CHAPTER - VI.

LOW PRESSURE BREAKDOWN IN GASES IN A UNIFORM HIGH FREQUENCY ELECTRIC FIELD (a) WITHOUT MAGNETIC FIELD (b) WITH A STEADY TRANSVERSE MAGNETIC FIELD.

INTRODUCTION.

The mechanism of breakdown of a gas at pressures less than a few microns of mercury has been under investigation for some time to bring to light its salient features. For breakdown measurements some of the workers used the external electrodes and measured the peak starting potential and others used internal electrodes. Early investigations of breakdown field strengths in gases at low pressures and high frequency from 1 to 100 Mc/sec were carried out by Gutton and Gutton (1924, 1928), Gutton (1930) and Kirschner (1925, 1930). Their main observation was that the breakdown field strength decreases with decreasing frequency to values as low as 10 v/cm until a cutoff frequency is reached where breakdown becomes a matter of chance even at a very high electric field strength. A latter investigation by Backmark and Bengston (1941) led to a theoretical analysis of the mechanism by Danielsson (1943) where he proposed that the breakdown is caused by the increase of electrons by the resonance of secondary electrons with the electric field. A few electrons present initially by natural causes were accelerated to one end electrode where they produced secondary electrons by impact. These secondaries were emitted in a reverse electric field which carried them to the opposite electrode in approximately half a cycle to produce another group of secondaries. As secondary emission yield is greater than unity, electrons are multiplied to a very large quantity in a very short duration and this results in the breakdown of the gas. This mechanism is called breakdown by secondary electron resonance. Gill and Von Engel (1948) made observations in the frequency range of 12 to 75 Mc/sec and observed using external electrodes that the peak starting field strength for high frequency uniform electric field at a pressure of few microns for gases like He, Hg, H_2 and air is independent of the nature of the gas and depends ~~microns~~ slightly on its pressure. They developed a theory postulated on the secondary electron resonance mechanism which was somewhat different in its mathematical formulation from that of Danielsson (1943), but it predicts a cutoff law relating cut-off frequency and electrode seperation. Hatch and Williams (1953, 1954) measured using internal electrodes the breakdown field strength in air and hydrogen at pressures of the order of 1 micron of Hg and exciting frequencies varying from 25 to 90 Mc/sec. By suddenly applying a high voltage and then lowering it slowly an upper breakdown curve has been observed which was then combined to the normal lower breakdown curve. Outside this closed curve, no discharge could be started. They extended their work (1958) by developing a theory assuming higher order modes besides the conventional half cycle one. The phenomena has been explained assuming the process of bunching the electrons in the multipacting discharge otherwise known as secondary electron resonance discharge by Miller and Williams (1962), Paschke (1961) and Hatch (1961). Though a consistent theory of the phenomena of secondary electron resonance has been developed by Gill and Von Engel (1948) it is worth while to investigate some of the consequences of the theory with regard to variation of the starting voltage using external electrodes and the cut-off frequency with the length of the discharge tube.

The effect of superimposing an external field upon this type of discharge was investigated by Kossel and Krebs (1954) though no quantitative explanation of the observed results was provided. It has been found that superimposing a d.c. electric field parallel to high frequency electric field, starting can be made more difficult. A small static magnetic field perpendicular to the high frequency electric field causes a general increase of starting potential and a lowering of the cut-off frequency without changing the nature of $(E-\lambda)$ curve, where E is the starting potential and λ is the wavelength of the applied r.f. field. At large magnetic field, starting potential becomes independent of frequency and for very low pressure (10^{-5} mm. Hg.) the discharge can be put out either by increasing the electric field or decreasing the magnetic field. Deb and Goswami (1964) made a theoretical approach to the phenomena when a steady magnetic field is placed perpendicular to high frequency electric field.

No systematic observation of the breakdown of gases controlled by secondary electron emission in a high frequency electric field in presence of an external d.c. magnetic field has so far been undertaken. The object of the present investigation is thus to study the effect of a transverse d.c. magnetic field on this type of breakdown with regard to starting field and the cut-off frequency. The theory of the previous workers has to be modified due to effects produced by the introduction of the magnetic field and it is presumed that these investigations may throw some light on the mechanism of such discharges.

E X P E R I M E N T A L   A R R A N G E M E N T.

The breakdown potential of the gas has been determined in the same way as has been done by Gill and Von Engel (1948). The source of radiofrequency electric field is a tuned plate tuned grid oscillator covering the frequency range of 4 Mc/ sec. to 30 Mc/sec in three stages. The out put of the oscillator which can be varied from 0 to 500 volts has been measured by a vacuum tube voltmeter; measurements of the breakdown voltage at the highest frequency and is limited by the radiofrequency voltage output of the oscillator. The cylindrical discharge tubes made of pyrex glass are properly cleaned and evacuated to 10^{-5} mm. of Hg. by an oil diffusion pump. The external electrodes which are perpendicular to the axis of the discharge tubes are connected to the radiofrequency source. Measurements have been taken in three discharge tubes of length 5 cm, 7cm and 15 cm (diameter of each tube is 3.5 cm.) to study the effect of the length of the discharge tube on the value of breakdown potential as well as on the cut-off frequency. Besides pure and dry air, hydrogen prepared by the electrolysis of barium hydroxide solution and dried by phosphorouspentoxide have been used as the dielectric media. No special attempt for the purification of the gases has been made as traces of impurity and nature of the gas media have practically no effect upon this type of discharge. All the measurements have been made at a pressure of 1.5 micron of mercury which has been measured by an Edward penning pirani vacuum gauge. The steady magnetic field has been provided by an electromagnet having flat pole pieces of face area 7.5 cm x 4.5 cm placed at right angles to the length of the tube. The experiment with transverse steady magnetic field has been performed with the tube of 5 cm. length only so that the tube remains well inside the magnetic pole pieces to ensure uniform magnetic field which has been measured by a calibrated fluxmeter. Except the external electrodes and the discharge tube the system has been properly grounded. As the voltage is gradually increased a faint glow appears at the breakdown point and simultaneously there is slight drop in the output voltage at the vacuum tube voltmeter. This drop in voltage is less and less marked as the cut-off frequency is approached.

RESULTS AND DISCUSSION.

PART (a) LOW PRESSURE BREAKDOWN IN A UNIFORM HIGH FREQUENCY ELECTRIC FIELD.

To start with, starting potentials have been measured in air and hydrogen in a discharge tube of length 15 cm to verify the argument that secondary electron resonance is independent of the nature of the gas. The solid curve in fig.25 represents the results in case of air and the circles on it are the observations with hydrogen. The identical nature of the two breakdown curves

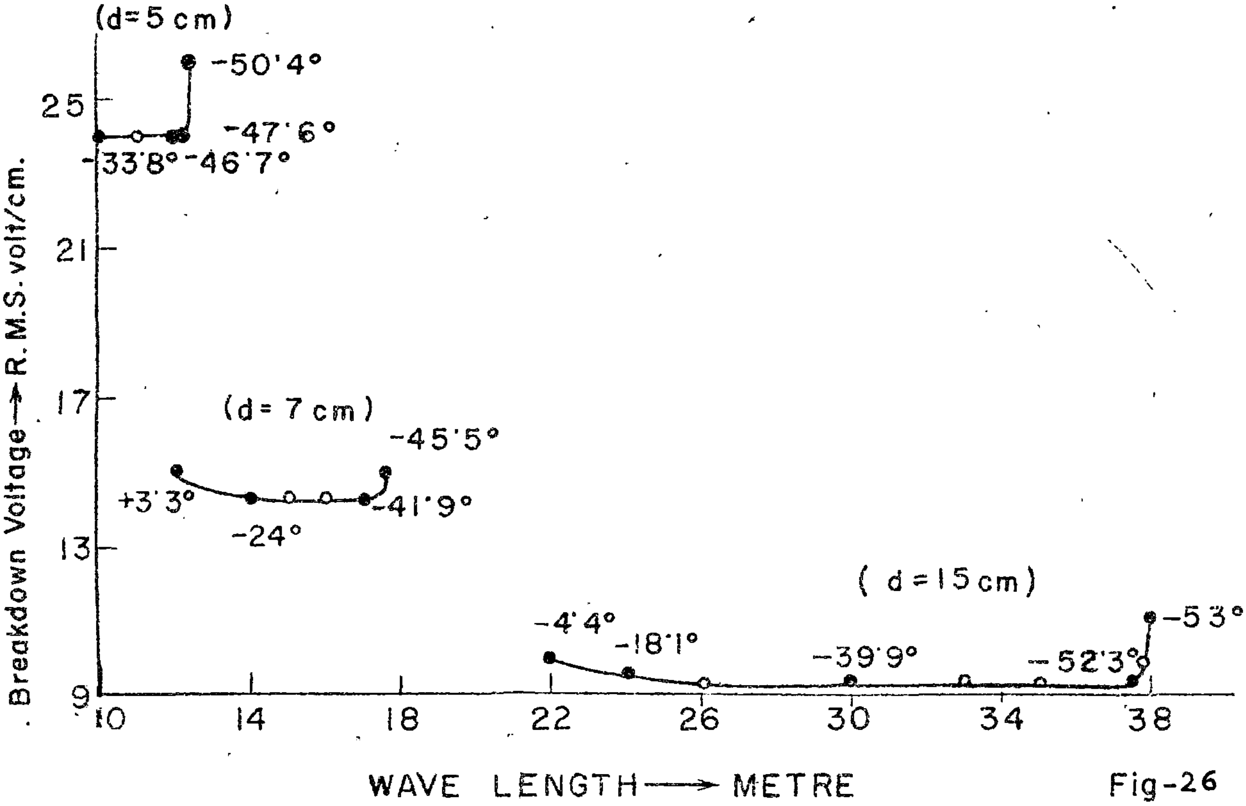
(d=5 cm)
-50'4°
-47'6°
-33'8°
-46'7°
(d= 7 cm)
-45'5°
+3'3°
-41'9°
-24°
(d=15 cm)
-4'4°
-18'1°
-39'9°
-52'3°
-53°
Breakdown Voltage → R. M. S. volt/cm.
25
21
17
13
9
10
14
18
22
26
30
34
38
WAVE LENGTH → METRE

Fig-26

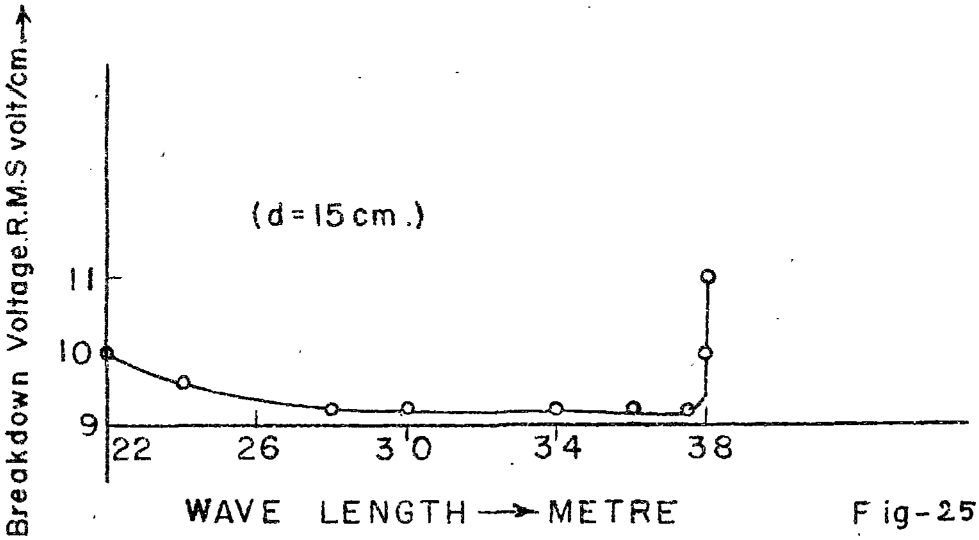
(d=15 cm.)
Breakdown Voltage.R.M.S volt/cm. →
11
10
9
22
26
3'0
3'4
38
WAVE LENGTH → METRE

Fig-25

indicates that the type of breakdown observed in the present experimental setup is independent of the nature of the gas and is controlled by secondary electron resonance. Starting potentials have been measured in three discharge tubes of length 5cm, 7 cm and 15 cm and the results have been plotted in fig.26. It is observed that the breakdown voltage is higher in tubes of shorter lengths and the cut-off wave length increases with the length of the discharge tube. Measurements towards the shorter wave length region could not be taken due to the limitation in the output power of the radiofrequency oscillator. It is evident that the curves are identical with those obtained by previous workers with the sharp cut-off towards the higher wavelength side.

The qualitative description of secondary electron resonance breakdown mechanism has been presented by Danielsson (1943), Gill and Von Engel (1948), Hatch and William (1953, 1954) and others almost in an identical way. At the start of a half cycle transit period, it is assumed that the electrons leave the emitting surface with energies of the order of 5 ev. At the end of the transit period most of these electrons arrive at the opposite end wall with energies of the order of 50 to 500 ev. The cyclic repetition of this process is referred to as resonance in this type of discharge. To proceed with a mathematical formulation of the process involved in the mechanism, some fundamental assumptions are necessary, though the extent to which many of these assumptions compensate in an undetermined manner the other processes occuring is not very clear. In order that this mechanism be operative it is necessary to assume that the electronic mean free path and wavelength of the applied high frequency field are both large compared to the electrode separation and both these assumptions are valid in the present experimental setup. For mathematical simplicity it is convenient to assume that all electrons have half cycle transit times, though according to Hatch and Williams (1954) this assumption is not an accurate representation of secondary emission characteristics but is very useful in getting a simple formulation of the problem and leads to good correlation between theory and observation. Further the secondary electron emission velocities are normal to the end surfaces and the electric field between the electrodes is uniform in space. Space charge effects are negligible and it is assumed that the electron arrival energies exceed the ionisation potential of the gas, and that a few electrons are produced randomly between the electrodes by natural processes.

Based on these assumptions Gill and Von Engel (1948) deduced that the breakdown field E is given by

$$E = \frac{\omega^2 d - \pi v_0 \omega}{\frac{e}{m}\left[\pi \cos\phi + 2\sin\phi\right]} \qquad \ldots(6.1)$$

where ω is the angular frequency of the applied radiofrequency field and ϕ is the phase angle of the emitted secondary electron with respect to the electric field, d the length of the discharge tube and v_0 the initial velocity of the electron. The velocity of the electron when it strikes the opposite end is given by

$$v = v_0 + \frac{2eE}{m\omega} \cos\phi \qquad \ldots(6.2)$$

if it is assumed, as has been done by Gill and Von Engel (1948), that $\frac{v}{v_0} = K$, a constant then

$$v = \frac{K}{K-1} \cdot \frac{2eE\cos\phi}{m\omega} \qquad \ldots(6.3a)$$

$$d = \frac{eE}{m\omega^2}\left[\pi \cdot \frac{K+1}{K-1} \cdot \cos\phi + 2\sin\phi\right] \qquad \ldots(6.3b)$$

$$E = \frac{\omega^2 d}{(e/m)\,\Phi} \qquad \ldots(6.3c)$$

$$\text{where} \quad \Phi = \frac{K+1}{K-1} \cdot \pi . \cos\phi + 2 \sin\phi \qquad \ldots(6.4)$$

Gill and Von Engel analysed their data using equations (6.3) and (6.4) and taking Φ as a parameter. Following Hatch and Williams (1954) the electron arrival energy can be expressed as

$$\mathcal{E} = \frac{1}{2} m v^2 \quad \text{where } \mathcal{E} \text{ is expressed in e.v.}$$

combining this with equations (6.3a) and (6.3c) we get for the frequency

$$f = \frac{(K-1)\ \Phi}{K \pi d \ \cos\phi} \cdot \left(\mathcal{E}/8m\right)^{1/2} \qquad \ldots(6.5)$$

From equation (6.5) it can be seen that for fixed values of " ω " and " d ", E becomes minimum for the maximum value of Φ. Maximising Φ with respect to ϕ gives the condition

$$\phi = \tan^{-1}\left(\frac{K-1}{K+2} \cdot \frac{2}{\pi}\right) \qquad \ldots(6.6)$$

Using equation (6.3), (6.4), (6.5), (6.6) and taking the value of K = 3, Hatch and Williams fitted their observation and obtained the value of ϕ as $-56° \leqslant \phi \leqslant 18°$. They obtained the linear portion of the curve where electric field remains almost constant by assuming a fixed particular electron arrival energy $\mathcal{E}$, which in their case was 60 e.v. By way of comparison, Danielsson (1943) assumed K = ∞ i.e. $v_0 = 0$ with $0 \leqslant \phi \leqslant 90°$ and found $\mathcal{E} = 80$ e.v. Gill and Von Engel (1948) found, by fitting their equations to data for external electrodes at a seperation of 3 cm, that for K = 4, $\mathcal{E} = 90$ e.v. and cut-off occured at an escape limiting value of ϕ = -58°. Their values of K of the order of 3 to 4 and electron arrival energies of the order of 60 to 90 e.v. are compatiable with known secondary electron emission energies and yield.

Following Hatch and Williams let us assume K = 3 then $\Phi = 2(\pi \cos\phi + \sin\phi)$ consequently equations (6.3c) and (6.4) can now be fitted to the experimental curve (fig. 26). Some points are chosen in each curve and the calculated values of corresponding ϕ are marked there(o).The values of ϕ so obtained are not much different from the values obtained by previous workers in all the three tube lengths of 15 cm, 7 cm and 5 cm respectively.

The cut-off occurs because at lower frequencies the electron must leave the wall in more negative phases in order that they take half a cycle to travel the length of the tube. For negative values of ϕ, the field, until it reverses, will oppose the motion of electrons in Z direction along the axis, say. Their initial velocity takes them a little distance against this force, and then they turn back, accelerating towards Z = 0, until the reversal of the field decelerates them and turns them finally in the proper direction. The largest permissible value of ($-\phi$) is that which just returns the electrons to the wall and hence the second turning point is $\frac{dZ}{dt} = 0$ at $Z = 0$. In more negative values of ϕ, the second turning point is theoretically at negative values of Z which means that the electrons are driven back to the wall at a very low speed and stick there, so that no multiplication can take place.

The cut-off frequency f_{co} can be obtained using equation (6.5) and assuming that the arrival energy of electron is critical i.e. secondary emission coefficient $\delta = 1$ and using the upper breakdown phase angle boundary value ϕ_{co} we get

$$f_{co} = \frac{(K-1)\left[\frac{K+1}{K-1}\pi \cos\phi_{co} + 2\sin\phi_{co}\right]}{K\pi d \cdot \cos\phi_{co}} \cdot \left(\frac{\mathcal{E}_{crit}}{8m}\right)^{1/2} \quad \text{....(7)}$$

For constant K, ϕ_{co} and $\mathcal{E}_{crit}$ the above relation becomes

$$f_{co} \times d = \text{constant} \quad \ldots(6.8)$$

where the constant of the relation (6.8) is obtained by fitting the experimental curve. Gill and Von Engel obtained the value of this constant from general similarity theorem to be 79. Hatch and Williams tried to fit the relation (6.8) to their work and those of Guttons (1924, 1930) and Gill and Von Engel. Results of Guttons fitted very well to the curve, but that of Gill and Von Engel and Hatch and Williams showd divergence from the predicted law. For length of the tube greater than 2 cm, the divergence increases with increase of length of the tube and shows a tendency of increase of the value of the constant. This is due to sidewall effect as shown by Chandrakar and Von Engel (1965) and Francis (1960) and Hatch and Williams (1954). From our observations, we have also calculated the values of ϕ_{co} using equation (6.4) and from equation (6.7) the values of $\mathcal{E}_{crit}$ for each length of the tube were also calculated. These quantities are shown in table I.

TABLE - I.

Length of discharge tube (d) cm.	Diameter of the Discharge tube cm.	Frequency at cut off f_{co} Mc/s.	$(f_{co} \times d)$	Breakdown voltage at cutoff point (E_{co}) volt/cm.	Phase angle at cut off (ϕ_{co}) degree	Arrival energy at cut off $\mathcal{E}_{crit}$ e.V.
15	3.5	8	120	9.3	-52.3°	102.5
7	3.5	17	119	14.3	-44.3°	73.5
5	3.5	24	120	24	-48.8°	88

From the table it can be seen that the values of ϕ_{co} are not much different from the observations of the previous authors. The values of ε_{crit} are also slightly different than the experimental value of energy of 80 e.v. for secondary emission coefficient $\delta = 1$. Fitting equation (6.8) to experimental results the value of the constant is found to be ≈ 120 for all the three lengths of the tube. This value, though much higher than the value predicted by Gill and Von Engel is yet not much uncommon as can be seen from the works of Hatch and Williams where for tube length of 4 cm, the result shows the value of the constant as 120. In our experiment the lengths are much higher than the length of the tubes taken by Hatch and Williams, but comparable with some of the observation tube lengths taken by Gutton and Gutton (1930) (1924) and Gill and Von Engel. In fig. (27) the cut-off law given by equation (6.8) is represented for the values of the constant 79 and 120 by solid curve and dotted line respectively and also the points of the different observations made previously along with our observations. It is found that for larger length of the tube, equation (6.8) fits in a better way with the observations if we take the value of the constant to be 120, whereas for smaller length, the value 79 is more reasonable. The reason for the increase of experimental ($f_{co} \times d$) value from the value of the constant 79 was ascribed by Hatch and Williams (1954) Francis (1960), Chandrakar & Von Engel (1965) and Gill & Von Engel (1948) to the fact that at the larger seperations the loss of electron to the side wall increases and most likely this loss will be more effective for smaller diameter of the tube. Chandrakar and Von Engel (1965) recently tested this assumption in their low pressure ring discharge experiment. At larger separation, the distortion of electric field may also be responsible for preventing some of the electrons from taking part in full half cycle of the transit and hence the loss may increase.

The breakdown voltage at cutoff V_{co} may be written by eliminating ω between (6.3c) and 96.5) and using the relation

$$\mathcal{E}_{crit} = mv^2/2$$

as

$$V_{co} = E_{co} \cdot d = \left(\frac{K-1}{K}\right)^2 \frac{\mathcal{E}_{crit}\left[\frac{K+1}{K-1}\pi \cos\phi_{co} + 2\sin\phi_{co}\right]}{2e\cos^2\phi_{co}} \qquad \ldots(6.9)$$

The relation (6.9) shows that it is independent of the wall seperation and applied frequency. This fact may help in studying the effect of electrode geometry on the mechanism. Hatch and Williams assumed ϕ_{co} and K as constant when relation (6.9) becomes.

$$V_{co} = C.\ \mathcal{E}_{crit} \qquad \ldots(6.10)$$

where 'C' is a constant the value of which is obtained by fitting with experimental data. In table II the values of V_{co} obtained experimentally, $\mathcal{E}_{crit}$ obtained from table (I) are given for the three lengths of the tube

TABLE - II.

Length of the tube (d) cm.	Breakdown voltage at cut-off E_{co} Volts/cm.	Energy of arrival at cutoff($\mathcal{E}_{crit.}$) e.v.	C Volts/ev.
15	9.3	102.5	1.36
7	14.3	73.5	1.36
5	24	88	1.36

The table clearly shows that the value of 'C' is a constant as predicted by relation (6.10) and when $\mathcal{E}_{crit}$ is expressed in e.v. and V_{co} in volt, the value of 'C' is 1.36.

It can thus be stated that the theory of Gill and Von Engel fits in with fair amount of success to our different observations. The yield of values of different unknown parameters involved in the theory for the process of fitting shows a fair amount of consistency among themselves for different lengths of the tube and when compared to the values obtained by previous workers.

PART (b) LOW PRESSURE BREAKDOWN BY A UNIFORM HIGH FREQUENCY ELECTRIC FIELD WITH A UNIFORM TRANSVERSE D.C. MAGNETIC FIELD.

The equation of motion of a secondary electron in presence of a transverse steady magnetic field and under the action of the oscillatory field

$$\vec{E}_x = E \sin(\omega t + \phi)$$

is given by

$$\frac{dv}{dt} = -\frac{e}{m}\left[\vec{E} + \vec{v} \times \vec{H}\right]$$

and hence

$$\frac{dv_x}{dt} = -\omega_H v_y + \frac{eE}{m}\sin(\omega t + \phi) \qquad \ldots(6.11)$$

$$\frac{dv_y}{dt} = \omega_H v_x$$

where $\omega_H = \frac{eH}{m}$ = cyclotron frequency.

From (6.11) $\frac{d^2 v_x}{dt^2} = -\omega_H^2 v_x + \frac{e\omega E}{m}.\cos(\omega t + \phi)$

The solution of the complementary function $\frac{d^2 v_x}{dt^2} + \omega_H^2 v_x = 0$

is $v_x = A \cos(\omega_H t) + B.\sin(\omega_H t)$

where A and B are two constants to be determined from the boundary conditions.

The particular integral

$$\frac{d^2 v_x}{dt^2} + \omega_H^2 v_x = \frac{e\omega E}{m}\cos(\omega t + \phi) \qquad \ldots(6.12)$$

has the solution

$$v_x = A' \cos(\omega t + \phi)$$

where A' is another constant

Putting this value of v_x to equation (6.12)

$$-A'\omega^2 \cos(\omega t + \phi) + \omega_H^2 A' \cos(\omega t + \phi) = \frac{e\omega E}{m}\cos(\omega t + \phi)$$

$$\therefore \quad A' = \frac{e\omega E}{m(\omega_H^2 - \omega^2)}$$

Hence the complete solution is

$$v_x = A\cos(\omega_H t) + B\sin(\omega_H t) + \frac{eE\omega\cos(\omega t+\phi)}{m(\omega_H^2-\omega^2)}$$

The boundary conditions are $v_x = v_0, \quad v_y = 0, \quad x = y = 0 \quad \text{at } t = 0.$

Now

$$\frac{dv_y}{dt} = \omega_H v_x = A\omega_H \cos(\omega_H t) + B\omega_H \sin(\omega_H t) + \frac{eE\omega\omega_H\cos(\omega t+\phi)}{m(\omega_H^2-\omega^2)}$$

$$\therefore \quad v_y = A\sin(\omega_H t) - B\cos(\omega_H t) + \frac{eE\omega_H\sin(\omega t+\phi)}{m(\omega_H^2-\omega^2)} + C$$

where C is a constant and let $\beta = (\omega_H^2 - \omega^2)$ **. Putting the value of** v_y **in equation (6.11) we get C = 0**

Hence

$$v_x = A\cos(\omega_H t) + B\sin(\omega_H t) + \frac{e\omega E}{m\beta}\cos(\omega t+\phi)$$

$$v_y = A\sin(\omega_H t) - B\cos(\omega_H t) + \frac{e\omega_H E}{m\beta}\sin(\omega t+\phi) \quad \text{...(6.13)}$$

Insertation of boundary conditions to equation (6.13) leads to the equation of velocity components and displacement components

$$v_x = \left[v_0 - \frac{eE\omega}{m\beta}\cos\phi\right]\cos(\omega_H t) + \frac{e\omega_H E}{m\beta}(\sin\phi)(\sin\omega_H t) + \frac{eE\omega}{m\beta}\cos(\omega t+\phi) \quad \text{...(6.14)}$$

$$v_y = \left[v_0 - \frac{eE\omega}{m\beta}\cos\phi\right]\sin(\omega_H t) - \frac{e\omega_H E}{m\beta}(\sin\phi)(\cos\omega_H t) + \frac{eE\omega_H}{m\beta}\sin(\omega t+\phi) \quad \text{...(6.15)}$$

$$x = \frac{1}{\omega_H}\left[v_0 - \frac{eE\omega}{m\beta}\cos\phi\right]\sin(\omega_H t) - \frac{eE\sin\phi}{m\beta}\cos(\omega_H t) + \frac{eE}{m\beta}\sin(\omega t+\phi) \quad \text{...(6.16)}$$

$$y = \frac{1}{\omega_H}\left[v_o - \frac{eE\omega \cos\phi}{m\beta}\right]\left[1 - \cos\omega_H t\right] + \frac{eE\omega_H}{m\omega\beta}\left[\cos\phi - \cos(\omega t + \phi)\right] - \frac{eE\sin\phi}{m\beta}\sin(\omega_H t)$$

...(6.17)

As before, let us assume that the tansit time of electron across the tube length is $t = \pi/\omega$, the half of the period of oscillatory field. Putting this value of t in equations (6.14) and (6.15) and taking the resultant velocity

$$v^2 = v_o^2 + 2\left(1 + \cos\frac{\omega_H}{\omega}\pi\right)\left[-\frac{v_o\omega eE}{m\beta}\cos\phi + \left\{\frac{eE\omega\cos\phi}{m\beta}\right\}^2 + \left\{\frac{eE\omega_H\sin\phi}{m\beta}\right\}^2\right] - \frac{2v_o eE\omega_H \sin(\frac{\omega_H}{\omega}\pi)\sin\phi}{m\beta}$$

...(6.18)

After Von Engel, let us assume that $v/v_o = K$ and hence

$$v^2 - v_o^2 = v^2\left(1 - \frac{1}{K^2}\right) \quad \text{and} \quad v_o = v/K$$

Hence

$$v^2\left(1 - \frac{1}{K^2}\right) = 2\left(1 + \cos\frac{\omega_H}{\omega}\pi\right)\left[-\frac{v\omega eE\cos\phi}{Km\beta} + \left\{\frac{eE\omega\cos\phi}{m\beta}\right\}^2 + \left\{\frac{eE\omega_H\sin\phi}{m\beta}\right\}^2\right] - \frac{2veE\omega_H\sin(\frac{\omega_H}{\omega}\pi)\sin\phi}{Km\beta}$$

... (6.19)

The electron is striking the other end with the velocity v given by equation (6.19) at an inclination angle θ to the axis of the cylinder. The angle θ is given by

$$\tan\theta = v_y/v_x$$

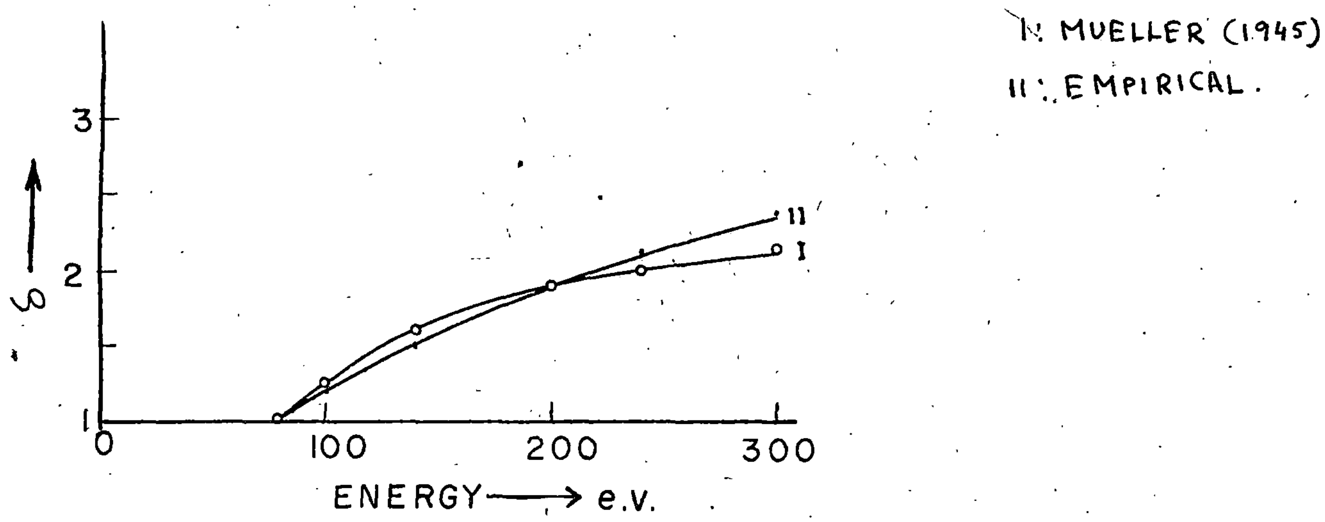
I: MUELLER (1945)
II: EMPIRICAL.
δ
3
2
1
0
100
200
300
ENERGY ⟶ e.v.
II
I

Fig.28

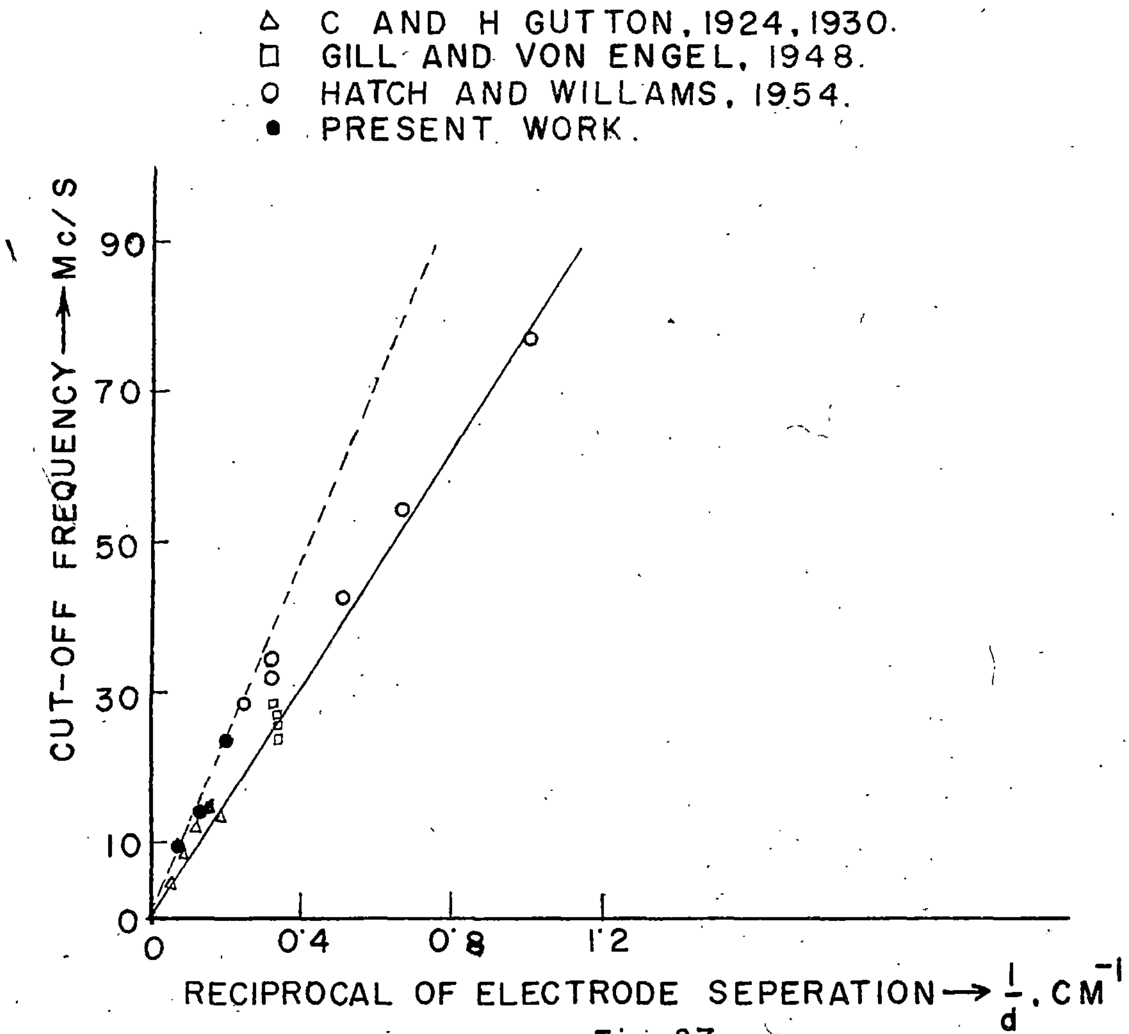
△ C AND H GUTTON, 1924, 1930.
□ GILL AND VON ENGEL, 1948.
○ HATCH AND WILLAMS, 1954.
● PRESENT WORK.
CUT-OFF FREQUENCY ⟶ Mc/S
90
70
50
30
10
0
0
0·4
0·8
1·2
RECIPROCAL OF ELECTRODE SEPERATION ⟶ 1/d, CM⁻¹

Fig-27

Bruining (1954) has shown that for certain metallic surfaces the relation between δ_θ and δ_0 is given by

$$\log \frac{\delta_\theta}{\delta_0} = C' \, (1 - \cos\theta)$$

where C' is a constant which is different for different surfaces and δ_θ is the maximum value of δ for an angle of incidence θ of primary electron of any energy and δ_0 = the maximum value of δ for normal incidence of primaries of the same energy. After Deb and Goswami (1964) we shall use this relation in case of glass, since no such relation between δ_θ and δ_0 is available for glass except a few experimental results of increase of δ_θ over δ_0. From the reported result of increase of by 50% over δ_0 for θ = 60° to θ = 0° in case of glass the value of the constant C' becomes approximately equal to unity, consequently

$$\delta_\theta / \delta_0 = \exp(1 - \cos\theta)$$

If n_θ and n_0 are the corresponding secondary yields of electrons for the same number of primaries, then

$$\frac{\delta_\theta}{\delta_0} = \frac{n_\theta}{n_0}$$

So it can be said that a particle with velocity smaller then the velocity of the normally incident particle is identical in its capacity to yield secondary electrons, when it hits the surface at a certain angle of inclination.

Mueller's (1945) results on hard pyrex glass is given in the fig. (28) which shows the average number of electrons released from the glass surface irrespective of the angle of emission as a function of the speed of the primaries. An empirical relation between energy of primary electrons in e.v. and δ, the yield can be represented by an equation

$$\delta = 1.2\,(\mathcal{E}/100)^{2/3}$$

This relation is fitted to the experimental curve which shows that the fitting is valid between 80 e.v. to 300 e.v. of energy of primary. In secondary resonance breakdown, the primary energies lie well within this limit. Hence we can utilise this relation between δ and $\mathcal{E}$.

Let $\mathcal{E}_{eff}$ be the effective energy of the particle which will yield the same number of secondaries, when it hits the surface normally as particle of energy $\mathcal{E}_o$ will yield hitting at an angle θ , we have then $\mathcal{E}_{eff} > \mathcal{E}_o$; the ratio of yield from Mueller's results fitted equation can then be written as

$$\frac{\delta_\theta}{\delta_o} = \left(\frac{\mathcal{E}_\theta}{\mathcal{E}_o}\right)^{2/3} = \frac{n_\theta}{n_o} = \exp\,(1-\cos\theta)$$

If v_{eff} = velocity of the primary corresponding to energy $\mathcal{E}_\theta$

v = velocity of primary corresponding to $\mathcal{E}_o$

then

$$\left(\frac{v_{eff}}{v}\right)^{4/3} = \exp\,(1-\cos\theta)$$

$$\therefore\; v_{eff} = v\,\exp\left[\tfrac{3}{4}(1-\cos\theta)\right]$$

Consequently we can state that the effect of hitting the surface with velocity v at an angle θ is equivalent to hitting the surface with velocity v_{eff} at normal incidence where v and v_{eff} are related by the above equation. Hence to account for the effect of hitting the surface obliquely, if we replace ν by $\nu_{eff}\{\exp -\frac{3}{4}(1-\cos\theta)\}$ in equation (6.19). We get the relation,

$$\nu_{eff}^2\left(1-\frac{1}{K^2}\right) = 2\left(1+\cos\frac{\omega_H}{\omega}\pi\right)\Bigg[-\frac{\nu_{eff}\,\omega\, e\, E\exp\{\frac{3}{4}(1-\cos\theta)\}}{Km\beta}$$

$$+\left\{\frac{eE\exp\{\frac{3}{4}(1-\cos\theta)\}\,\omega\cos\phi}{m\beta}\right\}^2+\left\{\frac{eE\exp\{\frac{3}{4}(1-\cos\theta)\}\,\omega_H\sin\phi}{m\beta}\right\}^2\Bigg]$$

$$-\frac{2\nu_{eff}\,eE\exp\{\frac{3}{4}(1-\cos\theta)\}\,\omega_H\sin\phi\,\sin(\frac{\omega_H}{\omega}\pi)}{Km\beta} \qquad \ldots(6.20)$$

From equation (6.20) we find that introducing ν_{eff} in place of ν changes the value of E to $E\exp\{\frac{3}{4}(1-\cos\theta)\}$. In other word, the effect of hitting obliquely at the surface can well be represented by replacing E by $E\exp\{\frac{3}{4}(1-\cos\theta)\}$ in any relation deduced from the solution of equation (6.11).

Deb and Goswami (1964) indicated that the general solution of equation (6.20) in simplified from is unobtainable. For large magnetic field, however, a simplified assumption may be taken, as was done by Deb and Goswami (1964) that the second turning point appears approximately after the completion of one full cycle of the cyclotron frequency i.e. when $\omega_H/\omega = 2n\pi$ where n is an integer. The justification of this assumption may be sought in the fact that the fraction of the phase that is not considered in this approximation occupies little portion of the half cycle of transit i.e. the distance between the end wall and position of electron, when it has executed a number of complete revolutions due to cyclotron rotation, is small in both positive and negative sides of the wall.

Applying this assumption to equation (6.20) we get

$$v_{eff}^2\left(1-\frac{1}{K^2}\right) = 4\left[-\frac{v_{eff}\,\omega e E \exp\{\frac{3}{4}(1-\cos\theta)\}}{K m\beta} + \left\{\frac{eE\exp\{\frac{3}{4}(1-\cos\theta)\}\,\omega\cos\phi}{m\beta}\right\}^2 + \left\{\frac{eE\exp\{\frac{3}{4}(1-\cos\theta)\}\,\omega_H \sin\phi}{m\beta}\right\}^2\right] \qquad \ldots(6.20a)$$

and displacements at $t = \pi/\omega$ from equation (6.16) and (6.17)

$$x = -\frac{2eE\sin\phi}{m\beta} \qquad \ldots(6.20b)$$

$$y = \frac{2eE\,\omega_H\cos\phi}{m\omega\beta} \qquad \ldots(6.20c)$$

The square of the resultant displacement

$$D^2 = x^2 + y^2 = \left(2eE/m\beta\right)^2\left[\frac{\omega_H^2\cos^2\phi}{\omega^2} + \sin^2\phi\right]$$

$$\therefore D = \frac{2eE}{m\beta}\left[\frac{\omega_H^2\cos^2\phi}{\omega^2} + \sin^2\phi\right]^{1/2} \qquad \ldots(6.21)$$

To calculate the resultant displacement when the electron strikes the opposite end, the value of the y- component of displacement at $t = \pi/\omega$ has been calculated from equation (6.20c) and the results are entered in the last column of the table III for different values of the magnetic field. The resultant displacement is $(x^2 + y^2)^{\frac{1}{2}}$ at $t = \pi/\omega$. Using the y- displacement and taking $x = 5$ cm. the resultant displacement for minimum and maximum values of the magnetic field, "D", lies between 5.297 cm and 5.702 cm. and the length of the tube is 5 cm. Consequently as a first approximation the resultant displacement can be taken to be equal to the length of the tube. Making this assumption and introducing the effect of oblique hitting of the electron to the end surface we get from equation (6.21) as

$$\frac{d.m.(\omega_H^2 - \omega^2).\omega}{2.e.E\ \Phi_H} = \exp\left[\frac{3}{4}(1-\cos\theta)\right]$$

where

$$\Phi_H = \left[\omega_H^2 \cos^2\phi + \omega^2 \sin^2\phi\right]^{1/2} \qquad \text{...(6.22)}$$

as $\frac{3}{4}(1-\cos\theta) < 1$ the equation can be written

$$\frac{d.m.(\omega_H^2 - \omega^2).\omega}{2eE\ \Phi_H} \simeq 1 + \frac{3}{4}(1-\cos\theta) \qquad \text{...(6.23)}$$

We have again

$$\tan\theta = -\frac{\omega_H}{\omega}\cot\phi$$

Therefore

$$\sec\theta = \frac{\Phi_H}{\omega\sin\phi}$$

or

$$\cos\theta = \frac{\omega\sin\phi}{\Phi_H}$$

Putting this value of $\cos\theta$ in equation (6.23)

$$\frac{d.m.(\omega_H^2 - \omega^2)\omega}{2eE\ \Phi_H} = 1 + \frac{3}{4}\left(1 - \frac{\omega\sin\phi}{\Phi_H}\right)$$

and carrying the simplification a few steps

$$\sin^2\phi(49\,\omega_H^2 - 40\,\omega^2) + 24\,\Lambda_H\,\omega\sin\phi + (16\Lambda_H^2 - 49\omega_H^2) = 0$$

where $\Lambda_H = \frac{m\,d\,(\omega_H^2 - \omega^2)\omega}{2eE}$

...(6.24)

Consequently, from equation (6.24), for different values of Λ_H, ω_H and ω, the value of $\sin\phi$ is obtained. Since equation (6.24) is quadratic in $\sin\phi$ so the phase angle values are chosen depending upon the portion of the curve under consideration. In fig. (29) experimental curves for magnetic field values of 18 gauss, 21 gauss, 30 gauss and 45 gauss are given upto the highest range of frequency at which the measurements are limited by the r.f. voltage limitation of the present experimental setup. In each curve some points are chosen and the values of ϕ from equation (6.24) are obtained for each point.

The limitation of r.f. out put voltage restricted us in obtaining the almost linear portion to cut-off point only but not the other end of the curves. Hence throughout our discussion, we have confined our discussion and fittings to the linear portion of the curves where energy is almost constant as shown by Hatch and Williams (1954).

From equation (6.20a) taking the assumption $\left(\frac{\omega_H}{\omega}\right)\pi = 2n\pi$ $(n = 1, 2, 3$ etc$)$ we get

$$v^2\left(1-\frac{1}{K^2}\right) = 4\left[\frac{-v\,eE\omega\cos\phi}{Km\beta} + \left\{\frac{eE\omega\cos\phi}{m\beta}\right\}^2 + \left\{\frac{eE\omega_H\sin\phi}{m\beta}\right\}^2\right]$$

or

$$v^2 + v\left[\frac{4}{1-\frac{1}{K^2}}\cdot\frac{eE\omega\cos\phi}{Km\beta}\right] - \left(\frac{4}{1-\frac{1}{K^2}}\right)\left[\left(\frac{eE}{m\beta}\right)^2\left(\omega^2\cos^2\phi + \omega_H^2\sin^2\phi\right)\right] = 0$$

This equation is solved for v as other quantities are known. The energy of arrival $\varepsilon_0 = \frac{mv^2}{2}$ is then calculated and using the relation

$$\varepsilon_{eff}/\varepsilon_0 = \exp\left\{\frac{3}{2}\left(1-\cos\theta\right)\right\}$$

the values of ε_{eff} for the cut-off points of each value of the magnetic field have been obtained. These values are shown in table III.

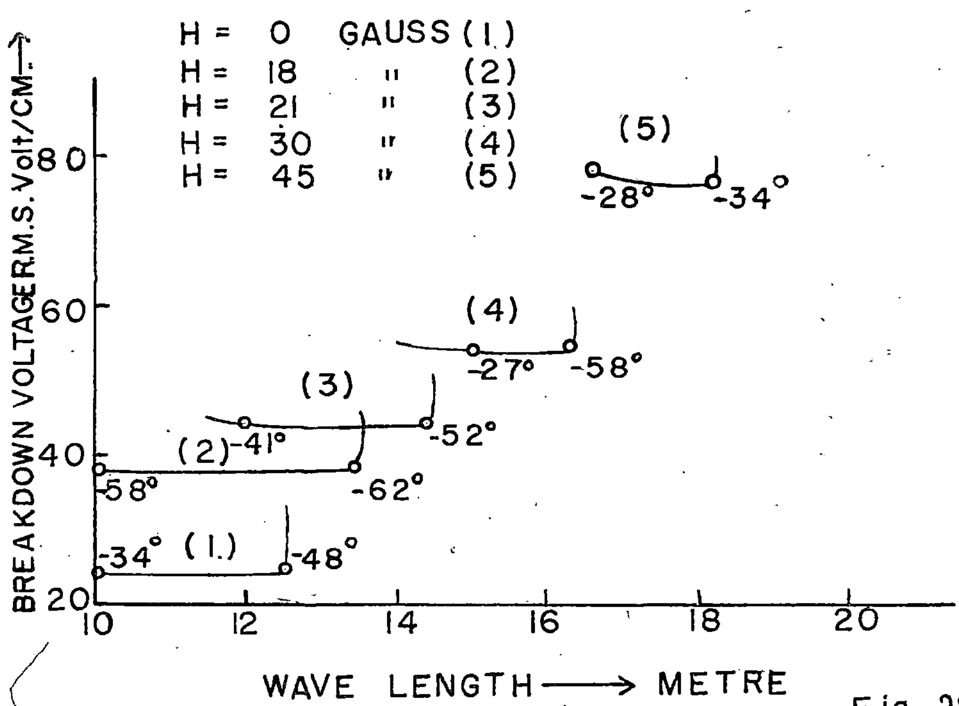
H = 0 GAUSS (1)
H = 18 " (2)
H = 21 " (3)
H = 30 " (4)
H = 45 " (5)
BREAKDOWN VOLTAGE R.M.S. Volt/CM.
80
60
40
20
10
12
14
16
18
20
(1)
-34°
-48°
(2)
-41°
-58°
-62°
(3)
-52°
(4)
-27°
-58°
(5)
-28°
-34°
WAVE LENGTH ⟶ METRE

Fig- 29

TABLE - III.

H Gauss	f_{co} Mc/s	E_{co} Volt/cm	$(-\phi_{co})$ degree	$v_{co}\times10^{-8}$ cm/sec	$\mathcal{E}$ original e.v.	$\mathcal{E}$ effective e.v.	$\mathcal{E}$ critical from table-I	Larmor radius cm.	γ displacement from equ. (6.20c)
18	22	38	62	4.3	52	88.4		1.327	1.75
21	21	44	52	3.57	35.85	86.04	88	.9445	2.228
30	18.5	54	58	3.1	27.04	73.35		.5741	2.601
45	16.5	76	34	1.84	8.52	38.08		.2272	2.747

From the table III, it is seen that the values of $\mathcal{E}_{original}$ is much smaller than the critical value of $\mathcal{E}$ for $\delta = 1$ and the value of $\mathcal{E}_{crit}$ calculated from the cut-off point of the curve of H = 0. But the value of $\mathcal{E}_{effective}$ for H upto 30 gauss is near or equal to $\mathcal{E}_{crit}$ for H = 0, though the value shows gradual decrease as H is increased. The deviation is not remarkable upto H = 30 gauss, but for H = 45 gauss, the value of $\mathcal{E}_{effective}$ is much smaller than the $\mathcal{E}_{crit}$. The reason of this deviation is due to different approximations made in calculating ϕ and v from comparison with the experimental results. It is evident from the results that these approximations do not much effect the results for small magnetic field, but for high magnetic field the different terms neglected in obtaining equation (6.21) and (6.20a) modified the values of ϕ and v to a large extent and consequently the deviation of the values of the different parameters. However, considering the limitations of our theory in explaining the mechanism of the discharge and the experimental results by the process of fitting it can be said that the agreement is fairly good at least for moderate values of the magnetic field. This treatment also shows that the mechanism of secondary electron resonance is still operative in original sense as the case of the breakdown of the gas, when the magnetic field is also present. It is also expected that if the original solutions of equation of motion of electron could be obtained, much better fitting of the experimental results and consequently more resonable values of the parameters could be obtained.

The values of the Larmor radius and those of y- displacement at $t=\pi/\omega$ have been numerically calculated for different values of the magnetic field and entered in the ninth and tenth column of table III respectively. The y- displacement when the electrons reach the opposite end for each of the values of the magnetic field is smaller than the diameter of the discharge tube (3.5 cm.), the Larmor radius in each case is much smaller than the radius of the tube for energies of electrons high enough to cause breakdown and the majority of electrons which are actually responsible for the continuense of the secondary electron resonance breakdown find ample free space during their transit between the end walls and are not lost due to collision with the side walls.

It is further observed from table III that an empirical relation between H, f_{co} and E_{co} can be obtained from the experimental data. The quantity $(H \cdot f_{co}/E_{co})$, where H is expressed in gauss, f_{co} in Mc/sec and E_{co} in volts/cm., is almost a constant as shown in table IV.

TABLE - IV.

H Gauss	f_{co} Mc/sec	E_{co} Volts/cm.	(Hf_{co}/E_{co}) Expt.	$(-\Phi_{co})$ in degree	θ_{co} in degrees	(Hf_{co}/E_{co}) Calc.
18	22	38	10.42	62	51	5.098
21	21	44	10.02	52	66	6.7
30	18.5	54	10.27	58	77	6.119
45	16.5	76	9.77	34	85	10.1

To test whether the theoretical analysis made above can explain the empirical relation observed we obtain from equation (6.22)

$$\frac{m.d.\,\omega_H^2\left(1-\frac{\omega^2}{\omega_H^2}\right).\omega}{2eE\,\omega_H\,\cos\Phi_{co}\left(1+\frac{\omega^2}{\omega_H^2}\tan^2\Phi_{co}\right)^{1/2}} = \exp\left\{\frac{3}{4}\left(1-\cos\theta_{co}\right)\right\}$$

as for the magnetic field used in this experiment $\frac{\omega^2}{\omega_H^2} << 1$

and $\frac{\omega^2}{\omega_H^2}\tan^2\Phi_{co} = \cot^2\theta_{co}$, we get

$$\frac{m.d.\,\omega_H\,\omega}{2eE\,\cos\Phi_{co}\left(1+\cot^2\theta_{co}\right)^{1/2}} = \exp\left\{\frac{3}{4}\left(1-\cos\theta_{co}\right)\right\}$$

or $$\frac{m\,d\,\omega_H\,\omega}{2eE} = \frac{\cos\Phi_{co}}{\sin\theta_{co}}\exp\left\{\frac{3}{4}\left(1-\cos\theta_{co}\right)\right\}$$

$$\therefore \frac{f_{co}\times H}{E_{co}} = 6{\cdot}37\,\frac{\cos\Phi_{co}}{\sin\theta_{co}}\exp\left\{\frac{3}{4}\left(1-\cos\theta_{co}\right)\right\} \text{ for } d=5\text{ cm.}$$

...(6.25)

where f_{co} is in Mc/sec H in gauss and E_{co} in volts/cm are expressed. In table IV the values of Φ_{co} and θ_{co} for different magnetic fields are given. Empirically we find that the r.h.s. of equation (6.25) is a constant and equal to 10, so we may say that θ_{co}, Φ_{co} and f_{co} adjust themselves in such a way that at the point of cut-off given by equation (6.25), the value of r.h.s. of equation remains constant for any magnetic field. To test the validity of this conclusion, the individual values of r.h.s. of equation (6.25) are calculated for each magnetic field and results entered in the last column of table IV. The table shows that the value of the constant compares reasonably with fair amount of agreement in the order of magnitude between the empirically determined value and the theoretical value of the last column. The discrepancy may be attributed to the different approximations taken and their validity during the theoretical deductions. Consequently, it can be concluded with reasonable agreement the cut-off relation

$$\frac{f_{co} \times H}{E_{co}} = \text{constant}$$

is true for moderate values of the magnetic field at the point of cut-off.

C O N C L U S I O N.

The phenomena of low pressure breakdown by secondary electron resonance oscillations has been explained in the light of the theory put forward by Gill and Von Engel and Hatch and Williams. The measurements without magnetic field lead to the conclusion that all the predictions of the theory of Gill and Von Engel (1948) can be extended for a wide range of the dimension of the discharge tube. The increase in the value of the constant ($f_{co} \times d$) at cut-off justifies to some extent the predicted reasoning of Hatch and Williams (1954) and supported recently by the work of Chandraker and Von Engel (1965) as due to side wall effect specially when the length of the discharge tube is large. For increase of the length of the discharge tube the constant increases which was also observed by previous workers for gaps shorter than those used in the present work. The quantity $V_{co}/\mathcal{E}_{crit}$ has been found to be a constant for all the three lengths of the discharge tube, a result which follows from the theoretical analysis.

The observations with magnetic field and the subsequent fitting of these observations to our theory yielded the values of the phase angle which are reasonable. Though our procedure has obtained a simplified form for the energy of arrival of the electrons, yet it gives results for the effective energy of arrival for moderate magnetic field with fair amount of accuracy.

The validity of the different assumptions made for the deduction of the simplified theory of breakdown with magnetic field are open to questions in the rigid theoretical sense. But in view of the fact that no conclusive theory showing the behaviour of the discharge for the continuous change of magnetic field can be deduced without making some simplifying assumptions and considering the usefulness of the theory in explaining the experimental results, it can be concluded that the assumptions can be regarded as valid in the range of the magnetic field studied in the present investigations.

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