CHAPTER: I

INTRODUCTION

1.1. Importance of Radiative transfer theory in Biosphere:

About 30% of the Earth's surface is covered by land and till date most part of this is vegetated. Rest is ocean. Thus, land surface and ocean processes are important components of the terrestrial climate system which essentially maintains nurtures and controls evolution in the life support systems for all living organisms in the earth. The only natural machine which supplies the driving energy to maintain the huge macroscopic or microscopic harmony between each and every element, living or non-living, throughout their existence, is sun. The bulk of the solar energy provided to the troposphere transits through to the lower boundary (atmosphere, oceans and continents) first and is made available to the atmosphere through the fluxes of sensible and latent heat and thermal radiation. Accurate descriptions of the interaction of radiation with surface vegetation, atmosphere and ocean processes require quantitative as well as qualitative information on fluxes of energy (radiation) and mass (water vapor and CO), which are strong functions of photosynthetic and evapotransportation rates. The energy received from the sum in the form of electromagnetic energy primarily drives the whole engine that sustains the biosphere through numerous kinds of interactions.

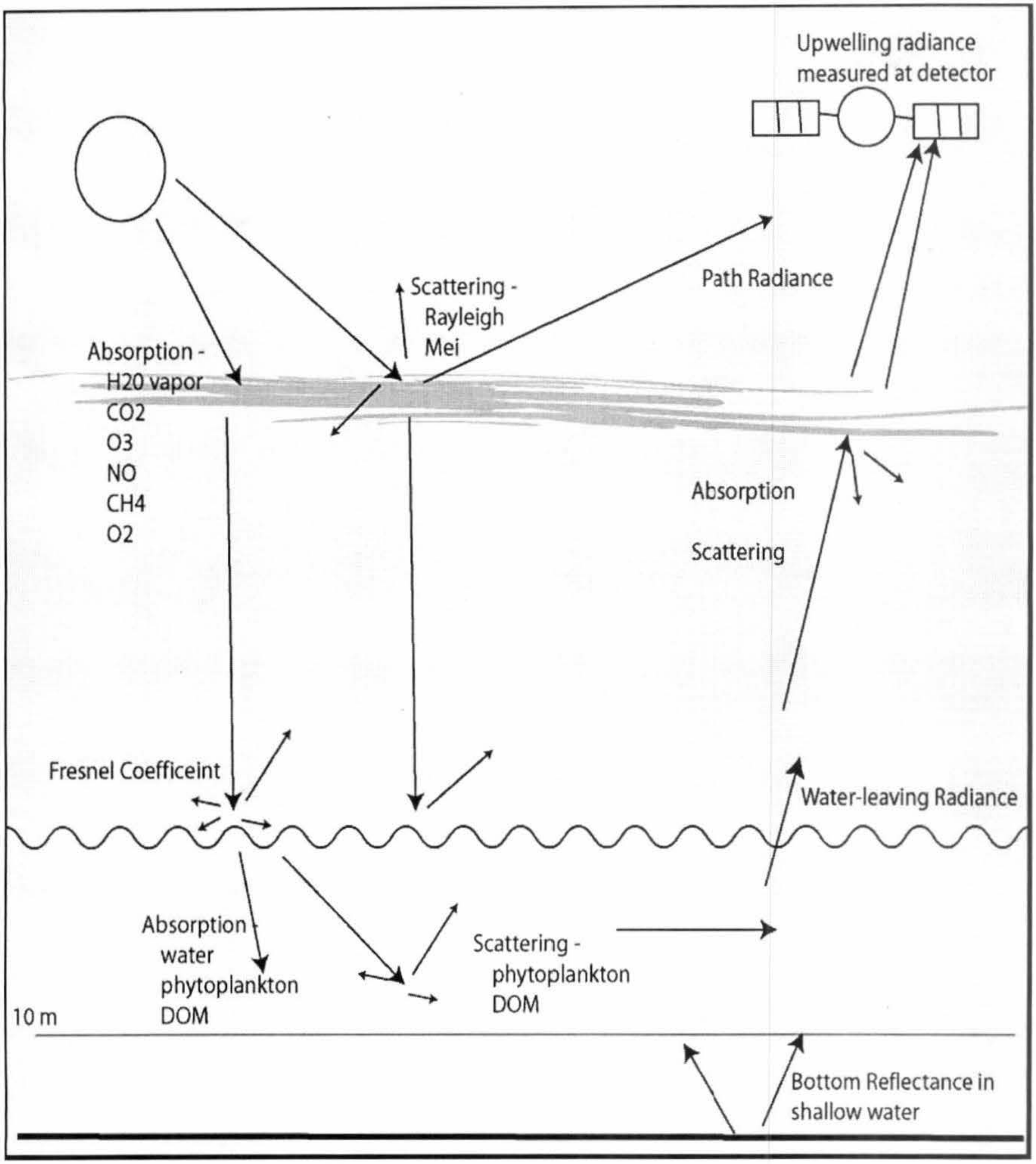
Solar radiation, as mentioned, is the primary energy source for the atmospheric general circulation and the hydrological cycle. As electromagnetic radiation (EMR) reaches the earth's surface, molecules and particles of the land, water, ocean and atmosphere environments interact with solar energy in the 400-2500nm spectral region through absorption, reflection, transmission and scattering. Some materials will reflect certain wavelengths of light, while other materials will absorb certain wavelengths. Many materials have unique patterns of reflectance and absorption across the electromagnetic spectrum. Before radiances are recorded at any sensor, the electromagnetic radiation has already passed through the atmosphere twice or more (sun to atmosphere then ocean and again after reflection to the sensor through atmosphere). During this process (Figure 1), EMR has been modified by the processes of scattering by air molecules and aerosols, oceanwater, oceanparticles and by absorption.

The processes of scattering require special citation. The incident solar radiation suffers scattering with appreciable changes in its initial direction of propagation several times before it reaches ocean surface. These scattered photons again interact with ocean water as well as suspended particles present within it. The science of atmospheric scattering is different from ocean scattering. However in general the scattering of light can be expressed mathematically in terms of phase functions. We have devoted one subsection to discuss the mathematical properties of phase functions in the next section. The main contributors to gaseous absorption are atmospheric water vapor, carbon dioxide, ozone, nitrous oxide, carbon monoxide, methane and oxygen (GAO, [1]). Major atmospheric bands, such as those of water vapor centered at approximately 940, 1140, 1380 and 1880 nm, the oxygen band at 760 nm, and the CO_2 band near 2006 nm can influence the relative brightness over an image. If the EMR interacts with the surface of the water, the reflection scattering and transmission of light across the air-water interface are governed by the Fresnel coefficient of reflection, which is modified by surface roughness, a function of wind speed (Hamilton et al, [2]). Photons can also penetrate the water column and can be absorbed by water itself, photosynthetic plant material, or dissolved organic substances. The fate of sunlight incident on the ocean or water surface may be represented by the diagram (Fig.2). Some fraction is reflected at the surface, the rest is refracted, scattered and finally absorbed in the water below. The diagram neglects diffuse illumination from the sky, waves, ripples, foam and the rest of the interesting detail just at the surface. We note that while the illumination of the surface may be a directed beam (as shown in Fig.2) during the middle of a bright day it will certainly be diffuse both on an overcast day and during sunrise and sunset. Just below the surface is a transition region in which the light experiences its first few scattering interactions with the water. Despite the sharp line shown dotted in the figure, the transition region actually has an indefinite lower boundary as the light distribution settles toward its asymptotic form. We shall also neglect this transition region in our calculational details but it is worth to mention that this region is likely to be narrower when the incident illumination is diffuse. Below the transition region lies the relatively orderly regime of downwelling light where many details of the surface illumination and sea state have been smoothed over and lost and the light is diffuse.

Multiple scattering effects by both photosynthetic and non photosynthetic particles can lead to increase probability of eventual absorption, or photons can be eventually ejected back through the surface (Hamilton et al, [2]). Upwelled light emerging from the surface carries information on all these processes. Because nearly all absorption takes place in the upper two to three attenuation lengths (a few to tens of meters), this information is confined to that region, and quantities derived from the water-leaving spectral radiance are generally referred to as surface values [72].

Transfer of solar radiation (Electromagnetic Radiation) and its spatial and temporal variations drive the general atmospheric and oceanic circulation and the hydrological cycle in the earth. The coupling between an atmospheric general circulation model (AGCM) and an oceanic general circulation model (OGCM) depends strongly on the radiative energy flow through the earth–atmosphere system. For the radiative energy budget near the surface the shortwave solar energy accounts for most of the heat flux transferred to the ocean. The solar radiation transferred into the upper-ocean layers affects the stability of the ocean mixed layer and the sea surface temperature. Consequently, the oceanic surface albedo (OSA) plays a key role in determining the energy flow exchange between atmosphere and ocean and so is an important issue for the coupling of atmosphere and ocean models.

One of the major sources of uncertainty in climate prediction lies in the radiative energy flow through the earth–atmosphere system and the radiative interactions between the atmosphere and the hydrosphere. The radiative energy budget is the most important component of the air–sea energy flux. In particular, the shortwave solar energy accounts for most of the heat flux transferred into the ocean. The absorption of solar radiation by the upper layers affects the stratification and stability of the ocean mixed layer, the sea surface temperature, and the general circulation of the ocean. Because solar radiation is the energy source for photosynthesis, it influences marine primary productivity directly, and impacts cascade throughout ocean ecosystems. The atmospheric radiation budget has long been recognized as fundamental to our understanding of the climate system ([3] Houghton et al. (1996)). Surface radiation measurements are essential for the validation of both radiative transfer models and flux retrieval algorithms using satellite data. Hence ground stations have been established to monitor the radiation budget. This has led to a series of comparisons between models and measurements (e.g., Kinne et al. [4]); Conant et al. [5]; Chou and Zhao [6]; Charlock and Alberta [7]; Kato et al. [8]). However, most measurements are over inhabited areas, and routine observations over the ocean are extremely limited. It is difficult to make accurate surface radiation measurements from a moving platform such as a ship or buoy.


Upwelling radiance measured at detector
Scattering -
Rayleigh
Mei
Path Radiance
Absorption -
H20 vapor
CO2
O3
NO
CH4
O2
Absorption
Scattering
Fresnel Coefficeint
Water-leaving Radiance
Absorption -
water
phytoplankton
DOM
Scattering -
phytoplankton
DOM
10 m
Bottom Reflectance in shallow water

(Fig.1)

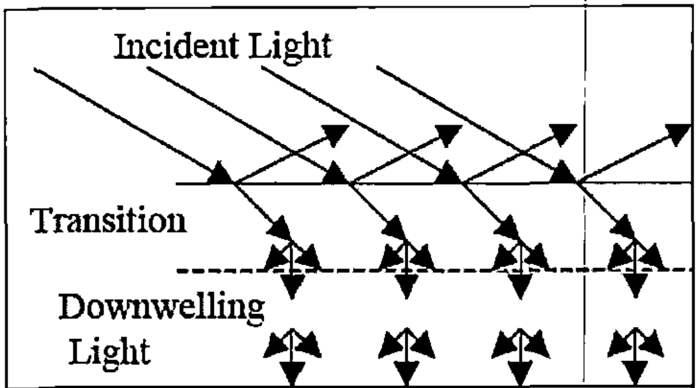


Fig (2)

1.2. What is radiative transfer?

The radiative transfer equation describes the propagation and scattering of electromagnetic radiation at any point inside an absorbing and scattering medium. Mathematically it consists of an integro-differential equation describing balance between the changes suffered by the energy (measured in terms of intensity) of radiations during its journey through any medium and the difference between the generation and loss of the energy. Since every medium (atmosphere, ocean, planetary-atmosphere, steller- atmosphere, etc.) have either some source of energy within or outside its periphery (like planets) this change in energy, if measurable or detectable, can be used to a large extent to retrieve the data containing informations about the nature and properties of the internal constituents of the prevailing medium through which the radiation have traversed. This is the key idea that drives the researcher to apply this equation in those fields where informations are to be collected remotely. The subject of radiative transfer was studied and formulated principally by astrophysicists for analysis of radiation coming from celestial bodies to retrieve information on the structure and constitution of the atmosphere of the astronomical objects. Soon it was realized that this equation is of much relevance to a variety of problems that arises in atmospheric sciences, nuclear reactor theory, optics, atmospheric-ocean processes and climatology or meteorology. Historically, it was developed in the early 1900's by astrophysicists and meteorolo-gists studying electromagnetic rotation in planetary, Stellar and terrestrial atmospheres (Schuster [41]). Later transfer theory became known by the name as transport theory in particle and Nano-paticle regime. Nowadays radiative transfer covers a variety of fields, including astrophysics, applied physics, optics, planetary sciences, atmospheric sciences, and meteorology, as well as various engineering disciplines.

Although the majority of the earth's surface is covered by water, in the early part of fifties of the last century, there had been remarkably little research done on the influence of various atmospheric and oceanic properties on the quantitative interaction between sunlight and the oceans. The path of a solar photon through the atmosphere and ocean can be very complex. As mentioned earlier, it may undergo absorption and multiple scattering by aerosols and atmospheric molecules, reflection and refraction at the ocean surface, and further absorption and multiple scattering by hydrosols and water molecules of the ocean. Additional complexity arises from scattering and absorption by the ocean floor, from waves on the ocean surface, and from the refraction and reflection (including total internal reflection at some angles) of the upwelling light at the ocean boundary. What is the distribution of the sunlight reflected from the ocean surface with waves? What is the angular distribution of the radiation within the ocean and what fraction of the radiation enters the upwelling stream and passes back into the atmosphere through the ocean surface? At what angles can an observer above the ocean surface see down into the ocean instead of seeing radiation reflected by the surface? Why can an observer in an airplane seated on the sunlit side easily see the shadows of isolated cumulus clouds upon the ocean, while an observer on the other side of the plane cannot see any trace of such shadows? A qualitative answer to some of these questions is given by Minnaert [264] in his delightful and provocative book on various types of phenomena about light and color. Both the theoretical and experimental knowledge about light in the oceans is reviewed later by Jerlov [265] and Jerlov and Nielsen [266] in their excellent books. The theory of radiative transfer in natural waters has been treated extensively (Preisendorfer [75, 348]; Prieur [349]). The aim of these studies is not only to describe natural and induced light fields under water but also to predict the composition of the water masses from their optical properties – the so-called inverse problems. Application of optical passive remote-sensing techniques concerns treatment of the Upwelled solar (IR) radiance and of irradiance reflectance, defined as the ratio of upwelling and downwelling irradiance. Various models have been developed and applied (Preisendorfer [75, 348]); Prieur [349]). Particularly attractive are irradiance models dealing with optical quantities that are relatively easy to measure as functions of depth and wave-length (see e.g., Tyler and Smith [350]; Spitzer and Wernand [351]). Two-flow (sunlight and fluorescence emissions) models have been previously applied and solved for homogeneous aquatic media (Preisendorfer [75, 348, 352]; Prieur [349]) including the fluorescence term (Kishino et al. [353]; Spitzer and Wernand [354]). The treatment described by Dirks, R. W. J., and Spitzer, D., [355] is restricted to the visible region (400 – 700 nm) and horizontally homogeneous media (i.e., horizontal irradiance variations being negligible compared to vertical variations). Calculations concern only subsurface phenomena and quantities, i.e., no atmospheric effects are included.

In general the transfer of radiation for all wavelengths that occurs in nature, fortunately obeys above mentioned wonderful mathematical equations which can be understood analyse and interpret qualitatively as well as quantitatively if appropriate boundary conditions can be formulated to obtain proper answers to the above mentioned questions. In remote sensing radiative transport theory basically serves as a mechanism for explaining the exchange of electro magnetic energy between the atmosphere and between the space and the earth atmosphere system. The radiation emitted by the atmosphere, ocean or in general target objects are identified and intercepted by the special purpose sensors stationed on board. The data collected through remote sensing are analyzed, interpreted and visualized suitably to make use of it. This technique is used as fundamental in the calculation of radiation energy budget of stars, planets, prediction of crop production, number counts and location of cattle and sheep, fish in ocean, atmospheric and ocean underwater fields, in the determination of reflecting properties of snows, ice, paint surfaces, photo emulsions, blood tissues viruses, bacterias, in the estimation of insulation properties of fiberglass materials, in the researches of thermal structure of blast furnaces and other high temperature machineries.

During the last six decades tremendous activities were undertaken by the scientific community to the development of solution of this RTE equation under different situations of practical interest. The mathematician relishes the technical challenge, whilst a representation of the radiation field is essential for the physicist wishing to understand the properties of stellar atmospheres. Schuster [41], Schwarzschild [88], Milne [51,51a], Eddington [50], Jeans [97], Hopf [52], Bronstein [78], Unsold [53] and others contributed substantially to the understanding of the theory of radiative transfer in connection with the problems of stellar atmospheres. One may find these initial investigations in the monographs stated above.

In the beginning most of the methods of solution were analytical involving beautiful mathematical techniques (Chandrasekhar [54], Sobolev [63], Busbridge [231] and Case & Zweifel [135]). In his landmark book, Chandrasekhar [54] presented the subject of radiative transfer in plane-parallel (one-dimensional) atmospheres as a branch of mathematical physics and developed numerous solution methods and techniques. The field of atmospheric radiation has evolved from the study of radiative transfer. It is now concerned with the study, understanding, and quantitative analysis of the interactions of solar and terrestrial radiation with molecules, aerosols, and cloud particles in planetary atmospheres as well as the surface on the basis of the theories of radiative transfer and radiometric observations made from the ground, the air, and space (Thomas and Stamnes [102] Liou [56, 57]). Fundamental understanding of radiative transfer processes is the key to understanding of the atmospheric greenhouse effects and global warming resulting from external radiative perturbations of the greenhouse gases and air pollution, and to the development of methodologies for inferring atmospheric and surface parameters by means of remote sensing.

Because of the diversity of topics that use radiative transfer theory, a wide range of solution methodologies of the radiative transfer equation (RTE) can be found in the literature. One can consult the monograph of Lenoble [58] and for radiative heat transfer Modest [114].The review article Stamens [59] is also very much informative.

1.3. Why radiative transfer theory for ocean is important?

There exists military and non-military importance of transport theory in ocean. We shall deal with non-military applications of this sub discipline. Photon absorption regulates photosynthesis which drives phytoplankton or primary production at the bottom of the food chain in ocean eco system. The correlation of optical radiance measurements from satellites with other ocean physics measurements (wave characteristics, temperature, salinity, etc.) is becoming more commonplace with recent enhanced efforts in ecological monitoring. Such applications include both remote and is situ sensing of ocean waters, typically done for a passive time-independent surface illumination, which is the emphasis of the discussion here. The solution of the time-dependent transport equation is needed, on the other hand, for active illumination imaging applications such as mine detection or communication by pulsed optical signal propagation.

The types of in-water problems being solved with the classic radiative transfer equation include (a) forward problems for determining the angle-dependent and angle-integrated light field, and (b) inverse problems for determining, for example, (c) the scattering and absorption properties in order to monitor the biological primary production of water or the ecology of coral reefs.

Good introductions to radiative transfer for oceanographic forward problem applications are available. The text by Kirk [323] is for people more interested in the biological aspects of radiative transfer. Mobley [222] gives details of how ocean optical transfer calculations can be done, as sell as a physicist's approach to biological oceanography. The text by Thomas and Stamnes [102] and the monograph by Walker [325] each contain an introduction to atmospheric and oceanic optics; with the former giving more details about atmospheric line-by-line radiative transfer calculations and the latter containing an extensive coverage of sea-surface and refracted light statistics. The monograph by Bukata et al. [321] pertains to water optical properties. For oceanographic inverse problem applications, an excellent review article is that by Gordon [322]. The key features (McCormick [49]) of optical transport in seawater are: Light in the 400 – 700 nm range is of primary interest, where most biological activity occurs and the optical transmission is the greatest; ocean waters are difficult to characterize because: a) they consist of water plus dissolved organic and inorganic matter not well-characterized as to type and location, b) the "microscopic cross sections," from which the effective absorption and scattering properties are determined, can only be idealized (e.g., with spherical or other regular shapes), c) the air-water interface condition is difficult to simulate because of refractive effects and surface waves; ocean waters usually are modeled with the plane parallel approximation for natural light illuminations because: a) the water column is assumed to be layered (i.e., there are minimal horizontal variations of the water constituents), b) the bottom is assumed flat and of uniform composition or very deep, c) the incident radiation from the atmosphere is assumed to be uniform over the sea surface.

The reasons ocean waters are difficult to characterize are:

A) The wavelength-dependent scattering of phytoplankton depends on the composition and particle size that can vary from approximately $0.7\,\mu m$ to $\sim 100\,\mu m$.

B) The suspended sediments are predominately scatterers, rather than absorbers, although strong absorption features have been observed in iron-rich sediment minerals.

C) Colored dissolved organic matter (CDOM) absorbs most strongly in ultraviolet wavelengths with various absorption peaks.

The similarities to other transport problems include: a) The linear Boltzmann equation for neutral particles is the governing equation, b) Active illumination problems (e.g., with a laser beam) usually are three-dimensional, c) The coefficients of individual, optically-active constituents of seawater are assumed additive. d) Phytoplankton and CDOM have the ability to fluoresce by re-emitting light within distinct wavebands that are somewhat longer than the absorbed light, which means a coupled wavelength analysis may be necessary. e) Raman (inelastic scattering events cause a wavelength increase (i.e., down-scattering in energy). f) Bioluminescence is analogous to an external source from neutron-gamma reactions in gamma transport or spontaneous fission in neutron transport analyses.

The differences from most other linear transport problems include: a) "Inverse Problems" (e.g., to characterize properties of medium) are very important, more so than for analyses of designed systems (e.g., nuclear engineering applications), b) a plane-parallel approximation usually is sufficient for waters illuminated by natural light so the radiance depends only on the water depth and the polar angle and azimuthal angle, c) the optical sensors most often used enable a simplified plane-parallel approximation so the azimuthal angular dependence need not be determined, d) The scattering is very strongly forward peaked (e.g., more like the scattering of atmospheric aerosols than neutrons), with peak forward-to-backward scattering ratios $O(10^5 - 10^6)$ (Petzold [324], Mobley [222]) the refractive index mis-match at the air-water interface causes angle-dependent internal reflection for some surface emerging directions, g) air-water interface waves are hard to characterize, h) problems with polarization effects have four radiance components for each wavelength.

It is worth emphasizing that ocean waters are dynamically changing systems that are hard to characterize, and hence the challenge of obtaining good input data for computations means the old adage "garbage in, garbage out" is a big concern. Furthermore, the environment for performing optical experiments in the field is often difficult, which leads to appreciable measurement uncertainties. For these reasons computational results are not needed with more precision than measurement uncertainties.

1.4. Why polarization:

The solar incident light interacts with all the components of the atmosphere-ocean system. Each phenomenon of scattering by molecules, aerosols, hydrosols and reflection over the sea surface introduces and modifies the polarization state of light. Therefore, the reflected and the transmitted solar radiation in any atmosphere-ocean system are polarized and contain embedded information about the intrinsic nature of aerosols and suspended matter in the ocean. All most all detail physical informations (i.e., size distribution, composition) about the particles present in the atmosphere-ocean system are available through the measurement and analysis of the spectral and angular polarization signature of the oceanic and atmospheric radiation.

For the description of fully polarized light propagating in a given direction Stokes vector convention is adopted in the literature. In the second chapter we have described in detail the Vector Radiative Transfer Equation (VRTE) for atmosphere –ocean system. There are four Stokes parameters which characterize the energy transported by the electromagnetic wave, its degree of polarization, the direction of polarization and the ellipticity. The first parameter can be any energetic quantity, as radiance, an irradiance, etc., and mathematically described as intensity intensity. The other three parameters are defined, as the first one from the two components of the electric vector on two arbitrary perpendicular axes in the wave plane. These quantities have the same energetic dimensions as intensity. Generally, a reference plane is chosen through the direction of propagation parallel to the reference plane and perpendicular to the same plane. The reference plane is taken as the vertical plane containing the direction of propagation.

It has been found that the principal reason for the greater effectiveness of remote sensing by means of polarization measurements is the significantly higher sensitivity of polarization features to particles size, shape and refractive index as a function of scattering angle and wavelength, than is the case for intensity measurements. The strength of polarization features has been widely demonstrated in the case of aerosol retrievals (Goloub et al., [9]; Chowdhary et al.,[10]); Li et al., [11]. However, retrievals of subsurface particulate matter properties using polarization and remotely sensed data have not been extensively studied yet. This is mainly because of practical difficulties in achieving reliable in-situ measurements. Most of the available observations were carried out decades ago Waterman, [13], Waterman [12]; Ivanoff and Waterman, [14]; Beardsley, [15]; Lundgren and Hojerslev, [16]; Voss and Fry, [17]. Another factor that contributes to reduce the number of studies about oceanic polarization is that most current methods of radiative transfer treat light as a scalar. As an example, the commonly used Hydrolight, CDISORT or Morel and Gentili's [370] Monte Carlo radiative transfer models do not account for the polarization of the oceanic radiation.

In general for the most practical situation consisting in coupling between atmospheric and ocean systems, the polarized radiative transfer equation has been developed and solved mainly to improve the modeling of bidirectional remote-sensing reflectance. The increasingly better performance of the new generations of ocean color sensors implies a more sophisticated description of the signal in which the full radiation field, including polarization is accounted for.

One such model (Chami, M. et al. [19]) of radiative transfer for the global ocean–atmosphere system has been developed (OSOA) which can predict the total and the polarized signals at the top of the atmosphere and at the ocean–atmosphere boundary. A previously developed radiative transfer code based on the successive-orders-of scattering was modified to account for various oceanic parameters. In the first part of the research article[19] a description of the new ocean– atmosphere radiative transfer model ,called the OSOA code, is given, where, in particular, attention is given to computation of the polarized component of the signal in the oceanic layer. The second part of the paper contains a nice description of the applications of the code to remote sensing of coastal and open ocean waters. The influence of marine particles on the polarization of water-leaving radiance is discussed. The radiance and the degree of polarization in the coupled ocean-atmosphere system helped in the study of the polarizing properties of the marine particles (namely phytoplankton and minerals) for different water conditions. Their analysis revealed that the use of the polarization of scattered energy in ocean color algorithms might significantly improve the retrieval of hydrosol properties, especially in coastal waters, [367].

Various techniques have been proposed to obtain numerical solutions of the equations of radiative transfer including multiple scattering. One of the most successful techniques, which have been used so far for modeling a system as complicated as coupled atmosphere-ocean system prevailing on earth, is the Monte-Carlo method. Later we have devoted a section on this method only.

Of the three fundamental properties of light intensity (or the rate of photon arrival), wavelength or spectrum (often interpreted as hue or color) and polarization, polarization is the least known to the general public. This is because humans are mostly insensitive to the polarization characteristics of light (although we use them in sunglasses, computer screens, digital displays, etc.). However, many animals, terrestrial and marine, are sensitive to the polarization of light and make use of this polarization sensitivity for a variety of tasks. In a simplified way, polarization can be described as the distribution of the planes of vibration (the orientation of polarization) of the electrical (or magnetic) fields of the electromagnetic waves within a light beam. Partially linearly polarized light can be conceived as a mixture of fully linearly polarized light, with the plane of vibration of its electric vector (e-vector) at a given orientation (angle), called the e-vector orientation, combined with fully depolarized light, having random e-vector orientations. The fraction of the fully polarized component is the partial polarization, often represented as a percentage (% polarization). Thus, linear polarization has three descriptors: intensity, partial polarization and orientation of polarization.

The distribution of polarized light in underwater condition is predominantly affected by the position of the sun (or the moon) in the sky, the optical properties of the water, the depth of viewing and reflections from surfaces, such as the sea floor or the surface of the water (Waterman, [12], Waterman [21]; Waterman and Westell, [22]; Tyler [23]; Timofeyeva, [24], Timofeyeva, V. A., [25]; Novales–Flamarique and Hawryshyn, [26]; Cronin. T., and Shashar, N. J., [27]). Measure-ments performed at depths of 5–6 m by Waterman [28] revealed that underwater there are two distinct polarization patterns, one inside Snell's window and one outside it. Generally, the polarization pattern inside Snell's window down to depths of a few meters is assumed to be determined by the same factors as those influencing the sky polarization. Therefore, sun position, amount of overcast, amount of atmospheric dust, the distance of the point observed from the zenith, multiple scattering, and depolarization due to anisotropy of air molecules will all influence the polarization pattern within Snell's window (Waterman [29], Eddington, A. S., [30]. [31] Horvath. G., and Varju D, [31]) modeled the underwater polarization pattern within Snell's window as it correlates to the celestial polarization pattern, taking into account refraction and repolarization of skylight at the air–water interface. However, due to the focusing and defocusing of sunlight by surface waves (Sehunck. H., [32]; Snyder and Dera, [33]; Stramska and Dickey, [34]; Maximov, [35]) and changes in polarization as the light propagates in water, certain distortions may well occur. Indeed, Cronin, T and Shashar, N. J. [27], measuring polarization at a depth of 15 m on a coral reef, found only small differences between the polarization patterns within Snell's window and outside it.

Underwater, factors such as turbidity, bottom reflection (Ivanoff and Waterman, [37]) and proximity to the shore line (Schwind, [38]) may diminish the percent polarization. In shallow waters, the percent polarization first decreases with depth (Ivanoff and Waterman [37]) and then reaches a depth-independent value (Timofeyeva, [40]). Assuming primarily Raleigh scattering, Waterman and Westell [22] proposed a model for the effect of the sun's position on the *e*-vector orientation outside Snell's window (see also illustration in Hawryshyn, [42]). However, with increasing depth, the pattern of *e*-vector orientation simplifies rapidly, tending to become horizontal everywhere (Waterman [12]; Tyler [23]; Timofeeva [24]). It also diverges from the predictions of Waterman and Westell's model (Waterman and Westell, [22]) suggesting an effect on polarization of other, non Raleigh, modes of scattering and of post-scattering processes.

1.5. Mathematical Methods in Treating Radiative Transfer Theory:

It is almost impossible to differentiate between mathematically purely analytical and purely numerical methods applied so far in the radiative transfer theory. Practically no method received attention until some numerically acceptable result is produced. In the beginning of investigation-era the analytical methods proposed were tested numerically to get first estimates of the outcome of the problem at hand. We shall now briefly describe the various important approaches made so far by eminent researchers to solve the RTE in different situations of practical and theoretical interest.

Principle of Invariance: The most fundamental characteristics of the radiation field in dispersive media such as stellar atmospheres, planetary atmospheres, planetary nebulae or 0cean are the diffuse radiation which arises from multiple scattering of radiation by the media. This has been studied through an approach called the principle of invariance, or invariant imbedding, due to Ambartzumian [62], Sobolev [63], Kourganoff [64], Wing [65], and Preisendorfer [66]. An important component of Chandrasekhar's work in radiative transfer was devoted to the development of the Invariant Principles. These principles were first introduced by Ambartzumian [62, 67].

Bellman and his collaborators have published several papers on this subject. The concept was developed by Sir George Stokes [47, 333] in his glass plate theory. In some remarkably simple papers, he derived the transmission and reflection factors when a ray of light passes through a system of glass plates. With such simple theoretical approach he was able to formulate the principle of invariance of reflectance when several glass plates are arranged parallel to each other, one on top of the other. He obtained difference equations for the reflection by a pile of identical glass plate and derived certain commutation relations for sets of glass plates. It is remarkable that he was able to obtain transmission and reflection factors which look similar to those obtained in more complicated media such as stellar atmospheres (Hottel, H. C., and Sarofim, [69]).

Ambartzumian's principle of invariance has been an essential tool for solving radiative transfer problems in semi-infinite homogeneous atmospheres and it was subsequently generalized by Chandrasekhar [54] to solve problems in finite homogeneous atmospheres. The idea of addition of layer of arbitrary optical thickness to a semi-infinite layer was proposed by Ambarzumian. The reflection characteristics remain invariant in such situations. This was taken up by Chandrasekhar and solved several problems which looked formidable until then. The emergent intensities are obtained through the H-functions in a semi-infinite medium and **X** and **Y** - function in finite medium. However, these are obtained by solving certain integral equations by iterative means. In plane parallel media, these **X** and **Y** functions have been tabulated for isotropic and anisotropic phase functions. The tables have a restricted use in general cases, such as inhomogeneous media or moving media. It would be ideal if one need not specify the thickness of the layers which are to be added. Layers with general properties can be added and their transmission and reflection properties can be calculated directly by utilizing what is known as Interaction Principle (see Redheffer, [71]; Preisendorfer, [66]; Grant and Hunt [68, 73]). The physical and geometrical properties of the layers would automatically determine the reflection and transmission properties. As there are no conditions imposed initially either on the size of the layer or its geometry or its physical properties, any in homogeneity can be introduced into the scheme at any point. One can easily obtain the internal radiation field in any given dispersive medium. This has been applied in non- stationary media such as the outer layers of supergiants stars, supernovae. The Interaction Principle which is a generalized form of the invariance principle is nothing but the manifestation of the conservation of the radiant flux. It balances the emergent radiation with that of the reflected and transmitted input radiation together with the internally generated radiation.

One of the most surprising long-term implications of the invariance principles has been their generalization and development into an entire mathematical field known as "invariant imbedding" by Bellman, R. E., Wing, G. M. [167]. This development was based on the recognition that the invariance principles had converted what was a boundary value problem (involving boundary conditions at two or more points) into an initial value problem (involving boundary conditions at a single point) through the introduction of the reflection and transmission functions and the integro-differential equations they satisfy. The invariant imbedding methods generalize this idea of transforming from a boundary value problem to an initial value problem to a much wider class of problems than just radiative transfer, including wave propagation and control theory, among others. This was an important practical advance, since initial value problems are generally much more numerically tractable than boundary value problems. In the introduction to their work, Bellman and Wing [167], after crediting earlier workers, clearly express the special influence of Chandrasekhar in the development of their methods.

Invariant Imbedding:

Invariant imbedding has been used extensively to solve astrophysical radiative transfer problems as well as in radiation dissymmetry calculations and radiative transfer in the atmosphere, ocean and atmosphere. This is the method of converting certain two-point boundary value problems to initial value problems. It is an outgrowth of the method of invariance, introduced by V.A. Ambarzumian and used successfully by Chandrasekhar's. in radiative transfer problems. Invariant imbedding is closely related to the sweep method of solving the differential equation. The radiative transfer equation may be classified as a linear transport equation that must be solved subject to boundary conditions at the top and the bottom of the medium. Mathematically, we refer to this problem as a linear two-point boundary value problem.. From a mathematical point of view this essentially amounts to the transformation of a "difficult" yet linear two-point boundary value problem into a set of "simpler" but partly nonlinear initial-value problems. From the general point of view this theory can be interpreted as the result of two different conceptual approaches to transport theory. One of these is considered as a physical approach whereby a particle counting procedure is developed to the basic physical concepts. These basic ideas of invariant imbedding can be traced back to an insight by the astrophysicist Ambarzumian [62]. The subsequent development of the theory and its application to radiative transfer problems was made by Chandrasekhar, Bellman, Preisendorfer, and others. The fullest exposition of invariant imbedding as applied to hydrologic optics is seen in Preisendorfer six-volume treatise Hydrologic optics (Preisendorfer [75]). They are applicable to the solution of the RTE including internal sources, depth-dependent inherent optical properties (IOP), arbitrary incident radiances, wind-blown air-water surfaces, and a finite or infinite-depth bottom. The only restriction is that of a plane parallel geometry. But there exist several simplicities. All quantities are computed with the same accuracy. In particular, there is no statistical noise in the numerical results. The methods are mathematically elegant and provide deep insights into the internal structure of radiative transfer theory. Many profound relationships are revealed among the building blocks of the theory. The methods are computationally efficient. The solution algorithms are fast and numerically stable. Moreover, computation time is a linear function of optical depth. The price one pays for the above benefits is mathematical complexity. Invariant imbedding methods require a considerable amount of mathematical development in going from the RTE to its solution, and the associated computer programming is much more tedious.

The first work with invariant imbedding in its title was by Bell man and Kalaba [76] where they combined the principles of invariance with the functional multistage processes of dynamic programming of Bellman [77]. A series of papers by Bellman and his co authors appeared showing the invariant imbedding formulation ands application to neutron transport problems with other particle transport processes. The summary paper is Bellman, Kalaba and Wing [39]. Rosescu [79] made a list of these extended works of Bellman as well as many other authors. An extended series of application papers were appeared (Dodson and Mingle [80], Mingle [81, 82], Timmons and Mingle [83], Kaiser and Mingle [84]), Shimizu and Mizuta [85]). Bellman et al. [86, 87] have studied the radiative transfer problem using the principles of invariant imbedding from the viewpoint of 'particle counting'. Addition of an infinitesimal layer and then counting the first order contributions to transmission and reflection leads to the derivation of integrodifferential equations for transmission and reflection coefficients in the limit as the layer thickness vanishes. The **X**-and **Y**-functions of Chandrasekhar are obtained in a similar way. They obtain a system of simultaneous non-linear ordinary differential equations by replacing the quadratures over angles by Gaussian sums. These are solved by the standard numerical methods such as fourth-order Adams–Moulton techniques or Runge-Kutta methods. Bellman, Kalaba, Prestrud [86] have tabulated the diffuse reflection function with fixed intervals of and for the albedo for single scattering. Kagiwada, et al. [89] has tabulated transmission and reflection factors for conservative isotropic scattering case. Kagiwada and Kalaba [90] estimated the local anisotropic function by using the invariance techniques. The first book containing considerable amount of numerical work was probably by Bellman, Kalaba and Prestrud [86] whereas the second book was by Mingle [92].

The second approach to invariant imbedding is a mathematical one that proceeded in parallel with the particle counting method. This approach transforms the basic transport equation into invariant imbedding form with the help of functional analysis. Wing [18] was first to proceed in this approach and details may be obtained from Bailey and Wing [48, 55, 94]. In addition books such as Bellman [95] and Denman [96] have sections based upon this mathematical procedure.

Discrete ordinate method:

The discrete ordinate method was first introduced by Wick, G. C. [168]. By 1944, fully analytic solutions of RTE had been presented by Chandrasekhar [91] for only a few problems e.g., the solution to the semi-infinite Milne problem by Wiener and Hopf [98], but these methods did not extend to all of the problems that were of interest to Chandrasekhar. This led him to adopt a scheme, introduced earlier by Wick, G. C. [168] that reduced an integrodifferential equation to an approximate, finite set of ordinary differential equations by the introduction of a quadrature scheme into the integral term. Because of Chandrasekhar's subsequent extensive development of this method, it is now often known as the Wick-Chandrasekhar discrete-ordinate method. These largely analytical calculations use the facts that (1) the transfer equation is linear in intensity-at least explicitly, and (2) the replacement of integral over angle / or frequency transforms it into a set of differential equations so that well known theorems of linear algebra and analysis can be applied to study the properties of these solutions. For these reasons Discrete-ordinate method (DOM) is regarded as one of the most useful and elegant approach. In most applications the angular dependency of the radiative transfer equation is discretized in the angular domain and the solution consists of a set of first-order differential equations. The solution of the radiative transfer equation can be derived explicitly and the intensity calculations do not depend on the total optical depth of cloud or aerosol layers. However the situation in particle transport theory is not so .The approaches for discretizing the direction variable in particle transport calculations are the discrete-ordinates method and function-expansion methods. Both approaches are limited if the transport solution is not smooth. Angular Discretization errors in the discrete-ordinates method arise from the inability of a given quadrature set to accurately perform the needed integrals over the direction ("angular") domain. Analytic two-stream and four-stream solutions can be derived in closed forms for cases of radiative transfer in atmospheric, astrophysical or oceanic contexts. Computational times are relatively small compared to other techniques. However, polarization effects are not included in the earlier works. Munch [100] applied this method in radiative transfer processes in stellar atmosphere considering broadening of spectral lines by electron scattering.

In a classic paper Chandrasekhar [93] explicitly formulated the equations of transfer for the two components of the polarized radiation field in a free-electron stellar atmosphere. The Thomson scattering of radiation by free electrons was recognized to be an important mechanism in the transfer of energy in a certain class of stars. This fact made necessary a more detailed description of the scattering laws in the formulation of the basic equations. In Chandrasekhar, S. [54] provided the new theory and presented an approximate solution for the outgoing angular distribution of the polarized light. In a following work, Chandrasekhar, S. [54] by passing to the infinite limit in a Wick-Chandrasekhar discrete-ordinate procedure, he was able to solve exactly for the laws of darkening in the Milne problem.

The discrete-ordinates method (DOM) has become one of the most popular methods for solving Boltzmann transport equations for radiation transfer and neutron transport. This is because the DOM can be accomplished to high-order accuracy, the derivation of DOM schemes is relatively simple, and the DOM is compatible with the finite-difference or finite-element schemes for specular or diffuse phenomena. Studies on almost every aspect of the DOM applicable to multidimensional radiative heat transfer have been reported. [168, 362-364] However, most of the previous DOM algorithms focused on the solution of steady-state RTE because the effect of time-dependent light propagation is negligible in traditional heat-transfer problems.

In most applicable astrophysical context the RTE in frequency domain is subject to the discretizationin either in angle and/or frequency variable. Intensive calculation in unpolarised regime with frequency dependence reveal the fact that general solution contains –in addition to terms which decreases sometimes exponentially with increasing optical depth particularly in finite media where coefficients are to be evaluated explicitly from the boundary conditions at two ends from a system of ill-conditioned algebraic equations. This disadvantages leads to the search for efficient algorithm for better numerical evaluation Mihalas, [173], Cannon, [174]. However despite several attempts several difficulties aroused particularly in the treatment of anisotropic media. By means of matrix formulation of discrete ordinate method (Wehrse [175], Wehrse & Kolkophen [176], Schmidt & Wehrse, [177]) the problem of increasing exponentials can be solved analytically and it was found that the resulting equations are numerically stable.

As indicated earlier computer implementations of this method were, however, plagued by numerical difficulties to such an extent that researchers made little use of it. Analytic two-stream (only two directions) solutions of the discrete ordinate equations have been worked out in detail and applied to a number of atmospheric and ocean scenarios (for a summary, see Thomas, G. E., Stamnes, K., [102]).The two-stream approximation dates back to 1905 when Schuster initiated the studies on the solution of RTE with an albedo in the context of simplest equation of radiative transfer A. Schuster [41] by considering the two directional streams of the radiation field there by converting the original integral equation into asset of two coupled differential equations with suitable breaking up of the boundary conditions. However Schwarzschild, K., [43, 74] considered the case of conserved scattering. An entertaining description of how the two-stream approximation may be used to explain numerous radiative transfer phenomena is found in Bohren, C. F., [20].

The Eddington approximation originated with A. S. Eddington [30]. The relationship between the two-stream and the Eddington approximation was discussed by Lyzenga, D. R., [133]. The accuracy of the Eddington and two-stream methods for anisotropic scattering was explored by W. J. Wiscombe and J. H. Joseph [178], who found that it was accurate for values of **g** less than **0.5**. This explains why the δ-Eddington and δ-two-stream methods are so valuable: The scaled asymmetry factor is always less than **0.5**. Attempts to combine two-stream solutions for several adjacent slabs with different optical properties date back more than thirty-nine years Shettle, E. P. and J. A. Weinman [326]. These solutions were intrinsically ill-conditioned, because the matrix (that had to be inverted to determine the constants of integration in the problem) contained a combination of very small and very large elements resulting from the negative and positive arguments of the exponential solutions. In the δ-Eddington method, subdivision of layers was employed to circumvent the ill-conditioning, but at the expense of increasing the computational burden substantially for thick layers (Wiscombe, W. U., [365]). The ill-containing problem was eliminated by a scaling transformation that removed the positive arguments of the exponential solutions (Stamnes, K. and P. Conklin [108]). This scaling transformation was eventually implemented into a general-purpose multistream (including two-stream) radiative transfer algorithm by Stamnes, K., S. C. Tsay, W. J. Wiscombe and K. Jayaweera [101]. The resulting code has been made generally available to interested users. A specific two-stream code that made use of this scaling transformation to remove the ill-conditioning has been developed (Toon, O. B, McKay, C. P., Ackerman, T. P., and Santhanam, K., [327]). Finally, a two-stream algorithm derived from the general-purpose multistream algorithm mentioned above has been extended for application to spherical geometry and to layers in which the internal source may vary rapidly (Kylling, A., K. Stamnes, and S. C. Tsay [328]). This two-stream code is generally available to interested users. Modern discussions of the two-stream method are due to Meador, W. E. and Weaver, W. R. [329]; Zdunkowski, W. G., Welch, R. M., and Korb, G., [330]; King, M. D., and Harshvardhan [331]; and Harshvardhan and King, M. D. [332].

A limited number of analytic 4-stream solutions have also developed Liou, K. N., [103]. With the advent of large and powerful computers in the 1970s, it became possible to develop multiple scattering RT models for multi-layer atmospheres. This was done for the 2N-stream plane-parallel discrete ordinate model by Knut Stamens and co-workers in a series of papers from 1980 onwards culminating in the release of the DISORT plane-parallel radiative transfer package in 1988 [101]. A new implement-tation of the discrete ordinate method for vertically inhomogeneous layered media which is free of these difficulties and to give a summary of its equations and its various advanced features DSORT (Stamens et al. [101]) was developed The resulting computer code represents the culmination of years of effort Stamnes et al. [104-109,101,110] to make it the finest available algorithm with the intension that the code be so well documented so versatile and error free so that other researchers can use it safely both in data analysis or as a component of large class of models. It solves the RTE for a scattering, absorbing, and emitting medium with an arbitrarily specified bidirectional reflectivity at the lower boundary. For multilayer applications one can see the article [241].

DISORT is the most widely used RT code available to the atmospheric community. It is a generic scattering formalism that does not require direct specification of atmospheric constituent inputs and their optical properties at the microphysical level. Instead, it is only necessary to specify three optical inputs for each layer - the total single scattering albedo, the vertical optical thickness and the total phase function moments.

The philosophy behind the DISORT work was to build a general-purpose and flexible radiative transfer package that could be used in a wide variety of atmospheric applications. The model applies to a plane-parallel medium and includes both thermal and solar beam sources. The model is called as a subroutine within an environment which the user tailors to his or her specific needs. The user will then create the DISORT inputs from the set of atmospheric constituents and parameters appropriate to the application. A pseudo-spherical version SDISORT has been developed [110], but unfortunately this has not been packaged as a general-purpose tool in the same manner as DISORT itself. The delta-M scaling is standard in DISORT. A second version of the code incorporates the single scatter correction procedure of Nakajima, T. and Tanaka, M., [112].

To extend the application of DISORT code for systems involving two or more media like atmosphere-ocean or atmosphere-cloud-ocean, Jin and Stamnes [113], developed CDISORT which account for the changes in the refractive indexes at the flat separating boundaries between the media. The study was extended for rough ocean surface.

On the basis of the linearization analysis, the numerical model LIDORT (Linearized Discrete Ordinate Radiative Transfer) has been developed and tested. The philosophy adopted for LIDORT is the same as that for DISORT - to make a general-purpose and flexible radiative transfer package that could be used in a wide variety of atmospheric applications, not just for simulating intensity, but also for generating weighting functions that are necessary in so many retrieval applications. Like DISORT, LIDORT is a subroutine called from a user-defined environment. LIDORT too is a scattering formalism; it does not need to know the number and nature of the atmospheric gases and particulates. Validation of LIDORT is straightforward: radiances may be compared directly with DISORT and SDISORT output, while weighting functions are validated using the finite-difference estimation and choosing optical depth small enough.

With the advent of the ultrafast laser and its broad applications in biomedical technologies, the study of time-dependent laser radiation transfer incorporating radiation propagation with the speed of light has become increasingly important. Recently the time-dependent DOM method has been explored to one-dimensional [118, 119] and multidimensional geometries. [120, 121] Yet the media were not characterized tissues. Fresnel's reflection was not taken into account, nor was the ultrafast laser pulse considered in these studies. Klose et al [122] used a DOM algorithm as a forward model for optical tomography. But the DOM there is time independent steady state and has only 24 discrete ordinates (equivalent to the S4 quadrature scheme) in the spatial angle direction. The DOM S4 method is easily subjected to the ray effect and false scattering as pointed out by Chai et al. [44] because of the limited number of discrete ordinates. Instead, high-order quadratures such as S8 (80 discrete ordinates) and S10 (120 discrete ordinates) are more commonly adopted in the radiation heat-transfer community. These high-order schemes have been applied to the study of ultrafast-laser-radiation transfer in anisotropically scattering, absorbing and emitting heterogeneous tissues in three dimensionally geometry. Guo, Z., and Kim, K., [123], Optical society of America.

Over the last fifty years polarized light travel through scattering media has been studied by the atmospheric optics and oceanography community in particular. An exact solution of the radiative transfer for a plane-parallel atmosphere with Raleigh scattering was derived by Chandrasekhar in [54]. More complicated geometries proved too complex to be solved analytically. He has also developed principle of invariance for both polarized and nonpolarised radiation field which serves as basic and fundamental in formulating physical interpretation of radiative transfer equation.

But if the discrete ordinate method has fallen out of favor for analytical radiative transfer, it remains to this day a very strong component of numerical work in stellar atmospheres and other astrophysical applications, because of its simplicity, accuracy, and adaptability to complex physical situations. In this way, the method continues to serve those seeking practical solutions to real physical problems, as Chandrasekhar himself was.

Cases Eigen function approach:

The discrete ordinate method was both Chandrasekhar's most transient and his most permanent contribution to the field. After DOM, the development of analytical radiative transfer rapidly moved toward full treatment of the angular dependence of the solutions, rather than discrete versions. This could already be seen in Chandrasekhar's own work, where he gradually (but not completely) shifted from the discrete ordinate method to the invariance principles to enable him to deduce analytical the structure of the solutions. Perhaps the most interesting development in this context was the singular eigenfunction method used in plasma physics by Van Kampen [134], and later applied to transfer theory by Case and others (see, e.g., Case & Zweifel [135]). In a sense, this is the true descendent of the discrete ordinate method, since it also starts by asking for solutions of exponential form, but now confronts the true nature of the continuous angular dependence in the scattering integral.

In this technique some singular eigenfunctions with specified eigenvalues are used to expand the unknown intensity. These normalized eigenfunctions are found to satisfy certain orthogonality and completeness conditions depending on the range of angular integrations. Analytical expression for singular and continuous eigenfunctions for both types of eigenvalues (for eigenvalues lying within the range of integration, another for lying outside the region) can be evaluated using orthogonality, normalization and completeness properties of the eigenfunctions. In radiative transfer this method of normal modes provides an elegant and systematic approach to the solution of one-dimensional, plane-parallel radiative transfer problems enabling the desired solution be written as a linear sum of the eigenfunctions of the homogeneous equation and a particular solution appropriate to the source function of interest. The solution to the problem is thus reduced to that of determining the unknown expansion coefficients appearing in the sum of elementary solutions These coefficients are determined by constraining the solution to meet the given boundary conditions and by then utilizing the orthogonality properties of these Case eigen functions. This procedure is completely analogous to the classical orthogonal expansion treatment of boundary value problems. This method is suitable when solution at any optical depth is sought. As a consequence of the method of singular Eigen solutions, a large class of problems in both radiative transfer and neutron-transport theory have become amenable to exact, closed form solution. The principal advantage of normal mode expansion technique is that solutions valid at any depth are obtained as well as results for various surface quantities. An additional merit is that determination of expansion coefficients requires minimum manipulations.

Case's normal-mode expansion technique Case, K. M., [136] is used to obtain solutions to the radiative transfer problem for an absorbing, emitting, isotropically scattering, nonisothermal gray medium bounded by specularly reflecting, diffusely emitting, gray parallel walls each held at uniform but different temperatures. Siewert, C. E and McCormick, N. J., [137] obtained a rigorous solution for an absorbing, emitting, anistropically scattering, semi-infinite medium with a linear source function and a free boundary. One of the earlier applications of the singular eigenfunction expansion technique to problems in finite geometry was made McCormick, N. J. and Mendelson, M. R. [138] who treated the slab albedo problem. Ferziger, J. H. and Simmons, G. M., [139] solved the radiative transfer problem for a non absorbing, non-emitting, perfectly scattering medium (or alternatively for a gray medium in radiative and local thermodynamic equilibrium) bounded by black heated parallel walls. Typical of transport problems with two boundaries, the results of Ferziger, J. H. and Simmons, G. M. [139] were not expressed in closed forms; however, they have shown that their analytical approximate solutions were highly accurate. Recently Heaslet, M. A. and Warming, R. F., [140] considered non-conservative radiative transfer in semi-infinite and finite media. The solution to the problem is thus reduced to that of determining the unknown expansion coefficients appearing in the sum of elementary solutions These coefficients are determined by constraining the solution to meet the given boundary conditions and by then utilizing the orthogonality properties of these Case eigen functions. This procedure is completely analogous to the classical orthogonal expansion treatment of boundary value problems.

In Case's eigen function approach ultimately one encounter a system of singular integral equations which are difficult to solve analytically and numerical implementations also require considerable cost. Siewert and Benoist [141] and Grandjean and Siewert [142] employed **Fn** method in which the unknown intensities appeared as integrand in the above mentioned equations is expanded in a series in angular variable with unknown coefficients. These coefficients are then determined from the resulting system of algebraic equations with coefficients which also involve some integration with singular eigenfunctions as integrand. These coefficients are found to obey certain recurrence relations from which they can be found easily. The values of the last coefficients are used to find the values of the expansion coefficients. However in doing so one has to choose discrete values for the continuous eigenvalues spectrum thus keeping the chance of affecting the numerical calculations in accuracy alive. Several authors Devaux and grandjean, [143], use several schemes to choose the eigenvalues depending upon the problems encountered. This method is considerably simpler than any other method. It has been shown that the desired numerical accuracy is achieved with few orders of terms in the expansion even in the cases of polarization studies in this method. (Siewert, C. E., [144] astrophysics & sp. science, Grandjean and Siewert, [145], Siewert, [144], Siewert and Benoist, [151], Siewert, (jqsrt), [150]. Despite its tremendous success in radiative transfer theory, neutron transport calculations and gas dynamics no attempt was seen so far as our knowledge is concerned to expand this technique in coupled atmosphere-ocean system.

Discrete Space Theory: As mentioned earlier Preisendorfer, R. W., [66], Grant, I. P. and Hunt, G. E., [73] generalizes the interactions principles into the invariance principles particularly in a finite medium. The basic idea of the interaction principle is to specify the radiation field in terms of the transmitted and reflected radiation at any given point in the medium.

Carlson [127] and Lathrop and Carlson [128] used a numerical version of the discrete ordinate technique in neutron reactor calculations. By integrating the radiative transfer equation over a finite volume in space coordinates and using the mean value theorem of integrals, we can develop difference equations that conserve flux. These difference equations are of quite general use in non-uniform media and curvilinear coordinate systems. This system of equations is solved by iterative methods. As these equations are the expression of the conservation of flux, invariance principles can be expected to be deduced from them. One needs to study the errors and stability factors of any system of equations. Carlson's S_n methods did not have a sell studied error and stability analysis. This can be overcome by rewriting equations in what is called 'invariant S_n' form. In this way, one can test the stability and estimate the errors due o truncation and round-off of the terms. The reflection and transmission operators can be expressed in the form of matrices. The matrix structure allows us to perform the desired analysis and to obtain an explicit solution which essentially expresses the results in terms of the Green's function of the transport operator which is related to the probability of quantum exit as defined and exploited by

Ueno [129]. The matrix structure is the discrete equivalent of the equation of Rybicki, G. B. and Usher, P. D., [130] and converges to it when we pass the limit of infinitesimally these segments. Discrete space theory Hunt, G. E., and Grant, I. P., [336] is actually a slight variation of doubling and matrix operator theory which will be described next.

The Doubling-Adding and the Matrix Operator Methods:

We shall now discuss a method that has been widely used to solve radiative transfer problems in planetary atmospheres. In this method doubling refers to how one finds the reflection and transmission matrices of two layers with identical optical properties from those of the individual layers, while Adding refers to the combination of two or more layers with different optical properties.

The doubling concept is rather old and seems to have originated with Stokes. It was rediscovered and put to practical use in atmospheric science by Van de Hulst and others.

The doubling concept seems to have originated in 1862 (Stokes, G., [333]). It was introduced into atmospheric physics one century later (Twomey, S., Jacobowitz, H., and Howell, J., [334]; Van de Hulst, H. C., and Grossman, K., [335]). The theoretical aspects as well as the numerical techniques have since been developed by a number of investigators; for references see, e.g., Wiscombe, W. J., [337].

The doubling method as commonly practiced today uses the known reflection and transmission properties of a single homogeneous layer to derive the resulting properties of two identical layers. To start the doubling procedure the initial layer is frequently taken to be thin enough that its reflection and transmission properties can be computed from single scattering. Repeated "Doublings" are then applied to reach the desired optical thickness. The division of an inhomogeneous slab into a series of adjacent sub-layers, each of which is homogeneous, (i.e., optical parameters do not vary with depth) but is principle different from all the others, is usually taken to be identical to that discussed previously for the discrete-ordinate method. The solution proceeds by first applying doubling to find the reflection and transmission matrices for each of the homogeneous layers, whereupon adding is subsequently used to find the solution for all the different layers combined.

The Spherical-Harmonics Method:

We have already discussed the doubling-adding method and how it is closely related to the discrete-ordinate method despite being seemingly quite different in concept. Another method that is closely related to the discrete-ordinate method is the spherical- harmonics method, which starts by expanding the intensity in Legendre polynomials in angular variables while the space dependency is expressed through some space dependent coefficients to be determined by a set of differential equations. This type of expansion was first suggested by Eddington in 1916 and led to the widely used Eddington approximation by retaining only two terms in the expansion. The spherical harmonics method (SHM) is the spectral analogue of the discrete ordinates method. Much in the way one uses sines and cosines to represent functions with Fourier transforms, one uses spherical harmonics (Legendre polynomials in their simplest form) to represent the specific intensity and scattering phase function. Just as the orthogonality of sines and cosines makes the Fourier transform useful for solving differential equations, the orthogonality of the spherical harmonics makes it easier to solve RTE equation. One of the most important developments are by Karp et al. [276]. A very nice review article was written by Karp [276a].

It can be shown that the Eddington and two-stream methods are closely related. Since the generalization of the two-stream and Eddington methods leads to the discrete-ordinate method and the spherical-harmonics method, respectively, it is perhaps not surprising to learn that these two latter methods are also closely related. One reason for this similarity is that the spherical-harmonics method relies on an expansion of the intensity in Legendre polynomials, while the discrete-ordinate method relies on using quadrature, which in turn is based on approximating the intensity with an interpolating polynomial that makes essential use of the Legendre polynomials to achieve optimum accuracy. The difference between the two methods lies mainly in the implementation of boundary conditions. As we have seen, this is quite straightforward in the discrete-ordinate approach, but it appears somewhat more cumbersome in the spherical-harmonics method in which moments of the intensity are specified at each boundary instead of specifying the intensity in discrete directions as is done in the discrete-ordinate method. In spite of this difficulty the spherical-harmonics method has been developed into a reliable and efficient solution technique.

Long before Chandrasekhar proposed DOM, Jeans [97] used what we call today SPH method. The key idea (1*, 2*) is to replace the intensity or radiance function and phase function in orthogonal polynomials such that the RTE is replaced by a set of differential equations which can be converted easily to matrix differential equation. Solutions come out as matrix exponentials which are difficult to calculate. However knowledge of physical problems and use of linear algebra [276] allows one to obtain numerically accurate solutions. Another approach is to expand the unknown intensity in a series Karp, A. H., [277] of the following form (1*)

$$I(z,\mu) = \sum_{n=0}^{n} f_n(z) P_n(\mu), \qquad (1^*)$$

$$P(\mu,\mu') = \sum_{k=0}^{K} \beta_k P_k(\mu) P_k(\mu'), \qquad (2^*)$$

$$I(z,\mu)=\sum_{k=0}^{K}\frac{2k+1}{2}P_k(\mu)\sum_{j=1}^{J}[A_j e^{-\lambda_j z}+(-1)^k B_j e^{-\lambda_j(z_0-z)}]g_k(\lambda_j), \quad (1^{**})$$

Where A_j and B_j are constants determined by the boundary conditions and $g_k(\lambda)$ are the Chandrasekhar polynomials defined by 3-term recurrence

$$(k+1)g_{k+1}(\lambda)=\frac{2k+1-\omega\beta_k}{\lambda}g_k(\lambda)-kg_{k-1}(\lambda), \quad (2^{**})$$

The λ_j in equation (1**) Are the inverse of the positive roots of $g_{k+1}(\lambda)=0$.

with unknown coefficients determined by boundary conditions. The Chandrasekhar polynomials are defined by three terms recurrence having the following structure. Substitution of (1*) in homogeneous RTE and use of orthogonality relations satisfied by Legendre polynomials and (2**) yield set of algebraic system of equations which can be solved to obtain the unknown coefficients. Particular solutions are given for solar illuminations Benassi, M. et al. [278], diffuse and specular reflective boundaries McCormick, N. J. and Siewert, C. E., [279] and thermal radiations Barichello, L. B. et al. [280], as well as for polarization problems Siewert, C. E. and McCormick, N. J., [281] and spherical symmetry Siewert, C. E. and Thomas, J. R. Jr. [282].

Several other solution methods in SHM have been proposed since the linear algebra technique is published. Unfortunately most of these methods do not achieve the numerical accuracy as the initial researchers achieved. However each of them [283, 284, 285] has some interesting perspective worthy of attention. A combination of spherical harmonics and discrete ordinate method was developed by Evans [250] in three dimensional atmosphere problem. In this research spherical harmonic representation of angular variable to reduce the memory requirements. The transfer equation is integrated along discrete ordinates through spatial grid to model streaming of radiations. The solution method is of type successive order of scattering approach with adaptive grid approach to improve accuracy.

Several papers have appeared that examine the mathematical and computational properties of some aspects of the SHM. In case of reflecting boundary Settle [286] improved the Dave's [285] approach to evaluate certain integrals involving Legendre polynomial as integrand by introducing a 5-term recurrence relation for the generalization of the integral and thus enabling any form of reflectivity to be expressed as a polynomial in polar direction. Gander [287] viewed the same integral as integration of a polynomial over a finite interval with a positive weight and can be evaluated with Gauss type integration rule. However this approach is proved to be unstable can be implemented using ORTHOPOL package Gautschi, E., [288].

Chandrasekhar polynomials defined by equation (2**) is another area of interest. Dehesa et al. [289] derived differential equations using Chandrasekhar polynomials and found solutions, in some restricted sense, in the form of polynomials of hypergeometric type. Certain identities are derived using the observation that Chandrasekhar polynomials are the eigenvector of some matrix similar to some tridiagonal matrix Siewert, C. E. and McCormick, N. J., [290]. Application to the case of azimuthal dependence requires great care especially to the higher order terms in Fourier expansion Garcia, R. D. M. and Siewert, C. E., [291].

While the basic model of radiative transfer is useful in many domains, there is a great deal of interest in including other effects. Nuclear engineers want to keep track of neutrons that change energy, the multi-group problem Siewert, C. E., [292]. Atmospheric scientists want to include polarization Garcia, R. D. M. and Siewert, C. E., [293]. Many physical problems are not well represented by plane parallel layers. Everyone would like to find an effective solution in two Evans, K. F., [297] or three dimensions Evans, K. F., [298]. Extension of this method to heat transfer in spherical media Li, W. and Tong, W., [294] was reported. Siewert and Thomas [295] reported a particular solution. Tine et al. [296] used this method in inhomogeneous media. There exists one article [164] that establishes the equivalence between discrete-ordinate and spherical harmonics method. Extension of spherical harmonics method in two and three dimension was developed and experimented with numerical evaluation of Fortran code TWOTRAN by Lathrop and coworkers [116, 117].

1.6. Numerical approaches:

Iteration Methods:

There are several methods of an iterative nature, such as "successive orders of scattering", "lambda iteration" (or Neumann series) or operator perturbation method and "Gauss-Seidel iteration" that have been important both for the development and under standing of multiple-scattering theories. The advantage is that these approaches are physically based and this allows for easy intuitive interpretation of the results. The disadvantages are that they apply only under restrictive conditions such as optically thin media and non-conservative scattering. So we did not try to revisit these methods.

The Feautrier Method:

Another class of methods was first introduced by Feautrier, P., [338]. Feautrier's approach, which has gained prominence and popularity in astrophysics, is based on using symmetric and anti-symmetric averages of the radiation field as the dependent variable. The resulting equations ate discretized in both angle and optical depth and solved numerically using finite-difference techniques. The method was originally used mainly for problems with isotropic scattering, but it has been generalized to apply also to problems with anisotropic scattering. Since then it has been used extensively (e.g., Mihalas, D., [339]; see also articles in Kolkofen, W., [340]). Despite its success in astrophysical radiative problems, it did not received attention in the atmospheric ocean literature.

Integral Equation Approach:
One may convert the integro-differential radiative transfer equation into a Fredholm-type integral equation commonly referred to as the Schwarzschild-Milne integral equation. This approach is particularly appealing in line transfer problems occurring in astrophysics in which isotropic scattering and complete frequency redistribution prevail, since the resulting integral equation becomes angle and frequency independent. Although it can be readily generalized to anisotropic scattering, this approach has not received much in the non-astrophysical literature. The integral equation approach is described by Chandrasekhar [346] and by Cheyney, H., and A. Arking [341], and several investigators have applied this method to solve a variety of problems (e.g., Anderson, D. E., [342]; Hummer, D. F. and Rybicki, G. B., [343]; Strickland, D. J., [344]; Strickland, D. J. and T. M. Donahue, [345]).

1.7. Work done on atmosphere- ocean system:
Radiative transfer in aquatic media is a mature discipline in itself with its own nomenclature, terminology, and methodology. We have assumed that we are dealing with media for which the index of refraction is constant throughout the medium. The coupled atmosphere-ocean system provides an important exception to this situation because we have to consider the change in the index of refraction across the interface between the atmosphere (with $\mathbf{m_r} \approx 1$) and the ocean (with $\mathbf{m_r} \approx 1.33$). We should also point out that radiative transfer in aquatic media is similar in many respects to radiative transfer in gaseous media. In pure aquatic media, density fluctuations lead to Rayleigh-like scattering phenomena. Turbidity (which formally is a ratio of the scattering from particles to the scattering from the pure medium) in an aquatic medium is caused by dissolved organic and inorganic matter acting to scatter and absorb radiation in much the same way aerosol and cloud "particles" do in the atmosphere.
We shall now try to review the basic state of mathematical researches done in radiative transfer processes in atmosphere-ocean system under various boundary conditions. The main difference in solving the radiative transfer equation consistently in such a coupled system, from a solution that treats only the atmosphere is caused by the refractive index change across the air– water interface. This refractive index discontinuity at the air–water interface changes the formulation of the radiative transfer equation and complicates the interface radiance continuity conditions and

therefore gives rise to a different solution of the radiative transfer equation. Several investigators have studied the transfer of radiation in coupled atmosphere ocean system.
For the plane-parallel geometry found in most hydrologic optics problems, there exist solution methods that are vastly more efficient than Monte Carlo simulation. We now begin the development of one of these analytical (meaning deterministic or non-statistical) methods for solving the RTE in one spatial dimension. Many such methods are available; they go by names such as discrete ordinates methods, spherical harmonics methods, iterative methods, matrix methods, and invariant imbedding methods. Van de Hulst [131] gave an excellent descriptive summary of the available solution methods, including the strengths and weaknesses of each. Some analytical methods are of great power and considerable generality. Others were developed for specific problems (such as Rayleigh scattering) and have found little or no application in hydrologic optics. Kattawar [132] has compiled 43 original papers on solution methods applicable to the plane-parallel geometry of interest here. His book is a good place to survey the richness of mathematical methods that has been brought to bear on solving the RTE.
Fraser and Walker [227] assumed a simple model of the ocean-atmosphere system (a standard gas on a smooth ocean) and reported the intensity and degree of polarization. For the same case of a smooth sea surface, Dave [228] and Kattawar et al. [229] conducted computations for more realistic atmospheric models. For a rough sea surface exhibiting the true complexity of the boundary conditions, Raschke [230], Plass et al. [216] and Quenzel and Kaestner [232] solved the problem, but neglected the polarization of diffuse radiation in the atmosphere is well as polarization of the reflected radiation. Ahmad and Fraser [233] and Takashima and Masuda [234] performed complete calculations accounting for the degree of polarization and also presented some limited comparisons. The difficulty of exact radiative transfer calculations for rough-ocean reflection is mainly numerical. Most radiative transfer calculations are made tractable by using Fourier series decomposition of the radiation field as a function of the azimuth. For the case of Lambertian ground or of a smooth sea surface, the boundary condition is compatible with this series expansion. On the other hand, this approach is not easy follow for the case of a rough ocean because of the complexity introduced by the wave slopes.

A detailed theoretical derivation of the radiative transfer equations and their solutions by discrete ordinate method for the case of flat ocean surface were given in Jin and Stamens [113]. The principal difficulty encountered in attempts to model radiative transfer throughout such a system with the discrete ordinate method originates from the bending or refraction of radiation across the interface between the media of different refractive properties. The Fresnel refraction and reflection will affect the form of the radiative-transfer equation and the particular solutions, and the continuity relations at the interface are totally different from the non refractive case. The atmosphere and the ocean are both assumed to be vertically stratified so that the optical properties depend only on the vertical coordinate. To account for the vertical inhomogeneity, the atmosphere and the ocean can be divided into any suitable number of horizontal layers, as required to resolve the vertical structure of the optical properties of each medium. In actual approach, the atmosphere and the ocean are divided into a suitable number of layers to adequately resolve the optical properties of each of the two media. Each layer is taken to be homogeneous, but the optical properties are allowed to vary from layer to layer. For a homogeneous medium, only one layer is required. At the interface between the ocean and the atmosphere (assumed to be flat), Fresnel's formula is used to compute the appropriate reflection and transmission coefficients, and Snell's law is applied to account for the refraction taking place there. The integral term in each of these azimuth-independent equations is then approximated by a Gaussian quadrature sum with terms (streams) in the atmosphere and terms in the ocean, so that there are streams in the refractive region of ocean that communicate directly with the atmosphere and streams in the total reflection region of the ocean. In this way the *integro*-differential equation is transformed into a system of coupled ordinary differential equations that is solved by the discrete ordinate method subject to appropriate boundary conditions. This method has the following unique features: (1) because the solution is analytic, the computational speed is completely independent of individual layer and total optical thickness, which may be taken to be arbitrarily large. The computational speed is directly proportional to the number of horizontal layers used to resolve the optical properties in the atmosphere and ocean. (2) Accurate Irradiances are obtained with just a few streams, which make the code very efficient. (3) Because the solution is analytic, radiances and irradiances can be returned at arbitrary optical depths unrelated to the computational levels. 4) The Discrete Ordinate method is essentially a matrix eigenvalues-eigenvector solution, from which the asymptotic solution is automatically obtained.

Cox and Munk [215] developed the statistical characteristics of reflection by wind driven ocean waves by modeling the sea surface as a collection of individual mirror like surface facets.Modelling the distribution of normals to each surface facets as wind speed dependent Gaussian function, roughness of the wind blown ocean surface have been successfully incorporated in_the radiative transfer problems in atmosphere-ocean system by many authors [215-220]. Extension to cases involving rough ocean surface using the method developed by Jin and Stamens [113] was made recently by Jin, Z., et al. [259]. Using computationally efficient DOM they successfully formulated analytical solutions of RTE in a coupled atmosphere ocean system with rough air-water interface and the results are compared with satellite observations. These studies reveal that surface roughness have significant effects on the upwelling radiations in the atmosphere and downwelling radiations in the ocean.

Radiative transfer calculations are used to quantify the effects of physical and biological processes on variations in the transmission of solar radiation through the upper ocean. Results indicate that net irradiance at 10 cm and 5 m can vary by 23 and 34 W m22, respectively, due to changes in the chlorophyll concentration, cloud amount, and solar zenith angle (when normalized to a climatological surface irradiance of 200 W m22). The thermal and dynamical evolution of the upper ocean is sensitive to the vertical distribution of the solar energy available for ocean radiant heating (Denman [185]; Simpson and Dickey, T. D., [186]; Charlock [187]; Kantha and Clayson [188]; Schneider et al. [189]; Brainerd and Gregg [190]; Ohlmann et al. [191]. A 10 W m22 change in the quantity of solar radiation absorbed within a 10-m layer can result in a temperature change of more than 0.68C month21. Simpson and Dickey, T. D., [186] reported a 0.58C change in mixed layer temperature over a 24-h period due to alteration of the solar attenuation coefficient. Such sensitivity to radiant heating processes demonstrates the need for upper ocean models that accurately represent the spatial and temporal variability in solar radiation transmission. Variations in solar transmission have been described primarily by Jerlov water type (Jerlov [192]), a subjective integer index used to indicate water turbidity, despite the continuous nature of solar attenuation (Kraus [193]; Paulson and Simpson [194]; Zaneveld and Spinrad [195]; Paulson and Simpson [196]; Woods et al. [197]; Simonot and Le Treut [198]). Models that rely upon continuous, measurable, physical and biological quantities on which solar transmission depends have been developed only recently (e.g., Morel [199]; Morel and Antoine [200]; Ohlmann et al. [201]). These models use the upper ocean chlorophyll concentration and, in one case, the cosine of the zenith angle of the in-water light field to describe solar attenuation. The models have been built upon existing bio-optical parameterizations because data sets with coincidentally measured optical, physical, and biological parameters are limited (Smith and Baker [202]; Morel [199]) To further improve ocean radiant heating rate parameterizations, a thorough understanding of relationships between solar transmission and the factors that regulate its variations must be developed. Chlorophyll concentration, cloud amount, solar zenith angle, and wind speed all influence solar transmission by altering the in-water solar flux divergence and the sea surface albedo. The quantity of attenuating materials, generally inferred from chlorophyll a concentration, has been shown to be the primary regulator of in-water solar transmission on mixed layer depth scales (Smith and Baker [202]; Siegel and Dickey [203]; Morel [199]; Lewis et al. [204]; Siegel et al. [205]; Ohlmann et al. [191]). However, the effect of chlorophyll biomass on solar transmission within the upper few meters (where a significant portion of solar energy exists outside the visible wavebands) is not well characterized. Clouds play a role in shaping the spectral composition of the incident irradiance (Nann and Riordan [206]; Ohlmann et al. [201]; Siegel et al. [207]) and influence the geometry of the incident light field (Liou [208]). In a later study by Siegel et al. [207] showed the radiant heating rate for the upper 10 cm of the ocean, normalized by the total incident irradiance, can decrease by 50% in the presence of clouds. Solar zenith angle can affect transmission through changes in the light field geometry. Dependence of the vertical decay of irradiance on sun angle has been illustrated for clear sky conditions using Monte Carlo simulations (Kirk [209]; Gordon [210]). Solar zenith angle and wind forcing of the sea surface have been shown to effect in-water radiative transfer through modification of the surface albedo (Payne [211]; Simpson and Paulson [214]; Katsaros et al. [212]; Preisendorfer and Mobley [213]. The relationships between solar transmission and chlorophyll concentration, cloud index, solar zenith angle, and wind speed must be quantified to determine the proper set of parameters for improved solar transmission parameterizations.

The HYDROLIGHT radiative transfer numerical model solves the radiance transfer equation for a plane parallel environment. A complete description of HYDROLIGHT is given in Mobley [222]. To solve the radiative transfer equation HYDROLIGHT discretizes the set of all directions into a finite set of quadrilateral regions, or quads, bounded by lines of constant directions both in azimuth and polar. Such a partitioning scheme is adequate for resolving changes in solar zenith angle and for the introduction of diffuse light due to clouds. When this directional discretization is applied to the radiance equation the fundamental quantity computed by HYDROLIGHT becomes the radiance averaged over each quad. Integration over all directions becomes a sum over all quads. Wavelength is similarly decomposed into finite wavelength bands. Invariant imbedding theory and Fourier analysis are used to reduce the set of equations for the quad- and band-averaged radiance to a set of Riccati differential equations governing transmittance and reflectance functions. Depth integration of Riccati equation by high order Runge-Kutta algorithm and incorporation of the boundary conditional the sea surface and bottom leads eventually to the radiances at desired levels A full description may be found in Mobley [221], Mobley and Preisendorfer [361]. Solution of these differential equations eventually gives the spectral radiance as a function of depth, direction, and wavelength. Inputs to HYDROLIGHT are absorption and scattering properties of the water column, which determine the beam attenuation coefficient and volume scattering function, the radiance distribution incident at the sea surface; and the wind speed, from which the sea surface roughness is computed. The standard version of HYDROLIGHT, which works from 350 to 700 nm, was modified to resolve the solar spectrum from 250 to 2500 nm. This requires the addition of absorption and scattering properties for the added ultraviolet and near-infrared wavebands. Total absorption and scattering are determined by summing the absorption and scattering coefficients for pure water and for chlorophyll biomass. A detailed comparison was described in by Mobley et al. [223].The chief advantage of this model is computational efficiency. Solution of the Riccati differential equations for radiance is an analytic process, and thus there are no Monte Carlo fluctuations in the computed radiances (except for a negligible amount introduced by the simulation of the sea surface). In particular, both upwelling and downwelling radiances are computed with the same accuracy. Moreover, computation time is a linear function of depth, so that accurate radiance distributions are easily obtained at great depths. Computation time depends only mildly on quantities such as the scattering-to-attenuation ratio, surface boundary conditions, and water stratification. The associated computer code is available and is documented by Mobley [224].

Energy transfer across the sea surface is crucial to the understanding of the general circulation of the ocean. Shortwave radiation from the sun contributes most the heat fluxes that penetrate the air-sea interface and are subsequently absorbed throughout the ocean mixed layer. Solar radiative transfer differs from other air-sea interaction processes such as wind stress, evaporation, precipitation, and sensible cooling that occur only at the sea surface. Ohlmann et al. [201] showed that the climatological value of solar flux penetrating the mixed layer can reach **40** $\mathbf{W\,m^{-2}}$ in the tropical regions and can produce a difference in the heating rate of a **20-m** mixed layer by about $0.33°C$ a month. The vertical distribution of solar flux also influences the stability and stratification of the mixed layer and the sea surface temperature. It is clear that a quantitative understanding of the solar flux profile is important to ocean model simulations.

The surface albedo, which includes the surface reflectivity and the upwelling radiation from the water surface, is critical to the energy budget in the atmospheric planetary boundary layer. However, the reflectivity of the wind-blown surface is difficult to evaluate. Mobley [222] showed that the surface reflectance may decrease by **50%** when the solar zenith angle (SZA) is $70°$ and the wind speed increases from **0** to $\mathbf{20\,m\,s^{-1}}$. The surface roughness also affects the upwelling radiation from the water surface and its determination requires accurate radiative transfer analysis. The impact of oceanic pigment of radiative transfer in the ocean is important, as discussed by Gordon et al. [238] and Morel [199]. A change of $\mathbf{0.10\,mg\,m^{-3}}$ in the phytoplankton concentration in the mixed layer can result in a corresponding change of the penetrative solar flux by about **10** $\mathbf{W\,m^{-2}}$ at a level of **20-m** depth (Siegel et al. [240]). Because solar radiation is the energy source for photosynthesis, it also directly affects the marine productivity.

A coupled atmospheric-ocean radiative transfer model based on analytic four stream approximation has been developed by Lee and Liou [366]. The objective of this study was to build an efficient coupled ocean-atmosphere radiative transfer model including consideration of the wind-blown sea surface. A typical coupled model either deals with radiative transfer in the atmosphere and ocean separately by considering one medium as the boundary condition for the other or is computationally expensive for solar flux calculation. The present coupled model is based on the delta-four-stream approximation, developed by Liou [225] and Liou et al. [226] that can provide an analytical solution for radiative flux calculation and at the same time maintain excellent accuracy. To include the effect of the wind-blown sea surface, a Monte Carlo method that simulates the traveling of photons is employed to calculate the surface reflectance and transmittance. Applying the results from the Monte Carlo simulation into the delta-four-stream approximation, radiative transfer in the atmosphere and the ocean can be treated simultaneously and consistently. The present model is computationally efficient and provides a physically consistent surface albedo and ocean heating rate profile in the mixed layer.

Various numerical models are now available for computing underwater irradiances and radiances to address a wide range of oceanographic problems. Using different levels of sophistication in simplifying various physical assumptions depending upon the physical situations various numerical schemes have been designed to solve RTE.

1.8. Monte Carlo Models:

A number of models, most of them based on the Monte Carlo technique, have been developed and are used for various studies of radiative transfer in the atmosphere-ocean system, [179-184]. For geometrics other than plane-parallel and/or media with irregular boundaries, Monte Carlo methods become attractive. In essence the Monte Carlo approach consists of simulating trajectories of individual photons using probabilistic methods and concepts such as those discussed in [6.13]. In order to get good statistics a large number or trajectories must be simulated. Such simulations can, in principle, yield very precise results. The accuracy is primarily limited by computer resources. Monte Carlo methods have been developed to a high degree of sophistication and used to solve a variety of radiative transfer problems in plane-parallel media as well as media with complicated geometrics. This approach has also been widely used to solve radiative transfer problems in the ocean including the coupled ocean-atmosphere problem in the presence of a non-planar (wavy) interface. Some of the advantages are: all relevant orders of multiple scattering are taken into account; accurate solutions can be obtained for optically deep layers that may be inhomogeneous; interior radiances can be calculated; waves on the ocean surface can be taken into account; highly asymmetric phase functions for the hydrosols and aerosols are easily incorporated into the theory.

The basic Monte Carlo method has been described by Plass and Kattawar [267, 268]. The method has been extended by them Plass, G. N and Kattawar, G. W., [269, 270, 271] to calculate the flux and radiance in an atmosphere-ocean system to include the Stokes vector so that the polarization and ellipticity of the radiation is obtained Kattawar, G. W. and Plass, G. N. and Guinn, J. A. Jr. [229] and to include the effect of waves on the ocean surface Plass, G. N., Kattawar, G. W. and Guinn, J. A. Jr. [216]. Raschke [273] has considered the effect of ocean waves. Gordon and Brown [274] have used Monte Carlo techniques to compute the radiation flux in calm ocean, but their calculations are for either an isotropic radiance distribution or a solar beam without sky radiation incident on the ocean; they did not couple the radiation fields of the atmosphere and ocean. In this article a particular model of the atmosphere ocean. We shall describe below in detail five Monte Carlo models in atmospheric-ocean RTE.

First Monte Carlo model is due to Gordon [300-303, 368]. This model simulates radiative transfer in both the ocean and the atmosphere, as coupled across a wind-roughened interface. The code is designed to simulate irradiances as a function of depth for computation of the irradiance reflectance and diffuse attenuation functions such. The nadir-viewing radiance is also computed as a source. The optical properties of the ocean are continuously stratified in the vertical. They can be specified as discrete values as a function of depth (with linear interpolation between the given depths) or determined from formulas. Separate scattering phase functions are used for the particles and for the water itself. Variants of this code have been used for a number of studies of radiative transfer in the ocean.

The sea-surface roughness is modeled using the Cox and Munk [215] surface slope distribution for a given wind speed. The effect of the surface roughness is not simulated exactly because the possibility of shadowing of one facet by another is ignored. Multiple scattering, however, is included: e.g., if a downward-moving photon in the atmosphere encounters the sea surface and is still moving downward after reflection, it will undergo a second interaction with the sea surface. One important aspect of this model is the proper use of photon weights to account for the fact that not all facets are oriented in such a manner as to be able to interact with an incident photon, i.e., facets with normals making an angle less than $90°$ to the direction of the incident photon.

The atmospheric part of the model consists of fifty **1**-km layers with both molecular and aerosol scattering. The vertical distribution of the optical properties is taken from Elterman, L., [305]. The aerosol phase function at the given wavelength is determined from Mie, G., [306] theory with Deirmendjian, D., [307] Haze **C** size distribution. When a photon interacts with the atmosphere, the scattering angle is chosen from either the molecular or aerosol phase functions based on the ratio of their scattering coefficients for the layer in which the interaction takes place.

When inelastic processes are to be included, the above code is operated at the excitation wavelength, to determine the excitation radiance distribution. This is used an input to a second Monte Carlo code that computes the light field at the wavelength of interest Ge, Y., Gordon, H. and Voss, K., [303]. As with the elastically scattered radiation, the goal is to determine the irradiances of the inelastically scattered radiation. This is considerable simplification because the solution can be effected by working with the azimuthally averaged radiance, i.e., only the azimuthally averaged radiative transfer equation need be solved.

The second model also simulates a coupled ocean-atmosphere system. The Monte Carlo code relies heavily on several variance-reducing schemes to increase computational efficiency. We give only a brief description of one of the most useful ones. The use of statistical weights allows us to treat each photon history as a packet of photons rather than as a single photon. Photons are never allowed to escape from the ocean-atmosphere system. The method of forced collisions is used, whereby we sample from biased distribution that ensures a collision along the path and the weight is then adjusted appropriately to unbias the result. This model can simulate inelastic scattering; the details are given in Kattawar, G. and Xu, X., [308]. The Monte Carlo method also been extended to include the full Stokes vector treatment of polarization [181, 309 – 311]; these papers show that substantial errors can occur if polarization is neglected.

The third Monte Carlo model is similar to those described by Plass and Kattawar [269 – 271] and by Gordon, H.R. and Brown, O. B., [274]. It is designed to simulate the radiance distribution at any level in the atmosphere and in the ocean. Between these two media, a wind-roughened interface is modeled with the isotropic Gaussian distribution of sea-surface slopes, as discussed under model MC1. The probability of occurrence of the various slopes is modified when considering nonvertically incident photons. This photon – facet interaction is modeled as in Plass et al., [216] it does not account for the possible occultation of a facet by an adjacent one. Transmitted and reflected photon packets resulting from interaction with the air-water surface are weighted according to Fresnel's law (including the possibility of total internal reflection). According to the problem under investigation, photon packets are introduced at the top of the atmosphere, or just above (or below) the ocean surface. For specific problems involving deep levels, packets can be reintroduced a intermediate depths inside the water body, according to a directional distribution that reproduces the downward radiance field as resulting from a previous Monte Carlo run. The bottom boundary is either an infinitely thick absorbing layer, in which photons are lost from the system, or a Lambertian reflecting bottom of a given albedo, from which weighted photon packets are reflected.

After each collision, the weight of each photon packet is multiplied by the local value of ω_0 that is pertinent to the altitude or the depth, to account for its partial absorption. A packet history is terminated when its weight falls below a predetermined value, typically 1×10^{-6}. For each collision a random number on the unit interval is compared with the local value of the ratio of the molecular scattering coefficient to the total scattering coefficient to determine if the scattering event will be of molecular type (air or water molecules) or is due to an aerosol or hydrosol particle. The appropriate phase function is then used to determine the scattering angle; the orientation of the scattering plane is chosen at random on the interval $(0,2\pi)$. The number of photons initiated depends on the single-scattering albedo value, so as to control the stochastic noise in the computed radiometric quantities (details can be found in Morel, A. and Gentili, B., [182, 312, 370]). The model is operated for its oceanic segment with the optical properties. For the atmospheric segment, fifty 1-km-thick layers are considered, with specified values for Rayleigh and aerosol scattering and for ozone absorption as in Elterman, L., [305]. The aerosol phase function (as computed by Mie scattering theory) for the maritime aerosol model defined by the Radiation Commission of the International Association of Meteorology and Atmospheric Physics is used; (see the models of Tanre et al. [313] and Baker, K. and Frouin, R., [314]).

This forth model is intended primarily for simulation of the radiance distribution above and just below the surface, and for simulation of irradiances with the first five mean free paths of the surface. The model based on techniques described by Kirk, J., [183]. The model atmosphere is composed of 50 layers, each characterized by separate Rayleigh and particulate scattering coefficients and an albedo of single scattering, as given by Elterman, L., [305]. Weighted photon beams are projected into the atmosphere from the atmosphere-space boundary, and a collision is forced somewhere in the atmosphere along this original trajectory. The attenuated beam, which is the weight of the original trajectory. Beam losses that are due to absorption and scattering take place at the point of collision. There the absorbed portion is lost and the scattered portion exists the collision point in another single, weighted beam. A random number is compared with the ratio of the Rayleigh scattering cross section to the total scattering cross section to determine the type of volume scattering function governing the scattering event. In the case of an aerosol scattering, a two-term Henyey-Greenstein phase function is used to determine the scattering angle Gordon, H. and Castano, D., [315]. Otherwise the angle is determined by a Rayleigh phase function Blattern, W. et al. [316]. Once the trajectory of the scattered portion of the beam is calculated, the distance from the point of collision to the next encountered interface (air-water or air-space) is determined. A new collision is forced somewhere along this trajectory, and the process is repeated until the weigh of the scattered portion of the beam falls below a preset minimum fraction of the original beam weight. This minimum traceable weight is set to 1×10^{-6} of the original beam weight for the simulations presented below.

Some of the scattered trajectories encounter the atmosphere-space boundary and are forgotten; the others impinge on the sea-surface. For the latter, the angle of incidence depends on the nadir angle of the ray and the slope of the sea surface. The directions of the reflected and refracted rays are determined geometrically and the weights of the rays are calculated from the Fresnel formula. Although wave shadowing is neglected, multiple surface interactions may occur. A reflected ray that is still projected downward, or a transmitted ray that is still projected upward, must encounter the sea surface again immediately, without an intervening trajectory. Ray trajectories resulting from reflection are followed in the original manner. Transmitted portions of the beams are followed similarly until encountering the bottom or the sea surface, or until they are diminished to less than the minimum traceable weight. Those beams striking the bottom are lost;

those beams that are incident upon the sea surface from below are again subjected to the reflection and transmission calculations.

The Naval Research Laboratory optical model (referred to as the NORDA or NOARL optical model in earlier publications) uses standard Monte Carlo techniques Gordon, H. [300]; Kattawar, G. W. and Plass, G. N. [271]; Kirk, J., [183]. At each scattering event, a random number is used to determine if the scattering is due to molecular water, quartzlike particulates, algae, or organic detritus; the volume scattering functions of these components are treated separately, rather than using an average volume scattering function. The model includes the effects of Raman scattering. If a photon collision results in inelastic scattering (as determined by comparing a random number to the appropriate optical properties of the medium), the wavelength is shifted by an amount corresponding to the mean wave-number shift of $3357\,\mathrm{cm}^{-1}$, corresponding to Raman scatter by water molecules. The finite bandwidth of the Raman-shifted light is taken into account by averaging over 10-nm bandwidths (roughly corresponding to current oceanographic instruments); details of this averaging are described in Stavn, R. and Weidemann, A., [317, 318]. For the simulation of problem Mobley, C., [224], below, it was assumed the Raman scattering occurs in a very narrow waveband. The photons are tallied into zonal bands, as is convenient for computation of irradiances and the nadir-viewing radiance. There is no atmosphere per se implemented in the model. Atmospheric transmittances of solar irradiance needed for simulations are obtained from the nonlayred atmospheric model of Brine, D. and Iqbal, M., [319]. The model determines the skylight radiance pattern from the empirical model of Harrison, A. and Coombes, C., [320]. The present version of the code handles only homogeneous waters. A comparative study on different approaches with numerical presentation is found in Mobley, C. D. et al. [223].

There exist more or less detail information on radiative transfer calculation with field experiment data in aquatic media as documented in Kinne, S. et al. [4]; Ramanathan, W. C. et al. [5] and Houghton, J. T. et al. [3], Dera, J., Marine Physics, [347].

1.9. Polarization consideration:

The solar incident light interacts with all the components of the atmosphere-ocean system. Each phenomenon of scattering by molecules, aerosols, hydrosols and reflection over the sea surface introduces and modifies the polarization state of light. Therefore, the reflected solar radiation is polarized and contains embedded information about the intrinsic nature of aerosols and suspended matter in the ocean. Most of the detailed physical information (i.e., size distribution, composition) about the particles present in the atmosphere-ocean system is available through the measurement and analysis of the spectral and angular polarization signature of the oceanic and atmospheric radiation. The principal reason for the greater effectiveness of remote sensing by means of polarization measurements is the significantly higher sensitivity of polarization features to particles size, shape and refractive index as a function of scattering angle and wavelength, than is the case for intensity measurements.

The polarization processes from the surface and atmosphere can be simulated using various radiative transfer models. Eddington's model computes the intensity accurately with fast speed. Surface polarization can be taken into account by using a different surface emissivity at vertical and horizontal polarization. This is adequate for clear and nonprecipitating conditions. To allow for all polarization processes in a fast forward model, Liu and Weng [235] develop a polarimetric two stream approximation. It is an extension of a scalar two stream model (Liou [208]). The radiative transfer model developed in these studies is applicable to a plane-parallel medium with randomly oriented cloud particles.

Various accurate Stokes vector radiative transfer models have been developed in the past. For example, the vector discrete ordinate radiative transfer method (VDISORT) was originally developed by Weng [237] and later improved by Schultz et al. [241]. A matrix operator method was used to derive a rigorous solution of the vector radiative transfer equation (Evans and Stephen [242]; Waterman [243]; Liu and Ruprecht [244]. A computationally efficient scalar two-stream model is utilized in some applications (Coakley and Chylek [245]; Kerschgens et al. [246]; Schmetz [247]; Weng and Grody [248]). However, it is found that these models are either computationally expensive or exclude the interaction among Stokes components.

Unfortunately, the radiative transfer equation (RTE), or the vector radiative transfer equation (VRTE) if polarization is involved, has no analytical solution for most realistic source medium configurations and boundary conditions. Therefore, numerical solutions become the only means to solve the RTE. Due to the complexity of the problem and limitation of the computational resources, the RTE is traditionally solved in a plane-parallel medium in which the optical properties are only allowed to be inhomogeneous in one dimension but kept homogeneous in the other two dimensions. Due to the rapid development of modern computer technology, solving the RTE in a three-dimensional (3D) turbid medium has recently become a very active research topic since most natural systems are 3D [249–258]. As mentioned earlier, among the plethora of research topics in radiative transfer theory, numerical solutions to the RTE in an optically coupled atmosphere–ocean system (AOS) is especially important Mobley, C. D. et al. [222–223] in the studies of the terrestrial atmosphere and oceans. Most of these methods use one-dimensional (1D) models for the AOS. Understandably, the 1D model fails when the system is not intrinsically 1D. A few examples of such cases Gordon, H. R., [261] are in simulating the radiance under a ship, in littoral zones or in a region Reinersman, P. N. and Carder, K. L., [262] where the hydrosol distribution is horizontally inhomogeneous. To study the inhomogeneity effect in the AOS, one has to use a 3D model. Currently, the Monte Carlo method is the only effective method to solve the RTE in a 3D AOS.

A hybrid method is developed by Zhai, Kattawar and Yang [263] to solve the vector radiative transfer equation (VRTE) in a three dimensional atmosphere–ocean system (AOS). The system is divided into three parts: the atmosphere, the dielectric interface, and the ocean. The Monte Carlo method is employed to calculate the impulse response functions (Green functions) for the atmosphere and ocean. The impulse response function of the dielectric interface is calculated by the Fresnel formulas. The matrix operator method is then used to couple these impulse response functions to obtain the vector radiation field for the AOS. The primary advantage of this hybrid method is that it solves the VRTE efficiently in an AOS with different dielectric interfaces while keeping the same atmospheric and oceanic conditions. For the first time, we present the downward radiance field in an ocean with a sinusoidal ocean wave.

1.10. Method employed in this work:

Most of the credit, in our opinion, for the introduction and development of the discrete-ordinates method in the general area of particle and radiative transport theory should go to Chandrasekhar [54], who in his fundamental work on radiative transfer did much to define the method as an effective computational tool. However this method, as used by Chandrasekhar had one difficult computational aspect that kept the approach from being used effectively past a certain order. This practical limitation generates from the required separation constants that are defined in terms of the zeros of a certain polynomial. Here we do not intend to review the numerous works devoted, in general, to discrete-ordinates methods, but some particularly important computational improvements to Chandrasekhar's original formulation may be mentioned. For example, Barichello and Siewert [164] under certain restrictions on the quadrature scheme found that the discrete-ordinates method is equivalent to the spherical-harmonics method (often used in radiative transfer and neutron transport theory). For these special quadrature schemes, where the equivalence holds between the spherical-harmonics method and the discrete-ordinates method, the separation constants can be computed as the eigenvalues of a tridiagonal matrix (a much easier task than finding zeros of polynomials). A second improvement they mention have to do with a scaling of the discrete-ordinates solution so as to avoid all positive exponentials that cause unnecessary "overows" in numerical calculations and have lead many formulations to fail. Finally, it was found that the use of half-range" quadrature schemes, as used have made the discrete-ordinates method a much more powerful technique, since boundary conditions in most radiative and neutron transport applications are typically of the half-range type. To complement Chandrasekhar's version of the method, the discrete-ordinates method has been combined with finite-difference techniques (Lewis and Miller, [146]) that are useful when the spatial dependence of the problem can not be treated analytically.

Modern analytical version (Barichello and Siewert, [147]) of the discrete-ordinates method used following facts. (a) It does not depend on any special properties of the quadrature scheme and (b) for many applications, such as the case of isotropic scattering considered, it has the separation constants defined as the eigenvalues of a matrix with special properties (diagonal matrix plus a rank-one update) so that the basic eigenvalues computation is of a type generally considered even easier than the one for a tridiagonal matrix. This paper has been developed in the context of non-coherent scattering for applications related to stellar atmospheres, and so in order to demonstrate the development of the method for reactor-physics applications. However a much simpler version of the method has been demonstrated in Barichello and Siewert, [375].

A nice and simple setting for the development of analytical discrete-ordinates method Fireland, W. A., [115] is discussed for two simple, but basic, problems related to the nuclear field, and in order to complete the work they briefed the various problems that have been solved using this version of the method. These remarks are noted just to indicate how the analytical version of the method discussed in the mentioned papers has been used for problems considerable more challenging than the specific problems used to illustrate the method. First of all in regard to other applications in the nuclear field, we note that solutions basic to fully-coupled multigroup neutron transport theory are reported by Siewert [161], and another paper (Garcia and Siewert, [152]) concerns the transport of neutral hydrogen atoms in hydrogen plasma. Continuing, we can point out those references Siewert [162,163, 374]; Barichello and Siewert, [147, 148, 155]; Barichello et al., [166]) which are devoted to solve various radiative transfer problems (grey and non grey models, one and multidimensional applications, scalar and vector problems and with and without the inclusion of polarization effects) in atmospheric sciences, and all the following references (Siewert, C. E., [157, 158, 159, 160, 165]; Barichello and Siewert [149, 156], Barichello et al. [46, 147]; Siewert, C. E. and Valougeorgis, D., [153, 154]) reported solutions to classical problems in the area of rarefied gas dynamics. In solving this broad class of problems, these researchers have found it very convenient to make use of quadrature schemes defined specifically for the considered application. In this way they have made use of a fundamental aspect of discrete-ordinates method, and so they have been able to deal efficiently with problems defined by difficult characteristic functions and with boundary conditions that are not continuous. To conclude, it should be noted that in implementing this algorithms for the considered problems, it is found that the analytical discrete-ordinates method to be concise, easy to implement and especially accurate.

To the best of our knowledge we are the first to apply this method for polarized radiative transfer problem in the coupled atmosphere-ocean system. We have developed the full theory taking into consideration to the following facts. (A) **Inhomogeneity in the underlying medium** (atmosphere and ocean). (B) In the boundary conditions we have taken care for **change in the refractive index in the air water interface** with proper formulation (C) **Total internal reflections at the air-water interface** for the out coming rays from ocean(However we did not use it in our numerical consideration for simplicity) (D) The **wind velocity dependent albedo functions** for ocean. (E) **Different source functions** for atmosphere and ocean.