Chapter 11

CONCLUDING REMARKS

A scale invariant analytical framework on a Cantor set like fractal subsets of R is presented. Since a Cantor set C is a totally disconnected, compact, perfect subset of R, the ordinary analysis of R can not be meaningfully extended over C, i.e., when a real variable x is assumed to live and undergo changes only over the points of C. The usual ultrametric structure of a Cantor set is extended to an inequivalent class of ultrametrics exploiting the concept of relative infinitesimals those are supposed to exist on the gaps of another Cantor set in the neighbourhood of 0. A variable living on a Cantor set are shown to undergo changes by smooth, inversion induced jumps. As a consequence, the derivative discontinuity of a Cantor function at a point of the Cantor set are removed, making the global variability over double logarithmic scales of such a function everywhere smooth. Since the ultrametric valuations of scale invariant infinitesimals turn out to be a Cantor function, the corresponding scale invariant analysis formulated over a Cantor set also happens to ascribe a smooth differentiable structure over the Cantor set. This is the main advantage of the present analysis over the other competing approaches in the literature [6, 11, 10, 12].

In this thesis we report on only a few specific applications of this formalism: namely, (i) the precise form of differential jump measure on a Cantor set is derived, (ii) the interesting possibility of growth of measure is explained exploiting the underlying reparametrisation invariance of the nontrivial valuation, (iii) the variability in a family of Cantor sets with identical Hausdorff dimension and thickness are shown to be exposed by an application of the higher order contribution in the scale invariant valuation, and finally (iv) the simplest scale invariant ODE $t\frac{dx}{dt} = x$ is shown to admit a novel class of solutions which can be modeled to exist on a Cantor set. Besides these, we also have studied a few applications of the novel solutions and related ideas in some wellknown longstanding problems such as (i) the issue of randomness and time asymmetry (ii) chaos threshold in a logistic map and (iii) the origin of the ubiquitous $1/f$ noise (signal).

The present study seems to open up a large number of interesting new areas of investigations in various interdisciplinary branches, such as analysis, geometry, number theory, nonlinear dynamics, to name a few. A few extensions of the present scale invariant formulation of analysis on a Lebesgue measure zero Cantor set are already studied recently, in the context of the ordinary Calculus on R, leading to an interesting new proof of the Prime Number Theorem [29]. The scale invariant extension of R leads to a new deformed space $\mathcal{R}$ of real numbers. An analysis of the diffusion equation on this deformed space is recently completed with a novel interpretation of the emergence of anomalous mean square fluctuations when a diffusive system is allowed to execute motion over infinitely long time scales [28]. Applications of similar ideas in various other well known differential equations as well as in dynamical system theories remain to be undertaken in future.

This thesis is prepared on the basis of the following publications:

[1] D P Datta and S Raut, The arrow of time, complexity, and the scale free analysis, *Chaos, Solitons & Fractals*, 28, 581-589, (2006).

[2] S Raut and D P Datta, Analysis on a fractal set, *Fractals*, 17(1), 45-52, (2009).

[3] S Raut and D P Datta, Nonarchimedean scale invariance and Cantor sets, *Fractals*, 18, 111-118, (2010).

[4] D P Datta, S Raut, A Raychaudhuri, Ultrametric Cantor sets and Growth of Measure, *p-Adic Numbers, Ultrametric Analysis and Applications*, Vol.3, No.1, pp.7–22, (2011).