Abstract

In this thesis, a scale invariant analysis for a Cantor set like fractal subset C of the real line R is developed using the concepts of relative infinitesimals and the associated nonarchimedean absolute values. The meaning and salient properties of the scale invariant nonarchimedean valuation are discussed in detail through various examples. The valuation is shown to be related to an appropriate Cantor function, which is then realized as a locally constant function defined over the ultrametric Cantor space. The formalism of calculus and a valued measure are introduced on such an ultrametric space. The increments on such an ultrametric space are mediated by inversions. The valued measure is shown to equal the finite Hausdorff measure of the original Cantor set. The ordinary limit $x \to 0$, $x \in C$ is shown to be given by a limit of the form $x \log x^{-1} \to 0$, when $x \in R$. Next, we study an interesting new phenomenon called the growth of measure, exploiting the reparametrisation invariance of a locally constant function. The phenomenon is explained explicitly by showing how a measure zero Cantor set may become a positive measure set. The role and meaning of a higher order valuation is explained by constructing a class of Cantor sets having identical Hausdorff dimension and thickness. Next, we study the relevance of a novel class of nonsmooth solutions of the scale invariant ODE $t\frac{dx}{dt} = x$ in the context of ultrametric Cantor sets. Some applications of the new class of solutions in the longstanding problems of time asymmetry, $1/f$ noise, origin of q-deformed exponential in the chaos threshold of the logistic maps are also discussed.