2.1 Conventionality of Simultaneity Thesis

In recent years some interesting (apparent) paradoxes in relativity theory such as the twin paradox, tippitop paradox, Selleri paradox and the likes have been successfully dealt with the conventionality of simultaneity (CS) thesis of SR. For example in one of the most cited paper on the twin paradox[1] a novel approach to understanding the twin paradox based on the conventionality of simultaneity has been presented providing a clearer way to settle the often discussed issue of twins relative aging. More recently some variants of this approach[2, 3] have been fruitfully used to resolve some other paradoxes found in the literature. The present dissertation also aims to discuss some counter-intuitive issues and their variants—from twin paradox to ultra high energy cosmic ray paradox and VSL and in its course, make use of the conventionality thesis of SR quite liberally, a brief review of the CS-thesis of SR therefore may be in order.

In Einstein's 1905 paper on the special theory of relativity it was indicated that the question of whether or not two spatially distant events were simultaneous did not necessarily have a definite answer, but instead depended on the adoption of a convention for its resolution. The convention in the definition of simultaneity is rooted in the conventionality of synchronization of clocks. The issue and role of conventionality concerning the synchronization of spatially distant clocks in a *given* inertial frame are much discussed in the literature[3, 4, 5]. The role of convention in the definition of the simultaneity of distant events (or the same thing in synchronizing spatially distant clocks) is one of the most debated issues in SR. The problem in synchronizing the distant clocks lies in the fact that in SR the spatially separated clocks in a *given* reference frame are synchronized by light signals, the one way speed (OWS) of light has to be known beforehand for the purpose. However to know the OWS one needs to have presynchronized clocks and the whole endeavour then ends up in a vicious circle which forces us to introduce some arbitrariness (within some limit) in assigning the value for the OWS of light. Einstein however chose to synchronize two spatially distant clocks by *stipulating* the equality of speed of the light in two opposite directions along the line joining the clock positions. This prescriptive assumption is known as the *standard synchronization* (or Einstein synchronization) convention in the literature. This standard synchronization procedure to synchronize clocks at different locations is but one of the several possible alternative conventions (termed as non-standard synchronization) and many of the results (or formulae) that he obtained depended on his special choice of synchronization. For example the issue of difference in judgments in regard to the simultaneity of two spatially distant events by different inertial observers in relative motion is also a matter of such a simultaneity convention.

Although Einstein gave indication of the problem, the role of convention in the procedure of the synchronization of clocks was exemplified specially by Reichenbach[6] in 1928 and later by Grünbaum[7]. These authors explained that the question of simultaneity of a pair of events within *one* inertial frame indeed contained an ineradicable element of convention which was linked to the assumption regarding the value for the OWS of light. To understand this point one may note that Einstein originally proposed that the criterion for the synchronization of any two spatially separated clocks be such that the time of arrival and the consequent reflection by a mirror at one clock position be determined by considering that the latter is halfway in time between the departure of the light signal and its arrival at the position of the other clock from where the light signal is sent out for synchronization. This criterion clearly is equivalent to the assumption that light has the same speed in all directions. Clearly because specifying a value for the OWS of light enables directly a simple light signal procedure for the synchronization of distant clocks, any prescription for OWS value(s) is equivalent to a convention for clock synchronization. It therefore follows that the specification of either distant simultaneity criterion or any assumption for the values of OWS of light can alike be referred to as a synchronization convention[8].

Einstein himself referred to the distant simultaneity criterion he proposed as a free stipulation for giving an empirical meaning of distant simultaneity[9], but the issue is whether other criteria leading to different one way speeds might not have been chosen without compromising on the empirical success of the theory. The conventionalist thesis holds that a range of choices are possible, all fully equivalent with respect to experimental outcome. According to the CS-thesis, any synchrony convention will be admissible so long as it is consistent with the round-trip principle, according to which the average speed of a light ray over any closed path has a constant value. It may not be out of place in this context to mention that one should restate the second relativity postulate (that is often found in text books) by replacing the phase "velocity of light" by the "TWS of light" or "round trip speed of light". A convention within the SR must be consistent with this round-trip principle since this principle is a consequence of the theory prior to adoption of any criterion for distant simultaneity and may in principle be tested with a single clock. According to CS-thesis the conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of SR.

The CS-thesis has attracted a considerable amount of discussions in the literature. Possibility of using synchronization convention other than that adopted by Einstein has also been much discussed. John Winnie[10] first studied the consequences of SR when no assumption regarding the OWS of light was made and then developed the so called ϵ-Lorentz transformations (using Reichenbach's notation) adopting non-Einstein one-way velocity assumption or non-standard synchronization convention in general. To understand the meaning of ϵ we may recall from our previous discussion that when two spatially separated clocks are synchronized using light signals it is not necessary to divide the difference of transmission time t_1 and reception t_3 of a signal back by two, as adopted by Einstein, in order to fix the time t_2 of the other clock. One may assume in general that

$$t_2 = t_1 + \epsilon(t_3 - t_1), \tag{2.1}$$

so that $0 < \epsilon < 1$. Note that Einstein's convention is equivalent to the assumption $\epsilon = 1/2$. In developing the ϵ-Lorentz transformation Winnie assumed a principle called "principle equal passage time". This was used in addition to the "Linearity principle" and the "Round-trip light principle". These principles were then shown to be independent of one-way velocity assumptions and thus may form the basis of SR without distant simultaneity assumptions. Ungar[11] extended Winnie's idea by considering a generalized Lorentz transformation group that does not embody Einstein's isotropy convention. The approach seems to be well suited for establishing the results of Winnie as well as some new results. However these discussions were confined to one-dimension only. Later it has been noted by some authors that at least a two–dimensional analysis is necessary. Otherwise the isotropy of one-way speed of light which follows from the modified second relativity postulate cannot be used and therefore some subtleties and richness of the relativistic physics[12] will have to be sacrificed.

In a series of important papers Mansouri and Sexl[13] developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity. They showed that two principal methods of synchronization could be considered: system internal and system external synchronization. Synchronization by the Einstein procedure (using the light signal) and that by slow clock transport (by collecting and synchronizing all clocks at a given locality and then after slowly transporting being them back at the respective space points of a given reference frame) turn out to be equivalent if and only if and only if the time dilation factor is given by Einstein result $(1-v^2/c^2)^{-1/2}$. The authors constructed an ether theory that maintains absolute simultaneity and was kinematically equivalent to SR.

Sjödin[14] developed the CS-thesis by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. Sjödin presented all logically possible linear transformations between inertial frames depending on physical behavior of scales and clocks in motion with respect to the so called "physical vacuum" and then examined Lorentz transformation in the light of true length contraction and time dilation. In his article Sjödin tried to separate the true effects and the effects due to synchronization convention. For this, the author considered two special cases: The Newtonian world– without any contraction of moving bodies and slowing down of moving clocks and Lorentzian world– with longitudinal contraction of moving bodies and slowing down of clocks. The author then used *standard synchrony* in the Newtonian world (This was later termed as Pseudo-standard synchrony by Ghosal, Mukhopadhyay and Chakraborty[12]) and got the transformations which were already derived by Zahar[15][1]. These transformations show that the (apparent) relativistic effects in the Newtonian world are only due to choice of special synchrony. But when Sjödin used absolute synchronization in the Lorentzian world, the relevant transformations were due to Tangherlini[16] which

[1] We shall later find the importance of this transformation in clarifying some counter-intuitive issues in SR.

showed the "real" effects. In this way Sjödin came to the conclusion that the confusion regarding the existence of the ether and the reality of length contraction/time dilation effects was mainly due to the mixing up of the effects arising out of synchronization and the real contraction of moving bodies and retardation of moving clocks.

We have already discussed that the conventionality thesis asserts that there can be a number of choices on the value of the OWS of light of which Einstein convention is just one. It is well known that in the relativistic world the transformation equations that follows from this choice is nothing but LT. Clearly, in a given kinematical world, different choices of the OWS of light may be made which will lead to different transformation equations. These equations although may be different outwardly, will predict the same kinematical world. In recent years these structurally different transformation equations have been found to give much insight into many conceptual issues including some interesting paradoxes in SR. (We have used some of these for the present investigation.) We give below some important transformation equations which explicitly incorporate the CS-thesis. These equations relate coordinates x, y, z and time t in an inertial frame Σ with those (x', y', z', t') in another inertial frame Σ'.

Winnie transformations:

Based on three synchrony independent principles "the round trip light principle, the principle of equal passage times and the linearity principle Winnie arrived at his following ϵ-Lorentz transformations (see Ref. [10] for the interesting derivation of these transformation equations).

$$\begin{aligned} x' &= \alpha^{-1}(x - \vec{v_\epsilon} t), \\ t' &= \alpha^{-1} t[2\vec{v_\epsilon} c^{-1}(1-\epsilon-\epsilon') + 1] - xc^{-2}[2c(\epsilon-\epsilon') + 4\vec{v_\epsilon}(\epsilon)(1-\epsilon)], \end{aligned} \tag{2.2}$$

where

$$\alpha = [(c - \vec{v_\epsilon}(2\epsilon - 1))^2 - \vec{v_\epsilon}^2]^{1/2}/\epsilon, \tag{2.3}$$

and ϵ and ϵ' are Reichenbach parameters in the two frames which are in relative motions. Recall that ϵ parameter(s) have already been defined by Eq.(2.1).

Note that the symbol $\vec{v_\epsilon}$ denotes the relative speed of Σ' with respect to Σ. There is a word of caution however; the vector sign does not imply that the transformation equations involve more than one dimension, the arrow sign only emphasizes the non-reciprocity of relative velocity when $\epsilon \neq 1/2$ (for non-standard synchronization). The equation could also have been written in terms of $\overleftarrow{v_\epsilon}$ which denotes the relative speed of Σ to Σ' and in general $\vec{v_\epsilon} \neq \overleftarrow{v_\epsilon}$.

Selleri transformations:

The general form of the transformation obtained by Selleri[17] following the CS-thesis approach is given by

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct) + e(y + z), \end{aligned} \tag{2.4}$$

where ϵ and e are two undetermined functions of relative velocity v and $\beta = v/c$ and also $R(\beta) = (1 - \beta)^{1/2}$. The demand of rotational invariance around x-axis gives $e = 0$, giving the final form of these transformation equations as

$$\begin{aligned} x' &= (x - \beta ct)/R(\beta) \\ y' &= y \\ z' &= z \\ t' &= R(\beta)t + \epsilon(x - \beta ct). \end{aligned} \tag{2.5}$$

The transformation Eqs.(2.2) and (2.5) represent the relativistic world.

An interesting consequence of these equations is that it allows for absolute synchronization ($\epsilon = 0$) and the consequent transformation equations for $\epsilon = 0$ are obtained as

$$\begin{aligned} x' &= \gamma(x - vt), \\ t' &= \gamma^{-1}t, \end{aligned} \tag{2.6}$$

which are known as Tangherlini transformations[16] or inertial transformations[17, 18, 19]. Note that although the above equations represent the relativistic world, simultaneity is not relative in character i.e it is absolute.

Zahar Transformation:

In the classical or Galilean world the question of clock synchronization by light signals is not an issue. Since there is no time dilation, clock transport synchronization holds without any ambiguity hence the transformation equations are the well known Galilean ones. However, if one tries to incorporate the light signal synchronization following Einstein's procedure (playfully say) one observes that the Galilean transformations are replaced by the Zahar transformation (named after E. Zahar who obtained these transformation equations originally in 1977[3, 12, 14, 15]).

$$\begin{aligned} x' &= x - vt, \\ t' &= \gamma^2(t - vx/c^2). \end{aligned} \tag{2.7}$$

There are some other interesting transformation equations as an outcome of the CS approach where synchronization is achieved by non-luminal signal (in general) following the standard synchronization procedure. The equations are quite general in nature in the sense that the world (classical or relativistic) is not specified beforehand. These transformations have been much helpful for our investigations reported in this thesis (specially in second part) hence the derivations will be given rather in some details at the end of this section.

2.2 Dealing with Myths and Paradoxes

There are many myths and paradoxes that still exist in SR. Much of these misconceptions concerning the relativity theory stems from overlooking of the role of conventionality in gradients of SR. Thus the CS-thesis often comes as an aid to understanding of these myths and counter-intuitive issues. In recent years some of them have been dealt with efficiently. Some of them as mentioned before, being Selleri paradox[3], Tippie Top paradox[2] and Twin paradox[1]. One myth most relevant for the present report will now be discussed in some detail, since this background will also prepare the reader for the material given in Chapter V.

In a recent paper Ralph Bairlein[20] addressed one myth or misconception concerning the low speed behavior of the Lorentz transformation. Much before, Ghosal et.al[21] discussed the same issue in the light of the CS-thesis[2]. The question is "Does SR goes over to Galilean relativity for relative speeds small compared to the speed of light in vacuum?". The myth is "yes" but this is not correct. In fact it can be shown that if the belief is taken to be true it would have led to an interesting fallacy which we shall discuss below. It will be argued that Galilean synchrony and Einstein Synchrony are different and we will show that small velocity approximation cannot alter the convention of distant simultaneity[21].

Consider two events $E_1 : (x_1, t_1)$ and $E_2 : (x_2, t_2)$ in an inertial frame S. Representing in a Minkoski diagram, the invariant interval between these two events

[2]In a private communication to my supervisor referring the myth Prof. Bairlein writes "... it seems that the physics community needs a reminder every twenty years or so that LTs do not reduce to GTs when the relative speeed of frames is small relative to c....".

is

$$\Delta s^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2 - c^2(\Delta \bar{t})^2, \tag{2.8}$$

where $(\Delta x_i)^2 = x_{i2} - x_{i1}$, $\Delta t = t_2 - t_1$ and bars represent the corresponding quantities in another references frame $\bar{S}$ moving relative to S with the uniform non-zero speed v. If v^2/c^2 is neglected and if it were true that LT goes over GT for $v^2/c^2 \to 0$, then one would usually expect the time to be absolute i.e

$$(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2. \tag{2.9}$$

This appears to be all very fine since it looks as if we are merely going from Minkowski metric to Euclidean metric. But this is only an illusion and students often make such a mistake. We will see that this leads to a contradiction since, according to GT

$$\bar{x} = x - vt, \quad \bar{y} = y, \quad \bar{z} = z, \quad \bar{t} = t, \tag{2.10}$$

so that

$$\Delta\bar{x} = \Delta x - v\Delta t, \quad \Delta\bar{y} = \Delta y, \quad \Delta\bar{z} = \Delta z, \quad \Delta\bar{t} = \Delta t, \tag{2.11}$$

and clearly, for any two non-simultaneous ($\Delta t \neq 0$) events, $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is not an invariant. The above fallacious situation can not be resolved unless one rejects the notion that alone the neglect of v^2/c^2 in LT leads to GT. Indeed, if v^2/c^2 is neglected in the Lorentz factor., the LT reduces to the approximate Lorentz transformation (ALT)[22]

$$\begin{aligned} \bar{x} &= x - vt, \\ \bar{t} &= t - vx/c^2, \end{aligned} \tag{2.12}$$

thus, for any pair of events

$$\begin{aligned} \Delta\bar{x} &= \Delta x - v\Delta t, \\ \Delta\bar{t} &= \Delta t - (v/c^2)\Delta x. \end{aligned} \tag{2.13}$$

Notice here that for any chosen spatial separation Δx between two events, we can take v sufficiently small, so that Δt becomes very large compared to $(v/c^2)\Delta x$ and hence the latter may be neglected implying $\Delta\bar{t} = \Delta t$. On the other hand, the approximation $v^2/c^2 << 1$ is certainly not dependent on the space time separation of two arbitrary and independent events. In fact, for any preassigned small value of v, one is free to consider a pair of sufficiently distant events so that one cannot ignore the $(v/c^2)\Delta x$ term in Eq.(2.13). Therefore absolute nature of distant simultaneity $(\Delta\bar{t} = \Delta t)$ can never be retrieved. That is, simultaneity is still relative. This is not surprising since we should realize that the relative character of distant simultaneity is the result of a synchronizing convention[3, 6, 7, 21]. A convention once chosen a priori is unlikely to change into a different one merely due to approximative assumption on the relative velocity alone.

Let us recall that the standard Einstein synchronization procedure requires spatially distant clocks to be so adjusted that in any given inertial frame the *to and fro* speeds of light appear to be the same and equal to the round trip speed of light. In this context it is now worthwhile to examine the nature of ALT (Eq.(2.12)) for all v. To do this, the velocity addition laws can be obtained from Eq.(2.12) as

$$\begin{aligned} \bar{W}_x &= (W_x - v)/(1 - vW_x/c^2), \\ \bar{W}_y &= W_y/(1 - vW_x/c^2), \\ \bar{W}_z &= W_z/(1 - vW_x/c^2). \end{aligned} \tag{2.14}$$

As expected, W_y and W_z do not transform as in SR. Now, if a light pulse is sent back and forth along the x-direction alone, that is,

$$\begin{aligned} W_x &= \pm c, \\ W_y &= W_z = 0, \end{aligned} \tag{2.15}$$

then the to and fro speed of light in $\bar{S}$, parallel to the direction of motion, is given by

$$c_{\parallel} = \pm c. \tag{2.16}$$

If, on the other hand, a light pulse is sent back and forth in S in such a direction that the signals travel back and forth only in the y-direction in $\bar{S}$, then

$$W_x = W_z = 0. \tag{2.17}$$

Now using the fact that $W_x^2 + W_y^2 = c^2$ in S, one obtains the speed of light in $\bar{S}$, perpendicular to the direction of motion, the value

$$c_{\perp} = \pm c/(1 - v^2/c^2)^{1/2}. \tag{2.18}$$

These results, i.e Eqs(2.16) and (2.18) certainly do not agree with the corresponding classical results unless $v = 0$ strictly. Furthermore, from Eqs(2.16) and (2.18) we see that the to and fro speeds are individually equal both in the longitudinal direction and in the transverse direction. In fact, it can be shown that the same conclusion holds also for any arbitrary direction in $\bar{S}$. This is precisely the *standard synchronization convention.* Thus Einsteinian synchrony inherent in LT is preserved (even under the approximation $v^2/c^2 << 1$). This is exactly in accordance with our earlier assertion.

However, one may still suspect whether the transformation Eq.(2.12) represents a Galilean world in essence, save the synchronization convention. In order to decide this, one must compare synchrony independent quantities obtained from Eq.(2.12) with those obtained from the usual Galilean transformations. One such quantity is the round-trip speed of any signal. In fact, two sets of transformations may appear structurally very different depending on the choice of synchrony, but when synchrony independent quantities are compared one might discover that they are essential same. In that case we say that these two transformations represent the same kinematical "World". From the Galilean transformation, it follows that two-way average speed of light in the direction parallel and perpendicular to the direction of relative motion are given respectively by

$$\overleftrightarrow{c}_{\parallel}= c(1 - v^2/c^2),$$
$$\overleftrightarrow{c}_{\perp}= c(1 - v^2/c^2)^{1/2}, \tag{2.19}$$

whereas we see from Eqs.(2.16) and (2.18) that they are given by

$$\overleftrightarrow{c}_{\parallel}= c,$$
$$\overleftrightarrow{c}_{\perp}= c(1 - v^2/c^2)^{1/2}, \tag{2.20}$$

Thus Eq.(2.12) for all v in general, does not represent a Galilean World (GW). Of course one may choose $v^2/c^2 << 1$ again in Eqs.(2.19) and (2.20), and it becomes clear that Eq.(2.12) represents GW approximately. But then there is a subtle point that must be carefully noted. The resulting GW is not a GW in totality but it is limited by the very approximation. To exemplify this point, consider the Tangherlini Transformations (TT), which represents an Einstein World (EW) with absolute (Galilean) synchrony[16]:

$$\bar{x} = (x - vt)/(1 - \beta^2)^{1/2},$$
$$\bar{t} = t(1 - \beta^2)^2, \tag{2.21}$$

with $\beta = v/c$.

Note that if $v^2/c^2 << 1$, the resulting transformations represent a GT in totality. This is expected because we mentioned before that any set of transformations depends structurally on the choice of synchrony. Since here we consider Galilean synchrony it is natural that under the condition $\beta^2 << 1$ it gives GT in totality. Obviously, this fact is absent in Eq.(2.12). Hence it proves again that a convention once chosen does not change into a different one due to an approximate assumption on the relativity velocity alone.

Thus we have demonstrated that the LT does not lead under the small velocity approximation to Galilean (absolute) synchrony. As a result, the GT for one-way velocities could not be obtained unless $v = 0$ strictly. However, Eq.(2.12) represents a GW only for small velocities but not for the entire velocity range, in contrast to the Tangherlini case just mentioned above.

Finally, one may raise the question whether it is at all possible to construct a transformation which represents a GW in totality having standard synchrony. Indeed, one may verify that the transformations due to Zahar and Sjodin[12, 14, 15] satisfies the above characteristics which are just complementary to those of TT.

$$\begin{aligned} \bar{x} &= (x - vt), \\ \bar{t} &= [t - (vx/c^2)]/(1 - v^2/c^2). \end{aligned} \tag{2.22}$$

It is evident that this transformations (ZT) reduces to ALT from Eq.(2.12) if the v^2/c^2 term is neglected. Note that here again the Poincare-Einstein synchrony is preserved.

Thus we see that LT under the small velocity approximation does not go over to GT but instead, it becomes, as it should be equivalent to ZT from Eq.(2.22) under the same approximation. In contrast, TT from Eq.(2.21) directly goes over to GT. Therefore, in order to fully comprehend the passage of SR to GR one should examine LT vis-a-vis ZT and GT vis-a-vis TT in the context of small speed approximation.

2.3 CS-thesis and Preferred Frame

In an interesting paper Ghosal et.al [12] dealt with the CS issue in a novel way by considering "Relativity in a substrate". Later Chakraborty[23] while putting it in the context of ether wrote "Sometimes in connection with the CS-thesis, the debatable issue of ether (as a *hypothetical* substrate providing a preferred inertial frame) often crops up[13, 14, 24]. But question have been raised whether considerations of synchronization alone can distinquish an ether frame or not[24, 25, 26]. As it stands now, as if the existence of a real physical ether as a preferred frame would have placed the CS-thesis on a stronger footing. In fact efforts are still on to give a physical support to this preferred frame of ether. (However for the understanding of CS-thesis at least, one can bypass the debate concerning the existence of ether by introducing at the out-set a real physical substrate (water for example) through which different inertial frames may be considered to be in relative motion). Given this perspective of confusion, misconstruction and polemics regarding the CS-thesis or SR for that matter, we are led to conclude that everything of SR is still not well understood. We therefore feel that it is necessary to provide some additional clarifications in this regard". It is to this task that the aforesaid paper addresses itself.

Before we discuss the context of the paper let us start with the following observations. In the standard formulation of SR light has two different roles to play. On the one hand it acts as a synchronizing agent, on the other hand it has invariant two-way-speed (TWS) in vacuum. The second role has a basis in the empirically verifiable property, but the first one is purely perspective in origin. In the derivation of the LT in the standard SR, these two roles are mixed up. The inseparability contributes to several misconceptions and prejudices in relativity theory. In order to separate these roles one may introduce non-luminal signal to synchronize clocks and re derive transformation transformation equations. The authors[12] consider reference frames submerged in a substrate. In order to derive the transformation equations, they propose to synchronize the clocks by some other signal (acoustic signal (AS)) which is a characteristic of the substratum. The authors first consider an acoustic wave generated at $t = 0$ at the common origin of the frames S_i and

S_k. In all other frames except for the frame S_0 which is at rest relativity to the substratum, the velocity of AS in the positive x-direction and negative x-direction will not be the same. Using the CS-thesis they define the synchronization of clocks so that these two velocities are equal in all frames although their values vary from frame to frame. This synchrony is called the *pseudo-standard synchrony* other than Einstein's standard synchrony. According to pseudo-standard synchrony, the one dimensional wave front equation will be

$$x_k^2 = a_{kx}^2 t_k^2, \tag{2.23}$$

where x_k's are co-ordinates of a frame S_k which is moving with respect to S_0 frame which is fixed in the substrate and a_{kx} is the TWS of the AS in the x- direction.

The acoustic wave front will not be spherical in frames other than in S_0 frame. Two-way-speed (TWS) of AS will not be the same in all directions, for example along the y-direction the wave front equation will be

$$y_k^2 = a_{ky}^2 t_k^2, \tag{2.24}$$

where a_{ky} is the TWS of AS along y-direction and may have different value from a_{kx}.

The Derivation of Transformation Equations:

In order to derive the transformation equations (TE) between two general inertial frame S_i and S_k one can use TE in the linear form as,

$$\begin{aligned} x_k &= \alpha_{ik}(x_i - v_{ik}t_i), \\ y_k &= y_i, \\ t_k &= \xi_{ik}x_i + \beta_{ik}t_{ik}. \end{aligned} \tag{2.25}$$

In the above equation v_{ik} is the velocity of S_k with respect to S_i and α_{ik}, ξ_{ik} and β_{ik} are constant that are to be determined by using pseudo-standard synchrony. Hence according to the chosen synchrony, one can set the condition

$$x_k^2 - a_{kx}^2 t_k^2 = \lambda_{ik}^2 (x_i^2 - a_{ix}^2 t_i^2), \tag{2.26}$$

where λ_{ik} is a sale factor that is independent of the space and time coordinates. Using Eqs.(2.25) and (2.26) one can obtain the transformation coefficients as

$$\alpha ik = \lambda_{ik}\gamma_{ik}, \tag{2.27}$$

$$\beta_{ik} = \alpha ik/\rho_{ik}, \tag{2.28}$$

$$\xi_{ik} = -\frac{\alpha_{ik}/\rho_{ik}}{v_{ik}/a_{ix}^2}, \tag{2.29}$$

with

$$\gamma_{ik}(1 - v_{ik}^2/a_{ix}^2)^{-1/2}. \tag{2.30}$$

and

$$\rho_{ik} = a_{kx}/a_{ix}. \tag{2.31}$$

Thus the transformation Eqs.(2.25) can be written as,

$$\begin{aligned} x_k &= \lambda_{ik}\gamma_{ik}(x_i - v_{ik}t_i), \\ t_k &= (\lambda_{ik}/\rho_{ik})\gamma_{ik}(t_i - v_{ik}x_i/a_{ix}^2). \end{aligned} \tag{2.32}$$

According to adopted synchrony the TWS of AS is isotropic in the preferred frame S_0 which is stationary with respect to the medium. In the general frame S_k it will not be isotropic. If the isotropic signal speed is a_0, one can write

$$a_x^2 + a_y^2 = a_0^2, \tag{2.33}$$

where a_x and a_y are the x and y components of the velocity of the wavefront along the direction.

The TWS in x- direction in S_k is given by

$$a_{kx} = \frac{\alpha_{0k}a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.34}$$

The TWS in y- direction in S_k is given by

$$a_{ky} = \frac{a_0(1 - v_{0k}^2/a_0^2)}{\beta_{0k} + \xi_{0k}v_{0k}}. \tag{2.35}$$

Also, the general transformation laws for any other signal whose isotropic TWS (equal to its OWS) in S_0 is a'_0 (which may differ from a_0) can be written as

$$a'_{kx} = \frac{\alpha_{0k}a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.36}$$

The TWS in y- direction in S_k is given by

$$a'_{ky} = \frac{a'_0(1 - v_{0k}^2/a_0'^2)}{\beta_{0k} + \xi_{0k}v_{0k}}, \tag{2.37}$$

where a'_{kx} and a'_{ky} are the TWS of the signal as measured from S_k in the longitudinal and in the transverse directions respectively. However it is clear that to arrive at these relations one assumes that with respect to S_0 under the chosen synchrony, the OWS of "other" signal is isotropic and hence is equal to its TWS. In other words it has been tacitly assumed that in S_0 the pseudo-standard synchrony with AS and with the "other" signal are equivalent.

Now using Eqs.(2.27), (2.34) and (2.35), after simplification reads

$$\lambda_{0k} = a_{kx}/a_{ky}. \tag{2.38}$$

Also

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}}\frac{a_{iy}}{a_{ix}}. \tag{2.39}$$

On putting the value of λ_{ik} the TE of Eq. (2.32) becomes

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})(a_{iy}/a_{ix})[(x_i - v_{ik}t_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}], \\ t_k &= (a_{iy}/a_{ky})[(t_i - (v_{ik}/a_{ix}^2)x_i)/(1 - v_{ik}^2/a_{ix}^2)^{1/2}]. \end{aligned} \tag{2.40}$$

With respect to preferred frame S_0 (where $a_{0x} = a_{0y} = a_0$) the TE from S_0 to any other inertial frame S_k is given by

$$\begin{aligned} x_k &= (a_{kx}/a_{ky})[(x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}], \\ t_k &= (a_0/a_{ky})[(t_0 - (v_{0k}/a_0^2)x_0)/(1 - v_{0k}^2/a_{0x}^2)^{1/2}]. \end{aligned} \tag{2.41}$$

In a lighter vein the authors term this set of transformation equations dolphin transformations (DT) as these TE perceived by intelligent dolphins. The DT is usable the space-time relations between two frames provided one knows the TWS of AS in these two frames. If one chooses light signal (vacuum) for synchronization of clocks instead of AS, by virtue of CVL postulate in SR

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c, \tag{2.42}$$

so that one would obtain the familiar LT. However in absence of any communication with the outside world, *apparently* c does not play any role in DT even though the dolphins live in the relativistic world where we know c plays a fundamental role! Indeed in the DT, c will appear as a *physical constant* through a_{kx} and a_{ky}. In order to make use of DT, the dolphins will have to measure the TWS of AS in S_k as a function of velocity v_{0k} and one can anticipate that they will eventually find that

$$\begin{aligned} a_{kx} &= a_{kx}(v_{0k}, c), \\ a_{ky} &= a_{ky}(v_{0k}, c), \end{aligned} \tag{2.43}$$

where c appears not as the speed of light but as some physical constant. If now the dolphins are able to communicate with the outside world and discover that their world admits an invariant speed c. Recall the formulas for two-way velocity transformation Eqs.(2.36) and (2.37) and put $a'_{kx} = a'_{ky} = a'_0 = c$. Now using Eqs(2.27-2.30) one may easily demonstrate that

$$\rho_{0k} = \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.44}$$

and

$$\lambda_{0k} = \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.45}$$

or by Eqs.(2.31) and (2.38)

$$a_{kx} = a_0 \frac{(1 - v_{0k}^2/a_0^2)}{(1 - v_{0k}^2/c^2)}, \tag{2.46}$$

and

$$a_{ky} = a_0 \frac{(1 - v_{0k}^2/a_0^2)^{1/2}}{(1 - v_{0k}^2/c^2)^{1/2}}. \tag{2.47}$$

Now inserting Eqs.(2.45) and (2.46) in Eq.(2.41) gives the DT for the relativistic world

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0)/(1 - v_{0k}^2/a_0^2)^{1/2}, \\ t_k &= (1 - v_{0k}^2/c^2)^{1/2}(1 - v_{0k}^2/a_0^2)^{-1}[t_0 - (v_{0k}/a_0^2)x_0]. \end{aligned} \tag{2.48}$$

There are important consequences of DT. These are the following:

1. The transformation equations contain TWS of synchronizing signal. The simultaneity is relative. Under this synchrony relative speeds are not symmetric in general.

2. a_0 is the speed of AS that is conventional. c appears as a physical constant - the TWS of light - and is not based on any convention. The factor $(1 - v_{0k}^2/c^2)^{1/2}$ is due to real effects. The other factor, $(1 - v_{0k}^2/a_0^2)$ arises from the synchronization procedure which is evident from the presence of the term a_0. Thus this clarifies that different synchronization procedure may not have relativity of simultaneity but they can predict length contraction and time dilation effects. From the DT, the length contraction factor (LCF) and time dilation factor (TDF) comes out to be

$$\begin{aligned} LCF &= (1 - v_{0k}^2/a_0^2)/(1 - v_{0k}^2/c^2)^{1/2}, \\ TDF &= (1 - v_{0k}^2/c^2)^{1/2}/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.49}$$

3. As we have mentioned earlier that light has two roles to play in SR. One is that its TWS in vacuum is constant and the other is that it is the synchronization agent in SR. These two roles are mixed up in standard SR. In the derivation of DT we see that these two roles are clearly split up.

Some important transformation equations in relativistic and classical worlds obtains by others can be obtained from DT by the choice and making use of the properties of the synchronization signal:

Lorentz transformation ***(Einstein synchrony and relativistic world)*:**
In the standard synchrony the synchronization agent is light. Putting $a_0 = c$ in DT one may obtain Lorentz transformation.

Tangherlini transformation ***(absolute synchrony and relativistic world)*:**
If in the *preferred frame* the speed of synchronization signals $a_0 \to \infty$ then we obtains (for $S_0 \to S_k$) the Tangherlini transformation

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma^{-1}t, \end{aligned} \tag{2.50}$$

Zahar transformation ***(Einstein synchrony and classical world)*:**
In the classical world the velocity addition law is the Galilean one. Then the TWS of AS is obtained to be

$$\begin{aligned} a_{kx} &= a_0(1 - v_{0k}^2/a_0^2), \\ a_{ky} &= a_0(1 - v_{0k}^2/a_0^2)^{1/2}. \end{aligned} \tag{2.51}$$

Inserting these expressions for a_{kx} and a_{ky} in the DT (and in particular in Eq.(2.41) we obtain DT in in classical world

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/a_0^2)x_0]/(1 - v_{0k}^2/a_0^2). \end{aligned} \tag{2.52}$$

In the standard synchrony, ($a_0 = c$) DT becomes Zahar transformation (ZT) as we have discussed earlier

$$\begin{aligned} x_k &= (x_0 - v_{0k}t_0), \\ t_k &= [t_0 - (v_{0k}/c^2)x_0]/(1 - v_{0k}^2/c^2). \end{aligned} \tag{2.53}$$

Galilean transformation ***(absolute synchrony and classical world)*:**
In this classical world if the synchronizing signal's speed is assumed to be arbitrarily large (hypothetically) so that one may put $a_0 \to \infty$ in Eq.(2.52), one retrieves the familiar form of GT.

Before we conclude this section it may be mentioned that the Dolphin transformation will be found to serve as a spring board for developing preferred frame theories which may be prompted by the possible violation of GZK limit by the ultra high energy cosmic rays or by the considerations of the variable speed of light in the context of cosmology (vide chapters VI and VII for details). Here I would like to point out that DT has been used earlier (by considering the cosmic microwave background as the substratum) to deal with the question of existence of non-zero photon rest mass (advocated by Narlikar, Pecker and Vigier[27]) by Ghosal and Karmakar[28].

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