CHAPTER - I

INTRODUCTION

1.1 Introductory remarks

The contents of this thesis are arranged in four chapters. Chapter I is of a review nature and deals with a general introduction to the thesis. Chapter II is devoted to the study of the unsteady flows of elastico-viscous liquid under various configurations. These problems are of general and continuing interest and are discussed here to study the flow-characteristics of elastico-viscous liquid under a variety of situations. It consists of two parts. In the first part of this chapter, an analysis is made to investigate the unsteady flow of a non-Newtonian fluid between two oscillating plates, while, in the second part, the oscillating flow of visco-elastic Oldroyd fluid in a long circular tube in presence of magnetic field is explained.

Chapter III is concerned with the unsteady flows of dusty fluid under several physical and geometrical circumstances. This chapter is divided into two sections. Section A deals with the dusty Newtonian fluid flows, while, in section B, the dusty non-Newtonian fluid flows are considered. These problems are becoming increasingly important due to their varied applications in the fields of science and technology. These studies reveal the detailed structure of the flow and estimate the surface characteristics, such as, skin-friction coefficients, particle velocity of the liquid, etc.. Section A is composed of two parts. In the first part, the rotational motion of a dusty viscous fluid contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface is analysed. The second part of this section is concerned with the study of the semi-infinite dusty viscous fluid flow in a rotating system when its horizontal boundary is suddenly stopped from its state of steady motion. Section B consists of three parts. In the first part, the unsteady flow of the dusty visco-elastic liquid between two oscillating plates is investigated. The second part of this section is concerned with the study of the unsteady axi-symmetric rotational flow of elastico-viscous fluid with a uniform distribution of dust particles. In the last part of this section, the unsteady flow of a dusty elastico-viscous liquid in the Ekman layer is analysed.

The last chapter i.e. chapter IV of the thesis is devoted to the study of the diffusion of the matter through the Newtonian fluid, the free convection and the mass transfer in the Newtonian fluid. It is divided into five parts. In the first part of this chapter, an exact analysis of dispersion of the solute in an electrically conducting fluid flowing between two parallel plates porous walled channel in the presence of a uniform transverse magnetic field is made by using a generalized dispersion model. Dispersion coefficients are evaluated as functions of time. Unsteady free convective flows in presence of mass transfer through a porous medium bounded by a vertical porous plate are subjects of discussion in the subsequent four parts of the last chapter. Part two and three are devoted to study the effects of magnetic parameter on the flow field while the effects of rotation are analysed in the last two parts.

Before we discuss various problems, we present below a brief survey of the literature on the non-Newtonian fluids, the dusty fluid motion, the diffusion/dispersion of the solutes in fluids flowing under the laminar-flow conditions, the free convection and the mass transfer in a porous medium with a view to presenting the work of this thesis in its proper perspective.

1.2 Non-Newtonian fluids

A 'Newtonian' fluid is one for which a linear relation exists between stress and the spatial variation of velocity. If changes in the fluid density are not important, the constant of proportionality is called viscosity, a characteristic constant of the material at a given temperature and pressure. So a 'Newtonian' fluid is characterised by a linear relation between stress and rate of strain of the form

$$p_{ij} = (\lambda\Delta - p) \delta_{ij} + 2\mu e_{ij} , \qquad (1.1)$$

$$e_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i}) , \qquad (1.2)$$

where p_{ij} , e_{ij} , P and Δ denote the stress tensor, strain-rate tensor, pressure and dilatation respectively; λ, μ are coefficients of the medium. Its theory has been extensively investigated during the last century. The relationship (1.1) explains reasonably well most of the phenomena like drag, lift, skin-friction, separation etc. occurring in the flows of fluids. However, it fails to explain the occurrence of Weissenberg effect [1], Merrington effect [2] and Poynting effect [3]. In order to explain these effects, the stress-rate of strain relation should be generalised so that the constitutive equation becomes non-linear. The fluids under this category are termed as non-Newtonian fluids. Non-Newtonian fluids form an extremely wide class of different materials, whose only common features are fluidity and a failure to obey Newton's viscous law [4]. The exceptions to Newton's viscous law are not of rare occurrence, in fact the so-called non-Newtonian fluids are to be found close at hand everywhere. The fluids like blood, honey, condensed milk, liquid lubricants, printing inks, starch, rasin pastes, plastics, high polymers, salad dressings, butter, whipped cream and doughs, egg white, paints, certain varieties of oil and many other materials of industrial importance fall under the category of non-Newtonian fluids [5]. But the behaviour of such fluids is not so readily amenable to theoretical analysis due to the non-linearity of the stress-strain rate relations governing the fluid model and the dependence of the coefficients occurring in them on physical properties of the fluids. Moreover, experiments are not many to throw sufficient light on the flow phenomena of this type of fluids.

The study of non-Newtonian fluids has become essntial due to their importance in modern industries. This has led to the formulation of various theories of non-Newtonian fluids. We present below a brief discussion of the constitutive equations for three models of non-Newtonian fluids, viz. i) Model B due to Oldroyd ii) Walters'liquid B' model iii) Kuvshiniski's model which are directly related to our present thesis.

(a) Elastico-viscous liquids (Model B due to Oldroyd)

The class of liquids which possesses a certain degree of elasticity in addition to viscosity is known as the elastico-viscous liquid. Thus, when an elastico-viscous liquid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to viscous dissipation. In a normal inelastic viscous liquid we are concerned only with the rate of strain, but in elastic liquids we can not neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the reverse flow that follows the removal of stress. Only in elastico-viscous liquid there is a degree of recovery from the strain when the stress is removed whereas in other fluids the whole strain remains.

Evidences of liquid elastcity in an 1.5 percent starch solution can be observed by Hess's experiment [6] and by the recoil of air bubbles in a mixture of Polymethyl Methacrylate and Cyclohexanone (made by dissolving 3 gms. perspex in 100 ml of solvent) contained in a bottle which has been suddenly turned and then brought to rest.

Oldroyd [7] formulated a non-linear theory of a class of isotropic incompressible elastico-viscous liquids with the following rheological equation of state between the stress tensor p_{ik} with the rate-of-strain tensor e_{ik} :

$$p_{ik} = - p\,\delta_{ik} + p'_{ik} \,, \qquad (1.3)$$

where

$$p'_{ik} + \lambda_1 \left(\frac{D}{Dt}\right) p'_{ik} + \mu_0 p'_{ij}\, e_{jk} - \mu_1 (p'_{ij} e_{jk} + p'_{jk} e_{ij})$$
$$+ \nu_1\, p'_{jl} e_{jl} \delta_{ik} = 2\,\eta_0 [e_{ik} + \lambda_2 \left(\frac{D}{Dt}\right) e_{ik} - 2\,\mu_2 e_{ij} e_{jk}$$
$$+ \nu_2\, e_{jl}\, e_{jl} \delta_{ik}], \qquad (1.4)$$

and e_{ij} given by the equation (1.2).

In equation (1.4), η_0, λ_1 and λ_2 denote the coefficient of viscosity, stress relaxation time and strain retardation time respectively. The other five material constants μ_0, μ_1, μ_2, ν_1 & ν_2 are all of dimensions of time. The quantity $\left(\frac{D}{Dt}\right)$ denotes the total derivative following a fluid element taking into account its translational and rotational motion. It may be noted that the memory of the liquid is accounted for through the stress relaxation time λ_1 and rate-of-strain retardation time λ_2 and the linearity of the Newtonian costitutive equation is broken through introduction of quadratic terms in the strain rate components and the product of the stress rate and strain rate components.

Oldroyd, in his paper [8], considered two particular types of liquid viz., liquid A and liquid B. General model of liquid B is goverened by equation (1.4) with

$$\left.\begin{array}{l} \eta_0 > 0,\ \lambda_1 = \mu_1 > \lambda_2 = \mu_2 \geq 0 \\ \mu_0 = \nu_1 = \nu_2 = 0 \end{array}\right\} \qquad (1.5)$$

It was observed by Oldroyd that the model represented by the constitutive equation (1.4) along with (1.5) exhibits Weissenberg climbing effect when sheared at a finite uniform rate between two co-axial cylinders and has a distribution of normal stress equivalent to an extra tension along streamlines with isotropic state of stress in the plane normal to the streamlines. Thus the constitutive equation (1.4) subject to (1.5) retains essentially the rehological properties of a liquid. This model can predict normal stress and time-dependent visco-elastic behaviour which are in accord with physical observation. However, this model fails to account for the variation of apparant viscosity with rate of shear. Further, the consitutive equation (1.4) along with (1.5) holds for low rates of shear.

It was experimentally found by Oldroyd, Strawbridge and Toms [9] that a solution of a mixture of Polymethyl Methacrylate in pyridine obeys the constitutive equation subject to (1.5) and for this solution λ_1 = 0.065 (sec.) and λ_2 = 0.015 (sec.) with ν_0 = 7.9 poises and density 0.98 gm/ml. Thus for a real elastico-viscous liquid, the restriction $\lambda_1 > \lambda_2$ is a valid one.

(b) <u>Walters' liquid B'</u>

The constitutive equation for Walters' liquid B' [10] (at small rates of shear) are given by

$$p_{ik} = - p\, g_{ik} + p'_{ik} \quad , \tag{1.6}$$

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi\,(t - t')\,\frac{dx^i}{dx'^m}\,\frac{dx^k}{dx'^r}\,e^{(1)mr}(x',t')\,dt' \tag{1.7}$$

where p'^{ik} is the deviatoric stress tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed co-ordinate system x^i, x'^i the position at time t' of the element which is instanteneously at the point x^i at time t, $e^{(1)}_{ik}$ the rate of strain tensor and

$$\psi(t - t') = \int_0^\infty \frac{N(\tau)}{\tau} e^{-(t-t')/\tau} d\tau, \quad (1.8)$$

$N(\tau)$ being the distributive function of the relaxation time τ. It may be noted that the elastico-viscous liquid of model B due to Oldroyd is the special case of Walters' liquid B', obtained by substituting

$$N(\tau) = \eta_0 \frac{\lambda_2}{\lambda_1} \delta(\tau) + \eta_0 \frac{\lambda_1 - \lambda_2}{\lambda_1} \delta(\tau - \lambda_1) \quad (1.9)$$

in equations (1.7) and (1.8). $\delta(\tau)$ denotes a Dirac delta function.

It has been shown by Walters [11] that in the case of liquids with short memories (i.e. short relaxation times), the equation of state can be simplified to

$$p'^{ik} = 2\eta e^{(1)ik} - 2K_0 \frac{\delta}{\delta t} e^{(1)ik}, \quad (1.10)$$

where η ($= \int_0^\infty N(\tau)\, d\tau$) is the limiting viscosity at small rates of shear, K_0 ($= \int_0^\infty \tau N(\tau)\, d\tau$) is the elastic coefficient and $\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor.

It is interesting to note that the mixture of Polymethyl Methacrylate in pyridine at 25° C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits well in the above model. For this mixture, the relaxation spectrum as by Walters is

$$\begin{aligned} N(\tau) &= \sigma \eta_0 \delta(\tau) + \frac{1-\sigma}{\beta} \eta_0, \quad (0 \le \tau \le \beta) \\ &= 0 \quad \text{for } \tau > \beta \end{aligned} \quad (1.11)$$

where $\sigma = 0.13$, $\eta_0 = 7.9$ poises (gm/cm.sec) and $\beta = 0.18$ sec. .

(c) Elastico-viscous fluid (model due to Kuvshiniski)

The constitutive equations for Kuvshiniski's liquid [12] are given by

$$\left(1 + \lambda \frac{D}{Dt}\right) p'_{ik} = 2\mu e_{ik} \qquad (1.12)$$

where

$$\frac{D}{Dt}(p'_{ij}) = \frac{\partial p'_{ij}}{\partial t} + v_m \frac{\partial p'_{ij}}{\partial x_m} \qquad (1.13)$$

and $$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) . \qquad (1.14)$$

The stress tensor is given by

$$p_{ij} = -p\,\delta_{ij} + p'_{ij} \qquad (1.15)$$

where p is static pressure, δ_{ij} the Kronecker delta and p'_{ij} is deviatoric stress tensor usually related to the rate of strain, e_{ij}, by the equation of state (1.12). Also, physically λ means, if the motion stops suddenly, the shear stress will decay as $e^{-t/\lambda}$; μ is coefficient of viscosity, v_i is the velocity of liquid particle.

1.3 Unsteady flow of elastico-viscous liquid

(a) Flow between parallel plates

An extensive liturature exits on flow of non-Newtonian fluids between two parallel plates as these problems are of vital importance in certain industrial processes. Sharma [13] considered plane Poiseuille flow through tube and case of a parallel plate visco-meter. Agarwal and Jain [14] extended the paper of Sharma [13] by introducing magnetic field in the system. He considered the fluid to satisfy the Oldroyd model. Frater [15] discussed the flow of an elastico-viscous liquid between torsionally oscillating discs. The flow of an elastico-viscous Walters' liquid B' near an oscillating infinite plate and between two infinite oscillating plates with a phase difference, with the same frequency and different amplitudes under a transverse magnetic field relative to the fluid and fixed relative to one of the plates was considered by Gulati [16]. The plane Poiseuille flow, plane Couette flow and the Couette flow of micropolar fluids was investigated by Rajagopalan [17]. Siddappa and Hegde [18] discussed the exact solution for oscillating motion of a visco-elastic fluid bounded by two infinite plates, one fixed and the other executing simple harmonic oscillations in its own plane. The case, in which both the plates oscillates was also analysed. Johri [19] discussed the problem of an elastico-viscous flow (Rivlin-Ericksen model) induced by circular oscillations of two infinite parallel discs. Later, Johri [20] studied the problem of unsteady slow flow of Oldroyd B-liquid between two infinite parallel discs. The flow was induced by the elliptic harmonic oscillations of the discs.

In part one of chapter II of the present thesis, an analysis is made on the flow characteristics of a certain class of non-Newtonian fluids, viz. elastico-viscous liquid proposed by Kuvshiniski [12], flowing between two oscillating parallel plates. Analytical expressions for velocity field and skin-friction on the upper plate wall are obtained by the technique of Laplace transform. The effects of elastic elements in the liquid, the amplitude of steady state oscillations and the phase difference of the velocity fields, the skin-friction at the upper plate are presented graphically/numerically.

(b) <u>Flow in a pipe due to applied pressure gradient</u>

Oscillating flow of viscous fluid in a long circular tube under the influence of a pressure gradient was investigated theoretically and experimentally by Richardson and Tyler [21] and theoretically by Sexl [22].

Many common liquids such as oils, certain paints, blood, polymer solutions, some organic liquids and many new materials of industrial importance exhibits both viscous and elastic properties. However, this sub-class shows diverse behaviour in response to applied stress. A number of rheological models were proposed to explain such a diverse behaviour. Many scientists carried out their research on flow problems in a tube under applied pressure gradient in different elastico-viscous liquid models. Flow of a Maxwell fluid through a tube under the application of an axial pressure gradient

(i) $-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ke^{\alpha^2 t}$ (ii) $-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ke^{-\alpha^2 t}$,

was considered by Soundalgekar [23]. In the first case formation of boundary layer was observed while the layer was not noticed in the second case. Both the cases could have been dealt by the author [23] as a single case. The unsteady flow of an Oldroyd fluid through a tube under a pressure gradient $-\frac{1}{\rho}\frac{\partial p}{\partial z} = a_0 + f(t)$ was studied by Ghosh [24]. The pulsating flow of a second-order fluid through a circular pipe was analysed by Ramacharyulu [25] while the flow problem for the same fluid under time-dependent exponentially increasing or decreasing pressure gradient was studied by Soundalgekar [26]. Later Soundalgekar [27] extended his work [26] in Walters' liquid B' under the same geometry and same physical situations. But none of the authors examined the effects of magnetic field in this geometry. In the part two of chapter II we study the flow behaviour of visco-elastic Oldroyd fluid in the presence of magnetic field in a long circular tube. Considering the flow field under the influence of an oscillating pressure gradient, the expression for velocity of liquid is obtained analytically. A comparative study for the effects of magnetic field in Newtonian and Oldroyd fluid is made.

1.4 Rotating fluid flows

The stimulus for scientific research on fluid system in rotating environments is originated from geophysical and fluid engineering applications. Many aspects of the motion of terrestrial and planetary atmospheres are influenced by the effects of rotation. The broad subjects of oceanography, meteorology, atmospheric science all contain some important and essential aspects of rotating flows. Rotating flow theory is utilised in detemining the viscosity of fluids and in the construction of turbines and other centrifugal machines. Also the study of flows through porous media in a rotating system is of considerable interest in many scientific and engineering applications. viz., to the petroleum engineers concerned with the movement of oil and gas through reservoir; to the hydrologist in his study of migration of underground water etc.. The complete literature pertaining to rotating fluids is enormous and an excellent review can be found in the monograph by Greenspan [28]. Rotation in a fluid system produces two effects, viz., the coriolis and the centrifugal forces, on the fluid particles. The balance between the coriolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. In considering flows in rotating environment we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the later case it is preferable to use an inertial co-ordinate system fixed in space. On the other hand the flow behaviour in the former case can be described in a co-ordinate system which rotates with the fluid, and in this frame of reference the fluid is at rest.

In a steadily rotating system, a balance is struck between coriolis and frictional forces in a thin layer over horizontal boundaries. This layer, called the Ekman layer, was first noticed by Ekman [29] and plays a very fundamental role in the rotating fluid flows. In this thesis we consider certain problems in rotating system due to their varied applications in the field of technology.

1.5 Dusty fluid flows

Fluid flows with particulate suspensions, when the suspended matter may consist of solid particles, liquid droplets, gas bubbles or combinations of these, are commonly termed as dusty gas flows or dusty fluid flows. They are also referred to as two-phase flows, since they involve a composite of two-phase or two materials with different distinguishable properties – one phase being the fluid medium which is a continuous phase and the other phase being the particulate suspensions which are scattered throughout the fluid medium and hence known as the dispersive phase or discrete phase or simply particulate phase.

The flows of fluids with suspended material particles abound in nature, classical examples being pollution of air and contamination of water. The earth's atmosphere is a predominantly gaseous envelope of air surrounding the earth and it contains solid particles and liquid droplets. Besides it is also being constantly polluted by a number of dust particles like carbon, sulpher and many other toxic elements which arise as inevitable consequence and natural by-products of industrialization.

Dusty gas flows assume importance in such engineering problems as fluidization (flow throuh packed beds), sedimentation, powder technology, flows in rocket tubes where small carbon or metalic fuel particles are present, aerosol filtration, gas purification and slurries. Further, problems concerned with atmospheric fallout, rain errosion of guided missiles and aircraft icing are some of the areas where the dynamics of dusty gases play a prominent role.

A knowledge of dusty fluid flows is useful to some extent in understanding the rheology of blood flows through capillaries, where red blood cells can be regarded as rigid particles embedded in the plasma. Another biological situation where the study of two-phase flows assumes importance is the phenomena of particle deposition in the respiratory tract.

The formulation of the fundamental equations of the dusty fluid flows is guided in a reasonably simple manner under certain basic assumptions : (i) the fluid is an incompressible viscous fluid; (ii) dust particles are spherical and undeformable, all having equal radius and mass ; (iii) the density of the dust particles is high compared with the fluid density and the mass fraction of the dust particles is not extremely high so that the volume occupied by the dust particles is negligible ; (iv) the volume-fraction of the dust particles is so small that the interaction between individual dust particles may be neglected and so the fluid phase contributes the entire pressure ; (v) dust particles form a cloud of pseudo-fluids with negligible viscosity; (vi) the thermal and gravitational forces are neglected ; (vii) the radius of the dust particles being small, the only force acting due to interaction between dust and fluid particles is the drag force given by Stokes' law, where it is also supposed that the Reynolds number of the relative motion of the dust and fluid is small compared to unity ; (viii) the distortion of the flow field around the dust particles is neglected.

Under these assumptions, following Saffman's formulation [30], the equations of continuity, equations of motion of viscous fluid and the dust particles are

$$u_{i,i} = 0, \tag{1.16}$$

$$\frac{\partial N}{\partial t} + (N v_i)_{,i} = 0 , \tag{1.17}$$

$$\rho \left(\frac{\partial u_i}{\partial t} + u_k u_{i,k} \right) = p_{ik,k} + KN (v_i - u_i), \tag{1.18}$$

$$m \left(\frac{\partial v_i}{\partial t} + v_k \, v_{i,k} \right) = K \, (u_i - v_i) \, , \qquad (1.19)$$

where u_i, v_i are the local velocity vectors of liquid and dust particles respectively, p_{ik} the stress tensor defined by equation (1.1) in which the rate of strain tensor e_{ik} defined in terms of the velocity field u_i by

$$e_{ik} = \frac{1}{2} \, (u_{i,k} + u_{k,i}) \, ,$$

ρ the density, K the Stokes resistance coefficient (for spherical particles of radius d, it is $6 \pi \mu d$), N the number density of a dust particle and m the mass of a dust particle.

Based upon the theoretical model proposed by Saffman [30], numerous authors investigated a number of dusty gas flow problems under several physical and geometrical circumstances. Kazakevich and Krapivin [31] and Sproull [32] performed experimental works on dusty fluid and observed that the aerodynamical resistance of the dusty gas is less than that of a clean gas. Soo [33,34,35,36] made pioneering works in the two-phase problems including the problems of dusty fluid flows. Marble [37], Pai [38], Jain [39] and Goddard [40] reviewed the subject of dynamics of dusty fluids. Rudinger and Chang [41] and Rudinger [42] considered the effects of volume-fraction of dust particles on the flow of dusty gases and also analysed unsteady two-phase flow. Wallis [43] studied one-dimensional two phase flow. Davidson [44] gave some features on the flow of fluid-solid mixtures. Torobin and Gauvin [45,46], Crooke [47], Hinch [48] and Purcell [49] put forward some noble ideas on the fundamental aspects of the flow of dusty fluids.

In section A of chapter III of the present thesis, we consider two problems on unsteady flow of dusty viscous liquid.

Among many fluid mechanical problems which are of direct technological interest, one of the most important classes is that involving the motion of particles in polymer solutions or melts and other non-Newtonian fluids. A considerable amount of both theoretical and experimental work in this line of study exists in literature. Leal [50] examined the slow motion of slender rod like axi-symmetric particles on a second order fluid while Goddard [51,52] discussed the stress field of slender and rod like particles respectively flowing in a non-Newtonian fluid. Recently, Leal [53] presented a report on the studies of motion of small particles in non-Newtonian fluids.

There is another class of flow problems which concerns with the study of motion of non-Newtonian fluids having suspension of undeformable small solid particles in it. Very little is known [54,55,56,57,58], so far as we are aware, about the dynamics of dusty non-Newtonian fluid although it has applications on the problems of transport of solid particles suspended in non-Newtonian fluids through pipes and channels, polluted oil extraction, polymer extrusion, paint spraying, boundary layer growth of non-Newtonian fluids having suspended dust particles, etc. and it is the subject of investigation presented in section B of chapter III of the thesis.

(a) **Dusty fluid flows through a circular cylinder or between two co-axial cylinders**

The classical problem on the laminar flow of a dusty gas between two rotating cylinders was investigated by Michael and Norey [59]. They assumed that the relaxation time of the dust, τ, to be small and obtained solutions by considering the ratio of the time scales, on which the gas velocity and the mass concentration of dust change, to be both large and small. Dusty fluid flows between two rotating co-axial cylinders were investigated by Nath [60] and Tiwari [61] under different physical situations. Gupta and Gupta [62] investigated the Couette flow of a dusty gas between two infinite co-axial cylinders. Crooke and Walsh [63] developed a method for the construction of solutions for the flow of a viscous, incompressible gas with suspended dust particles when the flow domain possesses special geometry. In first, second and third sections, they derived a set of linear partial differential equations representing two-dimensional flow and obtained solutions to those equations for rectangular and circular geometries. In the fourth section, they presented the general solution for two-dimensional flow through arbitrary cross-sections in terms of eigen function expansions for the geometry of these cross-sections. Later, oscillatory flow of a dusty viscous fluid in a cylinder was investigated by Arunachalam et al. [64].

In part one (section A) of chapter III, the rotational motion of a dusty viscous fluid contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface is considered. This paper extends the analysis of

Bhattacharyya [65] to observe the effects of dust particles on the fluid flow.

Dusty non-Newtonian fluid flows between two oscillating cylinders were investigated by Mukherjee et al. [57] and Mukherjee and Maiti [58]. In part two (section B) of chapter III we investigate the flow of elastico-viscous liquid embedded with particles in an oscillating cylinder. Explicit expressions are obtained for the velocities of liquid and dust particles by the technique of Laplace transform. Numerical computation of the velocity fields are carried out for different values of mass concentration, relaxation time of dust particles and elastic elements in the liquid.

(b) Dusty fluid flows in a rotating system

The study of dusty fluid flows in a rotating system is of interest to many research workers in fluid mechanics due to its varied practical importance. These studies have some bearing on the problems of atmospheric pollution. Oscillatory flow of a dusty gas in a rotating system was considered by Datta and Jana [66]. Unsteady flow of a dusty gas due to non-torsional oscillation of a plate in a rotating frame was investigated by Jana and Datta [67]. An exact solution for the flow of an incompressible viscous dusty gas induced by two infinitely extended parallel plates when the lower plate is oscillating harmonically and the upper one is at rest in a rotating frame of reference was studied by Mitra [68]. In the second part (section A) of chapter III, the unsteady flow of semi-infinite dusty fluid on a horizontal plate in a rotating frame is considered when the horizontal plate is impulsively brought to rest from the state of uniform motion. Due to complexity of the problem, we restrict ourselves to the case of large time only.

Boundary layer flow of viscous fluid over a flat plate in a rotating frame of reference has gained considerable attention due to its varied and wide applications in the areas of geophysics and astrophysics. Gupta and Pop [69] and Gupta [70] analysed the boundary layer growth in a liquid with suspended particles.

But these investigations are confined to the cases of dusty viscous fluid flows. However, the study of dusty non-Newtonian fluid flows over a flat plate in a rotating system is of considerable importance in various technical problems. In the third part of chapter III (section B), we extend the analysis of Gupta and Pop [69] to cover a wider class of elastico-viscous liquid, viz. Walters' liquid B ' (short memories). The initial value problem is solved by Laplace transform technique and the qualitative features of the unsteady boundary layer with particular reference to the effect of rotation through inertial oscillations and the establishment of the Ekman boundary layer are discussed. It is found that the frequency of inertial oscillations decreases with the increase of either mass concentration or elastic element.

(c) <u>Dusty non-Newtonian fluid flows between parallel plates</u>

The problem of unsteady flow of liquid between parallel plates is of common interest as it has varied applications in many engineering and technological problems. Moreover, the equations of motion are greatly simplified so that one can find an exact solution of the problem. Vimala [71] studied the flow of dusty gas induced by two infinite flat plates oscillating in their own planes. Subsequently, Mukherjee et al. [72], Mathur et al. [73] and Kishore and Pandey [74] analysed the flow of a dusty gas between two parallel flat plates under different conditions. Recently, Mitra [68] discussed the oscillatory flow of a dusty gas between two parallel flat plates in a rotating frame while Mitra and Bhattacharyya [75] investigated the unsteady flow of a dusty gas between two parallel flat plates, one being at rest and other begins to oscillate harmonically in its own plane. However, so far as we are aware, the work on dusty non-Newtonian fluid flows between parallel plates is not reported in literature.

In part one of chapter III (section B) of the present thesis, we discuss the unsteady flow of dusty elastico-viscous liquid between two oscillating parallel flat plates. The analytical expressions for the velocities of the dusty liquid and dust particles are obtained by the technique of Laplace transform. The effects of the elastic element in the liquid, the mass concentration and the relaxation time of the dust particles on the velocity fields, the skin-friction at the lower plate and the volume flow rate in between the plates presented numerically/graphically. It is found that both skin-friction at the lower plate wall and volume flow rate in between the plates increase in the presence of the elastic element in the liquid.

1.6 Unsteady convective diffusion

We frequently encounter with the problems of dispersion of soluble matter in laminar flow in chemical industries, tracer analysis, chromatography and physiological systems. When a solute is released in a solvent which flows steadily under laminar conditions through a circular tube, it spreads out longitudinally about a plane moving at the mean speed of the flow under the combined effect of lateral molecular diffusion and longitudinal convection, and longitudinal molecular diffusion. Extensive investigations were made, both theoretically and experimentally, by many authors to analyse the dispersion of soluble matter in laminar flow.

Taylor [76] investigated the dispersion of a soluble matter in an incompressible viscous fluid flowing in a circular tube under laminar conditions considering the unsteady convective diffusion in a steady flow by the dispersion model

$$\frac{\partial C_m}{\partial t} + u_m \frac{\partial C_m}{\partial x} = K \frac{\partial^2 C_m}{\partial x^2} , \qquad (1.20)$$

where t is time, x is the axial co-ordinate, C_m is the area average concentration, u_m is the average velocity and K is the dispersion coefficient which depends on the physical parameters but not on t or x. The analysis of Taylor [76] on his conceptual dispersion model is applicable only for large values of time t. On the other hand, Lighthill [77], neglecting axial molecular diffusion, obtained an exact solution of the unsteady convective diffusion equation which is asymptotically valid for small t. To be more precise, Taylor's dispersion model is valid if the time after the injection of the solute exceeds $0.5a^2/D$ ('a' is the tube radius, D the molecular diffusivity) while Lighthill's model takes account of the initial action of diffusion in front of the concentration distribution and is valid for time less than about 0.1 a^2/D. However, Gill [78] and Gill and Ananthakrishnan [79] obtained an exact solution for the local concentration using the series expansion of the dispersion model. Later, Gill and Sankarasubramanian [80] constructed a generalised dispersion model for the above steady flow, which is valid for all t, by involving an infinite set of time-dependent coefficients and obtained exact solution for the local concentration C. Since K is a function of time t in this model, this might account for the considerable amount of scatter which is generally found in experimental data on dispersion. Subsequently, using the generalised dispersion model, they [81,82,83] studied, respectively, the dispersion of a non-uniform slug, non-uniformly distributed time-variable continuous source and the interphase mass transfer in fully developed time-dependent flow. Recently, Krishnamurthy and Subramanian [84] formulated convective diffusion theory for the predictive modelling of field-flow fractionation columns used for the separation of colloidal mixture. Jayaraj and Subramanian [85] used the truncated versions of generalised dispersion theory to study the relaxation phenomena in field-flow-fractionation. Annapurna and Gupta [86] and Gupta [87] analysed the unsteady magnetohydrodynamic convective diffusion in electrically conducting fluid flowing in a parallel plate channel.

The first paper of the last chapter of this thesis deals with an exact analysis of the dispersion of solute in an elctrically conducting fluid flowing between two parallel plate porous channel in the presence of uniform transverse magnetic field. Using a generalised dispersion model, which is valid for all time after the injection of solute in the flow, the dispersion coefficients as functions of time are evaluated. The behaviour of the coefficients in the dispersion equation is explained on physical grounds and also numerical calculations are carried out to get a physical insight into the problem. The results of the present investigation are likely to have applications to situations where tracers are used for measuring the flow rate in a porous-walled channel.

1.7 Free convection and mass transfer in a porous medium

Fluid flow due to density differences in the force field is generally called free convection. Such external forces are gravity forces, and the density difference, a very simple case, is the result of the temperature drop between the solid surface and the fluid. Free convection flow is not of rare occurrence in nature. In fact trade winds are due to convection currents set up in the atmosphere due to unequal heating. Also land and sea breezes arise in a similar manner. Studies on free convection have long considered the problem of unsteady free convection flow past an infinite vertical plate as one of the fundamental problem in heat transfer owing to its practical applications. Free convection effects on the Stokes problem for an infinite vertical plate was investigated by Soundalgekar [88]. This problem is better known as

Stokes problem for the vertical plate. Later, Pop and Soundalgekar [89] investigated the free convection flow past an accelerated vertical infinite plate. However, in nature, along with the free convection currents caused by the temperature differences, the flow is also affected by chemical composition differences and gradients or by material or phase constitutions. This can be seen in our everyday life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by the temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as the mass transfer flow. When a mixture of gases or liquids is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from a region of high concentration to regions of low concentration. There is also a mass transfer associated with convection in which mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level. Due to applications in various technological problems and in agricultural science, effects of mass transfer on the unsteady free convective flow past an infinite porous plate with constant or variable suction were studied by Soundalgekar [90], Soundalgekar and Wavre [91,92], Soundalgekar [93] and Raptis et al. [94].

Flows through porous media are very much prevalent in nature and threfore, the study of flow through porous media has become of principal interest in many scientific and engineering applications. Also, porous media are very widely used for a heated body to keep its temperature. To make the heat insulation of the surface more effective it is necessary to study the free convection and mass transfer effects on the flow through porous medium. A number of workers studied both steady/unsteady free convection and mass transfer flows in a rotating/non-rotating systems through porous medium bounded by an infinite vertical porous plane. Raptis et al. [95,96] investigated the steady two-dimensional free convection and mass transfer flow of an incompressible viscous fluid through a porous medium bounded by a vertical infinite porous plate. Raptis et al. [95] investigated the problem when the plate was subjected to constant temperature while in a follow up paper Raptis et al. [96] investigated the case when the plate was subjected to a constant heat flux. Effects of Grashof number, modified Grashof number and permeability of the porous medium on the velocity and rate of heat transfer were discussed when the surface was subjected to a constant suction velocity. But in both the papers, free-stream velocity was considered to be zero. Unsteady free convective flow and mass transfer through a porous medium bounded by an infinite vertical limiting surface with constant suction and time-dependent temperature was studied by Raptis [97]. In further follow up programmme, Raptis [98] considered the effects of free convection and mass transfer flow through a very porous medium bounded by an infinite vertical porous plate when there was free-stream velocity. The porous plate was subjected to a constant suction, the temperature and the species concentration at the plate were considered to be constant and the flow was steady.

It is known that MHD flows have received considerable attention because of their practical applications. On the other hand the significance of suction/injection for the boundary layer control in the field of aerodynamics and space science is well recognised. A theoretical analysis of steady free convective and mass transfer flow was presented by Raptis and Kafousias [99] when a viscous, incompressible and electrically conducting fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite vertical porous plate. A magnetic field of uniform strength was applied perpendicular to the plate and constant heat flux at the plate was assumed. In part two and three of chapter IV, we have considered two problems on the free convection and mass transfer through porous medium bounded by an infinite vertical porous plate in presense of transverse magnetic field. In part two, the temperature at the plate is considered to vary with time about a non-zero constant mean while it is considered to be constant at the far field. In part three, we consider a situation where the concentration fluctuates with time about a non-zero constant mean. Analytical expressions for the velocity, temperature and concentration fields are obtained. The influence of the various parameters entering into the problem on the flow are discussed in both part two and part three.

The study of flows through porous media in a rotating system is of considerable interest in many scientific and engineering applications; viz., to the petroleum engineer concerned with the movement of oil and gas through reservoir; to the hydrologist in his study of migration of underground water etc. A number of workers [100,101,102,103] considered both steady and unsteady free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate for a fluid rotating with constant angular velocity in different physical situations. Raptis [100] considered the steady case when there was a constant heat flux at the plate while the effects of Hall current was observed by Raptis and Ram [101]. In a recent paper, Kumar and Varshney [102] considered the unsteady free convection flow of an incompressible, viscous fluid through a porous medium past an oscillating porous plate in a rotating system. Temperatures at the plate and at the far field were assumed to be constant (but different). In a more recent paper, Mahato and Maiti [103] considered a similar problem where temperature at the plate was assumed to be fluctuating with time. In part four of chapter IV, we consider free convection and mass transfer flow in a porous medium bounded by infinite vertical plate in a rotating system when there is constant heat flux at the vertical wall and species concentration oscillates with time. Expressions for the velocity, temperature and concentration fields are obtained with the help of perturbation technique and the effects of the various parameters on the velocity field are discussed. In a physically realistic situation we may assume fluctuations in velocity in far field flow region. Unsteady free convection flow and mass transfer of viscous fluid through a porous medium bounded by an infinite vertical porous plate in a rotating system when there is an oscillating free-stream velocity, is the subject of discussion in the last part of the last chapter of the thesis.

References

[1] Weissenberg, K. (1947) Nature, Vol.159,310.

[2] Merrington, A. C. (1943) Nature, Vol.152,663.

[3] Poynting, J. H. (1909) Proc. Roy. Soc., Lond., Vol. A82, 546.

[4] Astarita, G. and Marrucci, G. (1974) Principles of non-Newtonian Fluid Mechanics, Mc Graw-Hill Book Company Ltd., London.

[5] Harris, J. (1977) Rheology and non-Newtonian Flow, Longman Inc., New York.

[6] Reiner, M. (1949) Deformation and Flow,Lewis,London.

[7] Oldroyd, J. G. (1958) Proc. Roy. Soc., London, Vol. A245, 278.

[8] Oldroyd, J. G. (1950) Proc. Roy. Soc.,London, Vol. A200, 523.

[9] Oldroyd, J. G., Strawbridge, D. J. and Toms, B. A. (1951) Proc. Phys., Soc., Vol. B64, 44.

[10] Walters, K. (1960) Quart. J. Mech. Appl. Math., Vol.13, 444.

[11] Walters, K. (1962) J. Mecanique, Vol. 1, 474.

[12] Kuvshiniski, E. V. (1951) J. Exptl. Theor. Phys. (USSR), Vol. 21, 88.

[13] Sharma, S. K. (1959) ZAMM, Vol. 39, 313.

[14] Agarwal, J. P. and Jain, M. K. (1962) ZAMP, Vol. 13, 152.

[15] Frater, K. R. (1964) J. Fluid Mech., Vol. 19, 175.

[16] Gulati, S. P. (1966) J. Phys. Soc., Japan, Vol.21,1411.

[17] Rajagopalan, Renuka. (1968) J. Indian Inst. Sci., Vol. 50, 57.

[18] Siddappa, B. and Hegde, S. (1972) Prog. Math., Vol. 6, 15.

[19] Johri, A. K. (1974) Agra Univ. Res. Sci. J., Vol.23, 15.

[20] Johri, A. K. (1978) Indian J. Pure Appl. Math., Vol.9, 481.

[21] Richardson, E. C. and Tyler, E. (1929) Proc. Phys.Soc., London, Vol.42,1.

[22] Sexl, T. (1930) Z. Phys., Vol. 61, 346.

[23] Soundalgekar,V. M.(1965) A.I.A.A. J., Vol. 3, 868.

[24] Ghosh, A. K. (1969) Acta Tech. Academiae Sci. Hungaricae, Vol. 66, 455.

[25] Ramacharyulu, N. Ch. P. (1971) Bull. Tech. Univ. Istanbul, Vol.24, 19.

[26] Soundalgekar,V. M.(1972) Indian J. Phys., Vol. 46, 250.

[27] Soundalgekar,V. M.(1973) Chem. Engng. Sci., Vol. 28, 654.

[28] Greenspan, H. P. (1968) The Theory of Rotating Fluids, Camb. Univ. Press.

[29] Ekman, V. W. (1905) Arch. Math. Astr. Phy., Vol. 2, 1.

[30] Saffman, P. G. (1962) J. Fluid Mech., Vol. 13, 120.

[31] Kazakevich, F. P. and Krapivin, A. M. (1958) Izv. Vyssh. Uchaben. Zavedenii, Energetika, Vol. 1, 101.

[32] Sproull, W. T. (1961) Nature, Vol. 190, 976.

[33] Soo, S. L. (1961) A.I.Ch.E.J., Vol. 7, 384.

[34] Soo, S. L. (1965) I and EC Fundamentals, Vol.4, 426.

[35] Soo, S. L. (1967) Fluid dynamics of multiphase systems, Blaisdell Pub. Co., Waltham, Mass.

[36] Soo, S. L. (1969) Appl. Sci. Res., Vol.21, 68.

[37] Marble, F. E. (1963) Proc. Fifth AGARD Combust. Propul. Colloquium, 175.

[38] Pai, S. I. (1970) Techn. Note - BN - 668, Institute for Fluid Dynamics and Applied Mathematics,University of Maryland.

[39] Jain, A. C. (1975) Dynamics of dusty gases, Engineering and Environmental Fluid Mechanics, Ed. A. C. Gupta and A. K. Gupta , I.I.T., Kanpur, 3.1.

[40] Goddard, J. D. (1977) Continuum models of discrete systems, June 26 - July 2, 605.

[41] Rudinger, G. and Chang, A. (1964) Phys. Fluids, Vol. 7, 1747.

[42] Rudinger, G. (1965) A.I.A.A. J., Vol.3, 1217.

[43] Wallis, G. B. (1969) One-dimensional two-phase flow, McGraw-Hill Book Comp., New York.

[44] Davidson, J. F. (1969) J. Fluid Mech., Vol. 39, 375.

[45] Torobin, L. B. and Gauvin, W. H. (1959) Can. J. Chem. Eng., Vol.37, 129, 167, 224.

[46] Torobin, L. B. and Gauvin, W. H. (1960) Can. J. Chem. Eng., Vol.38, 142, 189.

[47] Crooke, P. S. (1975) ZAMM, Vol.55, 272.

[48] Hinch, E. J. (1977) J. Fluid Mech., Vol.83, 695.

[49] Purcell, E. M. (1978) J. Fluid Mech., Vol.84, 551.

[50] Leal, L. G. (1975) J. Fluid Mech., Vol.69, 305.

[51] Goddard, J. D. (1976) J. Fluid Mech., Vol.78, 177.

[52] Goddard, J. D. (1976) J. Non-Newtonian Fluid Mech., Vol.1, 1.

[53] Leal, L. G. (1979) J. Non-Newtonian Fluid Mech., Vol.5, 33.

[54] Srivastava, L. P. (1971) Istanbul Teknik Universitesi Bulteni, Vol.24, 19.

[55] Sharma, C. L. and Dube, S. N. (1976) Bull. de L' Acad. Polo. des Sci. Series des Sci. Tech., Vol. XXIV, 145.

[56] Bagchi, S. and Maiti, M. K. (1980) Acta Ciencia Indica, Vol. VIm, 130.

[57] Mukherjee, S., (1984) Indian J. Technol., Vol.22, 41.
Maiti, M. K.
and
Mukherjee, S.

[58] Mukherjee, S. (1986) Indian J. Technol., Vol.24, 698.
and
Maiti, M. K.

[59] Michael, D. H. (1968) Quart. J. Mech. and Appl. Math., Vol.21, 375.
and
Norey, P. W.

[60] Nath, G. (1970) Proc. Nat. Acad. Sci., India, Vol.40(A), 257.

[61] Tiwari, V. B. (1973) J. Sci. Res., Vol.23, 145.

[62] Gupta, R. K. (1977) Proc. Ind. Nat. Sci. Acad., Vol.43A, 56.
and
Gupta, S. C.

[63] Crooke, P. S. (1974) Powder Technology, Vol.9, 111.
and
Walsh, R. A.

[64] Arunachalam, P.V., (1976) Proc. 21st. ISTAM, 185.
Naidu, K. B.
and
Nirmala, C.

[65] Bhattacharyya, P. (1981) Indian J. Math., Vol.23, 65.

[66] Datta, N. and Jana, R. N. (1976) Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie,Vol. 20, 71.

[67] Jana, R. N. and Datta, N. (1977) Bull. Math. de la Soc.Sci.Math.de la R.S. de Roumanie, Vol 21, 309.

[68] Mitra, P. (1981) Mechanique Appliquee, Vol. 26, 47.

[69] Gupta, A. S. and Pop, I. (1975) Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie, Vol. 19, 291.

[70] Gupta ,A. S. (1977) Analde Universitatii, Bucurasti, Mathematica, Vol. 26, 45.

[71] Vimala, C. S. (1972) Def. Sci. J., Vol.22, 231.

[72] Mukherjee, S. R., Tewari, V. D. and Bhattacharjee, S. (1973) Prog. of Maths., Vol. 7, 37.

[73] Mathur, A. K., Sacheti, N. C. and Bhatta, B. S. (1976) The Mathematics Education, Vol. 10.

[74] Kishore, N. and Pandey, R. D. (1977) J. Scientific Res., Vol. XXVIII(2), 151.

[75] Mitra, P. and Bhattacharyya, P. (1982) Rev. Roum. Sci. Techn. – Mec. Appl., Vol.27, 57.

[76] Taylor, G. I. (1953) Proc. Roy. Soc., Lond., Vol. A219, 186.

[77] Lighthill, M. J. (1966) J. Inst. Math. Appls., Vol.2, 97.

[78] Gill, W. N. (1967) Proc. Roy. Soc., Lond., Vol.A298, 335.

[79] Gill, W. N. and Ananthakrishnan, V. (1967) A. I. Ch. E., Vol. 13, 801.

[80] Gill, W. N. and Sankarasubramanian, R. (1970) Proc. Roy. Soc., Lond., Vol.A316, 341.

[81] Gill, W. N. and Sankarasubramanian, R. (1971) Proc. Roy. Soc., Lond., Vol.A322, 101.

[82] Gill, W. N. and Sankarasubramanian, R. (1970) Proc. Roy. Soc., Lond., Vol.A327, 191.

[83] Sankarasubramanian, R. and Gill, W. N. (1973) Proc. Roy. Soc., Lond., Vol.A333, 115.

[84] Krishnamurthy, S. and Subramanian, R. S. (1977) Seperation Science, Vol.12, 347.

[85] Jayaraj, K. and Subramanian, R. S. (1978) Seperation Science and Technology, Vol.13, 791.

[86] Annapurna, N. and Gupta, A. S. (1979) Proc. Roy. Soc., Lond., Vol.A367, 281.

[87] Gupta, P. S. (1980) Chem. Engg. Commun., Vol.7, 301.

[88] Soundalgekar,V.M. (1977) Trans. ASME J. Heat Transfer, Vol.99, 499.

[89] Pop, I. and Soundalgekar, V. M. (1980) ZAMM, Vol.60, 167.

[90] Soundalgekar,V.M. (1976) Proc. Indian Acad. Sci., Vol.84A, 194.

[91] Soundalgekar,V.M. and Wavre, P. D. (1977) Int. J. Heat Mass Transfer, Vol.20,1363.

[92] Soundalgekar,V.M. and Wavre, P. D. (1977) Int. J. Heat Mass Transfer, Vol.20,1375.

[93] Soundalgekar,V.M. (1979) Trans. ASME J. Appl. Mech., Vol.46, 757.

[94] Raptis, A. A., Perdikis, C. and Tzivanidis, G. J. (1981) J. Eng. & Appl. Sci., Vol.1, 139.

[95] Raptis, A., Tzivanidis, G. and Kafousias, N. (1981) Lett. Heat Mass Transfer,Vol.8,417.

[96] Raptis, A. A., Kafousias, N. G. and Massalas, C. V. (1982) ZAMM, Vol.62, 489.

[97] Raptis, A. A. (1983) Energy Research, Vol.7, 385.

[98] Raptis, A. (1984) Arch. Mech., Vol.36, 307.

[99] Raptis, A. and Kafousias, N. (1982) Can. J. Phys., Vol.60, 1725.

[100] Raptis, A. (1983) Int. Comm. Heat Mass Transfer, Vol.10, 141.

[101] Raptis, A. and Ram, P. C. (1984) Int. Comm. Heat Mass Transfer, Vol.11, 385.

[102] Kumar, Komal. and Varshney, C. L. (1987) Reg. J. Energy Heat Mass Transfer, Vol.9, 1.

[103] Mahato, J. P. and Maiti, M. K. (1988) Indian J. Technol., Vol.26, 255.