Chapter VII

A Simple Minded VSL Cosmology

7.1 Introduction

In recent years a large number of papers[1, 2, 3, 4, 5, 6] have appeared in the literature in which the possibility of variation of speed of light with respect to time (and often with respect to space) has been investigated in the context of cosmology. The idea of variable speed of light (VSL) has gained a considerable popularity since it has been argued that cosmological models which assume large increase in the speed of light in the early universe might solve some cosmological problems[2, 3, 4, 5, 7, 8, 9, 10, 11, 12] and therefore they can be considered as an alternative to inflationary models[13, 14, 15].

On the empirical side, from the analyses of absorption systems in the spectra of distant quasars, the cosmological evolution of the fine structure constant α seems to be a distinct possibility. The many multiplet technique of Webb et.al[16, 17] involves studying relativistic transitions to different ground states using absorption lines in the spectra of quasi stellar objects (QSO) at medium red-shifts. These absorption lines are obtained from heavy elements in distant gas cloud (absorbed system) along the sight-lines of background QSOs. These studies offer one of the earliest evidences that α might change with cosmological time[16, 17, 18]. The group continues to study the possible variation of α with improved precision[19, 20, 21] and the trend of results indicate that the value of α was lower in the past. Many authors tend to relate this varying α to a varying speed of light[1,2][1, 2, 3, 22, 23].

[1] Indeed Joao Magueijo has clarified the meaning of a varying c by dispelling the myth that the constancy of the speed of light is a logical necessity. For a meaningful definition and discussion of variation of the dimensional constant c the reader is referred to the interesting and spirited article by the author[22].

[2] Another group (Srianand et.al)[24] who worked on UVES on the very large telescope in Chile however claimed a "null result". According to the team the relative variation of α $(\Delta\alpha/\alpha)$ must be less than 0.6 part per million. Murphy et.al[25] however revised the results of Srianand et.al

Coming back to the theory, several authors have found a connection between VSL and theories of quantum gravity. "Doubly special relativity" (DSR) seems to have emerged as a VSL effective model of quantum space-time with observational implications for ultra high energy cosmic rays (UHECR) paradox[22, 26, 27, 28]. In the last chapter we have presented a heuristic modification of the relativistic kinematics. It has been shown that the absence of Greisen-zatespin-Kuzim (GZK) limit[29] in the UHECR paradox can be resolved in terms of a non-preferred frame effect of the solar system for its motion with respect to the rest frame of the universe (the cosmic substratum)[30]. We have dubbed this novel theory as also a DSR one from a sense different from that of the currently known DSR theories. Interestingly we find that this neo-DSR (NDSR) theory can also lead to a VSL cosmology after proper interpretation. In the present chapter we will develop a VSL theory on the basis of the new kinematics proposed earlier. The purpose of this endeavour will however be limited to show that the theory can accommodate the Webb et.al's and other's result concerning the cosmological variation of the fine structure constant.

At present no attempt will be made to suggest a direction along which the gravitational field equation of GR is to be modified as a consequence of the deformed Lorentzian kinematics proposed. Neither we will attempt for now how the consequent VSL scenario would be able to solve the cosmological problems usually tackled by inflation. The present exercise will help clarify how a modification of the relativistic kinematics prompted by the UHECR paradox can go hand in hand with the idea of VSL. Indeed the later emerges naturally from the modified "second relativity postulate"[3] (MSRP) proposed in the earlier chapter in connection with

and demonstrated some "simple flaws" of their data analysis technique. We shall therefore assume for present that there has been a detectable evidence for the cosmological variation of α.

[3]By second relativity postulates (SRP) we mean Eq.(7.5) of the last chapter, which is an outcome of CVL postulate of Einstein in conjunction with the principle of relativity (see Sec.(6.2)

the resolution of the (apparent) violation of the GZK limit observed in the ultra high energy cosmic ray spectrum.

The basic idea is as follows. As already pointed out, some studies on distant quasars indicate deviations in the value of the fine structure constant from its laboratory value. Some authors tend to attribute this variation of dimensionless α parameter to the variation of the speed of light c as a function of red-shift of quasars. According to some popular interpretations, this red-shift dependence of the speed of light ($c = c(z)$) is viewed as the time dependence of the same, since the red-shift as is believed is caused by the expansion of the universe and hence through Hubble's law z measures the distance of a galaxy. Again since light takes finite time to reach us, increasing z implies going back more to the past. Webb et.al's results thus are interpreted to signify the variation of c with respect to time. This chain of arguments can be summarized by saying that light propagation was faster in the past. The present endeavour allows us to look it rather differently. The red-shift of a QSO (or to be precise, that of the absorber system) is directly connected with its recession speed and hence any variation (possible or observed) of the speed of light with red-shift can then be interpreted as a variation in the value for the same as a function of the recession speeds of the frames of reference (QSO's) in which the measurements of the fine structure constant are referred to. This is precisely in consonance with the deformed relativistic kinematics developed in the last chapter in connection with the UHECR paradox. Indeed it appears that a VSL kind of cosmology emerges naturally from the NDSR proposed earlier by us, which was successful in dealing with a problem appearing elsewhere in physics. In the next section we develop this basic idea with a brief recapitulation of the NDSR discussed in connection with the UHECR paradox. The penultimate section will be

of chapter VI for details)

devoted to discussing Webb et.al and other's observations vis-a-vis our proposals. In the last section we will summarize the whole idea where some final remarks will be made in favour of our proposal.

7.2 The New Kinematics

In chapter VI we developed in the context of the UHECR paradox, a novel kinematics based on the notion of the existence of a preferred frame, which is believed to be the one at rest with the cosmic microwave background radiation. The solar system moves through this CMBR frame at a speed of approximately $10^{-3}c$. We have seen how the UHECR paradox can be explained in terms of a non-preferred frame effect of the solar system according to the kinematics proposed therein. In dealing with a preferred frame it has been found advantageous to start from the Dolphin transformations developed by Ghosal et.al[31]. The general form of which has been given by Eqs.(6.3) and (6.4) of the last chapter. We reproduce these equations by replacing a_0, a_{kx}, a_{ky} in it by c_0, c_{kx} and c_{ky} where c_0 is the isotropic speed of light in the preferred frame Σ_0 and c_{kx} and c_{ky} represent the two-way speeds of the same along the x and the y directions respectively.

$$x_k = (c_{kx}/c_{ky})(1 - u_{0k}^2/c_0^2)^{-1/2}(x_0 - u_{0k}t_0), \tag{7.1}$$

$$t_k = (c_0/c_{ky})(1 - u_{0k}^2/c_0^2)^{-1/2}(t_0 - u_{0k}x_0/c_0^2). \tag{7.2}$$

Recall that x_0, t_0 and x_k, t_k refer to space-time coordinates as measured with respect to the CMBR frame Σ_0 and any moving frame Σ_k respectively and u_{0k} denotes the relative velocity of Σ_k with respect to Σ_0. In the previous equations (Eqs.(6.3) and (6.4)) a_0 denotes the isotropic one-way or two-way "acoustic" signal speed whereas a_{kx} and a_{ky} are TWS' of the synchronizing signal in Σ_k parallel and perpendicular to its direction of motion respectively.

Note that up to this point no explicit assumptions regarding the behavior of the speed of light as a function of the speed u_{0k} of the reference frame Σ_k with respect to Σ_0 have been made. In the case of optical signal synchronization, one may thus write generally

$$c_{kx} = c_{kx}(u_{0k}), \tag{7.3}$$

$$c_{ky} = c_{ky}(u_{0k}). \tag{7.4}$$

Note that one recovers from Eqs.(7.1) and (7.2) the familiar relativistic transformations if one explicitly uses the CVL postulate:

$$c_{kx}(u_{0k}) = c_{ky}(u_{0k}) = c_0. \tag{7.5}$$

One may now ask "what happens if the CVL postulate is approximately correct?" We have already discussed that this CVL postulate may be minimally modified by preserving the isotropy ingredient of the relativity theory while assuming the TWS of light to vary only in a minute way. In particular we have assumed

$$c_{kx} = c_{ky} \approx c_0[1 + \eta(u_{0k})], \tag{7.6}$$

where $\eta(u_{0k})$ is assumed to take *very small* values so that the assumption does not contradict SR in the tested domain. The last equation represents our MSRP as explained earlier. The specific form for η has been assumed to be

$$\eta = \alpha u_{0k}^2 / 2c_0^2. \tag{7.7}$$

We have shown that an assumption of a very small value $\eta \approx 1.71 \times 10^{-22}$ with respect to the solar system ($u_{0k} = 10^{-3}c_0$) can resolve the Fly Eye's detection of 3.2×10^{20} eV particle which apparently crosses the GZK limit. What is crucial here is that the dependence of the speed of light on the speed of the reference frame apparently finds sanction in the context of the resolution of the UHECR paradox.

7.3 MSRP, Quasar Absorption Spectra and VSL

In order to accommodate Webb et.al's results (following their spectral studies of distant quasars) of possible time variation of fine structure constant in the light of VSL, we propose to recast Eq.(7.6) in a new form:

$$c(u_{0k}) = c_0[1 + (u_{0k}/\xi)^{2\mu}]^{1/2}, \tag{7.8}$$

where ξ is a new velocity parameter and μ is a dimensionless constant of the theory. Note that the existence of two velocity parameters (c_0 and ξ) in the theory (instead of one in the usual relativity theory) allows us to call it a DSR theory of a new variant (dubbed as NDSR in the previous section).

We now introduce two dimensionless parameters

$$u_k = u_{0k}/c_0, \tag{7.9}$$

and

$$w = \xi/c_0, \tag{7.10}$$

and rewrite Eq.(7.8) as

$$c(u_k) = c_0[1 + (u_k/w)^{2\mu}]^{1/2}. \tag{7.11}$$

Note that according to Eq.(7.6) of the previous chapter $\mu = 1$, but for now we do not hand put any value for it. In what follows we will be interested in relating observations in three reference frames— the preferred frame Σ_0, the frame of the solar system Σ_s, and that of the distant gas clouds (along the line of sight of background quasars) Σ_q and accordingly we use subscripts "0", "s" and "q" for relevant quantities in these frames respectively. For example, we may write appropriately in Σ_s and Σ_q as

$$c_s = c_0[1 + (u_s/w)^{2\mu}]^{1/2}, \tag{7.12}$$

and

$$c_q = c_0[1 + (u_q/w)^{2\mu}]^{1/2}. \tag{7.13}$$

One may now connect, in a simple minded way, the red-shift of distant quasars with theirs recession speeds using relativistic formula for Doppler effect. If z denotes the red-shift parameter, one can write in a straight forward way for the recession speed u_k of a quasar (or absorber system):

$$u_k = c_0[((1+z)^2 - 1)/((1+z)^2 + 1)]. \tag{7.14}$$

Note that the red-shift parameter is defined as

$$z = -\Delta\nu/\nu_0, \tag{7.15}$$

where $\Delta\nu$ denotes the frequency shift while ν_0 represents the proper frequency of the source. In arriving at Eq.(7.14) we have made use of the standard relativistic formula,

$$\nu = \nu_0[(1 - u_k/c_0)/(1 + u_k/c_0)]^{1/2}, \tag{7.16}$$

where ν denotes the observed frequency.

We now wish to write Eq.(7.11) in a different form so that it can directly relate $\Delta c/c_0$ with the speed of concerned reference frame. Writing $c - c_0 = \Delta c$ one thus obtains

$$\Delta c/c_0 \approx \frac{1}{2}(u_k/w)^{2\mu}. \tag{7.17}$$

In writing the above approximate form we have made use of the fact that $\Delta c/c_0 << 1$ in relevant situations. Referring to the two frames Σ_s and Σ_q we can write the above equation as

$$(\Delta c/c_0)_s = \frac{1}{2}(u_s/w)^{2\mu}, \tag{7.18}$$

and

$$(\Delta c/c_0)_q = \frac{1}{2}(u_q/w)^{2\mu}, \tag{7.19}$$

which are nothing but the approximate forms of Eqs.(7.12) and (7.13). Given that the recent measurements and claims for variation of fine structure constants from the studies of spectra of distant quasars can be modeled by a scenario in which only the speed of light varies, one may ask the following questions. Are these observed values for $\Delta\alpha/\alpha = -(\Delta c/c_0)_q$ consistent with the value for η $(= (\Delta c/c_0)_s)$ used earlier in the context of our attempt to resolve the UHECR paradox? In other words we ask keeping Eqs.(7.18) and (7.19) in mind, what values of w and μ are consistent with $\Delta\alpha/\alpha$ values obtained from the recent quasar studies given the constraint $(\Delta c/c_0)_s \approx 1.71 \times 10^{-22}$ imposed by the UHECR data. To answer this question we proceed as follows. We mainly use the most widely quoted non-null results for $\Delta\alpha/\alpha$ obtained by a group and the corresponding red-shifts[16, 20, 21, 25]. For the latter we take the average value of the red-shift range provided by the authors. Below in Table (7.1) we quote some of the representative results reported by the group at different times. Up to the third row the entries correspond to measurements by essentially one group using the Keck/HIRES instrument. Fourth row however refers to the group who claims null result for variation of α following observation with UVES spectrograph on VLT[24]. The entries in row 5 correspond to results following a reanalysis of the data of Ref.[24].

Table 7.1: Values of parameters of the theory from the relative variation of α measurements.

Group	Red-shift range	z_{avg}	$\Delta\alpha/\alpha = -\Delta c/c$	u_q	w	2μ
Murphy et.al (2001a) (Keck/HIRES)	0.5-3.5	2	-0.72×10^{-5}	0.800	5.60	5.726
Murphy et.al (2003) (Keck/HIRES)	0.2-3.7	1.95	-0.543×10^{-5}	0.793	5.91	5.96
Murphy et.al (2004) (Keck/HIRES)	0.2-4.2	2.2	-0.573×10^{-5}	0.822	6.11	5.67
Srianand et.al (2004) (VLT/UVES)	0.4-2.3	1.55	-0.06×10^{-5}	0.733	9.06	5.42
Murphy et.al (2007) (VLT/UVES)	0.4-2.3	1.35	-0.64×10^{-5}	0.693	4.78	5.83

The table is self explanatory. The entries in column 1 refer to the names of the groups and their instruments (telescope and the spectrometer systems). In column 3 we give the average red-shift z_{avg} from the given red-shift range (column 2) provided by the groups. The recession speeds u_q have been calculated using the formula given in Eq.(7.14). These have been provided in column 4. The quoted central values $\Delta\alpha/\alpha$ are entered in column 5. From the entries w and 2μ have been calculated using Eqs.(7.18) and (19), and are displayed in column 6 and 7 respectively.

Although it would have been appropriate to use a particular value of $\Delta\alpha/\alpha$ for a given absorber with its corresponding red-shift value and use the data to obtain w and μ , we have opted for the average or mean values of z and $\Delta\alpha/\alpha$ for the fact that since the error margins for individual results are quite high, there is no point in giving any extra credence to a particular obtained value for $\Delta\alpha/\alpha$ against the corresponding red-shift from a given set of observations. However, although the correctness of the theory should demand a unique set of values for w and μ, variations in the values of these parameters corresponding to different set of observations have been made explicit. These variations only reflect the fact that outcome of the measurements of fine structure constants from quasar absorption spectra has not yet reached the desired confidence level.

Assuming any particular set of values for w and μ one may study the nature of variation of the speed of light $\Delta c/c_0 (= \Delta\alpha/\alpha)$ as a function of red-shift by using Eq.(7.14). In Fig.(7.1a) we display the theoretical curves for variations of α against red-shift (for $w = 5.6, 2\mu = 5.726$). Fig.(7.1b) and (7.1c) provides the same for other two values of the paired parameters w and μ (corresponding to rows 2 and 3). No graph has however been drawn for the data entered in row 4 since the claimed constraints therein for $|\Delta\alpha/\alpha|$ is one order less than those of the other entries. Indeed Murphy et.al revised Srianand et.al's null result derived from VLT/UVES quasar absorption spectra and after correcting the "flawed analysis" of the latter, concluded that the same data gives a weighted mean value $\Delta\alpha/\alpha = (-0.640 \pm 0.360) \times 10^{-5}$. The $\Delta\alpha/\alpha - z$ variation following this revised analysis is shown in Fig.(7.1d). The corresponding variation of c have been displayed in Fig.(7.2a-d). (For some data points see Fig.(7.3) which has been reproduced from Refs.[1, 22]).

Pending any rebuttal of Murphy et.al's claim one may assume that variation of fine structure constant with red-shift to be a reality. However if in the ultimate analysis the claim of Ref.[21] proves to be correct, the possibility of α-parameter variation (or variation of the speed of light) may still be a possibility as the authors' findings also allow for a slight relative variation of α (up to 0.6 per million). If however one would still like to believe that Srianand et.al analysis indeed puts stronger constraints (near null) on the α-variation (or c-variation), we can still

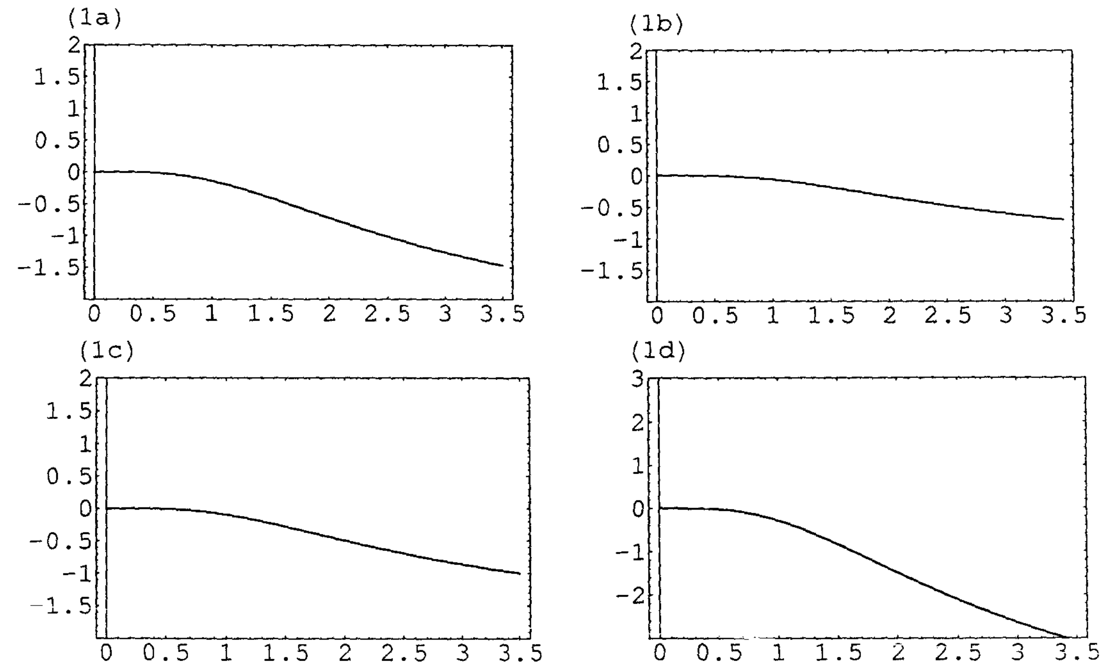

Figure 7.1: Theoretical curve showing the variation of $\Delta\alpha/\alpha \times 10^{-5}$ (vertical axes) with red-shift z (horizontal axes) in the simple minded VSL model for (a) $w = 5.60$ and $2\mu = 5.726$ (obtained from Murphy et.al 2001)[16] (b) $w = 5.91$ and $2\mu = 5.96$ (obtained from Murphy et.al 2003)[20] (c) $w = 6.11$ and $2\mu = 5.67$ (obtained from Murphy et.al 2004)[21] (d) $w = 4.78$ and $2\mu = 5.83$ (obtained from Murphy et.al 2007)[25].

develop our NDSR of the last chapter to accommodate Srianand et.al's claim. The importance of exploring this possibility lies in the fact that the current observational status regarding the α-variation in respect of accuracy, confidence level etc. allows us to doubt any claim or counter claim (with certainty) regarding the issue.

Recall that in the previous chapter we proposed a c-variation kinematics which enabled us to suggest a resolution of the UHECR paradox. The original suggestion required the c-variation in the following form:

$$c_k = c_0(1 + u_k^2/\xi^2)^{1/2}. \tag{7.20}$$

If we identify ξ with w of the present theory, we can see (by comparing Eq.(7.20) with Eq.(7.8)) that the above equation is equivalent to the assumption $\mu = 1$. We can also write the equation (with $\xi = w$) in a more aesthetic form:

$$c_k = c_0/(1 - u_k^2/w^2)^{1/2}, \tag{7.21}$$

since $u_k^2/w^2 << 1$ in the relevant situations.

Note the existence of *two* velocity parameters c_0 and w which allowed us to term the theory (in a lighter vein though) a doubly special relativistic one in a sense different from the current DSR theories. In the present chapter we have called it neo-DSR or NDSR. In the frame of reference of the absorber systems we replace the subscript k by q and write

$$c_q = c_0/(1 - u_q^2/w^2)^{1/2}, \tag{7.22}$$

which gives,

$$\Delta c/c_0 = (c_q - c_0)/c_0 = \frac{1}{2}(u_q/w)^2. \tag{7.23}$$

One thus is able to calculate $\Delta\alpha/\alpha = -\Delta c/c_0$ from the known value of w and u_q. The recession speed of the absorber system can be obtained from its red-shift value while w is already known from the cosmic ray spectrum data. The value of w has already been quoted as

$$w = 5.4 \times 10^7. \tag{7.24}$$

From this value of w and u_q from Srianand et.al's data (see Table (7.1)), which is $0.733c_0$, one obtains

$$\Delta\alpha/\alpha = -1.097 \times 10^{-16}, \tag{7.25}$$

which is surely consistent with Srianand et.al's findings.

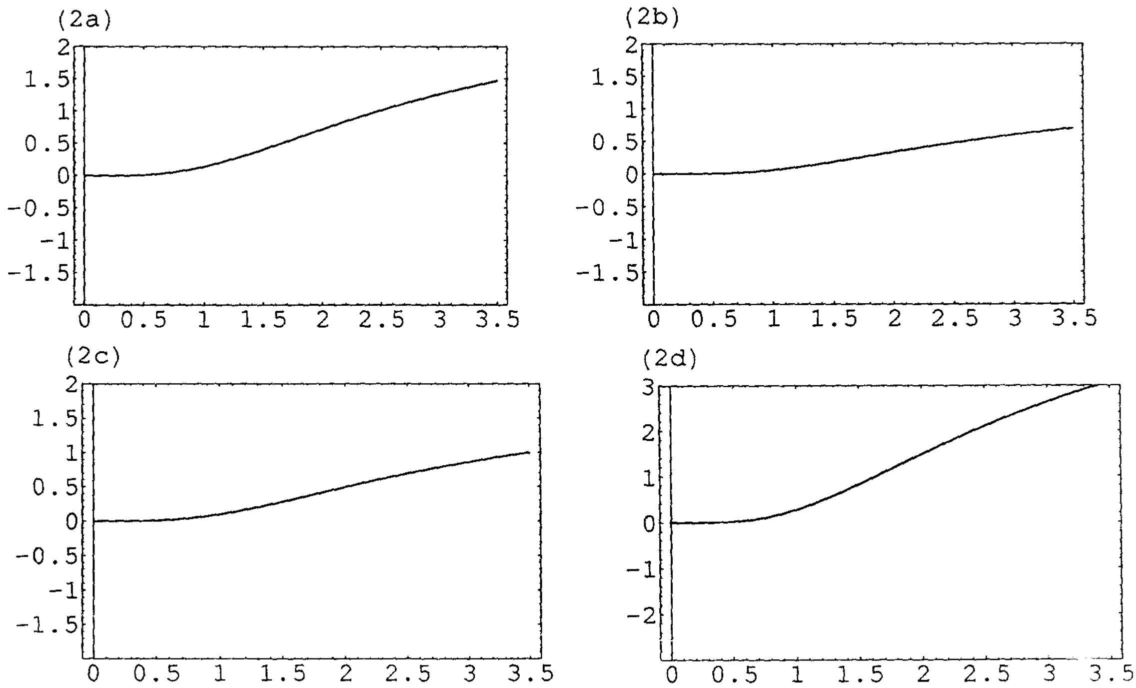

Figure 7.2: Theoretical curve showing the variation of $\Delta c/c_0 \times 10^{-5}$ (vertical axes) with red-shift z (horizontal axes) in the simple minded VSL model for (a) $w = 5.60$ and $2\mu = 5.726$ (obtained from Murphy et.al 2001)[16] (b) $w = 5.91$ and $2\mu = 5.96$ (obtained from Murphy et.al 2003)[20] (c) $w = 6.11$ and $2\mu = 5.67$ (obtained from Murphy et.al 2004)[21] (d) $w = 4.78$ and $2\mu = 5.83$ (obtained from Murphy et.al 2007)[25].

7.4 Summary and Conclusion

Let us now summarize our proposal for a VSL theory. The effort to resolve the UHECR paradox has guided us to assume a c-variation as a function of the velocity of the (inertial) frame of reference relative to the so called cosmological preferred (CMBR) frame, the specific form of function being

$$c_k = c_0[1 + (u_k/w)^{2\mu}]^{1/2}, \tag{7.26}$$

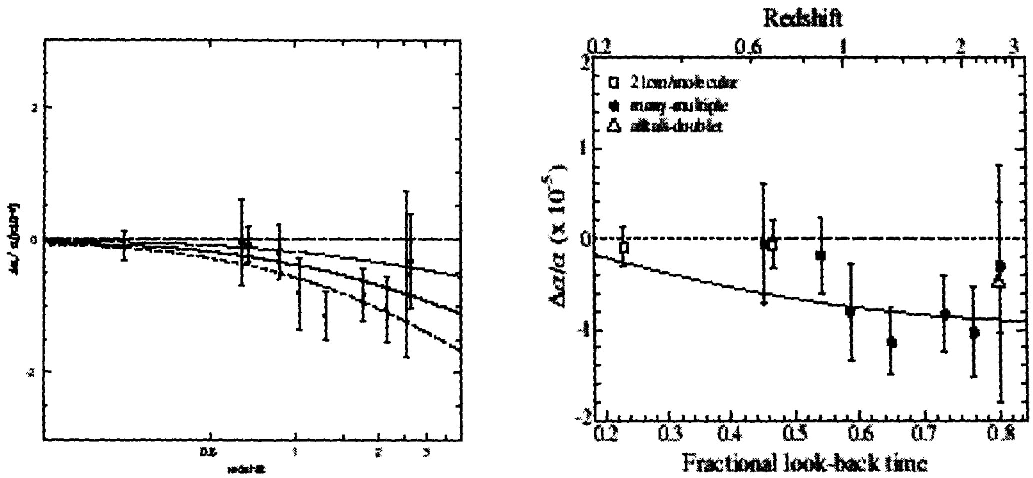

Figure 7.3: The data points are the absorption line in quasar (QSO) results from changing α. The solid line shows theoretical predictions in several varying α model[1,22].

where c_k is the speed of light measured with respect to Σ_k. As the recession velocities of distant galaxies can directly be linked to its age (look back time), one is free to interpret the proposed c-variation (as a function of recession speed) as the time variation of the speed of light. Clearly however the term 'VSL' in the present context has intrinsically a different connotation. This difference in connotation allows us to keep some basic principles of physics intact. For example, the existence of uniformly moving isolated particle is one of the fundamental notion of both classical mechanics and relativity theory; but the usual VSL theory requires that a photon or light pulse should accelerate on its own. In order to obtain a remedy of this unwanted feature Stepanov[32] has to assume a *spatial dependence* in addition to the temporal dependence of c. Clearly the present VSL is devoid of such malady. With respect to any inertial frame a light pulse will remain unaccelerated although its value differs in different inertial frames (with respect to cosmological rest frame its value is given by $c = c_0 = 3 \times 10^8$ m/s, the usual value for the "speed of light" in free space).

From the observational point of view, the idea of VSL seems to be supported by some recent observations of variation of fine structure constant from the study of absorption systems in the spectra of distant quasars, Paul Davies and collaborators[33] have suggested that in principle it is possible to disentangle which of the dimensional constants (the elementary charge e, Planks constant h, and the speed of light c of which the fine structure constant is composed) is responsible for the variation. They have argued that the black hole thermodynamics favours theories in which c decreases with time. Some authors however have disputed this claim[34, 35] not intending to delve into the controversy on the issue we have directly proposed the scenario in which we assume that only the speed of light varies. Originally this is done in the context of the UHECR paradox, but now we find that the proposed theory has the ability to account for the observed variations of fine structure constant. It is however too early to say a final word regarding the values of the parameters w and μ. The range of the values for these has been tabulated. With increased accuracy and more reliable data from quasar spectra in the near future will hopefully narrow down our choices. We do not wish to term this whole exercise of obtaining a theory capable of explaining the UHECR paradox and VSL as merely a phenomenological one. Although Einstein once apparently lamented that "... a physical theory can be satisfactory only when it builds up its structures from *elementary* foundation..."[36, 37] and his own SR failed on that count, the truth is – the apparent weak point of his "principle theory " (as opposed to "constructive theory") to our mind is indeed the strength of one most celebrated physical theories of science i.e relativity. One should not compare the situation with classical thermodynamics (a "principle theory") vis-a-vis the statistical mechanics (a "constructive theory"), in which case the macro (average property of a system) is governed by the properties of the micro structure of a system (working substance of a heat engine for example) and not the other way round. In case of relativity one obtains physics on the basis of the principle of relativity and the whole micro world seems to adjust itself to this principle only. The constructive accounts of relativistic effects (length contraction and time dilation for example) on the other hand still illusive.

What about the present theory? It definitely is not an example of a "constructive theory" which is built on some elementary foundations (a quantum gravity kind of explanation for the specific forms for the c-variation Eqs.(7.11) and (7.22) might enable us to say so). Is it a principle theory? Apparently no. But we would be inclined to say that it is much akin to a "principle theory" in its characteristics. We have already mentioned in the last chapter that one of the operative inputs of the principle of relativity being that the speed of light is the (i) same and (ii) isotropic with respect to all inertial observers.

What we have assumed so far amounts to saying that the principle of relativity holds only approximately which has led to our MSRP. The theory is still in the making. The time has not yet come to enable one to give a *principled* basis for a definite form of the equation

$$c = c(u_k), \tag{7.27}$$

for example, Eqs.(7.11), (7.22) or any other possible forms of MSRP, but that it is approximately a constant function has already given us a lot. However we have a long way to go. Surely there will be more questions prompted by our proposal than it has endeavoured to answer; nevertheless the reader can consider it just to be a humble beginning of a long voyage.

Acknowledgment: This work with coauthorship of S. K. Ghosal and D. Das is under publication.

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