Chapter 8

Core-Envelope Model for Compact Stars

8.1 Introduction

Macroscopic features of a supercompact star such as mass and radius are dependent crucially on the composition of the stellar material. To construct a stellar model we use an equation of state (EOS), i.e., $p = p(\rho)$, where ρ and p denote energy-density and pressure of the stellar material, respectively. To determine the mass and radius of the star one integrates the Tolman-Oppenheimer-Volkoff (TOV) equations numerically, using appropriate boundary conditions. However, a case of special interest occurs if the interior of a star has a deconfined quark core surrounded by an envelope of normal baryonic matter. In such a case it is not be possible to use a single EOS to describe the entire star and the situation becomes much more complicated. In this case, the construction of a stellar model demands two input parameters, namely central density and density at the junction and the equations of state of the two layers composed of two different kinds of material.

The possibility of a quark phase in the interior of a compact star has already been pointed out independently by Witten [25] and Farhi and Jaffe [26]. This possibility has generated speculations of an entirely new class of compact stars known as strange stars. In chapter 1 we have noted the suggestions by various investigators that some pulsars could be strange stars, e.g., 4U 1820-30 (Bombaci, 1997 [130]) Her X-1 (Dey *et al*, 1998 [42]), , SAX J 1808.4-3658 (Li *et al*, 1999 [40]), 4U 1728-34 (Li *et al*, 1999 [43]) and PSR 0943+10 (Xu *et al*, 1999 [45]). However, whether strange stars are bare or they have a normal nuclear crust has remainsd an unsolved issue till date [30], [160], [161]. Most of the strange star calculations are based on a single EOS, although there are suggestions that a strange star can have a low density crust if a sufficient gap of electron layer is assumed between the core and the nuclear crust [54, 91, 120]. A compact star may have other exotic components too. For example, Horvath and Pacheco [72] have claimed that Her X-1 is a star composed of a mixture of quark-diquark matter. In the model bosonic diquarks have been described by a self-interacting complex scalar field Φ and a potential of the form $\frac{\Lambda}{4}|\Phi|^4$, where Λ is the coupling constant. The calculation was based on the earlier work in this field by Kaup [56], Ruffini and Bonazzola [57], Colpi *et al* [58]. Kastor and Traschen [76] explored the possibility of a compact star having quark-diquark core surrounded by a low-density envelope of nucleon. Drago & Lavagno [164] presented a new model for compact stars having deconfined quark cores surrounded by a less dense mixed phase.

Following these developments, it will be intereseting to explore an alternative method to construct a model for a star having two layers; a deconfined quark core enveloped by a normal baryonic matter. This can easily be implemented using the solution of Mukherjee *et al* [67] under Vaidya-Tikekar model [63]. The solution has already been discussed in chapter 2. A parameter λ, in this model specifies the equation of state(EOS) of the star. Earlier, the Vaidya-Tikekar [63] model was used to construct a core-envelope model where an anisotropic solution for the core region was assumed [65]. Using TOV equations, Lindblom [77] analysed the features of the mass-radius curve of a relativistic stellar model constructed from an EOS with a first order phase-transition. In our work, without using TOV equations, we can construct a model exhibiting a phase transition, by choosing different values of the parameter λ.

As of now, we do not have enough experimental support to predict exactly at which point a hadronic phase transition occurs. Weber [7] gave a wide range for such a phase transition ($\rho_{DC} \approx 0.28 - 0.98$ GeV fm^{-3}). In a recent CERN pre-print, Heinz [152] analysed quark-gluon phase transition and showed that an energy density of about 0.6 Gev fm^{-3} is needed to make this phase transition possible. As we are yet to reach a concensus on this issue, we in our calculation, will consider 0.6 GeV fm^{-3} as a representative value for such a phase transition. Although we shall not discuss the explicit nature of the exotic phases for the two layers, the effect of a phase transition can easily be seen from the change in softness/stiffness of the EOS of the entire body.

8.2 Core-envelope model

We start with a static, spherically symmetric star having a line element of the form

$$ds^2 = -e^{2\gamma(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{8.1}$$

where,

$$e^{2\mu} = \frac{1 + \lambda r^2/R^2}{1 - r^2/R^2}, \tag{8.2}$$

and

$$e^{\gamma} = \psi(z) = A\left[\frac{\cos[(n+1)\zeta + \delta]}{n+1} - \frac{\cos[(n-1)\zeta + \delta]}{n-1}\right]. \tag{8.3}$$

All the parameters have been defined in chapter 2. For a given mass and radius of this class of stars, we have shown in Table 8.1, values of $\frac{dp}{d\rho}$ for different choices of the parameter λ. It is evident that at a small λ, the relevant stellar matter has almost a radiation like behaviour which becomes less stiff as λ increases. Thus the solution can be utilized to construct models for stars having concentric layers of different material components distinguished by the parameter λ.

λ	$R\ (km)$	δ	A	$\left(\frac{dp}{d\rho}\right)_{r=0}$	$\left(\frac{dp}{d\rho}\right)_{r=b}$
2	20.224	2.23367	0.9332	0.3418	0.3518
5	27.545	2.41532	1.6913	0.2714	0.2847
10	36.627	2.49738	2.9926	0.2479	0.2624
20	50.073	2.5449	5.61083	0.2361	0.2511
50	77.488	2.57512	13.4818	0.2291	0.2446
100	108.779	2.58577	26.6039	0.2268	0.2424
200	153.264	2.59118	52.8501	0.2256	0.2413

Table 8.1: Values of $\frac{dp}{d\rho}$ for different choices of the parameter λ for a star of mass $M = 0.88\ M_{\odot}$ & radius $b = 7.7\ km$.

For simplicity we consider a two-fluid model where the inner core and the outer crust of the star will be described by two values of the parameter λ. We consider a star of mass M and radius b km, whose core and envelope regions are defined as :

- Core: $0 \leq r \leq a$
- Envelope: $a \leq r \leq b$

Let λ_C and λ_E specify the equations of state of the core and envelope regions, respectively. For the solution in the core region $(0 \leq r \leq a)$, we write the constants as A_C, δ_C and R_C, while for the envelope region $(a \leq r \leq b)$, constants are A_E, δ_E and R_E.

8.2.1 Boundary conditions

We apply the following boundary conditions in our model: at $r = a$,

$$e^{\gamma_C(r=a)} = e^{\gamma_E(r=a)} \tag{8.4}$$

$$e^{\mu_C(r=a)} = e^{\mu_E(r=a)} \tag{8.5}$$

$$p_C(r = a) = p_E(r = a) \tag{8.6}$$

and at the boundary of the star, $r = b$,

$$e^{2\gamma_E(r=b)} = \left(1 - \frac{2M}{b}\right) \tag{8.7}$$

$$e^{2\mu_E(r=b)} = \left(1 - \frac{2M}{b}\right)^{-1} \tag{8.8}$$

$$p_E(r = b) = 0. \tag{8.9}$$

8.3 Numerical analysis

To get a qualitative feeling of the model, we now consider a couple of simple cases where we have assumed two stars of different masses and radii undergoing a phase transition.

8.3.1 Case I

Let us consider a star of mass $M = 1\ M_\odot$ and radius $b = 7.5\ km$ which has a deconfined core surrounded by a less dense hadronic matter. We also assume that the deconfinement occurs at $\rho_{dc} = 0.6\ GeV\ fm^{-3}$. From Table 8.1, we find that a larger value of λ gives a much softer EOS. Hence, we assume that the core region of the star is described by a value greater than the value of λ at the envelope region. To be specific, we take $\lambda_C = 100$ and $\lambda_E = 2$, arbitrarily. Note that although we are treating λ_C and λ_E as free parameters in this model, these values are fixed once we identify the exact composition of the two layers.

We now satisfy the boundary conditions (8.4)-(8.9) numerically and find that the junction will be at a radial distance $a = 6.20193\ km$. The other parameters are determined as $R_C = 96.9532\ km$, $R_E = 17.7912\ km$, $\delta_C = 2.51408$, $\delta_E = 2.17076$, $A_C = 25.229$ & $A_E = 0.900734$. The resulting EOS is shown in Fig.8.1.

We note that at $r = 6.20193\ km$ (junction), pressure is continuous, but there is a sharp discontinuity in energy-density which clearly suggests a phase transition. Also the slope of the core region EOS is ≈ 0.26 whereas its corresponding value at the envelope region is ≈ 0.38. The central density of the star of such a configuration is ≈ 0.972 $GeV\ fm^{-3}$. If the star would have been composed of only one kind of matter described by $\lambda = 2$, the central density would have been ≈ 0.858 $GeV\ fm^{-3}$. This shows that a higher value of λ at the core of the star increases the central density of the star considerably.

8.3.2 Case II

In [162], it has been shown that the EOS obtained for $\lambda = 100$, $R = 108.779$ km and $\delta = 2.58577$ in the model discussed in chapter 2, matches accurately with the EOS for a quark-diquark mixture obtained by Horvath and Pacheco [72]. In [162], it has been pointed out that energy contributions form diquarks should vanish when pressure goes to zero which was not considered in [72]. Motivated by this observation we now construct another model in which we make an ad hoc choice for the inner core, viz. $\lambda_C = 100$, $R_C = 108.779$ km and $\delta = 2.58577$. For the less dense outer layer we arbitrarily take $\lambda_E = 5$. The envelope may have a quark-hadron mixed phase, a purely hadronic phase or any other exotic phase. Note that, as we have already assumed the values of two unknown parameters, we do not make additional assumption for mass and radius.

Matching conditions, then yield, $R_E = 26.858$ km, $\delta_E = 2.41552$, $A_C = -9.6654$ and $A_E = 1.69184$. The mass and radius in this case turn out to be ≈ 0.857 $M_\odot$ and ≈ 7.5 km, respectively. The resulting EOS is shown in Fig.8.2.

Form Fig8.2, we find that at a distance of ≈ 4.42 km form the centre and energy density of ≈ 0.6 $GeV\ fm^{-3}$, a phase transition occurs. The slope of the core EOS is ≈ 0.23 whereas the slope of the envelope EOS is ≈ 0.28. The central density of the star is ≈ 0.772 $GeV\ fm^{-3}$. If the entire star would have been composed of the matter described by $\lambda = 5$, the central density would have been ≈ 0.753 $Gev\ fm^{-3}$.

Alternatively, if the star would have been composed entirely one kind of matter specified by $\lambda = 100$, the total mass and radius of the star would have been 0.88 $M_{\odot}$ and 7.7 km, respectively.

8.4 Discussions

We have studied compact stars having two layers - a highly dense core surrounded by a comparatively less dense envelope. This is a simplified model in the sense that in our model we have considered only two zones. In principle the model can also be extended to the case of a star having more than two concentric layers of different composition. Our approach may be contrasted with that of Kastor and Traschen [76] who used two equations of state to integrate the TOV equations.

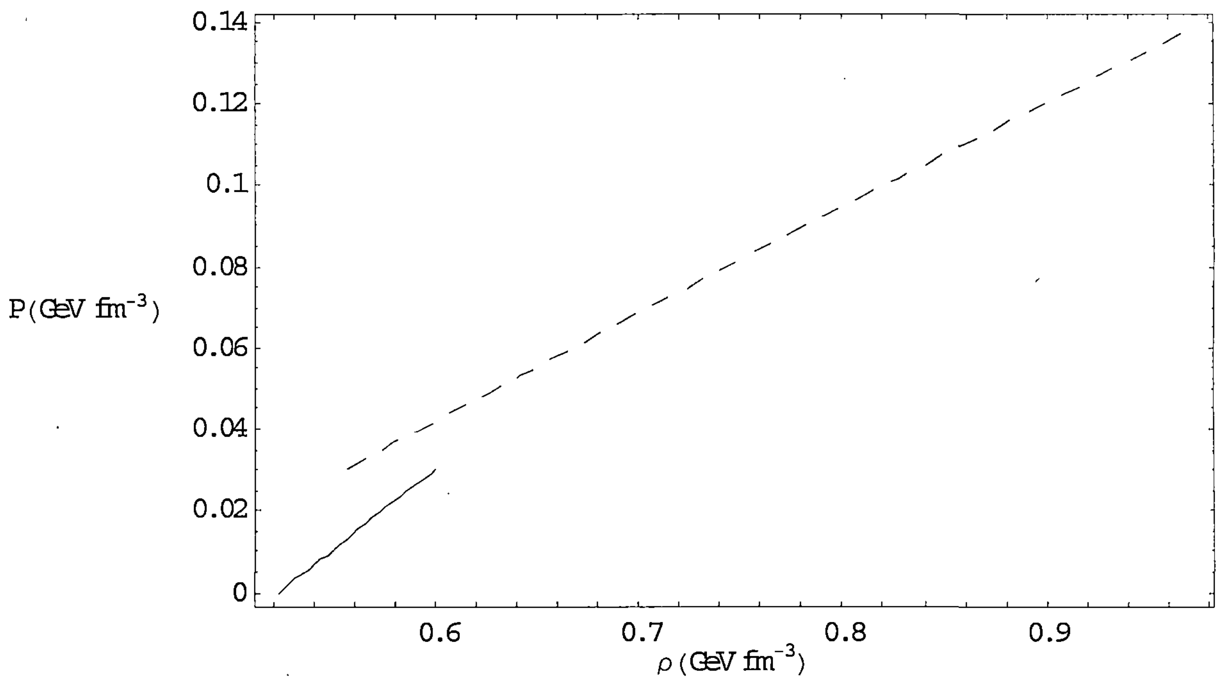

Figure 8.1: Equation of state for the core region with $\lambda = 100$ (dashed line) and EOS for the envelope with $\lambda = 2$ (solid line).

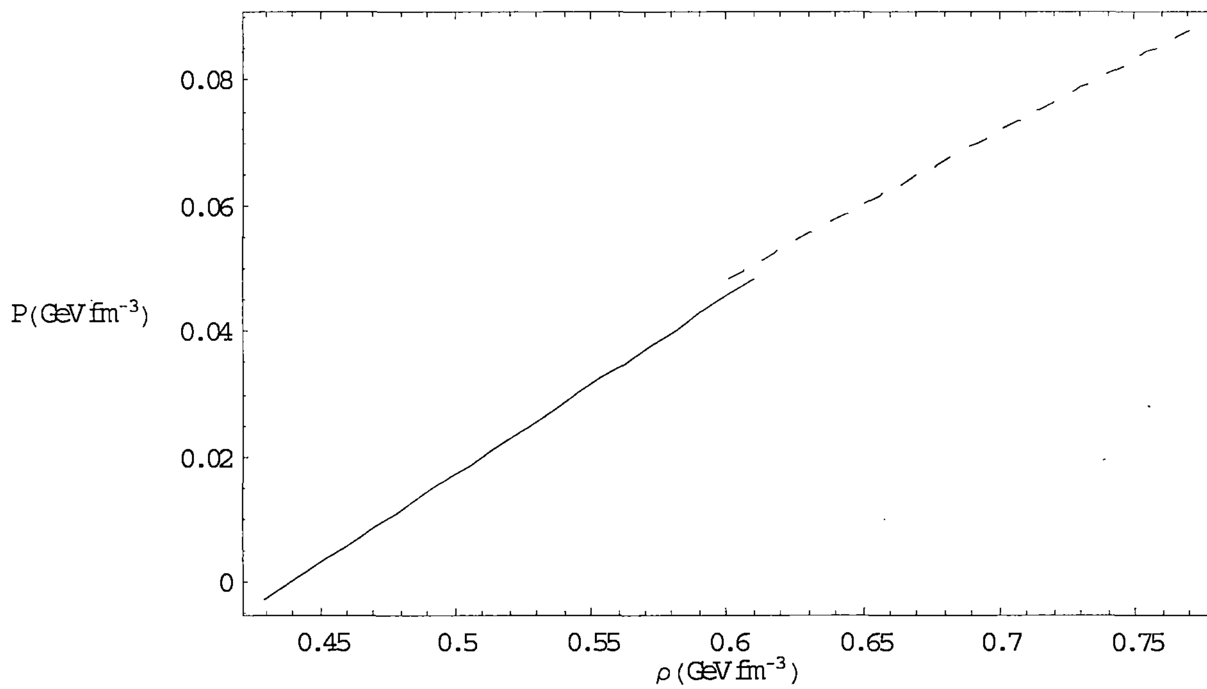

Figure 8.2: Equation of state for the core region with $\lambda = 100$ (dashed line) and EOS for the envelope region with $\lambda = 5$ (solid line).