Chapter 6

Baryonic Tully-Fisher test of Grumiller's modified gravity model

6.1 Introduction

Several astrophysical observations and specially the observation of flat rotation curve of galaxies lead to the hypothesis of dark matter. However, despite several efforts so far there is no direct evidence of dark matter particles, nor their existence is predicted by any standard theoretical model of particle physics. Consequently many alternative explanations of flat rotation curve of galaxies exist in the chapter including modification of gravitational law at large distances [240], [241] or even modification of Newton's laws of dynamics [29].

Grumiller proposed a quantum motivated theory of gravity that aims to explain the galactic flat rotation in terms of a Rindler acceleration term without the need of any dark matter [31], [242]. Assuming spherical symmetry, Grumiller considered the most general form of metric in four dimensions

$$ds^2 = g_{\alpha\beta}(x^\mu)dx^\alpha dx^\beta + \Phi^2(x^\mu)\left(d\theta^2 + sin^2\theta d\varphi^2\right), \ \alpha, \beta, \mu = 0, 1 \qquad (6.1)$$

where $g_{\alpha\beta}(x^\mu)$ is a two dimensional metric and the surface radius $\Phi^2(x^\mu)$ is a 2-dimensional dilaton field. To obtain $g_{\alpha\beta}(x^\mu)$ and $\Phi^2(x^\mu)$ Grumiller considered the most general two dimensional renormalizable gravitational theory of the form

$$S = -\int \sqrt{-g}[\Phi^2 R + 2\partial\Phi^2 - 6\Lambda\Phi^2 + 8a\Phi + 2]d^2x \,, \qquad (6.2)$$

which contains two fundamental constants, Λ and a, the cosmological constant and a Rindler acceleration, respectively. The specialty of the gravitational theory driven by the above action is that it gives a standard Newtonian kind of potential, and the theory has no curvature singularities at large $\Phi(x^\mu)$. The solution of the two dimensional fields $g_{\alpha\beta}(x^\mu)$ and $\Phi(x^\mu)$ are given by

$$g_{\alpha\beta}dx^\alpha dx^\beta = -B(r)dt^2 + \frac{dr^2}{B(r)}, \qquad (6.3)$$

$$\Phi^2(x^\mu) = r^2, \qquad (6.4)$$

where
$$B(r) = 1 - \frac{2M}{r} - \Lambda r^2 + 2ar, \qquad (6.5)$$

M is a constant of motion. When $\Lambda = a = 0$, the above solution reduces to the Schwarzschild solution and for $M = \Lambda = 0$ the solution becomes the 2-dimensional Rindler metric. The above solutions are mapped into the four dimensional world through equation (6.1).

The theory has found to explain the rotation curves of spiral galaxies well [243]. By fitting the rotation curves of eight galaxies of The HI Nearby Galaxy Survey (THINGS) [244] the Rindler acceleration term was found as $a \sim 3 \times 10^{-11} \ m \ s^{-2}$ [243]. When a larger sample (thirty galaxies) of rotation curves were considered the fitting of the data by the Rindler acceleration was found not very good [246], [247] but the goodness of fitting with the Grumiller's theory was still found comparable to that using standard Navarro-Frenk-White (NFW) profile [238], [245]. The fitted Rindler acceleration parameter, however, exhibit considerably large spread, at least one order of magnitude with mean around $3 \times 10^{-11} \ m \ s^{-2}$ [247].

The rotation velocity of galaxies is known to relate with their (galaxies) luminosity [232]. The optical Tully-Fisher relation, however, shows break; the relation is not universal for bright and faint galaxies [231]. Instead galactic rotation velocity is found to exhibit universal relation with the total baryonic mass (M) of the galaxy with the form $M \propto v_{rot}^4$ [231].

In this chapter we would like to test the Grumiller theory against baryonic Tully-Fisher relation and subsequently we shall estimate the Rindler acceleration parameter in the framework of Grumiller's model using observed total baryonic mass versus rotation velocity data for a sample of sixty galaxies.

6.2 Rotation velocity as a function of baryonic matter in Grumiller theory

For the metric given by equation (6.1) with equation (6.3) the expression of rotation velocity (v_{rot}) of galaxies is given by,

$$v_{rot}^2 = \frac{rB'(r)}{2B(r)} \tag{6.6}$$

where $B'(r)$ signifies the derivative with respect to r, r is the co-ordinate distance from galactic centre. For the solution of B(r) given by equation (6.5) the rotation velocity becomes

$$v_{rot}^2 \approx (\frac{m}{r} - \Lambda r^2 + ar)^{1/2} \tag{6.7}$$

Because of very small magnitude of Λ we henceforth ignore the corresponding term in the expression of rotation velocity. The observed rotation velocity in galaxies is in general not strictly constant even at large distances but often has some weak dependence on radial distance. The rotation velocity in Grumiller gravity is also not exactly flat (constant) at large r but slowly increases with r. So an obvious question is what value of rotation velocity will be considered for testing the Tully-Fisher relation. For Grumillers theory we consider (local) extremum value of rotation velocity. The radial distance (r_e) at which rotation velocity reaches its extremum value can be obtained by differentiating equation (6.7) with respect to r and equating it to zero which gives

$$r_e^2 \simeq \frac{m}{a} \tag{6.8}$$

Inserting it to equation (6.7), we get,

$$v'^4_{rot} = 4am, \tag{6.9}$$

where $v')$ denotes the extremum rotation velocity. The above expression shows that Grumiller's theory correctly describe the baryonic Tully-Fisher relation, at least at the theoretical level.

To match with the observed rotation curve feature a power-law generalization of the Rindler modified Newtonian potential $(-M/r + ar^n)$ is proposed in the chapter [247]. Such a power law generalization modifies the equation (6.9) as

$$v'^4_{rot} \propto m^{\frac{n}{n+1}} \tag{6.10}$$

In the above case baryonic mass is not strictly proportional to fourth power of rotation velocity but varies as $m \propto v'^{4(n+1)/2n}_{rot}$.

6.3   Estimation of Rindler acceleration parameter from observed rotation velocity vs Mass data

In this section our objective is to estimate the Rindler acceleration parameter from observed rotation velocity vs baryonic mass data for a sample of disk galaxies. We use the compiled data of Sanders and MacGaugh [235] as given in Table 6.1 that include the early works of many good astronomer.

The major luminous matter components in a typical spiral galaxy are stars and gas. Accordingly the total mass of the galaxy is considered as sum of the stellar mass and gas mass. In the used sample the mass is estimated through photometry, particularly using redder passbands as tracer. The HI thickness method was used for measuring the rotation velocity. The details of the data used and procedure of estimation of mass and rotating velocity are discussed in [231], [235].

The equation (6.9) is used to estimate the Rindler acceleration parameter a from the observed data. We fit the observed rotation velocity versus baryonic mass data by the Tully-Fisher relation (equation (6.9)) using the χ^2 goodness-of-fit test. The

fitting gives $a = (3.81 \pm 0.01) \times 10^{-11} \ ms^{-2}$ with reduced $\chi^2 = 2.0$. The fitted curve is shown in figure (6.1).
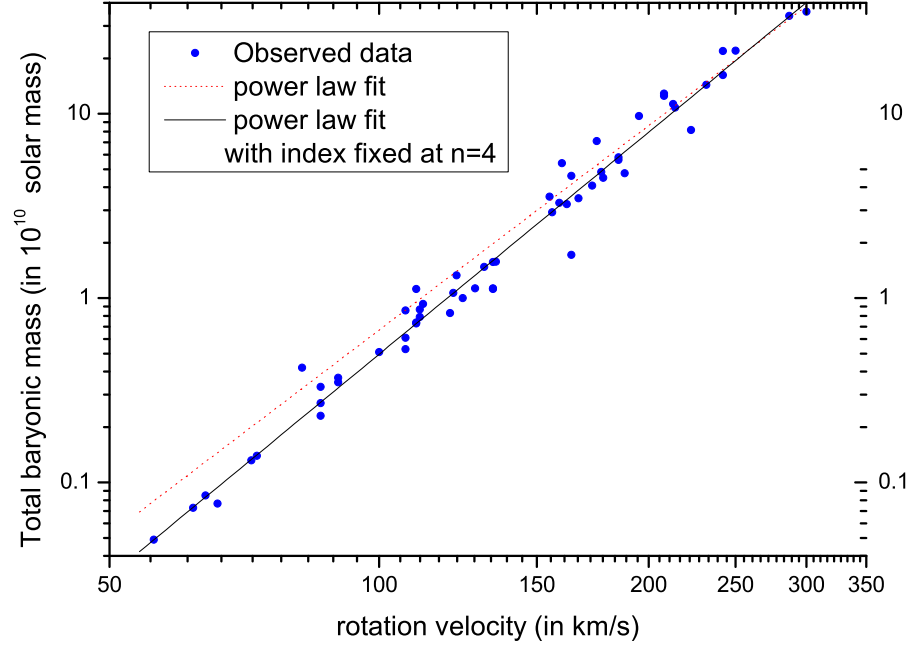

FIGURE 6.1: Variation of observed total baryonic mass with rotation velocity. The filled (blue) circle represent the observed data, solid (black) line gives the fitting of the data for standard Rindler acceleration (power index fixed at 4) and the dotted (red) line shows the fitting of the data with generalized Rindler acceleration under Grumiller's modified gravity model.

The estimated values of a for individual galaxies are given in the last column of the Table 6.1. It has a small spread, ranges from $1.99 \times 10^{-11} \ ms^{-2}$ for UGC 6446 to $7.79 \times 10^{-11} \ ms^{-2}$ for NGC 3949 with mean value $3.8 \times 10^{-11} \ ms^{-2}$ and standard deviation 0.90. The frequency distribution of estimated a for the sample of sixty galaxies is shown in figure (6.2).

We also fit the observed rotation velocity versus baryonic mass data for the modified Tully-Fisher relation (equation (6.10)) led by power-law generalization of the Rindler modified Newtonian potential using the χ^2 goodness-of-fit test which is also depicted in figure (6.2). In this case the fitted value of the parameters are found $n = 1.19$ and $a = 9.08 \times 10^{-11} \ ms^{-2}$ with reduced $\chi^2 = 1.77$.

TABLE 6.1: Galaxy data

| Galaxy	$V_{rot}(km\,s^{-1})$	$M_{stellar}(10^{10}M_\odot)$	$M_{gas}(10^{10}M_\odot)$	$a(10^{-11})(ms^{-2})$
UGC 2885	300	30.8	5	4.19
NGC 2841	287	32.3	1.7	3.70
NGC 5533	250	19	3	3.29
NGC 6674	242	18	3.9	2.90
NGC 3992	242	15.3	0.92	3.92
NGC 7331	232	13.3	1.1	3.73
NGC 3953	223	7.9	0.27	5.61
NGC 5907	214	9.7	1.1	3.60
NGC 2998	213	8.3	3	3.37
NGC 801	208	10	2.9	2.69
NGC 5371	208	11.5	1	2.77
NGC 5033	195	8.8	0.93	2.75
NGC 3893	188	4.2	0.56	4.86
NGC 4157	185	4.83	0.79	3.86
NGC 2903	185	5.5	0.31	3.73
NGC 4217	178	4.25	0.25	4.13
NGC 4013	177	4.55	0.29	3.76
NGC 3521	175	6.5	0.63	2.44
NGC 4088	173	3.3	0.79	4.06
NGC 3877	167	3.35	0.14	4.13
NGC 4100	164	4.32	0.3	2.90
NGC 3949	164	1.39	0.33	7.79
NGC 3726	162	2.62	0.62	3.94
NGC 6946	160	2.7	2.7	2.25
NGC 4051	159	3.03	0.26	3.60
NGC 3198	156	2.3	0.63	3.74
NGC 2683	155	3.5	0.05	3.01
NGC 3917	135	1.4	0.18	3.89
NGC 4085	134	1	0.13	5.28
NGC 2403	134	1.1	0.47	3.80
NGC 3972	134	1	0.12	5.33
UGC 128	131	0.57	0.91	3.68
NGC 4010	128	0.86	0.27	4.40
F568-V1	124	0.66	0.34	4.38
NGC 3769	122	0.8	0.53	3.08
NGC 6503	121	0.83	0.24	3.71
F568-3	120	0.44	0.39	4.63
NGC 4183	112	0.59	0.34	3.13
F563-V2	111	0.55	0.32	3.23
F563-1	111	0.4	0.39	3.56
NGC 1003	110	0.3	0.82	2.42
UGC 6917	110	0.54	0.2	3.66
UGC 6930	110	0.42	0.31	3.71
M 33	107	0.48	0.13	3.98
UGC 6983	107	0.57	0.29	2.82
NGC 247	107	0.4	0.13	4.58
NGC 7793	100	0.41	0.1	3.63
NGC 300	90	0.22	0.13	3.47
NGC 5585	90	0.12	0.25	3.28
NGC 55	86	0.1	0.13	4.40
UGC 6667	86	0.25	0.08	3.07
UGC 2259	86	0.22	0.05	3.75
UGC 6446	82	0.12	0.3	1.99
UGC 6818	73	0.04	0.1	3.76
NGC 1560	72	0.034	0.098	3.77
IC 2574	66	0.01	0.067	4.56
DDO 170	64	0.024	0.061	3.66
NGC 3109	62	0.005	0.068	3.75
DDO 154	56	0.004	0.045	3.72
DDO 168	54	0.005	0.032	4.26
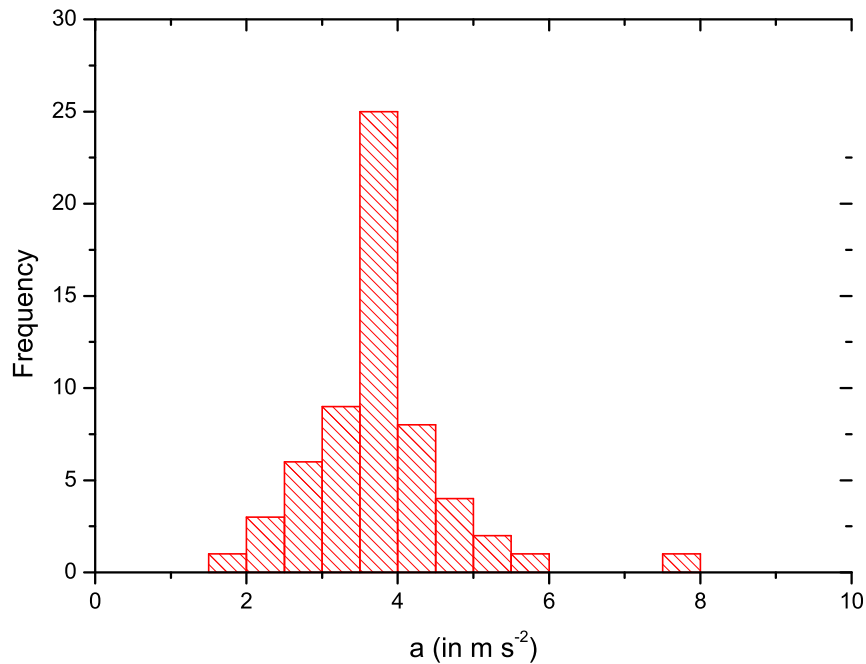
FIGURE 6.2: Frequency distribution of estimated Rindler acceleration parameter.

6.4 Discussion and conclusion

The Rindler parameter was estimated in [243] by fitting rotation curves of eight galaxies of The HI Nearby Galaxy Survey (THINGS) and the fitted mean value of the Rindler acceleration parameter was found $a \sim 3 \times 10^{-11} \ ms^{-2}$. However, when a larger sample of galaxies were considered for analysis the spread in the value of acceleration parameter becomes quite large and thereby the validity of the Grumiller model is questioned [247]. In contrast the Rindler acceleration parameter as estimated in the present chapter using the rotation velocity versus total baryonic mass data of a sample of sixty galaxies exhibits relatively small spread. The mean value is, however, nearly the same to that obtained by fitting rotation curves [247]. As stated already the rotation velocity in Grumiller's theory (equation (6.7)) is not flat but slowly diverges asymptotically which is not in accordance with the observed behaviour in typical rotation curves where rotation velocity is found to decrease slowly at large radial distances [237]. This seems the main reason of poor description of rotation velocity curves by the Grumiller's model. While describing the observed rotation velocity versus baryoinc mass data

we have considered extrema values of rotation velocity thereby taking out the radial dependence of rotation velocity.

It was found in [247] that the goodness of fits of rotation curves are better in the generalized Rindler acceleration model than that of the standard Rindler acceleration model. However, the power law index n was found to vary substantially (from 0.2 to 3.3) to describe the observer rotation curves [247], which is against the universality of the baryonic Tully-Fisher relation as may be noted from equation (6.10). The power law generalization is thus not suitable for Tully-Fisher feature unless power law index is kept fixed and universal for all galaxies. Since the form of the Grumiller's solution (equation (6.3) and (6.5)) is the same to the vacuum (static spherically symmetric) solution of Weyl gravity [30], [117] the present findings are also applicable to Weyl gravity.

The criterion of the stability of orbits in Grumiller's modified gravity/conformal gravity leads to testable upper limit on the size of the galaxies [143]. Future observations on last stable orbit in galaxies is expected to provide an important test of the Grumiller's model/conformal gravity prediction.

In conclusion we demonstrate that Grumiller's modified gravity model correctly reproduces the baryonic Tully-Fisher relation at theoretical level. We fit the observed total baryonic mass versus rotation velocity data for a sample of sixty galaxies by Grumiller's model and estimate the value of Rindler acceleration parameter. The mean value of so obtained Rindler parameter is found consistent with that estimated from fitting of rotation velocity curves of disk galaxies.