtained from eq. (2.13). It is to noted that this solution satisfies the HH no boundary conditions viz., $a(0) = 0$, $\dot{a}(0) = 1$. One can choose a path along the τ^{Re} axis to $\tau = \frac{\pi}{2H}$, the solution describes half of the Euclidean de Sitter instanton S^3. Analytical continuation of the metric (2.7) to the *Lorentzian region*, $x_1 \to \frac{\pi}{2} + i\sigma$, gives

$$ds^2 = d\tau^2 + a^2(\tau)\left[-d\sigma^2 + \cosh^2\sigma \; d\Omega_2^2\right], \tag{2.15}$$

which is a spatially inhomogeneous de Sitter like metric. However, if one sets $\tau = it$ and $\sigma = i\frac{\pi}{2} + \chi$, the metric under Wick rotation becomes,

$$ds^2 = -dt^2 + b^2(t)\left[d\chi^2 + sinh^2\chi \; d\Omega_2^2\right], \tag{2.16}$$

where $b(t) = -ia(it)$ leading to a metric corresponding to open universe. Thus it describes the creation of an open inflationary universe. The real part of the Euclidean action corresponding to the solution calculated by following the complex contour of τ suggested by BH is given by

$$I_{S^3}^{Re} = -\frac{\pi}{2H_o^4}\left[24\alpha H_o^4 + 4H_o^2 - \Lambda\right]. \tag{2.17}$$

With the chosen path for τ, the solution describes half the de Sitter instanton with S^4 topology, joined to a real Lorentzian hyperboloid of topology $R^1 \times S^3$. It can be joined to any boundary satisfying the condition $a_{\partial M} > 0$. For $a_{\partial M} > H_o^{-1}$, the wave function oscillates and predicts a classical spacetime. We note the following:

- For $\beta = 0$ one obtains $H_0^2 = \frac{\Lambda}{3}$ and $R = 4\Lambda$ which correspond to the result obtained by Paul *et. al.* [132].

- For $\beta \neq 0$ instanton solutions are permitted even without a cosmological constant where $H_0^2 = \frac{1}{12\sqrt{\beta}}$. The new gravitational instanton solutions obtained here are permitted even without a cosmological constant for $N \geq 2$ in the polynomial Lagrangian density $f(R)$.

2.2.2 $S^1 \times S^2$-Topology

Let us consider $S^1 \times S^2$-topology which can accommodate a pair of black holes. The corresponding ansatz for $(1+1+2)$ dimensions is given by

$$ds^2 = d\tau^2 + a^2(\tau)dx^2 + b^2(\tau)d\Omega_2^2, \tag{2.18}$$

where $a(\tau)$ is the scale factor corresponding to x-direction and $b(\tau)$ is the scale factor of the two sphere given by

$$d\Omega_2^2 = d\theta^2 + sin^2\theta d\phi^2.$$

The scalar curvature is given by

$$R = -\left[2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 2\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 4\frac{\dot{a}\dot{b}}{ab}\right]. \tag{2.19}$$

The Euclidean action (2.6) becomes

$$\begin{aligned} I_E &= -\frac{\pi}{2}\int\left[f(R)ab^2 - \beta\left(R + 2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 4\frac{\dot{a}\dot{b}}{ab} + 2\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right)\right]d\tau \\ &\quad -\frac{1}{8\pi}\int_{\delta M} d^3x\sqrt{h}Kf'(R). \end{aligned} \tag{2.20}$$

One can determine β as is done before and thereby eq. (2.20) reduces to

$$\begin{aligned} I_{S^1\times S^2} = -\frac{\pi}{2}\int_{\tau=0}^{\tau_{\delta M}} ab^2 d\tau & \left[f(R) - f'(R)\left(R - 4\frac{\dot{a}\dot{b}}{ab} - 2\frac{\dot{b}^2}{b^2} - \frac{2}{b^2}\right)\right. \\ & \left. - 2\dot{R}f''(R)\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\right] - \pi\left[(\dot{a}b^2 + 2ab\dot{b})f'(R)\right]_{\tau=0}. \end{aligned} \tag{2.21}$$

Variation of the action with respect to the scale factors a, b and scalar curvature R, gives the following field equations

$$f''(R)\left[R + 2\frac{\ddot{a}}{a} + 4\frac{\ddot{b}}{b} + 4\frac{\dot{a}\dot{b}}{ab} + 2\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right] = 0, \tag{2.22}$$

$$\begin{aligned} f'(R)\left[R+4\frac{\ddot{b}}{b}+2\left(\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}\right)\right]+2f''(R)\left(\ddot{R}+2\dot{R}\frac{\dot{b}}{b}\right)+ & \\ 2\dot{R}^2 f'''(R)-f(R) &= 0, \end{aligned} \quad (2.23)$$

$$\begin{aligned} f'(R)\left[R+2\frac{\ddot{a}}{a}+2\frac{\dot{a}\dot{b}}{ab}+2\frac{\dot{b}}{b}\right]+2f''(R)\left[\dot{R}\left(\frac{\dot{b}}{b}+\frac{\dot{a}}{a}\right)+\ddot{R}\right] & \\ +2f'''(R)\dot{R}^2-f(R) &= 0. \end{aligned} \quad (2.24)$$

Let us now consider $f(R) = R + \alpha R^2 + \beta R^3 - 2\Lambda$. The eqs. (2.22)-(2.24) permit an instanton solution which is given by

$$a = \frac{1}{H_o}\sin(H_o\tau), \; b = H_o^{-1}, \; R = 4H_o^2, \quad (2.25)$$

where H_o satisfies the following constraint equation

$$16\beta H_o^6 - H_o^2 + \Lambda = 0. \quad (2.26)$$

We note the following:

- when $\Lambda = 0$ with $H_o^4 = \frac{1}{16\beta}$ and $\alpha = -3\sqrt{\beta}$ one obtains instanton for a positive coupling constant β,
- when $\Lambda \neq 0$ with $\alpha = \pm 3\sqrt{\beta}$, instanton solutions are permitted for a positive coupling constant β in the presence of α which is either positive or negative,
- when $\beta = 0$, $\Lambda = H_o^2$ corresponds to the solution obtained by Paul et al.[132] and
- $\alpha = 0$ and $\beta = 0$ reduces to $\Lambda = H_o^2$ which corresponds to BH solution [129] where a non vanishing cosmological constant is essential for gravitational instanton solution.

The above solution satisfies the HH boundary conditions $a(0) = 0$, $\dot{a}(0) = 1$, $b(0) = b_o$, $\dot{b}(0) = 0$. Analytic continuation of metric (2.18) to *Lorentzian region* *i.e.*, using *Wick rotations* $\tau \to it$ and $x \to \frac{\pi}{2} + i\sigma$, one gets

$$ds^2 = -dt^2 + S(t)^2\, d\sigma^2 + H^{-2}\, d\Omega_2^2, \tag{2.27}$$

where $S(t) = -ia(it)$. In this case the analytic continuation of time and space do not give an open inflationary universe as is permitted in S^3-topology discussed above. The corresponding Lorentzian solution is given by

$$a(\tau^{\text{Im}})|_{\tau^{\text{Re}}=\frac{\pi}{2H_o}} = H_o^{-1} cosh H_o \tau^{\text{Im}}$$
$$b(\tau^{\text{Im}})|_{\tau^{\text{Re}}=\frac{\pi}{2H_o}} = H_o^{-1}$$

In this case the spacelike sections can be visualized as three spheres of radius a with a hole of radius b $(= H_o^{-1})$ punched through the north and south poles. The physical interpretation of the solution is that of two-spheres containing two black holes at the opposite ends. The black holes have the radius H_o^{-1} which accelerates away from each other with the expansion of the universe. The real part of the action can now be determined following the contour approach as was suggested by BH [129], and the real part of the action given by

$$I^{Re}_{S^1 \times S^2} = -\frac{\pi}{H_o^4}\left[8\alpha H_o^4 + 4H_o^4 - 3\Lambda\right], \tag{2.28}$$

where H_o^2 satisfies the constraint

$$16\beta H_o^6 - H_o^2 + \Lambda = 0. \tag{2.29}$$

Thus the gravitational instanton solution (2.25) describes a universe with two black holes at the poles of a two-sphere. However, unlike the BH instanton solutions, we found that the instantons are permitted even without cosmological constant. It may be mentioned here that contribution of the surface term due to the a non-linear gravitational action is zero. The non zero contribution arises from the volume integral.

2.3 Estimation of the Probability for PBH

In the previous section we have determined the actions for inflationary universe with or without a pair of primordial black holes. The estimated value of the actions will be used here to compare the probability measure. The probability for nucleation of universe without PBH is given by

$$P_{S^3} \sim e^{\frac{\pi}{H_o^4}(24\alpha H_o^4 + 4H_o^2 - \Lambda)}. \tag{2.30}$$

For an inflationary universe with a pair of black holes the corresponding probability of nucleation can be calculated from the action (2.21) which is given by

$$P_{S^1 \times S^2} \sim e^{\frac{2\pi}{H_o^4}(8\alpha H_o^4 + 4H_o^2 - 3\Lambda)}. \tag{2.31}$$

We note the following:

- When $\alpha = 0$, $\beta = 0$, one recovers the result obtained by Bousso and Hawking

$$P_{S^3} \sim e^{\frac{3\pi}{\Lambda}},\ P_{S^1 \times S^2} \sim e^{\frac{2\pi}{\Lambda}}. \tag{2.32}$$

  Thus with a positive cosmological constant the probability for a universe with PBH is less than that without PBH.

- When $\alpha \neq 0$, $\beta = 0$, one determines $H_o^2 = \Lambda$ and the probabilities are

$$P_{S^3} \sim e^{\frac{3\pi}{\Lambda} + 24\pi\alpha},\ P_{S^1 \times S^2} \sim e^{\frac{2\pi}{\Lambda} + 16\pi\alpha}. \tag{2.33}$$

  In this case with a positive cosmological constant and a positive α (> 0), de Sitter universe is more probable [132]. Negative values of $\alpha < -\frac{1}{8\Lambda}$ could lead to an interesting possibilities as the probability for a universe with PBH in this case is more. However, the case $\alpha < 0$ leads to a classical instability in R^2-theory.

- When $\alpha \neq 0$, $\beta \neq 0$ one obtains instanton solutions even without a cosmological constant where $H_o^2 = \sqrt{\frac{1}{144\beta}}$, the corresponding probability measures are

$$P_{S^3} \sim e^{8\pi(3\alpha+6\sqrt{\beta})}, \; P_{S^1\times S^2} \sim e^{16\pi(\alpha+6\sqrt{\beta})}. \tag{2.34}$$

It is evident that one requires $\beta > 0$ to obtain instanton but α may pick up either positive or negative values. A universe without PBH is more probable when $\alpha > -2\sqrt{\beta}$ or $|\alpha| < 2\sqrt{\beta}$. In this case interesting possibilities emerges when $\alpha < -2\sqrt{\beta}$ or $|\alpha| > 2\sqrt{\beta}$.

2.4 Discussions

The probability for a universe with a pair of primordial black holes is estimated in a modified theory of gravity where the gravitational Lagrangian density is a combination of non-linear terms in Ricci scalar (R) upto cubic order. The Euclidean solutions corresponding to the modified field equations admit gravitational instantons which are employed to evaluate the gravitational action. Using gravitational instanton solutions obtained in the two topologies considered here: a universe with (i) $R \times S^3$-topology and a universe with (ii) $R \times S^1 \times S^2$-topology, we determine the gravitational action thereafter probability for quantum creation of inflationary universe is estimated. It is found that the probability of a universe with $R \times S^3$-topology turns out to be much lower than that of a universe with topology $R \times S^1 \times S^2$ in the R^2-theory unless $\alpha < -\frac{1}{8\Lambda}$. It may be mentioned here that one gets a regular instanton in S^4-topology if there are no black holes. The existence of black holes restricts such a regular topology. The result obtained here on the probability of creation of a universe with a pair of primordial black holes is found to be strongly suppressed if one extends the theory with R^3-term in the action with the constraints (i) $\alpha > -2\sqrt{\beta}$ or (ii) $|\alpha| < 2\sqrt{\beta}$ when $\Lambda = 0$. We obtain new solutions in R^3-theory which permits de Sitter instantons with S^3 and $S^1 \times S^2$ topologies even without a cosmological constant. Thus cosmological constant is not essential in a modified theory of gravity for obtaining gravitational instantons as was required in Einstein gravity. The probability of PBH pair creation suppresses under a suitable limiting values of the coupling parameters of the gravitational action. It is interesting to note further that analytical continuation of a $R \times S^3$ metric considered here to Lorentzian region leads to a universe with open 3-space. Thus Hawking-Turok [133, 134] type open inflationary universe is also permitted in the modified gravity. A detail study of an open inflationary universe will be discussed in Chapter- 4. Thus in the case of a polynomial Lagrangian in R, quantum creation of PBH seems to be suppressed in the minisuperspace model for some restrictions of the parameters of the gravitational action which are discussed here.

Chapter 3

Primordial Black Holes Creation Probability in Multidimensional Universe with Non-linear Scalar Curvature Terms Using Gravitational Instantons

3.1 Introduction

At high enough energy scale Einstein field equation in four dimensions may not give a correct predictions of the universe. Such an energy scale perhaps was available shortly after the big bang. Thus a more general theory is needed which at low energy reproduces the Einstein's theory. In this context it is important to consider a multidimensional universe. The motivation of studying higher dimensions is that most of the theories of particle interactions, including String theory requires space-time dimensions more than the usual four for their consistent formulation. It is therefore considered essential to check if consistent cosmological or astrophysical solutions, which can accommodate these theories, are also allowed.It was Kaluza and Klein who first independently tried to unify gravity with electromagnetic interaction introducing an extra dimension. But the initial approach to unify the forces failed. However, the success of Superstring theory/M-theories revived the studies in higher dimensional framework which has been considerably generalized [135]. In modern higher dimensional scenario, the fields of the standard model are considered to be confined to $(3+1)$ dimensional hyper-surface (referred to as 3-Brane) embedded in a higher dimensional space-time but the gravitational field may propagate through the bulk dimensions perpendicular to the brane which is referred to as braneworld [136]. In a multidimensional universe the gravitational sector of the action may also be modified by considering non-linear terms of scalar curvature (R) like the four dimensional theory.

In the previous chapter PBH are considered in a four dimensional universe. The technique adopted will be extended to a multidimensional universe with a polynomial Lagrangian for gravitational sector. Using the prescription of Hartle-Hawking no boundary proposal [128] the probability of a universe with PBHs pair will be estimated. First the wave function of the universe (Ψ) will be determined from a saddle point approximation to the path integral. As was discussed earlier we consider the spacelike section characterized by (i) $R \times S^d$ topology and (ii) $R \times S^1 \times S^d$ topology. The former topology represents a universe without a pair of PBH and the later topology with a pair of PBH. We obtain Euclidean solutions of the modified Einstein field equation corresponding to each of the universes, which can be analytically continued to match a boundary ∂M of the appropriate topology. The wave function of the universe in the semiclassical approximation is given by eq. (2.4). We use eq. (2.5) to evaluate the probability measure for each type of universe.

Günther *et al.* [137] studied multidimensional gravitational models with non-linear scalar curvature terms R^{-1} and R^4. The study reveals the existence of at least one minimum of the effective potential for the volume moduli of the internal spaces with an emphasis to obtain stabilization. The stabilization of the internal space is important because the extra dimensional space components should be static or nearly static at least from the time of primordial nucleosynthesis. In contrast to R^2-models, the R^4-model shows a rich substrate of the stability region in parameter space which minimally depends on the total space-time dimensions of the bulk space. In this chapter, we consider a modified gravitational action described by a Lagrangian polynomial in R containing non-linear terms up to R^4 in higher dimensional universe in order to probe the early universe.

3.2 Gravitational Instantons with or without PBH

Let us consider a higher dimensional Euclidean gravitational action given by

$$I_E = -\frac{1}{16\pi}\int d^D x\sqrt{g} f(R) - \frac{1}{8\pi}\int_{\partial M} d^{(D-1)}x\sqrt{h} K f'(R), \tag{3.1}$$

where g is the determinant of D-dimensional Euclidean metric, R is the Ricci scalar, Λ is the cosmological constant, and $K = h^{ab}K_{ab}$ is the trace of the second fundamental form of the boundary ∂M in the metric. We consider a polynomial function in Ricci scalar $f(R) = \sum_i \lambda_i R^i$, in particular we choose $\lambda_o = -2\Lambda, \lambda_1 = 1, \lambda_2 = \alpha, \lambda_3 = \beta, \lambda_4 = \gamma$, so that $f(R) = R + \alpha R^2 + \beta R^3 + \gamma R^4 - 2\Lambda$.

3.2.1 S^{D-1}- Topology

In this section, we study vacuum solutions of the field equation corresponding to non-linear higher dimensional action given by eq. (3.1). We look for a gravitational instanton solution with space like section S^{D-1}. We choose the D-dimensional metric ansatz given by

$$dS^2 = d\tau^2 + a^2(\tau)d\Omega_d^2, \tag{3.2}$$

where $d = D-1$, a is the scale factor of a D dimensional universe and $d\Omega_d^2$ is a line element of unit $(D-1)$-sphere. The scalar curvature is given by

$$R = -\left[2d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right)\right], \tag{3.3}$$

where the overdot denotes differentiation with respect to τ. We use the constraint through a undetermined Lagrangian multiplier $\tilde{\beta}$ and rewrite the action (3.1) as

$$\begin{aligned} I_E &= -V_o \int \left[f(R)a^d - \tilde{\beta}\left(R + 2d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right)\right)\right] d\tau - \\ & \frac{1}{8\pi}\int_{\delta M} d^{(D-1)}x\sqrt{h}f'(R), \end{aligned} \tag{3.4}$$

where $V_o = \frac{1}{16\pi}\frac{2\pi^{\frac{(d+1)}{2}}}{\Gamma\left(\frac{(d+1)}{2}\right)}$. Variation of the action with respect to R, determines $\tilde{\beta}$, which is

$$\tilde{\beta} = a^d f'(R). \tag{3.5}$$

Substituting the above constraint in the action we get

$$\begin{aligned} I_E &= -V_o \int_{\tau=0}^{\tau_{\delta M}} a^d \left[f(R) - f'(R)\left(R - d(d-1)\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)\right) + \right. \\ & \left. 2d\dot{R}f''(R)\frac{\dot{a}}{a}\right] d\tau + \left[2dV_o\dot{a}a^{d-1}f'(R)\right]_{\tau=0}^{\tau_{\delta M}}, \end{aligned} \tag{3.6}$$

Varying the above action with a and R respectively, we get

$$\begin{aligned} & f'(R)\left[2(d-1)\frac{\ddot{a}}{a} + (d-1)(d-2)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) + R\right] \\ & + f''(R)\left[(d-1)\frac{\dot{a}}{a}\dot{R} + 2\ddot{R}\right] + 2f'''(R)\dot{R}^2 - f(R) = 0, \end{aligned} \tag{3.7}$$

$$f''(R)\left[2d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) + R\right] = 0. \tag{3.8}$$

The above field equations admits an instanton solution given by

$$a = \frac{1}{H_o}\,\sin\left(H_o\tau\right), \tag{3.9}$$

where H_o can be determined from the constraint equation

$$\begin{aligned} & d(d-1)H_o^2 + \alpha d^2(d+1)(d-3)H_o^4 + \beta d^3(d+1)^2(d-5)H_o^6 + \\ & \gamma d^4(d+1)^3(d-7)H_o^8 - 2\Lambda = 0. \end{aligned} \tag{3.10}$$

Thus $H_o = f(d, \alpha, \beta, \gamma, \Lambda)$ determined by five parameters. We note that

(i) when $D = 4$, $H_o = f(\beta, \gamma, \Lambda)$;

(ii) when $D = 6$, $H_o = f(\alpha, \gamma, \Lambda)$;

(iii) when $D = 8$, $H_o = f(\alpha, \beta, \Lambda)$.

The above instanton solutions satisfy the HH boundary condition *viz.*, $a(0) = 0$, $\dot{a}(0) = 0$. Now one can choose a path along the $\tau^{\rm Re}$ axis to $\tau = \frac{\pi}{2H}$, the solutions describe half of the Euclidean de Sitter instanton in S^{D-1} -topology. Analytic continuation of the metric (3.2) to Lorentzian region $x_1 \to \frac{\pi}{2} + i\sigma$ gives

$$ds^2 = d\tau^2 + a^2(\tau)\left[-d\sigma^2 + cosh^2\sigma \, d\Omega_{d-2}^2\right], \tag{3.11}$$

which is a de Sitter like metric. However, if one sets $\tau = it$ and $\sigma = \frac{i\pi}{2} + \chi$, the metric becomes

$$ds^2 = -dt^2 + S^2(t)\left[d\chi^2 + sinh^2\chi \, d\Omega_{d-2}^2\right], \tag{3.12}$$

where $S(t) = -ia(it)$. The above metric describes an open universe. Thus, the creation of an open inflationary universe may be realized in this case. It may be pointed out here that a closed universe created quantum mechanically may be realized as an open inflationary universe under a Wick rotation. Since it is not yet known whether our universe is exactly flat, it may be interesting to study an open universe. The real part of the Euclidean action corresponding to the solution calculated by following the complex contour of τ suggested by BH is considered here which determines

$$\begin{aligned} I_E^{Re} &= -\frac{V_o I_d}{H_o^{d+1}}\left[d(d+1)H_o^2 + \alpha d^2(d+1)^2 H_o^4 + \beta d^3(d+1)^3 H_o^6 + \right. \\ &\left. \gamma d^4(d+1)^4 H_o^8 - 2\Lambda\right], \end{aligned} \tag{3.13}$$

where $I_d = \int_0^{\frac{\pi}{2H_o}} \sin^d y \, dy$, denoting $y = H_o \tau$. The value of the integral I_d depends on the dimensions of the universe. For odd dimensions (D),

$$I_d = \frac{d-1}{d}\frac{d-3}{d-2}........\frac{3}{4}\frac{1}{2}\frac{\pi}{2}, \quad for \;\; d \;\; even, \tag{3.14}$$

and for even dimensions (D),

$$I_d = \frac{d-1}{d}\frac{d-3}{d-2}.......\frac{4}{5}\frac{2}{3}1, \quad for \;\; d \;\; odd. \tag{3.15}$$

With the chosen path for τ, the solution describes half the de Sitter instanton in a higher dimensional universe with S^d topology, joined to a real Lorentzian hyperboloid of $(R^1 \times S^d)$ -topology. It can be joined to any boundary satisfying the condition $a_{\partial M} > 0$. For $a_{\partial M} > H_o^{-1}$, the wave function oscillates and predicts a classical space-time.

3.2.2 $S^1 \times S^{D-2}$- Topology

We consider vacuum solution of the field equation corresponding to the action (3.1) and look for a universe with $S^1 \times S^{D-2}$ -spacelike sections. The above topology accommodates a pair of black holes. The corresponding metric ansatz for $D = 1 + 1 + d$ dimensions is given by

$$ds^2 = d\tau^2 + a^2(\tau)dx^2 + b^2(\tau)d\Omega_d^2, \tag{3.16}$$

where $a(\tau)$ is a scale factor for one direction and $b(\tau)$ is the scale factor for the $d = D - 2$-sphere given by the metric

$$d\Omega_d^2 = dx_1^2 + sin^2x_1dx_2^2 + sin^2x_1sin^2x_2dx_3^2 + ... \; d \; space.$$

The scalar curvature is given by

$$R = -\left[2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 2d\frac{\dot{a}\dot{b}}{ab}\right]. \tag{3.17}$$

Using the above constraint (3.17) the Euclidean action (3.1) may be rewritten as

$$I_E = -V_o'\int\left[f(R)ab^d - \tilde{\beta}\left(R + 2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 2d\frac{\dot{a}\dot{b}}{ab}\right)\right] d\tau - \frac{1}{8\pi}\int_{\delta M} d^{(D-1)}x\sqrt{h}f'(R), \qquad (3.18)$$

where $V_o' = \frac{1}{16\pi}\frac{2\pi^{\frac{(D+1)}{2}}}{\Gamma\left(\frac{(D+1)}{2}\right)}$. The undetermined multiplier $\tilde{\beta}$ is obtained by varying the action and inserting the corresponding $\tilde{\beta}$ in the action, we finally obtain

$$I_E = -V_o'\int_{\tau=0}^{\tau_{\delta M}} ab^d\left(f(R) - f'(R)\left(R - d(d-1)\frac{\dot{b}^2}{b^2} - 2d\frac{\dot{a}\dot{b}}{ab} - d(d-1)\frac{1}{b^2}\right)\right) d\tau - V_o'\int_{\tau=0}^{\tau_{\delta M}} 2\dot{R}f''(R)\left(\frac{\dot{a}}{a} + d\frac{\dot{b}}{b}\right) d\tau + 2ab^d f'(R)V_o'\left[\frac{\dot{a}}{a} + d\frac{\dot{b}}{b}\right]_{\tau=0}^{\tau_{\delta M}}. \qquad (3.19)$$

Varying the action with respect to a, b and R respectively, we get the following field equations :

$$f''(R)\left[R + 2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + 2d\frac{\dot{a}\dot{b}}{ab} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right] = 0, \qquad (3.20)$$

$$f'(R)\left[R + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right] + 2f''(R)\left(\ddot{R} + 2d\dot{R}\frac{\dot{b}}{b}\right) + 2\dot{R}^2 f'''(R) - f(R) = 0, \qquad (3.21)$$

$$f'(R)\left[R + 2(d-1)\frac{\ddot{b}}{b} + 2(d-1)\frac{\dot{a}\dot{b}}{ab} + (d-1)(d-2)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 2\frac{\ddot{a}}{a}\right] + 2f'''(R)\dot{R}^2 + 2f''(R)\left[(d-1)\dot{R}\frac{\dot{b}}{b} + \ddot{R} + \dot{R}\frac{\dot{a}}{a}\right] - f(R) = 0. (3.22)$$

We now look for instanton solutions from the above field equations which is given by

$$a = \frac{1}{H}\sin(H\tau),\ b = \sqrt{d-1}\ H^{-1},\ R = (2+d)H^2 \qquad (3.23)$$

where H satisfies the following constraint equation

$$d\ H^2+\alpha(2+d)(d-2)H^4+\beta(2+d)^2(d-4)H^6+\gamma(2+d)^3(d-6)H^8-2\Lambda=0. \quad (3.24)$$

It is evident that H depends on the parameters $\alpha, \beta, \gamma, \Lambda$ *i.e.*, $H = f(d, \alpha, \beta, \gamma, \Lambda)$ for a given extra space dimension (d). The above solution satisfies the HH boundary conditions $a(0) = 0$, $\dot{a}(0) = 0$, $b(0) = 0$, $\dot{b}(0) = 0$. Analytic continuation of metric (3.16) to *Lorentzian region i.e.*, using *Wick rotations* $\tau \to it$ and $x \to \frac{\pi}{2} + i\sigma$, one gets

$$ds^2 = -dt^2 + S(t)^2\ d\sigma^2 + H^{-2}\ d\Omega^2_{d-2}, \quad (3.25)$$

where $S(t) = -ia(it)$. In this case it is evident that the analytic continuation of time and space is not an open universe. The space-time now represents an anisotropic universe. The corresponding Lorentzian solution is given by

$$a(\tau^{\rm Im})|_{\tau^{\rm Re}=\frac{\pi}{2H}} = H^{-1} cosh H\tau^{\rm Im},$$
$$b(\tau^{\rm Im})|_{\tau^{\rm Re}=\frac{\pi}{2H}} = H^{-1}.$$

Its spacelike section represents $(D-2)$-spheres of radius a with a hole of radius $b\ (= H_o^{-1})$ punched through the north and south poles. The physical interpretation of the solution is that of $(d-1)$-spheres containing two black holes, they accelerate away from each other with the expansion of the universe, at the opposite ends. The real part of the action may now be determined following the contour approach suggested by BH, which is

$$I^{Re}_{S^1\times S^d} = -\left[\frac{V_o'(d-1)^{d/2}}{H^{d+2}}\left((2+d)H^2+\alpha(2+d)^2H^4+\beta(2+d)^3H^6+\gamma(2+d)^4H^8-2\Lambda\right)\right], \quad (3.26)$$

where $d = D-2$. The solution (3.23) describes a universe with two black holes at the poles of a $(D-2)$-sphere. It may be pointed out here that the non-zero contribution to the action comes from the surface terms only.

3.3 Evaluation of Probability for PBH

The probability for creation of a higher dimensional de Sitter universe in $f(R)$-theory may now be obtained using gravitational instantons following the technique adopted in the previous sections. The probability for nucleation of a higher dimensional universe without a PBH pair is

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[D(D-1)H_o^2+\alpha D^2(D-1)^2H_o^4+\beta D^3(D-1)^3H_o^6+\gamma D^4(D-1)^4H_o^8-2\Lambda\right]\right]}, \quad (3.27)$$

where H_o satisfies the constraint eq. (3.10). However, the probability of nucleation of an inflationary universe with a pair of black holes is obtained from action (3.19), which is

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[DH^2+\alpha D^2H^4+\beta D^3H^6+\gamma D^4H^8-2\Lambda\right]\right]}, \quad (3.28)$$

where H satisfies constraint eq. (3.24).

Special Cases : For $D \geq 4$ we note the following :
(i) $\alpha = \beta = \gamma = 0$, the probabilities are given in (3.27) and (3.28) reduce to

$$P_{S^{D-1}} \sim e^{\left[4V_o I_{D-1}(D-1)^{D/2}\left(\frac{D-2}{2\Lambda}\right)^{(D-2)/2}\right]}, \quad (3.29a)$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[4V_o'\left(\frac{(D-3)(D-2)}{2\Lambda}\right)^{(D-2)/2}\right]}. \quad (3.29b)$$

We recover the probabilities obtained by Bousso and Hawking in Einstein theory when $D = 4$, which is

$$P_{S^3} \sim e^{3\pi/\Lambda}, \quad P_{S^1\times S^2} \sim e^{2\pi/\Lambda}. \quad (3.30)$$

In the above de Sitter universe is more probable for a positive cosmological constant.

(ii) For $\alpha = \beta = 0,\ \gamma \neq 0$, the corresponding probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-6D(D-1)H_o^2+16\Lambda}{D-8}\right)\right]}, \tag{3.31a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-6DH^2+16\Lambda}{D-8}\right)\right]}, \tag{3.31b}$$

where H_o and H are determined from eqs.(3.10) and (3.24) respectively for $D \neq 8$. In four dimensions the probabilities reduce to

$$P_{S^3} \sim e^{\left[\frac{\pi}{3H_o^4}(9H_o^2-2\Lambda)\right]}, \quad P_{S^1\times S^2} \sim e^{\left[\frac{2\pi}{H^4}(3H^2-2\Lambda)\right]}. \tag{3.32}$$

For $\Lambda = 0$ and $D \neq 8$, we evaluate the probabilities which are

$$P_{S^{D-1}} \sim e^{\left[12V_o I_{D-1}(D(D-1))^{D/2}(8-D)^{(D-8)/6}(\frac{\gamma}{D-2})^{(D-2)/6}\right]}, \tag{3.33a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[12V_o' D^{D/2}(8-D)^{(D-8)/6}(D-3)^{(D-2)/2}(\frac{\gamma}{D-2})^{(D-2)/6}\right]}. \tag{3.33b}$$

We note the following:

(a) a new gravitational instanton solution in higher dimensions is obtained for a R^4-theory with a negative coupling parameter (i.e., $\gamma < 0$) for $D > 8$, and that with a positive (i.e., $\gamma > 0$) for $D < 8$, (b) de Sitter universe is more probable for $D \geq 3$. We also note that for $D = 8$, an instanton solution with $H_o^2 = \frac{\Lambda}{21}$ in S^7-topology and with $H^2 = \frac{\Lambda}{3}$ in $S^1 \times S^6$-topology are permitted. The probabilities of PBH for $D = 8$ dimensions are

$$P_{S^7} \sim e^{\left[\frac{1372\pi^3}{15\Lambda^3}\left(27+2048\gamma\Lambda^3\right)\right]}, \ P_{S^1\times S^6} \sim e^{\left[\frac{100\pi^3}{21\Lambda^3}\left(27+2048\gamma\Lambda^3\right)\right]}. \tag{3.34}$$

It is evident that the probabilities are equal if (a) $|\gamma| = \frac{27}{2048\Lambda^3}$ and $\Lambda > 0$ with negative γ and (b) $\gamma = \frac{27}{2048\Lambda^3}$ and $\Lambda < 0$ with positive γ. A new gravitational instanton is permitted even without a cosmological constant in $D = 8$.

(iii) For $\alpha \neq 0,\ \beta = \gamma = 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-2D(D-1)H_o^2+8\Lambda}{D-4}\right)\right]}, \tag{3.35a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-2DH^2+8\Lambda}{D-4}\right)\right]}. \tag{3.35b}$$

Thus PBH pair creation is possible without a cosmological constant in higher dimensions also. Taking $\Lambda = 0$, the probabilities are estimated which are given by

$$P_{S^{D-1}} \sim e^{\left[\frac{4V_o I_{D-1}}{(D-2)^{(D-2)/2}}(D(D-1))^{D/2}(4-D)^{(D-4)/2}\right]}, \tag{3.36a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[4V_o'\left(\frac{D-3}{D-2}\right)^{(D-2)/2}D^{D/2}(4-D)^{(D-4)/2}\right]}. \tag{3.36b}$$

It is noted that the instanton solutions considered here for the two different topologies are physically sensible when $\alpha < 0$ for $D > 4$ and $\alpha > 0$ for $D < 4$. We note that in higher dimensions with space-time dimensions $D \geq 3$ the probability of nucleation of a S^{D-1}-topology is found more than that of a universe with $S^1 \times S^{D-2}$ topology. In four dimensions the probabilities reduce to

$$P_{S^3} \sim e^{[24\pi\alpha+3\pi/\Lambda]},\quad P_{S^1\times S^2} \sim e^{[16\pi\alpha+2\pi/\Lambda]}. \tag{3.37}$$

A theory with a positive cosmological constant and $\alpha > 0$, leads to a universe without PBH, which is more probable. Though a negative $\alpha < -\frac{1}{8\Lambda}$ leads to greater probability for a universe with PBH, the negative values of α lead to a classical instability in R^2-theory [132].

(iv) For $\alpha = \gamma = 0,\ \beta \neq 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-4D(D-1)H_o^2+12\Lambda}{D-6}\right)\right]}, \tag{3.38a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-4DH^2+12\Lambda}{D-6}\right)\right]}, \tag{3.38b}$$

with $D \neq 6$. The probabilities in $D = 4$ dimensions may now be obtained as

$$P_{S^3} \sim e^{\left[\frac{\pi}{H_o^4}(H_o^2-\Lambda)\right]}, \quad P_{S^1\times S^2} \sim e^{\left[\frac{2\pi}{H^4}(4H^2-3\Lambda)\right]}. \tag{3.39}$$

We note that PBH pair creation is permitted even without a cosmological constant. The corresponding probabilities are

$$P_{S^{D-1}} \sim e^{\left[8V_o I_{D-1}\left(\frac{\beta}{D-2}\right)^{(D-2)/4}(D(D-1))^{D/2}(6-D)^{(D-6)/4}\right]}, \tag{3.40a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[8V_o'(D-3)^{(D-2)/2}\left(\frac{\beta}{D-2}\right)^{(D-2)/4}D^{D/2}(6-D)^{(D-6)/4}\right]}. \tag{3.40b}$$

The interesting result is that a universe with space-time dimensions (a) $D > 6$ admits an instanton solution with $\gamma < 0$ and (b) $D < 6$, with a positive β. The probability measure for pair creation in an inflationary universe is found more compared to that in a universe without primordial black holes.
Let us now consider $D = 6$ dimensional universe. In this case $H_o^2 = \frac{\Lambda}{10}$ in S^5-topology and $H^2 = \frac{\Lambda}{2}$ in $S^1 \times S^4$-topology. The corresponding probabilities are

$$P_{S^5} \sim e^{\left[\frac{200\pi^2}{3\Lambda^2}\left(1+27\beta\Lambda^2\right)\right]}, \; P_{S^1\times S^4} \sim e^{\left[\frac{48\pi^2}{5\Lambda^2}\left(1+27\beta\Lambda^2\right)\right]}. \tag{3.41}$$

It is evident that a de Sitter universe without a pair of PBH is more probable.
(v) For $\Lambda = 0$, the probability measure in the two topologies under consideration are given by

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2H_o^4+4\beta D^3(D-1)^3H_o^6+6\gamma D^4(D-1)^4H_o^8}{D-2}\right]\right]}, \tag{3.42a}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2H^4+4\beta D^3H^6+6\gamma D^4H^8}{D-2}\right]\right]}. \tag{3.42b}$$

In $D = 4$ dimensions the probabilities reduce to

$$P_{S^3} \sim e^{\left[\pi(24\alpha+576\beta H_o^2+10368\gamma H_o^4)\right]}, \; P_{S^1\times S^2} \sim e^{\left[\pi(16\alpha+128\beta H^2+768\gamma H^4)\right]}. \tag{3.43}$$

The following points are noted:

(a) For $\gamma = 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2 H_o^4 + 4\beta D^3(D-1)^3 H_o^6}{D-2}\right]\right]}, \tag{3.44a}$$

$$P_{S^1 \times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2 H^4 + 4\beta D^3 H^6}{D-2}\right]\right]}, \tag{3.44b}$$

one obtains gravitational instanton solution even in the absence of Λ [138].

(b) For $\beta = 0$, and $\Lambda = 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2 H_o^4 + 6\gamma D^4(D-1)^4 H_o^8}{D-2}\right]\right]}, \tag{3.45a}$$

$$P_{S^1 \times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2 H^4 + 6\gamma D^4 H^8}{D-2}\right]\right]}. \tag{3.45b}$$

For $D = 4$

$$P_{S^3} \sim e^{\left[3\pi(8\alpha + 3(128\gamma)^{1/3})\right]}, \quad P_{S^1 \times S^2} \sim e^{\left[2\pi(8\alpha + 3(128\gamma)^{1/3})\right]}. \tag{3.46}$$

Here $H_o^2 = \sqrt[3]{\frac{1}{3456\gamma}}$ and $H^2 = \sqrt[3]{\frac{1}{128\gamma}}$, correspond to new gravitational instanton solutions. In this case a physically interesting instanton solution is obtained for a positive value of γ. For $\alpha < 0$ and if $|8\alpha| < 3(128\gamma)^{1/3}$, de Sitter universe without PBH is more probable. However interesting possibilities emerges when $|8\alpha| > 3(128\gamma)^{1/3}$.

3.4 Discussions

The probability of a pair of primordial black holes in a non linear theory of gravity in a multidimensional universe is estimated. An action with a polynomial function of Ricci scalar R as $f(R) = \sum_i \lambda_i R^i$, $i = 0, 1, 2, 3, 4$ is taken up to obtain gravitational instanton solutions in a multidimensional universe. The Euclidean action is then evaluated using instanton solutions for the two topologies: (i) a universe with $R \times S^d$-topology and (ii) a universe with $R \times S^1 \times S^d$-topology in higher dimensions. The former admits an inflationary universe without PBH and later a pair of PBH in a higher dimensional universe. The constraints on the the coupling parameters for a physically realistic instanton solutions are obtained. One interesting point is that in $f(R)$ gravity model, gravitational instanton solutions are permitted even without a cosmological constant. The probability of creation of a universe with a pair of PBH is found to be strongly suppressed if one considers a polynomial function of $f(R)$ gravity model upto R^4-term with the constraints $|8\alpha| < 3(128\gamma)^{1/3}$ or $8\alpha > -3(128\gamma)^{1/3}$. The dimensional dependence on probability of PBH pairs is studied also. A class of new gravitational instanton solutions is obtained here which are relevant for cosmological model building.

Chapter 4

Open Inflationary Scenario of the Universe Using Gravitational Instantons in R^4-Gravity

4.1 Introduction

In modern cosmology inflation is one of the essential ingredients to construct cosmological models. A number of inflationary models came up in the last three decades to address different issues in Cosmology and Particle Physics. All inflationary scenario of the early universe are derived out of an inflaton field guided by a potential that is available in the standard model of Particle Physics. The recent astronomical observations led to the fact that the total energy density might be very close to its critical value where the density parameter has the value $\Omega_o = 1.02 \pm 0.04$ [139]. Although the observations point toward a flat universe, it is interesting to look for an alternative point of view having marginally open or closed model that might have emerged from an inflationary phase in the early era as the density parameter is not yet exactly known. In cosmology it is always possible to make inflation short and density parameter (Ω) different from unity by fine-tuning the parameters; however the homogeneity and isotropy of the observable part of the universe in that case remain unclear. This view was modified after it was discovered that there exist some special classes of inflaton effective potentials which may lead to a nearly homogeneous open universe with $\Omega_0 \leq 1$ at the present epoch. The potentials in the above should have a metastable minimum followed by a small slope region that permits a *slow-roll* inflation. The inflaton field is supposed to be initially trapped in the false vacuum leading to an inflationary phase that describes an almost de Sitter expansion with small quantum fluctuations leading to the observed universe. The inflaton field undergoes a quantum tunneling nucleating a bubble within which the inflaton field slowly rolls down to the true vacuum. The interior of such a bubble looks like an open universe [140]. A large number of works in the literature [141] - [172] appeared considering open or closed inflationary universe model in the context of Einstein theory of relativity, Jordan-Brans-Dicke theory, Randall-Sundrum type Brane model, tachyonic-field model and Gauss-Bonnet Brane cosmology. Alternatively, Hawking and Turok (henceforth, HT) [133, 134] proposed that an open universe may be realized from nothing even without passing through an intermediate stage of false vacuum inflation and tunneling. Using no boundary proposal, Hawking and Turok [128] described a useful technique to obtain an open inflationary universe that may be realized via a singular instanton in a minisuperspace model in the presence of an inflaton field. The HT instanton solution can further be used in a gravitational action to determine the probability of a universe that permits a homogeneous open universe. The HT-instanton has a singular boundary which is timelike in the Lorentzian region. The Euclidean action corresponding to the instanton is integrable and the boundary term is found to be finite. In spite of its novelty, several aspects of the mechanism adopted here have been criticized and further justified.

Vilenkin [154] raised the question about the unusual feature of the instanton indicating that it is singular as both the curvature and scalar field diverge at one point. Using an instanton where Hawking-Turok type singularity exists, it is argued that the instanton may lead to an immediate decay state of a flat space, in contradiction to observation. Later Unruh [155] provided clarification on HT instanton as has been obtained and pointed out that a universe with an arbitrarily large section of open universe that is homogeneously closed off by a naked timelike singularity permits HT instanton solution rather than for a homogeneous universe only. A precise examination reveals that such a singularity behaves as a reflecting boundary for scalar and tensor cosmological perturbations [156, 157]. Wu [158] proposed that the singular instantons should be treated as constrained solutions that are not the stationary points of the gravitational action. Linde [159, 160] subsequently proposed that one should change the 'wrong' sign of the gravitational action for evaluating the contribution of an instanton in quantum cosmology. The probability of quantum creation of a class of classical solutions led to $P \sim e^{+S}$, where S represents the Euclidean action on the trajectory describing the nucleation of a universe and this is very different from the one which is obtained with the help of the Hartle and Hawking (henceforth, HH) proposal. It has been argued [161, 3, 162, 163] further that the HH wavefunction may be important to evaluate the probability of quantum fluctuations of a universe after its birth instead evaluating the creation of a universe. It has been noted that if the HH prescription is used for describing the creation of all other versions of open universe scenario, it does not work well [163]. However, a serious problem [163] arises in calculating the probability of creation of a universe with Linde's prescription. If the sign of the action for both the gravitational instanton and fluctuations from which the open universe is assumed to emerge is reversed, the later is left unsuppressed, and the description of the spacetime as a classical background with small fluctuations breaks down. Again, if the sign of the action for the background is reversed leaving the sign for the fluctuations unchanged, which may be fixed by the usual Wick rotation, one gets a physically realistic situation. But the above approach voilates the general coordinate invariance since there is no invariant way to split a given four-geometry into background and perturbations. Using Linde's prescription, Bousso and Hawking showed [164] that the creation of a universe with large number of black holes is favoured resulting in a universe without a radiation-dominated era. Later, Garriga [165] obtained a gravitational instanton solution in five dimensions that admits an open universe in four dimensions similar to that obtained by Hawking and Turok. It is shown that a flat space with a compact extra dimension may be sufficiently long lived if the size of the extra dimension is taken large enough compared to the Planck length in four dimensions. The singularity present in cosmological instantons of Hawking-Turok type is resolved by a conformal transformation [166] where the conformal factor has a linear zero of co-dimension 1 and in higher dimensions within the context of M-(or string) theory [167]. Similar types of singular instanton solutions may be found when a regular instanton in higher dimensions is compactified to four dimensions.

Using a quadratic Lagrangian in Ricci scalar it has been shown [168] in four dimensions that the field equations admit both kinds of singular and non-singular instantons. Paul *et al.* [169] noted existence of both the singular HT instanton and non-singular de Sitter instanton in a modified theory of gravity described by quadratic terms of Ricci Scalar in the Einstein gravitational action. In the case of cubic Lagrangian such instanton solutions are found to exist [168] even in the absence of a cosmological constant. It is also known [45, 170, 171] that the modified gravitational action can be transformed into theories of self interacting scalar fields minimally coupled to Einstein gravity using conformal transformation. It has been argued that the cubic term in Einstein gravity represents a generic perturbation of the polynomial type to the action and then one can determine the constraints on the parameters for the existence of both singular and non-singular instanton solutions. Recently non-linear Lagrangian (for example, $\sim R^4$ term in the action) is considered to study different issues in cosmology [137, 172]. Such a non-linear theory arises for "low-energy" approximation to a more fundamental theory which may bring out new possibilities for extracting the observed cosmic acceleration. In a multidimensional universe curvature terms of the forms R^{-1} and R^4 are also considered in the gravitational action to study AdS solution and stability of extra dimensions. It is known that the curvature inverse term might have played an important role for late universe evolution in order to accommodate an accelerating universe while R^4-term is important for cosmological evolution of the early universe. In this chapter gravitational instanton solutions in the presence of a nonlinear term of the form R^4 to the Einstein gravitational action with cosmological constant will be explored. The singular instantons obtained in the conformally equivalent picture will be taken up here to obtain open inflationary universe.

4.2 Field Equations and Gravitational Instanton in R^4 Gravity

We consider Euclidean gravitational action (2.6) which is given by

$$I_E = -\frac{1}{16\pi}\int d^4x\sqrt{g}f(R) - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}Kf'(R).$$

Let us consider a polynomial function $f(R)$ which is given by

$$f(R) = R + \gamma R^4 - 2\Lambda. \tag{4.1}$$

Using the above polynomial function in the action we explore general solutions which admits $O(4)$ invariance property in the spacetime. We consider Euclidean metric given by

$$dS^2 = d\sigma^2 + a^2(\sigma)d\Omega_3^2, \tag{4.2}$$

where $d\Omega_3^2 = d\psi^2 + sin^2(\psi)\ d\Omega_2^2$ is the metric on the three-sphere, $d\Omega_2^2$ is the metric of 2-sphere and $a(\sigma)$ represents the radius of S^3 latitude of the S^4-topology, σ is the Euclidean time.

The relevant field equation obtained from the action (2.6) using the metric

(4.2) is

$$2f'''(R)\dot{R}^2+2f''(R)\left[\ddot{R}+2\frac{\dot{a}}{a}\dot{R}\right]+f'(R)\left[4\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}-\frac{2}{a^2}+R\right]-f(R)=0, \quad (4.3)$$

where the prime represents the derivative with respect to σ. It admits a de Sitter instanton solution which is given by

$$a(\sigma)=\frac{1}{H_o}\sin H_o\sigma,$$

where H_o satisfies a constraint equation given by

$$2\gamma {R_o}^4 - R_o + 4\Lambda = 0 \quad (4.4)$$

with $R_o = 12H_o^2$. The above instanton solution satisfies HH no boundary conditions *viz.*, $a(0) = 0$, $\dot{a}(0) = 1$, $\dot{R} = 0$, $R \neq 0$. The de Sitter instanton solution emerges for any real positive root of the above equation. We note the following :
(i) For $\Lambda = 0$, one gets $R = 12H_o^2 = \frac{1}{\sqrt[3]{2\gamma}}$, which is physically realistic for $\gamma > 0$.
(ii) For $\Lambda \neq 0$ and a large R_o, one obtains an asymptotic value $R_o = \left(-\frac{2\Lambda}{\gamma}\right)^{\frac{1}{4}}$, which is relevant for (a) $\Lambda < 0$ and $\gamma > 0$ or (b) $\Lambda > 0$ and $\gamma < 0$.

The probability of creation of a de Sitter universe is now estimated using the gravitational instantons. The Euclidean action then corresponds to the dominant saddle point of the path integral *i.e.*, the classical solution satisfying the Hartle-Hawking boundary conditions [128]. The probability measure in this case becomes $P \sim e^{-2I_E^{Re}}$, where the subscript *Re* stands for the real part of the Euclidean action corresponding to the dominant saddle point of the path integral. Considering $f(R)$ as given by eq. (4.1) we integrate the general action given by eq. (2.6) over half of the S^4. The Euclidean action yields

$$I_E = -\left[\frac{3\pi}{2{H_o}^2} - \frac{\pi\Lambda}{3{H_o}^4}\right] \quad (4.5)$$

where H_o^2 is determined from eq. (4.4). The probability for the de Sitter instanton is

$$P \sim exp\left[\frac{3\pi}{H_o^2}\left(1 - \frac{2\Lambda}{9H_o^2}\right)\right].$$

It is evident that the probability of a de Sitter instanton is large if $R_o > \frac{8\Lambda}{3}$. We note the following :

(a) For $\gamma = 0$, Bousso and Hawking [129] instanton solution is recovered.

b) For $\Lambda = 0$ and $\gamma \neq 0$, we obtain Euclidean action: $I_E = -\frac{3\pi}{2{H_o}^2}$, where ${H_o}^2 = R = \frac{1}{12\sqrt[3]{2\gamma}}$. It is a new instanton solution which is interesting. Unlike Bousso and Hawking, in the modified theory of gravity de Sitter instanton is realizable even in the absence of a cosmological constant. In this case the probability of creation of open universe is found to depend on the coupling parameter γ of the R^4 term in the gravitational action. The open universe may be visualized here under the Wick rotation of both time and space coordinates which leads to a Coleman-de Luccia type instanton [140].

4.3 Creation of Open Universe

It is known that an open universe may be realized for an inflaton field potential having a metastable minimum (a false vacuum) followed by a small slope region allowing slow-roll inflation. By adjusting the duration of slow-roll epoch it is possible to arrange the spatial curvature of the present universe which is of the order the Hubble radius [173]. However, it is not essential for a universe to evolve from a false vacuum in the framework of Hartle-Hawking prescription. In the above the quantum fluctuations are computed by continuing the field and metric perturbation modes from the Euclidean region. According to Hartle-Hawking the universe originated from a state where fluctuations are at the minimal level to begin with. Hawking-Turok (HT) considered the path integral technique to determine the probability in the case of Euclidean Einstein gravity coupled to a scalar field ϕ, with potential $V(\phi)$, which is assumed to have a true minimum with $V(\phi) = 0$. Using a homogeneous space-time described by an $O(4)$ symmetric

metric given by eq. (4.2), HT obtained gravitational instanton. The scalar field and the metric solutions are chosen so that near the point $\sigma = 0$, the scale factor $a(\sigma) \sim \sigma$. This ensures that the point $\sigma = 0$ is a regular point in the Euclidean space-time. In general, HT solution is such that $a(\sigma)$ increases with σ and thereafter decreases to zero but this time it approaches to a singularity with $a(\sigma) \to 0$ (a second singular point). To study HT instanton we consider Euclidean action with a self-interacting field (ϕ) which is given by

$$I = -\int \sqrt{g}\, d^4x \left[\frac{R}{16\pi} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]. \tag{4.6}$$

The following field equations are obtained

$$3\left(\frac{a'^2}{a^2} - \frac{1}{a^2}\right) = 8\pi\left[\frac{1}{2}\phi'^2 - V(\phi)\right], \tag{4.7}$$

$$\phi'' + 3\frac{a'}{a}\phi' = \frac{\partial V}{\partial \phi}, \tag{4.8}$$

where prime $()'$ denotes derivative with respect to σ. Since the point $\sigma = 0$ is regular, we must have $\phi' = 0$ at that point. The Einstein equation corresponding to the action (4.6) can be written as

$$R = 8\pi\left((\partial\phi)^2 + 4V(\phi)\right), \tag{4.9}$$

which leads to the Euclidean action

$$I_E = -\int d^4x\sqrt{g}V = -\pi^2\int d\sigma a(\sigma)^3 V(\phi), \tag{4.10}$$

when integrated over half of the S^4-section. Here HT solution is singular but the potential $V(\phi)$ diverges logarithmically at the singularity. The volume measure $b^3(\sigma)$ vanishes linearly so the Euclidean action is perfectly convergent. To obtain an open universe the Euclidean solution is first matched across the $\psi = \frac{\pi}{2}$ surface. At this surface the extrinsic curvature of the slice is zero, except at $\sigma = \sigma_f$, the singularity. Analytic continuation of the metric (4.2) in Lorentzian region for $\psi = \frac{\pi}{2} + i\tau$ gives

$$ds^2 = d\sigma^2 + a^2(\sigma)\left[-d\tau^2 + cosh^2\tau \; d\Omega_2^2\right]. \tag{4.11}$$

The above metric resembles a spatially inhomogeneous de Sitter metric. However, setting $\sigma = it$ and $\tau = i\pi/2 + \chi$, it yields the Lorentzian metric

$$ds^2 = -dt^2 + b^2(t)\left[d\chi^2 + sinh^2\chi \; d\Omega_2^2\right], \tag{4.12}$$

where $b(t) = -ia(it)$. It describes an open inflationary universe which will be taken up in the next section.

4.4 Gravitational Instantons in the Conformally Equivalence Picture

It is known [170, 171] that the theories of gravity with higher order Lagrangian are conformally equivalent to Einstein gravity with a matter sector containing a minimally coupled self-interacting scalar field (ϕ) with an effective potential $V(\phi)$. A conformal equivalence picture may be obtained here considering a conformal transformation of the form given by

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{4.13}$$

where $\ln\Omega = \frac{\phi}{\sqrt{6}} = \frac{1}{2}\ln|f'(R)|$, the prime denoting a derivative with respect to R. Using the above transformation the action given by eq. (2.6) reduces to

$$I = -\int \sqrt{\tilde{g}} d^4x \left[R(\tilde{g}) - \frac{1}{2}\left(\tilde{\partial}\phi\right)^2 - V(\phi)\right], \tag{4.14}$$

with $V(\phi) = \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}\left[R\frac{\partial f}{\partial R} - f(R)\right]$. Therefore one can determine the corresponding potential knowing a precise form of $f(R)$. For example, if we consider

$f(R) = \sum_{n=0}^{N} \lambda_n R^n$ i.e., for a general polynomial Lagrangian of order n, it is extremely complicated to express the potential in a known form. But one can determine qualitatively an asymptotic behaviour of $V(\phi)$ for a specific choice of N. In our case we choose $N = 4$ and $\lambda_o = -2\Lambda$, $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 0$ and $\lambda_4 = \gamma$. Since the dimension(D) of the space-time considered here is less than $2N$, the potential is found to have a decaying tail [170] which is evident from the figs. (4.1) and (4.2). It is known that a qualitative behaviour of $V(\phi)$ does not change even for $N > 4$, thus we consider here a specific action with $N = 4$. The dynamical equations corresponding to the action given by eq. (4.14) can be derived using the following variations:

$$(i) \quad \frac{\partial I}{\partial \tilde{g}^{\mu\nu}} = 0,$$

$$(ii) \quad \frac{\partial I}{\partial \phi} = 0.$$

Choosing $\tilde{g}$ to be given by the metric $ds^2 = d\tilde{\sigma}^2 + a^2(\tilde{\sigma})(d\psi^2 + sin^2\psi \; d\Omega_2^2)$, we obtain the following field equations

$$3\left(\frac{a'^2}{a^2} - \frac{1}{a^2}\right) = \frac{1}{2}\phi'^2 - V(\phi), \tag{4.15}$$

$$\phi'' + 3\frac{a'}{a}\phi' = \frac{\partial V}{\partial \phi}, \tag{4.16}$$

where prime denotes derivative with respect to $\tilde{\sigma}$ and $8\pi G = 1$. The scalar field potential corresponding to the polynomial function given by eq. (4.1) takes the form:

$$V(\phi) = \frac{3}{8}\frac{e^{-2\sqrt{\frac{2}{3}}\phi}}{(4\gamma)^{1/3}}\left(e^{\sqrt{\frac{2}{3}}\phi} - 1\right)^{4/3} + \Lambda e^{-2\sqrt{\frac{2}{3}}\phi}. \tag{4.17}$$

The field equation admits a de Sitter instanton solution given by

$$a(\tilde{\sigma}) = \frac{1}{H_o}\sin H_o\tilde{\sigma},$$

$$\phi = \phi_o = \sqrt{\frac{2}{3}}\; \ln(1 + 256\gamma\Lambda^3) \tag{4.18}$$

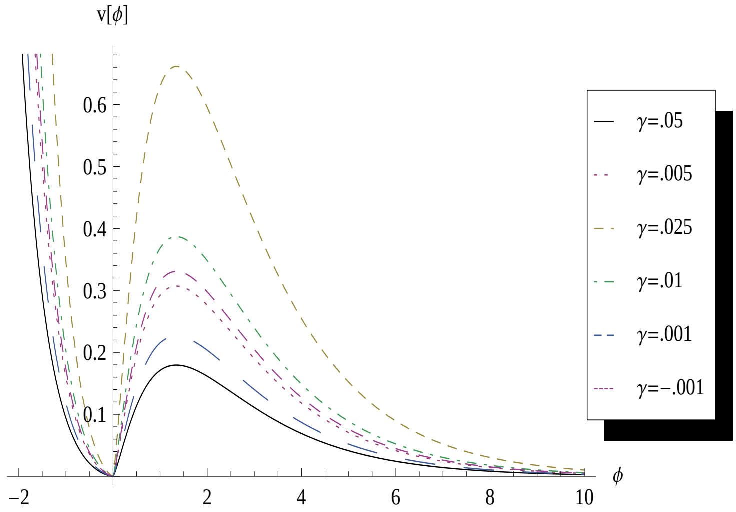


Figure 4.1: $V(\phi)$ Vs. ϕ curve for $\Lambda = 0$

with $V(\phi_o) = 3H_o^2 = \left[\frac{\Lambda(1+384\gamma\Lambda^3)}{1+256\gamma\Lambda^3}\right]$. The above solution is realized with Λ and γ satisfying the constraints
(i) for $\gamma > 0$ with $\Lambda > 0$ or (ii) for $\gamma < 0$ with $0 < \Lambda < \frac{1}{256|\gamma|}$.
The corresponding action evaluated for this case found to have same value as that obtained in eq. (4.5).

We plot the potential for different values of γ and Λ which are shown in figs. (4.1) and (4.2). In the absence of a cosmological constant the shape of the potential is shown in fig. (4.1). It has a maximum at $\phi = \phi_m$, at $\phi = 0$ and at $\phi \to \infty$ the potential vanishes once again. In the case of a non-zero positive cosmological constant, the shape of the potential is shown in fig. (4.2) which has a local minimum at $\phi = 0$, but $V|_{\phi=0} > 0$. In this case it is evident that a barrier exists between the first zero and the second zero of the potential which permits tunneling. The height of the potential barrier decreases as one picks up increasing values of γ, however, the barrier height vanishes at some value of γ shown in the figure. The potential attains a maximum at a positive value of ϕ and $V(\phi) \to 0$ as $\phi \to \infty$. The shape of the potential is interesting and may be important for studying quantum creation of early universe.

We now consider singular HT instantons in the framework of a conformally equivalent picture. The general features of the scalar field potential $V(\phi)$ given

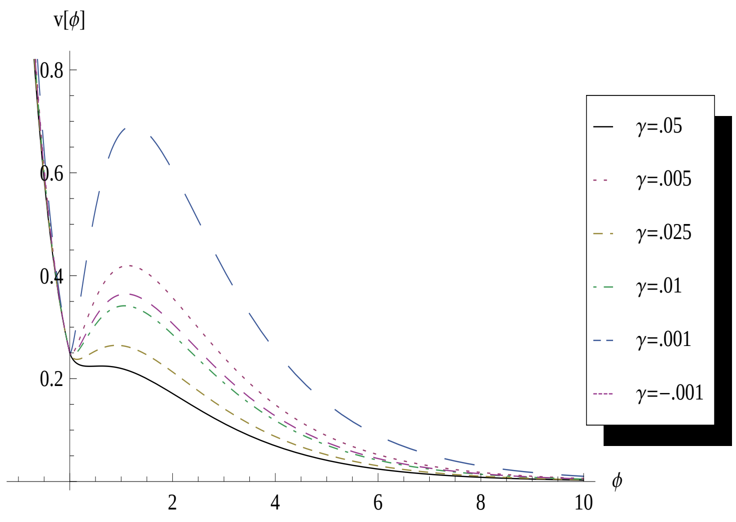


Figure 4.2: $V(\phi)$ Vs. ϕ curve for $\Lambda > 0$

by eq. (4.17) is important to realize the HT-instanton. We note the following properties of the potential:

(i) There are two values of ϕ (namely, ϕ_+ and ϕ_-), for which $V(\phi) = 0$. These are given by

$$\phi_\pm = \sqrt{\frac{3}{2}} \ln \left[1 \pm \frac{8}{3} \left(\frac{3}{2} \gamma' \Lambda^3 \right)^{1/4} \right], \tag{4.19}$$

where $\gamma' = -\gamma < 0$ requires $\Lambda < 0$ and $\gamma' > 0$ requires $\Lambda > 0$.

(ii) For a positive cosmological constant $\Lambda = \frac{1}{8\gamma^{1/3}}$, the scalar field potential has a maximum for $\gamma > 0$ at $\phi = \phi_m = \sqrt{\frac{3}{2}}\ \ln{(m^3 + 1)}$,

$$m = \tfrac{1}{3} \left[-1 - \tfrac{2}{(17+3\sqrt{13})^{\frac{1}{3}}} + (17 + 3\sqrt{33})^{\frac{1}{3}} \right].$$

It has a decaying tail at large ϕ.

(iii) For $\Lambda = 0$, the potential $V(\phi) \to 0$ at two values of ϕ namely, (a) when $\phi \to 0$ and (b) $\phi \to \infty$. In the case of a positive cosmological constant i.e., $\Lambda > 0$, the potential has a tail with $V(\phi) \to 0$ at $\phi \to \infty$ only.

In the R^2-theory [92], however, it is found that $V(\phi) \to -\frac{1}{8\alpha'}$ as $\phi \to \infty$ even if $\Lambda \neq 0$. A different form of potential compared to the R^4-theory where $V(\phi) \to 0$ as $\phi \to \infty$ for non zero Λ is possible. So, in all the cases with or without cosmological constant $V(\phi) \to 0$ for a large value of ϕ.

The choice of initial conditions however plays a dominant role in the evolution of the instantons. Let us assume that at $\tilde{\sigma} = 0$ (which is a nonsingular point), $\phi(o) = \phi_+$. Consequently $\phi'(0) = 0$ and we have $\frac{dV}{d\phi}|_{\phi_+} = -\frac{\left(\frac{2}{3}\gamma\Lambda^3\right)^{1/4}}{1+\frac{8}{3}\left(\frac{2}{3}\gamma\Lambda^3\right)^{1/4}}$. So the potential has non zero slope at $\tilde{\sigma} = 0$. Otherwise we would obtain the $O(5)$ invariant instanton. Also the potential has a negative gradient at $\tilde{\sigma} = 0$. Since $\tilde{\sigma} = 0$ is a nonsingular point and $a(\tilde{\sigma}) \sim \tilde{\sigma}$ at small $\tilde{\sigma}$, the manifold looks locally like R^4 in spherical polar coordinates. If we approach forward in $\tilde{\sigma}$, $a(\tilde{\sigma})$ decelerates and it's velocity $a'(\tilde{\sigma})$ changes sign. At $\tilde{\sigma} = \tilde{\sigma}_f$, $a(\tilde{\sigma})$ becomes zero once again and $\phi'(\tilde{\sigma})$ goes to infinity as one approaches $\tilde{\sigma}_f$. To determine the conditions for the existence of a singular HT instanton in the theory, we assume that close to the singularity $\tilde{\sigma}_f$ i.e, $\tilde{\sigma}_f - \tilde{\sigma} < 1$

$$\phi = q \ln\left(\tilde{\sigma}_f - \tilde{\sigma}\right), \tag{4.20}$$

$$a \sim \left(\tilde{\sigma}_f - \tilde{\sigma}\right)^n \tag{4.21}$$

then the field eqs. (4.15) and (4.16) determine

$$q = \sqrt{\frac{3}{2}} \tag{4.22}$$

for $n < 1$ and it yields

$$n = \frac{1}{3}\left[1 + \frac{3 + 8\Lambda(4\gamma)^{1/3}}{6(4\gamma)^{1/3}}\right]. \tag{4.23}$$

In this case HT singular instanton exists for $n = \frac{3}{4}$ yielding

$$\Lambda = \frac{15(4\gamma)^{1/3} - 6}{16(4\gamma)^{1/3}}. \tag{4.24}$$

For a physically relevant solution one obtains the following: (i) $\Lambda < 0$ for $\gamma > 0$ and (ii) $\Lambda > 0$ with $\gamma > \frac{2}{125}$ or $\gamma < 0$ which makes them rather special. In this case the scalar field ϕ may be driven up the potential and become stabilized at ϕ_m, giving the singular HT solution or they may end up in a singular state at

$\tilde{\sigma} = \tilde{\sigma}_f$, giving the singular HT instanton.

4.5 Early Inflation

The universe created via gravitational instantons is important for describing the observed universe also. In a conformally equivalent picture we obtain a scalar field potential which permits such singular instantons. It is therefore important to explore whether the potential admits sufficient inflation for a successful inflationary scenario. In this section using the above potential we explore its importance for a late inflating phase. However, late universe is described in the Lorentzian time. The Euclidean field equation with a single minimally coupled scalar field with potential $V(\phi)$ given in eqs. (4.15) and (4.16) with proper gravitational unit ($8\pi G = M_P^{-2}$) may now reduce to the form

$$H^2 = \frac{1}{3M_P^2}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right], \tag{4.25}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0, \tag{4.26}$$

for a Lorentzian time scale where a dot represents the Lorentzian time derivative and M_P is the Planck mass. Introducing slow roll approximation the above field equations reduce to the following forms:

$$H^2 \approx \frac{V(\phi)}{3M_P^2}, \tag{4.27}$$

$$3H\dot{\phi} \approx -V'(\phi), \tag{4.28}$$

where the primes are derivatives with respect the scalar field ϕ and the conditions for obtaining inflation is ensured from the following slow-roll parameters ϵ and η given by

$$\epsilon(\phi) << 1, \quad |\eta(\phi)| << 1 \tag{4.29}$$

γ	ϕ_{end}	Range of $\phi_{initial}$ value
0.05	0.517469	1.255091-1.25509149
1	0.47285	1.23850205-1.2385021018
2.5	0.44506	1.3105279-1.31053074
5	0.415152	1.2502127-1.250213
7.5	0.392246	1.215426791027-1.21542691085
10	0.372691	1.21960385-1.2196047

Table 4.1: Range of $\phi_{initial}$ values for sufficient inflation for $\gamma > 0$, $\Lambda = 0$

$\|\gamma\|$	ϕ_{end}	Range of $\phi_{initial}$ value
0.05	0.563997	1.268001916-1.26800207
1	0.600327	1.28100275-1.28100295
2.5	0.619666	1.2874230503-1.2874230543
5	.638047	1.32171445-1.32171498
7.5	0.650578	1.32016599-1.3201668
10	0.660285	1.31401037-1.314012

Table 4.2: Range of $\phi_{initial}$ values for sufficient inflation for $\gamma < 0$, $\Lambda = 0.01$

which are defined by

$$\epsilon(\phi) = \frac{{M_P}^2}{2}\left(\frac{V'}{V}\right)^2, \tag{4.30}$$

$$\eta(\phi) = {M_P}^2\frac{V''}{V}. \tag{4.31}$$

The number of e-foldings N is an important parameter which indicates the amount of inflation that occurs normally to solve some of the outstanding problems of cosmology. Using slow-roll approximation we determine number of e-folding which is given by

$$N \approx \frac{1}{{M_P}^2}\int_{\phi_{end}}^{\phi_{initial}}\left(\frac{V}{V'}\right)d\phi, \tag{4.32}$$

where $\phi_{initial}$ and ϕ_{end} denote the initial and final values of inflaton field. To obtain the sufficient inflation one should take this number $N \geq 62$. This determines the end values of the field corresponding to different values of γ in the action ($\epsilon \sim 1$). Here we note a range of initial values of scalar field permitted to begin with to get sufficient inflation which are tabulated in Tables-(4.1) and (4.2) respectively for various Λ.

4.6 Discussions

In a modified theory of gravity considering a Lagrangian density which is a polynomial of fourth degree in Ricci scalar including a cosmological constant Λ we obtain both singular and non-singular gravitational instanton solutions that are relevant for cosmological model building. We note that the gravitational action permits de Sitter instanton solution even in the absence of a cosmological constant. We determine the sign of the coupling parameter γ with R^4 in the action relevant to a realistic instanton solution. The probability of creation of a de Sitter instanton depends on the coupling parameter γ. We note that in the case of non-singular instanton one requires either (i) $\Lambda < 0$ for $\gamma > 0$ or (ii) $\Lambda > 0$ for a γ having either a lower bound $\gamma > \frac{2}{125}$ or a negative value ($\gamma < 0$). In the conformally equivalence picture, we determine the constraints for the existence of both the singular and non-singular cosmological instantons. A singular instanton is permitted with a negative Λ when $\gamma > 0$ and with a positive Λ when $\gamma < 0$ or $\gamma > \frac{2}{125}$. We obtain interesting scalar field potentials in the conformally equivalent theory which are shown in figs. (4.1) and (4.2). The potential obtained in fig. (4.2) for $\Lambda > 0$ may be useful for obtaining Coleman-de Luccia type instantons [140] as it has both true and false vacuum. However, we discuss only the singular instantons corresponding to the potentials drawn in figs. (4.1) and (4.2) in the Euclidean gravity. The scalar field potentials are relevant for realizing singular instanton solutions. This aspect has been studied in detail in sec. 4.4. We also study the possibility of obtaining a universe with sufficient inflation after the quantum creation via gravitational instantons. The slow-roll parameters are determined and found that a sufficient inflation may be achieved for a realistic cosmology at late time. In the presence of cosmological constant we determine the range of initial values of the scalar field for a given ϕ_{end}. A suitable range of values which accommodates sufficient inflation are tabulated in Tables (4.1) and (4.2) respectively. It is evident from the Table-(4.1) that as γ increases the final value of the field decreases along with a decrease of the initial value of the field. However for a negative γ it is evident from Table-(4.2) that the final values of the field increases with an increase in $|\gamma|$. We note that a region of parameter space exists for a given γ and Λ for which the universe created after tunneling or via HT instanton admits a suitable inflationary scenario good enough to solve the horizon and the flatness problems satisfactorily. It may be pointed out here that the criterion for the first order transition and decay rate employing the negative mode of instanton has been studied by Lee *et al.* [174].

Chapter 5

Holograpahic Dark Energy Model with Modified Generalized Chaplygin Gas

5.1 Introduction

Contemporary cosmological observations have revealed a new and somewhat unexpected accelerated expansion of the universe. In the last few years, a volume of literature came out considering different types of exotic matters as possible candidates for the dark energy. Chaplygin gas is considered one such candidate for dark energy. A brief discussion on Chaplygin gas and its EoS is discussed in chapter-1. To accommodate dark energy various modified equations of state are also considered in the literature [39, 40]. However, we consider here a modified generalized Chaplygin gas (in short, MCG) [39] its equation of state is given in (1.6),

$$p = B\rho - \frac{A}{\rho^{\alpha}}.$$

The EoS of MCG has three free parameters satisfying the conditions A, $B > 0$ and $0 < \alpha \leq 1$.

Recently, holographic principle is incorporated in cosmology to track the dark energy content of the universe. The basic ideas of holographic principle were introduced by 't Hooft [69], Susskind [75] and Busso [175]. Later Cohen *et al.* [176], Li [76], Hsu [73], Huang [177], others [77, 178], employed holographic dark energy models to explain the current acceleration of the universe.

5.2 Modified Generalized Chaplygin Gas in FRW Universe

The Einstein's field equation is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\, T_{\mu\nu}, \tag{5.1}$$

where $R_{\mu\nu}$ is the Ricci tensor which depends on the metric and its derivatives, R is the Ricci scalar and $T_{\mu\nu}$ is the energy momentum tensor. We consider Robertson-Walker metric given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right], \tag{5.2}$$

where $a(t)$ is the scale factor of the universe with cosmic time t. The coordinates r, θ and ϕ are known as comoving coordinates. Einstein field equation allows one to determine the scale factor provided the matter content of the universe is specified. The constant k in the metric (5.2) describes the geometry of the spatial section of space time, with closed, flat and open universes corresponding to $k = +1, 0, -1$ respectively. Using the metric (5.2) and the energy momentum tensor for isotropic fluid $T^{\mu}_{\mu} = (-\rho,\ p,\ p,\ p)$, where ρ and p are energy density and pressure respectively, the Einstein's field equation (5.1) can be written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\rho, \tag{5.3}$$

where $M_P^2 = 8\pi G$ and H is the Hubble parameter. The energy conservation equation is

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0. \tag{5.4}$$

Using the energy conservation equation for modified generalized Chaplygin gas given by eq. (1.6), we determine the energy density

$$\rho = \left(\frac{A}{1+B} + \frac{C}{a^{3(1+B)(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}, \tag{5.5}$$

where C is an arbitrary integration constant. Let us now define the following

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}},\ \Omega_m = \frac{\rho_m}{\rho_{cr}},\ \Omega_k = \frac{k}{a^2H^2} \tag{5.6}$$

where $\rho_{cr} = 3M_P^2H^2$, Ω_Λ, Ω_m and Ω_k represents density parameter corresponding to Λ, matter and curvature respectively. We assume here that the origin of dark energy is a scalar field. In this case, using Barrow's scheme [78], we determine the energy density and pressure as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{1}{\alpha+1}}, \tag{5.7a}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-\frac{A}{B+1} + B\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}, \tag{5.7b}$$

where $n = 3(1+B)(1+\alpha)$. It is now simple to derive the scalar field potential and its kinetic energy term, which are given by

$$V(\phi) = \frac{\frac{A}{B+1} + \frac{1-B}{2}\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}, \tag{5.8}$$

$$\dot{\phi}^2 = \frac{(B+1)\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}. \tag{5.9}$$

5.3 Holographic Dark Energy in MCG

In a FRW universe we now consider a non-flat universe with $k \neq 0$ and use the holographic dark energy density which is

$$\rho_\Lambda = 3c^2M_P^2L^{-2}, \tag{5.10}$$

where c is the speed of light and L is the cosmological length scale for tracking the field corresponding to holographic dark energy in the universe. The parameter L is defined as

$$L = a(t)r(t), \tag{5.11}$$

where $r(t)$ is relevant to the future event horizon of the universe. Using Robertson-Walker metric one gets [178]

$$L = \begin{cases} \frac{a(t)\sin\left[\sqrt{|k|}R_h(t)/a(t)\right]}{\sqrt{|k|}}, & \text{for k=+1} \\ R_h(t), & \text{for k=0} \\ \frac{a(t)\sinh\left[\sqrt{|k|}R_h(t)/a(t)\right]}{\sqrt{|k|}}, & \text{for k=-1} \end{cases} \tag{5.12}$$

where R_h represents the event horizon which is given by

$$R_h = a(t)\int_t^\infty \frac{dt'}{a(t')} = a(t)\int_o^{r_1} \frac{dr}{\sqrt{1-kr^2}}. \tag{5.13}$$

Here R_h is measured in r direction and L represents the radius of the event horizon measured on the sphere of the horizon. Using the definition of $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_P^2H^2$, one can derive [177]

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}}. \tag{5.14}$$

Using eqs.(5.12)-(5.13), we determine the rate of change of L with respect to t and it is given by

$$\dot{L} = \begin{cases} \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}}\cos\left(\frac{\sqrt{|k|}R_h}{a(t)}\right), & \text{for k=+1} \\ \frac{c}{\sqrt{\Omega_\Lambda}} - 1, & \text{for k=0} \\ \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}}\cosh\left(\frac{\sqrt{|k|}R_h}{a(t)}\right), & \text{for k=-1} \end{cases} \tag{5.15}$$

Using eqs. (5.10)-(5.15), it is possible to construct the required equation for the holographic energy density ρ_Λ, which is given by

$$\frac{d\rho_\Lambda}{dt} = -2H\left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X)\right]\rho_\Lambda, \tag{5.16}$$

where we use the notation

$$f(X) = \frac{1}{\sqrt{|k|}} cosn(\sqrt{|k|}X) = \begin{cases} \cos(X), & \text{for k=+1} \\ 1, & \text{for k=0} \\ \cosh(X), & \text{for k=-1} \end{cases} \tag{5.17}$$

and $X = \frac{R_h}{a(t)}$. The energy conservation equation becomes

$$\frac{d\rho_\Lambda}{dt} + 3H(1+\omega_\Lambda)\rho_\Lambda = 0, \tag{5.18}$$

which is used to determine the the equation of state parameter

$$\omega_\Lambda = -\left(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X)\right). \tag{5.19}$$

Let us now establish the correspondence between the holographic dark energy and the modified generalized Chaplygin gas energy density. The corresponding energy density may be obtained using the equation of state given by eq. (5.5). Again the equation of state parameter using eq. (1.6) can also be re-written as

$$\omega = \frac{p}{\rho} = B - \frac{A}{\rho^{\alpha+1}}. \tag{5.20}$$

Hence we can determine dark energy fields using eqs.(5.7), (5.19) and (5.20). We obtain

$$A = (3c^2 M_P^2 L^{-2})^{\alpha+1}\left[B + \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X)\right], \tag{5.21a}$$

$$C = (3c^2 M_P^2 L^{-2})^{\alpha+1} a^n \left[1 - \frac{3B+1}{3(B+1)} - \frac{2\sqrt{\Omega_\Lambda}}{3(B+1)c} f(X)\right], \tag{5.21b}$$

The scalar field potential becomes

$$V(\phi) = 2c^2 M_P^2 L^{-2}\left[1 + \frac{\sqrt{\Omega_\Lambda}}{2c} f(X)\right], \tag{5.22}$$

and the corresponding kinetic energy of the field is given by

$$\dot{\phi}^2 = 2c^2 M_P^2 L^{-2}\left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X)\right]. \tag{5.23}$$

Considering $X(= \ln a)$, we transform the time derivative to the derivative with logarithm of the scale factor, which is the most useful function in this case. We get

$$\phi' = M_P \sqrt{2\Omega_\Lambda \left(1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X)\right)} \tag{5.24}$$

where $()'$ prime represents derivative with respect to x. Thus the evolution of the scalar field is given by

$$\phi(a) - \phi(a_o) = \sqrt{2} M_P \int_{\ln a_o}^{\ln a} \sqrt{\Omega_\Lambda \left(1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X)\right)} dx. \tag{5.25}$$

5.4 Squared Speed for Holographic Dark Energy Fluid

The dark energy equation of state parameter given by eq. (5.19) reduces to

$$\omega_\Lambda = -\frac{1}{3}\left(1 + \frac{2}{c}\sqrt{\Omega_\Lambda}\cos X\right), \tag{5.26}$$

where $y = \frac{R_H}{a(t)}$. The minimum value it can take is $\omega_{min} = -\frac{1}{3}\left(1 + 2\sqrt{\Omega_\Lambda}\right)$ and one obtains a lower bound $\omega_{min} = -0.9154$ for $\omega_\Lambda = 0.76$ with $c = 1$. Considering a variation of the state parameter with respect to $y = \ln a$, one obtains [177]

$$\omega'_\Lambda = -\frac{\sqrt{\Omega_\Lambda}}{3c}\left[\frac{1 - \Omega_\Lambda}{1 - \gamma a} + \frac{2\sqrt{\Omega_\Lambda}}{c}\left(1 - \Omega_\Lambda \cos^2 X\right)\right]. \tag{5.27}$$

Let us now introduce the squared speed of holographic dark energy fluid as

$$v_\Lambda^2 = \frac{dp_\Lambda}{d\rho_\Lambda} = \frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda} = \frac{p'_\Lambda}{\rho'_\Lambda}, \tag{5.28}$$

where the variation of eq. (5.20) with respect to X is given by

$$p'_\Lambda = \omega'_\Lambda \rho_\Lambda + \omega_\Lambda \rho'_\Lambda. \tag{5.29}$$

Using the eqs.(5.27) and (5.29) we get

$$v_\Lambda^2 = \omega'_\Lambda \frac{\rho_\Lambda}{\rho'{}_\Lambda} + \omega_\Lambda$$

which now becomes

$$v_\Lambda^2 = -\frac{1}{3} - \frac{2}{3c}\sqrt{\Omega_\Lambda}\cos X + \frac{1}{6c}\sqrt{\Omega_\Lambda}\left[\frac{\frac{1-\Omega_\Lambda}{1-\gamma a} + \frac{2}{c}\sqrt{\Omega_\Lambda}\left(1-\Omega_\Lambda\cos^2 X\right)}{1-\frac{\Omega_\Lambda}{c}\cos X}\right]. \tag{5.30}$$

The variation of v_Λ^2 with Ω_Λ is shown in fig. (5.1) for different X values. It is found that for a given value of c, a, γ, the model admits a positive squared speed for $\Omega_\Lambda > 0$. However, Ω_Λ is bounded below otherwise instability develops. We note also that for $\frac{(2n+1)\pi}{2} < y < \frac{(2n+3)\pi}{2}$, (where n is an integer) no instability develops. Fig. (5.1) is plotted for $n = 0$, it is evident that for $y \leq \frac{\pi}{2}$ and $y \geq \frac{3\pi}{2}$, the squared speed for holographic dark energy becomes negative which led to instability. But for the region $\frac{\pi}{2} < y < \frac{3\pi}{2}$ with $n = 0$ no such instability exists. It is also found that for $y = 0$ *i.e.*, in flat case the holographic dark energy model is always unstable [179].

5.5 Discussions:

Holographic dark energy model in an FRW universe is studied with a scalar field which describes the modified generalized Chaplygin gas (MCG). In the case of a flat FRW-universe, the corresponding potential for MCG is $V = \frac{1-B}{2}\ \rho + \frac{A}{\rho^\alpha}$.

When $\rho \to \infty$ i.e., $a \to 0$ one obtains the following : (i) $V(\phi) \to \infty$ for $B \neq 1$, (ii) $V(\phi) \to 0$ for $B = 1$. However, when $a \to \infty$ i.e., $\phi \to 0$, the potential asymptotically attains a constant value $(\phi) \to \left(\frac{A}{B+1}\right)^{\frac{1}{1+\alpha}}$. We obtain the evolution of the holographic dark energy field and the corresponding potential in the framework of modified GCG in a non flat universe. We note that in the closed model the holographic dark energy is stable for a restricted domain of the values of Ω_Λ. It is also observed that inclusion of a barotropic fluid in addition to modified Chaplygin gas does not alter the form of potential and evolution of the holographic dark energy field but the parameter C in the equation of state becomes proportional to a^n with $n = 3(1+B)(1+\alpha)$. Thus the contribution of the holographic dark energy is more if $(B \neq 0)$ compared to the case when one considers $B = 0$ as was obtained in [180]. It is also noted that the form of the potential does not differ on adding a barotropic fluid although it changes the overall holographic dark energy density. The holographic dark energy is found stable for a restricted domain of the values of Ω_Λ in a closed model of the universe.

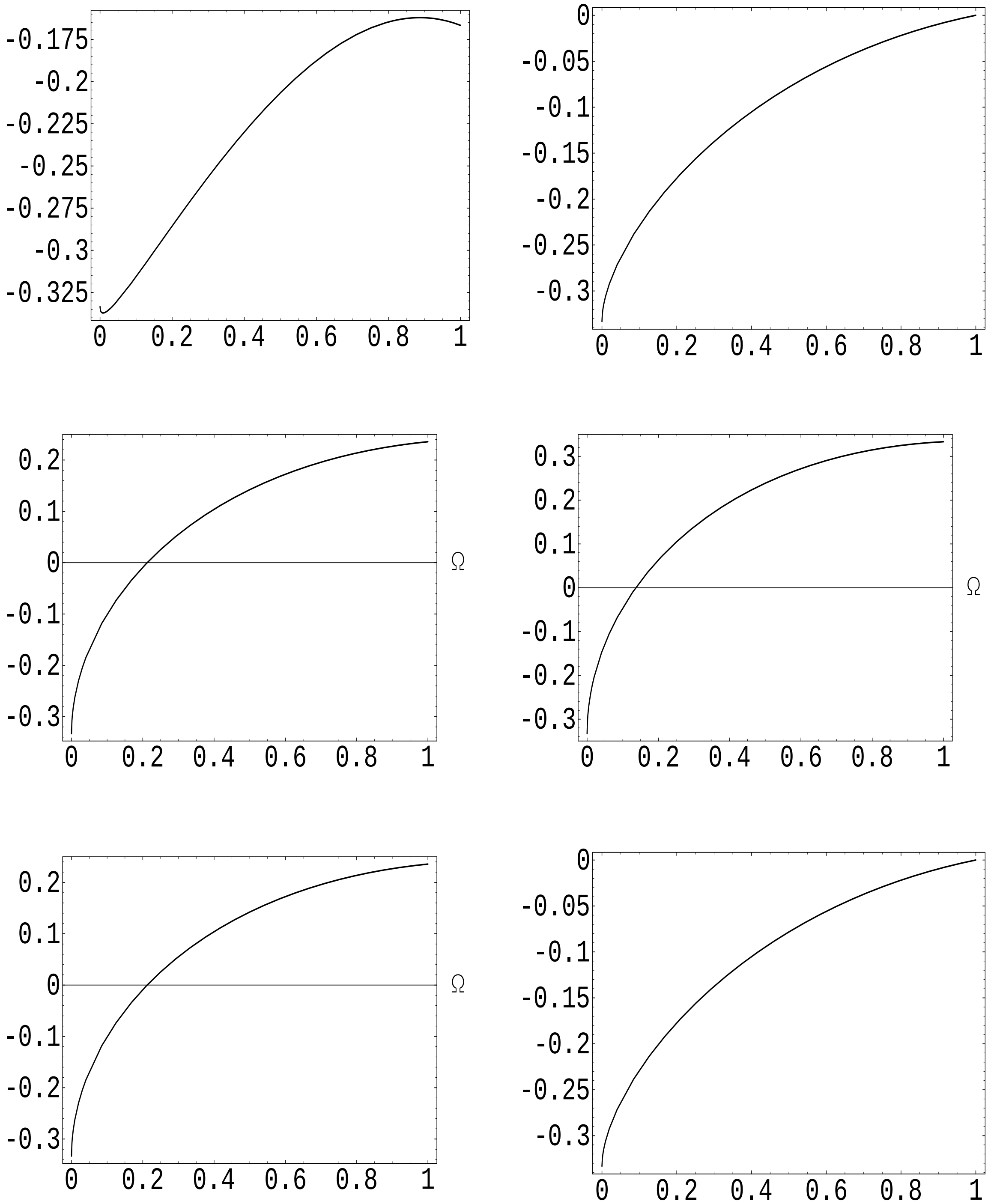


Figure 5.1: The plot of v_Λ^2 versus Ω_Λ for different values of X with $c = 1$, $\gamma = 1/3$ and $a = 1$, in the first array the figures are for $y = \frac{\pi}{3}$ and $y = \frac{\pi}{2}$, in the second array for $y = \frac{1.5\,\pi}{2}$, $y = \pi$ and in the third array for $y = \frac{2.5\,\pi}{2}$, $y = \frac{3\pi}{2}$.

Chapter 6

Hořava-Lifshitz Gravity with Modified Chaplygin Gas and Constraints on its B Parameter

6.1 Introduction

Recently Hořava proposed a theory of gravity, which is renormalizable in the ultraviolet (UV) region, with higher spatial derivatives in the context of four dimensional quantum gravity [98, 99]. Due to it's novel features, there has been a large amount of effort in examining and extending the theory itself in different areas of theoretical physics. Apart from the foundational and conceptual issues of Horava-Lifshitz (henceforth, HL) gravity and its associated cosmology, many interesting cosmological implications are found in HL gravity. Cosmological models with generalized Chaplygin gas (GCG) are introduced to address the recent acceleration of the universe [181]. GCG behaves as a pressureless fluid at the early stages of evolution of the universe, and as a cosmological constant at late times. Recently, a modified form of Chaplygin gas has been considered in cosmology which is suitable for describing dark energy [182]. The modified Chaplygin gas (MCG) is more general and contains three free parameters. The idea is to interpolate states of standard fluids at high pressures and at high energy densities to a constant negative pressure at low energy densities [183]. In this chapter we construct the scenario of MCG in HL gravitational background and determine the limits of the parameters that are permitted by observations. The equation of state parameter of the total cosmic fluid defined by, $w(z) = \frac{p_{tot}}{\rho_{tot}}$, is evaluated at different redshifts (z). The suitability of the cosmologies is then tested using $\mu \left[= 5\log_{10}\left(\frac{c}{H_o(1+z)}\int_0^z \frac{dz'}{E(z')}\right) + 25\right]$ vs z data for closed universe and open universe respectively which then compared with the curve obtained from union compilation data [184].

6.2 Hořava-Lifshitz Gravitational Theory:

The dynamical variables in HL gravity are similar to the Arnowitt-Deser-Misner (in short, ADM) formalism of Einstein gravity and they are namely, the lapse function N, shift vector N_i, and the spatial metric g_{ij}. In terms of these variables the metric is given by

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \tag{6.1}$$

where $N_i = g_{ij}N^j$. The scaling transformation of the co-ordinates is written as: $t \to l^3 t$ and $x^i \to l x^i$. The Einstein-Hilbert action in the ADM formalism is given by

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{g}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda), \tag{6.2}$$

where G is Newton's constant and K_{ij}, the extrinsic curvature, is defined by

$$K_{ij} = \frac{1}{2N}(\dot{g_{ij}} - \nabla_i N_j - \nabla_j N_i)$$

with covariant derivatives defined with respect to the spatial metric g_{ij}. The action of HL gravity consists of kinetic and potential terms which can be written as

$$S_g = S_K + S_V = \int dtd^3x\sqrt{g}N(L_K + L_V). \tag{6.3}$$

The former is given by

$$S_K = \int dt d^3x \sqrt{g} N \left[\frac{2(K_{ij}K^{ij} - \lambda K^2)}{\kappa^2} \right], \tag{6.4}$$

where λ is the coupling constant and $\kappa^2 = 8\pi G$ Using symmetry property of the Lagrangian L_V, it can be noted that the number of invariants, one should actually consider in the action to begin with is drastically reduced [98]. The symmetry is known as detailed balance (which follows from condensed matter systems) and it requires that the part L_V should be derivable from a superpotential W [185]. The potential term in this detailed-balance form is given by

$$S_L = \int dt d^3x \sqrt{g} N E_{ij} \mathcal{G}_{ijkl} E^{kl}, \tag{6.5}$$

where the inverse of the 'generalized De Witt metric' or 'super-metric' $\mathcal{G}^{ijkl}$ is written as

$$\mathcal{G}_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk}) + \frac{\lambda}{1-3\lambda} g_{ij}g_{kl}, \tag{6.6}$$

and it depends on an arbitrary real dimensionless coupling constant λ. E^{ij} can be defined by

$$E^{ij} = \frac{2}{w^2}C^{ij} - \mu \left(R^{ij} - \frac{1}{2}Rg^{ij} + \Lambda_w g^{ij} \right), \tag{6.7}$$

where

$$C^i_j = \nabla_k \left(R_{li} - \frac{Rg_{li}}{4} \right) \epsilon^{klj}$$

is the Cotton tensor and the covariant derivatives are determined with respect to the spatial metric g_{ij}, ϵ^{ijk} is a totally antisymmetric unit tensor. The variables κ, w and μ are constants. Under the detailed balance condition the full action of HL gravity is given by

$$S_g = \int dtd^3x\sqrt{g}N\left[\frac{2(K_{ij}K^{ij}-\lambda K^2)}{\kappa^2}+\frac{\kappa^2 C_{ij}C^{ij}}{2w^4}\right.$$
$$-\frac{\kappa^2\mu\epsilon^{ijk}R_{il}\nabla_j R^l_k}{2w^2\sqrt{g}}+\frac{\kappa^2\mu^2 R_{ij}R^{ij}}{8}$$
$$\left.-\frac{k^2\mu^2}{8(3\lambda-1)}\left[\frac{(1-4\lambda)R^2}{4}+\Lambda R-3\Lambda^2\right]\right]. \tag{6.8}$$

In order to incorporate the matter components one needs to add a cosmological stress-energy tensor to the gravitational field equations, that recovers the usual general relativity formulation in the low-energy limit [100, 186, 111]. This matter-tensor is a hydrodynamical approximation that leads to the existence of ρ_m and p_m in Friedmann equation. Here ρ_m, represents the total matter energy density, that accounts for both the baryonic ρ_b as well as that of the dark matter ρ_{dm}, including the normal matter and p_m represents pressure. In the simplest version of the theory in [98] the lapse function is assumed as just a function of time *i.e.*, $N = N(t)$. Setting aside this '*projectability*' restriction, one can assume the homogeneous and isotropic universe described by the FRW metric with $N = 1$, $g_{ij} = a^2(t)\gamma_{ij}$, $N^i = 0$, where

$$\gamma_{ij}dx^i dx^j = \frac{dr^2}{1-kr^2}+r^2 d\Omega_2^2. \tag{6.9}$$

Here $k = -1, 1, 0$, corresponds to open, closed and flat universe respectively. Varying N and g_{ij}, one obtains the Friedmann equations:

$$\begin{aligned} H^2 &= \frac{\kappa^2}{6(3\lambda-1)}(\rho_m+\rho_r) \\ &+\frac{\kappa^2}{6(3\lambda-1)}\left[\frac{3\kappa^2\mu^2k^2}{8(3\lambda-1)a^4}+\frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right] \\ &-\frac{\kappa^4\mu^2\Lambda k}{8(3\lambda-1)^2a^2}, \end{aligned} \tag{6.10}$$

$$\begin{aligned}\dot{H}+\frac{3H^2}{2} &= -\frac{\kappa^2}{4(3\lambda-1)}(\rho_m\omega_m+\rho_r\omega_r)\\ &-\frac{\kappa^2}{4(3\lambda-1)}\left[\frac{\kappa^2\mu^2k^2}{8(3\lambda-1)a^4}-\frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right]\\ &-\frac{\kappa^4\mu^2\Lambda k}{16(3\lambda-1)^2a^2},\end{aligned} \tag{6.11}$$

where $H=\frac{\dot{a}}{a}$. In the above, the term proportional to a^{-4} may be considered as the usual "dark radiation term", characteristics of the HL Cosmology [185, 187] and Λ is the cosmological constant. The conservation equation for matter is:

$$\dot{\rho_m}+3H(\rho_m+p_m)=0, \tag{6.12}$$

and that for radiation is:

$$\dot{\rho_r}+3H(\rho_r+p_r)=0. \tag{6.13}$$

We consider here matter and radiation without interaction. One can set the following expressions in order to require the equation (6.10) similar to that of the standard Friedmann equations:

$$G_{cosmo}=\frac{\kappa^2}{16\pi(3\lambda-1)}, \tag{6.14a}$$

$$\frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)^2}=1, \tag{6.14b}$$

$$G_{grav}=\frac{\kappa^2}{32\pi}. \tag{6.14c}$$

G_{cosmo} is the cosmological Newton's constant, G_{grav} is the gravitational Newton's Constant.

6.3 EoS for Modified Chaplygin Gas:

Recently the modified Chaplygin gas (MCG) is widely used in cosmology [182]. It contains one more free parameter (B) than that of GCG. The corresponding cosmological model with MCG is consistent with Gravitational lensing test [188, 189] and Gamma-ray bursts [190]. The equation of state (in short, EoS) for the MCG as in (1.6) is:

$$p = B\rho - \frac{A}{\rho^{\alpha}},$$

where A, B, α are positive constants with $0 \leq \alpha \leq 1$. Using equation for energy conservation the energy density of MCG is given by

$$\rho = \left[\frac{A}{1+B} + \frac{C}{a^{3n}}\right]^{\frac{1}{1+\alpha}}, \tag{6.15}$$

where C is an arbitrary constant and $n = (1+B)(1+\alpha)$. The above equation can also be written as

$$\rho = \rho_0 \left[A_S + \frac{1-A_S}{a^{3n}}\right]^{\frac{1}{1+\alpha}}, \tag{6.16}$$

where $A_S = \frac{A}{1+B}\frac{1}{\rho_0^{\alpha+1}}$ and $\frac{a}{a_0} = \frac{1}{1+z}$, z is redshift parameter. We choose $a_0 = 1$ for convenience. The above MCG model reduces to GCG model when one set $B = 0$.

6.4 Observational Constraints on EoS Parameters:

In GTR the cosmological scenario with MCG is studied and the probable constraints on EoS parameters have been determined using observational data [191, 192]. We consider here cosmological models with MCG in the Horava-Lifshitz gravity and determine the EoS parameters using the recent observational data. For this we use data from Observed Hubble Data (OHD), BAO peak parameter and CMB shift parameter.

6.4.1 Constraints Obtained from Detailed Balance

Using eq. (6.14), the Friedmann equation can be rewritten as:

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_c + \rho_r) + \left[\frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2}\right] - \frac{k}{a^2}, \tag{6.17}$$

$$\dot{H} + \frac{3}{2}H^2 = -4\pi G(p_c + \frac{1}{3}\rho_r) - \left[\frac{k^2}{4\Lambda a^4} - \frac{3\Lambda}{4}\right] - \frac{k}{2a^2}, \tag{6.18}$$

where ρ_b, ρ_c and ρ_r represent energy densities of baryon, MCG and radiation respectively. We use the following dimensionless density parameters:

- for matter component

$$\Omega_i \equiv \frac{8\pi G}{3H^2}\rho_i$$

- for curvature

$$\Omega_k \equiv -\frac{k}{H^2 a^2}$$

- for cosmological constant

$$\Omega_0 \equiv \frac{\Lambda}{2H_0^2}$$

We also use another dimensionless expansion rate which is defined as:

$$E(z) \equiv \frac{H(z)}{H_0}. \tag{6.19}$$

Using the above parameters the Friedmann equation (6.17) can be rewritten as

$$\begin{aligned} E^2(z) &= \Omega_{b0}(1+z)^3 + \Omega_{c0}F(z) + \Omega_{r0}(1+z)^4 \\ &\quad +\Omega_{k0}(1+z)^2 + [\Omega_0 + \frac{\Omega_{k0}^2(1+z)^4}{4\Omega_0}] \end{aligned} \tag{6.20}$$

where

$$F(z) = \left[A_S + \frac{1-A_S}{a^{3(1+B)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{6.21}$$

At the present epoch $E(z=0)=1$, which leads to

$$\Omega_{b0} + \Omega_{c0} + \Omega_{r0} + \Omega_{k0} + \Omega_0 + \frac{\Omega_{k0}^2}{4\Omega_0} = 1 \tag{6.22}$$

where Ω_{b0}, Ω_{c0}, Ω_{r0}, Ω_{k0} represent the present day baryon, MCG, radiation, curvature energy density respectively. Here Ω_0 is the energy density associated with the cosmological constant. The last term in eq. (6.22) corresponds to dark radiation, which is a characteristic feature of the Horava-Lifshitz theory of gravity. The dark radiation component may be important during nucleosynthesis. Thus a suitable bound from Big Bang Nucleosynthesis(BBN) may be incorporated in the EoS. If the upper limit on the total amount of Horava-Lifshitz dark radiation that permits during BBN era is expressed by the parameter ΔN_ν, which represents the effective neutrino species [193, 194], then one obtains a constraint [112] which is given by:

$$\frac{\Omega_{k0}^2}{4\Omega_0} = 0.135\Delta N_\nu \Omega_{r0}. \tag{6.23}$$

The BBN upper limit on ΔN_ν is $-1.7 \leq \Delta N_\nu \leq 2.0$, following [194, 195]. A negative value of ΔN_ν is usually associated with models involving decay of a massive particles which we have not considered here. Again $\Delta N_\nu = 0$ corresponds to the zero curvature scenario (a non-interesting case since Horava-Lifshitz cosmology with zero curvature becomes indistinguishable from ΛCDM [187, 185]) which is not considered here. Thus ΔN_ν satisfies an inequality $0 \leq \Delta N_\nu \leq 2.0$.

For the numerical analysis in the model, the following parameters namely, Ω_{b0}, Ω_{c0}, Ω_{r0}, Ω_{k0}, Ω_0, ΔN_ν, H_0, A_S, B, α are important. We fix some of the parameters using the best-fit values from 7 year WMAP data [196]. The fixed parameters are $\Omega_{m0}(\equiv \Omega_{b0} + \Omega_{c0})$, Ω_{b0}, H_0, Ω_{r0} and the corresponding values of the parameters are chosen as follows: $\Omega_{m0} = 0.27$, $\Omega_{b0} = 0.04$, $H_0 = 71.4\ Km/sec/Mpc$, $\Omega_{r0} = 8.14 \times 10^{-5}$. Therefore it is necessary to determine six free parameters namely Ω_{k0}, Ω_0, A_S, B,α, ΔN_ν. Using equation (6.23) in equation (6.22), one obtains

$$\begin{aligned}\Omega_0(k, \Delta N_\nu, A_S, \alpha) = 1 - \Omega_{m0} - (1 - 0.135\Delta N_\nu)\Omega_{r0} \\ -0.73(k)\sqrt{\Delta N_\nu}\sqrt{\Omega_{r0} - \Omega_{m0}\Omega_{r0} - \Omega_{r0}^2}\end{aligned} \tag{6.24}$$

and

$$\Omega_{k0}(\Delta N_\nu, A_S, \alpha) = \sqrt{0.54\Delta N_\nu \Omega_{r0}\Omega_0(k, \Delta N_\nu, A_S, \alpha)} \tag{6.25}$$

which may be used for closed and open universe depending on the value of k. It reduces to determine four free parameters namely, A_S, B, α, ΔN_ν. To determine the constraints on the parameters of the MCG, we have taken three different values of α (α=0.999, 0.500, and 0.001) within the range $0 \leq \alpha \leq 1$ for both closed and open universe. For each of these values of α we determine the best-fit values of the rest three free parameters (*i.e.*, A_S, B, ΔN_ν). Then at the best-fit values of ΔN_ν, we plot contours for the parameters A_S, B at different confidence limits for a given α. From the contours of A_S, B drawn at different values of α and best-fit ΔN_ν it is possible to determine range of values of the B-parameter of the MCG in HL gravity for open and close universe.

6.5 Numerical Analysis to Determine Constraints on the EoS Parameters

We use three types of data to constrain the parameters of the MCG. Stern data set for $(H-z)$-data has been used along with BAO peak parameter and CMB shift parameter. The Chi-square minimization technique has been used here to determine the constraints.

A. H-z data as a tool for constraining

The best-fitted parameters of the model considered here can be obtained by minimizing the chi-square function which is defined as

$$\chi^2_{OHD}(H_0, A_S, B, \alpha, z) = \sum \frac{(H(H_0, A_S, B, \alpha, z) - H_{obs}(z))^2}{\sigma_z^2} \tag{6.26}$$

where $H_{obs}(z)$ is the observed Hubble parameter at redshift z and σ_z^2 is the associated error with that particular observation. Here $(H(z)-z)$ data is obtained from Stern Data analysis [197]. 12 data points of $H(z)$ at redshift z are used to constrain EoS for the MCG.

B. BAO Peak Parameter as a tool for constraining

A model independent Baryon Acoustic Oscillation (in short, BAO) peak parameter can be defined for low redshift (z_1) measurements as

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{\left[E\left(z_1\right)\right]^{\frac{1}{3}}}\left[\frac{\int_0^{z_1}\frac{dz}{E(z)}}{z_1}\right]^{\frac{2}{3}} \tag{6.27}$$

where Ω_m is the matter density parameter for the Universe. A detailed description of the above defined parameter and related approximations can be found in [198]. The *chi*- square function in this case can be defined as

$$\chi^2_{BAO} = \frac{\left(\mathcal{A} - 0.469\right)^2}{\left(0.017\right)^2} \tag{6.28}$$

Here we have used the measured value for $\mathcal{A} = 0.469 \pm 0.017$ as obtained in [198] from the Sloan Digital Sky Survey (in short, SDSS) data for Luminous Red Galaxies (in short, LRG) survey.

C. CMB Shift Parameter as a tool for constraining

Here the CMB shift parameter is defined as

$$R = \sqrt{\Omega_m}\int_0^{z_{ls}}\frac{dz}{E(z)} \tag{6.29}$$

where z_{ls} is the z at last scattering. The WMAP7 data gives us $R = 1.726 \pm 0.018$ at $z = 1091.3$ [196]. The Chi-square function for this case is defined as

$$\chi^2_{CMB} = \frac{(R - 1.726)^2}{(0.018)^2} \tag{6.30}$$

Joint analysis with $(H - z) + BAO + CMB$

We perform a likelihood analysis using the above three data. The total Chi-square function for joint analysis is defined as

$$\chi^2_{tot} = \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{CMB} \tag{6.31}$$

Model	B	A_S	ΔN_ν
$\alpha = 0.999$	0.003745	0.062817	0.232994
$\alpha = 0.500$	0.016592	0.110548	0.099996
$\alpha = 0.001$	0.006192	0.052076	0.807051

Table 6.1: Best-fit values of MCG parameters in closed universe ($k = 1$)

Model	A_S	ΔN_ν
$\alpha = 0.999$	0.013749	0.066159
$\alpha = 0.500$	0.010766	0.080758
$\alpha = 0.001$	0.007317	0.076100

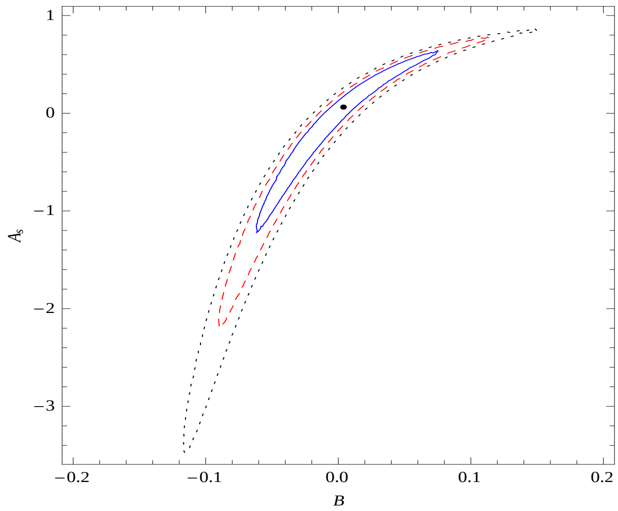
Table 6.2: Best-fit values for GCG ($B = 0$) parameters in closed universe ($k = 1$)

The statistical analysis with χ^2_{tot} gives the bounds on the parameter of the model specially on B.

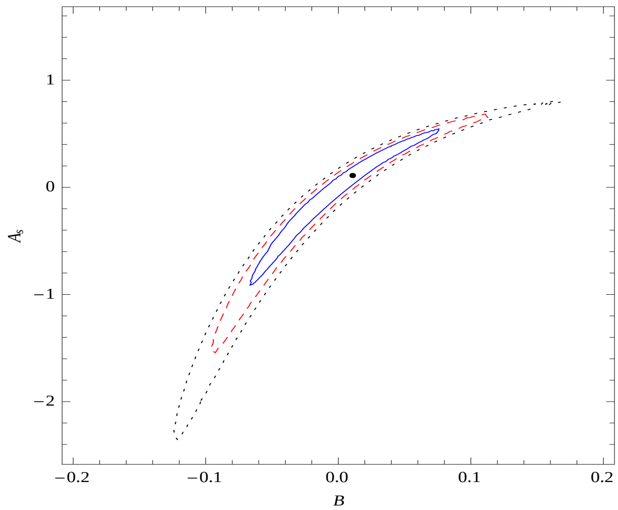
In fig. (6.1) we present likelihood contours between A_S, B for closed universe and that for open universe in fig. (6.2). They are drawn for three different values of α (α=0.999, 0.500, and 0.001) at the best-fit value of ΔN_ν. The best-fit values of the parameters of MCG are presented in Table-(6.1) for an open universe and in Table-(6.3) for a closed universe. Similarly the best-fit values of the parameters of GCG for an open universe and a closed universe are presented in Table-(6.2) and Table-(6.4) respectively. We note the following for closed universe:

- Fig. (6.1(a)) is plotted for $\alpha = 0.999$ for closed universe. We note that at 66.7% confidence limit the acceptable range for B lies from -0.06297 to 0.07725 and that for A_S lies from -1.206 to 0.6321. For 95.5% confidence limit, the bound on B widens and it takes values from -0.08998 to 0.1146 and A_S takes -2.175 to 0.7623. At 99.73% the same is from -0.1144 to 0.1531 for B and -3.478 to 0.8347 for A_S. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.232994 (Table-(6.1)).

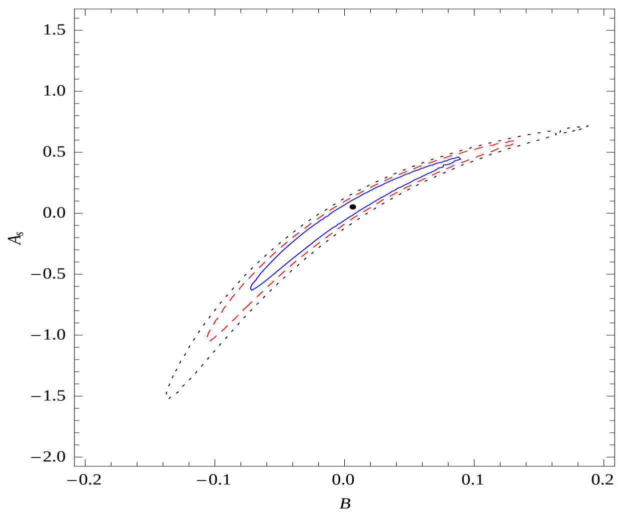
- From fig. (6.1(b)) which is plotted for $\alpha = 0.5000$ for closed universe, we note that the acceptable range for B at 66.7% confidence limit is from -0.0654 to 0.07854, for A_S it is from -0.9372 to 0.5396. At 95.5% confidence limit, the acceptable range for B lies between -0.09513 to 0.1158 and for A_S that is in between -1.544 to 0.6846. At 99.73% the same is from -0.1234 to 0.1673



(a) For $\alpha = 0.999$



(b) For $\alpha = 0.500$



(c) For $\alpha = 0.001$

Figure 6.1: Constraints for a closed universe from OHD + SDSS + CMB Shift data [66.7%, 95.5% and 99.73% likelihood contours are represented by solid, dashed and dotted line respectively].

Model	B	A_S	ΔN_ν
$\alpha = 0.999$	0.007498	0.107866	0.100055
$\alpha = 0.500$	0.010499	0.110576	0.100002
$\alpha = 0.001$	0.016478	0.114298	0.100005

Table 6.3: Best-fit values of MCG parameters in open universe ($k = -1$)

Model	A_S	ΔN_ν
$\alpha = 0.999$	0.016779	0.085888
$\alpha = 0.500$	0.013136	0.100004
$\alpha = 0.001$	0.009011	0.099961

Table 6.4: Best-fit values for GCG ($B = 0$) parameters in open universe ($k = -1$)

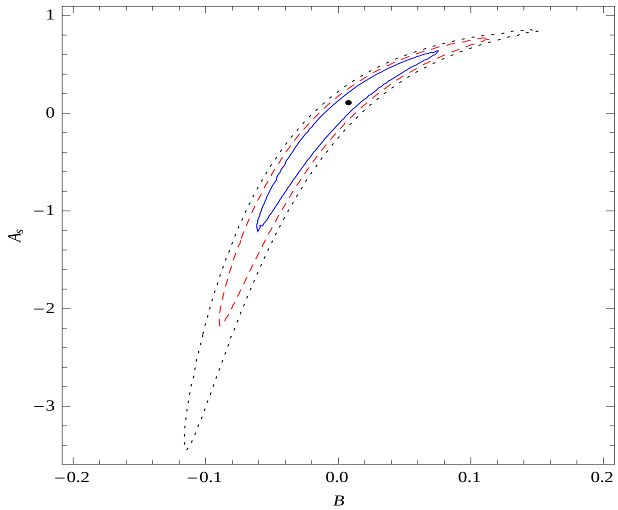
for B and from -2.374 to 0.7901 for A_S. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.0999996 (Table-(6.1)).

- Fig. (6.1(c)) is plotted for $\alpha = 0.001$ for closed universe. It is evident that the acceptable limit for B we get at 66.7% confidence limit, is -0.07244 to 0.08984. For A_S, the above range is from -0.6392 to 0.4532. At 95.5% confidence limit, the values of B lies between -0.1081 to 0.1334 and A_S is in between -1.043 to 0.5838. At 99.73% the acceptable range lies between -0.1358 to 0.1901 for B and -1.53 to 0.7025 for A_S. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.807051 (Table-(6.1)).

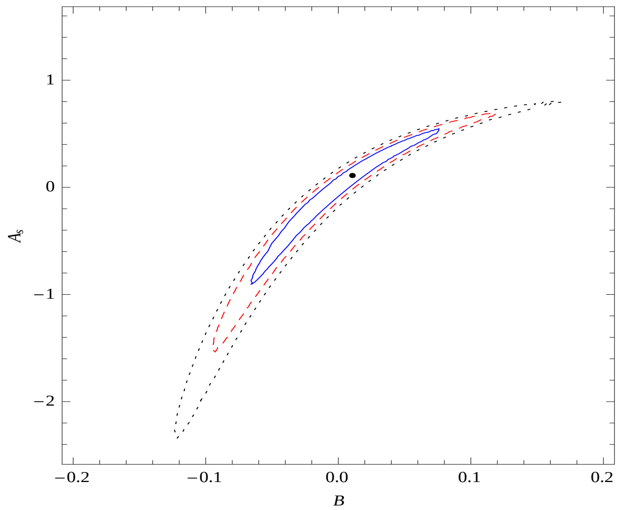
Hence for closed universe considering all the above figures, we note that at 66.7% confidence limit the acceptable range for B lies between -0.07244 to 0.08984 and the parameter A_S lies in the range -1.206 to 0.6321. At 95.5% confidence limit, B lies in the range -0.1081 to 0.1334 and A_S is in the range -2.175 to 0.7623. At 99.73% confidence limit, B lies in the range -0.1358 to 0.1901 with A_S is in the range -3.478 to 0.8347.

For open universe we note the following:

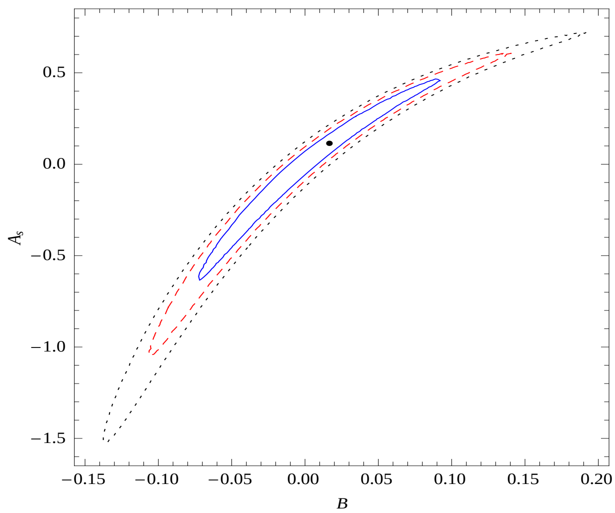
- From fig. (6.2(a)) which is plotted for $\alpha = 0.999$ for open universe scenario, we note that at 66.7% confidence limit the acceptable range for B lies between -0.0604 to 0.07725, and A_S lies between -1.191 to 0.6321, at 95.5% confidence limit B lies in the range -0.0887 to 0.1133 and A_S lies in the range -2.175 to 0.7623 and at 99.73%, B lies in the range -0.1157 to 0.1519



(a) For $\alpha = 0.999$



(b) For $\alpha = 0.500$



(c) For $\alpha = 0.001$

Figure 6.2: Constraints for open universe from OHD + SDSS + CMB Shift data [66.7%, 95.5% and 99.73% likelihood contours are represented by solid, dashed and dotted line respectively].

and A_S lies in the range -3.449 to 0.8492. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.100055 (Table-(6.3)).

- Fig. (6.2(b)) is plotted for $\alpha = 0.5000$ for open universe, in this case at 66.7% confidence limit the acceptable range for B becomes -0.06426 to 0.07725, for A_S it is -0.8977 to 0.5264. At 95.5% confidence limit B is in between -0.09513 to 0.1223 and A_S is in between -1.531 to 0.6846. At 99.73% it is -0.1234 to 0.1673 for B and -2.348 to 0.8033 for A_S. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.100002 (Table-(6.3)).

- Fig. (6.2(c)) is plotted for $\alpha = 0.001$ for open universe, we note that at 66.7% percent confidence limit, the acceptable range for B lies between -0.07117 to 0.09177, that of A_S lies between -0.6336 to 0.4598. At 95.5% confidence limit B lies between -0.1072 to 0.1430 and A_S lies between -1.049 to 0.6114. At 99.73%, B lies between -0.1387 to 0.1942 and A_S lies between -1.535 to 0.7231. The best-fit value for the effective neutrino parameter ΔN_ν is equal to 0.100005 (Table-(6.3)).

Now taking into the account all the above figures for open universe, we note that at 66.7% confidence limit, the acceptable range for B lies in the range -0.07117 to 0.09177, and A_S lies in the range -1.191 to 0.6321. At 95.5% confidence limit B lies from -0.1072 to 0.1430, and A_S lies from -2.175 to 0.7623. Again B lies in the range -0.1387 to 0.1942 and A_S lies in the range -3.449 to 0.8492 at 99.73% confidence limit.

Moreover comparing the acceptable ranges for different confidence levels obtained above for the closed and open universes, it is found that for a closed universe at 66.7% confidence limit B lies between -0.07244 to 0.08984 compared to -0.07117 to 0.09177 that in the case of an open universe. At 95.5% confidence limit B lies in the range -0.1081 to 0.1334 for a closed universe compared to that of -0.1072 to 0.1430 in the case of an open universe. For a closed universe, B, satisfies the limit $-0.1358 < B < 0.1901$ and that for an open universe it becomes $-0.1387 < B < 0.1942$ in 99.73% confidence level.

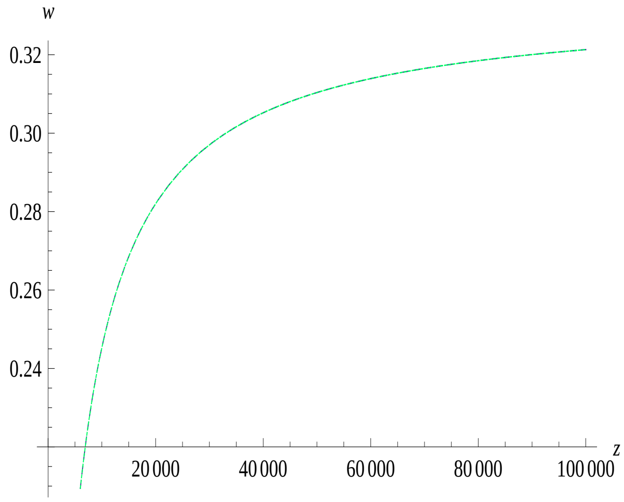


Figure 6.3: Plot of EoS parameter $\omega(z)$ vs. redshift parameter (z) for closed (dotted line) and open (solid line) universe.

6.6 Viability of MCG in HL Gravity

In the previous section of this chapter we present the cosmological model of MCG in the framework of HL theory of gravity and we determine the allowed ranges of the values of the parameter B using the recent astrophysical and cosmological observational data. Now we discuss some of the cosmological implications of this model. Specially we note that the evolution of EoS parameter of the total cosmic fluid of the universe, defined as $w(z) = \frac{p_{tot}}{\rho_{tot}}$, with the total pressure and energy density given by

$$p_{tot} = p_c + \frac{1}{3}\rho_r + \frac{2}{\kappa^2}\left[\frac{k^2}{\Lambda a^4} - 3\Lambda\right], \tag{6.32}$$

$$\rho_{tot} = \rho_c + \rho_b + \rho_r + \frac{2}{\kappa^2}\left[\frac{3k^2}{\Lambda a^4} + 3\Lambda\right]. \tag{6.33}$$

Replacing the scale factor $a(t)$ by the redshift parameter (z) and using the density parameter (Ω) and the Hubble parameter (H) as a function of redshift, one can obtain the total equation of state parameter as a function of redshift parameter i.e., $\omega(z)$.

In fig. (6.3), we plot the evolution $\omega(z)$ with z. We observe that at high redshift *i.e.*, at very early times the EoS value attains $\frac{1}{3}$ for both closed and open universe, this is expected since the radiation dominates in that epoch. In the

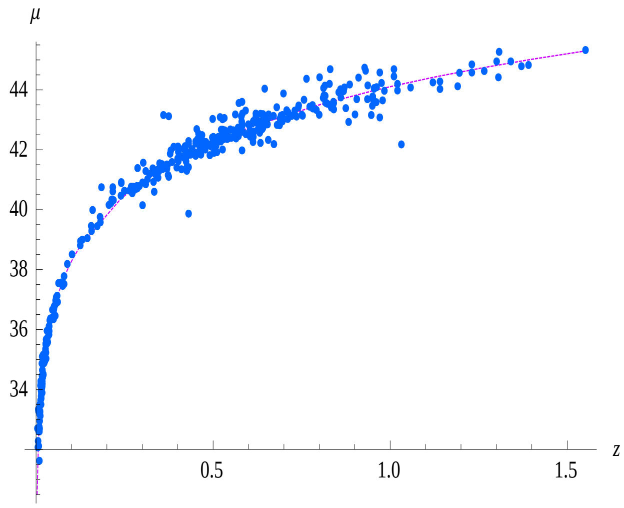


Figure 6.4: Plot of μ *vs.* z for a closed universe using μ values from the Union compilation data and the best-fit values obtained in the model.

intermediate redshift it behaves as dust for quite a long time. It is observed that state parameter picks up negative values at small redshift *i.e.*, at an epoch of very recent past. In the case of closed or open universe the present value of the state parameter is found to be negative (-0.7) that admits the present acceleration of the universe. So the evolution of EoS in the framework of HL theories of gravity is consistent with the thermal history of our universe.

To check the validity of our model we use the best-fit values of the parameters of the MCG to find supernovae magnitudes (μ) at different redshift z and plot μ vs z curves in fig. (6.6) for closed universe. We compare this with original curves of Union Compilation data [184] and observe an excellent agreement. In case of open universe, we note similar behaviour of universe.

6.7 Discussion

Cosmological models with modified Chaplygin gas in the framework of the HL gravity are investigated here. Using observational data namely, $H(z) - z$ (OHD) data taken from Stern data set, Baryon Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB), observed bound from Big Bang Nucleosynthesis

(BBN) we constrain the EoS parameters of MCG. Adopting numerical technique we determine a range of suitable values of the EoS parameter B for a given set of values of α, A_S. The analysis has been carried out here both in the cases of open and closed universe at 66.7%, 95.5% and 99.73% confidence limit. It is found that the range of parameter B in the case of a closed universe is less than that of an open universe at all confidence levels. It is also noted that the range of admissible values of B is small which may be negative as well. The negative value of B leads to MCG corresponding to exotic matter. The range of values of B obtained here is in conformity with that obtained in Ref. [199].

The variation of total equation of state parameter $w(z)$ with the redshift z in the case of open and closed universe are plotted in fig. (6.3) which is found to admit the thermal history of our universe. From the above curve it is also evident that at high redshift *i.e.*, at very early times, the equation of state parameter attains a value close to $\frac{1}{3}$, indicating radiation dominated era. At the intermediate redshifts, MCG behaves like dust and this dust dominated phase continues for quite a long period of time.

The best-fit values of the parameters of MCG both in the case of closed and open universes are presented in Tables-(6.1)-(6.3) respectively and those for GCG are presented in Tables-(6.2)-(6.4) respectively. From the tables it is evident that the values of the parameters A_S, ΔN_ν are found to be smaller in GCG model than that in MCG indicating dependence of B parameter.

The effective Neutrino parameter is then determined in MCG model as well as in GCG model. The parameter A_S which is related to the speed of sound is found to have small values. Models with MCG are found to permit positive values for B when fitted with observational data. Thus it appears from our analysis that the models with MCG is better than that with GCG in the framework of HL theory of gravity. In the model with MCG, the best-fit value for neutrino parameter ΔN_ν is found to take values less than one. The deviation of the parameter from standard value may be due to the thermal history of the universe at the epoch such as the low reheating temperature [200]. The low value of the parameter signifies that the radiation-matter equality, attained at earlier epoch, which on the other hand leads to an increase of matter energy density.

Using the best-fit values in closed universe, the supernovae magnitudes (μ) vs redshift (z) curve is plotted in fig. (6.4) and then compared with Union Compilation Data. Similar curves can be drawn in an open universe. It is found that both the cases agree quite satisfactorily. Thus cosmological models with MCG in the framework of HL gravity are successful in describing the observational predictions. The analysis carried out here shows the edge of MCG over GCG in the context of HL gravity. Earlier in the Einstein-frame, the parameters of MCG are obtained ([191, 192]) for a viable cosmological models, however, in the present case the EoS parameters of the MCG is determined in the HL-gravity using observational data. However, the present analysis does not enlighten the conceptual issues in HL-gravity. The neutrino parameter evaluated in HL-gravity with MCG is found to be very small, this feature needs to be studied in details.

Chapter 7

Kaluza-Klein Cosmology with Interacting Holographic Generalized Chaplygin Gas

7.1 Introduction

Recently the idea of Holographic Dark Energy (HDE) is incorporated in cosmology to build an interacting dark energy model. HDE is also taken up in KK cosmology [95] to construct a higher dimensional cosmological model. In this chapter we reconstruct the HDE model with Generalized Chaplygin Gas (GCG) in a flat higher dimensional universe. The GCG is considered as dark energy interacting with dark matter in the framework of compact five dimensional cosmology.

7.2 Higher Dimensional Field Equation

The Einstein field equation is given by

$$R_{AB} - \frac{1}{2}g_{AB}R = \kappa T_{AB} \tag{7.1}$$

where A and B runs from 0 to 4, R_{AB} is the Ricci tensor, R is the Ricci scalar and T_{AB} is the energy-momentum tensor. The 5-dimensional spacetime metric of

KK- cosmology is given by [201] :

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left(1-kr^2\right)d\psi^2\right], \tag{7.2}$$

where $a(t)$ denotes the scale factor and $k = 0, 1, -1$ represents the curvature parameter for flat, closed and open universe. We consider a cosmological model where KK universe is filled with a perfect fluid defined by the following energy-momentum tensor:

$$T_{AB} = (p+\rho)U_A U_B - g_{AB}p.$$

We consider p_Λ, ρ_Λ for dark energy and p_m, ρ_m for dark matter (where p denotes pressure and ρ denotes energy-density) with $p = p_\Lambda + p_m$ and $\rho = \rho_\Lambda + \rho_m$. The Einstein's field equation for the metric given by (7.2) becomes:

$$\rho = 6\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}, \tag{7.3}$$

$$p = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2} - 3\frac{k}{a^2}, \tag{7.4}$$

where $\kappa = 1$. For simplicity, we consider a flat universe, *i.e.*, $k = 0$. The above equations (eqs. (7.3) and (7.4)) reduce to:

$$\rho = 6\frac{\dot{a}^2}{a^2}, \tag{7.5}$$

$$p = -3\frac{\ddot{a}}{a} - 3\frac{\dot{a}^2}{a^2}. \tag{7.6}$$

The Hubble parameter is defined as $H = \frac{\dot{a}}{a}$. $T^{AB}_{;B} = 0$ yields the continuity equation which is given by

$$\dot{\rho} + 4H(\rho + p) = 0 \tag{7.7}$$

Using the equation of state $p = \omega\rho$, the equation of continuity becomes

$$\dot{\rho} + 4H\rho(1+\omega) = 0. \tag{7.8}$$

Let us consider two different kinds of fluids with total energy density $\rho = \rho_\Lambda + \rho_m$, where ρ_Λ corresponds dark energy and ρ_m for matter in the form of cold Dark Matter (CDM) with an EoS parameter $\omega_m = 0$. The conservation equation holds separately for p_Λ, ρ_Λ and p_m, ρ_m. As we consider here interacting dark energy, the continuity equations are modified accordingly. The equations are

$$\dot{\rho_m} + 4H\rho(1 + \omega_m) = Q \tag{7.9}$$

$$\dot{\rho_\Lambda} + 4H\rho(1 + \omega_\Lambda) = -Q \tag{7.10}$$

where Q denotes the interaction between dark energy and dark matter.

7.3 Interacting Holographic GCG model

We consider the interaction term Q of the form $Q = \Gamma\rho_\Lambda$ and denote the ratio of the energy densities with r, *i.e.*, $r = \frac{\rho_m}{\rho_\Lambda}$. This gives the decay of GCG into CDM and Λ is the decay rate. Following reference [202] we define effective EoS parameters as follows:

$$\omega_\Lambda^{eff} = \omega_\Lambda + \frac{\Gamma}{4H} \;\; and \;\; \omega_m^{eff} = -\frac{1}{r}\frac{\Gamma}{4H} \tag{7.11}$$

The continuity equations are

$$\dot{\rho_m} + 4H\rho(1 + \omega_m^{eff}) = 0, \tag{7.12}$$

$$\dot{\rho_\Lambda} + 4H\rho(1 + \omega_\Lambda^{eff}) = 0. \tag{7.13}$$

In terms of Hubble parameter the Friedmann equation in flat KK universe can be rewritten as:

$$H^2 = \frac{1}{6}\left(\rho_\Lambda + \rho_m\right), \tag{7.14}$$

where we consider $M_p^2 = 1$. Let us now define density parameters as:

$$\Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \tag{7.15}$$

where $\rho_{cr} = 6H^2$, Ω_m and Ω_Λ correspond to density parameters for matter and cosmological constant respectively. The eq. (7.14) gives:

$$\Omega_m + \Omega_\Lambda = 1. \tag{7.16}$$

Using eqs. (7.15) and (7.16), one obtains:

$$r = \frac{1 - \Omega_\Lambda}{\Omega_\Lambda} \tag{7.17}$$

The energy density of GCG is obtained as

$$\rho_\Lambda = \left[A + \frac{B}{a^{4(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}. \tag{7.18}$$

The corresponding EoS parameters are given by:

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -\frac{A}{\rho_\Lambda^{\alpha+1}} = -\frac{A}{A + \frac{B}{a^{4(1+\alpha)}}}, \tag{7.19}$$

$$\omega_\Lambda^{eff} = -\frac{A}{A + \frac{B}{a^{4(1+\alpha)}}} + \frac{\Gamma}{4H}. \tag{7.20}$$

For holographic correspondence of GCG in KK cosmology we obtain from ([95]), the energy density in a flat KK universe which is given by:

$$\rho_\Lambda = 3c^2\pi^2 L^2. \tag{7.21}$$

The infrared cutoff of the universe L in the flat KK universe is equal to the apparent horizon which coincides with Hubble horizon [203]. So we can write as in [95]:

$$r_a = \frac{1}{H} = r_H = L. \tag{7.22}$$

The decay rate is now given, as in [204],:

$$\Gamma = 4b^2(1+r)H. \tag{7.23}$$

Using eq. (7.23) in eq. (7.20) we obtain

$$\omega_\Lambda^{eff} = \frac{b^2-2}{1+\Omega_\Lambda}. \tag{7.24}$$

The correspondence between holographic dark energy and GCG in KK model enables us to write from eqs. (7.18) and (7.21):

$$\left[A + \frac{b}{a^{4(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} = 3c^2\pi^2 L^2. \tag{7.25}$$

Also from eqs. (7.20) and (7.21) one can compute A which is given by

$$A = \frac{b^2 - 2\Omega_\Lambda}{\Omega_\Lambda(1+\Omega_\Lambda)}(3c^2\pi^2 H^{-2})^{1+\alpha}. \tag{7.26}$$

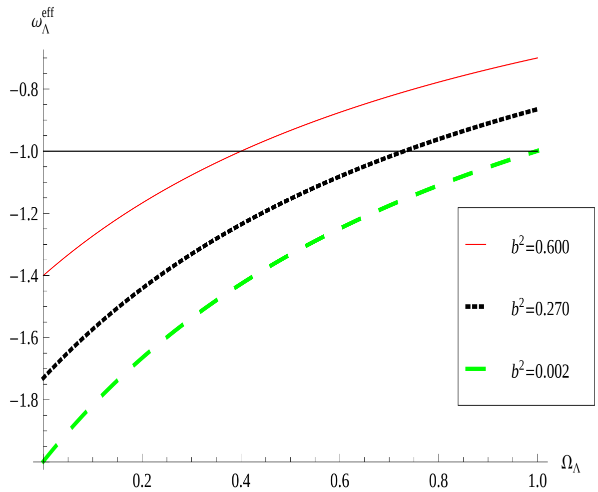
Using eq. (7.26) in eqs. (7.18) and then comparing with equation (7.21) one obtains B, which is given by

$$B = \frac{\Omega_\Lambda^2 - \Omega_\Lambda - b^2}{\Omega_\Lambda(1+\Omega_\Lambda)}(3c^2\pi^2 H^{-2}a^4)^{(1+\alpha)}. \tag{7.27}$$

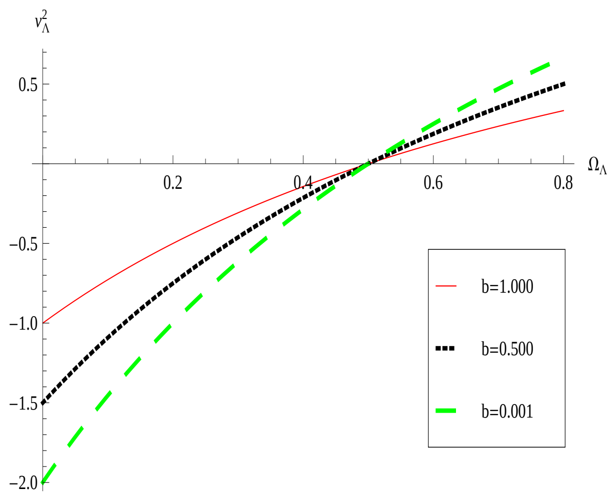
7.4 Squared Speed of Chaplygin Gas and Stability of the Model

Stability of the GCG model may be analyzed determining the squared speed of sound for GCG which is defined as

$$v_g^2 = \frac{dp_\Lambda}{d\rho_\Lambda}. \tag{7.28}$$



(a) Variation of ω_Λ^{eff}



(b) Variation of Squared speed for GCG (v_Λ)

Figure 7.1: Variation of (a) effective equation of state parameter for GCG and (b) squared speed for GCG with Ω_Λ for different choices of b^2.

The GCG model is unstable if $v_g^2 < 0$ [202]. In our model of holographic interacting GCG we obtain:

$$v_\Lambda^2 = \frac{dp_\Lambda}{d\rho_\Lambda} = \frac{\dot{p}}{\dot{\rho}} \tag{7.29}$$

In this case $\dot{p}$ is given by:

$$\dot{p} = \dot{\omega}_\Lambda^{eff} \rho_\Lambda + \omega_\Lambda^{eff} \dot{\rho_\Lambda} \tag{7.30}$$

where the over dot implies differentiation with respect to time. So the squared speed becomes:

$$v_\Lambda^2 = \omega_\Lambda^{eff} + \dot{\omega}_\Lambda^{eff} \frac{\rho_\Lambda}{\dot{\rho_\Lambda}} \tag{7.31}$$

Using equation (7.24) one obtains:

$$\dot{\omega}_\Lambda^{eff} = -\frac{b^2 - 2}{(1 + \Omega_\Lambda)^2} \dot{\Omega_\Lambda} \tag{7.32}$$

$\dot{\Omega_\Lambda}$ is obtained from the equation (7.15) and equation (7.21). Using the relation $L = \frac{1}{H}$, we get

$$\dot{\Omega_\Lambda} = -2c^2\pi^2 \frac{\dot{H}}{H^5}. \tag{7.33}$$

The expressions for $\dot{\omega}_\Lambda^{eff}$ and $\dot{\omega}_\Lambda^{eff}$ are used in eq. (7.31) which gives

$$v_\Lambda^2 = \omega_\Lambda^{eff} - \frac{c^2\pi^2(b^2 - 2)}{H^4(1 + \Omega_\Lambda)} = \omega_\Lambda^{eff}(1 - 2\Omega_\Lambda). \tag{7.34}$$

We use the observational predicted value $\Omega_\Lambda \approx 0.73$. From eq. (7.34) we determine $v_\Lambda^2 = -0.46\omega_\Lambda^{eff}$. For dark energy $\omega^{eff} < 0$ which ensures that $v_\Lambda^2 > 0$. Thus the GCG model is stable at the present epoch.

7.5 Discussion

In this chapter a holographic dark energy model is studied with an interacting generalized Chaplygin Gas (GCG) in the framework of higher dimensions. Sharif and Khanum obtained an interacting dark energy models in the framework of compact KK cosmology. It was shown that generalized second law of thermodynamics (GSLT) holds good at all time in the above model [95]. Considering interacting dark energy described by GCG it is found that GSLT is also valid. We determine the decay rate of GCG into CDM in a higher dimensional universe. For $\omega_\Lambda^{eff} = -1$ at the present epoch one obtains $b^2 = 0.27$. However, for $b^2 < 0.27$, we note that the dark energy behaves like phantom. Similarly for different values of Ω_Λ at different epochs it is possible to determine b^2. In fig. (7.1(a)) effective equation of state parameter for dark energy is plotted with Ω_Λ. It is evident that as the parameter b increases the value of ω_Λ^{eff} increases for a given Ω_Λ. Thus the dark energy might have evolved from a phantom phase at early universe. In fig. (7.1(b)) we plot the squared speed of sound with Ω_Λ for various values of b^2. The stability of the interacting HDE model is ensured from the positivity of squared speed of sound. From fig. (7.1(b)), it is evident that the squared speed of sound in the case of GCG is positive for the range of values of $\Omega_\Lambda \geq 0.5$ with a positive b^2. It is evident from fig. (7.1b) that a HDE model with interacting GCG is stable in KK cosmology for $\Omega_\Lambda \geq 0.5$ otherwise it is unstable. Thus we note a different result in a higher dimensional universe from that one obtained in GTR [202] where it is found that interacting GCG is always unstable.

Chapter 8

Cosmologies with Interacting Fluids in Modified $F(R)$ Hořava-Lifshitz Gravity

8.1 Introduction

Recent cosmological observations predict that the matter in the present universe is made up of a mixture of fluids including usual matter. It is therefore important to study cosmologies with interacting cosmic fluids. In the standard cosmological model, cold dark matter (in short, CDM), dark energy, baryon-photon fluid, neutrinos are usually considered to be decoupled. However, in the early universe these fluids might be coupled through annihilation and/or scattering processes. A number of literatures [205, 206] appeared where a detail primordial interactions of these fluids are considered. Although a cosmological model with perfect fluids without interaction is predictive, it is important to probe cosmologies containing fluids that can exchange energy among themselves. This type of interacting mixture of fluids may be important to explore the late universe particularly, the late accelerating phase of the universe [27, 207],

It is known that some of the recent observed features of the universe are not properly understood within the framework of standard cosmologies. The recent predictions from precise data from cosmological and astronomical observations have provided room for cosmologists to take up alternative theories of gravity for a viable cosmological model building. In this chapter a modified theory of gravity particularly HL gravity in the presence of interacting fluids is considered to study the evolution of the universe. For simplicity we consider a universe described by power-law expansion. In general, the power-law cosmology is described by a scale factor, $a(t) \propto t^n$. The evolution of the universe can be described by Hubble parameter $H = \frac{\dot{a}}{a}$ and deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$. An accelerated universe corresponds to $n > 1$ [208]. An expanding universe with constant velocity is represented for $n = 1$ and a contracting universe for $n < 0$. The advantage of power law expansion is that it solves satisfactorily the horizon and the flatness problems. However it is obvious from these models that inflation never ends because the slow-roll parameter(ϵ) is determined by n which is a constant [209]. Nevertheless such a scaling is a generic feature in a class of models that attempt to solve dynamically the cosmological constant problem. Still it is interesting to probe a universe with power law expansion for understanding the evolutionary history.

It is known that HL gravity reduces to that of an Einstein theory of gravity with a non-vanishing cosmological constant in the IR limit but with improved UV behaviours. A spatially-flat FRW cosmology in the framework of HL gravity brings out the fact that the background cosmology is same as that one obtains in the usual GTR. The integration constant obtained from the action in deriving the field equation may be useful here for describing dark matter [210, 211]. Chaichian *et. al.,* [111] proposed a modification of Hořava-Lifshitz gravity that can be easily related to a non-linear $F(R)$- gravity on the spatially-flat FRW background for a special choice of parameters. Cosmological model with an interacting mixture of two fluids in the framework of the modified Hořava-Lifshitz $F(R)$-gravity considering power-law expansion of the universe will be explored here.

8.2 Modified Hořava-Lifshitz $F(R)$-Gravity

The gravitational action of a standard $F(R)$ theory can be written as

$$S_{F(R)} = \int d^4x\sqrt{-g}F(R), \tag{8.1}$$

where $F(R)$ is the function of scalar curvature R. Recently Kluson [212] proposed an extension of $F(R)$-gravity to a Hořava-Lifshitz theory [98] by introducing the action given by

$$S_{F(R)} = \int d^4x\sqrt{g^{(3)}}NF(R_{HL}), \tag{8.2}$$

where $R_{HL} \equiv K^{ij}K_{ij} - \lambda K^2 - E^{ij}G_{ijkl}E^{kl}$, $\sqrt{-g} = \sqrt{g^{(3)}}$. Here λ is a real constant in the expression of inverse of the the generalized de Witt metric (6.6) which is given by

$$\mathcal{G}_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk}) + \frac{\lambda}{1-3\lambda}g_{ij}g_{kl},$$

where E^{ij} can be defined by (6.7). Assuming that the lapse function, the dynamical variable of HL gravity, is just a function of time *i.e.*, $N = N(t)$ and considering the FRW universe with a flat spatial part given by

$$ds^2 = -N^2dt^2 + a(t)^2\sum_{i=1,2,3}(dx^i)^2, \tag{8.3}$$

one can obtain the scalar curvature R, which is given by

$$R = \frac{12H^2}{N^2} + \frac{6}{N}\frac{d}{dt}\left(\frac{H}{N}\right) = -\frac{6H^2}{N} + \frac{6}{a^3N}\frac{d}{dt}\left(\frac{Ha^3}{N}\right), \quad R_{HL} = \frac{(3-9\lambda)H^2}{N^2}. \tag{8.4}$$

Chaichian *et. al.*, [111] proposed a new and modified HL-like $F(R)$ gravity given by

$$S_{F(\tilde{R})} = \int d^4x\sqrt{g^{(3)}}NF(\tilde{R}). \tag{8.5}$$

where $\tilde{R} \equiv K^{ij}K_{ij} - \lambda K^2 + 2\mu\nabla_\mu(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu) - E^{ij}G_{ijkl}E^{kl}$. In the FRW universe with flat spatial geometry, $\tilde{R}$ becomes

$$\tilde{R} = \frac{(3-9\lambda)H^2}{N^2} + \frac{6\mu}{a^3 N}\frac{d}{dt}\left(\frac{Ha^3}{N}\right) = \frac{(3-9\lambda+18\mu)H^2}{N^2} + \frac{6\mu}{N}\frac{d}{dt}\left(\frac{H}{N}\right). \quad (8.6)$$

We use the FRW equation in Hořava-Lifshitz like gravity as obtained in [111] :

$$F(\tilde{R}) - 6(1-3\lambda+3\mu)H^2 + \mu H^2 F'(\tilde{R}) + 6\mu H\frac{dF'(\tilde{R})}{dt} - \rho - \frac{C}{a^3} = 0, \quad (8.7)$$

where C is the integration constant. One can set $C = 0$, though C may not vanish always in local region [210]. In the region where $C > 0$, the last term of the above equation *i.e.,* $\frac{C}{a^3}$ may be considered as the representative of dark matter.

Now we consider matter which is composed of two different types of fluids with energy densities ρ_1 and ρ_2 whose equations of states are $p_1 = \omega_1\rho_1$, $p_2 = \omega_2\rho_1$. The two fluids are considered interacting through the interaction term Q, the corresponding conservation equations are

$$\dot{\rho_1} + 3H(\rho_1 + p_1) = Q, \quad (8.8a)$$

$$\dot{\rho_2} + 3H(\rho_2 + p_2) = -Q. \quad (8.8b)$$

In the above $Q > 0$ represents a transfer of energy from ρ_2 to ρ_1 and $Q < 0$ represents a transfer of energy from ρ_1 to ρ_2. However, non-interacting fluids are represented by $Q = 0$. In the later case the fluids satisfy the standard conservation equation separately. Using eqs. (8.8a) and (8.8b) in eqs. (8.7), it is possible to determine energy densities of the two interacting fluids which are given by

$$\rho_1 = \frac{1}{3(\omega_2-\omega_1)}\Big[3(1+\omega_2)F(\tilde{R}) + A_1F'(\tilde{R}) + B_1F''(\tilde{R}) + D_1F'''(\tilde{R}) - \frac{CH}{a^3}(2+3\omega_2)\Big], \quad (8.9)$$

where

$$
\begin{aligned}
A_1 &= \frac{1}{H}(\dot{\tilde{R}} - 6\mu\ddot{H}) - 6\dot{H}\{2(1-3\lambda+3\mu) + 3(1+\omega_2)\mu\} - 18H^2(1+\omega_2)(1-3\lambda+3\mu), \\
B_1 &= 6\dot{\tilde{R}}H\{-(1-3\lambda+3\mu) + 3(1+\omega_2)\mu\} + 6\mu\ddot{\tilde{R}}, \\
D_1 &= 6\mu\dot{\tilde{R}}^2
\end{aligned}
\tag{8.10}
$$

and

$$
\begin{aligned}
\rho_2 \;=\; & -\frac{1}{3(\omega_2-\omega_1)}\Big[3(1+\omega_1)F(\tilde{R}) + A_2F'(\tilde{R}) + B_2F''(\tilde{R}) + D_2F'''(\tilde{R}) - \\
& \frac{CH}{a^3}(2+3\omega_1)\Big]
\end{aligned}
\tag{8.11}
$$

where

$$
\begin{aligned}
A_1 &= \frac{1}{H}(\dot{\tilde{R}} - 6\mu\ddot{H}) - 6\dot{H}\{2(1-3\lambda+3\mu) + 3(1+\omega_1)\mu\} - 18H^2(1+\omega_1)(1-3\lambda+3\mu), \\
B_1 &= 6\dot{\tilde{R}}H\{-(1-3\lambda+3\mu) + 3(1+\omega_1)\mu\} + 6\mu\ddot{\tilde{R}}, \\
D_1 &= 6\mu\dot{\tilde{R}}^2
\end{aligned}
\tag{8.12}
$$

As the field equations are complicated, we consider a simple case to study cosmology. For simplicity we consider a power-law evolution of the universe given by

$$a(t) = t^n \tag{8.13}$$

where n is a constant. The deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1-n}{n}.$$

An accelerating universe corresponds to $q < 0$.

8.3 Power-Law Cosmology in HL-Like Gravity with Interacting Fluids in Flat Universe

In this section we consider a flat universe with a mixture of two interacting fluids. For a power law evolution described by eq. (8.13), the energy densities given by eqs. (8.9) and (8.11) are

$$\rho_1 = \frac{1}{3(\omega_2 - \omega_1)}\left[3(1+\omega_2)F(\tilde{R}) + aF'(\tilde{R}) + bF''(\tilde{R}) + dF'''(\tilde{R}) + \frac{3C\omega_2}{t^{3n}}\right], \tag{8.14}$$

where

$$\begin{aligned} a &= \frac{6}{t^2}\left[n - 3n^2(1+\omega_2) + 3\lambda\{3n^2(1+\omega_2) - n\} + \mu\{9n + 3\omega_2 - 9n^2(1+\omega_2) - 4\}\right], \\ b &= \frac{108\mu}{t^3}(-n + 3n\lambda + 6\mu n)(1+\omega_2) - \frac{36n}{t^4}\left(-12\mu n + 12\mu\lambda n + 12\mu^2\right. \\ &\left. -3n^2\lambda + 9n^2\lambda^2 + 18\mu\lambda n^2 + n - 3n\lambda + 2\mu\right), \\ d &= \frac{216}{t^3}(-n + 3n\lambda + 6n\mu - 2\mu)^2. \end{aligned} \tag{8.15}$$

and

$$\rho_2 = -\frac{1}{3(\omega_2 - \omega_1)}\left[3(1+\omega_1)F(\tilde{R}) + a'F'(\tilde{R}) + b'F''(\tilde{R}) + d'F'''(\tilde{R}) + \frac{3C\omega_1}{t^{3n}}\right], \tag{8.16}$$

where

$$\begin{aligned} a' &= \frac{6}{t^2}\left[n - 3n^2(1+\omega_1) + 3\lambda\{3n^2(1+\omega_1) - n\} + \mu\{9n + 3\omega_1 - 9n^2(1+\omega_1) - 4\}\right], \\ b' &= \frac{108\mu}{t^3}(-n + 3n\lambda + 6\mu n)(1+\omega_1) - \frac{36n}{t^4}\left(-12\mu n + 12\mu\lambda n + 12\mu^2\right. \\ &\left. -3n^2\lambda + 9n^2\lambda^2 + 18\mu\lambda n^2 + n - 3n\lambda + 2\mu\right), \\ d' &= \frac{216}{t^3}(-n + 3n\lambda + 6n\mu - 2\mu)^2. \end{aligned} \tag{8.17}$$

If $\lambda = \mu = 1$ and $C = 0$, eq. (8.7) reduces to that one obtains in the case of standard $F(R)$-gravity [213]. A detailed description for power-law scaling Friedmann-Robertson-Walker universe with two interacting fluid is studied in Ref. [214].

It can also be seen that in case of $\mu = 0$ the scalar curvature $\tilde{R}$ in eq. (8.6) reduces to R_{HL} in eq. (8.4). The corresponding action given by eq. (8.5) is then identical with the action given by eq. (8.2). Hence $\mu = 0$ version corresponds to a degenerate limit of the Hořava-Lifshitz-like $F(R)$-gravity [111]. Here we have considered $\mu = 0$, $C = 0$ and the function $F(\tilde{R}) = R_{HL} - 2\Lambda$ to obtain cosmological models. The energy densities of the two fluids are given by

$$\rho_1 = \frac{1}{(\omega_2 - \omega_1)} \left[\frac{3n}{t^2}(3\lambda - 1)\{-2 + 5n(1+\omega_2)\} - 2\Lambda(1+\omega_2) \right]. \tag{8.18a}$$

$$\rho_2 = -\frac{1}{(\omega_2 - \omega_1)} \left[\frac{3n}{t^2}(3\lambda - 1)\{-2 + 5n(1+\omega_1)\} - 2\Lambda(1+\omega_1) \right]. \tag{8.18b}$$

The interaction term Q in this case takes two forms given by

$$Q = \frac{3n(3\lambda-1)}{(\omega_2-\omega_1)t^3}\{-2+3n(1+\omega_1)\}\{-2+5n(1+\omega_2)\} - \frac{6n\Lambda(1+\omega_1)(1+\omega_2)}{(\omega_2-\omega_1)t} \tag{8.19a}$$

or

$$Q = -\frac{3n(3\lambda-1)}{(\omega_2-\omega_1)t^3}\{-2+3n(1+\omega_2)\}\{-2+5n(1+\omega_1)\} - \frac{6n\Lambda(1+\omega_1)(1+\omega_2)}{(\omega_2-\omega_1)t} \tag{8.19b}$$

Consequently the Q-term can be re-written as,

$$\begin{aligned} Q &= \frac{H}{n}\left[\{-2+3n(1+\omega_1)\}\rho_1 - \frac{4\Lambda(1+\omega_2)}{(\omega_2-\omega_1)}\right] \\ &= \frac{H}{n}\left[\{-2+3n(1+\omega_2)\}\rho_2 - \frac{4\Lambda(1+\omega_1)}{(\omega_2-\omega_1)}\right]. \end{aligned} \tag{8.20}$$

Thus Q depends not only on the expansion rate of the universe but also on densities. We note that the evolution of Q in our case contains additional terms compared to that obtained in Ref. [214], which are useful for viable cosmologies.

In the absence of a cosmological constant, we note the following:

- $\rho_1 = 0$ when $n = \frac{2}{5(1+\omega_2)}$ when $\rho_2 = -\frac{12(3\lambda-1)}{5(1+\omega_2)(1+\omega_1)t^2}$.
- $\rho_2 = 0$ when leads to $n = \frac{2}{5(1+\omega_1)}$ where $\rho_1 = \frac{12(3\lambda-1)}{5(1+\omega_2)(1+\omega_1)t^2}$.

In the HL gravity the parameter λ lies in the range $\frac{1}{3} \leq \lambda \leq 1$. Based on observations of supernova, the baryon acoustic oscillation (BAO) and the cosmic microwave background (CMB), the present value of λ lies in the range $|\lambda_0 - 1| < 0.002$ [112]. Therefore the factor $(3\lambda - 1)$ occurring in the field equation is a positive quantity.

The detailed relationship between the state parameters of the equations of state for interacting cosmic fluids and the power-law scale factor is obtained here. It is found that the requirement the simultaneous fulfillment of the conditions $\rho_1 \geq 0$, $\rho_2 \geq 0$, leads to the following inequalities for $\omega_1 < \omega_2$:

$$Case\ (i) \quad \frac{2}{5(1+\omega_2)} < n < \frac{2}{5(1+\omega_1)},\ (\omega_2 > -1), \tag{8.21a}$$

$$Case\ (ii) \quad -\infty < n < \frac{2}{5(1+\omega_1)},\ \frac{2}{5(1+\omega_2)} < n < \infty,\ (\omega_1 < -1,\ \ \omega_2 > -1), \tag{8.21b}$$

$$Case\ (iii) \quad \frac{2}{5(1+\omega_2)} < n < \frac{2}{5(1+\omega_1)},\ (\omega_2 < -1), \tag{8.21c}$$

and for $\omega_1 > \omega_2$

$$Case\ (i) \quad \frac{2}{5(1+\omega_1)} < n < \frac{2}{5(1+\omega_2)},\ (\omega_1 > -1), \tag{8.22a}$$

$$Case\ (ii) \quad -\infty < n < \frac{2}{5(1+\omega_2)},\ \frac{2}{5(1+\omega_1)} < n < \infty,\ (\omega_2 < -1\ \ \omega_1 > -1), \tag{8.22b}$$

$$Case\ (iii) \quad \frac{2}{5(1+\omega_1)} < n < \frac{2}{5(1+\omega_2)},\ (\omega_1 < -1), \tag{8.22c}$$

In the above it is evident that a realistic solution is obtained for $\omega_1 \neq -1$ and $\omega_2 \neq -1$. For $n > 0$, eqs. (8.21a) and (8.22a) represent configurations which include two interacting fluids obeying the dominant energy condition (DEC), eqs. (8.21b) and (8.22b) represent configurations where one interacting fluid obeys DEC while the other is phantom fluid, and eqs. (8.21c) and (8.22c) represent two interacting phantom fluids.

In the next section we consider two interacting fluids with known EoS.

8.3.1 Dust ($w_1 = 0$) and perfect fluid ($w_2 \neq 0$) interaction

In this section, we consider two interacting fluids dust and barotropic fluid. For this we set $w_1 = 0$ and $\Lambda = 0$, in eqs (8.18a) and (8.18b), the corresponding energy densities for dust and perfect fluid becomes

$$\rho_{dust} = \frac{1}{\omega_2}\left[\frac{3n}{t^2}(3\lambda - 1)\{-2 + 5n(1+\omega_2)\}\right], \tag{8.23}$$

$$\rho_{pf} = -\frac{1}{\omega_2}\left[\frac{3n}{t^2}(3\lambda - 1)(-2 + 5n)\right], \tag{8.24}$$

where $p_{dust} = 0$ and $p_{pf} = \omega_2 \rho_{pf}$. The interaction term Q in this case is given by

$$Q = \frac{3n(3\lambda - 1)}{\omega_2 t^3}(3n-2)\{-2 + 5n(1+\omega_2)\} \tag{8.25}$$

The conditions $\rho_{dust} \geq 0$ and $\rho_p f \geq 0$ leads to the following:

$$Case\ (i) \quad \frac{2}{5(1+\omega_2)} < n < \frac{2}{5}, \ (\omega_2 > 0), \tag{8.26a}$$

$$Case\ (ii) \quad -\infty < n < \frac{2}{5(1+\omega_1)}, \ \frac{2}{5} < n < \infty, \ (-\infty < \omega_2 < -1), \tag{8.26b}$$

$$Case\ (iii) \quad \frac{2}{5} < n < \frac{2}{5(1+\omega_1)}, \ (\omega_2 < 0), \tag{8.26c}$$

We shall now consider a specific value for ω_2, say $\omega_2 = \frac{1}{3}$. Both the energy densities are positive in the interval $\frac{3}{10} < n < \frac{2}{5}$, they are equal at $n = \frac{12}{35}$. Hence in the dust-radiation interacting case permits a decelerated universe. The radiation is found to dominate over dust if $\frac{3}{10} < n < \frac{12}{35}$ and dust dominates for $n > \frac{12}{35}$ (see fig. (8.1)). Here in case of the permissible range of values of n, Q is positive *i.e.*, there exists a transfer of energy from radiation to dust (see fig. (8.2)).

The plot in fig. (8.3) shows the behaviour of the energy densities for the interacting dust-phantom fluid plotted for $\lambda = 1.2$. Here we consider cosmic fluid with equation of state $p = -\frac{4}{3}\rho$ for phantom fluid. The dust-phantom fluid

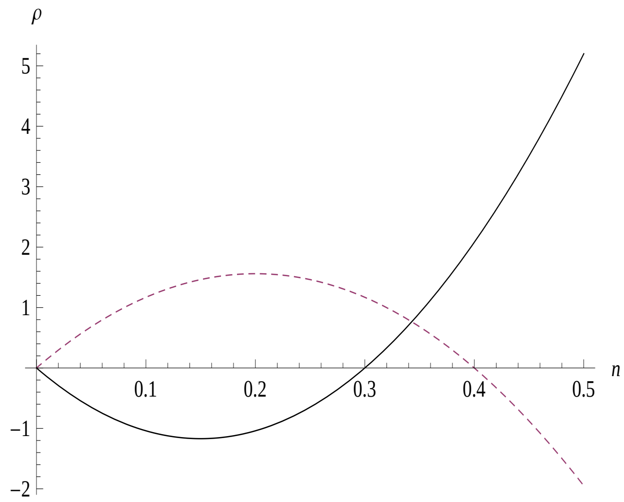


Figure 8.1: Variation of energy densities ρ_{dust} and ρ_{rad} are plotted as a function of n with $\lambda = 1.2$. The solid line represents the variation of ρ_{dust} and the the dotted one represents the same for ρ_{rad}.

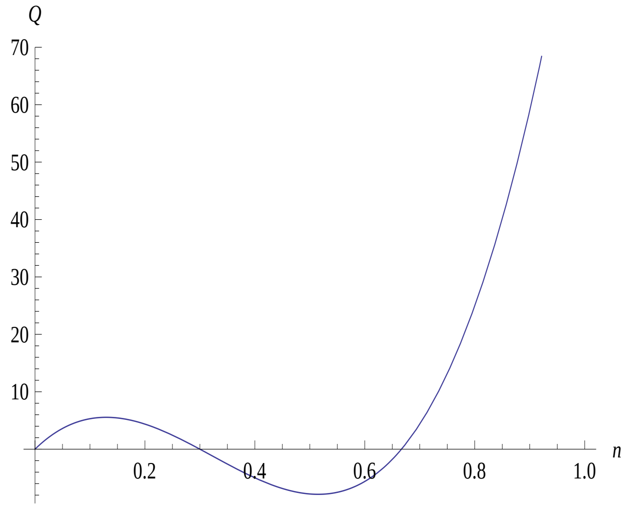


Figure 8.2: Variation of the interaction term Q with n for dust-radiation interaction.

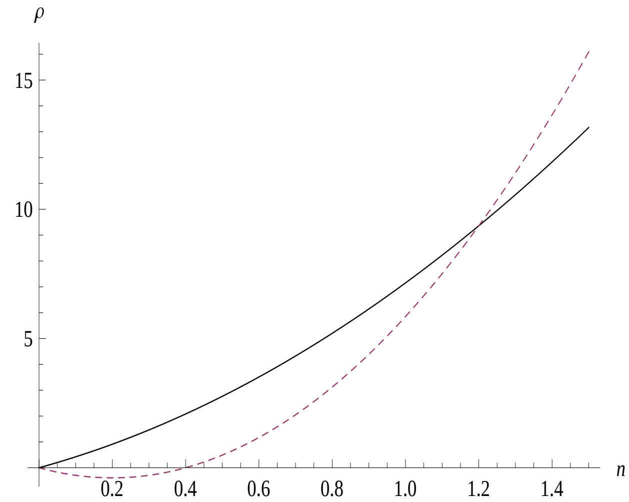


Figure 8.3: Variation of energy densities ρ_{dust} and ρ_{phf} are plotted as a function of n with $\lambda = 1.2$. The solid line represents the variation of ρ_{dust} and the the dotted one represents the same for ρ_{phf}.

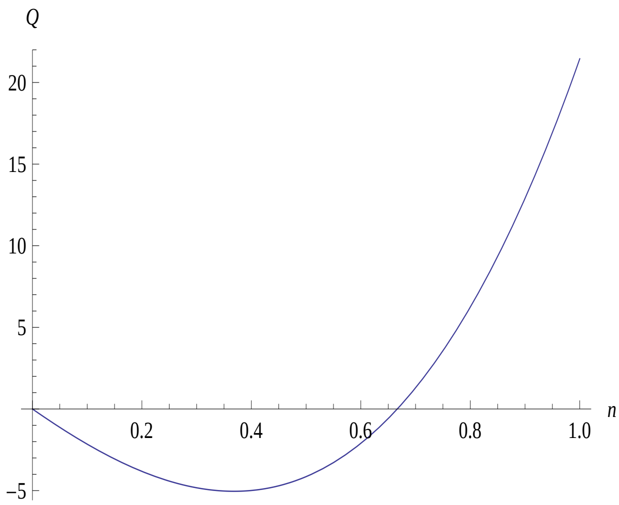


Figure 8.4: Variation of the interaction term Q with n for dust-phantom fluid interaction.

interaction considered here leads to $n \geq \frac{2}{5}$ for positive energy density. The dust and phantom fluid densities are equal when $n = \frac{6}{5}$. The dust is found to dominate in the interval $\frac{2}{5} < n < \frac{6}{5}$. Thus in this case both accelerated and decelerated expansion of the universe can be accommodated. The Phantom fluid is found to dominate for $n > \frac{6}{5}$ which leads to an accelerated expansion.

The interaction term Q is negative, which means a transfer of energy from the dust to phantom fluid which arises during $0 < n < \frac{2}{3}$. If $n > \frac{2}{3}$ there will be a transfer of energy from the phantom fluid to dust (see fig. (8.4)).

8.3.2 Phantom fluid ($w_1 = -\frac{4}{3}$) and perfect fluid ($w_2 \neq 0$) interaction

Let us now consider a composition of two interacting fluids, phantom with perfect fluid. For phantom fluids, using $\omega_1 = -4/3$ in eqs. (8.18a) and (8.18b), in the absence of Λ the energy densities are given by

$$\rho_{phf} = \frac{1}{(\omega_2 + \frac{4}{3})}\left[\frac{3n}{t^2}(3\lambda - 1)\{-2 + 5n(1 + \omega_2)\}\right], \tag{8.27}$$

$$\rho_{pf} = \frac{1}{(\omega_2 + \frac{4}{3})t^2}\left[\frac{n}{t^2}(3\lambda - 1)(6 + 5n)\right], \tag{8.28}$$

with equations of state $p_{phf} = -\frac{4}{3}\rho_{phf}$ and $p_{pf} = \omega_2 \rho_{pf}$. The interaction term in this case is given by

$$Q = -\frac{3n(n+2)(3\lambda - 1)}{(\omega_2 + 4/3)t^3}\{-2 + 5n(1 + \omega_2)\}. \tag{8.29}$$

For $\rho_{phf} \geq 0$ and $\rho_{pf} \geq 0$, the following cases are noted:

$$Case(i) \quad -\infty < n < -\frac{6}{5}, \quad \frac{2}{5(1 + \omega_2)} < n < \infty, \ \ (\omega_2 > -1), \tag{8.30a}$$

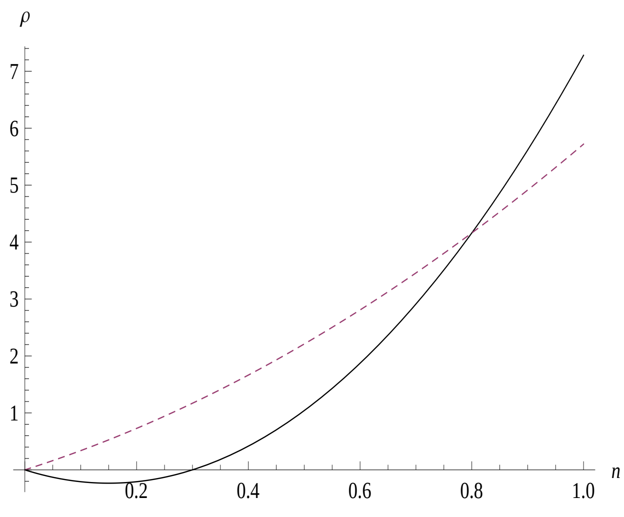


Figure 8.5: Variation of energy densities ρ_{phf} and ρ_{rad} are plotted as a function of n with $\lambda = 1.2$. The solid line represents the variation of ρ_{phf} and the the dotted one represents the same for ρ_{rad}

$$Case(ii) \quad \frac{2}{5(1+\omega_2)} < n < -\frac{6}{5}, \; (-\frac{4}{3} < \omega_2 < -1), \tag{8.30b}$$

$$Case(iii) \quad -\frac{6}{5} < n < \frac{2}{5(1+\omega_2)}, \; (\omega_2 < -\frac{4}{3}), \tag{8.30c}$$

Let us consider a composition of interacting phantom fluid with radiation. The energy densities corresponding to the phantom fluid and radiation are given by:

$$\rho_{phf} = \frac{6n}{5t^2}(3\lambda - 1)(10n - 3), \tag{8.31}$$

$$\rho_{radiation} = -\frac{3n}{5t^2}(3\lambda - 1)(5n + 6). \tag{8.32}$$

The interaction term in this case is given as

$$Q = -\frac{6n(n+2)}{5t^3}(3\lambda - 1)(10n - 3). \tag{8.33}$$

In fig. (8.5), the behaviour of the energy densities of the interacting phantom fluid-radiation is plotted for $\lambda = 1.2$. In this case the interaction of phantom fluid with radiation demands $n \geq \frac{3}{10}$ with positive energy densities. Hence the above

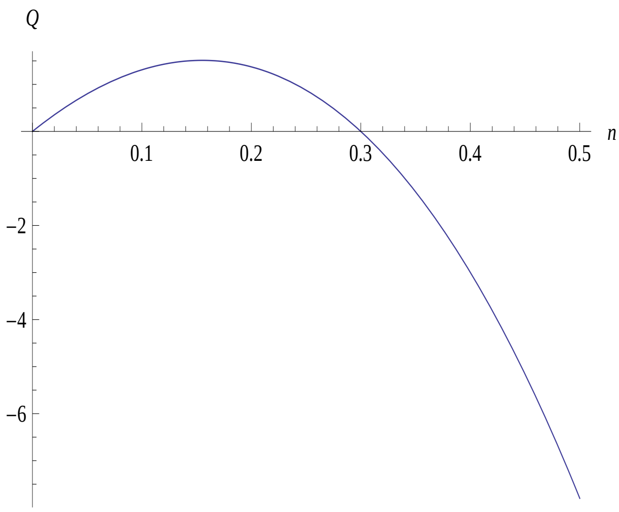


Figure 8.6: Variation of the interaction term Q with n for phantom fluid-radiation interaction.

interaction accommodates both decelerated and accelerated expansion. An accelerated expansion is obtained where the phantom fluid dominates over radiation. The interaction term Q is negative (see fig. (8.6)), which means a transfer of energy from the phantom fluid to radiation, however, for the intervals $0 < n < \frac{3}{10}$ and $n > \frac{3}{10}$, a transfer of energy from radiation to phantom fluid results.

8.3.3 Effective vacuum energy ($w_1 = -1$) and perfect fluid ($w_2 \neq 0$) interaction

We consider here interaction between a perfect fluid with a cosmological constant or vacuum energy satisfying equation of state $p_v = -\rho_v$. Hence putting ($w_1 = -1$) in eqs. (8.18a) and (8.18b), the energy densities are given by

$$\rho_v = \frac{3n}{(\omega_2 + 1)t^2}\left[(3\lambda - 1)\{-2 + 5n(1 + \omega_2)\}\right] \tag{8.34}$$

$$\rho_{pf} = -\frac{6n}{(\omega_2 + 1)t^2}(3\lambda - 1). \tag{8.35}$$

The interaction term is given by

$$Q = -\frac{6n(3\lambda - 1)}{(\omega_2 + 1)t^3}\{-2 + 5n(1 + \omega_2)\}. \tag{8.36}$$

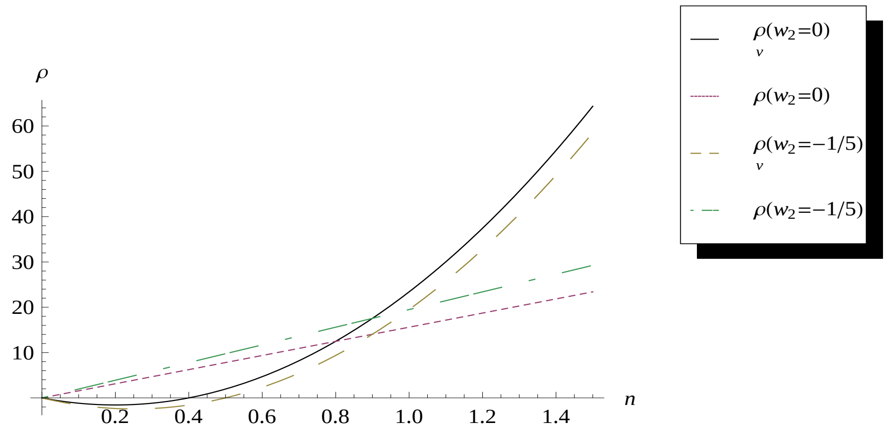


Figure 8.7: Plot of ρ_v and ρ_{pf} with n for $\lambda = 1.2$.

To satisfy the conditions $\rho_v \geq 0$ and $\rho_{pf} \geq 0$ simultaneously we obtain the following constrains on n:

$$n > \frac{2}{5(\omega_2 + 1)} > 0, \quad for \quad \omega_2 > -1; \tag{8.37a}$$

$$n < \frac{2}{5(\omega_2 + 1)} < 0, \quad for \quad \omega_2 < -1. \tag{8.37b}$$

Again from eq. (8.36), one obtain

$$0 < n < \frac{2}{5(\omega_2 + 1)}, \quad for \quad \omega_2 > -1, \tag{8.38a}$$

$$\frac{2}{5(\omega_2 + 1)} < n < 0, \quad for \quad \omega_2 < -1, \tag{8.38b}$$

for a positive Q The interacting fluids in this case permits accelerated universe for $-1 < \omega_2 < -1/5$, decelerated expansion for $\omega_2 > -1/5$.

In fig. (8.7), variation of energy densities of the interacting 'vacuum energy' and perfect fluid with EoS $\omega_2 = 0$ and $-1/5$ are plotted with different n *i.e.*, expansion. Here one obtains both decelerated and accelerated expansion. The 'vacuum energy' always dominates over the perfect fluid for accelerated expansion. A transfer of energy from 'vacuum energy' to the perfect fluid is permitted.

8.3.4 The effective fluid interpretation

In this section we determine necessary conditions for which the interacting fluids are considered equivalent to an effective fluid. We introduce an effective pressure p_{eff} defined as

$$p_{eff} = p_1 + p_2 = \omega_1 p_1 + \omega_2 p_2, \tag{8.39}$$

with equation of state

$$p_{eff} = \gamma\rho = \gamma(\rho_1 + \rho_2), \tag{8.40}$$

where γ is an effective state parameter. From the above equations a relation with n can be obtained which is given by

$$\gamma = -1 - \frac{2}{5n}, \tag{8.41}$$

Thus effectively it reduces to a single fluid model with composition of two interacting perfect fluids. Although the equation of state of the associated effective fluid is not based on the interactions of the physical particles [215], we intend to study the conditions under which the two interacting fluids are equivalent to an effective fluid. In Hořava-Lifshitz $F(R)$-gravity theory, γ is found independent of time. It is evident that the effective state parameter γ attains $\gamma \to -1$ for $n \to \pm\infty$.

8.4 Discussion

In a modified $F(R)$ Hořava-Lifshitz gravity we study power law evolution of the universe with interacting mixture of two fluids . The range of values of the state parameters for the interacting fluids are determined for a viable cosmological scenario. The two fluids model considered here satisfy WEC *i.e.*, $\rho_1 \geq 0, \rho_1 + p_1 \geq 0$, $\rho_2 \geq 0, \rho_2 + p_2 \geq 0$ simultaneously. We have considered here the most general modification of Hořava-Lifshitz like gravity proposed by Chaichian *et. al.,* [111], which is similar to the $F(R)$-gravity with a spatially-flat FRW cosmology for a special choice of parameters. In all the cases considered here, it is noted that both the energy densities of the fluids decreases as $\frac{1}{t^2}$. The ratio of the energy densities becomes a constant and it agrees with the coincidence problem. We note that in Hořava-Lifshitz like $F(R)$ gravity with flat spatial space it admits existence of an interacting two fluids with positive energy densities. It admits both accelerated and decelerated expansion. However, we note that a dust-radiation composition of interacting fluid does not permit an accelerating universe. The power-law inflationary universe is permitted for two interacting fluids, in which one component may be dark energy and the other phantom fluid. Thus the values of n and λ, are important to identify the nature of interacting fluids. Since the present observational data suggests that our universe is undergoing an accelerated expansion, the constraints imposed on the equation of state parameters is important to construct a cosmological model.

The effective EoS parameter are obtained which determines the rate of expansion of the universe with interacting fluids. The composition of fluids may be predicted from the knowledge of EoS of the interacting fluids for a known scale factor *i.e.,* (n) or Hubble parameter (H).

Chapter 9

Concluding Remarks and Future Work

In recent times the precision experiments in cosmology and in astronomy opened up a new vista of research to understand both the early and late universe. It is now generally accepted that the present universe emerged out from an inflationary phase in the early epoch. The need of matter different from perfect fluid in the Einstein gravity becomes essential to accommodate such phase. Consequently theories of Particle Physics that are relevant at high enough energies are taken into account to realize inflation. Thus a semiclassical theory of gravity, where geometry is classical and matter is described by quantum fields, is important. Although existence of inflationary solution was known in GTR, the efficacy of the theory is realized only after the seminal work of Guth [11]. Before Guth, an inflationary universe solution by considering a modification to the gravitational action including a R^2 term was obtained by Starobinsky [9]. Subsequently inflationary universe scenario in the R^2-gravity in the presence of inflaton field is also obtained by Kofman *et. al.* [10]. Inflation not only solves some of the outstanding problems of Cosmology but also it opens up new avenues in the interface of Particle Physics and Cosmology. A number of inflationary universe came up in the last 30 years in the framework of different theories of particle interactions. It is not yet known properly how and when the universe entered into the phase of inflation. But the idea is attractive to understand the evolution of the observed universe. The density perturbation for large scale structure formation of the universe in the framework of inflation gets support from Cosmic Background Explorer (COBE). The recent precision measurements from supernova legacy data and WMAP opened up another interesting window in understanding the universe. When the present observational data are analyzed in the framework of Big Bang model it came out that 73 % density of the universe is due to some fluid having negative pressure which is known as dark energy. As a result the present universe is accelerating. The cause of this acceleration is not known yet. Consequently cosmological models are explored employing different modified theories of gravity which can accommodate the late accelerating phase. It is also known that the normal matter fields available in the standard model of particle physics fails to account the issue. This is due to the fact that the matter with EoS at the present epoch points to a EoS parameter value *i.e.*, $\omega \rightarrow -1$ which is not attainable by normal particle physics fields. Consequently exotic matter namely, phantom, tachyon, quintessence, K-essence, chemeleon, chaplygin gas etc. are considered in the literature. It is known that Primordial Black Holes may have a sensitive signature for primordial cosmological structure formation. It is, therefore, necessary to incorporate a suitable modification either to the gravitational sector or to the matter sector to address different issues in cosmology. However, it is not yet known precisely which modifications work well. A volume of literature came out incorporating both the ideas to build a consistent theory for the evolution of the universe. Some specific issues relevent for cosmological models are also investigated in the above theories that incorporates the quantum nature of matter or both matter and gravity in the early universe.

In *chapter 2*, the probability of pair creation of primordial black holes is estimated in a modified theory of gravity where Lagrangian density contains nonlinear terms of Ricci scalar upto cubic term in 4 dimensions. It is noted that unlike BH a cosmological constant is not essential to realize PBH. In the R^2-theory, the probability of a universe without a pair of PBHs turns out to be much lower than that of a universe with a pair of PBHs unless $\alpha < \frac{1}{8\Lambda}$. In the case of a polynomial

Lagrangian in R, quantum creation of PBH seems to be suppressed in the minisuperspace cosmology for some restrictions of the parameters of the gravitational action.

In *chapter 3*, the PBH pair creation probability is estimated in multidimensional universe with polynomial Lagrangian with Ricci scalar. The motivation for considering a multidimensional universe is the fact that the some of the particle interactions need dimensions more than the usual four for its consistency. In the framework of modified gravity in higher dimensions we obtain gravitational instantons which are useful to understand the quantum creation of the universe. A class of new gravitational instantons relevant for cosmological model building are obtained. We note dimensional dependence of the gravitational instanton and determined limitations of different parameters of the gravitational action to realize such instantons in non-linear theory of gravity.

The gravitational instantons are also employed to study creation of an open universe. It is interesting to note that analytic continuation of a $R \times S^3$ metric to Lorentzian region leads to an open 3-space. For this purpose a special class of inflaton effective potential, which must have a metastable minimum followed by a slope region for a slow-roll inflation, has to be introduced to achieve a nearly homogeneous open universe with $\Omega_o \leq 1$ at the present epoch. Hawking and Turok provided a technique for creating such open inflationary universe described by a singular instanton obtained in a minisuperspace model with an inflation field and the corresponding potential does not require any special property required for open universe. In *chapter 4* singular HT instanton and non-singular de Sitter instanton in the framework of Einstein-Hilbert action with R^4 term are obtained. Subsequently, using conformal transformation the scalar field potential is obtained in the framework of Euclidean gravity and it is noted that the potential permits a suitable range of the initial values of scalar field that can accommodate sufficient inflation.

The matter sector of gravity is also modified with exotic kind of fluid to account for a late accelerating universe. Out of many versions of exotic fields, Chaplygin gas is considered to be one of the promising candidate. The Chaplygin gas is used in cosmology to understand evolution of the universe which however ruled out by observation. Subsequently generalized Chaplygin gas (GCG) and then modified GCG (MCG) are proposed in the literature. In *chapter 5* using the technique adopted by Barrow to incorporate Chaplygin gas in FRW universe, we derive the corresponding scalar field potential and kinetic energy term in the case of modified GCG. Recently a new form of dark energy based on holographic principle is used in cosmology. The correspondence between the holographic dark energy and modified GCG energy density is established. The evolution of the holographic dark energy field and the corresponding potential in the framework of modified GCG for non-flat universe are studied. We have also noted that the holographic dark energy is suitable for a restricted domain of the density parameter of the cosmological constant Ω_Λ in a closed model of the universe.

In *chapter 6* an alternative theory of gravity proposed by Hořava which is a renormalizable theory with higher spatial derivatives in four dimensional quantum gravity is considered . The HL gravity reduces to Einstein gravity with a non-vanishing cosmological constant in IR but with improved UV behaviours. In the framework of this theory of gravity the cosmological scenario of MCG is considered. Using observational data, we determine the constraints of EoS parameters for MCG and the corresponding range of values of the parameter B in both the open universe and closed universe. It is noted that the range of values of B is less in a closed universe compared to that in an open universe. The plot showing the variation of total EoS parameter $\omega(z)$ with redshift z admits the thermal history of the evolution of the universe and ensures the viability of the model. We have compared the ranges of the parameters for MCG and GCG in cases of both the open and closed universes. It is found that models with MCG in the context of HL gravity are better than that of GCG and more physically relevant. The suitability of the model is tested by comparing the plot of the supernovae magnitudes (μ), obtained using the best-fit values, vs. redshift z and the same obtained from union compilation data. Thus, it is evident from our study that models with MCG in the framework of HL gravity gets an experimental support though the analysis does not enlighten the conceptual issues in HL gravity.

In *chapter 7* we reconstructed the HDE with GCG in KK cosmology. The evolution of modified HDE and the effective EoS parameter are obtained here. We note that the dark energy might have evolved from a phantom phase at early universe. It is also noted that HDE model for an interacting GCG is unstable in standard GTR, but in KK cosmology a stable model for the same is possible at the present epoch.

In *chapter 8*, we explore cosmological model with an interacting mixture of two fluids in the framework of the modified Hořava-Lifshitz $F(R)$-gravity taking into account power-law scale factors of the universe. The two interacting fluids considered here satisfy the Weak energy condition simultaneously. It is noted that the energy densities of the interacting fluids decreases as $\frac{1}{t^2}$. Hence the ratio the energy densities becomes a constant and this agrees with the so-called coincidence problem. We note that Hořava-Lifshitz like gravity with flat spatial space admits existence of an interacting composition of two fluids, having a positive energy densities which permits both accelerating and decelerating expansion of the universe. The constraints imposed on the EoS parameters permits to construct a viable cosmological scenario admitting late acceleration.

To conclude, we investigate different theories of gravity where the gravitational or matter sectors of Einstein gravity are modified. A class of cosmological solutions is obtained in the framework of modified theories of gravity which are new and interesting. We also obtain a class of new gravitational instanton solutions that are relevant for understanding creation of the universe leading to an inflationary universe. Different cosmological issues are investigated and found dependence of the coupling parameters of the action in the case of modified gravity with polynomial Lagrangian density in R (upto 4th power). The cosmological models obtained in the modified theories of gravity accommodate a late accelerating phase. The inflationary universe with PBH pair may be useful for describing the dark matter which is also explored. It is possible to replace the dark energy with the higher derivative terms in the gravitational action. The results we obtain in the thesis are useful for describing both the early and late universe. Cosmological models are constructed with MCG in modified theories of gravity. The EoS parameters of MCG are constrained using observational data. In the observational sector of cosmology we see a great progress in recent times. It is expected that newer information will be derived from space based missions *e.g.*, the WMAP, PLANCK and ground based accelerator Large Hadron Collider (LHC) in future which will be helpful to select a consistent scenario of the universe. In future our understanding of the universe will be enriched more than that what we have now both theoretically and observationally.

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