Abstract

In modern cosmology it is accepted that the present universe emerged from an inflationary phase in the early era and it is passing through an accelerating phase. However, the early and late universe evolve in a different way which cannot be described by Einstein's theory of General Relativity (GR). Consequently, there was a spurt in activities to construct the early universe with inflation by modifying the gravitational sector or the matter sector of the Einstein field equations in the last 40 years. The standard model of cosmology is found to suffer from the initial singularity problem. Different cosmological scenarios are proposed to remove the initial singularity. The emergent Universe (EU) model is one such cosmology where a non-linear equation of state (nEoS) is employed to describe a universe without an initial singularity. The late universe is not only expanding but also accelerating. This observation made it to conceive the idea of Dark Energy (DE). The DE is a pressureless fluid, which is not yet known. This is a new and interesting field of research to know DE. Consequently, modification of the matter sector or the gravitational sector in GR is considered to describe DE. The role of interactions among the components of the cosmic fluid is also considered to explore the dynamics of the evolution of the present universe. The gravitational wave speed puts a strict constraint on the scalar field which will be taken into account in constructing cosmological models. Wormholes (WH) are the topological passageways that connect two distant regions of the same universe or two entirely different universes. Morris and Thorne obtained traversable WH solutions in GR with exotic matter. In the modified $f(R,T)$ gravity, WH solutions can be obtained even without exotic matter. Therefore the existence of traversable WHs can be explored in other theories of gravity. Primordial Black Holes (PBH) are considered as a candidate for dark matter. The PBHs are explored in the modified gravity in the presence of nEoS. The thesis is comprised of seven chapters as follows:

- In **Chapter 1** a brief review of the standard model and other cosmological models are discussed. The aim of the work and methodology are also discussed.
- In **Chapter 2**, the Emergent Universe (EU) model is presented with a non-linear viscous fluid in GR. The flat EU model proposed by Mukherjee *et.al*. [1] is a singularity

free cosmological model obtained in GR with a nEoS given by $p = A\rho - B\rho^{\frac{1}{2}}$, where A and B are two EoS parameters. In this chapter we consider nEoS described by $p = A\rho + f(\rho)$, where $f(\rho)$ contains non-linear components that describe viscous fluid. The viscous fluid described by Truncated Israel-Stewart (TIS) theory is considered to explore the cosmological model and its stability. The model parameters are constrained using the Observed Hubble Data ($H(z) - z$) and the distance modulus versus redshift ($\mu(z) - z$) plot obtained from the UNION compilation data. A class of EU solutions obtained here are found to be consistent with the present observations and cannot be ruled out.

- In **Chapter 3**, Rényi holographic dark energy (RHDE) model is considered in a higher dimensional universe. RHDE with interacting and non-interacting fluids are considered in Kaluza-Klein (KK) theory. The non-interacting scenario naturally leads to the late accelerating phase of the universe, which is different from that obtained in the four dimensional standard Holographic dark energy. In the later case, the late time accelerating phase of the universe where it can be realized only in the presence of interacting fluids. In the case of interacting RHDE, though an accelerating universe at late time is permitted, it fails to attain the observed universe. We analyze both the models with statefinder and *Om* diagnostics and found that the non-interacting model is viable in the light of recent cosmological observations. The stability of the cosmological models are also studied.
  We also study the cosmological model with RHDE in a flat higher dimensional Friedmann Robertson Walker (FRW) universe. It is found that the extra dimensions played an important role in the evolution of the universe and its transition from a decelerating to an accelerating phase. The statefinder diagnostics is also applied to test the cosmology and is found to follow the ΛCDM model.

- In **Chapter 4**, modified $f(R)$ gravity coupled with Gauss-Bonnet (GB) terms in the gravitational action is considered to probe the late-time universe. The dynamical role of the GB terms coupled with a dilaton field is explored for Case (I): $f(R) = R + \gamma R^2 - \lambda \left(\frac{R}{3m_s^2}\right)^{\delta}$ where γ, λ and δ are arbitrary constants and Case (II): $f(R) = R$. We consider a linear interaction among the cosmic fluids consisting of non-relativistic matter which includes cold dark matter, radiation and DE. The field equations are highly non-linear, therefore we adopt numerical techniques to study the dynamics of the universe. Defining a new density parameter Ω_H, which is a ratio of the dark energy density to the present day energy density of non-relativistic matter, we study the evolution of the late universe. In the first case GB terms coupled with a free scalar field and in the second case GB terms coupled with a scalar field in a self interacting potential are considered. Using the PLANCK 2018 mission predictions the cosmological models are analyzed. A non-singular universe with cosmological parameters having oscillations in the past for a given strength of the interaction is observed in model-I. It is found that DE oscillation results when the $f(R)$ gravity is dominant over the GB terms in the gravitational action. In model-II, we do not find oscillations of the cosmological parameters. The range of values of the strengths of interactions are determined considering the speed of the gravitational wave (GW) equal to unity.

- In **Chapter 5**, static traversable Morris-Thorne (MT) wormholes is obtained in the modified $f(R,T)$ gravity, where T is the trace of the energy-momentum tensor. Considering $f(R,T) = R + \alpha R^2 + \lambda T^{\beta}$, where α, β and λ are coupling constants, MT wormhole solutions with normal matter are found for a given shape function. Two different values of the exponent β in $f(R,T)$ gravity is considered to obtain WHs. The energy conditions are tested at the throat and away from the throat of the WH. The coupling parameters α and λ in the gravitational action play an important role in determining the matter components that thread the WH. For a given λ, WHs are found to exist in the presence of exotic matter at the throat when $\alpha < 0$, but without exotic matter when $\alpha > 0$. The later result is new in $f(R,T)$ gravity. The shape functions are considered for realizing the WHs with or without exotic matter. Modified $f(R,T)$ gravity permits WHs with normal matter which is not permitted in GR. The no-go theorem for WHs in GR is valid also in $f(R,T)$ gravity. A class of WH solutions is found to exist in this case with anisotropic fluid for $\lambda \neq -8\pi G$. Flat asymptotic regions with anisotropic fluids cannot be realized when $\lambda = -8\pi G$. The hybrid shape function considered here is found to satisfy all the energy conditions indicating the existence of WHs even with normal matter for $\lambda \to 0$.

- In **Chapter 6**, the evolution of PBHs is probed in the modified $f(Q)$ gravity for matter accreting from the cosmic fluid surrounding the PBHs. Considering nEoS $p = f(\rho)$, the accretion of matter into PBHs is also analyzed. Two different branches of the nEoS is considered here namely, (i) modified Chaplygin gas (MCG) and (ii) fluid with dissipative effects. The mass evolution of the PBH and the evolutionary features are also analyzed. In the $f(Q)$ gravity, PBHs are found to gain mass in the early epoch finally attaining a saturated mass. This is different from that of GR as well as $f(\mathscr{T})$ gravity (with $\mathscr{T}$ being the torsion scalar) where the PBHs initially attain mass but finally, the PBH mass decreases. The evolution of PBHs in $f(Q)$-gravity with or without the MCG is studied and found that for PBHs attain maximum mass with MCG, which may provide a clue to the super-massive black holes.

- Finally, in **Chapter 7** the concluding remarks and a future plan of research to be undertaken are given.