C H A P T E R - I.

A NEW APPROACH TO LARGE DEFLECTION ANALYSIS OF HEATED SANDWICH PLATES.

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A NEW APPROACH TO LARGE DEFLECTION ANALYSIS OF HEATED SANDWICH PLATES *

A B S T R A C T

This Chapter represents non-linear analysis of sandwich plates under thermal load. A new approach is followed. Analysis of rectangular sandwich plates has been carried out in detail. Numerical results have been computed, plotted graphically and compared with other known results. To test the accuracy of the method, corresponding classical Equations of sandwich plates given in the appendix have also been solved and these results have been shown side by side for comparison.

GOVERNING EQUATIONS

First we posit a rectangular co-ordinate system x, y, z as shown in Figure 1. For the sake of simplicity we consider a sandwich plate with an isotropic core as well as isotropic upper and lower faces of identical thickness; while the faces respond to the bending and membrane actions of the plate, the core is assumed to transfer only shear deformations. Moreover, the equal thickness (t) of upper and lower faces is sufficiently thin in comparison with the core thickness ($h \gg t$) to ignore a variation of stress in the thickness direction of the faces.

Under thermal loading the total strain energy of the entire sandwich plate is represented as [156]

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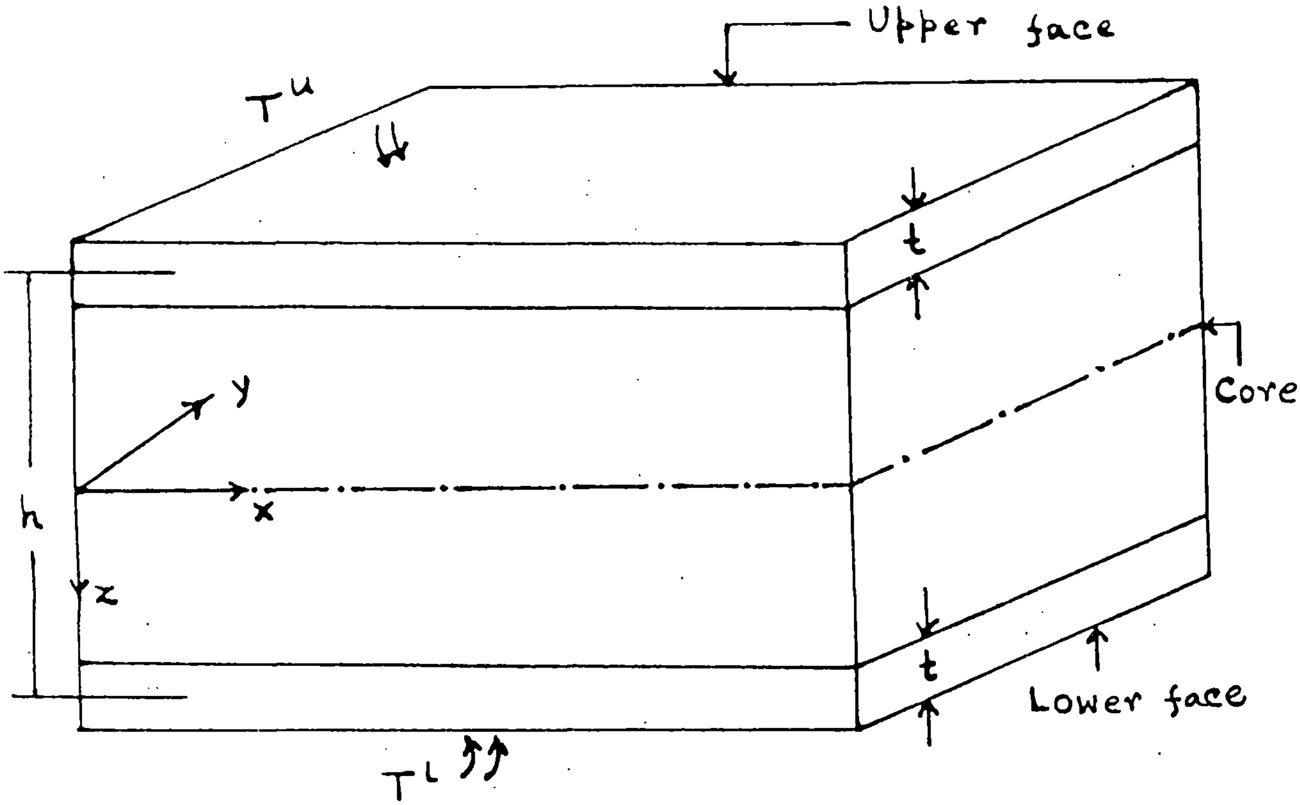

Fig. 1. Sandwich Plate

$$V_1 = \frac{E^f t}{1-\nu^{f2}} \Big[\varepsilon_x^{m2} + \varepsilon_y^{m2} + 2\nu \varepsilon_x^m \varepsilon_y^m + \frac{1}{4}\Big\{\Big(\frac{\partial r}{\partial x}\Big)^2 + \Big(\frac{\partial s}{\partial y}\Big)^2 + 2\nu^f \frac{\partial r}{\partial x}\cdot\frac{\partial s}{\partial y}\Big\} + \frac{(1-\nu^f)}{2}\Big\{\gamma_{xy}^{m2} + \frac{1}{4}\Big(\frac{\partial r}{\partial y} + \frac{\partial s}{\partial x}\Big)^2\Big\}\Big] + \frac{hG^c}{2}\Big[\Big(\frac{r}{h} - \frac{\partial w}{\partial x}\Big)^2 + \Big(\frac{s}{h} - \frac{\partial w}{\partial y}\Big)^2\Big] - \frac{2E^f\alpha^f t}{(1-\nu^f)}\Big[T^m\big(\varepsilon_x^m + \varepsilon_y^m\big) + \frac{f}{4}\Big(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\Big)\Big], \quad ..\ (1)$$

where $T^m = 1/2\ (T^u + T^l)$ and $f = T^u - T^l$.

The first two invariants of the average face strains are conventionally defined as [156]

$$I_1^m = \varepsilon_x^m + \varepsilon_y^m, \quad I_2^m = \varepsilon_x^m \varepsilon_y^m - 1/4\ \gamma_{xy}^{m2} \quad ..\ (2)$$

By virtue of equation (2), equation (1) becomes

$$V_1 = \frac{E^f t}{1-\nu^{f2}}\Big[I_1^{m2} - 2(1-\nu^f) I_2^m + \frac{1}{4}\Big\{\Big(\frac{\partial r}{\partial x}\Big)^2 + \Big(\frac{\partial s}{\partial y}\Big)^2 + 2\nu^f \frac{\partial r}{\partial x}\cdot\frac{\partial s}{\partial y}\Big\} + \frac{(1-\nu^f)}{8}\Big(\frac{\partial r}{\partial y} + \frac{\partial s}{\partial x}\Big)^2\Big] + \frac{hG^c}{2}\Big[\Big(\frac{r}{h} - \frac{\partial w}{\partial x}\Big)^2 + \Big(\frac{s}{h} - \frac{\partial w}{\partial y}\Big)^2\Big] - \frac{2E^f\alpha^f t}{(1-\nu^f)}\Big[T^m I_1^m + \frac{f}{4}\Big(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\Big)\Big] \quad ..\ (3)$$

Using the modified strain energy expression proposed by Banerjee [224], equation (3) can be rewritten as

$$V_1 = \frac{E^f t}{1-\nu^{f2}} \left[I_1^{m\,2} + \frac{\lambda}{4}\left\{ \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right\}^2 + \right.$$

$$+ \frac{1}{4}\left(\frac{\partial r}{\partial x}\right)^2 + \frac{1}{4}\left(\frac{\partial s}{\partial y}\right)^2 + \frac{\nu^f}{2}\cdot\frac{\partial r}{\partial x}\cdot\frac{\partial s}{\partial y} + \frac{(1-\nu^f)}{8}\left(\frac{\partial r}{\partial y} + \frac{\partial s}{\partial x}\right)^2 \Big] +$$

$$+ \frac{h G^c}{2}\left[\left(\frac{r}{h} - \frac{\partial w}{\partial x}\right)^2 + \left(\frac{s}{h} - \frac{\partial w}{\partial y}\right)^2 \right] -$$

$$- \frac{2 E^f \alpha^f t}{1-\nu^f}\left[\frac{f}{4}\left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\right) + I_1^m T^m + \right.$$

$$\left. + \frac{\sqrt{\lambda}}{2}\cdot T^m \frac{(1-\nu^f)}{\sqrt{1-\nu^{f2}}}\left\{ \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right\} \right], \qquad .. (4)$$

where $$I_1^m = \frac{\partial}{\partial x}\left(u^u + u^l\right) + \nu^f \frac{\partial}{\partial y}\left(v^u + v^l\right) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\nu^f}{2}\left(\frac{\partial w}{\partial y}\right)^2 \qquad .. (5)$$

and λ is a constant depending on the Poisson's ratio of the plate, materials [224].

Executing the variational calculus so as to minimize the total potential energy of the present elastic system of the sandwich plate, we arrive at the following differential equations :

$$\frac{\partial I_1^m}{\partial x} - (1+\nu^f)\alpha^f \frac{\partial T^m}{\partial x} = 0 \qquad .. (6.1)$$

$$\frac{\partial I_1^m}{\partial y} - (1+\nu^f)\alpha^f \frac{\partial T^m}{\partial y} = 0 \qquad .. (6.2)$$

$$\frac{E^f t}{2(1-\nu^{f2})}\left[\frac{\partial^2 r}{\partial x^2} + \nu^f \frac{\partial^2 s}{\partial x \partial y} - (1+\nu^f)\alpha^f \frac{\partial f}{\partial x} \right] + $$

$$+ \frac{E^f t}{4(1+\nu^f)}\left(\frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 s}{\partial x \partial y}\right) - G^c\left(\frac{r}{h} - \frac{\partial w}{\partial x}\right) = 0 \qquad .. (7)$$

$$\frac{E^f t}{2(1-\nu^{f2})}\left[\frac{\partial^2 s}{\partial y^2} + \nu^f \frac{\partial^2 r}{\partial x \partial y} - (1+\nu^f)\alpha^f \frac{\partial f}{\partial y} \right] + $$

$$+ \frac{E^f t}{4(1+\nu^f)}\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 r}{\partial x \partial y}\right) - G^c\left(\frac{s}{h} - \frac{\partial w}{\partial y}\right) = 0 \qquad .. (8)$$

$$\frac{2E^f t}{(1-\nu^{f2})} I_1^m \left(\frac{\partial^2 w}{\partial x^2} + \nu^f \frac{\partial^2 w}{\partial y^2} \right) + hG^c \nabla^2 w - G^c \left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y} \right) +$$

$$+ \frac{E^f t \lambda}{(1-\nu^{f2})} \left[\left\{ \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right\} \nabla^2 w + 2 \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + \right.$$

$$\left. + 2 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2} + 4 \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \cdot \frac{\partial^2 w}{\partial x \partial y} \right] -$$

$$- \frac{2E^f \alpha^f t T^m}{(1-\nu^f)} \left[\left(\frac{\partial^2 w}{\partial x^2} + \nu^f \frac{\partial^2 w}{\partial y^2} \right) + (1-\nu^f) \sqrt{\frac{\lambda}{(1-\nu^{f2})}} \nabla^2 w \right] = 0 , \qquad .. (9)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

According to equations (6.1) and (6.2) we can write

$$I_1^m - (1 + \nu^f) \alpha^f T^m = \text{constant} = A \qquad .. (10)$$

for an immovable edge.

Differentiating equations (7) and (8) with respect to x and y respectively and after simplification, we obtain

$$\frac{E^f f}{2(1-\nu^{f2})} \left[\nabla^2 \left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y} \right) - (1+\nu^f) \alpha^f \nabla^2 f \right] -$$

$$- \frac{G^c}{h} \left(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y} - h \nabla^2 w \right) = 0 \qquad .. (11)$$

We may eliminate terms relating r and s from equations (9) and (11) and consequently we obtain the following differential equation expressed only in terms of the deflection w with the help of equation (10) :

$$\frac{2E^f t A}{(1-\nu^{f^2})} \nabla^2 \left(\frac{\partial^2 w}{\partial x^2} + \nu^f \frac{\partial^2 w}{\partial y^2} \right) + h G^c \nabla^4 w +$$

$$+ \frac{E^f t \lambda}{(1-\nu^{f^2})} \nabla^2 \left[\left\{ \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right\} \nabla^2 w + 2 \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + \right.$$

$$\left. + 2 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2} + 4 \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \cdot \frac{\partial^2 w}{\partial x \partial y} \right] -$$

$$- \frac{2E^f t \alpha^f T^m \sqrt{\lambda}}{\sqrt{(1-\nu^{f^2})}} \nabla^4 w - (1+\nu^f) \alpha^f G^c \nabla^2 f -$$

$$- \frac{4G^c A}{h} \left(\frac{\partial^2 w}{\partial x^2} + \nu^f \frac{\partial^2 w}{\partial y^2} \right) - \frac{2\lambda G^c}{h} \left[\left\{ \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right\} \nabla^2 w \right.$$

$$\left. + 2 \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + 2 \left(\frac{\partial w}{\partial y} \right)^2 \frac{\partial^2 w}{\partial y^2} + 4 \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \cdot \frac{\partial^2 w}{\partial x \partial y} \right] +$$

$$+ \frac{4G^c \alpha^f T^m \sqrt{\lambda(1-\nu^{f^2})}}{h} \nabla^2 w = 0. \qquad \text{.. (12)}$$

It is to be noted that for <u>movable edge</u> conditions

$$I_1^m - \frac{(1+\nu^f)\ \alpha^f T^m}{2} = 0 \qquad \text{..(12.1)}$$

<u>EXAMPLE OF ANALYSIS</u>

As an illustration, we consider the large deflection of a simply supported rectangular sandwich plate (a x b), due only to the temperature difference between the upper and lower faces with constrained in-plane displacements at the boundaries. The boundary conditions are formulated as follows [156]:

At $x = 0$ and a, $w = 0$, $M_x = 0$, $u^u + u^l = 0$

$$v^u + v^l = 0, \quad s = v^u - v^l = 0 \qquad \text{..(13.1)}$$

At $y = 0$ and b, $w = 0$, $M_y = 0$, $u^u + u^l = 0$

$$v^u + v^l = 0, \quad r = u^u - u^l = 0, \qquad ..(13.2)$$

where M_x and M_y denote bending moments.

In the present example, the following temperature distributions at each face are assumed [156]:

$$T^m = \frac{T^u + T^l}{2} = \bar{T}^m = \text{constant.}$$

$$f = T^u - T^l = f_o \operatorname{Sin} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}, \qquad ..(14)$$

where $\bar{T}^m$ and f_o are constants.

For simply supported edge conditions, we assume w in the following form :

$$w = w_o \operatorname{Sin}\left(\frac{\pi x}{a}\right) \operatorname{Sin}\left(\frac{\pi y}{b}\right) \qquad ..(15)$$

If we now integrate equation (10) over the entire plane area of the plate, we get

$$\int_0^a \int_0^b I_1^m \, dx \, dy - (1 + \nu^f) \alpha^f \int_0^a \int_0^b T^m \, dx \, dy = abA \qquad ..(16)$$

After evaluating the integral we get

$$A = \frac{1}{8} w_o^2 \pi^2 \left(\frac{1}{a^2} + \frac{\nu^f}{b^2} \right) - (1 + \nu^f) \alpha^f T^m \qquad ..(17)$$

This determines A.

We are interested in the normal displacement w only. Therefore, the in-plane displacements of the upper and lower faces, namely u^u, u^l, v^u, v^l, have been eliminated through integration by assuming suitable expressions for these displacements compatible with their boundary conditions.

Let us now pay attention to the final equation (12). Putting equation (15) into this equation, remembering equation (17) and applying Galerkin's principle we arrive at the following cubic equation determining the deflection $w_o(x, y)$ for immovable edges :

$$\Bigg[\frac{\pi^2}{4}\left(1+\nu^f\frac{a^2}{b^2}\right)+\frac{4\lambda\pi^4E^f th}{G^c a^2(1-\nu^{f2})\Theta}\Bigg\{\frac{3}{64}\left(1+\frac{a^4}{b^4}-\frac{a^2}{b^2}\right)+$$

$$+\frac{3}{32}\left(1+\frac{a^4}{b^4}\right)-\frac{1}{16}\left(\frac{a^2}{b^2}\right)\Bigg\}+\frac{\lambda\pi^2}{8\Theta}\Bigg\{3\left(1+\frac{a^2}{b^2}\right)+$$

$$+6\frac{\left(1+\frac{a^4}{b^4}\right)}{\left(1+\frac{a^2}{b^2}\right)}-4\frac{\left(\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)}\Bigg\}\Bigg]\left(\frac{w_o}{h}\right)^3+\Bigg[\frac{\pi^2}{\Theta}\left(1+\frac{a^2}{b^2}\right)$$

$$-2\left(1+\nu^f\right)\alpha^f T^m\left(\frac{a^2}{b^2}\right)-\frac{4}{\Theta}\sqrt{\frac{\lambda(1-\nu^f)}{(1+\nu^f)}}\cdot\left(1+\nu^f\right)\alpha^f T^m\left(\frac{a^2}{h^2}\right)-$$

$$-\frac{2E^f th\pi^2}{(1-\nu^{f2})G^c a^2\Theta}\left(1+\frac{a^2}{b^2}\right)\sqrt{\frac{\lambda(1-\nu^f)}{(1+\nu^f)}}\cdot\left(1+\nu^f\right)\alpha^f T^m\left(\frac{a^2}{h^2}\right)\Bigg]\left(\frac{w_o}{h}\right)+$$

$$+\frac{(1+\nu^f)\alpha^f f_o\left(\frac{a^2}{h^2}\right)}{\Theta}=0, \qquad ..(18)$$

where

$$\Theta=2\left[\frac{1+\nu^f\left(\frac{a^2}{b^2}\right)}{1+\left(\frac{a^2}{b^2}\right)}\right]+\frac{E^f th\pi^2}{(1-\nu^{f2})G^c a^2}\left[\frac{1+(1+\nu^f)\frac{a^2}{b^2}+\nu^f\frac{a^4}{b^4}}{\left(1+\frac{a^2}{b^2}\right)}\right]$$

Utilizing the same procedure for movable edges, the following equation is obtained with the help of equation (12 a) :

$$\Bigg[\frac{4\lambda\pi^2E^f th}{G^c a^2(1-\nu^{f2})\Theta}\Bigg\{\frac{3}{64}\left(1+\frac{a^4}{b^4}-\frac{a^2}{b^2}\right)+\frac{3}{32}\left(1+\frac{a^4}{b^4}\right)-$$

$$-\frac{1}{16}\left(\frac{a^2}{b^2}\right)\Bigg\}+\frac{\lambda\pi^2}{8\Theta}\Bigg\{3\left(1+\frac{a^2}{b^2}\right)+6\frac{\left(1+\frac{a^4}{b^4}\right)}{\left(1+\frac{a^2}{b^2}\right)}$$

$$- 4\frac{\left(\frac{a^2}{b^2}\right)}{\left(1+\frac{a^2}{b^2}\right)}\Bigg\}\Bigg]\left(\frac{W_0}{h}\right)^3 + \Bigg[\frac{\pi^2}{\Theta}\left(1+\frac{a^2}{b^2}\right) - (1+\nu^f)\alpha^f T^m\left(\frac{a^2}{h^2}\right)$$

$$- \left(\frac{4}{\Theta}\right)\sqrt{\frac{\lambda(1-\nu^f)}{(1+\nu^f)}}\cdot(1+\nu^f)\alpha^f T^m\left(\frac{a^2}{h^2}\right) - \frac{2E^f t h \pi^2}{(1-\nu^{f2})G^c a^2\Theta}\left(1+\frac{a^2}{b^2}\right)\cdot$$

$$\cdot\sqrt{\frac{\lambda(1-\nu^f)}{(1+\nu^f)}}\cdot(1+\nu^f)\alpha^f T^m\left(\frac{a^2}{h^2}\right)\Bigg]\left(\frac{W_0}{h}\right) + \frac{(1+\nu^f)\alpha^f f_0\left(\frac{a^2}{h^2}\right)}{\Theta} = 0$$

..(19)

NUMERICAL CALCULATIONS

Tables 1 and 2 show numerical results of the maximum deflections of a rectangular sandwich plate with immovable as well as movable edges obtained respectively from equation (18) and (19) where geometrics of plates and material constants are identical to those utilized in the investigation of Nowinski and Ohnabe [128], namely

$a = 0.254$ m, $t = 0.635 \times 10^{-3}$ m

$h = 1.7135 \times 10^{-2}$ m,

$E^f = 7347.201 \times 10^6$ Kg / m^2

$G^c = 4218.4884 \times 10^3$ Kg / m^2

$\nu = 0.3$ and $\lambda = 0.09$ [224]

T A B L E - 1.

Simply supported (immovable edge) : $(1+\nu^{f})\,\alpha^{f}T^{m}\left(\frac{a^{2}}{h^{2}}\right) = 1$

$\frac{b}{a}$	$\frac{f_o}{T^m}$	$\frac{w_o}{h}$, Known Value [156] (equation 31)	$\frac{w_o}{h}$, Known Value [156] (equation 32)	$\frac{w_o}{h}$, Calculated Value	$\frac{w_o}{h}$, Obtained from classical equations given in the Appendix.
1	-5	0.447	0.473	0.493	0.515
	-8	0.538	0.571	0.620	0.684
	-10	0.585	0.622	0.694	0.77
2	-5	0.641	0.635	0.700	
	-8	0.750	0.745	0.813	
	-10	0.810	0.805	0.871	
3	-5	0.688	0.668	0.745	
	-8	0.818	0.779	0.850	
	-10	0.875	0.835	0.908	

TABLE - 2.

Simply supported (movable edge).

$$(1+\nu^{f})\,\alpha^{f}T^{m}\left(\frac{a^{2}}{h^{2}}\right) = 1$$

$\frac{b}{a}$	$\frac{f_o}{T^m}$	$\frac{w_o}{h}$, Calculated Value	$\frac{w_o}{h}$, Obtained from Classical equations
1	-5	0.442	0.402
	-8	0.669	0.610
	-10	0.795	0.730
2	-5	0.825	
	-8	1.098	
	-10	1.241	
3	-5	0.925	
	-8	1.190	
	-10	1.325	

OBSERVATION AND CONCLUSION :

The classical equations given in the Appendix are based on displacement formulations. The solution has been obtained by solving the differential equation for the in-plane displacements u and v completely. Therefore, the numerical results obtained from classical equations are believed to be the most accurate. Thus the accuracy of the other results has been tested by comparing with those obtained from the classical equations.

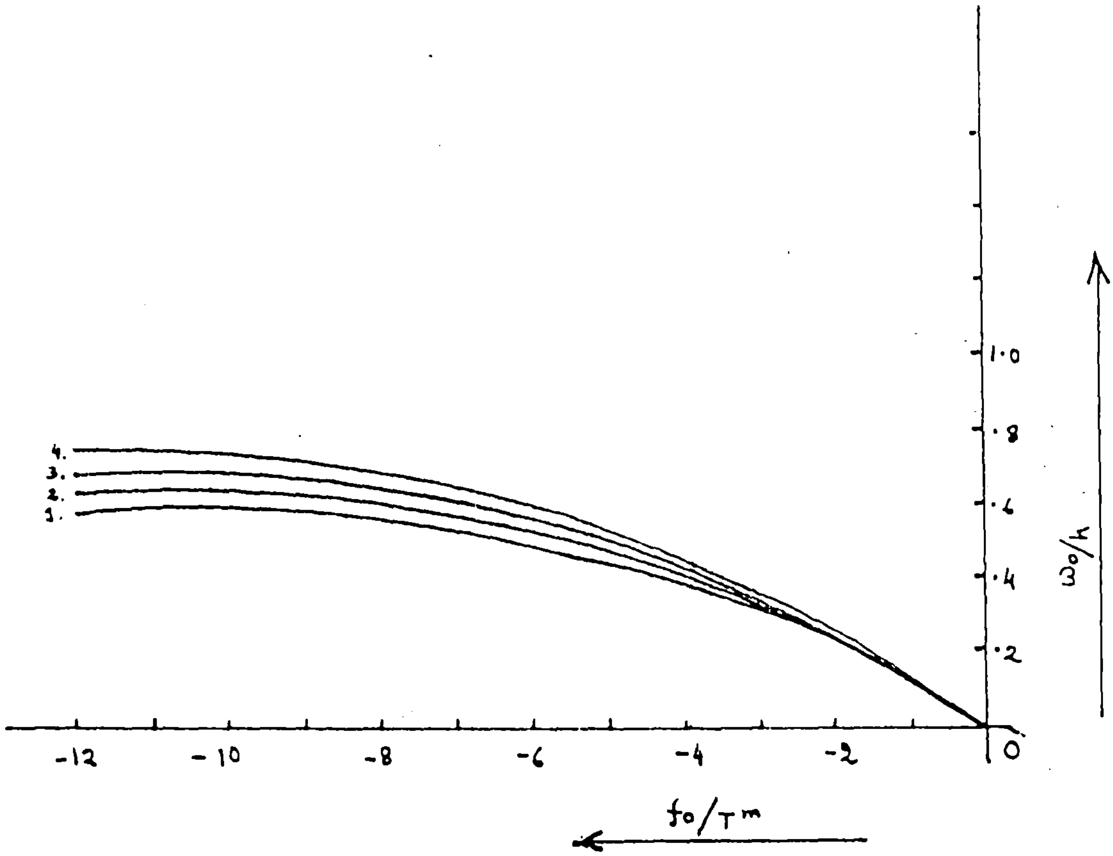

Fig. 2. Immovable edge (f_o/T^m vs w_o/h), $b/a = 1$;

1, from Berger's equation (31) [156];

2, from Kamiya's equation (32) [156];

3, from Calculated value;

4, from Classical equation.

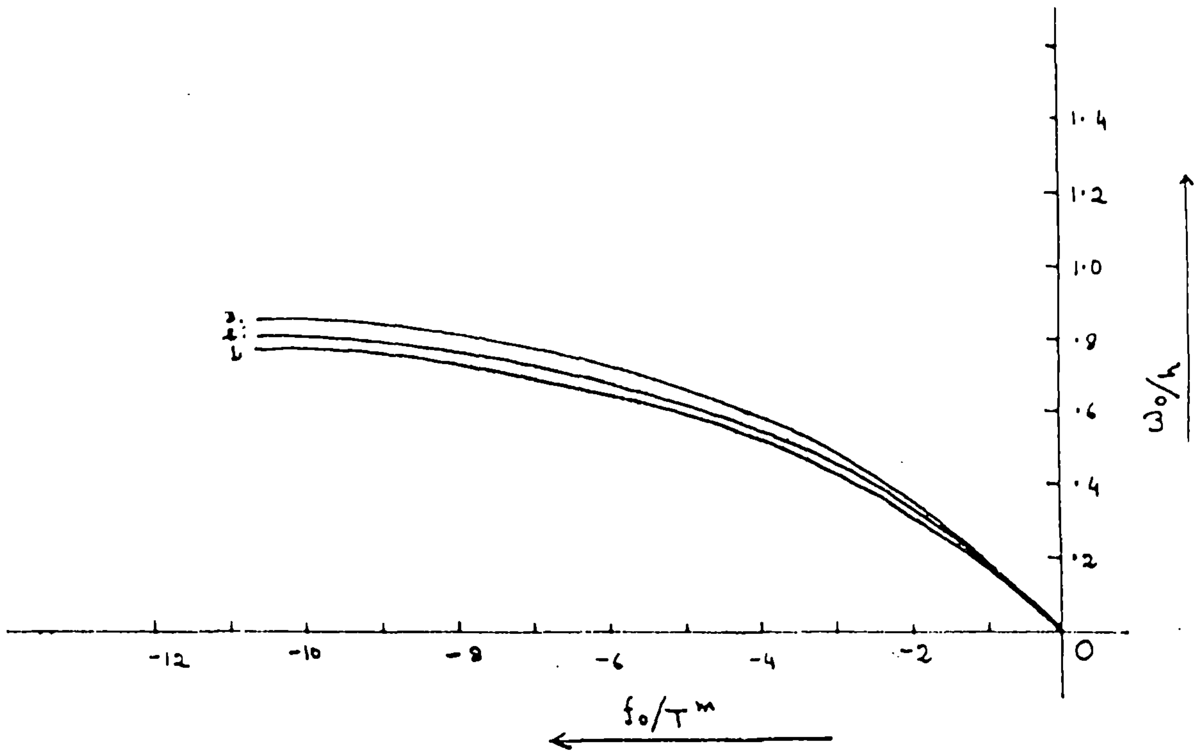

Fig. 3. Immovable edge (f_0/T^m vs w_0/h), $b/a = 2$;
1, from Kamiya's equation (32) [156];
2, from Berger's equation (31) [156];
3, from Calculated value.

It appears that deviations of Kamiya's results (equations (31), (32)) from those of classical equations given in the Appendix are due to the fact that Kamiya has utilized Berger's approximation (31) while equation (32) of Kamiya's paper has also incorporated approximation, due to the application of Ritz - Galerkin principle. It is to be noted that Berger's approximation fails completely for movable edge conditions.

The proposed differential equations are uncoupled and thus simple, computational labour is minimised for the simplified form of the differential equation. Results for movable as well as for immovable edges seem to be sufficiently accurate from the practical point of view.

From Figures 2 and 3, it is clear that as the ratio b/a increases, the results of Berger come close to those of the present study.

The proposed differential equations can be utilized to study the non-linear behaviours of sandwich plates of any shape, namely circular, elliptic, triangular etc. with the proper choice of the normal displacement w. In this study rectangular shape has been chosen for the sake of comparison with other known results.

Comparative study of the results obtained from the present analysis and from the classical equations shows that non-linear behaviours of sandwich plates with movable as well as immovable edges under thermal loading can be predicted with ease and accuracy by using the proposed differential equations.

APPENDIX :

Alwan [76] proposed differential equations for large deflections of sandwich plates with orthotropic core in terms of Airy stress function. Now the displacement formulations of the isotropic sandwich plates under thermal loading take the following forms :

$$\frac{\partial^2 u}{\partial x^2} + \left(\frac{1-\nu^f}{2}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{1+\nu^f}{2}\right)\frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial w}{\partial x}\cdot\frac{\partial^2 w}{\partial x^2} - \left(\frac{1-\nu^f}{2}\right)\frac{\partial w}{\partial x}\cdot\frac{\partial^2 w}{\partial y^2} - \left(\frac{1+\nu^f}{2}\right)\cdot\frac{\partial w}{\partial y}\cdot\frac{\partial^2 w}{\partial x \partial y} \quad ..(20.1)$$

$$\frac{\partial^2 v}{\partial y^2} + \left(\frac{1-\nu^f}{2}\right)\frac{\partial^2 v}{\partial x^2} + \left(\frac{1+\nu^f}{2}\right)\frac{\partial^2 u}{\partial x \partial y} = -\frac{\partial w}{\partial y}\cdot\frac{\partial^2 w}{\partial y^2} - \left(\frac{1-\nu^f}{2}\right)\frac{\partial w}{\partial y}\cdot\frac{\partial^2 w}{\partial x^2} - \left(\frac{1+\nu^f}{2}\right)\frac{\partial w}{\partial x}\cdot\frac{\partial^2 w}{\partial x \partial y} \quad ..(20.2)$$

$$-\frac{E^f \alpha^f}{D(1-\nu^f)}\nabla^2 M_T + \left(1 - D_1\frac{\partial^2}{\partial x^2} - D_1\frac{\partial^2}{\partial y^2}\right)\Delta\Delta w = \frac{1}{D}\left[1 - \left(D_1 + \frac{2D_1}{1-\nu^f}\right)\frac{\partial^2}{\partial x^2} - \left(D_1 + \frac{2D_1}{1-\nu^f}\right)\frac{\partial^2}{\partial y^2} + \frac{2D_1D_1}{(1-\nu^f)}\Delta\Delta\right]\cdot\left(\frac{\partial^2 F}{\partial y^2}\cdot\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\cdot\frac{\partial^2 w}{\partial x \partial y}\right) \quad ..(20.3)$$

Where

$$\frac{\partial^2 F}{\partial y^2} = \frac{2E^f t}{1-\nu^{f2}}\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \nu^f\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right\} - \frac{\alpha^f T^m (1+\nu^f)}{4}\right] \quad ..(20.4)$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{2E^f t}{1-\nu^{f2}}\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \nu^f\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right\} - \frac{\alpha^f T^m (1+\nu^f)}{4}\right] \quad ..(20.5)$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{E^f t}{1+\nu^f}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right] \quad ..(20.6)$$

F being the Airy's stress function and

$$M_T = \frac{f(h + 2t)^2}{12}, \text{ where } f = T^u - T^l$$

Here u, v, w are displacement components along with x, y, z directions respectively and

$$D_1 = \frac{(1 - \nu^f) D}{2 h G^c}, \qquad D = \frac{E^f t (h + t)^2}{2(1 - \nu f^2)}$$

Let us now analyse the case of a simply supported square plate of side "a" under thermal loading. For this purpose we solve first the equations for u and v.

Let $w = w_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{a}$

be the deflection function which clearly satisfies the simply supported edge conditions. Putting the value of w in equation (20.1) and (20.2) the solutions of u and v can be obtained in the following form :

$$U = Ax + C_1 \sin \frac{2\pi x}{a} + C_2 \sin \frac{2\pi x}{a} \cos \frac{2\pi y}{a} \qquad ..(21.1)$$

$$V = By + C_3 \cos \frac{2\pi x}{a} \sin \frac{2\pi y}{a} + C_4 \sin \frac{2\pi y}{a} \qquad ..(21.2)$$

where A and B are to be determined from the prescribed boundary conditions. Now, for <u>immovable</u> edge conditions A = B = 0, and A and B are determined from the following boundary conditions of <u>movable</u> edges

$$\int_0^a \left\{ \left. \frac{\partial^2 F}{\partial x^2} \right|_{y=a} \right\} dy - \frac{E^f t}{(1 - \nu^{f\,2})} (1 + \nu^f) \alpha^f T^m \int_0^a dy = 0$$

..(22.1)

$$\int_0^a \left\{ \frac{\partial^2 F}{\partial y^2} \bigg|_{x=a} \right\} dx - \frac{E^f t}{(1-\nu^{f2})} (1+\nu^f) \propto^f T^m \int_0^a dx = 0$$

..(22.2)

Equations (22.1) and (22.2) yield

$$A = B = -\frac{W_o^2 \Pi^2}{8\ a^2} + \frac{3}{4} \propto^f T^m$$

Here C_1, C_2, C_3, C_4 are given by

$$C_1 = C_4 = -\frac{W_o^2 \Pi (1-\nu^f)}{16\ a}$$

$$C_2 = C_3 = \frac{W_o^2 \Pi}{16\ a}$$

Putting $W = W_o \ \text{Sin} \frac{\Pi x}{a} \ \text{Sin} \frac{\Pi y}{a}$,

Values of u and v obtained from equation (21.1) and (21.2), using equation (20.4), (20.5) and (20.6), we finally obtained the following cubic equations after applying Galerkin's technique :

$$\frac{(7-2\nu^f)\Pi^4 h^3}{16 a^2} \Bigg[\frac{(1+\nu^f)}{(h+t)^2} + \frac{E^f t \Pi^2}{2a^2 h (1-\nu^{f2})} \Bigg\{ \frac{(1+\nu^f)(3-\nu^f)}{G^c} +$$

$$+ \frac{(3-2\nu^f) E^f t \Pi^2 (h+t)^2}{(7-2\nu^f) a^2 h G^{c2}} \Bigg\} \Bigg] \left(\frac{W_o}{h}\right)^3 +$$

$$+ \frac{\Pi^4 h}{a^2} \Bigg[1 + \frac{E^f t \Pi^2 (h+t)^2}{2a^2 h (1+\nu^f) G^c} - \frac{1}{2}(1+\nu^f) \propto^f T^m \Bigg\{ \frac{a^2}{(h+t)^2 \Pi^2} +$$

$$+ \frac{E^f t}{2(1-\nu^{f2}) h} \left(\frac{(3-\nu^f)}{G^c} + \frac{E^f t \Pi^2 (h+t)^2}{a^2 h G^{c2} (1+\nu^f)} \right) \Bigg\} \Bigg] \left(\frac{W_o}{h}\right)$$

$$+ \frac{\Pi^2 (h+2t)^2 h^2}{12 t (h+t)^2 a^2} (1+\nu^f) \propto^f f_o \left(\frac{a^2}{h^2}\right) = 0$$

..(23.1)

for the <u>immovable</u> edge and

$$\frac{(3-2\nu^f)\,\pi^2 h^3}{16a^2}\left[\frac{(1+\nu^f)}{(h+t)^2}+\frac{E^f t\,\pi^2}{2a^2 h(1-\nu^{f2})}\left\{\frac{(1+\nu^f)(3-\nu^f)}{G^c}-\right.\right.$$

$$\left.\left.-\frac{(1+2\nu^f)\,E^f t\,\pi^2 (h+t)^2}{(3-2\nu^f)\,a^2 h\, G^{c\,2}}\right\}\right]\left(\frac{W_0}{h}\right)^3+$$

$$+\frac{\pi^4 h}{a^2}\left[1+\frac{E^f t\,\pi^2 (h+t)^2}{2a^2 h\, G^c (1+\nu^f)}+(1+\nu^f)\,\alpha^f T^m\left\{\frac{a^2}{(h+t)^2\pi^2}+\right.\right.$$

$$\left.\left.+\frac{E^f t}{2(1-\nu^{f2})h}\left(\frac{3-\nu^f}{G^c}+\frac{E^f t\,\pi^2 (h+t)^2}{a^2 h\, G^{c\,2}(1+\nu^f)}\right)\right\}\right]\left(\frac{W_0}{h}\right)+$$

$$+\frac{\pi^2 (h+2t)^2 h^2}{12t(h+t)^2 a^2}\,(1+\nu^f)\,\alpha^f f_0\left(\frac{a^2}{h^2}\right)=0$$

..(23.2)

for the <u>movable</u> edge.

Equations (23.1) and (23.2) are now ready for numerical computation.