CHAPTER - IV

CONVECTIVE DIFFUSION

A N D

FREE CONVECTION AND MASS TRANSFER

EXACT ANALYSIS OF UNSTEADY MHD CONVECTIVE DIFFUSION IN A POROUS CHANNEL*

4.1 Introduction

Taylor [1] investigated the dispersion of soluble matter in a solvent flowing under laminar conditions in a circular tube. The analysis of Taylor on his conceptual model is applicable only for large values of time t. On the other hand, Lighthill [2], neglecting axial molecular diffusion, obtained an exact solution of the unsteady convective diffusion equation which is asymptotically valid for small t. However, recently Gill and Sankarasubramanian [3,4,5,6] developed a generalized dispersion model to study the unsteady convective diffusion systems, which are valid for all t, by involving an infinite set of time-dependent coefficients that can be determined from first principles and obtained exact solution for the local concentration C. Krishnamurthy and Subramanian [7], using generalized dispersion model, formulated convective diffusion theory for the predictive modelling of field-flow fractionation columns used for the seperation of colloidal mixture.

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The purpose of the present investigation is to study the dispersion of soluble matter in an incompressible electrically conducting viscous fluid in a porous-walled parallel plate channel permeated by transverse magnetic field. We desire to obtain the dispersion coefficients in MHD flow in a porous-walled channel valid for all time t and also to find the dependencies of these coefficients on the cross-flow Pe'clet number P and the magnetic parameter M, defined for the problem considered. The results of the present investigation are likely to have applications to situations where tracers are used for measuring the flow rate in a porous-walled channel, in drilling technology and in environmental pollution problems.

4.2 Mathematical formulation

We consider the laminar flow under a uniform pressure gradient Pr of an electrically conducting incompressible viscous fluid (of conductivity σ and permeability μ_e) between two infinite electrically non-conducting porous-walled parallel plate channel permeated by transverse magnetic field H_o along y-axis. This flow, studied by Hartmann [8], has a velocity u(y) along x-axis (parallel to the plates) given by,

$$u(y) = Z\left[M \coth M - \frac{M \cosh (My/b)}{\sinh M}\right], \tag{4.1}$$

$$Z = \frac{Pr}{\mu_e^2 \sigma H_o^2}, \quad M = \mu_e H_o b(\sigma/\rho\nu)^{1/2}. \tag{4.2}$$

The average velocity $\bar{u}$ of the flow is

$$\bar{u} = \frac{1}{2b}\int_{-b}^{b} u(y)dy = Z\ (M \coth M - 1). \tag{4.3}$$

In addition to this, some fluid is injected with a velocity v in the lower porous plate and removed the same at the opposite plate at the same rate.

If a solute diffuses in the above fully developed flow, the concentration C(t,x,y) of solute satisfies

$$\frac{\partial C}{\partial t} + u(y)\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] \tag{4.4}$$

along with the following initial and boundary conditions

$$\left.\begin{aligned} C(0,x,y) &= C_o \quad \text{for} \quad |x| \le \tfrac{1}{2}x_s \\ &= 0 \quad \text{for} \quad |x| > \tfrac{1}{2}x_s \end{aligned}\right\}, \tag{4.5}$$

$$-D\frac{\partial C(t,x,\pm b)}{\partial y} + vC(t,x,\pm b) = 0, \tag{4.6}$$

where D is the molecular diffusivity assumed to be independent of C, x_s is the length of the slag input of solute and since the amount of solute is finite

$$C(t,\infty,y) = 0. \tag{4.7}$$

Since there is no net solute flux at the walls, the total mass of the solute in the channel is conserved and the equation (4.6) is consistent with this statement.

Introducing the dimensionless variables

$$\theta = \frac{C}{C_o},\ U(y) = \left[\frac{u(y)}{\bar{u}} - 1\right],\ X = \frac{Dx}{b^2\bar{u}},$$

$$Y = \frac{y}{b},\ P = \frac{bv}{D},\ P_e = \frac{b\bar{u}}{D},\ T = \frac{Dt}{b^2}$$

in equations (4.4) – (4.7) we get

$$\frac{\partial\theta}{\partial T} + U(y)\frac{\partial\theta}{\partial X} + P\frac{\partial\theta}{\partial Y} = \left[\frac{1}{P_e^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right] \tag{4.8}$$

with the conditions

$$\left.\begin{aligned}\theta(0,X,Y) &= 1 \quad \text{for} \quad |X| \le \tfrac{1}{2}X_s \\ &= 0 \quad \text{for} \quad |X| > \tfrac{1}{2}X_s\end{aligned}\right\}, \tag{4.9}$$

$$\frac{\partial\theta(T,X,\pm1)}{\partial Y} = P\theta(T,X,\pm1), \tag{4.10}$$

$$\theta(T,\infty,Y) = 0. \tag{4.11}$$

Define a new axial co-ordinate moving with the average velocity of flow in the dimensionless form as

$$X_1 = X - T. \tag{4.12}$$

4.3 Solution

The solution of equations (4.8) – (4.11) are formulated as a series expansion in $\frac{\partial^k\theta_m}{\partial X_1^k}$ such that

$$\theta = \sum_{k=0}^{\infty} f_k(T,Y)\frac{\partial^k\theta_m}{\partial X_1^k}, \tag{4.13}$$

where $$\theta_m = \frac{1}{2}\int_{-1}^{1}\theta\, dY. \tag{4.14}$$

Upon substituting equation (4.13) into (4.8), after transforming to (T,X_1,Y) coordinate system and integrating equation (4.8) with respect to Y from −1 to 1 followed by the use of equation (4.10), we obtain generalised dispersion equation for $\theta_m\ (T,X_1)$ as

$$\frac{\partial \Theta_m}{\partial T} = \sum_{i=1}^{\infty} K_i(T) \frac{\partial^i \Theta_m}{\partial X_1^i} , \qquad (4.15)$$

where

$$K_i(T) = \frac{\delta_{i2}}{P_e^2} - \frac{1}{2} \int_{-1}^{1} U(Y) f_{i-1}(T,Y) dY, \quad (i = 1,2,3,\ldots) \qquad (4.16)$$

and the differential equations for the function $f_k(T,Y)$ defined in equation (4.14) are

$$\frac{\partial f_k}{\partial T} + P \frac{\partial f_k}{\partial Y} = \frac{\partial^2 f_k}{\partial Y^2} - U f_{k-1} + \frac{1}{P_e^2} f_{k-2} - \sum_{i=1}^{k} K_i f_{k-i} ,$$

$$(k=0,1,2,\ldots\ldots) \qquad (4.17)$$

with $f_{-1} = f_{-2} = 0$.

The initial and boundary conditions on the functions $f_k(T,Y)$ are

$$\frac{\partial f_k}{\partial Y} - P f_k = 0 \text{ at } Y = \pm 1, \qquad (4.18)$$

$$f_k(0,Y) = 0 , \qquad (4.19)$$

$$\int_{-1}^{1} f_k \, dY = 2\delta_{k0}, \quad (k = 0,1,2,\ldots). \qquad (4.20)$$

The function $f_o(T,Y)$ is independent of the velocity field and can be easily solved. The equation for $f_o(T,Y)$ is derived from equation (4.17) by setting $k = 0$:

$$\frac{\partial f_o}{\partial T} + P \frac{\partial f_o}{\partial Y} = \frac{\partial^2 f_o}{\partial Y^2} , \qquad (4.21)$$

$$\frac{\partial f_o}{\partial Y}(T,\pm 1) = P f_o(T, \pm 1), \qquad (4.22)$$

$$f_o(0,Y) = 0, \qquad (4.23)$$

$$\int_{-1}^{1} f_o(T,Y)dY = 2. \tag{4.24}$$

Using the method of separation of variables the solution of equation (4.21) and (4.22), (4.23), (4.24) are obtained as

$$f_o(T,Y) = \sum_{n=o}^{\infty} A_n e^{-\lambda_n^2 T} . \phi_n(Y), \tag{4.25}$$

$$\lambda_n^2 = b_n^2 + \frac{P^2}{4} , \tag{4.26}$$

$$b_o^2 = \frac{-P^2}{4} \quad , \; b_n^2 = \frac{n^2 \pi^2}{4} \; , \; n = 1,2,\ldots\ldots \tag{4.27}$$

and

$$\phi_n(Y) = e^{(P/2)Y} (\cos b_n Y + H_n \sin b_n Y) \tag{4.28}$$

and

$$\left.\begin{aligned} H_n &= -\frac{2b_n}{P} \quad (n \text{ odd}), \\ &= \frac{P}{2b_n} \quad (n \text{ even}). \end{aligned}\right\} \tag{4.29}$$

When n = 0, equation (4.28) may be written as

$$\phi_o(Y) = e^{PY}. \tag{4.30}$$

The expansion coefficients A_n are obtained by using the orthogonality property of the set of eigen function ϕ_n and

$$\begin{aligned} A_n &= \frac{1}{\lambda_n^4} (P^2 b_n \sin b_n \cosh \frac{P}{2}), \qquad (n \text{ odd}) \\ &= \frac{1}{\lambda_n^4} (2P b_n^2 \cos b_n \sinh \frac{P}{2}), \qquad (n \text{ even}). \end{aligned} \tag{4.31}$$

It is noted that A_o is independent of the initial distribution and is given as

$$A_o = \frac{P}{\sinh P} . \tag{4.32}$$

Asymptotically as $T \longrightarrow \infty$, $f_o(\infty,Y)$ approaches a steady state distribution given by

$$f_o(\infty,Y) = \frac{P}{\sinh P} e^{PY} . \tag{4.33}$$

From equation (4.16), we have

$$K_1(T) = -\frac{1}{2}\int_{-1}^{1} U(Y) f_o(T,Y)\, dY$$

$$= K_1(\infty) + \frac{1}{2}\,\frac{M/\sinh M}{M\coth M-1} \sum_{n=1}^{\infty} A_n\, e^{-\lambda_n^2 T} . C_n , \tag{4.34}$$

where

$$K_1(\infty) = -\frac{1}{(M\coth M-1)} + \frac{1}{2}\,\frac{MP/\sinh P}{(M\coth M-1)\sinh M}$$

$$\times \left\{ \frac{\sinh(P+M)}{(P+M)} + \frac{\sinh(P-M)}{(P-M)} \right\} \tag{4.35}$$

and

$$C_n = -\frac{2Mb_n}{P} \sin b_n \left[\frac{\cosh(\frac{P}{2}+M)}{(\frac{P}{2}+M)^2+b_n^2} - \frac{\cosh(\frac{P}{2}-M)}{(\frac{P}{2}-M)^2+b_n^2} \right], \text{(n odd)}$$

$$= M\cos b_n \left[\frac{\sinh(\frac{P}{2}+M)}{(\frac{P}{2}+M)^2+b_n^2} - \frac{\sinh(\frac{P}{2}-M)}{(\frac{P}{2}-M)^2+b_n^2} \right], \qquad \text{(n even)}.$$

From equation (4.34), it is interesting to note that $K_1(T)$ depends on T even though velocity field is independent of T.

Putting k=1 in equation (4.17) we get an expression for f_1 as

$$\frac{\partial f_1}{\partial T} + P\,\frac{\partial f_1}{\partial Y} = \frac{\partial^2 f_1}{\partial Y^2} - \left[\, U(Y) + K_1(T)\right] f_o(T,Y) \qquad (4.36)$$

along with the conditions

$$\frac{\partial f_1}{\partial Y}(T,\pm\ 1) = Pf_1(T,\pm\ 1) \qquad (4.37a)$$

$$\text{and} \int_{-1}^{1} f_1 dY = 0,\ f_1(0,Y) = 0. \qquad (4.37b)$$

Using Duhamel's theorem, solution for equation (4.36) is written as

$$f_1(T,Y) = f_1(\infty,Y) + \sum_{n=1}^{\infty} S_n(T)\ \phi_n(Y), \qquad (4.38)$$

where

$$f_1(\infty,Y) = L_1 + L_2 e^{PY} + \frac{L_3}{P}\,(Y - \frac{1}{P})e^{PY} - \frac{L_4 e^{PY}}{M^2 - P^2}\,(\cosh MY - \frac{P}{M}\sinh MY), \qquad (4.39)$$

$$S_n(T) = -\frac{1}{E_n}\left[\sum_{s=1}^{\infty} \left(F\, A'_s\, d_{sn} + 2GB_{sn}(e^{-b_{ns}T} - e^{-\lambda_n^2 T})\right) + \sum_{s=1}^{\infty}\ \sum_{m=1}^{\infty} D_{smn}\ \overline{e}^{\lambda_n^2 T}\left\{ \overline{e}^{(\lambda_n^2 + \lambda_m^2)} - 1\right\}\right] + \sum_{m=1}^{\infty} B_m e^{-\lambda_m^2 T} - \frac{B'_n}{E_n}\, e^{-\lambda_n^2}\, T, \qquad (4.40)$$

$$L_3 = \frac{P}{\sinh P}\left\{ K_1(\infty) + \frac{1}{M\coth M - 1}\right\}, \qquad (4.41)$$

$$L_4 = \frac{PM/(M\coth M - 1)}{\sinh M\ \sinh P}, \qquad (4.42)$$

$$L_1 = \frac{e^{-P}}{P}\left[\frac{L_3}{P} + \frac{L_4}{M^2 - P^2}(M \sinh M - P \cosh M)\right], \quad (4.43)$$

$$B_m = \frac{1}{1+G_m^2}\left[\frac{2L_1P}{\lambda_m^2} + \frac{2L_3}{P\lambda_m^2}\cos b_m \sinh\frac{P}{2}\right.$$

$$-\frac{L_4}{M^2 - P^2}\left\{\left(1-\frac{P}{m}\right) 4M \cos b_m \sinh\left(\frac{P}{2} + M\right)\right.$$

$$\left.\left.-\left(1 + \frac{P}{m}\right)\frac{4M \cos b_m \sinh\left(\frac{P}{2} - M\right)}{(P - 2M)^2 + 4b_m^2}\right\}\right], \text{ (m even)}$$

$$= \frac{1}{1 + G_m^2}\left[\frac{4b_m L_1}{\lambda_m^2}\sin b_m \cosh\frac{P}{2} - \frac{4L_3 b_m}{\lambda_m^2 P^2}\sin b_m \cosh\frac{P}{2}\right.$$

$$+\frac{L_4}{M^2-P^2}\left[\frac{\left(1-\frac{P}{M}\right)8Mb_m \sin b_m \cosh\left(\frac{P}{2} + M\right)}{P\left\{(P + 2M)^2 + 4b_m^2\right\}}\right.$$

$$\left.+\left(1 + \frac{P}{M}\right)\frac{8M b_m \sin b_m \cosh\left(\frac{P}{2} - M\right)}{P\left\{(P - 2M)^2 + 4b_m^2\right\}}\right], \text{ (m odd)} \quad (4.44)$$

$$L_2 = -\frac{L_1P}{\sinh P} + \frac{L_3}{P}\left(\frac{2}{P} - \coth P\right) + \frac{L_4P}{(M^2 - P^2)^2}$$

$$\times \left\{\left(M + \frac{P^2}{M}\right)\sinh M \coth P + 2P \cosh M\right\}, \quad (4.45)$$

$$E_n = (1 + G_n^2), \quad (4.46)$$

$$F = \frac{1}{(M \coth M - 1)} + K_1(\infty), \quad (4.47)$$

$$G = \frac{1}{2}\frac{M/\sinh M}{M \coth M - 1}, \quad (4.48)$$

$$A'_s = -\frac{U_s}{\lambda_s^2}\ , \qquad (4.49)$$

$$\left.\begin{aligned} d_{ns} &= (1 + G_n^2) \text{ if } n = s \\ &= 0 \qquad\quad \text{ if } n \neq s \end{aligned}\right\}, \qquad (4.50)$$

$$B_{ns} = \frac{A_n\ f_{ns}}{\lambda_n^2}\ , \qquad (4.51)$$

$$f_{ns} = 4M \sinh M \left[\frac{1 + G_n\ G_s}{M^2 + (b_n - b_s)^2} - \frac{1 - G_n\ G_s}{M^2 + (b_n + b_s)^2} \right],$$

(n odd, s odd)

$$= 4M \sinh M \left[\frac{1 + G_n\ G_s}{M^2 + (b_n - b_s)^2} + \frac{1 - G_n\ G_s}{M^2 + (b_n + b_s)^2} \right],$$

(n even, s even)

$$= -\ 4 \cosh M \left[\frac{(b_n + b_s)(1 - G_n G_s)}{M^2 + (b_n + b_s)^2} + \frac{(b_n - b_s)(1 + G_n G_s)}{M^2 + (b_n - b_s)^2} \right],$$

(n odd, s even)

$$= -\ 4 \cosh M \left[\frac{(b_n + b_s)(1 - G_n G_s)}{M^2 + (b_n + b_s)^2} - \frac{(b_n - b_s)(1 + G_n G_s)}{M^2 + (b_n - b_s)^2} \right]$$

(n even, s odd) (4.52)

$$b_{ns} = (\lambda_n^2 + \lambda_s^2), \qquad (4.53)$$

$$U_s = -\frac{1}{(1 + G_s^2)} \left[\left\{\frac{1}{M \coth M - 1} + K_1(\lambda)\right\} \sum_{n=1}^{\infty} A_n\ e^{-\lambda_n^2 \lambda} . d_{ns}\right.$$

$$- \frac{MP/\sinh P \sinh M}{(M \coth M - 1)} e_s - \frac{M/(M \coth M - 1)}{\sinh M} \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \lambda} . f_{ns} \Big] ,$$

(4.54)

$$e_s = \frac{2Mb_s}{P} \sin b_s \left\{ \frac{\cosh (\frac{P}{2} - M)}{(\frac{P}{2} - M)^2 + b_s^2} - \frac{\cosh (\frac{P}{2} + M)}{(\frac{P}{2} + M)^2 + b_s^2} \right\}, \text{ (s odd)}$$

$$= M \cos b_s \left\{ \frac{\sinh (\frac{P}{2} + M)}{(\frac{P}{2} + M)^2 + b_s^2} - \frac{\sinh (\frac{P}{2} - M)}{(\frac{P}{2} - M)^2 + b_s^2} \right\} , \text{(s even)}$$

(4.55)

$$D_{nms} = - G \frac{A_n B_m C_n d_{ms}}{(\lambda_m^2 + \lambda_n^2)} , \tag{4.56}$$

$$B'_n = \frac{2 P G e_n}{\sinh P} . \tag{4.57}$$

Now setting k = 2 in equation (4.16) and using (4.38) we get the expression for $K_2(T)$ as

$$K_2(T) - \frac{1}{P_e^2} = K_2(\infty) + \frac{1}{2} \frac{M/\sinh M}{M \operatorname{Coth} M - 1} \sum_{n=1}^{\infty} S_n(T) C_n , \tag{4.58}$$

where

$$K_2(\infty) = - \frac{1}{2(M \coth M - 1)} \Big[2L_2 \frac{\sin P}{P} + \frac{2L_3}{P^2} (\cosh P - \frac{2}{P} \sinh P)$$

$$+ \frac{L_4}{M^2 - P^2} (\frac{P}{M} - 1) . \frac{\sinh (P + M)}{(P + M)} - \frac{L_4}{M^2 - P^2} (\frac{P}{M} + 1) . \frac{\sinh (P - M)}{(P - M)}$$

$$- \frac{L_2 M}{\sinh M} \left\{ \frac{\sinh (P + M)}{(P + M)} + \frac{\sinh (P - M)}{(P - M)} \right\}$$

$$- \frac{L_3 M}{P \sinh M} \left\{ \frac{\cosh (P + M)}{(P + M)} - \frac{\sinh (P + M)}{(P + M)^2} + \frac{\cosh (P - M)}{(P - M)} \right.$$

$$- \frac{\sinh (P - M)}{(P - M)^2} \Bigg\} + \frac{L_3 M}{P^2 \sinh M} \left\{ \frac{\sinh (P + M)}{(P + M)} + \frac{\sinh (P - M)}{(P - M)} \right\}$$

$$+ \frac{L_4 M/\sinh M}{2(M^2 - P^2)} \left\{ \frac{\sinh (P+2M)}{(P + 2M)} + \frac{\sinh (P-2M)}{(P - 2M)} + \frac{2 \sinh P}{P} \right\}$$

$$- \frac{L_4 P/\sinh M}{2(M^2 - P^2)} \left\{ \frac{\sinh (P+2M)}{(P + 2M)} - \frac{\sinh (P - 2M)}{(P - 2M)} \right\} \Bigg] . \qquad (4.59)$$

Gill and Sankarasubramanian [3] have shown that the equation (4.15) can be truncated after the term involving K_2 without causing serious error. So the resulting model for the mean concentration θ_m can be written as

$$\frac{\partial \theta_m}{\partial T} = K_1(T) \frac{\partial \theta_m}{\partial X_1} + K_2(T) \frac{\partial^2 \theta_m}{\partial X^2} . \qquad (4.60)$$

Since a slug is being considered, θ_m will have to satisfy

$$\left. \begin{aligned} \theta_m(0, X_1) &= 1, \ |X_1| \le \tfrac{1}{2} X_s \\ &= 0, \ |X_1| > \tfrac{1}{2} X_s \end{aligned} \right\} \qquad (4.61)$$

$$\theta_m(T, \infty) = 0. \qquad (4.62)$$

The solution of equation (4.60) satisfying initial and boundary conditions is given as

$$\theta_m(T, X_1) = \frac{1}{\sqrt{\pi X_1}} e^{-\xi^2/4\zeta} , \qquad (4.63)$$

where

$$\xi(T, X_1) = X_1 + \int_0^T K_1(\tau)\, d\tau ,$$

$$\zeta = \int_0^T K_2(\tau)\, d\tau .$$

4.4 Discussion

Following Gill and Sankarasubramanian [3], we confine ourselves to calculate the dispersion coefficients upto the order K_2. Higher order dispersion coefficients may be determined by adopting the foregoing procedure but the computation will be more and more complicated. Moreover, K_3 and higher order dispersion coefficients decrease in magnitude at a very rapid rate.

For small p, equation (4.35) may be approximated as

$$K_1(\infty, P) \approx \frac{1}{(M \coth M - 1)}\left(MP \coth M + \frac{P^2}{3} + \frac{P^2}{2M^2}\right). \quad (4.64)$$

Equation (4.64) shows that as $P \longrightarrow 0$ i.e. the case of no cross-flow at channel walls, $K_1(\infty)$ becomes zero and which is the expected result.

For large P, equation (4.35) is approximated as

$$K_1(\infty) \approx \frac{1}{(M \coth M-1)}\left[M\left(1+\frac{M^2}{P^2}\right) \coth M-1 - \frac{M^2}{P}\right]. \quad (4.65)$$

The present analysis reveals that the dimensionless solute concentration in the porous-walled channel depends on three pertinent flow parameters, viz. P (cross-flow Pe'clet number), P_e(axial Pe'clet number) and M (magnetic parameter). Equation (4.60) clearly shows that the solute concentration is convected downstream in the porous-walled channel with a time-dependent velocity $- K_1(T)$ and spreads axially with respect to its centre of gravity with a time-dependent dispersion coefficient $K_2(T)$. To get a physical insight into the problem, numerical calculations are carried out of the equations (4.34), (4.35) and (4.59).

Figure 4.1 shows that the transient approach of $K_1(T)$ to its steady state value for various representative values of P (fixed M) and M (fixed P). Figure 4.1 depicts that $K_1(T)$ increases as T increases and approaches to the steady state nearly at time T = 5 for different values of M and P. Figure 4.1 also clearly shows that the relaxation time required for $K_1(T)$ to reach its asymptotic steady state decreases with increasing P and M. As expected $K_1(T)$ is equal to zero for P = M = 0 at all time.

Figure 4.2 shows the graph of asymptotic $K_1(\infty)$ against P for different values of M. It reveals that $K_1(\infty)$ increases with increasing values of both P and M.

Figure 4.3 shows the behaviour of $K_2(\infty)$ as a function of P for various values of M and it can be remarked from the figure that $K_2(\infty)$ decreases as P increases with fixed M and $K_2(\infty)$ increases as M increases with fixed P.

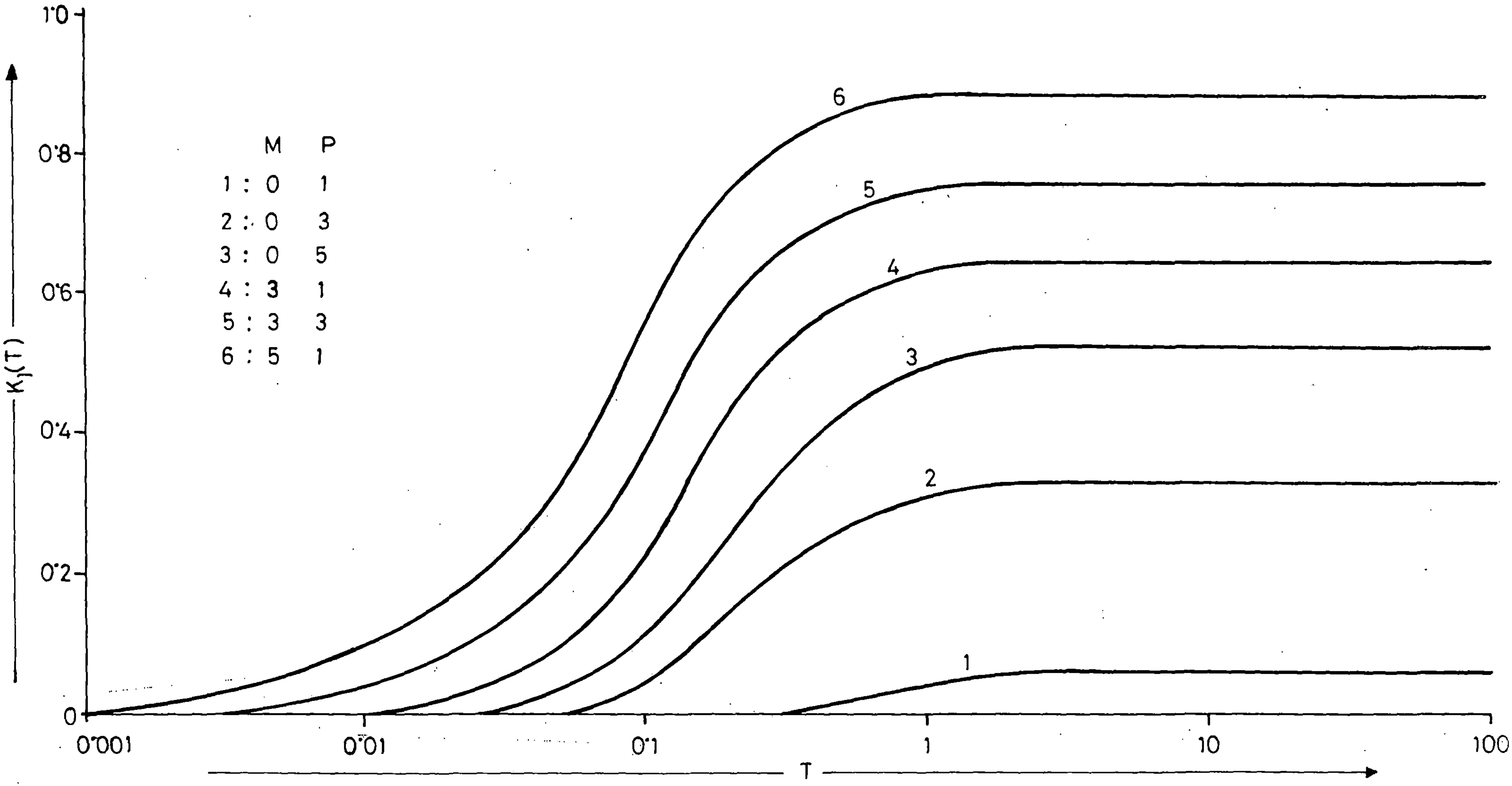

Fig. 4·1. Plot of the dimensionless convective coefficient $K_1(T)$ against T for different values of P and M.

(Semi-log).

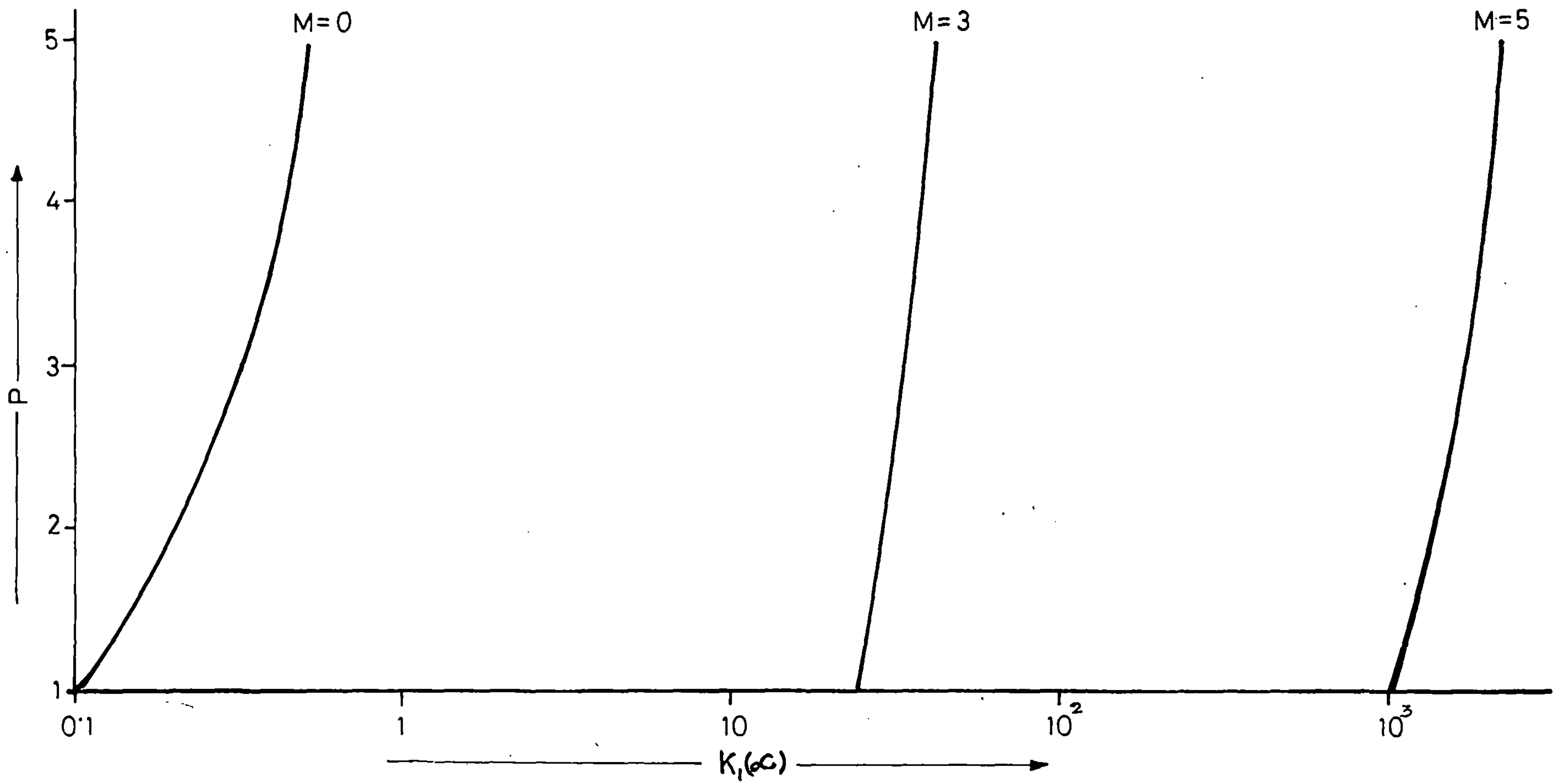

Fig. 4·2. Plot of the dimensionless steady-state convective coefficient $K_1(\infty)$ against P for different values of M.

(Semi-log)

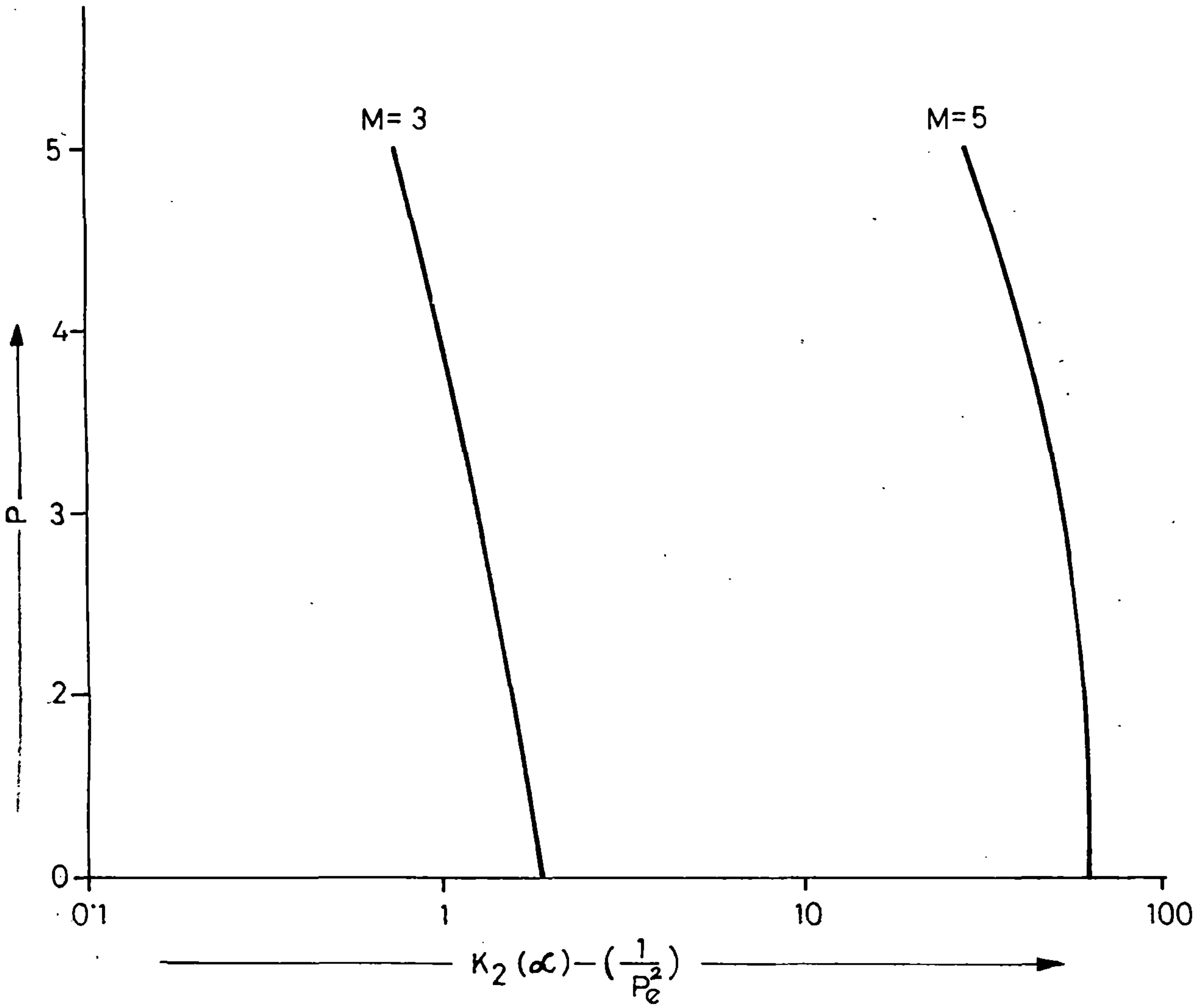

Fig. 4·3. Plot of the steady-state dispersion coefficient $K_2(\alpha) - (\frac{1}{P_e^2})$ against P for M = 3, 5.

(Semi - log).

UNSTEADY FREE CONVECTION MHD FLOW PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION AND MASS TRANSFER *

4.5 Introduction

There are many transport processes occurring in nature due to temperature differences. This difference causes the density difference. This can be seen in our every day life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as mass transfer flow.

The Stokes problem for a vertical plate was solved by Soundalgekar [9] by taking into account the presence of free convection currents. The effect of mass transfer on the free convective flow in Stokes problem for an infinite vertical plate was studied by Georgantopoulos et al. [10]. In these papers the

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plate was assumed to be impermeable and fluid was non-conducting. It is known that mhd flows have received considerable attention [11, 12, 13, 14, 15] because of their practical applications. Therefore it is of interest to make an investigation in order to analyse the effects of magnetic field on the free convection flow with mass transfer of an electrically conducting viscous fluid past a vertical plate subjected to a constant suction.

4.6 Mathematical formulation

We consider the unsteady flow in a porous medium of an electrically conducting viscous fluid past an infinite vertical porous plate. The temperature at the plate varies with time about a non-zero constant mean while the temperature at the free stream is constant. Also the species concentration on the plane surface and at the free stream are constant. The x-axis is taken along the plate in vertically upward direction and z-axis is taken normal to the plate and pointing towards the medium. A magnetic field of uniform strength is applied transversely in the direction of flow. The governing equations for flow are given by

$$\frac{\partial w}{\partial z} = 0, \qquad (4.66)$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = g\beta\,(T - T_\infty) + g\beta^*\,(C - C_\infty) + \upsilon\,\frac{\partial^2 u}{\partial z^2}$$

$$- \frac{\upsilon}{K}\,u - \frac{\sigma B_o^2}{\rho}\,u\ , \qquad (4.67)$$

$$\frac{\partial T}{\partial t} + w\,\frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p}\,\frac{\partial^2 T}{\partial z^2}\,, \tag{4.68}$$

$$\frac{\partial C}{\partial t} + w\,\frac{\partial C}{\partial z} = D\,\frac{\partial^2 C}{\partial z^2}\,, \tag{4.69}$$

where u, w are the corresponding velocity components along and perpendicular to the plate, υ the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion, T the fluid temperature, T_∞ the fluid temperature at infinity, K^* the thermal conductivity, ρ the density of fluid, C_p the specific heat at constant pressure, K the permeability of porous medium, σ the electrical conductivity, B_o the magnetic induction, D the diffusivity, C the species concentration. The equation of continuity (4.66) gives

$$w = \text{Constant} = -\,w_o\,, \tag{4.70}$$

where w_o is the steady constant velocity of suction at the surface. The relevant boundary conditions are

$$\left.\begin{array}{l} u = 0,\ T = T_w + \epsilon\,(T_w - T_\infty)e^{i\omega t},\ C = C_w \text{ at } z = 0 \\ u \longrightarrow 0,\ T \longrightarrow T_\infty,\ C \longrightarrow C_\infty \text{ as } z \longrightarrow \infty \end{array}\right\}. \tag{4.71}$$

In a physically realistic situation we cannot ensure perfect insulation in any experimental set-up. There will always be some fluctuation in the temperature. The plate temperature is assumed to vary harmonically with time. Since ϵ is very small, the plate temperature varies only slightly from the mean value T_w.

Introducing the following non-dimensional parameters

$$\bar{z} = \frac{w_o z}{\upsilon}, \ \bar{u} = \frac{u}{w_o}, \ \bar{t} = \frac{t w_o^2}{\upsilon}, \ \bar{T} = \frac{T - T_\infty}{T_w - T_\infty} .$$

$$\bar{C} = \frac{C - C_\infty}{C_w - C_\infty}, \ S_c = \frac{\upsilon}{D} \text{ (Schmidt number)},$$

$$P = \frac{\mu C_p}{K^*} \text{ (Prandtl number)}, \ \bar{K} = \frac{w_o^2 K}{\upsilon^2} \text{ (permeability}$$

$$\text{parameter)}, \ M = \frac{\mu\sigma B_o^2}{w_o^2} \text{ (magnetic parameter)},$$

$$G_r = \frac{\nu g \beta (T_w - T_\infty)}{w_o^3} \text{ (Grashof number)}, \ G_m = \frac{\nu g \beta^* (C_w - C_\infty)}{w_o^3}$$

(modified Grashof number) in equations (4.66) and (4.71) we get (dropping bars)

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} = G_r T + G_m C + \frac{\partial^2 u}{\partial z^2} - \frac{u}{K} - Mu, \tag{4.72}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{P} \frac{\partial^2 T}{\partial z^2} , \tag{4.73}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} . \tag{4.74}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} u = 0, \ T = 1 + \epsilon \, e^{i\omega t}, \ C = 1 \text{ at } z = 0 \\ u \to 0, \ T \to 0, \ C \to 0 \text{ as } z \to \infty \end{array}\right\} . \tag{4.75}$$

4.7 Solution of the problem

In order to obtain a solution of the above coupled non-linear system of equations, we expand u, T and C in the following manner,

$$u = u_o + \epsilon\, e^{i\omega t}\, u_1 + \ldots\ldots \tag{4.76}$$

$$T = T_o + \epsilon\, e^{i\omega t}\, T_1 + \ldots\ldots \tag{4.77}$$

$$C = C_o + \epsilon\, e^{i\omega t}\, C_1 + \ldots\ldots \tag{4.78}$$

Substituting equations (4.76) – (4.78) into equations (4.72) – (4.74), equating harmonic and non-harmonic terms, we get

$$u_o'' + u_o' - (M + \frac{1}{K})\, u_o = - G_r\, T_o - G_m\, C_o\ , \tag{4.79}$$

$$u_1'' + u_1' - (M + \frac{1}{K} + i\omega)\, u_1 = - G_r T_1 - G_m C_1\ , \tag{4.80}$$

$$T_o'' + PT_o' = 0, \tag{4.81}$$

$$T_1'' + PT_1' - i\omega PT_1 = 0, \tag{4.82}$$

$$C_o'' + S_c\, C_o' = 0, \tag{4.83}$$

$$C_1'' + S_c\, C_1' - i\omega S_c\, C_1 = 0, \tag{4.84}$$

with the corresponding boundary conditions

$$\left.\begin{array}{l} u_o\,(0) = 0,\ T_o\,(0) = 1,\ C_o\,(0) = 0 \\ u_1\,(0) = 0,\ T_1\,(0) = 1,\ C_1\,(0) = 0 \\ u_o(\infty) \longrightarrow 0,\ T_o\,(\infty) \longrightarrow 0,\ C_o\,(\infty) \longrightarrow 0 \\ u_1(\infty) \longrightarrow 0,\ T_1\,(\infty) \longrightarrow 0,\ C_1\,(\infty) \longrightarrow 0 \end{array}\right\} . \tag{4.85}$$

Thus the solution of the problem is

$$u = L_1 e^{-M_2 z} + L_2 e^{-Pz} + L_3 e^{-S_c z}$$

$$+ \epsilon(e^{-M_4 z} - e^{-P_1 z}) e^{i\omega t} \tag{4.86}$$

$$T = e^{-Pz} + \epsilon\, e^{-P_1 z} \cdot e^{i\omega t} , \tag{4.87}$$

$$C = e^{-S_c z} , \tag{4.88}$$

where

$$L_1 = \frac{G_r}{P^2 - P - M_1} + \frac{G_m}{S_c^2 - S_c - M_1} ,$$

$$L_2 = -\frac{G_r}{P^2 - P - M_1} ,$$

$$L_3 = -\frac{G_m}{S_c^2 - S_c - M_1} ,$$

$$L_4 = \frac{G_r}{P_1^2 - P_1 - M_2} ,$$

$$P_1 = \frac{1}{2}\left(- P + \sqrt{P^2 + 4i\omega P}\right) ,$$

$$M_1 = (M + \frac{1}{K}),$$

$$M_2 = \frac{1}{2}(1 + \sqrt{1 + 4M_1}) ,$$

$$M_3 = (M + \frac{1}{K} + i\omega),$$

$$M_4 = \frac{1}{2}(1 + \sqrt{1 + 4M_3}).$$

4.8 Discussion

In order to point out the effects of the permeability parameter K, Grashof number G_r, modified Grashof number G_m, magnetic parameter M and Schmidt number S_c on the velocity field, numerical calculations are carried out for $t = \frac{\pi}{4}$, $\omega = 2$ and $\epsilon=0.1$ and are depicted in figure 4.4. Two values of Schmidt number S_c

(= 0.30 and 0.60) are considered here to represent species H_e and H_2O in air. The Grashof number G_r represents the effect of free convection currents and the case $G_r > 0$ corresponds physically to an externally cooled surface. The velocity variations in the case of externally cooled surface are shown in drawings 4 and 5 of figure 4.4. We conclude from these curves that greater cooling of the surface (as G_r increases from 3 to 5) results in an increase in velocity. It is seen from drawings 3 and 4 in figure 4.4 that an increase in permeability parameter K increases the velocity. It is observed from drawings 1, 2 and 3 that velocity is much influenced by mass transfer parameter G_m and magnetic parameter M. The velocity increases with the increase in modified Grashof number G_m while decreases with the increase in magnetic parameter. Also an increase in Schmidt number S_c leads to decrease the fluid velocity.

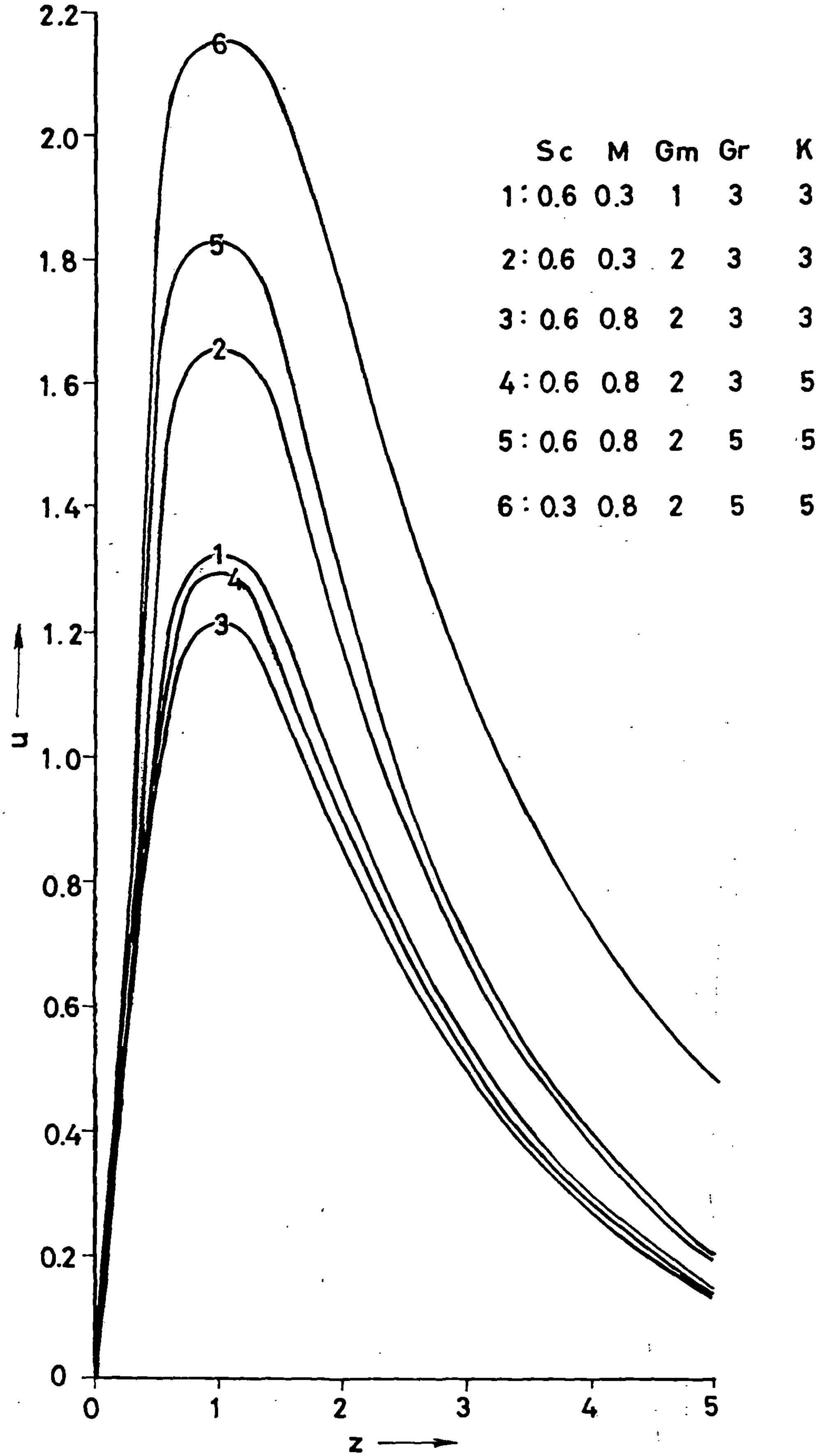

Fig.- 4.4. u against z when $\omega = 2$, $\epsilon = 0.1$, $t = \pi/4$

PART THREE

EFFECTS OF MAGNETIC FIELD ON THE FREE CONVECTION FLOW AND MASS TRANSFER THROUGH A POROUS MEDIUM*

4.9 Introduction

Flows which arise in fluids due to interaction of the force of gravity and density difference caused by the simultaneous diffusion of thermal energy and chemical species, is known as free convection and mass-transfer flow. On the other hand results of the effects of magnetic field on the flow in presence of free convection and mass-transfer are also useful in steller atmosphere. For this reason, Georgantopoulos and Nanousis [16] studied the effects of mass transfer on the free convection flow of an electrically conducting viscous fluid past an impulsively started infinite vertical surface, when the magnetic Reynolds number of the flow was small. Raptis [17] considered exactly the same problem, when the magnetic Reynolds number of the flow was not small. Raptis and Kafousias [18] analysed the steady free convective and mass transfer flow in an electrically conducting fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Hence it is of interest to

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make an investigation in order to analyse the effects of magnetic field on the free convective flow with mass transfer when the concentration of species oscillate in time about a constant mean. The plate is subjected to a constant normal suction velocity and the heat flux at the plate is also constant. Analytical expressions for the velocity field, concentration field and temperature field are given and numerical calculations are carried out to enable a discussion of the results, which are shown on graphs.

4.10 Mathematical analysis

We consider the unsteady flow of viscous fluid through a porous medium bounded by a vertical porous plate in the presence of free convection and mass transfer. Magnetic field of uniform strength is applied transversely to the direction of flow. The concentration of species is considered to be oscillating with time about a constant mean when the heat flux at the plate wall is constant. The x-axis is taken along the plate in the direction opposite to the direction of gravity and y-axis is taken to be normal to the plate pointing towards the medium.

Under the above consideration, the governing equations of motion are

$$\left(\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y}\right) = g \beta^* (C - C_\infty) + g\beta (T - T_\infty)$$

$$+ \frac{\upsilon}{K^*} (U - u) + \frac{\sigma B^2}{\rho} (U - u) + \upsilon \frac{\partial^2 u}{\partial y^2} , \qquad (4.89)$$

$$\frac{\partial C}{\partial t} - v_o \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} , \tag{4.90}$$

$$- v_o \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} , \tag{4.91}$$

with the boundary conditions

$$\left.\begin{aligned} & u = 0,\ \frac{\partial T}{\partial y} = -\frac{q}{K},\ C = C_v(1 + \epsilon\, e^{i\omega t}) \text{ at } y = 0, \\ & u = U,\ T = T_\infty,\ C = C_\infty \quad \text{as } y \longrightarrow \infty , \end{aligned}\right\} \tag{4.92}$$

where u is the velocity component in x-direction, T the temperature of the fluid, T_∞ the temperature of fluid far away from the plate, β the volumetric coefficient of thermal expansion, C the species concentration, C_∞ the species concentration far away from the plate, β^* the volumetric coefficient of expansion with concentration, υ the kinematic viscosity of the fluid, K^* the permeability of the porous medium, K the thermal conductivity, C_p the specific heat at constant pressure and D the molecular diffusivity, q the constant heat flux per unit area and v_o is the suction velocity.

In river bed, we may come across some situation where fluctuations in concentration is possible in presence of earth's magnetic field. The concentration at the plate z = 0 is assumed to vary harmonically with time. Since ϵ is very small, the concentration at the plate varies only slightly from the mean value.

The non-dimensional quantities introduced in the above equations are as follows

$$\bar{u} = \frac{u}{U}\ ,\ \bar{y} = \frac{y\ v_o}{\upsilon}\ ,\ \bar{t} = \frac{t\ v_o^2}{\upsilon}\ ,\ \bar{w} = \frac{w\upsilon}{v_o^2}\ ,$$

$$S_c = \frac{\upsilon}{D}\ \text{(Schmidt number)},\quad \bar{K}^* = \frac{v_o^2\ K^*}{\upsilon^2}\ ,$$

$$G_r = \frac{g\beta q v^2}{v_O U\ K}\ \text{(Grashof number)},\ \theta = \frac{T - T_\infty}{qv}\ K\ v_O$$

$$\bar{C} = \frac{C - C_\infty}{C_w - C_\infty}\ ,\quad G_m = \frac{\nu g\beta^*(C_w - C_\infty)}{\nu_O^2 \rho}\ \text{(modified}$$

Grashof number), $M^2 = \dfrac{\sigma\ B^2\upsilon}{v_o^2\ \rho}$ (magnetic parameter),

$P = \dfrac{\mu\ C_p}{K}$ (Prandtl number).

Substituting the above non-dimensional quantities into equations (4.89) – (4.91) we get (on dropping bars)

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y} + G_m\ C + G_r\ \theta + \left\{ \frac{1}{K^*} + M^2 \right\} (1-u) + \frac{\partial^2 u}{\partial y^2}\ , \tag{4.93}$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial y} + \frac{1}{S_c}\ \frac{\partial^2 C}{\partial y^2}\ , \tag{4.94}$$

$$\frac{d^2\theta}{dy^2} + P\ \frac{d\theta}{dy} = 0 \tag{4.95}$$

and the boundary conditions (4.92) become

$$\left.\begin{aligned} &u = 0,\ C = 1 + \frac{C_v}{C_w - C_\infty}\ \epsilon\ e^{i\omega t},\ \frac{\partial\theta}{\partial y} = -1 \text{ on } y = 0 \\ &u = 1,\ C = 0,\ \theta = 0 \text{ as } y \longrightarrow \infty\ . \end{aligned}\right\} \tag{4.96}$$

Solution of equation (4.95) is obtained with the help of boundary condition in (4.96) as

$$\theta = \frac{1}{P} e^{-Py}. \tag{4.97}$$

In order to solve equations (4.93) and(4.94) we use the method of perturbation, considering ϵ to be very small.

$$\text{Let } u = u_o + \epsilon u_1 e^{i\omega t}, \tag{4.98}$$

$$C = C_o + \epsilon C_1 e^{i\omega t}. \tag{4.99}$$

Substituting equations (4.98) and (4.99) in equations (4.93) and (4.94) we get

$$\frac{d^2 u_o}{dy^2} + \frac{du_o}{dy} - L u_o + \frac{G_r}{P} e^{-Py} + (L + G_m C_o) = 0, \tag{4.100}$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - (L + i\omega)u_1 + G_m C_1 = 0, \tag{4.101}$$

$$\frac{d^2 C_o}{dy^2} + S_c \frac{dC_o}{dy} = 0, \tag{4.102}$$

$$\frac{d^2 C_1}{dy^2} + S_c \frac{dC_1}{dy} - i\omega S_c C_1 = 0 \tag{4.103}$$

with the boundary conditions

$$u_o = 0,\ u_1 = 0,\ C_o = 1,\ C_1 = \frac{C_v}{C_v - C_\infty} \text{ on } y = 0, \tag{4.104}$$

$$u_o = 1,\ u_1 = 0,\ C_o = 0,\ C_1 = 0 \text{ as } y \longrightarrow \infty \tag{4.105}$$

where $L = (M^2 + \frac{1}{K^*})$.

Thus the solution of the problem is

$$u = 1 + A_1 e^{-B_1 y} + A_2 e^{-Py} + A_3 e^{-S_c y} + \epsilon A_4 e^{-(B_2+B_3)y} e^{i\omega t} \tag{4.106}$$

and

$$C = e^{-S_c y} + \epsilon \, e^{i\omega t} \, . \, \frac{C_v}{C_v - C_\infty} \, e^{-B_3 y} \, , \qquad (4.107)$$

where

$$A_1 = \frac{G_r}{P(P^2 - P - L)} + \frac{G_m}{S_c^{\,2} - S_c - L} - 1,$$

$$A_2 = -\frac{G_r}{P(P^2 - P - L)} \, ,$$

$$A_3 = -\frac{G_m}{S_c^{\,2} - S_c - L} \, ,$$

$$A_4 = \frac{G_m \, C_v}{\left\{ B_3^{\,2} - B_3 - (L + i\omega) \right\} (C_v - C_\infty)} \, ,$$

$$B_1 = \frac{1 + \sqrt{1 + 4L}}{2},$$

$$B_2 = \frac{1 + \sqrt{1 + 4(L + i\omega)}}{2},$$

$$B_3 = \frac{S_c + \sqrt{S_c^{\,2} + 4 \, i\omega \, S_c}}{2} \, .$$

4.11 Discussion

Numerical calculations are carried out and are depicted in figure 4.5 to observe the effects of various parameters G_m, M and K^* on the velocity profile. It is seen from the figure that velocity increases with the increasing value of both G_m and K^* while it decreases as magnetic parameter increases.

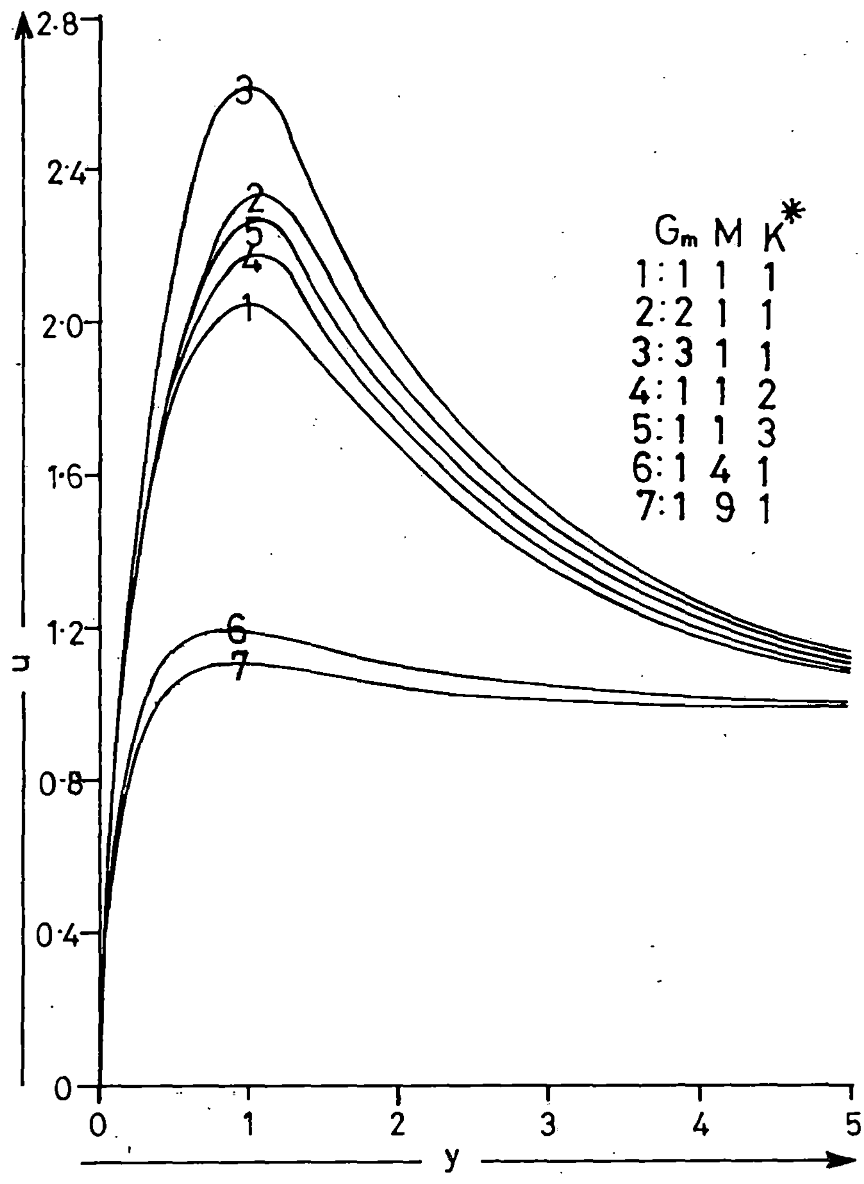

Fig. 4·5. Non-dimensional velocity u against y when $Sc = 0{\cdot}6$, $\omega = 2$, $t = \pi/4$, $P = 0{\cdot}71$, $\epsilon = 0{\cdot}1$ & $Gr = 4$.

MASS TRANSFER AND FREE CONVECTION FLOW PAST A VERTICAL POROUS PLATE IN A ROTATING LIQUID*

4.12 Introduction

The investigation of the flow through a porous medium is important to understand the character of the porous medium and to make it more effective. Therefore the study of flows through porous media has become principal interest in many scientific and engineering applications. In nature, the flow of fluids is caused not only by the temperature differences but also by concentration. So in recent years analytical solutions to such problems of flow have been presented by many authors. Gebhart and Pera [19] studied the laminar fluid flows due to the interaction of the forces of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species. Raptis [20] analysed the unsteady free convection and mass transfer flow of a viscous fluid through a porous medium due to the fluctuation of the surface temperature about a constant non-zero mean value. Recently Mahato and Maiti [21] analysed the unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium, bounded by an infinite vertical porous surface, in a rotating system.

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In the present work we study the unsteady free convection flow and mass transfer through a porous medium for a rotating fluid bounded by an infinite vertical porous plate where there is constant suction and constant heat flux. The concentration on the surface fluctuates with time about a non-zero constant mean and the concentration at the free stream is constant. The analytical expressions for the velocity, temperature and concentration are obtained. The influence of the various parameters entering into the problem on the velocity field are discussed. This study is likely to have some bearing on the geophysical and fluid engineering problems.

4.13 Mathematical analysis

We consider the unsteady flow of an incompressible viscous fluid by the presence of free convection and mass transfer through a porous medium bounded by an infinite vertical porous plate. We also consider a cartesian co-ordinate system rotating uniformly with the fluid in a rigid state of rotation with a constant angular velocity Ω about z-axis and the vertical plate is assumed to coincide with the plane $z = 0$ and z-axis is normal to the plate and pointing towards the medium.

The equations which govern the problem are :

Momentum equations

$$\rho \left(\frac{\partial u}{\partial t} - 2\Omega v + w \frac{\partial u}{\partial z}\right) = (T - T_\infty)\beta g \rho + \mu \frac{\partial^2 u}{\partial z^2} - \frac{\mu}{K} u$$

$$+ \rho g \beta^* (C - C_\infty) , \qquad (4.108)$$

$$\rho \left(\frac{\partial v}{\partial t} + 2\Omega u + w \frac{\partial u}{\partial z} \right) = \mu \frac{\partial^2 v}{\partial z^2} - \frac{\mu}{K} v . \qquad (4.109)$$

Diffusion equation

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} \qquad (4.110)$$

Energy equation

$$w \frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p} \frac{\partial^2 T}{\partial z^2} \qquad (4.111)$$

where u, v, w are the velocity components in x, y and z directions respectively, T the temperature of the fluid, T_∞ the temperature of the fluid far away from the plate, β the volumetric coefficient of thermal expansion, C the species concentration, C_∞ the species concentration far away from the plate, β^* the volumetric coefficient of expansion with concentration, μ the viscosity of the fluid, K the permeability of the porous medium, K^* the thermal conductivity, ρ the density, C_p the specific heat at constant pressure and D the chemical molecular diffusivity.

The boundary conditions of the problem are

$$u = 0 = v, \frac{\partial T}{\partial z} = - \frac{q}{K^*}, C = C_v(1 + \epsilon e^{i\alpha t}), \text{ at } z = 0 \qquad (4.112)$$

$$u, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty \qquad (4.113)$$

where q is the constant heat flux per unit area at the plate.

In river bed, we may come across some situation where fluctuations in concentration is possible. The concentration at the plate z = 0 is assumed to vary harmonically with time. Since ϵ is very small, the concentration at the plate varies only slightly from the mean value C_v. For constant suction, we have $w = -w_o$.

Using $W = u + iv$ and introducing the following non-dimensional quantities

$$\bar{u} = \frac{u}{w_o}, \quad \bar{v} = \frac{v}{w_o}, \quad \bar{z} = \frac{w_o z}{\nu}, \quad \theta = \frac{(T-T_\infty)\, K^* \, w_o}{q\nu},$$

$$S_c = \frac{\nu}{D}, \quad G_r = \frac{g\nu^2 q\beta}{K^* \, w_o^4}, \quad G_m = \frac{\nu g \beta^* (C_w - C_\infty)}{W_o^3},$$

$$\bar{K} = \frac{w_o^2}{\nu^2} K, \quad \bar{\alpha} = \frac{\alpha\nu}{w_o^2}, \quad \bar{C} = \frac{C - C_\infty}{C_v - C_\infty}, \quad E = \frac{\Omega\nu}{w_o^2},$$

$$\bar{t} = \frac{t w_o^2}{\nu}, \qquad p = \frac{\mu C_p}{K^*}$$

in equations (4.108) – (4.111) (dropping bars) we get

$$\frac{\partial W}{\partial t} + 2iEW - \frac{\partial W}{\partial z} - \frac{\partial^2 W}{\partial z^2} + \frac{W}{K} - G_m\, C - G_r\, \theta = 0, \qquad (4.114)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \qquad (4.115)$$

$$\frac{d^2\theta}{dz^2} + P\frac{d\theta}{dz} = 0 \qquad (4.116)$$

and the boundary conditions are

$$W = 0, \ \frac{d\theta}{dz} = -1, \ C = 1 + \epsilon \frac{C_w}{C_w - C_\infty} e^{i\alpha t} \text{ on } z = 0, \qquad (4.117)$$

$$W \rightarrow 0,\ \theta \rightarrow 0,\ C \rightarrow 0 \text{ as } z \rightarrow \infty . \qquad (4.118)$$

To solve the system of equations (4.114) and (4.115) we assume

$$W = W_o(z) + \epsilon\, W_1\, e^{i\alpha t} + \ldots\ldots , \qquad (4.119)$$

$$C = C_o + \epsilon\, C_1\, e^{i\alpha t} + \ldots\ldots . \qquad (4.120)$$

Substituting (4.119) and (4.120) in equations (4.114) and (4.115) we get

$$W_o'' + W_o' - MW_o + G_m\, C_o + \frac{G_r}{P}\, e^{-Pz} = 0 , \qquad (4.121)$$

$$W_1'' + W_1' - (M + i\alpha)\, W_1 + G_m\, C_1 = 0 , \qquad (4.122)$$

$$C_o'' + S_c C_0' = 0 , \qquad (4.123)$$

$$C_1'' + S_c C_1' - i\alpha\, S_c\, C_1 = 0 , \qquad (4.124)$$

where

$$\theta = \frac{1}{P}\, e^{-Pz} \text{ and } M = 2iE + \frac{1}{R} .$$

The boundary conditions (4.117) and (4.118) now become

$$W_o = W_1 = 0,\ C_o = 1,\ C_1 = \frac{C_W}{C_w - C_\infty} \text{ on } z = 0 , \qquad (4.125)$$

$$W_o = W_1 = 0,\ C_o = 0,\ C_1 = 0 \text{ as } z \rightarrow \infty . \qquad (4.126)$$

Thus the solution of the problem is

$$W(z,t)=R_1\, e^{-L_1 z}+R_2\, e^{-Pz}+R_3\, e^{-S_c z}+\epsilon\, e^{i\alpha t}\, R_4\, (e^{L_2 z}+e^{-L_3 z}) , \qquad (4.127)$$

$$C(z,t) = e^{-S_c z} + \epsilon\, e^{i\alpha t} \frac{C_w}{C_w - C_\infty} e^{-L_3 z}, \qquad (4.128)$$

where

$$R_1 = \frac{G_r}{P(P^2 - P - M)} + \frac{G_m}{S_c^2 - S_c - M},$$

$$L_1 = \frac{1 + \sqrt{1 + 4M}}{2},$$

$$R_2 = -\frac{G_r}{P(P^2 - P - M)},$$

$$R_3 = -\frac{G_m}{S_c^2 - S_c - M},$$

$$R_4 = \frac{G_m C_w}{(C_w - C_\infty)(L_3^2 - L_3 - (M + i\alpha))},$$

$$L_2 = -\frac{1 + \sqrt{1 + 4(M + i\alpha)}}{2},$$

$$L_3 = \frac{S_c + \sqrt{S_c^2 + 4\, i\alpha\, S_c}}{2}.$$

4.14 Discussion

It is seen from equation (4.127) that the steady state flow has multiple-layer character. The first layer is due to the temperature distribution, the second layer is due to the concentration of the species and the third layer is due to the porosity of the medium (modified by rotation). It can be also remarked that as the porosity (K) of the medium increases the depth of penetration increases.

In order to have a physical insight into the problem, graphs are drawn for the primary and secondary velocity components for different values of E, K and G_m for fixed values of P, $\in$, α and S_c. We see from the figures (Figs. 4.6 and 4.7) that u increases as G_m increases while it decreases as rotation parameter E increases and v follows the same as u in both the cases of increasing G_m ad E. The primary velocity u decreases as porosity K increases while reverse effect is seen for secondary velocity v.

Figure 4.8 shows the variation of concentration of species with z for different values of Schmidt number S_c. It clearly shows that the concentration of species is more near the vertical porous plate and decreases slowly as it moves far away from the plate.

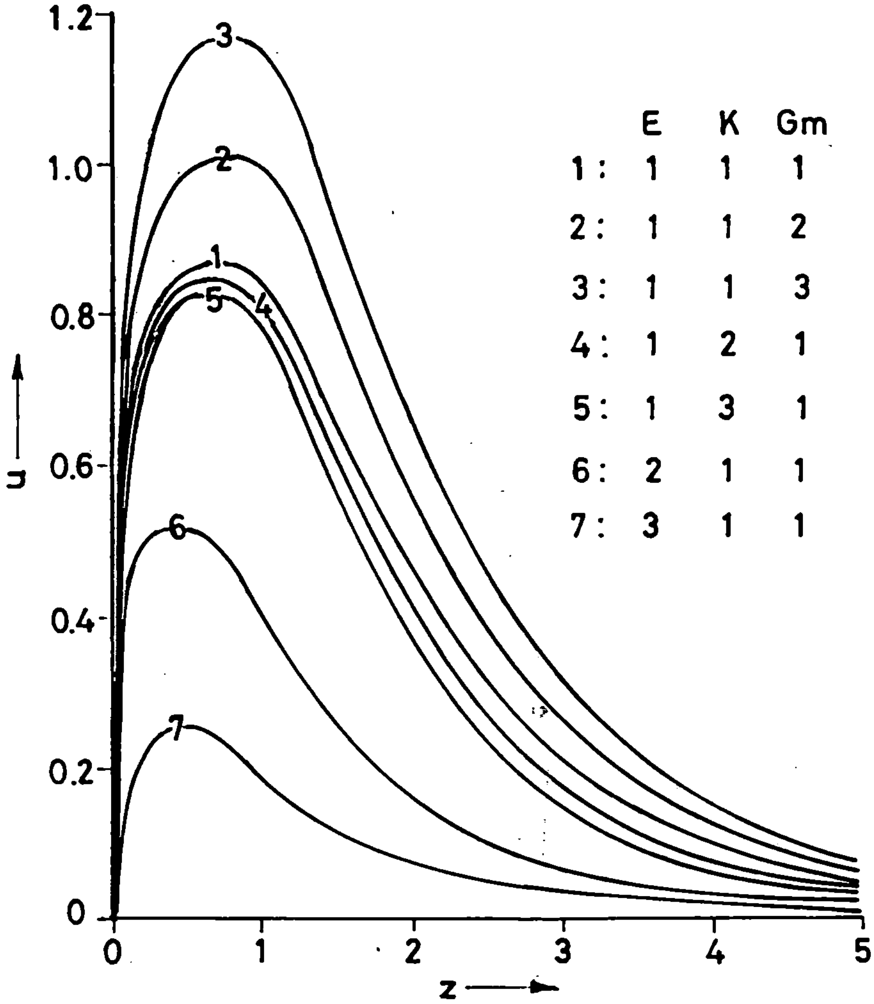

Fig.-4.6. Primary velocity profiles (u) when Sc = 0.60, P = 0.71, ϵ = 0.01 & α = 2.00.

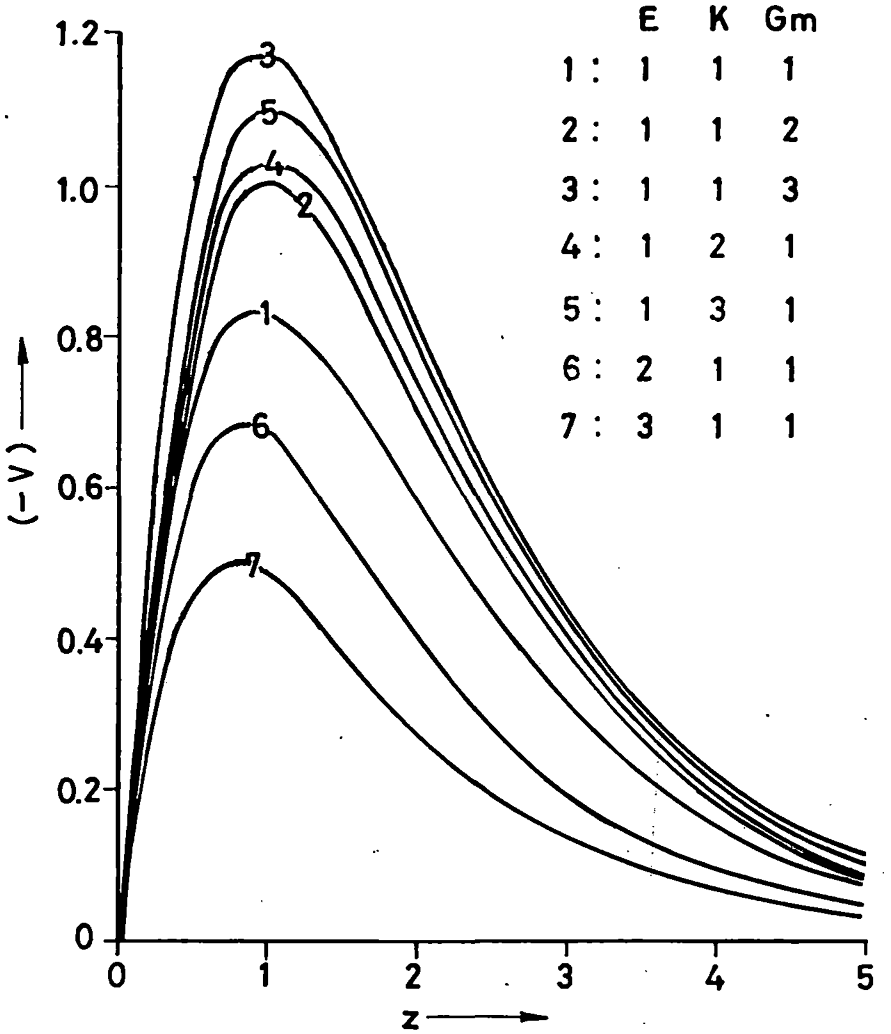

Fig.- 4.7. Secondary velocity profiles (v) when Sc = 0.60, P = 0.71, ϵ = 0.01, α = 2.00.

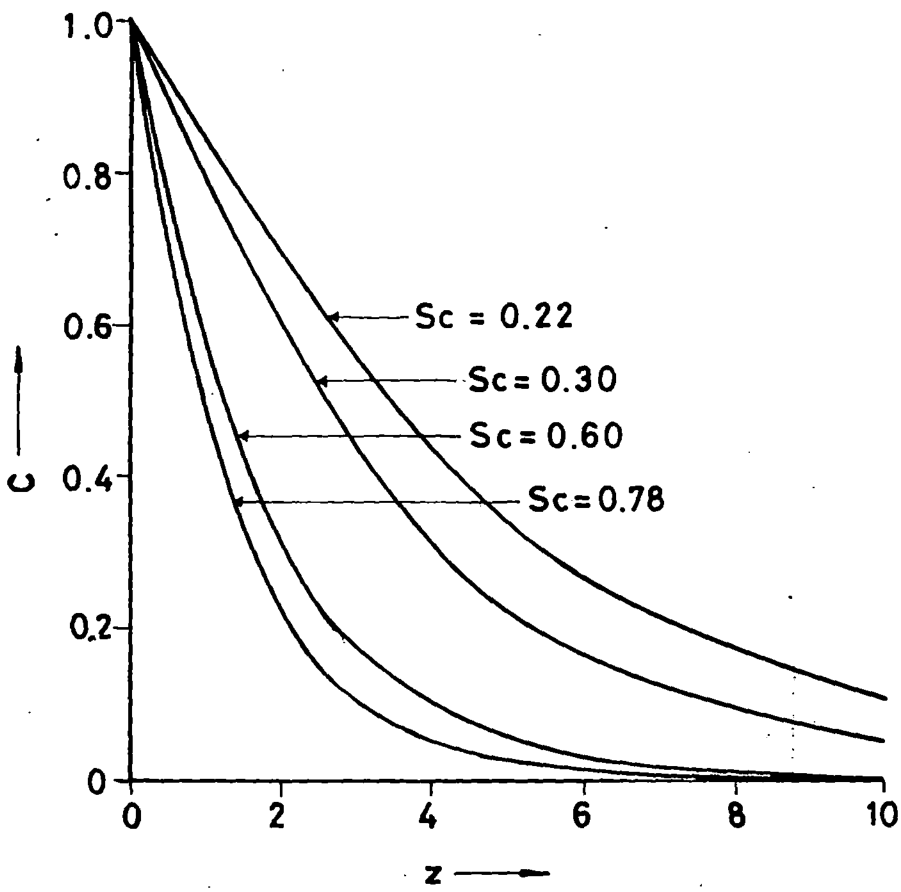

Fig.-4.8. Concentration (C) for different values of Sc when $\alpha=2$

PART FIVE

FREE CONVECTION FLOW AND MASS TRANSFER THROUGH A POROUS MEDIUM IN A ROTATING SYSTEM

4.15 Introduction

In recently published papers by Raptis et al. [22, 23], Raptis [24] and Raptis and Perdikis [25], an analytical study of the free convection flow and mass transfer through a very porous medium bounded by an infinite porous plate was given. It is also known that the flow through a porous medium in a rotating system, is one of the most considerable and contemporary subjects, because it finds great applications in geothermy, geophysics, petroleum industry and ground water technology. Raptis [26] studied steady free convection and mass transfer through a porous medium bounded by an infinite vertical porous plate for a fluid rotating with a constant angular velocity when the heat flux at the plate was constant. Very recently Mahato and Maiti [21] investigated the unsteady free convective flow and mass transfer in a rotating porous medium bounded by an infinite porous plate when the temperature at the plate was fluctuating with time.

In the present work an attempt is made to study the unsteady free convection flow and mass transfer during the motion of viscous fluid through a porous medium bounded by an infinite porous plate in a rotating system when there is an oscillating free stream velocity. In a physically realistic situation we may assume fluctuations in velocity far field flow region. The effects of various parameters on the primary and secondary velocity profiles and temperature distribution are analysed.

4.16 **Mathematical analysis**

We consider that the vertical infinite porous plate rotates in unison with viscous semi-infinite fluid occupying the porous region with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. Certesian coordinate system is chosen such that x, y axes respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface z = 0 while z-axis is normal to it. The fluid is subjected to constant suction along the plate z = 0 with velocity $w = -w_o$. Since the plate is infinite in length, all the physical quantities, except pressure p, are functions of z and t only. Consequently the equations expressing the conservation of momentum and energy and mass transfer are

$$\frac{\partial q}{\partial t} - w_o \frac{\partial q}{\partial z} + 2i\Omega q = \frac{\partial q^*}{\partial t} + 2i\Omega q^* + g\beta\,(T - T_\infty) + g\beta^*(C - C_\infty)$$

$$+ \frac{1}{\rho}\frac{\mu}{K}\,(q^* - q) + \frac{\mu}{\rho}\,\frac{\partial^2 q}{\partial z^2}\ , \qquad (4.129)$$

$$\frac{\partial T}{\partial t} - w_o \frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p}\,\frac{\partial^2 T}{\partial z^2}\ , \qquad (4.130)$$

$$\frac{\partial C}{\partial t} - w_o \frac{\partial C}{\partial z} = D\,\frac{\partial^2 C}{\partial z^2}\ , \qquad (4.131)$$

with the boundary conditions

$$\left.\begin{aligned} &q = 0,\ T = T_v,\ C = C_v \quad \text{at } z = 0 \\ &q = q^* = q'\ (1 + \epsilon\, e^{-i\alpha t}),\ T = T_\infty,\ C = C_\infty \text{ as } z \to \infty \end{aligned}\right\} \tag{4.132}$$

where $q = u + iv$, u and v are velocity components in the directions of x, y directions respectively; ρ the density; g the acceleration due to gravity; β the coefficient of volume expansion; T the temperature of the fluid; T_v is the surface temperature; T_∞ the fluid temperature far away from the plate; K the permeability coefficient of the medium; K^* the thermal conductivity and C_p the specific heat at constant pressure.

We introduce the following non-dimensional quantities

$$\bar{z} = \frac{w_o\ z}{\upsilon}\ ,\ \bar{t} = \frac{w_o^2\ t}{\upsilon}\ ,\ \bar{q} = \frac{q}{q'}\ ,\ \bar{\alpha} = \frac{\upsilon\alpha}{w_o^2}\ ,$$

$$\bar{K} = \frac{w_o^{\ 2}}{\upsilon^2} K,\ \bar{T} = \frac{T - T_\infty}{T_v - T_\infty}\ ,\ G_r = \frac{\nu g\beta(T_w - T_\infty)}{q'\ w_o^{\ 2}}$$ (Grashof number), $$G_m = \frac{\nu g\beta^*(C_w - C_\infty)}{w_o^{\ 2}\ q'}$$ (modified Grashof number),

$$\bar{q}^{\ *} = \frac{q^*}{q'}\ ,\ E\ \frac{\Omega\upsilon}{w_o^{\ 2}}$$ (rotation parameter),

$$\bar{C} = \frac{C - C_\infty}{C_v - C_\infty}\ ,\ P_r = \frac{\mu\ C_p}{K^*}$$ (Prandtl number),

$$S_c = \frac{\upsilon}{D}$$ (Schmidt number).

In view of the above non-dimensional quantities, equations (4.129)' – (4.132) reduce to (dropping the dashes)

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + 2iEq = \frac{\partial q^*}{\partial t} + 2iEq^* + G_r\, T + G_m\, C + \frac{\partial^2 q}{\partial z^2} + \frac{1}{K}\,(q^* - q), \tag{4.133}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{P_r}\ \frac{\partial^2 T}{\partial z^2}\ , \tag{4.134}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c}\ \frac{\partial^2 C}{\partial z^2}\ , \tag{4.135}$$

with boundary conditions

$$\left.\begin{aligned} &q = 0,\ T = 1,\ C = 1 \text{ at } z = 0 \\ &q = q^* = 1 + \epsilon\, e^{-i\alpha t},\ T = 0,\ C = 0 \text{ as } z \rightarrow \infty \end{aligned}\right\} . \tag{4.136}$$

To solve equations (4.133) – (4.135) under the boundary conditions (4.136), we assume

$$q = q_o(z) + \epsilon\ q_1(z)\ e^{-i\alpha t}\ , \tag{4.137}$$

$$T = T_o(z) + \epsilon\ T_1(z)\ e^{-i\alpha t}\ , \tag{4.138}$$

$$C = C_o(z) + \epsilon\ C_1(z)\ e^{-i\alpha t}\ . \tag{4.139}$$

Substituting (4.137) – (4.139) in (4.133) – (4.135) and separating the harmonic and non-harmonic terms, we have

$$q_1'' + q_1' + q_1\ (i\alpha - 2iE - \frac{1}{K}) = (i\alpha - 2iE - \frac{1}{K}), \tag{4.140}$$

$$q_o'' + q_o' - q_o\ (2iE + \frac{1}{K}) = -2iE - G_r\ e^{-P_r z} - G_m\ e^{-S_c z} - \frac{1}{K}\ , \tag{4.141}$$

$$T_1'' + P_r\ T_1' + i\alpha P_r\ T_1 = 0, \tag{4.142}$$

$$T_o'' + P_r\ T_o' = 0\ , \tag{4.143}$$

$$C_1'' + S_c C_1' + i\alpha S_c C_1 = 0 , \tag{4.144}$$

$$C_o'' + S_c C_o' = 0 . \tag{4.145}$$

Changed boundary conditions are

$$q_o = 0, \; q_1 = 0, \; T_o = 1, \; T_1 = 0, \; C_o = 1, \; C_1 = 0 \text{ at } z = 0, \tag{4.146}$$

$$q_o = 1, \; q_1 = 1, \; T_o = 0, \; T_1 = 0, \; C_o = 0, \; C_1 = 0 \text{ as } z \longrightarrow \infty . \tag{4.147}$$

On solving equations (4.140) - (4.145) with the boundary conditions in (4.146) and (4.147), we get the solutions for the velocity profile, temperature distribution and mass concentration as

$$q = 1 - L_1 e^{-P_r z} - L_2 e^{-S_c z} + L_3 e^{-R_1 z} + \epsilon [1 - e^{-M_2 z}] e^{-i\alpha t}, \tag{4.148}$$

$$C = e^{-S_c z}, \tag{4.149}$$

$$T = e^{-P_r z} , \tag{4.150}$$

where

$$L_1 = \frac{G_r}{(P_r - R_1)(P_r - R_2)} ,$$

$$L_2 = \frac{G_m}{(S_c - R_1)(S_c - R_2)} ,$$

$$L_3 = L_1 + L_2 - 1 ,$$

$$R_1, R_2 = \frac{1}{2} [1 \pm \sqrt{1 + 4 M_1}] ,$$

$$M_2 = \frac{1}{2} [1 + \sqrt{1 - 4(i\alpha - M_1)}] ,$$

$$M_1 = \frac{1}{K} + 2iE.$$

The real and imaginary part of q give the velocity expressions in the directions of x and y respectively. We are not presenting the mathematical expressions of u and v as discussions are made through numerical results.

4.17 Discussion

Equation (4.148) gives the expression of the composite velocity distribution and this exhibits the existence of multiple boundary layer. The steady state field is controlled by three layers viz. thermal layer $[O\ (1/P_r)]$, concentration layer $[O\ (1/S_c)]$ and suction layer (modified by rotation). Thermal layer is due to interaction of the thermal field and velocity field while the concentration layer arises due to the interaction of the concentration field and the velocity field.

Primary velocity u and secondary velocity v are depicted in figures. 4.9 and 4.10 for different values of G_r, G_m, S_c, K, E when $\alpha = 2$, $\epsilon = 0.1$, $P_r = 0.71$, $t = \pi/4$.

The Prandtl number P_r is taken equal to 0.71, which corresponds to air. Schmidt number $S_c = 0.6, 0.3$ are chosen in such a way as to represent H_2O vapour and H_e respectively at low concentration in air at approximately 25°C and 1 atm.

Figure 4.9 depicts that primary velocity decreases with increase in permeability parameter K, Grashof number G_r and Schmidt number S_c while it increases with increasing modified Grashof number G_m and rotation parameter E. This shows that permeability of porous medium exert retarding influence on the primary flow but rotation parameter enhance the velocity.

Figure 4.10 reveals that secondary velocity decreases as permeability parameter K, rotation parameter E and Schmidt number S_c increase and it increases as Grashof number G_r and modified Grashof number G_m increase.

Figure 4.11 shows the variation of temperature for different fluids. For mercury, the temperature remains almost stationary near the plate. The slope of the curve increases in case of air while in case of water the curve becomes very steep. This behaviour may be explained as induction of heat is quicker in case of small Prandtl number than those having large Prandtl number, as such water is more effective to maintain the temperature of its surroundings for a long time than air or mercury.

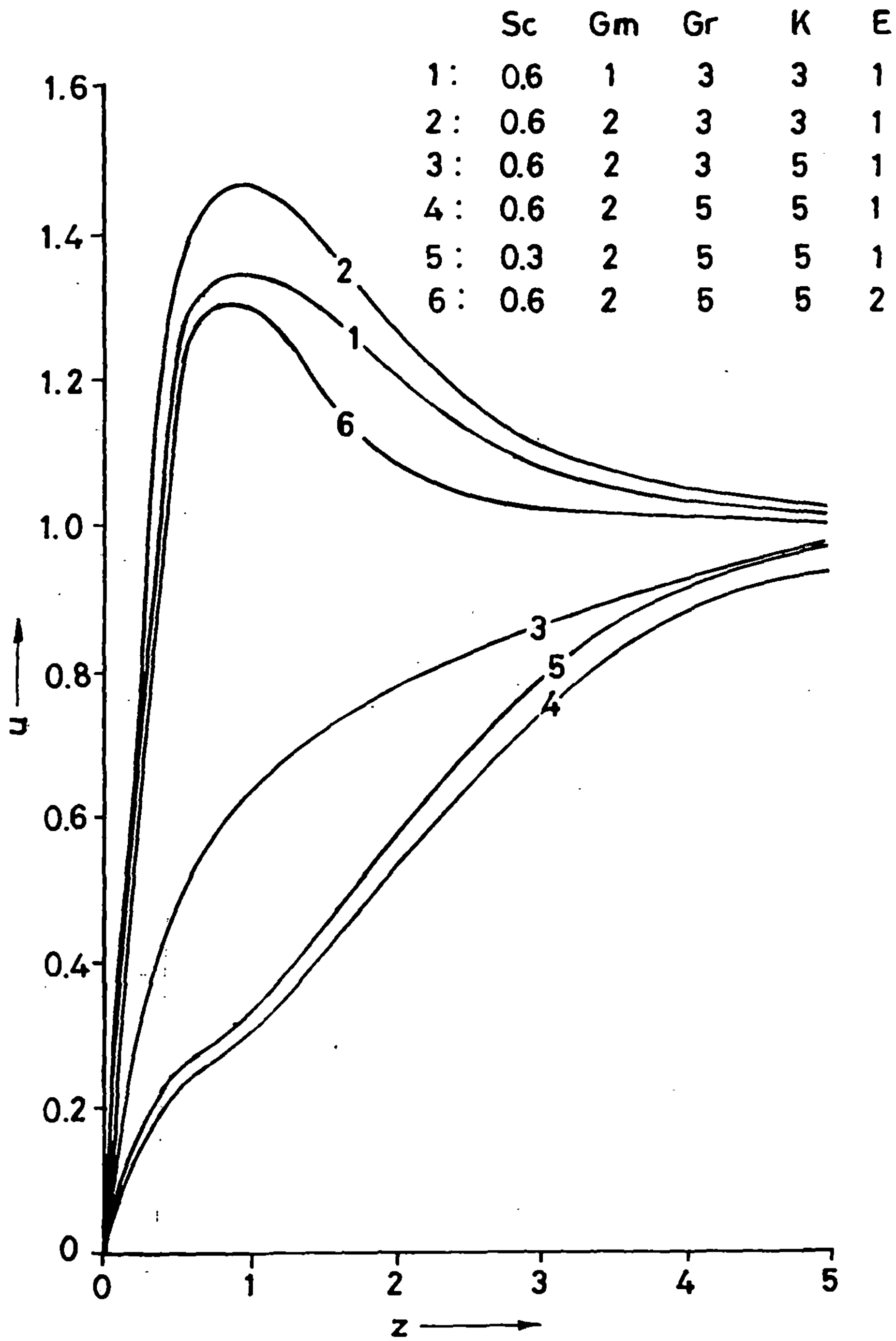

Fig.- 4.9. Plot of primary velocity profile u against z when $P_r = 0.71$, $\alpha = 2$, $\epsilon = 0.1$, $t = \pi/4$.

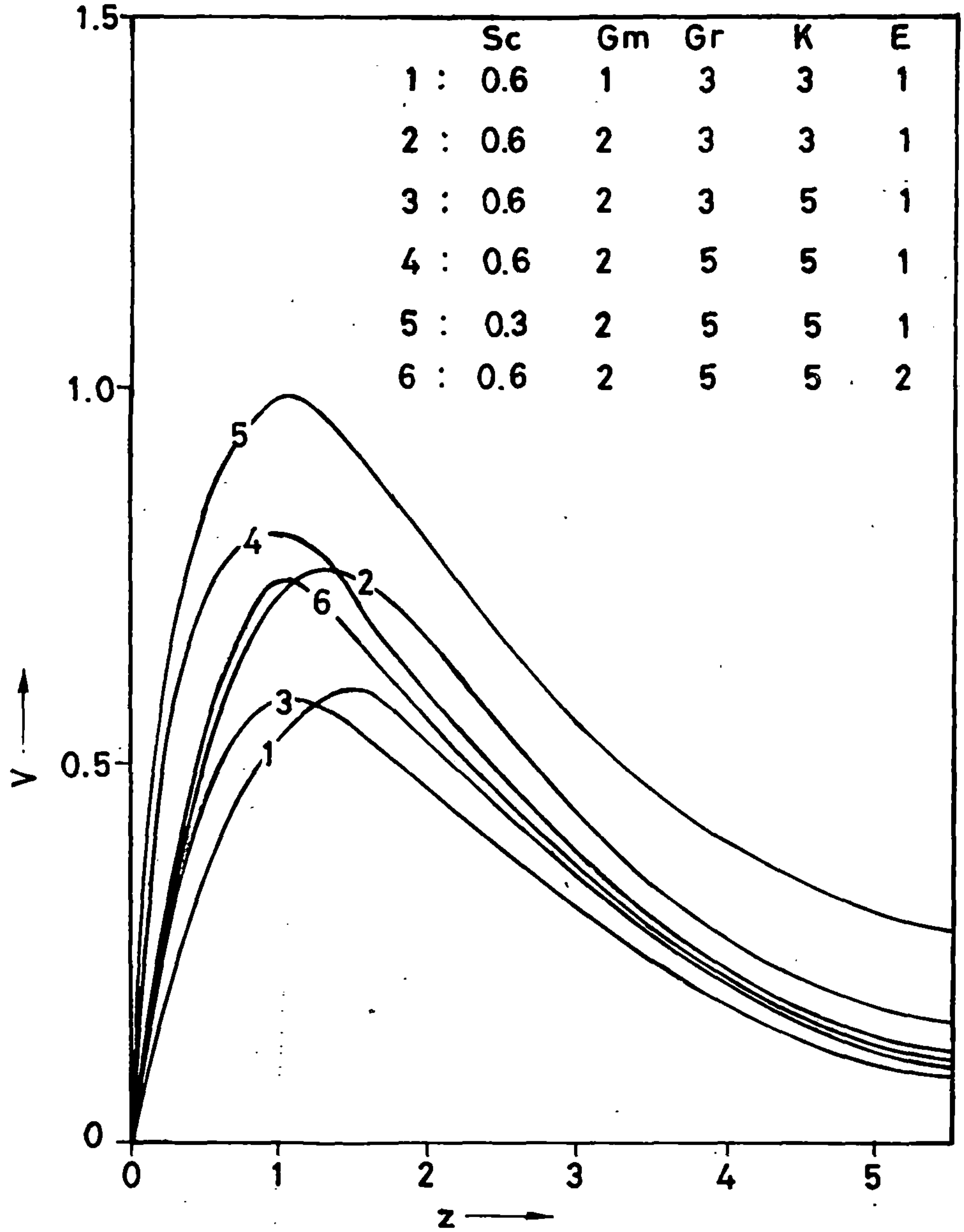

Fig.- 4.10. Plot of secondary velocity profile v against z when $P_r = 0.71$, $\alpha = 2$, $\epsilon = 0.1$, $t = \pi/4$.

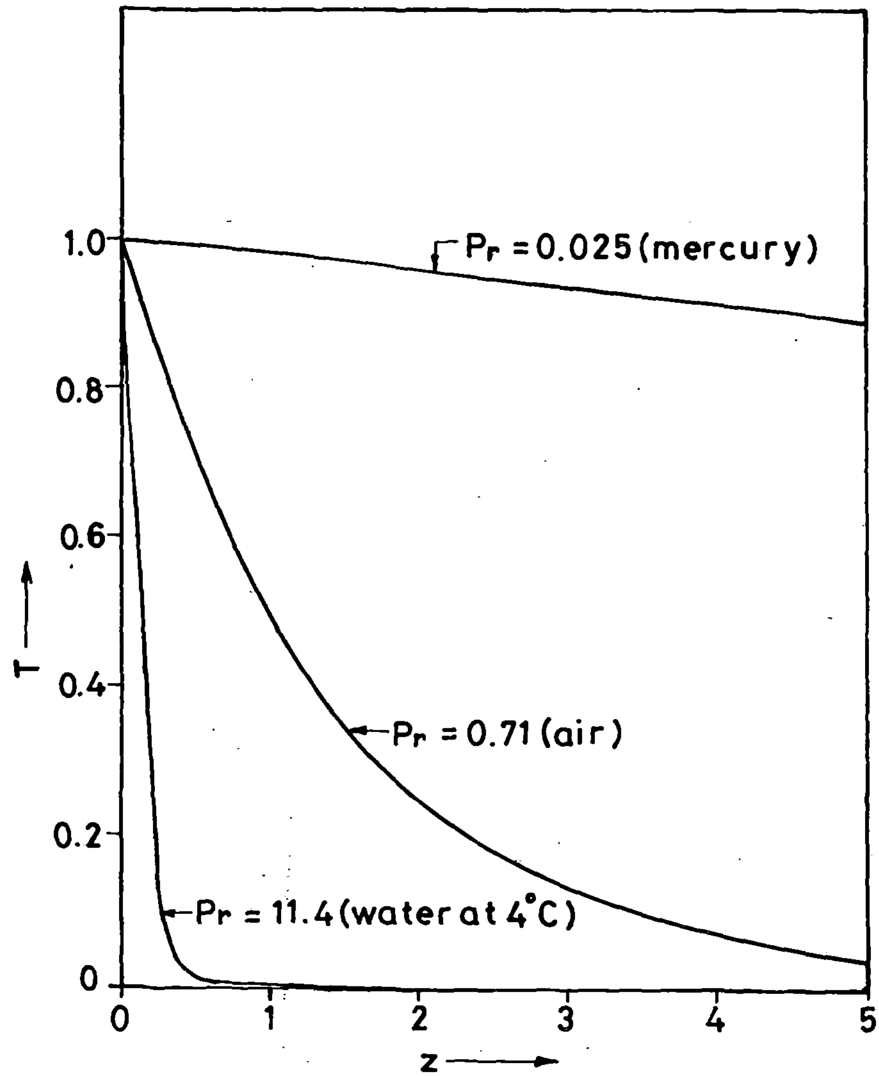

Fig.- 4.11. Plot of temperature (T) against z for different values of P_r.

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