THIS THESIS CONTAINS SOLUTIONS OF SOME NEW PROBLEMS OF LAMINAR FLOW OF VISCOUS INCOMPRESSIBLE FLUID, DIFFERENT TYPES OF THESE PROBLEMS HAVE BEEN SPLIT UP INTO SIX CHAPTERS.

chapter - I

The First chapter deals with problem of laminar flow of viscous incompressible fluid of finite depth resting on a rigid plane base. Three problems have been included in this chapter.

In the first paper, we study the laminar motion of viscous incompressible fluid standing on a rigid plane base under the action of time-varying pressure gradient. When the flow is unsteady, pressure gradient is a function time. Laminar flow of viscous fluid through tubes of circular section under time-dependent pressure gradient has been treated by Sexl[22](1930), Pai[13](1955) and Mithal[12](1960). Satya Prakash[20](1967) has discussed the non-steady parallel flow through a straight channel due to the action of pressure gradient decaying exponentially with time.

In the present problem, the fluid is of finite height and has a free surface. Three types of pressure gradients viz. (i) impulsive (ii) defined by Heaviside function of time (iii) periodic for a finite time are chosen.

The nature of the velocity distribution for the latter case has been graphically represented and it has been found that the velocity attains its maximum value when $t = \frac{2.18}{\omega}$ Sec. approximately and after this time it decreases exponentially.

The Second paper is Concerned with the pulsating flow of viscous incompressible fluid of finite depth resting on a rigid base due to a certain periodic shearing force superposed on a Constant force applied on the free surface. The problem of pulsating flow of viscous fluid under time-dependent function superposed on a Constant pressure gradient has received the attention of several investigators. Uchida[29](1956) has considered the pulsating flow superposed on steady laminar motion in a pipe of circular cross-section. Adopting the same procedure Bhattacharyya[3](1968) solved the problem of flow of two incompressible immiscible fluids between two parallel plates Verma[30](1960) considered the flow of viscous liquid under exponential pressure gradient superposed on steady laminar flow between Coaxial Cylinders.

In the present paper, the method is applied to determine the unsteady flow of viscous liquid set up by a periodic shearing force superposed on a Constant surface force. When the pulsation ceases to act, the solution agrees with that known for steady laminar flow due to the action of a constant shearing stress applied on the free surface in the presence of a

Constant pressure gradient. For simple pulsation, the flow pattern is shown graphically and some physical features of motion are pointed out. The total flux and skin friction on the boundary have also been numerically evaluated.

The problem of unsteady flow of a viscous incompressible fluid due to a moving boundary has been solved in the third paper. The flow near an infinite wall executing harmonic oscillations parallel to itself was first treated by G. Stokes[27](1851). The same problem has been discussed in detail in the classical treatises of Lamb[11](1959), Schlichting[21] and Pai[13]. If the velocity of the oscillating wall is assumed to be $U_0 \cos nt$, the velocity profile has the form of a damped oscillation of amplitude $U_0 e^{-y\sqrt{n/2\nu}}$ in which a fluid layer at a distance y from the wall has a phase lag $y\sqrt{\frac{n}{2\nu}}$ with respect to the motion of the wall.

In this problem we have assumed that the fluid is at rest on a rigid boundary. The boundary is made to move by applying different types of forces viz (i) impulsive (ii) force defined by sinusoidal function of time and acting for a finite period (iii) Constant force acting for a short interval (iv) a force exponentially decaying with time upto a finite period respectively. Retarding force on the boundary per unit area has been calculated.

The second chapter is devoted to the solution of the problem of flow of viscous incompressible fluid through tubes whose cross-sections are circles and sectors of circles, due to the action of pressure gradient. This Chapter contains three papers.

In the first paper, we have studied the problem of flow of viscous incompressible fluid through the annulus of two concentric circular cylinders due to the presence of pressure gradient. The pressure gradient is assumed to be a suitable function of time.

The flow of viscous fluid through the tubes of circular cross-sections has been treated by many investigators. Grace[7] (1928) has studied the oscillatory motion of viscous liquid in a long straight circular tube and discussed the velocity distribution over a right section of the tube. He has pointed out the difference in the velocity distribution in the case of oscillatory motion of a viscous fluid as compared with that which occurs when the liquid is in steady motion. Sexl[22] (1930) has investigated the motion of a viscous liquid through a circular tube under the influence of a periodic pressure gradient. The unsteady flow of viscous liquid through a circular tube under the presence of a time-varying pressure gradient has been investigated by Mithal[12] (1960). Such a flow under exponentially increasing or decreasing pressure gradient was analysed by Sanyal[19](1956).

In the first paper, we have chosen the pressure gradient as (i) an exponential function of time and (ii) a damped harmonic force respectively. The general solution for the problem which has been derived by using the method of Laplace transform, involves Bessel function of 1st and 2nd kind of zero order. Making the radius of the inner cylinder zero in the general expression for velocity distribution, we can directly obtain the flow of viscous liquid through a circular tube and it is found that the distribution of velocity agrees to the result already derived by Grace[7](1928) when the pressure gradient is taken of the form $ce^{i\omega t}$ and in case of time-decaying pressure gradient, the distribution of velocity coincides with that of the viscous liquid through a long circular tube and this has been derived by Mithal[12](1960).

For very slow oscillatory motion of a viscous liquid through the annular region between two concentric circular cylinders, the amplitude of the velocity distribution resembles the solution for the steady motion of the viscous liquid through the annulus of two concentric circular cylinders, the velocity being in the same phase with that of the disturbing force. When the pressure-gradient is taken to be $c\,e^{-\lambda t}$, gradual decrease in the fluid~~s~~ velocity is observed as t increases and ultimately vanishes when t approaches to infinity. Assuming Ω =0 in the general expression for velocity distribution, we have the unsteady flow of viscous liquid through the annulus of two concentric circular cylinders with constant pressure gradient and it has been found that after an infinite time being elapsed, the flow coincides with the well known poisueilli flow between two cylinders. [Lamb[11] - P-586]

The Second paper of this chapter is concerned with the flow of viscous liquid through the annular region between two coaxial circular cylinders under the presence of pressure gradient P($1-e^{-\alpha t}$) which means that pressure gradient increases with time and ultimately it becomes constant when t approaches infinity. The method that has been adopted here is exactly similar to that used in the first paper of this chapter. In case of high viscous fluid the increase in the pressure gradient causes the increase in the velocity distribution and for sufficiently large value of t, the fluid velocity coincides with that of the fluid through two coaxial circular cylinders due to a constant pressure gradient [Lamb[11] - P-586].

In the third paper, we have discussed the unidirectional flow of viscous fluid between two infinite concentric tubes with sectors of circles as cross sections, under the presence of pressure gradient.

In a recent paper, Rawat(1970)[17] has obtained the solution of the problem of uni-directional flow along the length of the tube whose cross-section is a sector of a circle. The solution of our problem has been derived in exact form by applying finite Hankel transform. It has been shown that same solution follows from our problem if the radius of the inner cylinder be made to approach Zero.

The third chapter deals with the problem of flow of viscous incompressible fluid through the annulus of two porous concentric circular cylinders. Two papers are included in this chapter.

Singh, G.S.[23] (1967) solved the problem of flow of visco-elastic Maxwell fluid through two porous concentric cylinders. He assumed the flow to be caused by pressure gradient of the form $Ke^{-\Omega t}\cos\beta t$. Devi Singh[24] (1964) considered the same problem under the action of exponential pressure gradient. Surja Prakash Rao[16] (1961) treated the periodic flow through an annulus of two porous concentric cylinders for ordinary viscous fluid.

The first paper of this chapter is concerned with the solution of the problem of unsteady flow of viscous incompressible fluid through the region bounded by two porous stationary concentric cylinders. The general solution of the problem has been obtained in an integral form by using the method of Laplace Transforms. In the cases of impulsive and periodic pressure gradient, the solution has been deduced in exact finite forms. It is found that on putting the suction parameter equal to Zero, the results of same flow problem between two impervious cylinders follow. When the pressure gradient is taken

$P(1 - e^{-\alpha t})$ which means that pressure gradient increases with time and ultimately it becomes constant when $t \to \infty$. The velocity for large viscous liquid is increasing with the increase in time and attains the steady state solution when t approaches infinity.

The second paper of this chapter is meant for study of the flow of viscous incompressible fluid set up by rotation of a porous cylinder within a fixed porous cylinder. The analogous problem for the case of non-porous cylinders has been treated by Sneddon[26](P-305). The general solution for the problem is obtained in terms of Bessel function of order n by applying finite Hankel Transform. The couples exerted on the inner cylinder are calculated for particular types of angular velocities imparted.

In the fourth chapter which consists of three papers, we study the motion of viscous heterogeneous fluid due to time - varying body forces of different types. Some simple cases of Continuous variation of density and viscosity are studied.

In the first paper, we consider the motion of a viscous incompressible heterogeneous fluid set up by a periodic body force. Previously Bhattacharyya[2] (1965) obtained solution for the motion of viscous heterogeneous fluid caused by transient shearing force applied on the surface.

In the second paper, we have considered the flow of viscous heterogeneous fluid through a circular tube due to body force acting along the axis of the tube. Two types of such forces are assumed viz (i) periodic and (ii) decaying exponentially with time. When the body force is taken to be a periodic function of time and density varies inversely as square of distance, viscosity remaining constant, the fluid velocity is found to be periodic with the phase angle different from that of the disturbing force. For very small oscillation, it has been found that the phase of the fluid velocity decreases as the radius of the cylinder increases from 0 to its natural value and at the central portion of the tube the phase difference attains the maximum value $\frac{\pi}{2}$, ahead of that of the disturbing force, while for a homogeneous fluid, the fluid velocity is periodic and has the same phase as that of the disturbing force. The nature of the fluid velocity for (i) heterogeneous and (ii) homogeneous fluid has been graphically evaluated in figure(1) and (2) respectively. When the time decaying body force $c\,e^{-\Omega t}$ is considered, damped oscillatory motion is generated in the fluid, the amplitude of the fluid velocity is decaying with the factor $e^{-\Omega t}$.

The third paper is devoted to the solution of the problem of laminar flow of viscous heterogeneous fluid through the annular space of two co-axial circular cylinders due to the body force acting on it in the direction of the common axis of the cylinders. When the density varies inversely as r^2, r denoting distance from axis, keeping viscosity constant, the distribution of the velocity is found to be harmonic, but has phase different from that of the disturbing force which is taken to be of the form e^{ipt} . When the radius of the inner cylinder tends to zero, the fluid velocity resembles that of the hetero - geneous fluid through a circular tube under the action of same body force $e^{i\beta t}$ (cf - paper II of this chapter). When both the density and viscosity vary inversely as r , the distribution of velocity has been calculated under the action of same body force. It is found that the velocity distribution is periodic and has phase different from that of the disturbing force.

This chapter consists of two papers. In the first paper we have considered the problem of slow steady flow of viscous incompressible fluid of finite depth standing over a Corrugated bed. The equation of the bed is given by the equation

$$y = h + \epsilon N(x)$$

where h is a constant and ϵ is the roughness parameter and it has been assumed that $\epsilon N(x) << h$

The method of solution is the same as that used by Citron[6] (1962) who has investigated the problem of slow steady flow of viscous fluid between two rotating concentric cylinders when the radii of the cylinders vary axially, the roughness parameter ϵ being small compared with the radii of the cylinders. Khamrui[8] (1963) has treated the flow of viscous fluid through a circular tube with axial roughness. The problem of slow steady flow of viscous liquid between two walls at slighty variable distance from each other has been solved by Bhattacharyya[1] (1965). Pandey[14] (1969) has studied the slow steady motion of a viscous incompressible fluid between two porous walls at slightly variable distance from each other with disturbed section.

When $\epsilon = 0$, the present problem reduces to that of two dimensional flow of viscous incompressible fluid of finite depth standing over a rigid plane base and leads to a known solution. The perturbation in the velocity components and pressure due to variation in depth of the fluid are then obtained from the Navier - Stokes' equation for slow and steady motion by the application of the method of Fourier transform with appropriate boundary conditions. Assuming $\epsilon N(x) = \epsilon \sin \frac{2\pi x}{\lambda}$, an exact solution has been obtained by making the use of a property of the Dirac Delta function.

In the second paper of this chapter, we have obtained solution to the problem of flow of viscous incompressible fluid through the annulus of two concentric circular cylinders due to the presence of pressure gradient, when the radii of the cylinders vary axially, the roughness parameter ϵ being small compared with the radii of the cylinders.

The sixth and the last chapter has been devoted to the solution of two dimensional problems of waves generated by surface load acting on the free surface of a semi infinite viscous incompressible fluid. Sneddon[26] (1951) has solved the problem of flow of semi infinite viscous incompressible fluid under the action of radially symmetric pressure distribution. Paramanik[15] (1972) has considered the two dimensional problem of waves generated by a moving oscillatory pressure distribution which is applied on the free surface of an infinitely deep viscous incompressible fluid.

In the first paper of this chapter, we have considered the two dimensional motion of waves generated by two types of pressure distributions ~~are~~ viz. (i) Oscillatory (ii) damping pressure. Displacement of the free surface has been calculated. Considering the effect of large viscosity, the waves generated due to above two pressure distributions have been calculated and it is seen that in the case of damping force decaying exponentially with time the motion becomes steady after a sufficiently long time.

In the second paper of this chapter we have studied the motion of a viscous incompressible fluid due to the actions of ring loads and disk loads applied on the surface. These loads act normally to the surface and expand radially at a constant rate over the surface. Expression for the surface elevation has been obtained.