CHAPTER - II

SOME ELASTODYNAMIC DIFFRACTION PROBLEMS INVOLVING GRIFFITH
IN ISOTROPIC ELASTIC MEDIUM

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DIFFRACTION OF SH-WAVES BY A GRIFFITH CRACK IN NONHOMOGENEOUS ELASTIC STRIP

1. INTRODUCTION

The natural or artificial materials are usually inhomogeneous; so in recent years great attention has been given to the study of diffraction of elastic waves by cracks or obstacles in inhomogeneous media in view of their application in fracture mechanics. Many problems have been solved involving one or more cracks in an infinite homogeneous elastic medium. Loeber and Sih (1960) and Mal (1970b) have studied the problem of diffraction of elastic waves by a Griffith crack in an infinite medium. The problem of finite crack at the interface of two elastic half-spaces has been discussed by Srivastava et al. (1980a) and Bostrom (1987). Singh et al. (1977, 1980) considered the problem of scattering of a SH-wave by cracks or strips in a nonhomogeneous infinite elastic medium. Papers involving cracks located in an infinitely long elastic strip are very few. The problem of an infinite elastic strip containing an arbitrary number of unequal Griffith cracks, located parallel to its surfaces and opened by an arbitrary internal pressure, has been treated by Adams (1980). Finite crack perpendicular to the surface of the infinitely long elastic strip has been studied by Chen (1978) (for an impact load) and by Srivastava et al. (1981) (for normally incident waves).

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Recently Shindo et al. (1986) considered the problem of impact response of a finite crack in an orthotropic strip.

In our paper, the diffraction of normally incident SH-waves by a Griffith crack situated in an infinitely long inhomogeneous elastic strip has been discussed. The shear modulus (μ) and the density (ρ) of the material have been assumed to vary in the vertical direction. Applying the Fourier transform, the mixed boundary value problem has been converted to the solution of dual integral equations. The dual integral equations have been finally reduced to a Fredholm integral equation of second kind by applying the Abel transform. Expressions for the stress intensity factor and crack opening displacement have been derived. The numerical values of stress intensity factor and crack opening displacement have been depicted by means of graphs to show the effect of material inhomogeneity.

2. FORMULATION OF THE PROBLEM

Consider the problem of diffraction of SH-waves by a Griffith crack in an inhomogeneous elastic strip of width $2h_1$. The crack is located in the region $-a \leq x_1 \leq a$, $-\infty < y_1 < \infty$, $z_1 = 0$ (fig.1). Normalizing all the lengths with respect to a and putting $x_1/a = x$, $y_1/a = y$, $z_1/a = z$, $h_1/a = h$ it is found that the location of the crack is $-1 \leq x \leq 1$, $-\infty < y < \infty$, $z = 0$ referred to a cartesian co-ordinate system (x, y, z). Let a plane harmonic SH-wave originating at $z = -\infty$ impinge on the crack normally to the x-axis. The variation of the shear

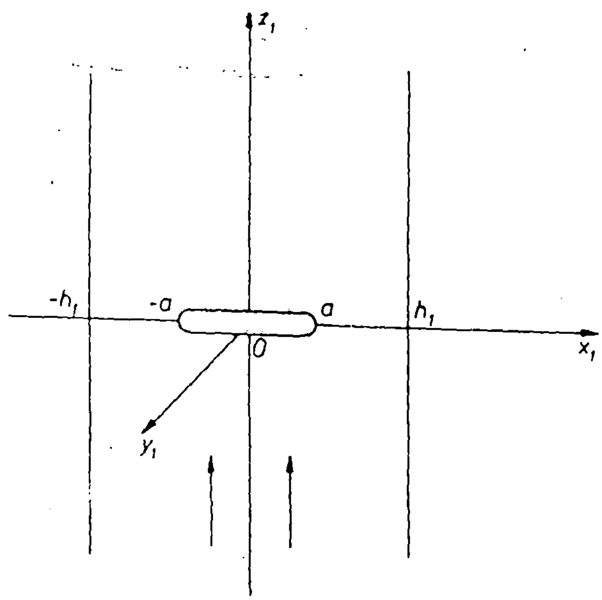

FIG. 1. Crack in the inhomogeneous strip.

modulus μ and the density ρ is taken in the vertical (z) direction in such a manner that the shear velocity $(\mu_o/\rho_o)^{1/2}$ is constant. The only non-vanishing y-component of the displacement which is independent of y is $v = v(x,z,t)$.

The equation of motion is given by

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) = \rho\frac{\partial^2 v}{\partial t^2} \tag{2.1}$$

If we consider $v(x,z,t)$ in the form

$$v(x,z,t) = \frac{W(x,z,t)}{\sqrt{\mu(z)}} \tag{2.2}$$

then (2.1) becomes

$$\mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2} \right) + \frac{1}{2}\left[\frac{1}{2\mu}\left(\frac{\partial\mu}{\partial z}\right)^2 - \frac{\partial^2\mu}{\partial z^2} \right]W = \rho\,\frac{\partial^2 W}{\partial t^2} \tag{2.3}$$

Putting $W(x,z,t) = F(x)G(z)e^{-i\omega t}$ and $\mu(z) = \mu_o f(z)$, $\rho(z) = \rho_o f(z)$ in equation (2.3) where μ_o, ρ_o are constants, such that $(\mu_o/\rho_o)^{1/2} = c_2$ is the shear wave velocity, it is found that $F(x)$ and $G(z)$ satisfy the following equations

$$\frac{\partial^2 F}{\partial x^2} + n^2 F = 0 \tag{2.4}$$

$$\frac{\partial^2 G}{\partial z^2} + \left(\frac{a^2\omega^2}{c_2^2} - b^2 - n^2\right)G = 0 \tag{2.5}$$

provided $f(z)$ is of the form

$$-\frac{1}{4}\left(\frac{\partial f}{\partial z}/f\right)^2 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial z^2}/f\right) = b^2 \tag{2.6}$$

where n and b are contants.

Let us assume $f(z)$ in the form

$$f(z) = \cosh^2(bz) \tag{2.7}$$

so that equation (2.6) is automatically satisfied.

Now the shear modulus $\mu(z)$ and density of the medium $\rho(z)$ are

$$\mu = \mu_o \cosh^2(bz) , \qquad \rho = \rho_o \cosh^2(bz) \tag{2.8}$$

Using equations (2.8), (2.2) and $W(x,z,t) = W(x,z)e^{-i\omega t}$, equation (2.1) takes the form

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2} + k^2 W = 0 \tag{2.9}$$

where

$$k^2 = (k_z^2 - b^2), \qquad k_z = \frac{a\omega}{c_2} .$$

The displacement component $v^{(i)}(x,z,t)$ and stress $\tau^{(i)}(x,z,t)$ due to incident wave are given by

$$v^{(i)}(x,z,t) = \frac{A_o \; e^{i(kz-\omega t)}}{\sqrt{\mu_o}\; \cosh(bz)} \tag{2.10}$$

and

$$\tau^{(i)}_{yz}(x,z,t) = A_o\sqrt{\mu_o}\;[ik\cosh(bz) - b\sinh(bz)]e^{i(kz-\omega t)}, \tag{2.11}$$

where A_o is a constant.

Henceforth the time factor $e^{-i\omega t}$ will be suppressed in the sequel.

Solution of equation (2.9) is

$$W(x,z) = \int_0^\infty B_1(\xi)e^{-\beta z}\cos(\xi x)d\xi + \int_0^\infty C_1(\zeta)\cosh(\alpha x)\sin(\zeta z)d\zeta , \tag{2.12}$$

where $\alpha = (\zeta^2-k^2)^{1/2}$, $\zeta > k$, $\qquad \beta = (\xi^2-k^2)^{1/2}$, $\xi > k$,

$\qquad = -i(k^2-\zeta^2)^{1/2}$, $\zeta < k$, $\qquad = -i(k^2-\xi^2)^{1/2}$, $\xi < k$.

Now displacement $v(x,z)$ and stresses $\tau_{yz}(x,z)$, $\tau_{xy}(x,z)$ due to scattered field are

$$v(x,z) = \frac{1}{\cosh(bz)}\left[\int_0^\infty B(\xi)e^{-\beta z} \cos(\xi x)d\xi + \int_0^\infty C(\zeta)\cosh(\alpha x)\sin(\zeta z)d\zeta\right] \tag{2.13}$$

$$\tau_{yz}(x,z) = -\mu_0 b \sinh(bz)\left[\int_0^\infty B(\xi)e^{-\beta z} \cos(\xi x)d\xi + \right.$$

$$\left. + \int_0^\infty C(\zeta)\cosh(\alpha x)\sin(\zeta z)d\zeta \right] + \mu_0 \cosh(bz) \times$$

$$\times \left[-\int_0^\infty \beta B(\xi)e^{-\beta z} \cos(\xi x)d\xi + \int_0^\infty \zeta C(\zeta)\cosh(\alpha x)\cos(\zeta z)d\zeta\right] \tag{2.14}$$

$$\tau_{xy}(x,z) = \mu_0 \cosh(bz)\left[-\int_0^\infty \xi B(\xi)e^{-\beta z} \sin(\xi x)d\xi + \right.$$

$$\left. + \int_0^\infty \alpha C(\zeta)\sinh(\alpha x)\sin(\zeta z)d\zeta \right] \tag{2.15}$$

where

$$B(\xi) = \frac{1}{\sqrt{\mu_0}} B_1(\xi) , \qquad C(\zeta) = \frac{1}{\sqrt{\mu_0}} C_1(\zeta).$$

The boundary conditions are

$$\tau_{yz}(x,0) = -\tau_0 , \qquad |x|\le 1 , \tag{2.16}$$

$$v(x,0) = 0 , \qquad 1\le|x|\le h , \tag{2.17}$$

$$\tau_{xy}(\pm h,z) = 0 , \qquad |z|<\infty , \tag{2.18}$$

where $\tau_0 = ikA_0\sqrt{\mu_0}$.

From the boundary condition (2.18) $C(\zeta)$ is found to be expressible in terms of $B(\xi)$ as follows :

$$C(\zeta) = \frac{2\zeta}{\pi\alpha\sinh(\alpha h)}\int_0^\infty \frac{\xi B(\xi)\sin(\xi h)}{\xi^2+\alpha^2} d\xi \tag{2.19}$$

Next, the use of equation (2.19) in the boundary condition (2.16) and (2.17) yields the following dual integral equations from which the unknown function $B(\xi)$ is to be determined :

$$\int_0^\infty \xi[1+M(\xi)]B(\xi)\cos(\xi x)d\xi = p(x) , \qquad |x|\le 1 \tag{2.20}$$

and

$$\int_0^\infty B(\xi)\cos(\xi x)d\xi = 0 , \qquad 1\le|x|\le h \tag{2.21}$$

where

$$M(\xi) = \left[\frac{\beta}{\xi} - 1 \right], \tag{2.22}$$

$$p(x) = \frac{\tau_o}{\mu_o} + \frac{2}{\pi}\int_0^\infty \frac{\zeta^2\cosh(\alpha x)}{\alpha\sinh(\alpha h)}\, d\zeta \int_0^\infty \frac{\xi B(\xi)\sin(\xi h)}{\xi^2+\alpha^2}\, d\xi \tag{2.23}$$

3. METHOD OF SOLUTION

In order to solve the dual integral equations (2.20) and (2.21), $B(\xi)$ is taken in the form

$$B(\xi) = \frac{\tau_o}{\mu_o}\int_0^1 t\phi(t)J_o(\xi t)dt , \tag{3.1}$$

so that equation (2.21) is automatically satisfied.

Substitution of the value of $B(\xi)$ from equation (3.1) in equation (2.20), yields a Fredholm integral equation of second kind

$$\phi(t) + \int_0^1 u\big[L_1(u,t) + L_2(u,t)\big]\phi(u)\, du = 1 , \tag{3.2}$$

where

$$L_1(u,t) = \int_0^\infty \xi M(\xi)J_o(\xi u)J_o(\xi t)\, d\xi , \tag{3.3}$$

$$L_2(u,t) = -\int_0^\infty \frac{\zeta^2 I_0(\alpha t) I_0(\alpha u) e^{-\alpha h}}{\alpha \sinh(\alpha h)} d\zeta . \qquad (3.4)$$

Using contour integration technique (Srivastava et al., 1980a) the infinite integral arising in the kernel $L_1(u,t)$ can be converted to a finite integral and is given by

$$L_1(u,t) = -ik^2\int_0^1 (1-\eta^2)^{1/2} J_0(k\eta t) H_0^{(1)}(k\eta u)\, d\eta, \quad u>t,$$

$$= -ik^2\int_0^1 (1-\eta^2)^{1/2} J_0(k\eta u) H_0^{(1)}(k\eta t)\, d\eta, \quad u<t \qquad (3.5)$$

Now

$$L_2(u,t) = \int_0^k \frac{\zeta^2 J_0(\alpha_1 t) J_0(\alpha_1 u)\; e^{i\alpha_1 h}}{\alpha_1 \sin(\alpha_1 h)} d\zeta - \int_k^\infty \frac{\zeta^2 I_0(\alpha t) I_0(\alpha u) e^{-\alpha h}}{\alpha \sinh(\alpha h)} d\zeta$$

$$= \int_0^k \frac{\zeta^2}{\alpha_1} J_0(\alpha_1 t) J_0(\alpha_1 u) \cot(\alpha_1 h) d\zeta + i\int_0^k \frac{\zeta^2}{\alpha_1} J_0(\alpha_1 t) J_0(\alpha_1 u) d\zeta$$

$$- \int_k^\infty \frac{\zeta^2 I_0(\alpha t) I_0(\alpha u) e^{-\alpha h}}{\alpha \sinh(\alpha h)} d\zeta ,$$

where $\alpha_1 = (k^2-\zeta^2)^{1/2}$.

Putting $\zeta^2 = k^2(1-y^2)$ in the first and second integrals and $\zeta^2 = k^2(1+y^2)$ in the third integral, it is found that

$$L_2(u,t) = k^2\Big[\int_0^1 (1-y^2)^{1/2} J_0(kyt) J_0(kyu) \cot(kyh) dy +$$

$$+ i\int_0^1 (1-y^2)^{1/2} J_0(kyt) J_0(kyu)\,dy -$$

$$- \int_0^\infty (1+y^2)^{1/2} I_0(kyt) I_0(kyu) e^{-kyh} \operatorname{cosech}(kyh)\,dy \Big] \qquad (3.6)$$

4. STRESS INTENSITY FACTOR AND CRACK OPENING DISPLACEMENT

From equation (2.14) the stress τ_{yz} on the plane z=0 can be written as

$$\tau_{yz}(x,0) = \mu_0 \Big[-\int_0^\infty \beta B(\xi)\cos(\xi x)\,d\xi + \int_0^\infty \zeta C(\zeta)\cosh(\alpha x)\,d\zeta \Big]. \qquad (4.1)$$

Substituting the value of $C(\zeta)$ and $B(\xi)$ from equations (2.19) and (3.1), the expression for the stress can finally be presented as

$$\tau_{yz}(x,0) = \frac{\tau_0 x}{(x^2-1)^{1/2}} \phi(1) + O(1), \qquad |x|>1.$$

Defining the stress intensity factor N by

$$N = \lim_{x\to 1+} \left| \frac{(x-1)^{1/2} \tau_{yz}(x,0)}{\tau_0} \right| ,$$

we obtain

$$N = \frac{1}{\sqrt{2}} |\phi(1)|. \qquad (4.2)$$

Now the crack opening displacement $\Delta v(x,0) = v(x,0+) - v(x,0-)$ can be obtained from equation (2.13) as

$$\Delta v(x,0) = 2\int_0^\infty B(\xi)\cos(\xi x)\,d\xi , \qquad |x|\le 1 ,$$

which, on substitution of the value of $B(\xi)$ from equation (3.1), takes the form

$$\Delta v(x,0) = \frac{2\tau_0}{\mu_0}\int_x^1 \frac{t\phi(t)}{(t^2-x^2)^{1/2}}\,dt\,, \qquad |x|\le 1 \tag{4.3}$$

5. NUMERICAL RESULTS AND DISCUSSION

Using the method of Fox and Goodwin (1953), the Fredholm integral equation given by equation (3.2) has been solved numerically for different values of the material inhomogeneity parameters. In this method the integral in equation (3.2) has been represented at first by a quadrature formula involving the values of the desired function $\phi(t)$ at the pivotal points inside the specified range of integration, and then converted to a set of simultaneous linear algebraic equations; their solutions yield the first approximations to the required pivotal values of $\phi(t)$. Applying the difference-correction technique, the first approximations have been improved. After solving the integral equation (3.2) numerically, the stress intensity factor N and the crack opening displacement $\mu_0\Delta v(x,0)/\tau_0$ have been calculated numerically and plotted separately against the dimensional frequency $k_2 (0.5\le k_2\le 1)$ and dimensionless distance $x (0\le x\le 1)$, respectively, for different values of the material inhomogeneity parameter b and strip width 2h.

In fig.2, the effect of the width of the strip on the stress

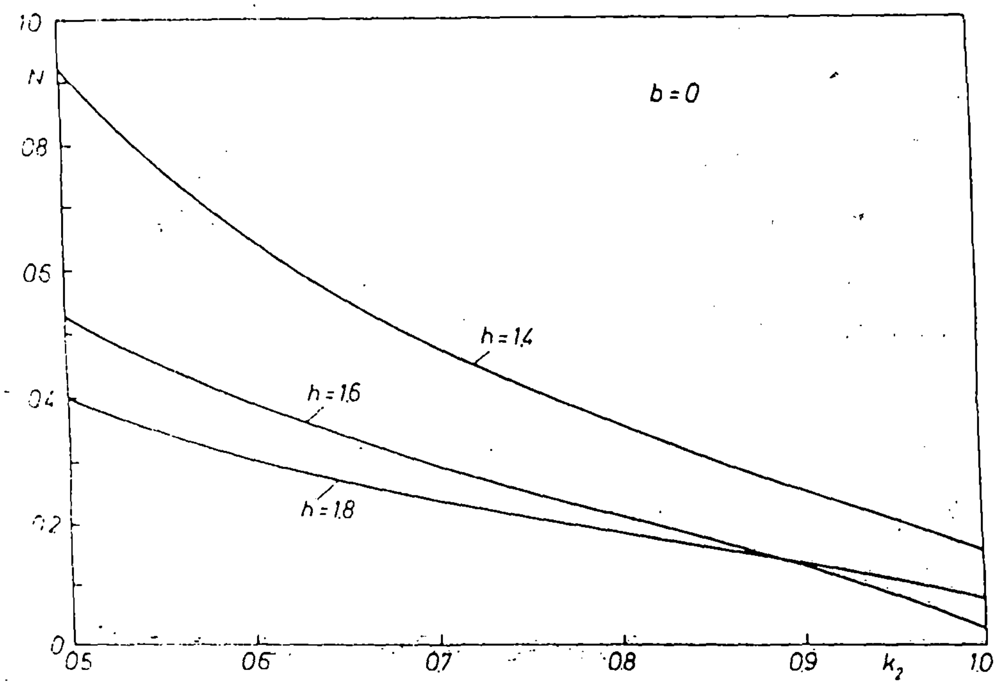

Fig. 2. Stress intensity factor N vs. dimensionless frequency k_2 for homogeneous medium ($b = 0$).

intensity factor for a homogeneous material has been shown; the effect of inhomogeneity of the material on the stress intensity factor for different widths of the strip has been depicted in figs.3-5.

It is found that in both the homogeneous and nonhomogeneous cases, the effect of the strip width decreases with the increase of the frequency, and the graphs of the stress intensity factor N become flat with the increase of strip width 2h. From fig.3 it is clear that the effect of inhomogeneity parameter b is prominent for low frequency k_2 and stress intensity factor is greater for higher values of the inhomogeneity parameter b.

In figs.4-8 the crack opening displacements against dimensionless distance x for different values of the material inhomogeneity parameter b and the strip width 2h have been illustrated by means of graphs. Case b=0 corresponds to the homogeneous case(fig.4). From figs.4-6 it is seen that for a fixed value of inhomogeneity parameter b, the crack opening displacement is greater for lower values of h when the frequencies are small, but the reverse effect is found for higher frequencies.

Next, in figs.7-8 we see that for a fixed value of h, the crack opening displacement is greater for higher values of the inhomogeneity parameter b when the frequencies are small, but for higher frequencies the effect is just reverse.

Finally it is found in all cases that the crack opening displacement reaches its maximum at about x=0, and then it gradually decreases and becomes zero at x=1.

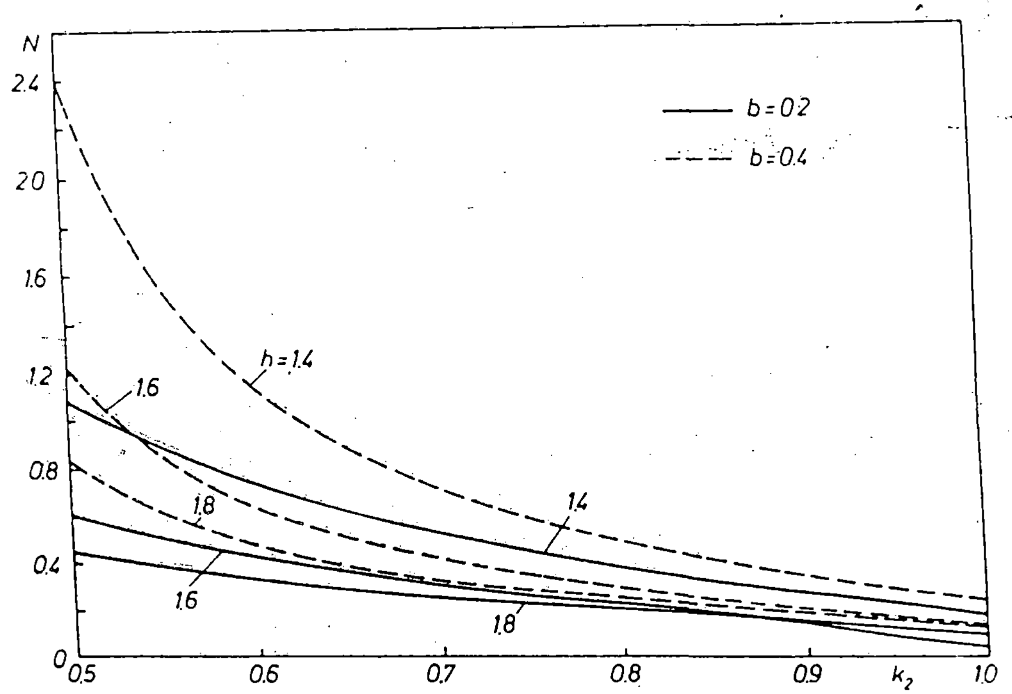

FIG. 3. Stress intensity factor N vs. dimensionless frequency k_2.

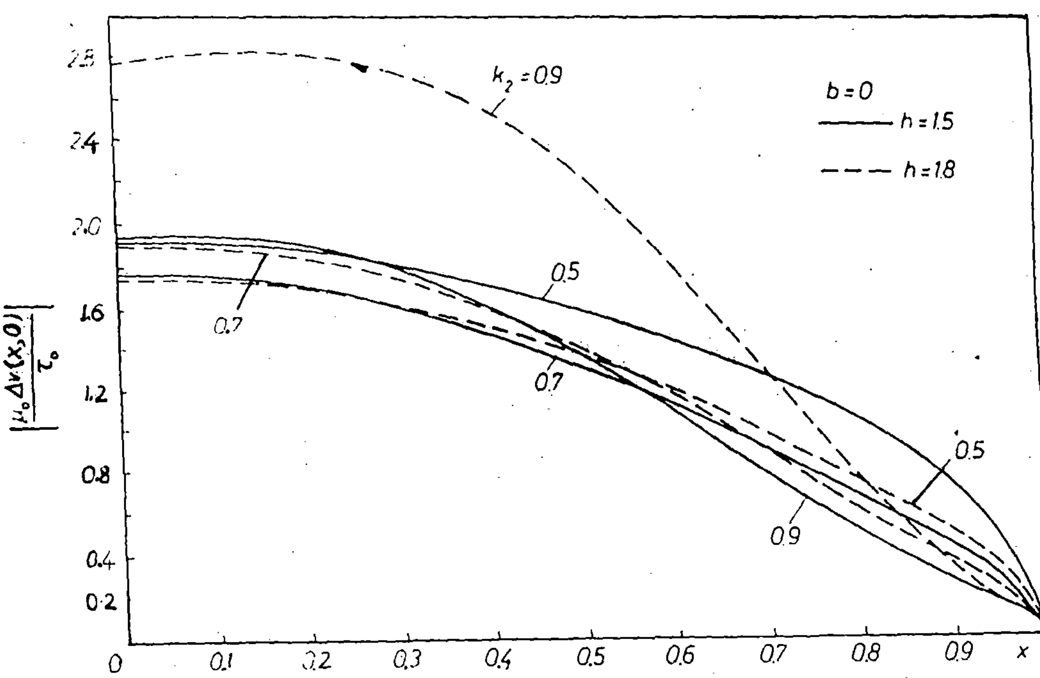

Fig. 4. Crack opening displacement vs. dimensionless distance $x (b = 0)$.

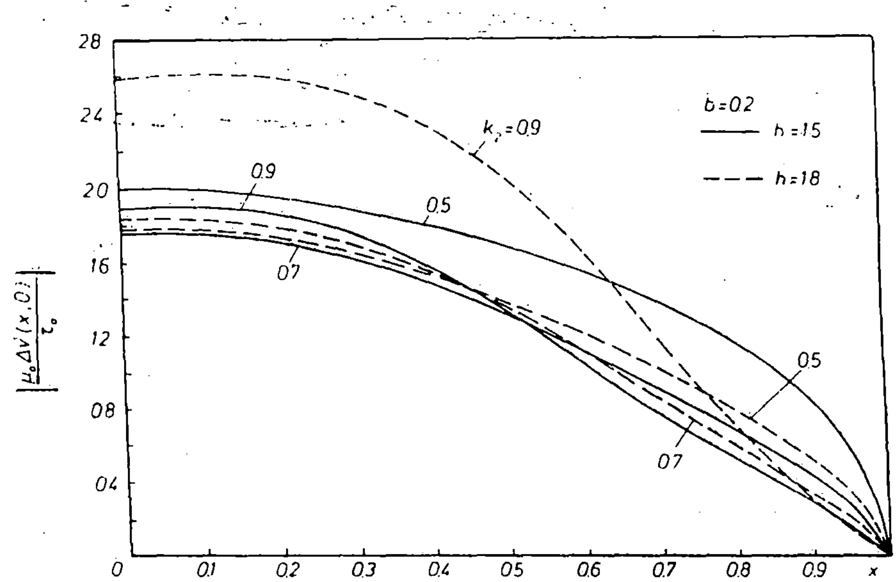

FIG 5. Crack opening displacement vs. dimensionless distance $x(b = 0.2)$.

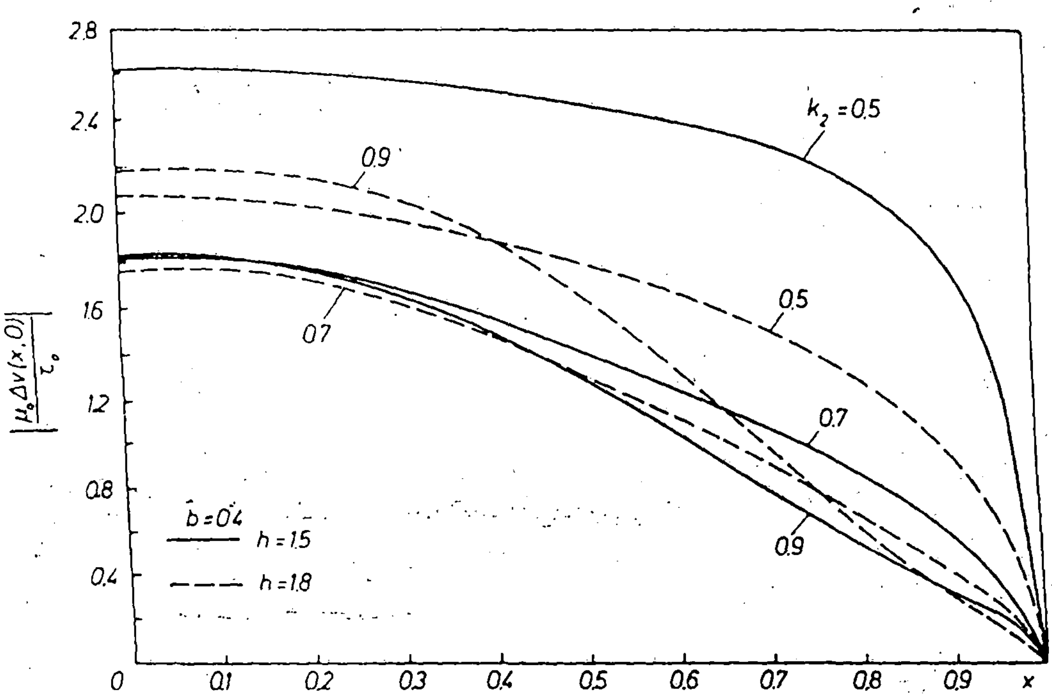

FIG. 6. Crack opening displacement vs. dimensionless distance $x(b = 0.4)$.

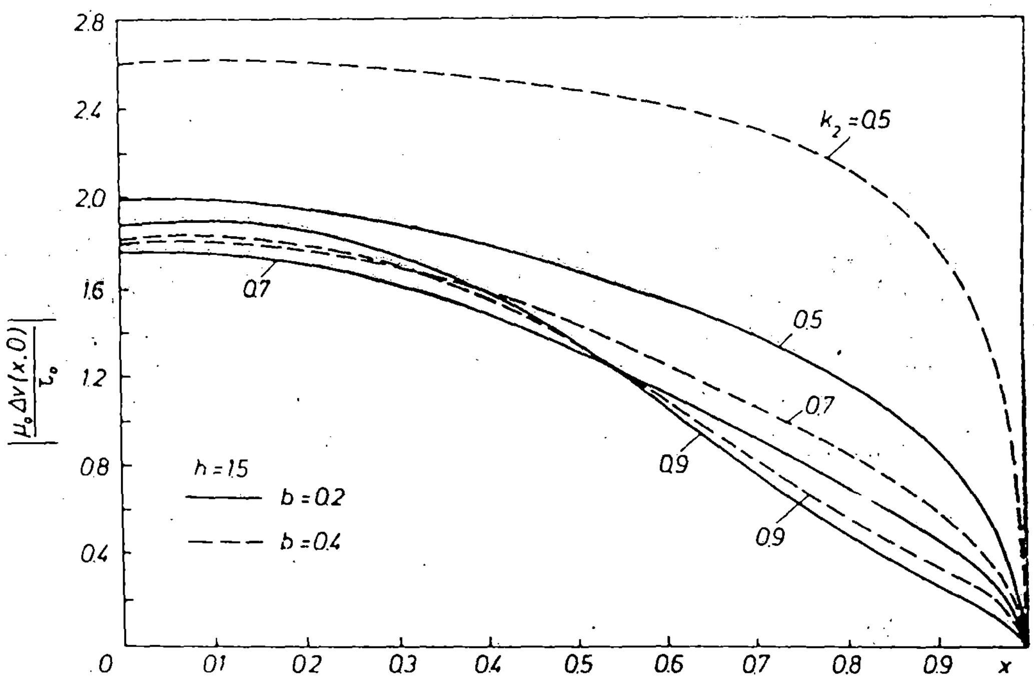

FIG. 7. Crack opening displacement vs. dimensionless distance x (h = 1.5).

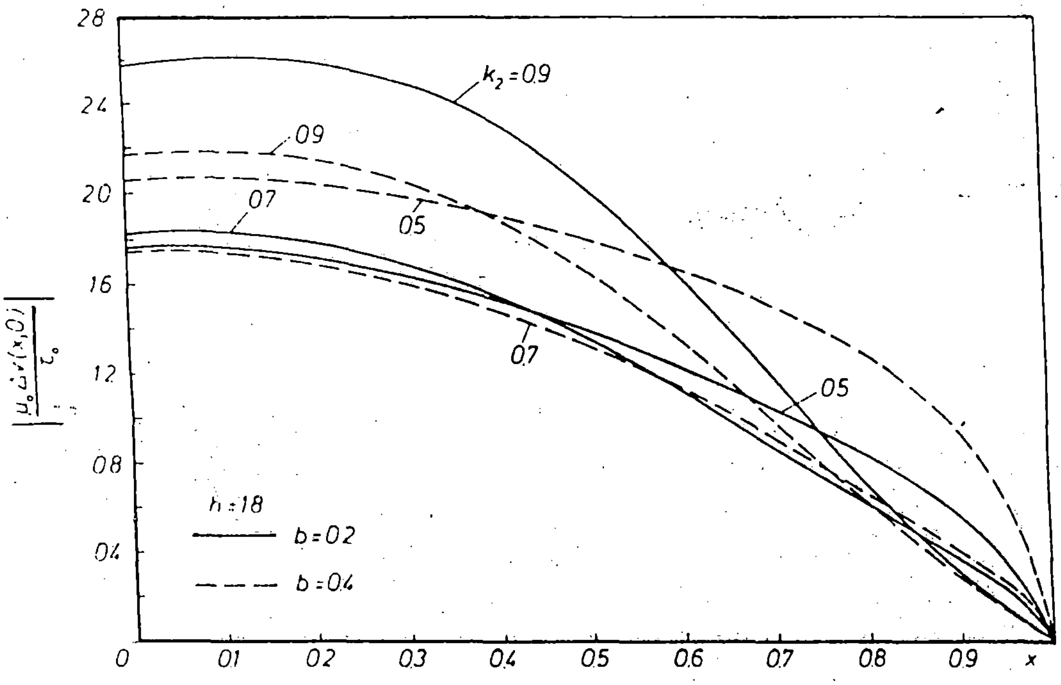

FIG. 8. Crack opening displacement vs. dimensionless distance x (h = 1.8).

INPLANE PROBLEM OF DIFFRACTION OF ELASTIC WAVES BY A PERIODIC ARRAY OF COPLANAR GRIFFITH CRACKS

1. INTRODUCTION

The problems involving cracks or inclusions in elastodynamics are of much importance in view of their application in geophysics and earthquake engineering. Uptil now many problems have been solved involving one or two cracks in an infinite homogeneous elastic medium. Loeber and Sih (1968) and Mal (1970b) have studied the problem of diffraction of elastic waves by a Griffith crack in an infinite medium. The problem of a finite crack at the interface of two elastic half spaces has been discussed by Srivastava et al (1980a) and Bostrom (1987). Finite crack perpendicular to the surface of the infinitely long elastic strip has been studied by Chen (1978) for impact load and by Srivastava et al. (1981) for normally incident waves. But elastodynamic problems involving two or more Griffith cracks have not yet received much attention. Jain and Kanwal (1972a) have studied the problem of scattering of elastic waves by two Griffith cracks for normally incident waves and the same problem has been considered by Itou (1980b) for impact load. Angel and Achenbach (1985) have studied the problem of reflection and transmission of elastic waves by a periodic array of cracks in an infinite isotropic medium. The problem of diffraction of

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SH-waves by a series of cuts in nonhomogeneous solid was investigated by De Sarkar (1983). The steady state vibration of an infinite isotropic medium with a periodic system of coplanar cracks has been discussed by Parton and Morozov (1978) using the method of the finite Fourier transforms to reduce the relevant mixed relations.

In our paper, the diffraction of normally incident time harmonic elastic waves by a periodic array of coplanar Griffith cracks in infinite elastic medium has been analyzed. Due to geometrical symmetry the problem has been reduced to the solution of the problem of a single crack in a strip whose boundaries are shear free and constrained in a way not to permit normal displacement. Applying Fourier transform the problem has been converted to the solution of dual integral equations. The dual integral equations finally have been reduced to a Fredholm integral equation of second kind by applying Abel's transform. Expressions for stress intensity factor and crack opening displacement have been derived in closed form. The numerical values of stress intensity factor and crack opening displacement have been presented graphically to bring out the salient features of the problem.

2. FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, linearly elastic, unbounded solid weakened by a infinite number of collinear cracks of equal length which are equally spaced on a line taken as the x_1-axis.

The length of each crack is 2a and the period of the crack-array is $2h_1$ as shown in fig.1. The cracks lie in the plane $x_2=0$ and extend to infinity in the x_3-direction which is perpendicular to the plane of the figure.

For convenience we make all the lengths dimensionless by writting

$$x_1/a=x, \quad x_2/a=y, \quad x_3/a=z, \quad h_1/a=h.$$

Let an incident time-harmonic body wave travel in the direction of the positive y-axis. The steady state term $e^{-i\omega t}$, which is common to all field variables, has been omitted in the sequel.

By simple symmetry considerations, the displacement and stress distribution due to the scattered field in the entire xy-plane can be derived by considering only the isotropic elastic strip $|x|\le h$ with a central crack $|x|\le 1$, $y=0$; the boundaries of the strip $x=\pm h$ being shear free and constrained in a way not to permit normal displacement.

The displacement components are

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial y}$$

and

$$v = \frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial x} \tag{1}$$

where ϕ and ψ are scalar and vector potentials satisfying the following equations :

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{a^2}{c_1^2}\frac{\partial^2\phi}{\partial t^2}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{a^2}{c_2^2}\frac{\partial^2\psi}{\partial t^2} \tag{2}$$

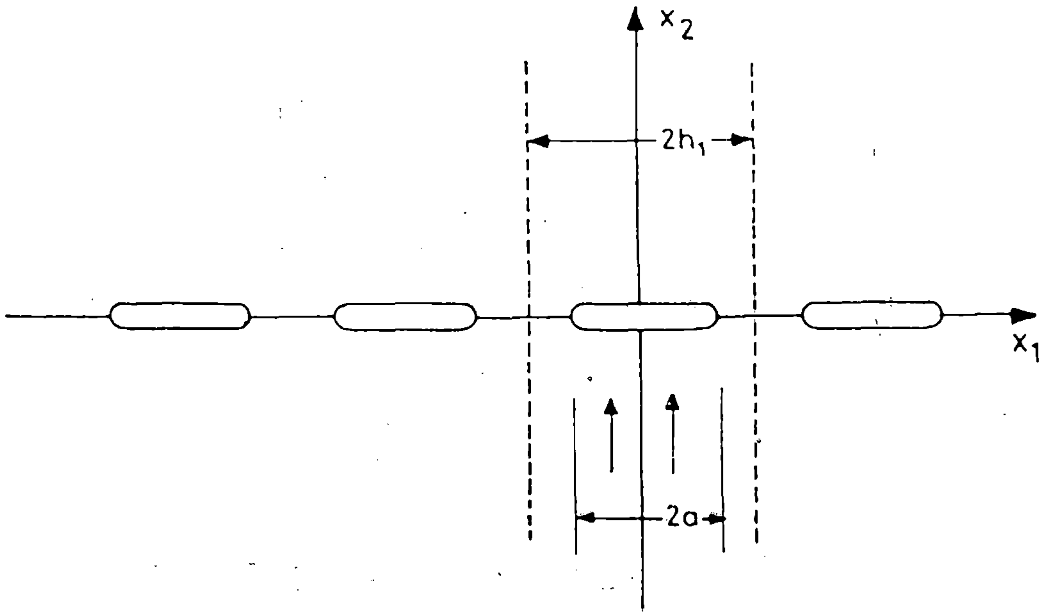

Fig. 1 Incidence of plane time - harmonic wave on a periodic array of cracks.

where $c_1 = \left(\frac{\lambda+2\mu}{\rho}\right)^{1/2}$ and $c_2 = \left(\frac{\mu}{\rho}\right)^{1/2}$ are the dilatational and shear wave velocities, λ, μ are the Lame's constant, ρ is the density of the material.

Therefore, substituting $\phi(x,y,t) = \phi(x,y)e^{-i\omega t}$ and $\psi(x,y,t) = \psi(x,y)e^{-i\omega t}$, our problem reduces to the solution of the equations

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + k_1^2\phi = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + k_2^2\psi = 0 \tag{3}$$

subject to the boundary conditions

$$\tau_{yy}(x,0) = -p(x) \ , \quad |x|<1 \tag{4}$$

$$\tau_{xy}(x,0) = 0 \ , \quad |x|\le h \tag{5}$$

$$v(x,0) = 0 \ , \quad 1\le|x|\le h \tag{6}$$

$$\tau_{xy}(\pm h,y) = 0 \ , \quad |y|<\infty \tag{7}$$

$$u(\pm h,y) = 0 \ , \quad |y|<\infty \tag{8}$$

where $k_i = a\omega/c_i$ $(i=1,2)$.

Solutions of the equations (3) are

$$\phi(x,y) = \sqrt{\frac{2}{\pi}}\left[\int_0^\infty A_1(\zeta)e^{-\alpha y}\cos\zeta x \, d\zeta + \int_0^\infty A_2(\xi)\cosh(\alpha_1 x)\cos\xi y \, d\xi\right]$$

and

$$\psi(x,y) = \sqrt{\frac{2}{\pi}}\left[\int_0^\infty B_1(\zeta)e^{-\beta y}\sin\zeta x \, d\zeta + \int_0^\infty B_2(\xi)\sinh(\beta_1 x)\sin\xi y \, d\xi\right] \tag{9}$$

where $A_1(\zeta)$, $A_2(\xi)$, $B_1(\zeta)$, $B_2(\xi)$ are constants and

$$\alpha = (\zeta^2 - k_1^2)^{1/2}, \quad \zeta > k_1 \qquad \beta = (\zeta^2 - k_2^2)^{1/2}, \quad \zeta > k_2$$

$$= -i(k_1^2 - \zeta^2)^{1/2}, \quad \zeta < k_1 \qquad = -i(k_2^2 - \zeta^2)^{1/2}, \quad \zeta < k_2$$

$$\alpha_1 = (\xi^2 - k_1^2)^{1/2}, \quad \xi > k_1 \qquad \beta_1 = (\xi^2 - k_2^2)^{1/2}, \quad \xi > k_2$$

$$= -i(k_1^2 - \xi^2)^{1/2}, \quad \xi < k_1 \qquad = -i(k_2^2 - \xi^2)^{1/2}, \quad \xi < k_2.$$

Now the stress τ_{xy} can be expressed as

$$a\tau_{xy}(x,y) = \sqrt{\frac{2}{\pi}}\left[-\mu\int_0^\infty \left\{-2\zeta\alpha A_1(\zeta)e^{-\alpha y} + (\zeta^2+\beta^2)B_1(\zeta)e^{-\beta y}\right\}\sin\zeta x \, d\zeta \; + \right.$$

$$\left. +\mu\int_0^\infty \left\{-2\xi\alpha_1 A_2(\xi)\sinh(\alpha_1 x) + (\xi^2+\beta_1^2)B_2(\xi)\sinh(\beta_1 x)\right\}\sin\xi y \, d\xi\right] \tag{10}$$

The boundary condition (5) yields

$$B_1(\zeta) = \frac{2\zeta\alpha}{\zeta^2+\beta^2} A_1(\zeta) \tag{11}$$

Assuming $\quad -\zeta A_1(\zeta) = A(\zeta), \quad \alpha_1 A_2(\xi) = C(\xi), \quad -\xi B_2(\xi) = D(\xi)$

and using the relation (11), expressions for displacements and stresses finally can be written as

$$u = \sqrt{\frac{2}{\pi}}\int_0^\infty \left[e^{-\alpha y} - \frac{2\alpha\beta}{2\zeta^2 - k_2^2} e^{-\beta y}\right] A(\zeta) \sin\zeta x \, d\zeta \; +$$

$$+ \sqrt{\frac{2}{\pi}}\int_0^\infty \left[C(\xi)\sinh(\alpha_1 x) + D(\xi)\sinh(\beta_1 x)\right]\cos\xi y \, d\xi \tag{12}$$

$$v = \sqrt{\frac{2}{\pi}}\int_0^\infty \left[e^{-\alpha y} - \frac{2\zeta^2}{2\zeta^2 - k_2^2} e^{-\beta y}\right]\alpha\zeta^{-1} A(\zeta) \cos\zeta x \, d\zeta \; -$$

$$- \sqrt{\frac{2}{\pi}} \int_0^\infty \left[\xi \alpha_1^{-1} C(\xi)\cosh(\alpha_1 x) + \beta_1 \xi^{-1} D(\xi)\cosh(\beta_1 x)\right] \sin\xi y \, d\xi \tag{13}$$

$$a\tau_{yy} = -\mu\sqrt{\frac{2}{\pi}} \int_0^\infty \left[(2\zeta^2 - k_2^2)e^{-\alpha y} - \frac{4\alpha\beta\zeta^2}{2\zeta^2 - k_2^2} e^{-\beta y}\right]\zeta^{-1} A(\zeta) \cos\zeta x \, d\zeta -$$

$$- \mu\sqrt{\frac{2}{\pi}}\int_0^\infty \left[\frac{(2\alpha_1^2 + k_2^2)}{\alpha_1} C(\xi)\cosh(\alpha_1 x) + 2\beta_1 D(\xi)\cosh(\beta_1 x)\right] \cos\xi y \, d\xi \tag{14}$$

$$a\tau_{xy} = -\mu\sqrt{\frac{2}{\pi}} \int_0^\infty \left[e^{-\alpha y} - e^{-\beta y} \right] 2\alpha A(\zeta) \sin\zeta x \, d\zeta -$$

$$- \mu\sqrt{\frac{2}{\pi}}\int_0^\infty \left[2\xi C(\xi)\sinh(\alpha_1 x) + \xi^{-1}(2\xi^2 - k_2^2) D(\xi)\sinh(\beta_1 x)\right] \sin\xi y \, d\xi \tag{15}$$

3. SOLUTION OF THE PROBLEM

The boundary conditions (4) and (6) yield the following two integral equations :

$$\int_0^\infty \frac{1}{\zeta}\left[1 + H(\zeta)\right] B(\zeta) \sin\zeta x \, d\zeta = R(x) \quad , \quad 0 \le |x| \le 1 \tag{16}$$

$$\int_0^\infty \frac{1}{\zeta} B(\zeta) \cos\zeta x \, d\zeta = 0 \quad , \quad 1 \le |x| \le h \tag{17}$$

where,

$$B(\zeta) = \frac{2\alpha(k_1^2 - k_2^2) A(\zeta)}{2\zeta^2 - k_2^2} \tag{18}$$

$$H(\zeta) = \frac{(2\zeta^2 - k_2^2)^2 - 4\alpha\beta\zeta^2}{2\alpha\zeta(k_1^2 - k_2^2)} - 1 \tag{19}$$

$$H(\zeta) \to 0 \quad \text{as} \quad \zeta \to \infty.$$

$$R(x) = \sqrt{\frac{2}{\pi}}\,\mu^{-1} a \int_0^x p(x)dx -$$

$$- \int_0^\infty \left[\frac{(2\alpha_1^2 + k_2^2)}{\alpha_1^2} C(\xi)\sinh(\alpha_1 x) + 2D(\xi)\sinh(\beta_1 x) \right] d\xi \tag{20}$$

Let us consider the solution of integral equations (16) and (17) in the form

$$B(\zeta) = \sqrt{\frac{\pi}{2}}\,\zeta \int_0^1 tf(t)J_0(\zeta t)dt \tag{21}$$

so that the integral equation (17) is automatically satiesfied. Now, substituting the value of $B(\zeta)$ from (21) in (16) and using Abel's transform we obtain the following Fredholm integral equation of second kind :

$$f(t) + \int_0^1 uf(u)L_1(t,u)\,du = Q(t) \tag{22}$$

where,

$$Q(t) = \frac{2a}{\mu\pi t}\frac{d}{dt}\int_0^t (t^2 - z^2)^{1/2} p(z)dz -$$

$$- \sqrt{\frac{2}{\pi}} \int_0^\infty \left[\alpha_1^{-1}(2\alpha_1^2 + k_2^2) I_0(\alpha_1 t)C(\xi) + 2\beta_1 I_0(\beta_1 t)D(\xi) \right] d\xi \tag{23}$$

and

$$L_1(t,u) = \int_0^\infty \zeta H(\zeta) J_o(\zeta u) J_o(\zeta t) d\zeta \tag{24}$$

From the boundary conditions (7) and (8), the unknown functions $C(\xi)$ and $D(\xi)$ can be found to be related to $B(\zeta)$ as :

$$C(\xi) = \frac{2}{\pi k_2^2 (k_1^2 - k_2^2) \sinh(\alpha_1 h)} \left[-\xi^2 \int_0^\infty g_1(\xi,\zeta) B(\zeta) d\zeta + \right.$$

$$\left. + \frac{(2\xi^2 - k_2^2)}{2} \int_0^\infty g_2(\xi,\zeta) B(\zeta) d\zeta \right] \tag{25}$$

$$D(\xi) = \frac{2}{\pi k_2^2 (k_1^2 - k_2^2) \sinh(\beta_1 h)} \left[\xi^2 \int_0^\infty g_1(\xi,\zeta) B(\zeta) d\zeta - \xi^2 \int_0^\infty g_2(\xi,\zeta) B(\zeta) d\zeta \right] \tag{26}$$

where,

$$g_1(\xi,\zeta) = \left\{ \frac{2\beta_1^2 + k_2^2}{\zeta^2 + \beta_1^2} - \frac{2\alpha_1^2 + k_2^2}{\zeta^2 + \alpha_1^2} \right\} \sin(\zeta h)$$

$$g_2(\xi,\zeta) = \left\{ \frac{2(\beta_1^2 + k_2^2)}{\zeta^2 + \beta_1^2} - \frac{2\alpha_1^2 + k_2^2}{\zeta^2 + \alpha_1^2} \right\} \sin(\zeta h) \tag{27}$$

Next, substituting the value of $B(\zeta)$ from (21) in the expressions of $C(\xi)$ and $D(\xi)$ given by (25) and (26) and using the result (Gradshteyn et al., 1965)

$$\int_0^\infty \frac{\zeta \sin(\zeta h) J_o(\zeta u)}{\zeta^2 + \alpha_1^2} d\zeta = \frac{\pi}{2} I_o(\alpha_1 u) e^{-\alpha_1 u}$$

$C(\xi)$ and $D(\xi)$ can be written in terms of $f(t)$ as

$$C(\xi) = \sqrt{\frac{\pi}{2}}\, \frac{1}{2(k_1^2-k_2^2)} \int_0^1 \left[(2\alpha_1^2+k_2^2)\, I_0(\alpha_1 u)\, e^{-\alpha_1 h} \right] \frac{uf(u)du}{\sinh(\alpha_1 h)}$$

$$D(\xi) = -\sqrt{\frac{\pi}{2}}\, \frac{\xi^2}{(k_1^2-k_2^2)} \int_0^1 \left[I_0(\beta_1 u)\, e^{-\beta_1 h} \right] \frac{uf(u)du}{\sinh(\beta_1 h)} \tag{28}$$

Using the above relations (28) in (23) we obtain

$$Q(t) = \frac{2a}{\mu\pi t}\frac{d}{dt}\int_0^t \sqrt{t^2-z^2}\; p(z)dz + \int_0^1 u\left[L_2(t,u)+L_3(t,u)\right] f(u)du \tag{29}$$

where,

$$L_2(t,u) = -\frac{1}{2(k_1^2-k_2^2)}\int_0^\infty \left[\alpha_1^{-1}(2\alpha_1^2+k_2^2)^2\, I_0(\alpha_1 t)\, I_0(\alpha_1 u)\, e^{-\alpha_1 h}\right] \frac{d\xi}{\sinh(\alpha_1 h)} \tag{30}$$

$$L_3(t,u) = \frac{2}{(k_1^2-k_2^2)}\int_0^\infty \left[\beta_1(\beta_1^2+k_2^2)\, I_0(\beta_1 t)\, I_0(\beta_1 u)\, e^{-\beta_1 h}\right] \frac{d\xi}{\sinh(\beta_1 h)} \tag{31}$$

Next substituting $Q(t)$ from (29) in (22) and assuming $p(x)=p_0$ and $f(t)= ap_0 g(t)/\mu$ we finally obtain the following Fredholm integral equation of second kind for the determination of $g(t)$:

$$g(t) + \int_0^1 ug(u)L(t,u)\; du = 1 \tag{32}$$

where $L(t,u) = L_1(t,u) - L_2(t,u) - L_3(t,u)$ (33)

and $L_1(t,u)$, $L_2(t,u)$ and $L_3(t,u)$ are given by (24),(30) and (31) respectively.

It is to be noted that the kernel $L_1(t,u)$ represented by the semi-infinite integral given by equation (24) has a slow rate of convergence. In order to make the numerical analysis easier, the semi-infinite integral has therefore been converted to finite integrals by using simple contour integration technique (Srivastava et al. 1980a) and is given by

$$L_1(t,u) = -\frac{ik_2^4}{2(k_2-k_1)}\left[\int_0^{\gamma}\frac{(2\eta^2-1)^2}{(\gamma^2-\eta^2)^{1/2}}J_0(k_2\eta u)H_0^{(1)}(k_2\eta t)d\eta\right. +$$

$$\left. + \int_0^1 4\eta^2(1-\eta^2)^{1/2}J_0(k_2\eta u)H_0^{(1)}(k_2\eta t)d\eta\right], \quad t>u \qquad (34)$$

where $\gamma=k_1/k_2$. The corresponding expression of $L_1(t,u)$ for $t<u$ can be obtained by interchanging t and u in (34).

4. STRESS INTENSITY FACTOR AND DISPLACEMENT

The normal stress $\tau_{yy}(x,y)$ in the plane y=0 in the vicinity of the crack tip can be found from equation (14) and is given by

$$\tau_{yy}(x,0) = -\mu\sqrt{\frac{2}{\pi}}\int_0^{\infty}B(\zeta)\cos\zeta x\, d\zeta + O(1), \qquad x>1$$

$$= -\frac{p_0 x}{\sqrt{x^2-1}}\,g(1) + O(1), \qquad x>1$$

Defining the stress intensity factor by

$$K = \underset{x \to 1+}{Lt} \left| \frac{\tau_{yy}(x,0)\sqrt{x-1}}{p_o} \right|$$

it is found that

$$K = \frac{|g(1)|}{\sqrt{2}} \tag{35}$$

Now the crack opening displacement $\Delta v(x,0) = v(x,0+) - v(x,0-)$ can be obtained from (13) as

$$\Delta v(x,0) = -\frac{k^2}{\sqrt{2\pi}(k_1^2-k_2^2)} \int_0^\infty \frac{1}{\zeta} B(\zeta)\cos(\zeta x)d\zeta \quad , \quad |x| \le 1$$

which on substitution of the value of $B(\zeta)$ from (21) takes the form

$$\Delta v(x,0) = \frac{ap_o}{\mu(1-\gamma^2)} \int_x^1 \frac{tg(t)\,dt}{(t^2-x^2)^{1/2}} \quad , \quad |x| \le 1 \tag{36}$$

5. NUMERICAL RESULTS AND DISCUSSION

Using the method of Fox and Goodwin (1953) the Fredholm integral equation given by equation (32) has been solved numerically for different values of dimensionless frequency k_2 and h, the separating distance of the cracks. At first the integral in (32) has been presented by a quadrature formula involving values of the desired function g(t) at pivotal points inside the specified range of integration and then converted to a set of linear algebraic simultaneous equations, solving which the first approximation to the required pivotal values of g(t) has been obtained. Applying difference-correction technique the first approximations has been improved. Standard numerical integration technique has been used to evaluate the kernals $L_1(t,u)$, $L_2(t,u)$ and $L_3(t,u)$ given by (34), (30) and (31). After solving the integral equation (32) numerically, the stress intensity factor K and the crack opening displacement $\mu\Delta v(x,0)/ap_0$ have been calculated numerically and plotted separately against dimensionless frequency k_2 $(0<k_2\leq1)$ and dimensionless distance $x(0\leq x\leq1)$ respectively for different values of h. The value of γ is taken to be $1/\sqrt{3}$. From fig.2 it is interesting to note that the number of oscillations in stress intensity factor K increases with the increase in the values of h. The crack opening displacement(fig.3) is greater for higher values of h and also for higher values of dimensionless frequency k_2.

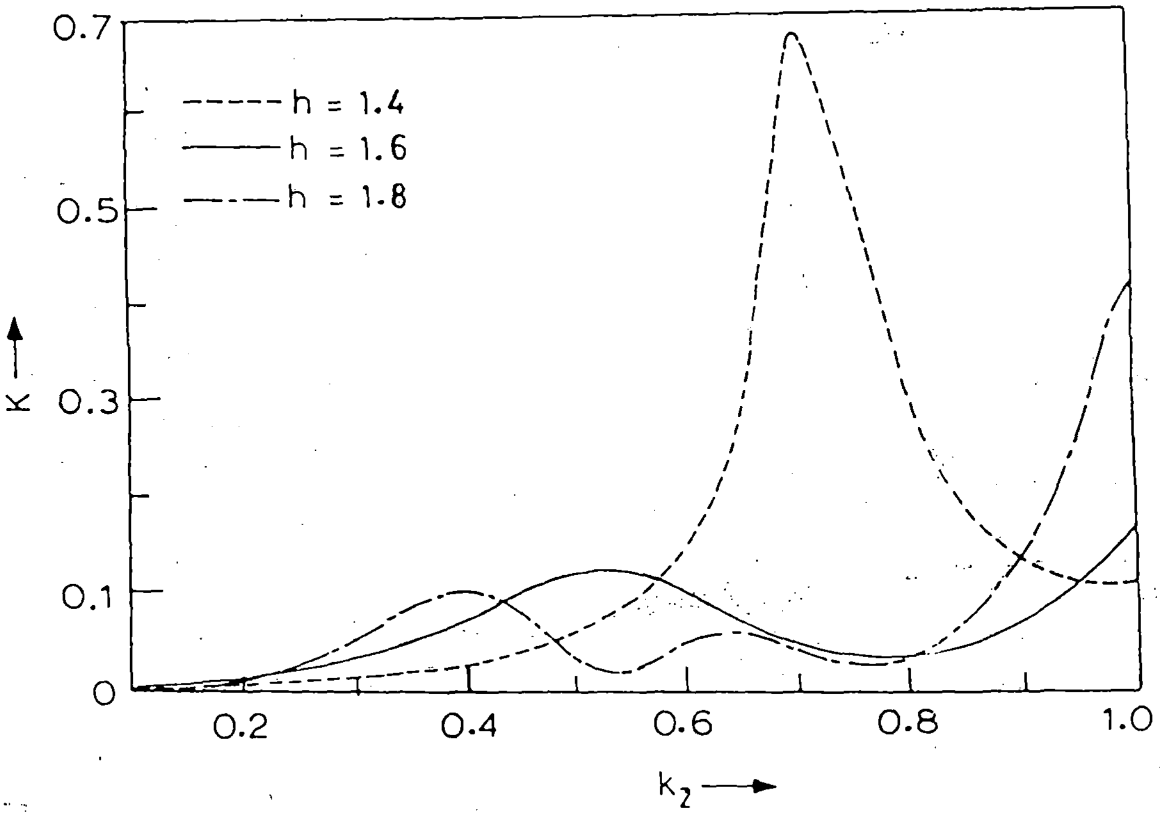

Fig. 2 Stress intensity factor K vs dimensionless frequency k_2

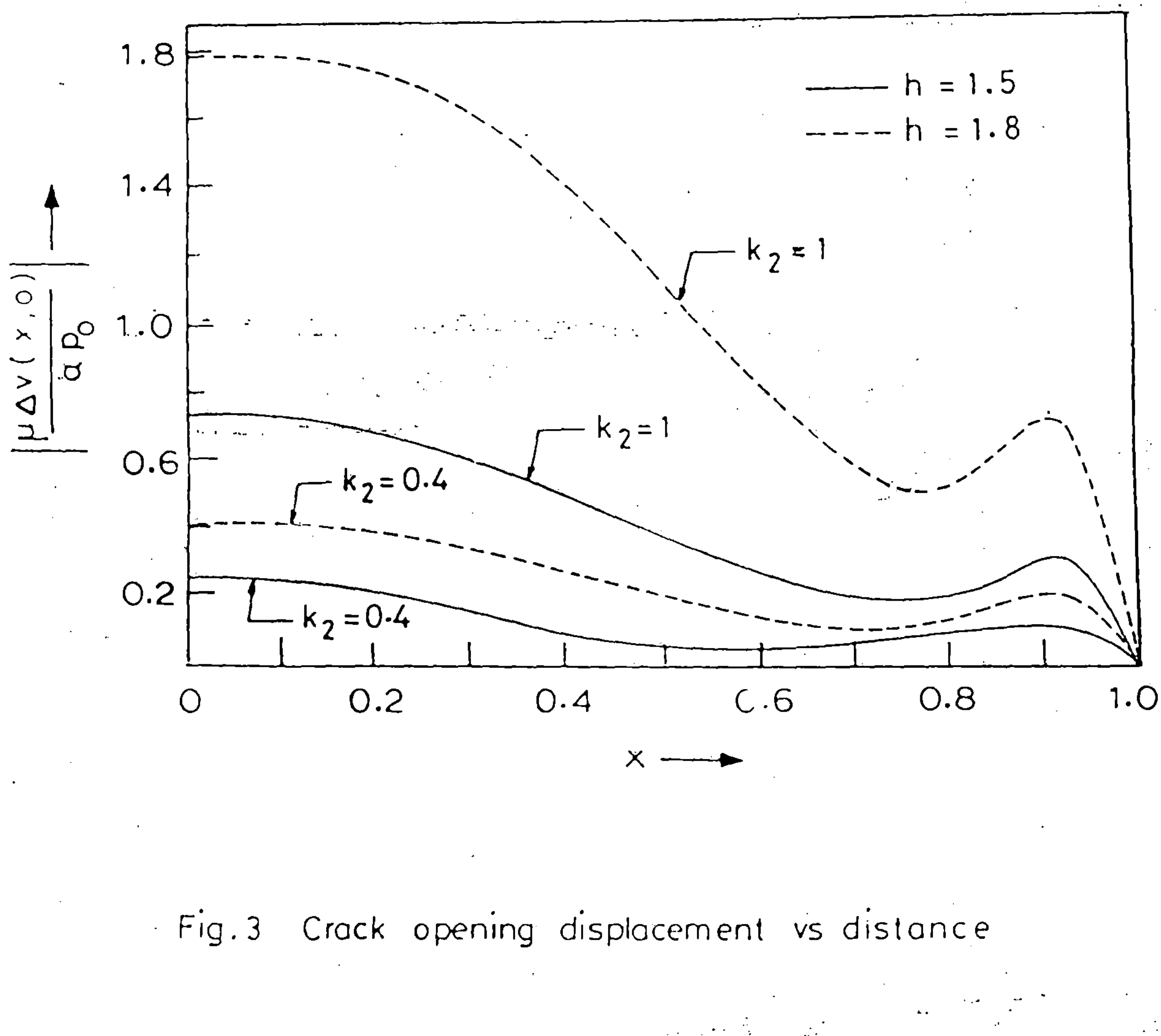

Fig. 3 Crack opening displacement vs distance

AN ELASTIC STRIP WITH THREE CO-PLANAR MOVING GRIFFITH CRACKS

1. INTRODUCTION

In fracture mechanics, the problem of diffraction of elastic waves by cracks of finite dimension in a strip of elastic material has been examined by several investigators. Sih and Chen (1972) investigated the problem of propagation of a crack of finite length in a strip under plane extension. Closed-form solutions for a finite length crack moving in a strip under anti-plane shear stress were obtained by Singh et al. (1981). Using a finite Hilbert transform technique developed by Srivastava and Lowengrub (1968), Lowengrub and Srivastava (1968-b)solved the static problem of distribution of stress and displacement in an infinitely long elastic strip containing two co-planar Griffith cracks. Recently, several dynamic problems of determining stress and displacement due to moving Griffith cracks have been solved by Das and Ghosh (1991, 1992a, 1992b, 1992c) and by Das (1993, 1992). Dhawan and Dhaliwal (1978) also solved the static problem of determining the stress distribution in an infinite transversely isotropic medium containing three co-planar Griffith cracks.

In this paper, the problem of propagation of three co-planar

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Griffith cracks in a fixed direction with constant velocity V in an infinitely long but finite width elastic strip is considered. Employing the Fourier integral transform, the problem when the lateral boundaries are assumed to be clamped and displaced by an equal amount has been reduced to solving a set of four integral equations which are solved using the finite Hilbert transform technique and Cook's result (1970) to derive the exact form of stress intensity factors and crack opening displacement. Numerical results for stress intensity factors are presented graphically to show their variations with crack speed, crack length and the separating distance between the cracks.

2. STATEMENT OF THE PROBLEM

Consider an infinitely long elastic strip occupying the region $-h \le y \le h$, weakened by three co-planar Griffith cracks moving steadily at a constant velocity V in the X-direction, referred to a fixed co-ordinate system (X,Y,Z) as shown in Fig.1.

In dynamic problems of anti-plane shear, the non-vanishing component of displacement W directed in the Z-direction satisfies the equation of motion :

$$W_{,XX} + W_{,YY} = \frac{1}{C_2^2} W_{,TT} \tag{1}$$

where $C_2 = (\mu/\rho)^{1/2}$ is the shear wave velocity, ρ is the material

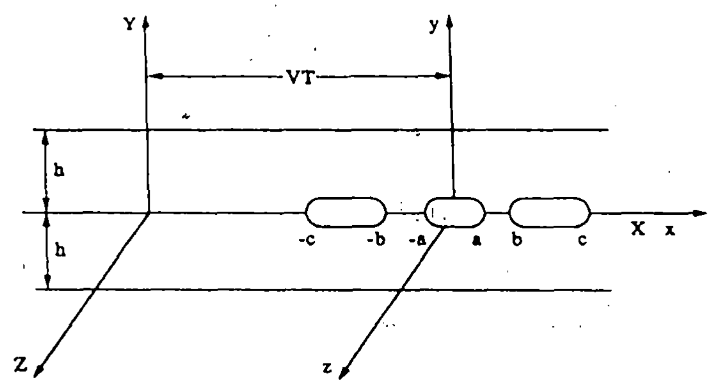

Fig. 1. Geometry and coordinate system

density and $W_{,X}$ represents partial derivatives of W with respect to X.

For cracks moving at a constant velocity V in the X-direction, it is convenient to introduce the Galilean transformation :

$$x=X-VT, \quad y=Y, \quad z=Z, \quad t=T \tag{2}$$

where (x,y,z) represents the translating co-ordinate system shown in Fig.1.

Let three co-planar Griffith cracks of finite length located along the X-axis be moving steadily with velocity V in the direction of the X-axis so that their positions referred to translating co-ordinates (x,y,z) are $-c<x<-b$, $-a<x<a$ and $b<x<c$ on $y=0$. The edges of the strip $y=\pm h$ are assumed to be clamped and displaced by an equal amount W_o, where W_o is a constant.

The boundary conditions of the proposed problem are

$$\sigma_{yz}(x,0) = 0, \quad |x|<a, \ b<|x|<c \tag{3}$$

$$W(x,\pm h) = \pm W_o , \quad -\infty<x<\infty \tag{4}$$

$$W(x,0) = 0 , \quad a<|x|<b , \ |x|>c. \tag{5}$$

In order to apply the integral transform technique it is required to solve a different but equivalent problem which can be obtained from the clamped strip problem (without any cracks) while the uniform strain is applied. The equivalent stress conditions on the cracks are

$$\sigma_{yz}(x,0) = -\frac{\mu W_o}{h}, \quad |x|<a , \ b<|x|<c \tag{6}$$

and the boundary conditions for the displacement are

$$W(x, \pm h) = 0 , \quad -\infty < x < \infty \tag{7}$$

$$W(x, 0) = 0 , \quad a < |x| < b , \quad |x| > c \tag{8}$$

In the moving co-ordinate system, the equation of motion becomes independent of time and takes the form

$$s^2 W_{,XX} + W_{,YY} = 0, \tag{9}$$

with $\quad s = \sqrt{(1 - V^2/C_2^2)}. \quad (10)$

Introducing

$$\overline{W}_c(\xi, y) = \int_0^\infty W(x, y)\cos(\xi x)dx$$

$$W(x, y) = \frac{2}{\pi}\int_0^\infty \overline{W}_c(\xi, y)\cos(\xi x)d\xi \tag{11}$$

in equation (3), the solution of equation (3) is obtained as

$$W(x, y) = \frac{2}{\pi}\int_0^\infty \left[C_1(\xi)e^{-\xi ys} + C_3(\xi)e^{\xi ys}\right]\cos(\xi x)d\xi , \tag{12}$$

with

$$\sigma_{yz}(x, y) = -\frac{2\mu s}{\pi}\int_0^\infty \xi\left[C_1(\xi)e^{-\xi ys} - C_3(\xi)e^{\xi ys}\right]\cos(\xi x)d\xi . \tag{13}$$

Using the expression for W(x,y) given in (6) in equation (9), it has been found that

$$C_1(\xi) = \frac{C(\xi)}{1 - e^{-2\xi hs}}$$

$$C_3(\xi) = -\frac{C(\xi)e^{-2\xi hs}}{1 - e^{-2\xi hs}} , \tag{14}$$

where the unknown function $C(\xi)$ is to be determined.

From conditions (8) and (10) it is determined that $C(\xi)$ satisfies the following quadruple integral equations

$$\int_0^\infty \xi C(\xi)\coth(\xi hs)\cos(\xi x)d\xi = \frac{\pi W_0}{2hs}, \quad x \in I_1, I_3 \qquad (15a,b)$$

and

$$\int_0^\infty C(\xi)\cos(\xi x)d\xi = 0, \quad x \in I_2, I_4, \qquad (16)$$

where

$$I_1=(0,a), \quad I_2=(a,b), \quad I_3=(b,c), \quad I_4=(c,\infty).$$

3. METHOD OF SOLUTION

In order to solve the quadruple integral equations given by equations (15) and (16), let us take

$$C(\xi) = \frac{1}{\xi}\int_0^a h(u)\sin(\xi u)du + \frac{1}{\xi}\int_b^c g(v^2)\text{sech}^2(ev)\sin(\xi v)dv, \qquad (17)$$

where $h(u)$ and $g(v^2)$ are the unknown functions to be determined from the boundary conditions of the proposed problem. Substituting the value of $C(\xi)$ given by (17) in (16) and using the following result :

$$\int_0^\infty \frac{\sin(\xi u)\cos(\xi x)}{\xi}d\xi = \begin{cases} \frac{\pi}{2}, & u>x>0 \\ \frac{\pi}{4}, & u=x>0 \\ 0, & x>u>0, \end{cases}$$

it is found that this choice of $C(\xi)$ leads to the condition

$$\int_b^c g(v^2)\mathrm{sech}^2(ev)dv = 0. \tag{18}$$

Rewriting equation (15a) as

$$\frac{d}{dx}\int_0^\infty C(\xi)\coth(\xi hs)\sin(\xi x)d\xi = \frac{\pi W_0}{2hs}, \quad x \in I_1 \tag{19}$$

and inserting the value of $C(\xi)$ from equation (17) in (19), it is found that $h(u)$ is the solution of the following singular integral equation :

$$\int_0^a h(u)\log\left|\frac{\tanh(ex)+\tanh(eu)}{\tanh(ex)-\tanh(eu)}\right|du = \pi f(x), \qquad x\in I_1. \tag{20}$$

with

$$f(x) = \int_0^x \left[\frac{W_0}{hs} - \frac{2}{\pi}\int_b^c \frac{eg(v^2)\mathrm{sech}^2(ex')\mathrm{sech}^2(ev)\tanh(ev)}{\tanh^2(ev) - \tanh^2(ex')}dv\right]dx',$$

where the following result (Gradshteyn et al., 1965) has been used:

$$\int_0^\infty \coth(\xi hs)\frac{\sin(\xi u)\sin(\xi x)}{\xi}d\xi = \frac{1}{2}\log\left|\frac{\tanh(ex)+\tanh(eu)}{\tanh(ex)-\tanh(eu)}\right|, \quad e=\frac{\pi}{2hs}. \tag{21}$$

Now using Cook's result (1970), the solution of (20) has been obtained with the aid of the following result :

$$\int_0^a \frac{\sqrt{[\tanh^2(ea)-\tanh^2(ex)]}\,e\,\mathrm{sech}^2(ex)}{[\tanh^2(ex)-\tanh^2(eu)][\tanh^2(ev)-\tanh^2(ex)]}dx$$

$$= -\frac{\pi}{2\tanh(ev)} \frac{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}}{\tanh^2(ev)-\tanh^2(eu)} \quad \text{for} \quad u\in I_1 \text{ and } v\in I_3,$$

$$h(u) = \frac{-2e\ \tanh(eu)\mathrm{sech}^2(eu)}{\pi[\tanh^2(ea)-\tanh^2(eu)]} \left[\frac{W_o}{hs}\int_o^a \frac{\sqrt{[\tanh^2(ea)-\tanh^2(ex)]}}{\tanh^2(ex)-\tanh^2(eu)}\, dx + \right.$$

$$\left. + \int_b^c \frac{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}}{\tanh^2(ev)-\tanh^2(eu)}\, g(v^2)\mathrm{sech}^2(ev)\ dv \right]. \qquad (22)$$

Substituting the resulting value of $C(\xi)$, obtained using equation (22) in equation (17), in condition (15b) and making use of the following results :

$$\int_o^a \frac{e\ \mathrm{sech}^2(eu)\tanh^2(eu)\ du}{[\tanh^2(eu)-\tanh^2(ex)][\tanh^2(ev)-\tanh^2(eu)]\sqrt{[\tanh^2(ea)-\tanh^2(eu)]}}$$

$$= \frac{\pi}{2[\tanh^2(ev)-\tanh^2(ex)]} \left[\frac{\tanh(ev)}{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}} - \right.$$

$$\left. - \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}\right],$$

$$\int_o^a \frac{e\ \mathrm{sech}^2(eu)\tanh^2(eu)\ du}{[\tanh^2(eu)-\tanh^2(ex)][\tanh^2(ey')-\tanh^2(eu)]\sqrt{[\tanh^2(ea)-\tanh^2(eu)]}}$$

$$= \frac{\pi}{2[\tanh^2(ex)-\tanh^2(ey')]} \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}},$$

for $x,v \in I_3$ and $y' \in I_1$,

it can be shown that $g(v^2)$ is the solution of the following singular integral equation :

$$\int_b^c \frac{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}}{\tanh^2(ev)-\tanh^2(ex)}\, e g(v^2)\operatorname{sech}^2(ev)dv$$

$$= \frac{\pi W_o}{2hs}\left[\frac{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}{\operatorname{sech}^2(ex)\tanh(ex)} + \frac{e}{\pi}\int_o^a \frac{\sqrt{[\tanh^2(ea)-\tanh^2(ey')]}}{\tanh^2(ex)-\tanh^2(ey')}\,dy'\right]$$

$$, \quad \text{for } x \in I_3. \qquad (23)$$

Using the finite Hilbert transform technique (Srivastava et al., 1968) and the following result :

$$\int_b^c \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ex)-\tanh^2(eb)}\right]} \times$$

$$\times \frac{2\operatorname{sech}^2(ex)\tanh(ex)\ dx}{[\tanh^2(ex)-\tanh^2(ey')][\tanh^2(ex)-\tanh^2(ev)]}$$

$$= -\frac{\pi}{e[\tanh^2(ev)-\tanh^2(ey')]}\sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ey')}{\tanh^2(eb)-\tanh^2(ey')}\right]},$$

the solution of equation (23) is found as

$$g(v^2) = -\frac{2eW_o}{\pi hs}\frac{\tanh^2(ev)\sqrt{[\tanh^2(ev)-\tanh^2(eb)]}}{\sqrt{\{[\tanh^2(ev)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(ev)]\}}}$$

$$\times\left[\int_b^c \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ex)-\tanh^2(eb)}\right]}\frac{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}{\tanh^2(ex)-\tanh^2(ev)}\,dx\ -\right.$$

$$\left. - \int_o^a \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ey')}{\tanh^2(eb)-\tanh^2(ey')}\right]}\frac{\sqrt{[\tanh^2(ea)-\tanh^2(ey')]}}{\tanh^2(ev)-\tanh^2(ey')}\,dy'\right] +$$

$$\frac{C_1 \tanh(ev)}{\surd\{[\tanh^2(ev)-\tanh^2(ea)][\tanh^2(ev)-\tanh^2(eb)][\tanh^2(ec)-\tanh^2(ev)]\}} \qquad (24)$$

Next substituting the value of $g(v^2)$ from equation (24) in equation (22) and finally using the following result :

$$\int_b^c \surd\left[\frac{\tanh^2(ev)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ev)}\right] \times$$

$$\times \frac{2\operatorname{sech}^2(ev)\tanh(ev)\,dv}{[\tanh^2(ev)-\tanh^2(eu)][\tanh^2(ex')-\tanh^2(ev)]}$$

$$= -\frac{\pi}{e[\tanh^2(eu)-\tanh^2(ex')]}\left[\surd\left[\frac{\tanh^2(eb)-\tanh^2(eu)}{\tanh^2(ec)-\tanh^2(eu)}\right] - \right.$$

$$\left. - \surd\left[\frac{\tanh^2(eb)-\tanh^2(ex')}{\tanh^2(ec)-\tanh^2(ex')}\right]\right],$$

for $u, x' \in I_1$,

$h(u)$ is derived in the form :

$$h(u) = -\frac{2eW_o}{h\pi s}\frac{\operatorname{sech}^2(eu)\tanh(eu)\surd[\tanh^2(eb)-\tanh^2(eu)]}{\surd\{[\tanh^2(ea)-\tanh^2(eu)][\tanh^2(ec)-\tanh^2(eu)]\}} \times$$

$$\times \left[\int_o^a \surd\left[\frac{\tanh^2(ea)-\tanh^2(ey')}{\tanh^2(eb)-\tanh^2(ey')}\right]\frac{\surd[\tanh^2(ec)-\tanh^2(ey')]}{\tanh^2(ey')-\tanh^2(eu)}\,dy' + \right.$$

$$\left. + \int_b^c \surd\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ex)-\tanh^2(eb)}\right]\frac{\surd[\tanh^2(ex)-\tanh^2(ea)]}{\tanh^2(ex)-\tanh^2(eu)}\,dx\right] -$$

$$\frac{C_1 \tanh(eu)\,\mathrm{sech}^2(eu)}{\sqrt{\{[\tanh^2(ea)-\tanh^2(eu)][\tanh^2(eb)-\tanh^2(eu)][\tanh^2(ec)-\tanh^2(eu)]\}}} \tag{25}$$

Substitution of the value of $g(v^2)$ from equation (24) in the condition (18) yields

$$C_1 = -\frac{2eW_o}{\pi hs}\Bigg[\int_b^c \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ex)-\tanh^2(eb)}\right]}\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}\ \times$$

$$\times\left\{\frac{\tanh^2(ex)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ex)}\times\Pi\left[\frac{\pi}{2},\frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ex)},q\right]/F(\frac{\pi}{2},q)+1\right\}dx\ +$$

$$+\int_0^a \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(es)}{\tanh^2(eb)-\tanh^2(es)}\right]}\sqrt{[\tanh^2(ea)-\tanh^2(es)]}\ \times$$

$$\times\left\{1-\frac{\tanh^2(eb)-\tanh^2(es)}{\tanh^2(ec)-\tanh^2(es)}\ \Pi\left[\frac{\pi}{2},\frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(es)},q\right]/F(\frac{\pi}{2},q)\right\}ds, \tag{26}$$

where $F(\phi,q)$ and $\Pi(\phi,n,q)$ are elliptic integrals of the first and third kinds respectively and

$$q = \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ea)}\right]}.$$

The relevant displacement and stress components in the plane of the crack can now be shown to be given by

$$W(x,0) = \int_x^a h(u)du\ , \qquad 0\le x\le a$$

$$= \int_x^c g(v^2)\cosh(ev)dv\ , \qquad b\le x\le c \tag{27}$$

and

$$[\sigma_{yz}(x,0)]_{a<x<b} = \frac{2\mu s}{\pi}\left[\int_0^a \frac{eh(u)\tanh(eu)du}{\tanh^2(ex)-\tanh^2(eu)} - \right.$$

$$\left. - \int_b^c \frac{eg(v^2)\tanh(ev)\mathrm{sech}^2(ev)}{\tanh^2(ev)-\tanh^2(ex)}\, dv\right]\mathrm{sech}^2(ex)$$

$$[\sigma_{yz}(x,0)]_{x>c} = \frac{2\mu s}{\pi}\left[\int_0^a \frac{eh(u)\tanh(eu)du}{\tanh^2(ex)-\tanh^2(eu)} + \right.$$

$$\left. + \int_b^c \frac{eg(v^2)\tanh(ev)\mathrm{sech}^2(ev)}{\tanh^2(ex)-\tanh^2(ev)}\, dv\right]\mathrm{sech}^2(ex). \qquad (28)$$

Now insertion of the values of h(u) and $g(v^2)$ as given by equations (25) and (24) in the expressions (28) yields, after some algebraic manipulations,

$$[\sigma_{yz}(x,0)]_{a<x<b} = \frac{2\mu eW_o}{\pi hs}\left[-\sqrt{\left[\frac{\tanh^2(eb)-\tanh^2(ea)}{\tanh^2(ec)-\tanh^2(ea)}\right]} \times \right.$$

$$\times \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}\left\{\int_0^a F_2(u,x)du + \int_b^c F_2(v,x)dv\right\} -$$

$$- \frac{2e[\tanh^2(ec)-\tanh^2(eb)]}{\pi}\left\{\int_0^a F_2(u',x)du'\int_0^a F_4(c,u)\; F_3(0,x,u)du + \right.$$

$$\left. + \int_b^c F_2(v,x)dv\int_0^a F_4(c,u)\; F_3(v,x,u)du\right\} +$$

$$+ \frac{\mu sh}{eW_o}\, C_1\left\{\frac{\pi}{2}\, \frac{1 - \tanh(ex)/\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}{\sqrt{\{[\tanh^2(eb)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(ea)]\}}} + \right.$$

$$+ e\int_0^a F_4(c,u)\; F_5(x,u)du\Bigg\} + \frac{e[\tanh^2(eb)-\tanh^2(ea)]}{\pi} \times$$

$$\times \Bigg\{\int_b^c F_2(v',x)dv'\int_b^c F_4(a,v)\; F_6(v',x,v)dv + \int_0^a F_2(u,x)du$$

$$\times\int_c^b F_4(a,v)\; F_6(u,x,v)dv - \frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(eb)-\tanh^2(ea)} \times$$

$$\times\int_0^a F_1(u,x)du\int_0^a F_4(c,u')\; F_9(u,u')du'\Bigg\} - \frac{\mu sh}{eW_o}\frac{C_1}{X_1} \times$$

$$\times \Bigg\{\frac{\pi}{2}\frac{\tanh(ec)}{\sqrt{[\tanh^2(ec)-\tanh^2(ea)]}} + e\tanh^2(ea)\int_b^c F_7(x,v)dv\Bigg\}\Bigg]\operatorname{sech}^2(ex)$$

and

$$[\sigma_{yz}(x,0)]_{x>c} = \frac{2\mu eW_o}{\pi hs}\Bigg[-\sqrt{\Bigg[\frac{\tanh^2(eb)-\tanh^2(ea)}{\tanh^2(ec)-\tanh^2(ea)}\Bigg]} \times$$

$$\times \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}\Bigg\{\int_0^a F_2(u,x)du + \int_b^c F_2(v,x)dv\Bigg\} -$$

$$- \frac{2e[\tanh^2(ec)-\tanh^2(eb)]}{\pi}\Bigg\{\int_0^a F_2(u',x)du'\int_0^a F_4(c,u)\; F_3(0,x,u)du +$$

$$+ \int_b^c F_2(v,x)dv\int_0^a F_4(c,u)\; F_3(v,x,u)du\Bigg\} +$$

$$+ \frac{\mu sh}{eW_o}C_1\Bigg\{\frac{\pi}{2}\frac{1 - \tanh(ex)/\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}}{\sqrt{\{[\tanh^2(eb)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(ea)]\}}} +$$

$$+ e\int_0^a F_4(c,u)\; F_5(u,x)du\Bigg\} - \frac{e[\tanh^2(eb)-\tanh^2(ea)]}{\pi} \times$$

$$\times \left\{\int_b^c F_2(v',x)dv' \int_b^c F_4(a,v)\ F_8(v',v,x)dv + \int_0^a F_2(u,x)du \right.$$

$$\times \int_b^c F_4(a,v)\ F_8(u,v,x)dv + \frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(eb)-\tanh^2(ea)} \times$$

$$\left. \times \int_0^a F_1(u,x)du \int_0^a F_4(c,u')\ F_9(u,u')du' \right\} + \frac{\mu sh}{eW_0} \frac{C_1}{X_1} \times$$

$$\times \left\{\frac{\pi}{2} \frac{\tanh(ec)}{\sqrt{[\tanh^2(ec)-\tanh^2(ea)]}} + e\tanh^2(ea) \int_b^c F_7(x,v)dv \right\} -$$

$$- \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ea)} \right]} \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ec)]}} \times$$

$$\times \left\{\int_0^a F_2(u,x)du + \int_b^c F_2(v,x)dv \right\}\Bigg] \operatorname{sech}^2(ex) \ , \qquad (29)$$

where

$$F_1(u,x) = \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eu)}{\tanh^2(eb)-\tanh^2(eu)} \right]} \frac{\tanh(eu)}{\tanh^2(ex)-\tanh^2(eu)}$$

$$F_2(v,x) = \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ev)}{\tanh^2(ev)-\tanh^2(eb)} \right]} \frac{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}}{\tanh^2(ev)-\tanh^2(ex)}$$

$$F_3(v,x,u) = \frac{\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ea)]}} \tan^{-1}\left\{\frac{\tanh(eu)}{\tanh(ex)} \times \right.$$

$$\left. \times \sqrt{\left[\frac{\tanh^2(ex)-\tanh^2(ea)}{\tanh^2(ea)-\tanh^2(eu)} \right]}\right\} - \frac{\tanh(ev)}{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}} \times$$

$$\times \tan^{-1}\left\{\frac{\tanh(eu)}{\tanh(ev)}\sqrt{\left[\frac{\tanh^2(ev)-\tanh^2(ea)}{\tanh^2(ea)-\tanh^2(eu)}\right]}\right\}$$

$$F_4(w,u) = \frac{\operatorname{sech}^2(eu)\tanh(eu)}{\sqrt{\{[\tanh^2(ew)-\tanh^2(eu)]^3[\tanh^2(eb)-\tanh^2(eu)]\}}}$$

$$F_5(u,x) = [2\tanh^2(eu)-\tanh^2(ec)-\tanh^2(eb)]\left\{\sin^{-1}\left(\frac{\tanh(eu)}{\tanh(ea)}\right) - F_3(0,x,u)\right\}$$

$$F_6(u,x,v) = \frac{\tanh(ex)}{\sqrt{[\tanh^2(ec)-\tanh^2(ex)]}} \times$$

$$\log\left|\frac{\tanh(ex)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}+\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(ex)]}}{\tanh(ex)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}-\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(ex)]}}\right|$$

$$- \frac{\tanh(eu)}{\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}} \times$$

$$\log\left|\frac{\tanh(eu)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}+\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}}{\tanh(eu)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}-\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}}\right|$$

$$F_7(x,v) = \tan^{-1}\left\{\sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ec)-\tanh^2(ev)}\right]}\sqrt{\left[\frac{\tanh^2(ev)-\tanh^2(eb)}{\tanh^2(eb)-\tanh^2(ex)}\right]}\right\}$$

$$\times \frac{\operatorname{sech}^2(ev)}{\sqrt{\{[\tanh^2(ev)-\tanh^2(ea)]^3\}}}$$

$$F_8(u,v,x) = -\frac{2\tanh(ex)}{\sqrt{[\tanh^2(ex)-\tanh^2(ec)]}}\tan^{-1}\left\{\frac{\tanh(ev)}{\tanh(ex)}\times\right.$$

$$\times \sqrt{\left[\frac{\tanh^2(ex)-\tanh^2(ec)}{\tanh^2(ec)-\tanh^2(ev)} \right]}\Bigg\} + \frac{\tanh(eu)}{\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}} \times$$

$$\log\left|\frac{\tanh(eu)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}+\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}}{\tanh(eu)\sqrt{[\tanh^2(ec)-\tanh^2(ev)]}-\tanh(ev)\sqrt{[\tanh^2(ec)-\tanh^2(eu)]}}\right|$$

$F_\rho(u,u') =$

$$\log\left|\frac{\tanh(eu)\sqrt{[\tanh^2(ea)-\tanh^2(eu')]}+\tanh(eu')\sqrt{[\tanh^2(ea)-\tanh^2(eu)]}}{\tanh(eu)\sqrt{[\tanh^2(ea)-\tanh^2(eu')]}-\tanh(eu')\sqrt{[\tanh^2(ea)-\tanh^2(eu)]}}\right|$$

and $\quad X_1 = \sqrt{\{[\tanh^2(eb)-\tanh^2(ex)][\tanh^2(ec)-\tanh^2(ex)]\}}.$

$$(30)$$

The dynamic stress intensity factors are defined by

$$N_a = \lim_{x\to a+} \sqrt{[2(x-a)]}[\sigma_{yz}(x,0)]_{a<x<b}$$

$$N_b = \lim_{x\to b-} \sqrt{[2(b-x)]}[\sigma_{yz}(x,0)]_{a<x<b}$$

$$N_c = \lim_{x\to c+} \sqrt{[2(x-c)]}[\sigma_{yz}(x,0)]_{x>c} \qquad (31)$$

Substitution of the results given by equations (29) in expressions (31) yields

$$N_a = \sqrt{\left[\frac{\tanh(ea)}{e}\right]}\left[-\sqrt{\left[\frac{\tanh^2(eb)-\tanh^2(ea)}{\tanh^2(ec)-\tanh^2(ea)}\right]}\frac{2eW_0}{\pi h}\left\{\int_0^a F_2(u,a)du + \right.\right.$$

$$\left. + \int_b^c F_2(v,a)dv\right\} -$$

$$- \left.\frac{\mu s C_1}{\sqrt{\{[\tanh^2(eb)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(ea)]\}}}\right]\operatorname{sech}(ea)$$

$$N_b = - \frac{\mu s C_1}{\sqrt{\{[\tanh^2(eb)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(eb)]\}}} \times$$

$$\sqrt{\left[\frac{\tanh(eb)}{e}\right]}\operatorname{sech}(eb)$$

$$N_c = \sqrt{\left[\frac{\tanh(ec)}{e}\right]}\left[- \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ea)}\right]}\frac{2eW_o}{\pi h}\left\{\int_o^a F_2(u,c)du + \right.\right.$$

$$\left. + \int_b^c F_2(v,c)dv\right\} +$$

$$+ \left.\frac{\mu s C_1}{\sqrt{\{[\tanh^2(ec)-\tanh^2(ea)][\tanh^2(ec)-\tanh^2(eb)]\}}}\right]\operatorname{sech}(ec)$$

(32a-c)

Again insertion of the values of $h(u)$ and $g(v^2)$, given by equations (24) and (25), in the expressions for displacements given by equations (27) yields

$$[W(x,0)]_{o\le x\le a} = - \frac{W_o}{h\mu\pi s}\left[\frac{2[\tanh^2(eb)-\tanh^2(ea)]}{\sqrt{[\tanh^2(ec)-\tanh^2(ea)]}} \times\right.$$

$$\times \left\{\int_b^c \Pi\left\{\lambda, \frac{\tanh^2(ev)-\tanh^2(eb)}{\tanh^2(ev)-\tanh^2(ea)}, q\right\} \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ev)}{\tanh^2(ev)-\tanh^2(eb)}\right]} \times\right.$$

$$\times \frac{dv}{\sqrt{[\tanh^2(ev)-\tanh^2(ea)]}} - \int_o^a \Pi\left\{\lambda, \frac{\tanh^2(eb)-\tanh^2(eu)}{\tanh^2(ea)-\tanh^2(eu)}, q\right\} \times$$

$$\times \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eu)}{\tanh^2(eb)-\tanh^2(eu)}\right]} \frac{du}{\sqrt{[\tanh^2(ea)-\tanh^2(eu)]}}\Bigg\}\Bigg] -$$

$$- \frac{C_1 F(\lambda, q)}{e\sqrt{[\tanh^2(ec)-\tanh^2(ea)]}}$$

and

$$[W(x,0)]_{b\leq x\leq c} = \left[\frac{2W_o}{h\mu\pi s}\left(\int_b^c \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ev)}{\tanh^2(ev)-\tanh^2(eb)}\right]} \times\right.\right.$$

$$\times \sqrt{[\tanh^2(ev)-\tanh^2(ea)]}\Bigg\{F(\lambda',q)+\frac{\tanh^2(ev)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ev)} \times$$

$$\times \Pi\left\{\lambda', \frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(ev)}, q\right\}\Bigg\}dv + \int_o^a \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(eu)}{\tanh^2(eb)-\tanh^2(eu)}\right]}\times$$

$$\times \sqrt{[\tanh^2(ea)-\tanh^2(eu)]}\Bigg\{F(\lambda',q)-\frac{\tanh^2(eb)-\tanh^2(eu)}{\tanh^2(ec)-\tanh^2(eu)} \times$$

$$\left.\left.\times \Pi\left\{\lambda', \frac{\tanh^2(ec)-\tanh^2(eb)}{\tanh^2(ec)-\tanh^2(eu)}, q\right\}\Bigg\}du\right) + \frac{C_1}{e} F(\lambda',q)\right] \times$$

$$\times \frac{1}{\sqrt{[\tanh^2(ec)-\tanh^2(ea)]}}, \qquad (33a,b)$$

where

$$\sin\lambda = \sqrt{\left[\frac{\tanh^2(ea)-\tanh^2(ex)}{\tanh^2(eb)-\tanh^2(ex)}\right]}, \quad \sin\lambda' = \sqrt{\left[\frac{\tanh^2(ec)-\tanh^2(ex)}{\tanh^2(ec)-\tanh^2(eb)}\right]}$$

and $F(\phi,q)$, $\Pi(\phi,n,q)$ and q have been defined earlier.

On putting b=c and simplifying, it may be noted that the results (33a) and (32a) become those given by equations (3.18) and (3.21) of Singh et al (1981) and for a=0 the results given by (32b), (32c) and (33b) coincide with those given by equations (4.21), (4.22) and (4.17) of Das and Ghosh (1991).

4. NUMERICAL RESULTS AND DISCUSSION

Numerical results for stress intensity factor at the tips of the cracks for different values of crack speed, crack length and the separating distance between the cracks are presented in this section. The crack length dependence of the stress intensity factors and its variations with V/C_2 are shown in Figs.2-5. It is shown in Figs.2 and 3 that stress intensity factors at the edges of the cracks decrease with an increase in the values of V/C_2 and have a prominent variation when $V/C_2 \to 1$. Variations of stress intensity factors at the edge x=a become more prominent than those at the tips x=b and x=c when the length of the inner crack increases.

Variations of stress intensity factors at the edges of the cracks with a/b for different values of c/b and those with b/a for different values of c/a are plotted in Figs.4 and 5, respectively. It is found that when the separating distance between the inner crack and outer pair of cracks decreases the stress intensity factors at the tips x=a and x=b become more prominent than that at the edge x=c.

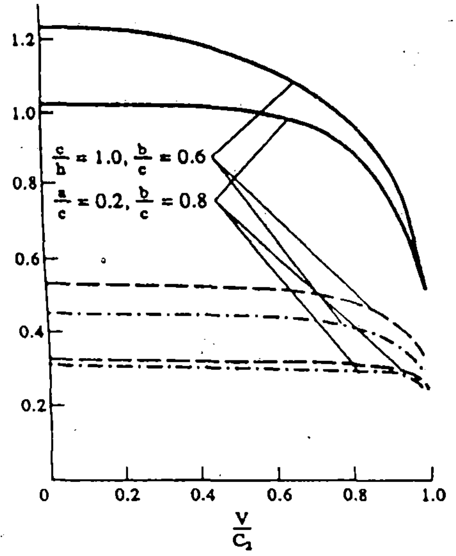

Fig. 2. Variations of stress intensity factors with V/C_2: (——) $hN_a/\mu W_0\sqrt{a}$; (– – –) $hN_b/\mu W_0\sqrt{b}$; (– · – · –) $hN_c/\mu W_0\sqrt{c}$.

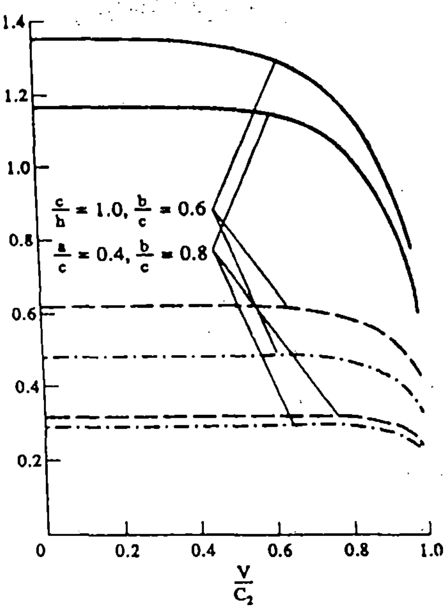

Fig. 3. Variations of stress intensity factors with V/C_2: (——) $hN_a/\mu W_0\sqrt{a}$; (– – –) $hN_b/\mu W_0\sqrt{b}$; (– · – · –) $hN_c/\mu W_0\sqrt{c}$.

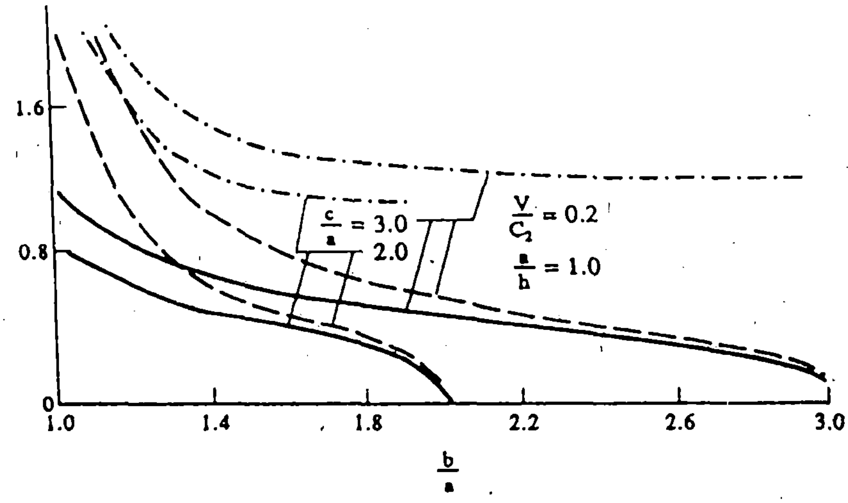

Fig. 4. Stress intensity factors vs b/a: (—·—·—) $hN_a/\mu W_0\sqrt{a}$; (– – –) $hN_b/\mu W_0\sqrt{b}$; (—– –) $hN_c/\mu W_0\sqrt{c}$.

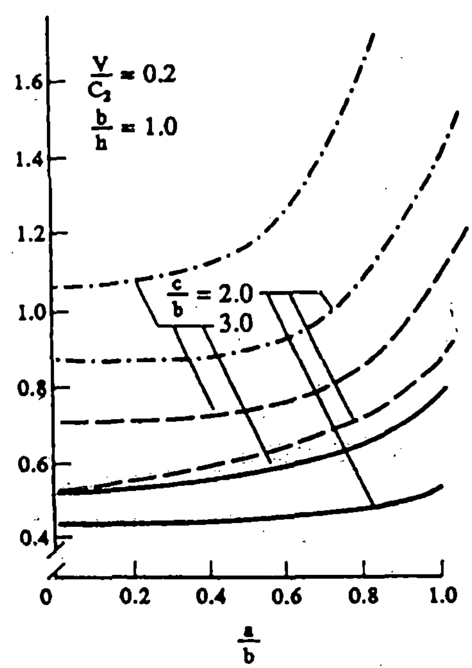

Fig. 5. Stress intensity factors vs a/b: (—·—·—) $hN_a/\mu W_0\sqrt{a}$; (– – –) $hN_b/\mu W_0\sqrt{b}$; (——) $hN_c/\mu W_0\sqrt{c}$.

FOUR COPLANAR GRIFFITH CRACKS MOVING IN AN INFINITELY LONG ELASTIC STRIP UNDER ANTIPLANE SHEAR STRESS

1. INTRODUCTION

In recent years, scattering of elastic waves by cracks of finite dimension in a strip of elastic material has been investigated by several investigators. The theory of cracks in 2-dimensional medium was first developed by Griffith (1920). Sih and Chen (1972) solved the problem of a uniformly propagating finite crack in a strip of isotropic material under plane extension. Singh et al. (1981) also studied the problem of propagation for a finite length crack moving in a strip under anti-plane shear stress and gave the closed form solution. In the above analysis, the usual method of solving mixed boundary value problems by integral transforms is to reduce the problem to a Fredholm integral equation of second kind and then proceed to its numerical solution.

As regards the crack problem research has been restricted mainly to the case of a single crack or a pair of cracks because of the severe mathematical complexity encountered in solving the problems of three or more cracks. Jain and Kanwal (1972a) solved the low frequency solution of diffraction of normally incident longitudinal waves by two co-planar Griffith cracks in an infinite isotropic elastic medium. Using a completely different technique Itou (1980b)solved the diffraction problem of elastic waves by two co-planar Griffith cracks in an infinite elastic medium. Problems on three coplanar Griffith cracks moving steadily in an elastic strip has been solved by Das and Sarkar (1993).

To the best knowledge of the authors, the problem of stress distribution around four co-planar Griffith cracks in a strip has not been investigated so far. In this paper we have considered the problem of propagation of four co-planar Griffith cracks moving steadily in an infinitely long finite width strip under antiplane shear stress. Cracks are assumed to be moving steadily along a fixed direction with a constant speed V less than the shear wave velocity in the medium. The application of two-dimensional Fourier transforms reduced this problem to that of solving a set of five integral equations with cosine kernel and weight function. Employing finite Hilbert transform technique (Srivastava et al., 1968), the closed form solutions are obtained when the lateral boundaries are subjected to shearing stresses. The dynamic stress intensity factors and the crack opening displacement have been evaluated numerically for various values of crack velocity and distance between the cracks and the results have been presented by means of graphs.

2. FORMULATION OF THE PROBLEM

We first consider a strip of elastic material occupying the region $-h' \le Y \le h'$ referred to a fixed co-ordinate system (X', Y', Z') as shown in fig.1. The strip extends from $-\infty$ to ∞ in X'-direction and contains four coplanar Griffith cracks such that these cracks are located in the region $-d' \le X' \le c'$, $-b' \le X' \le -a'$, $a' \le X' \le b'$, $c' \le X' \le d'$, $|Z'| < \infty$, $Y' = 0$ moving at a constant speed v in the X'-direction.

In dynamic problem of antiplane shear, there exists a single non-vanishing component of displacement $W = W(X', Y', t)$ in the Z'-direction. The corresponding stress components are

$$\sigma_{X'Z'} = \mu \frac{\partial W}{\partial X'} \quad , \quad \sigma_{Y'Z'} = \mu \frac{\partial W}{\partial Y'} \tag{2.1}$$

where μ is the shear-modulus of elastic material.

The two dimensional wave equation for $W(X', Y', t)$ is given by

$$\frac{\partial^2 W}{\partial X'^2} + \frac{\partial^2 W}{\partial Y'^2} = \frac{1}{c_2^2} \frac{\partial^2 W}{\partial t^2} \tag{2.2}$$

where $c_2 = (\mu/\rho)^{1/2}$ is the shear wave velocity and ρ is the density of the material.

Using Galilean transformation, $x' = X' - Vt$, $y' = Y'$, $z' = Z'$, $t' = t$ where (x', y', z') represents the translating co-ordinate system as shown in fig.1 and also normalizing all the lengths with respect to 'd'' so that $x' = d'x$, $y' = d'y$, $a' = ad'$, $b' = bd'$, $c' = cd'$, $h' = d'h$, $W = d'w$, equation (2.2) reduces to

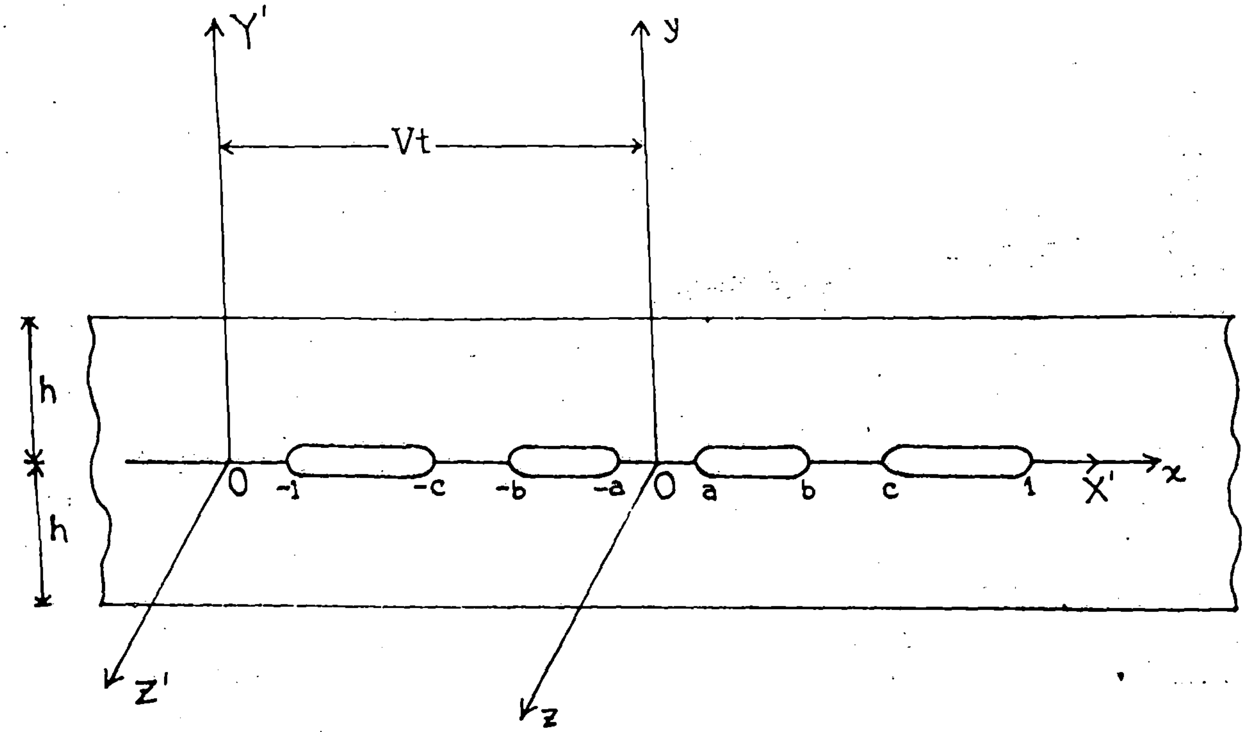

Fig.1. Geometry of the cracks.

$$s^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \tag{2.3}$$

with

$$s^2 = 1 - v^2/c_2^2. \tag{2.4}$$

Since the geometry of the problem is symmetric about the y-axis, so introducing Fourier cosine transform

$$A_1(\xi) = \int_0^\infty A(x)\cos(\xi x)\, dx$$

and

$$A(x) = \frac{2}{\pi}\int_0^\infty A_1(\xi)\cos(\xi x)\, d\xi$$

we obtain the solution of equation (2.3) as

$$w(x,y) = \pm\frac{2}{\pi}\int_0^\infty \left[A_1(\xi)\exp(-\xi|y|s) + A_2(\xi)\exp(\xi|y|s)\right]\cos(\xi x)\, d\xi \quad (y \gtrless 0) \tag{2.5}$$

with

$$\sigma_{yz}(x,y) = -\frac{2\mu s}{\pi}\int_0^\infty \left[A_1(\xi)\exp(-\xi|y|s) - A_2(\xi)\exp(\xi|y|s)\right]\xi\cos(\xi x)\, d\xi \quad (y \gtrless 0) \tag{2.6}$$

where s is the positive root of equation (2.4) and $A_1(\xi)$, $A_2(\xi)$ are the unknown functions to be determined.

In our case uniform shearing stress p is applied to the upper and lower boundaries $y=\pm h$ of the strip. The equivalent problem in our case involves the application of the shear stress $-p$ to the crack faces at $y=0$. Accordingly, the boundary conditions are

$$\sigma_{yz}(x,\pm h) = 0 , \qquad 0<x<\infty \tag{2.7}$$

$$w(x,0) = 0 , \qquad x \in I_1,\ I_3,\ I_5 \tag{2.8a-c}$$

$$\sigma_{yz}(x,0) = -p , \qquad x \in I_2,\ I_4 \tag{2.9a-b}$$

where $I_1=(0,a)$, $I_2=(a,b)$, $I_3=(b,c)$, $I_4=(c,1)$, $I_5=(1,\infty)$.

3. SOLUTION OF THE PROBLEM

Using the expression for $w(x,y)$ from (2.5) in (2.7) it has been found that

$$A_1(\xi) = \frac{A(\xi)}{1 + \exp(-2\xi hs)}$$

and

$$A_2(\xi) = \frac{A(\xi)\exp(-2\xi hs)}{1 + \exp(-2\xi hs)}$$

where $A(\xi)$ is to be determined from the boundary conditions. With the help of boundary conditions (2.8) and (2.9) $A(\xi)$ is found to satisfy the following set of five integral equations

$$\int_0^\infty A(\xi)\cos(\xi x)d\xi = 0 , \qquad x \in I_1,\ I_3,\ I_5 \tag{3.1a-c}$$

and

$$\int_0^\infty \xi H_1(\xi hs)A(\xi)\cos(\xi x)d\xi = \frac{\pi p}{2\mu s} , \qquad x \in I_2,\ I_4 \tag{3.2a-b}$$

where

$$H_1(\xi hs) = \frac{1 - \exp(-2\xi hs)}{1 + \exp(-2\xi hs)} = \tanh(\xi hs) \qquad (3.3)$$

In order to solve the set of five integral equations given by equations (3.1) and (3.2), let us take

$$A(\xi) = \frac{1}{\xi}\int_a^b g(u^2)\cosh(eu)\sin(\xi u)du + $$
$$+ \frac{1}{\xi}\int_c^1 h(v^2)\cosh(ev)\sin(\xi v)dv. \qquad (3.4)$$

In equation (3.4), $g(u^2)$ and $h(v^2)$ are unknown functions to be determined from the boundary conditions and $e = \frac{\pi}{2hs}$.
Using the following result (Gradshteyn et al., 1965)

$$\int_0^\infty \frac{\sin(\xi u)\cos(\xi x)}{\xi} d\xi = \begin{cases} \frac{\pi}{2}, & u>x>0 \\ 0, & x>u>0 \end{cases}$$

it is found that the choice of $A(\xi)$ satisfies equations (3.1a,c) if $g(u^2)$ and $h(v^2)$ satisfy

$$\int_a^b g(u^2)\cosh(eu)du = 0 \qquad (3.5a)$$

and

$$\int_c^1 h(v^2)\cosh(ev)dv = 0 \qquad (3.5b)$$

Now equations (3.2a-b) may be written in the form

$$\frac{d}{dx}\int_0^\infty \tanh(\xi hs)A(\xi)\sin(\xi x)d\xi = \frac{\pi p}{2\mu s}, \quad x \in I_2, I_4 \qquad (3.6a\text{-}b)$$

Substitution of equation (3.4) in (3.6a) and use of the following result (Das, 1992)

$$\int_0^{\infty} \xi^{-1}\tanh(\xi hs)\sin(\xi u)\sin(\xi x)d\xi = \frac{1}{2}\log\left|\frac{\sinh(ex)+\sinh(eu)}{\sinh(ex)-\sinh(eu)}\right|$$

yields

$$\int_a^b \frac{eg(u^2)\sinh(2eu)}{\sinh^2(eu)-\sinh^2(ex)}\,du + \int_c^1 \frac{eh(v^2)\sinh(2ev)}{\sinh^2(ev)-\sinh^2(ex)}\,dv \tag{3.7}$$

$$= \frac{\pi p}{\mu s\cosh(ex)}, \qquad x \in l_2 .$$

Substituting cosh(eu) = U, cosh(ev) = S equation (3.7) is found to reduce to the form

$$\int_A^B \frac{UG(U^2)}{U^2-X^2}\,dU = \frac{\pi}{2}F(X), \qquad (A<X<B) \tag{3.8}$$

where $X = \cosh(ex)$, $A = \cosh(ea)$, $B = \cosh(eb)$, $C = \cosh(ec)$, $D = \cosh(e)$, $g(u^2) = G(U^2)$, $h(v^2) = H(S^2)$ and

$$F(X) = \frac{p}{\mu sX} - \frac{2}{\pi}\int_C^D \frac{SH(S^2)}{S^2-X^2}\,dS \tag{3.9}$$

Using the finite Hilbert transform technique (Srivastava et al., 1968) the solution of equation (3.8) is

$$G(U^2) = -\frac{2}{\pi}\sqrt{\frac{U^2-A^2}{B^2-U^2}}\int_A^B \sqrt{\frac{B^2-X^2}{X^2-A^2}}\left[\frac{p}{\mu sX} - \frac{2}{\pi}\int_C^D \frac{SH(S^2)}{S^2-X^2}\,dS\right]\times$$

$$\times\frac{X\,dX}{X^2-U^2} + \frac{B_1}{\sqrt{(U^2-A^2)(B^2-U^2)}}, \qquad (A<U<B) \tag{3.10}$$

The constant B_1 is to be determined from equation (3.5a). Using the result

$$\int_A^B \sqrt{\frac{B^2-X^2}{X^2-A^2}} \frac{X\ dX}{(U^2-X^2)(S^2-X^2)} = \frac{\pi}{2} \sqrt{\frac{S^2-B^2}{S^2-A^2}} \frac{1}{S^2-U^2}$$

equation (3.10) can be rewritten as

$$G(U^2) = \frac{2p}{\pi\mu s} \sqrt{\frac{U^2-A^2}{B^2-U^2}} \int_A^B \sqrt{\frac{B^2-X^2}{X^2-A^2}} \frac{dX}{U^2-X^2} - \frac{2}{\pi} \sqrt{\frac{U^2-A^2}{B^2-U^2}} \times$$

$$\times \int_C^D \sqrt{\frac{S^2-B^2}{S^2-A^2}} \frac{SH(S^2)}{S^2-U^2} dS + \frac{B_1}{\sqrt{(U^2-A^2)(B^2-U^2)}}, \quad (A<U<B) \qquad (3.11)$$

Substitution of expression for $A(\xi)$ from (3.4) in (3.66) yields with aid of (3.11) the following singular integral equation involving $H(S^2)$

$$\int_C^D \sqrt{\frac{S^2-B^2}{S^2-A^2}} \frac{SH(S^2)}{S^2-U^2} dS = \frac{\pi}{2}\left[\frac{p}{\mu sX} \sqrt{\frac{X^2-B^2}{X^2-A^2}} - \frac{2pA^2}{\pi\mu sBX^2} \times \right.$$

$$\left. \times \left\{ \left(\frac{X^2-B^2}{X^2-A^2}\right) \Pi\left(\frac{\pi}{2}, \frac{X^2(B^2-A^2)}{B^2(X^2-A^2)}, q\right) - \frac{B^2}{A^2} F\left(\frac{\pi}{2}, q\right) \right\} + \frac{B_1}{X^2-A^2} \right]$$

(3.12)

where $q = \frac{\sqrt{B^2-A^2}}{B}$ and $F(\phi,k)$, $\Pi(\phi,n,k)$ are elliptic integrals of first and third kind respectively.

While deriving equation (3.12), the following results have been made use of.

$$\int_A^B \sqrt{\frac{U^2-A^2}{B^2-U^2}} \frac{U\,dU}{(X^2-U^2)(S^2-U^2)} = \frac{\pi}{2(V^2-X^2)} \left[\sqrt{\frac{X^2-A^2}{X^2-B^2}} - \sqrt{\frac{S^2-A^2}{S^2-U^2}} \right]$$

$$\int_A^B \sqrt{\frac{U^2-A^2}{B^2-U^2}} \frac{U\,dU}{(X^2-U^2)} \int_A^B \sqrt{\frac{B^2-Z^2}{Z^2-A^2}} \frac{dZ}{(U^2-Z^2)} = -\frac{\pi}{2BX^2} \sqrt{\frac{X^2-A^2}{X^2-B^2}} \left[A^2 \left(\frac{X^2-B^2}{X^2-A^2}\right) \Pi - B^2 F \right]$$

and

$$\int_A^B \frac{U\,dU}{(X^2-U^2)\sqrt{(U^2-A^2)(B^2-U^2)}} = \frac{\pi}{2\sqrt{(X^2-A^2)(X^2-B^2)}} \qquad (C<X<D).$$

Again, using finite Hilbert transform technique (Srivastava et al., 1968) it is found that

$$H(S^2) = -\frac{2}{\pi} \sqrt{\frac{(S^2-A^2)(S^2-C^2)}{(S^2-B^2)(D^2-S^2)}} \left[\frac{p}{\mu s} \left\{ \int_C^D \sqrt{\frac{(D^2-X^2)(X^2-B^2)}{(X^2-C^2)(X^2-A^2)}} \times \right.\right.$$

$$\left.\left. \times \frac{dX}{(X^2-S^2)} - \int_C^D \sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(Y^2-A^2)(C^2-Y^2)}} \frac{dY}{(S^2-Y^2)} \right\} - \frac{\pi}{2} \sqrt{\frac{D^2-A^2}{C^2-A^2}} \frac{B_1}{(S^2-A^2)} \right] +$$

$$+ \frac{B_2 \sqrt{S^2-A^2}}{\sqrt{(S^2-B^2)(S^2-C^2)(D^2-S^2)}} \qquad (C<S<D) \qquad (3.13)$$

where we have used

$$\int_C^D \sqrt{\frac{D^2-X^2}{X^2-C^2}}\ \frac{X\,dX}{(X^2-A^2)(X^2-S^2)} = -\frac{\pi}{2}\sqrt{\frac{D^2-A^2}{C^2-A^2}}\ \frac{1}{(S^2-A^2)}$$

the constant B_z occuring in (3.13) is to be determined using the condition given by equation (3.5b).

Next, substituting the value of $H(S^2)$ from equation (3.13) in equation (3.11) and using the following results

$$\int_C^D \sqrt{\frac{S^2-C^2}{D^2-S^2}}\ \frac{S\,dS}{(S^2-U^2)(X^2-S^2)} = -\frac{\pi}{2}\sqrt{\frac{C^2-U^2}{D^2-U^2}}\ \frac{1}{(X^2-U^2)}$$

$$\int_C^D \sqrt{\frac{S^2-C^2}{D^2-S^2}}\ \frac{S\,dS}{(S^2-A^2)(S^2-U^2)} = -\frac{\pi}{2(U^2-A^2)}\left[\sqrt{\frac{C^2-A^2}{D^2-A^2}} - \sqrt{\frac{C^2-U^2}{D^2-U^2}}\right]$$

$$\int_C^D \frac{S\,dS}{(S^2-U^2)\sqrt{(S^2-C^2)(D^2-S^2)}} = \frac{\pi}{2\sqrt{(C^2-U^2)(D^2-U^2)}} \qquad (A<U<B).$$

$G(U^2)$ may be written in the following form

$$G(U^2) = \frac{2}{\pi}\sqrt{\frac{U^2-A^2}{B^2-U^2}}\ \frac{p}{\mu s}\left[\frac{(B^2-U^2)}{BU^2(U^2-A^2)}\left\{A^2\Pi\left(\frac{\pi}{2},\ \frac{X^2(B^2-A^2)}{B^2(X^2-A^2)},\ q\right] + \right.\right.$$

$$\left. + (U^2-A^2)F(\frac{\pi}{2},\ q)\right\} + \frac{1}{B}F(\frac{\pi}{2},\ q) - \sqrt{\frac{C^2-U^2}{D^2-U^2}}\left\{\int_C^D \sqrt{\frac{(D^2-X^2)(X^2-B^2)}{(X^2-C^2)(X^2-A^2)}} \times\right.$$

$$\times \frac{dX}{(X^2-U^2)} + \int_A^B \sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(Y^2-A^2)(C^2-Y^2)}} \frac{dY}{(Y^2-U^2)} \Bigg\} + \int_A^B \sqrt{\frac{(B^2-Y^2)}{(Y^2-A^2)}} \frac{dY}{(Y^2-U^2)} \Bigg] +$$

$$+ \sqrt{\frac{(D^2-A^2)(C^2-U^2)}{(C^2-A^2)(D^2-U^2)}} \frac{B_1}{\sqrt{(U^2-A^2)(B^2-U^2)}} - \frac{B_2 \sqrt{U^2-A^2}}{\sqrt{(B^2-U^2)(C^2-U^2)(D^2-U^2)}}$$

$$(A<U<B) \qquad (3.14)$$

To determine the values of the unknown constants B_1 and B_2, we substitute $H(S^2)$ and $G(U^2)$ given by (3.13) and (3.14) in (3.5a,b) and obtain

$$B_1 = \frac{p}{\mu s} \left\{ \frac{K_3(K_{1,2}-K_{1,1}) - K_6(K_{1,3}+K_{2,3})}{RK_4K_6 + K_3K_5} \right\} \qquad (3.15a)$$

$$B_2 = \frac{p}{\mu s} \left\{ \frac{RK_4(K_{1,1}-K_{1,2}) - K_5(K_{1,3}+K_{2,3})}{RK_4K_6 + K_3K_5} \right\} \qquad (3.15b)$$

where

$$K_{1,1} = \int_C^D M_1(X)dX \int_C^D \frac{M_2(S)}{X^2-S^2} dS \qquad (3.16)$$

$$K_{1,2} = \int_A^B M_1(Y)dY \int_C^D \frac{M_2(S)}{S^2-Y^2} dS \qquad (3.17)$$

$$K_{1,3} = \int_C^D M_1(X)dX \int_A^B \frac{M_2(U)}{X^2-U^2} dU \qquad (3.18)$$

$$K_{2,3} = \int_A^B M_1(Y)dY \int_A^B \frac{M_2(U)}{Y^2-U^2} dU \qquad (3.19)$$

$$K_3 = \frac{\pi}{2} \int_A^B \frac{M_2(U)}{C^2 - U^2} \, dU \quad , \quad K_4 = \int_A^B \frac{M_2(U)}{U^2 - A^2} \, dU \qquad (3.20)$$

$$K_5 = R \int_C^D \frac{M_2(S)}{S^2 - A^2} \, dS \quad , \quad K_6 = \frac{\pi}{2} \int_C^D \frac{M_2(S)}{S^2 - C^2} \, dS \qquad (3.21)$$

$$M_1(T) = \sqrt{\frac{(D^2 - T^2)(T^2 - B^2)}{(T^2 - C^2)(T^2 - A^2)}} \quad , \quad M_2(T) = \sqrt{\frac{(T^2 - A^2)(T^2 - C^2)}{(T^2 - B^2)(D^2 - T^2)}} \, \frac{T}{\sqrt{T^2 - 1}} \qquad (3.22)$$

and

$$R = -\frac{\pi}{2} \sqrt{\frac{D^2 - A^2}{C^2 - A^2}} \qquad (3.23)$$

4. STRESS INTENSITY FACTORS

The corresponding displacement and stress components in the plane of the cracks may be written as

$$w(x,0) = \frac{1}{e} \int_x^B \frac{UG(U^2)}{\sqrt{U^2 - 1}} \, dU \quad , \quad x \in l_2$$

$$= \frac{1}{e} \int_x^D \frac{SH(S^2)}{\sqrt{S^2 - 1}} \, dS \quad , \quad x \in l_4 \qquad (4.1a,b)$$

and

$$\left[\sigma_{yz}(x,0)\right]_{0<x<a} = -\frac{2\mu s X}{\pi} \left[\int_A^B \frac{UG(U^2)}{U^2 - X^2} \, dU + \int_C^D \frac{SH(S^2)}{S^2 - X^2} \, dS \right] \qquad (4.2a)$$

$$\left[\sigma_{yz}(x,0)\right]_{b<x<c} = \frac{2\mu s X}{\pi}\left[\int_A^B \frac{UG(U^2)}{X^2-U^2}\,dU - \int_C^D \frac{SH(S^2)}{S^2-X^2}\,dS\right] \qquad (4.2b)$$

$$\left[\sigma_{yz}(x,0)\right]_{x>1} = \frac{2\mu s X}{\pi}\left[\int_A^B \frac{UG(U^2)}{X^2-U^2}\,dU + \int_C^D \frac{SH(S^2)}{X^2-S^2}\,dS\right] \qquad (4.2c)$$

With the aid of the results given by equations (3.13) and (3.14) the expressions (4.2a-c) yield after some algebraic manipulation, the results

$$\left[\sigma_{yz}(x,0)\right]_{0<x<a} = \frac{2\mu s}{\pi}\left[F_1(X) - F_2(X) - F_3(X) + F_4(X) - F_5(X) - F_6(X) - F_7(X) - F_8(X)\right] \qquad (4.3a)$$

$$\left[\sigma_{yz}(x,0)\right]_{b<x<c} = \frac{2\mu s}{\pi}\left[F_1(X) - F_2(X) + F_3(X) + F_4(X) - F_5(X) - F_6(X) - F_7(X) - F_8(X)\right] \qquad (4.3b)$$

$$\left[\sigma_{yz}(x,0)\right]_{x>1} = \frac{2\mu s}{\pi}\left[F_1(X) - F_2(X) + F_3(X) + F_4(X) - F_5(X) - F_6(X) - F_7(X) + F_8(X)\right] \qquad (4.3c)$$

where

$$F_1(X) = \frac{2pX}{\pi\mu s}\int_C^D \sqrt{\frac{(D^2-Y^2)(Y^2-B^2)}{(Y^2-C^2)(Y^2-A^2)}}\left[\frac{\pi}{2(Y^2-X^2)}\left\{\sqrt{\frac{Y^2-A^2}{Y^2-B^2}} - \sqrt{\frac{A^2-X^2}{B^2-X^2}}\right\}\sqrt{\frac{C^2-B^2}{D^2-B^2}} + I_{A,C}^{B,D}(X,Y)\right] dY \qquad (4.4a)$$

$$F_2(X) = \frac{2pX}{\pi\mu s}\int_A^B \sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(C^2-Y^2)(Y^2-A^2)}}\left[\frac{\pi}{2(Y^2-X^2)}\times\right.$$

$$\left.\times\sqrt{\frac{(C^2-B^2)(A^2-X^2)}{(D^2-B^2)(B^2-X^2)}} + L_{A,C}^{B,D}(X,Y)\right] dY \qquad (4.4b)$$

$$F_3(X) = \frac{B_1 X}{X_1}\left[\frac{\pi}{2}\sqrt{\frac{(C^2-B^2)}{(D^2-B^2)}} + J_{A,C}^{B,D}(X)\right]\sqrt{\frac{(D^2-A^2)}{(C^2-A^2)}} \qquad (4.4c)$$

$$F_4(X) = B_2 X\left[\frac{\pi}{2\sqrt{(C^2-B^2)(D^2-B^2)}}\left\{1-\sqrt{\frac{(A^2-X^2)}{(B^2-X^2)}}\right\} + K_{A,C}^{B,D}(X)\right] \qquad (4.4d)$$

$$F_5(X) = \frac{2pX}{\pi\mu s}\int_C^D \sqrt{\frac{(D^2-Y^2)(Y^2-B^2)}{(Y^2-C^2)(Y^2-A^2)}}\left[\frac{\pi}{2(Y^2-X^2)}\times\right.$$

$$\left.\times\sqrt{\frac{(D^2-A^2)(C^2-X^2)}{(D^2-B^2)(D^2-X^2)}} - L_{C,A}^{D,B}(X,Y)\right] dY \qquad (4.4e)$$

$$F_6(X) = \frac{2pX}{\pi\mu s}\int_A^B \sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(C^2-Y^2)(Y^2-A^2)}}\left[\frac{\pi}{2(Y^2-X^2)}\left\{\sqrt{\frac{C^2-X^2}{D^2-X^2}}\right.\right. -$$

$$\left.\left. - \sqrt{\frac{C^2-Y^2}{D^2-Y^2}}\right\}\sqrt{\frac{D^2-A^2}{D^2-B^2}} + I_{C,A}^{D,B}(X,Y)\right] dY \qquad (4.4f)$$

$$F_7(X) = \frac{B_1 X}{(A^2-X^2)} \left[\frac{\pi}{2} \sqrt{\frac{(D^2-A^2)}{(D^2-B^2)}} \left\{ \sqrt{\frac{C^2-X^2}{D^2-X^2}} - \sqrt{\frac{C^2-A^2}{D^2-A^2}} \right\} + \right.$$

$$\left. + I^{D,B}_{C,A}(X,A) \right] \sqrt{\frac{(D^2-A^2)}{(C^2-A^2)}} \qquad (4.4g)$$

$$F_8(X) = \frac{B_1 X}{X_2} \left[\frac{\pi}{2} \sqrt{\frac{(D^2-A^2)}{(D^2-B^2)}} - J^{D,B}_{C,A}(X) \right] \qquad (4.4h)$$

$$I^{Q,S}_{P,R}(X,Y) = \int_P^Q \left(\frac{S^2-R^2}{Y^2-X^2}\right) \left\{ \sqrt{\frac{(Y^2-P^2)}{(Y^2-Q^2)}} \tan^{-1} \sqrt{\frac{(U^2-P^2)(Y^2-Q^2)}{(Q^2-U^2)(Y^2-P^2)}} - \right.$$

$$\left. - \sqrt{\frac{(P^2-X^2)}{(Q^2-X^2)}} \tan^{-1} \sqrt{\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}} \right\} \frac{U\, dU}{\sqrt{(R^2-U^2)(S^2-U^2)^3}} \qquad (4.4i)$$

$$L^{Q,S}_{P,R}(X,Y) = \int_P^Q \left(\frac{S^2-R^2}{Y^2-X^2}\right) \left\{ \sqrt{\frac{(P^2-X^2)}{(Q^2-X^2)}} \tan^{-1} \sqrt{\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}} + \right.$$

$$\left. + \frac{1}{2} \sqrt{\frac{(Y^2-P^2)}{(Q^2-Y^2)}} \log \left| \frac{\sqrt{(U^2-P^2)(Q^2-Y^2)} - \sqrt{(Q^2-U^2)(Y^2-P^2)}}{\sqrt{(U^2-P^2)(Q^2-Y^2)} + \sqrt{(Q^2-U^2)(Y^2-P^2)}} \right| \right\} \times$$

$$\times \frac{U\, dU}{\sqrt{(R^2-U^2)(S^2-U^2)^3}} \qquad (4.4j)$$

$$J^{Q,S}_{P,R}(X) = \int_P^Q \left\{ \tan^{-1}\sqrt{\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}} \right\} \frac{U(S^2-R^2)\,dU}{\sqrt{(R^2-U^2)(S^2-U^2)^3}} \qquad (4.4k)$$

$$K^{Q,S}_{P,R}(X) = \int_P^Q \left\{ \tan^{-1}\sqrt{\frac{(U^2-P^2)}{(Q^2-U^2)}} - \sqrt{\frac{(P^2-X^2)}{(Q^2-X^2)}} \times \right.$$

$$\left. \times \tan^{-1}\sqrt{\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}} \right\} \frac{U(2U^2-R^2-S^2)\,dU}{\sqrt{(R^2-U^2)^3(S^2-U^2)^3}} \qquad (4.4l)$$

$$X_1 = \sqrt{(A^2-X^2)(B^2-X^2)} \quad , \qquad X_2 = \sqrt{(C^2-X^2)(D^2-X^2)} \ . \qquad (4.4m)$$

The dynamic stress intensity factors are given by

$$N_a = \underset{x \to a-}{\mathrm{Lt}} \sqrt{2(a-x)} \left| \frac{\sigma_{yz}(x,0)}{p} \right|_{0<x<a} \qquad (4.5a)$$

$$N_b = \underset{x \to b+}{\mathrm{Lt}} \sqrt{2(x-b)} \left| \frac{\sigma_{yz}(x,0)}{p} \right|_{b<x<c} \qquad (4.5b)$$

$$N_c = \underset{x \to c-}{\mathrm{Lt}} \sqrt{2(c-x)} \left| \frac{\sigma_{yz}(x,0)}{p} \right|_{b<x<c} \qquad (4.5c)$$

$$N_1 = \underset{x \to 1+}{\mathrm{Lt}} \sqrt{2(x-1)} \left| \frac{\sigma_{yz}(x,0)}{p} \right|_{x>1} \qquad (4.5d)$$

With the aid of the results given by (4.3) in (4.5) it follows that

$$N_a = - \frac{\mu s\sqrt{A}}{\sqrt{e(A^2-1)^{1/2}(B^2-A^2)}} B_1 \qquad (4.6a)$$

$$N_b = -\frac{\mu s\sqrt{B}}{\sqrt{e(B^2-1)^{1/2}}}\left[-\frac{2p}{\pi\mu s}\sqrt{\frac{(B^2-A^2)(C^2-B^2)}{(D^2-B^2)}}\left\{\int_A^B G_1(Y)dY + \right.\right.$$

$$\left.\left. +\int_C^D G_1(Y)dY\right\} + \sqrt{\frac{(C^2-B^2)(D^2-A^2)}{(B^2-A^2)(C^2-A^2)(D^2-B^2)}}\,B_1 - \sqrt{\frac{(B^2-A^2)}{(C^2-B^2)(D^2-B^2)}}\,B_2\right]$$

(4.6b)

$$N_c = -\frac{\mu s\sqrt{C(C^2-A^2)}}{\sqrt{e(C^2-1)^{1/2}(C^2-B^2)(D^2-C^2)}}\,B_2 \qquad (4.6c)$$

$$N_1 = -\frac{\mu s\sqrt{D}}{\sqrt{e(D^2-1)^{1/2}}}\left[-\frac{2p}{\pi\mu s}\sqrt{\frac{(D^2-A^2)(D^2-C^2)}{(D^2-B^2)}}\left\{\int_A^B G_2(Y)dY + \right.\right.$$

$$\left.\left. +\int_C^D G_2(Y)dY\right\} + \sqrt{\frac{(D^2-C^2)}{(D^2-B^2)(C^2-A^2)}}\,B_1 + \sqrt{\frac{(D^2-A^2)}{(D^2-C^2)(D^2-B^2)}}\,B_2\right]$$

(4.6d)

where

$$G_1(Y) = \frac{\sqrt{(D^2-Y^2)}}{\sqrt{(Y^2-A^2)(Y^2-B^2)(Y^2-C^2)}} \qquad (4.7a)$$

$$G_2(Y) = \frac{\sqrt{(B^2-Y^2)}}{\sqrt{(Y^2-A^2)(C^2-Y^2)(D^2-Y^2)}} \qquad (4.7b)$$

The crack opening displacements are obtained by using the expressions for $G(U^2)$ and $H(S^2)$ from equations (3.14) and (3.13) in equations (4.1a,b).

Again letting $a \to 0$ and simplifying, it may be noted that the results (4.6b), (4.6c) and (4.6d) become those given by equations (3.16) of Das (1993).

5. NUMERICAL RESULTS

The numerical values of stress intensity factors (SIF) N_a, N_b, N_c and N_1 given by (4.6a-d) at the tips of the crack have been plotted against crack speed (V/c_2) for different values of crack lengths, separating distances of the cracks and strip width(h).

Keeping the length of the outer cracks and distance between inner and outer cracks fixed (b=0.6, c=0.8) SIFs at the tips of the cracks have been plotted against crack speed ($0.1 \le V/c_2 < 1$) for different lengths of the inner cracks (a=0.2, 0.4) and strip width (h=1,3,5). It is found from the graphs (fig.2-5) that SIFs increase rapidly as $V/c_2 \to 1$ and with the decrease in the value of inner crack length i.e. with the increase in the value of the distance between inner cracks the value of SIF decreases.

When lengths of the outer cracks and the distance between inner cracks are kept fixed (a=0.2, c=0.8) it is noted from the graphs (fig.6-9) that with the increase in the value of b (0.4, 0.6) i.e. with the decrease in the value of the distance between inner and

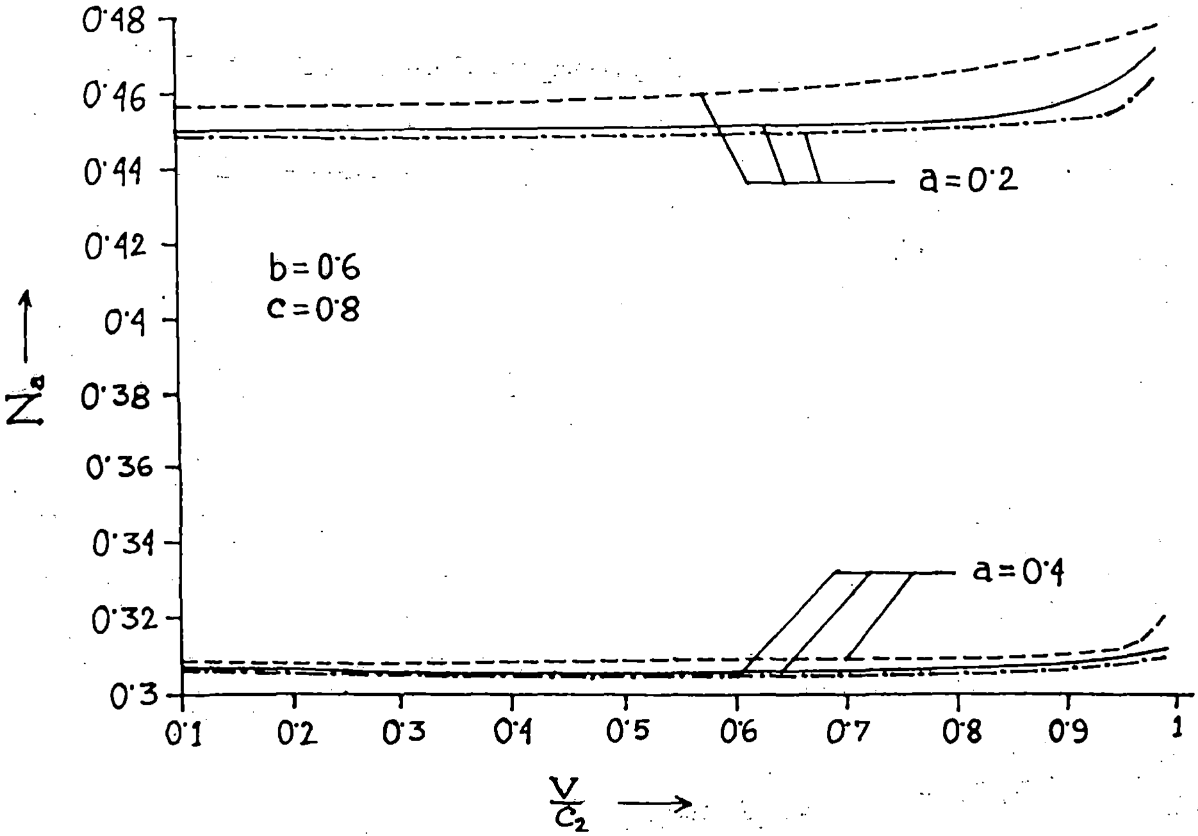

Fig.2. Stress intensity factor N_a vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

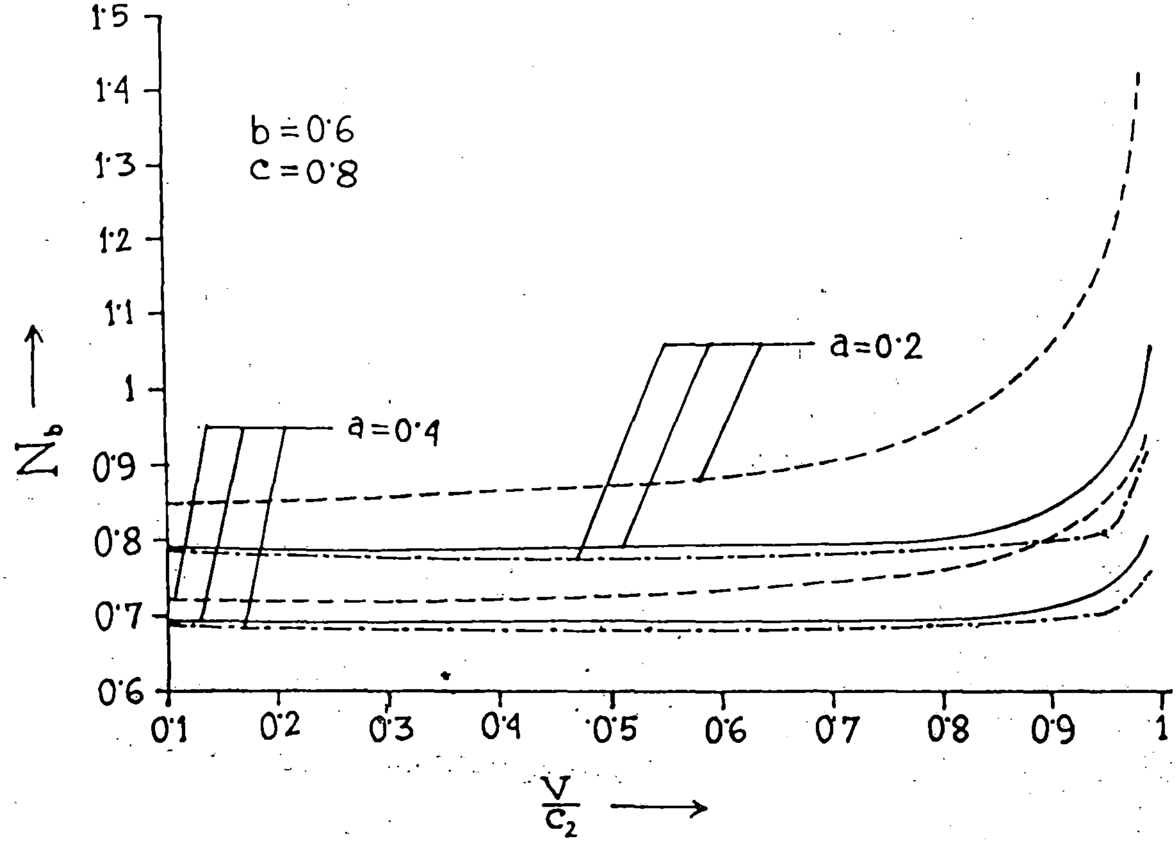

Fig. 3. Stress intensity factor N_b vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

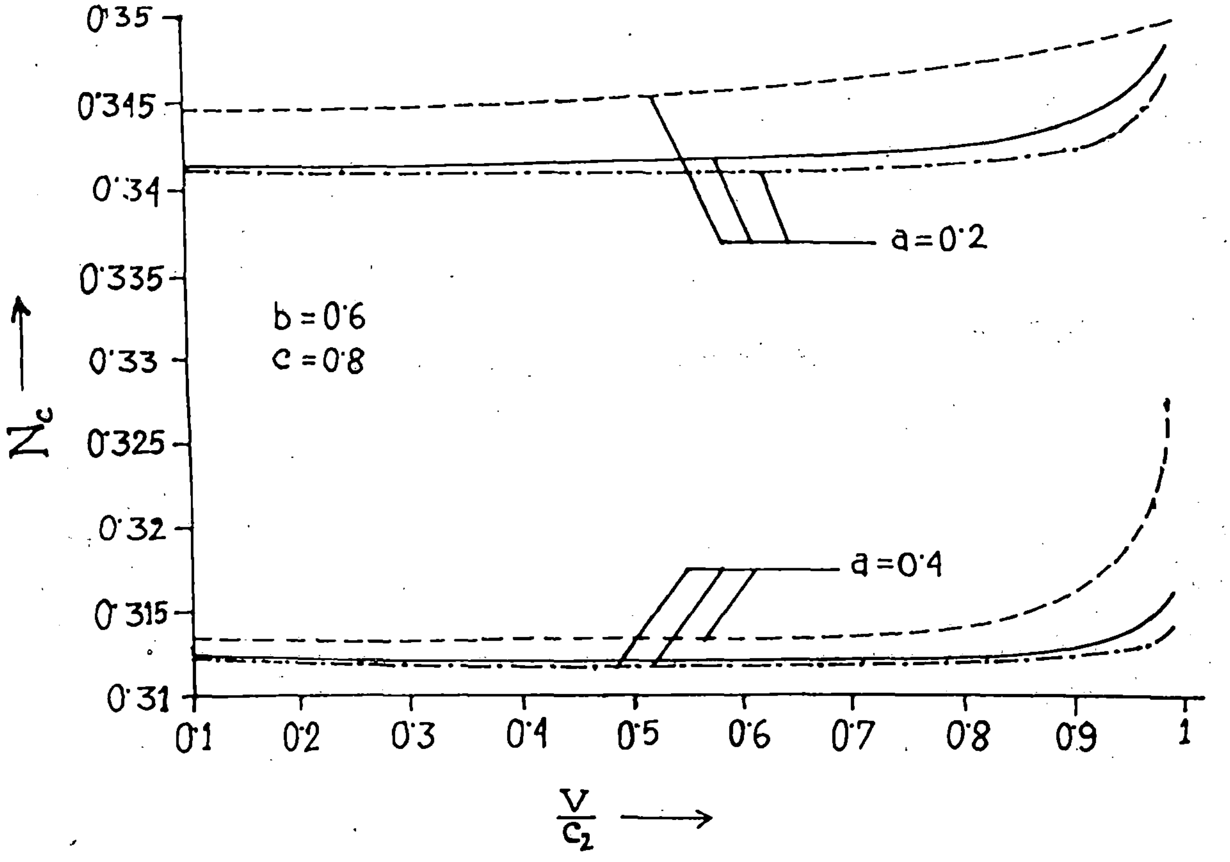

Fig. 4. Stress intensity factor N_c vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

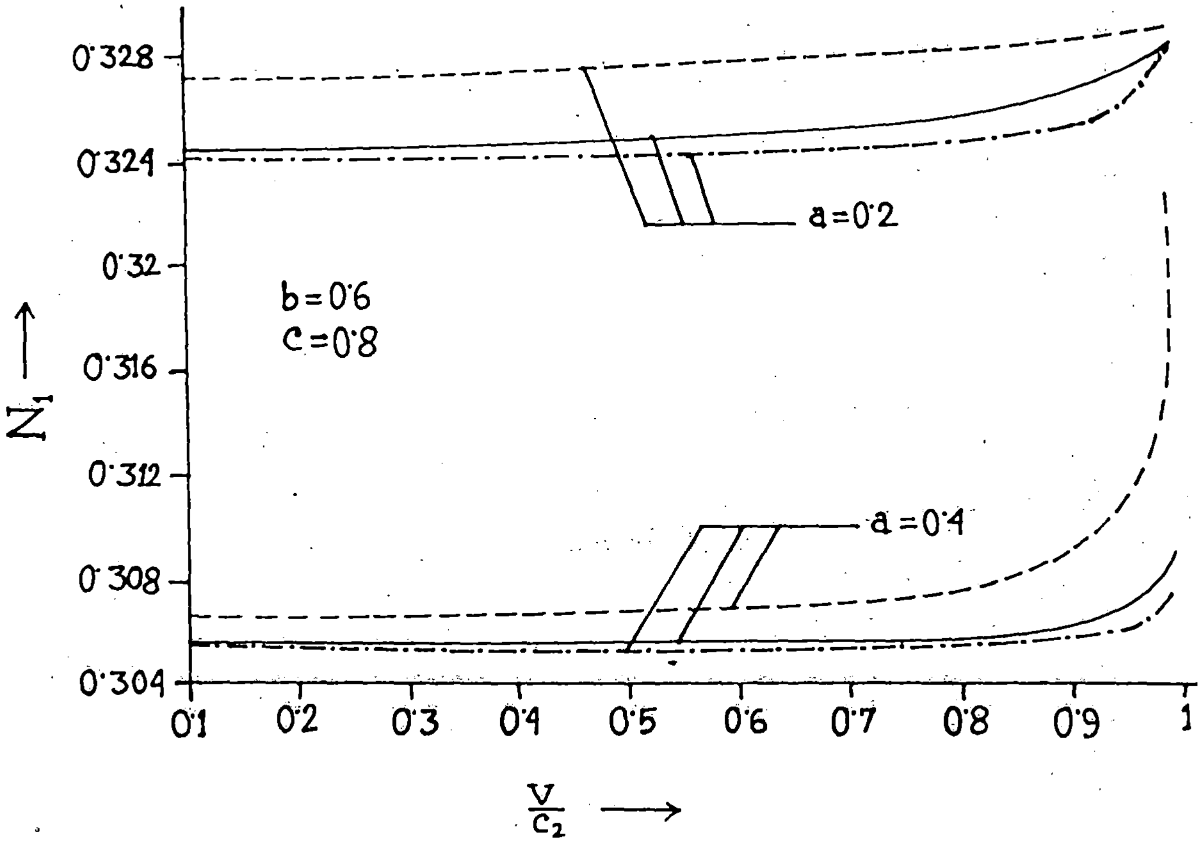

Fig. 5. Stress intensity factor N_1 vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

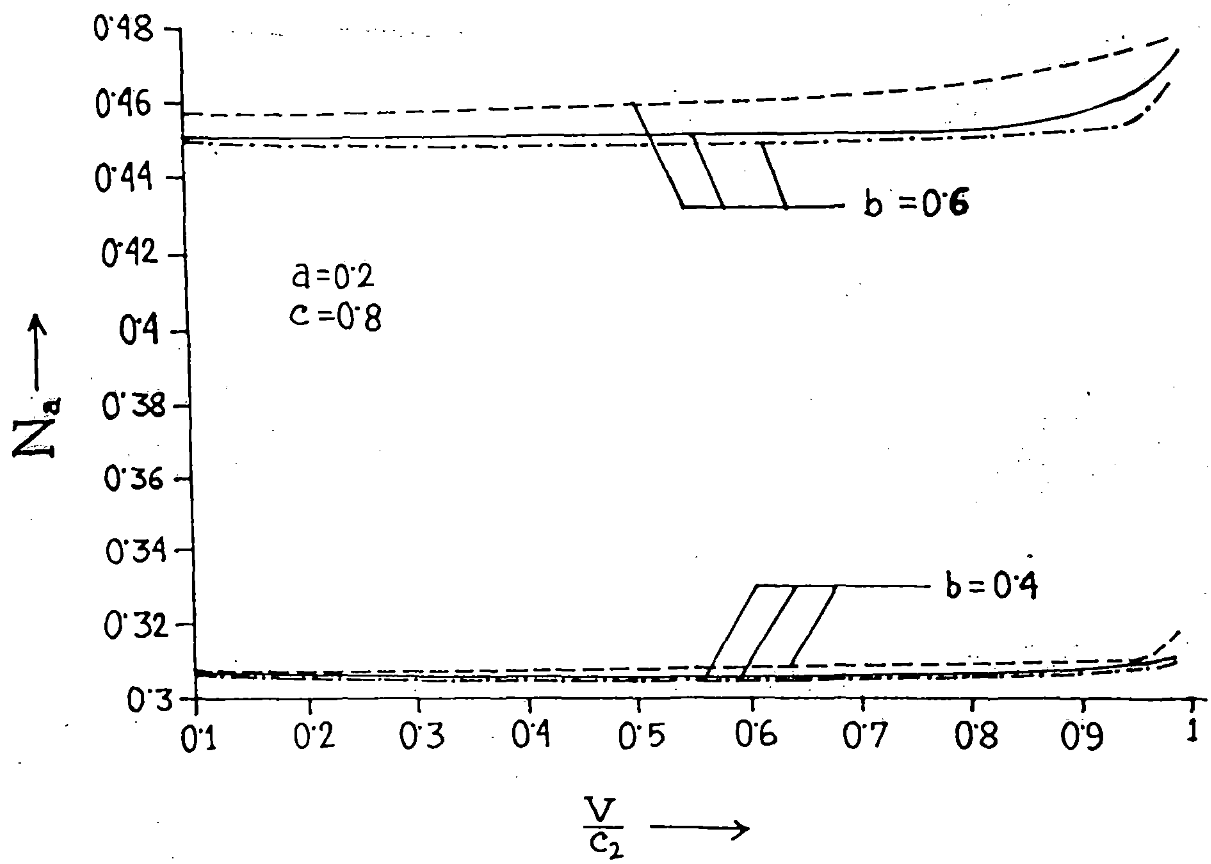

Fig.6. Stress intensity factor N_a vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

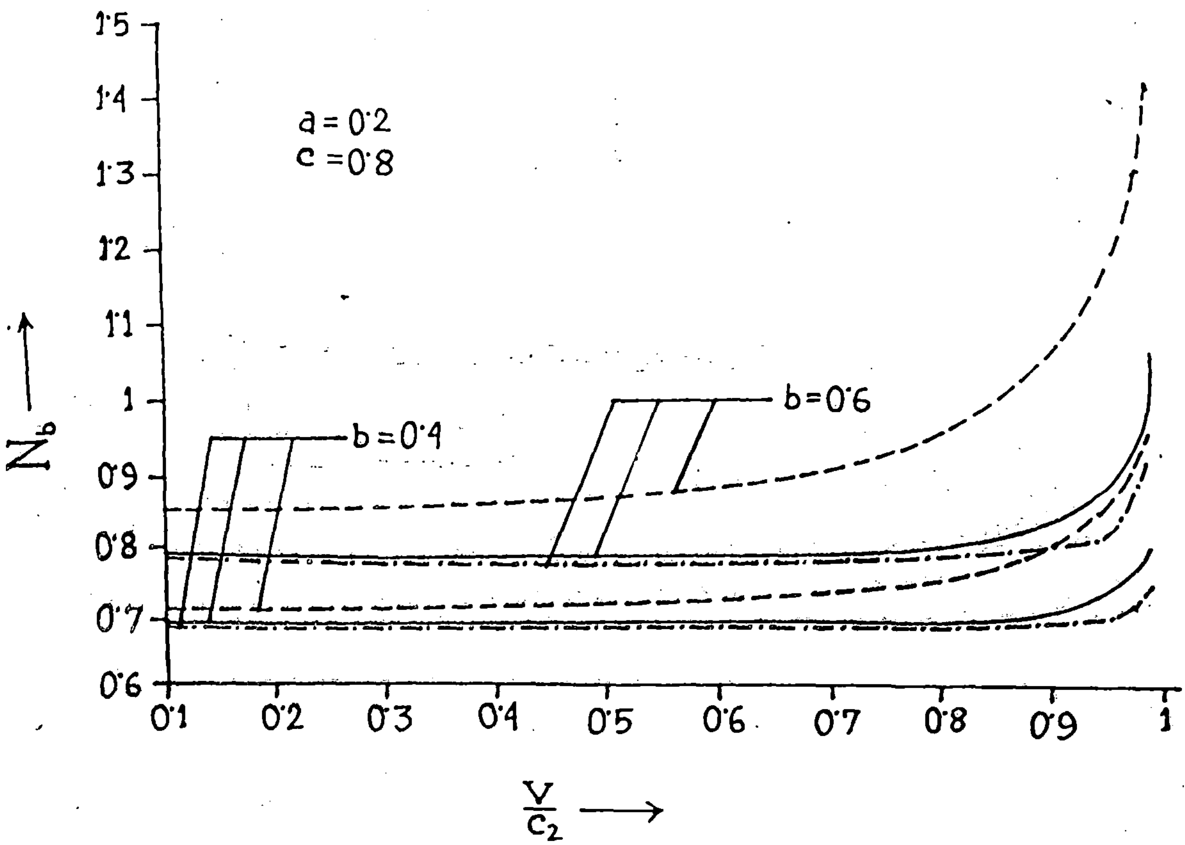

Fig.7. Stress intensity factor N_b vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

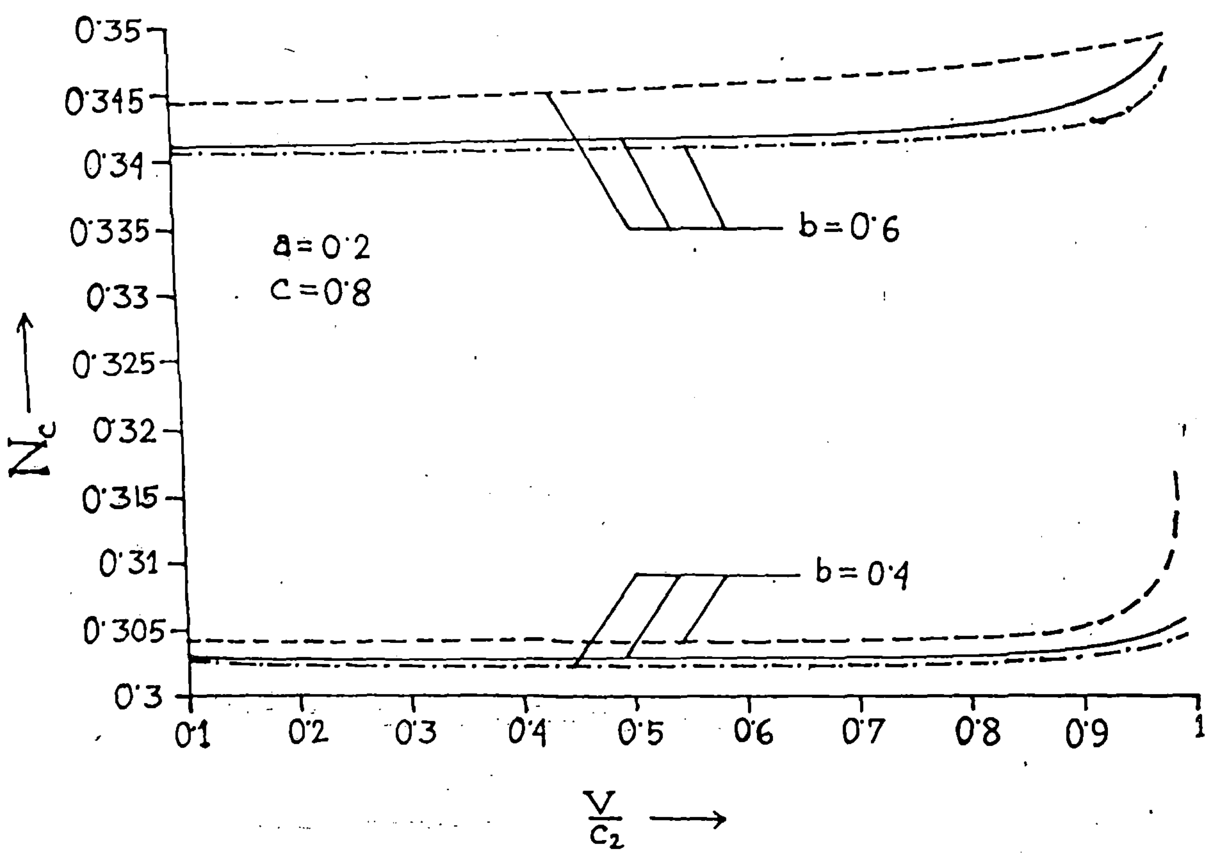

Fig. 8. Stress intensity factor N_c vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

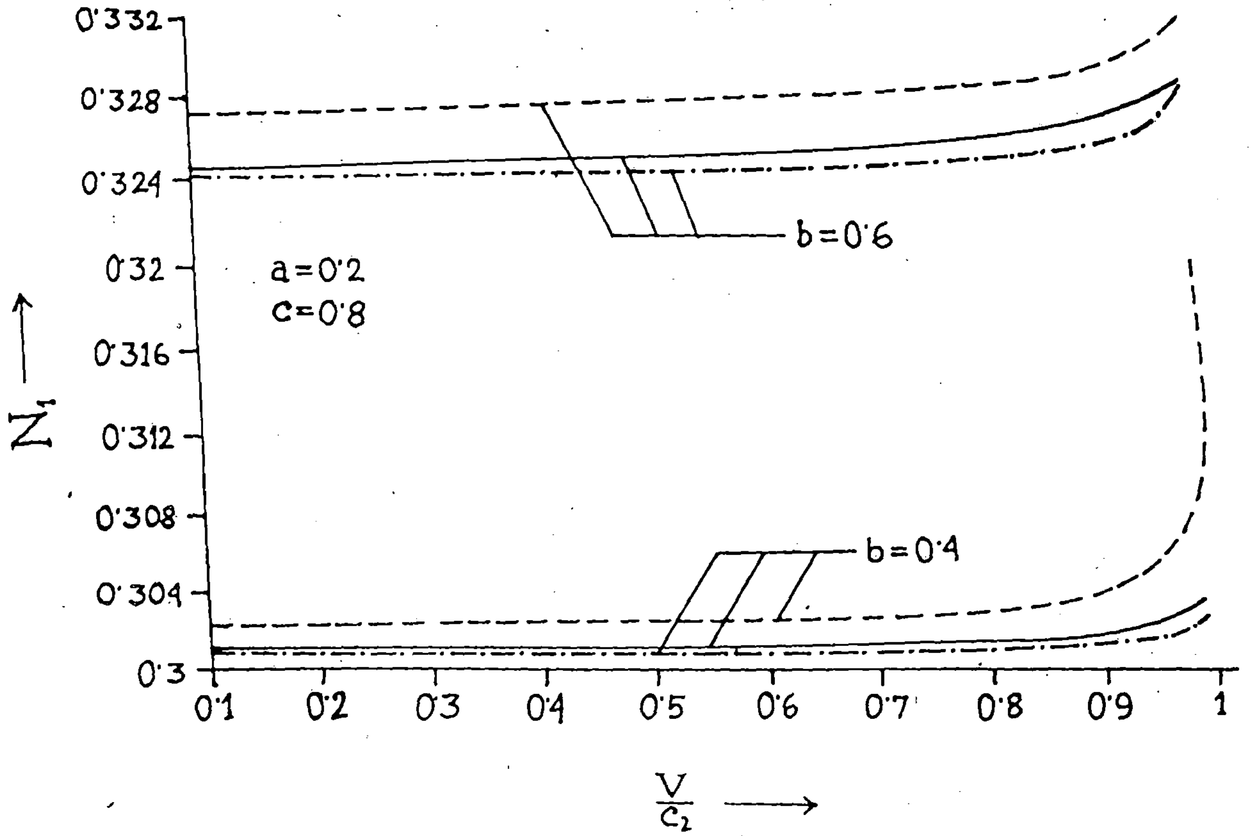

Fig. 9. Stress intensity factor N_1 vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

outer cracks SIF increases.

Next, keeping the lengths of the inner cracks fixed (a=0.2, b=0.4), it is seen from the graphs (fig.10-13) that the value of SIF N_b is higher for higher values of c(0.6, 0.8). But the nature is opposite in case of N_a, N_b and N_1.

In all the cases mentioned above the SIFs increase with the increase in the value of V/c_2 gradually at a slow rate in the begining but increase rapidly as $V/c_2 \to 1$. Also the value of SIFs are higher for lower values of h in these cases.

The nature of SIFs, when plotted against 'a' are exhibited in fig.14-17. In fig.14 for fixed strip width (h=2) SIFs have been plotted against 'a' for different values of V/c_2(0.1,0.8). From the graph it is found that SIF N_b firstly increases with the increase in the value of 'a', attains a maximum and then decreases rapidly whereas SIFs N_b and N_1 decrease gradually with the increase in the value of 'a'. Further it is found that SIFs are higher for higher values of V/c_2.

In fig.15-17, SIFs have been plotted against 'a' for different values of h(1,2,3,4) when V/c_2 is kept fixed. With the increase in the value of 'a', N_b first increases and then decreases sharply. But N_c and N_1 decrease gradually (fig.16-17) with the increase in the value of 'a'. In all the cases (fig.15-17) values of SIFs are found to be higher for lower values of h as expected from the physical standpoint.

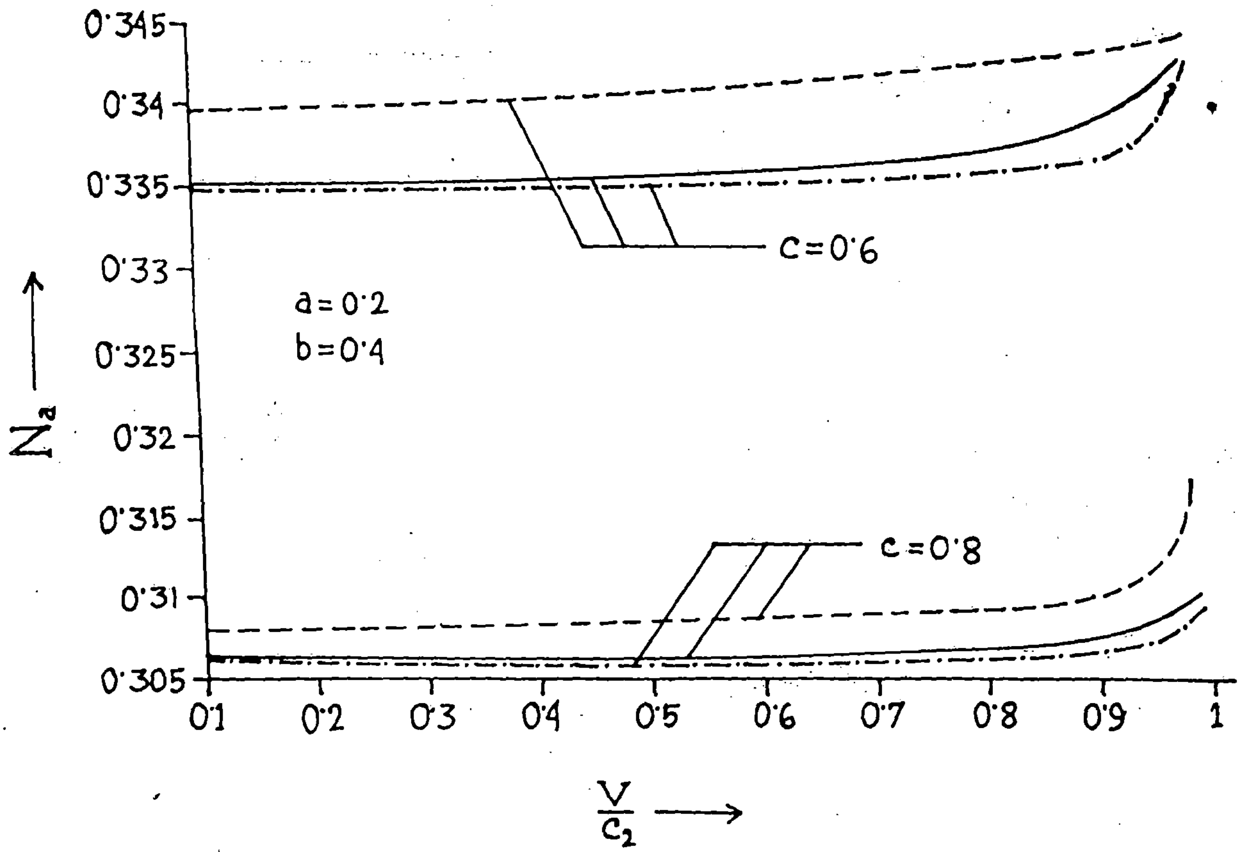

Fig.10. Stress intensity factor N_a vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

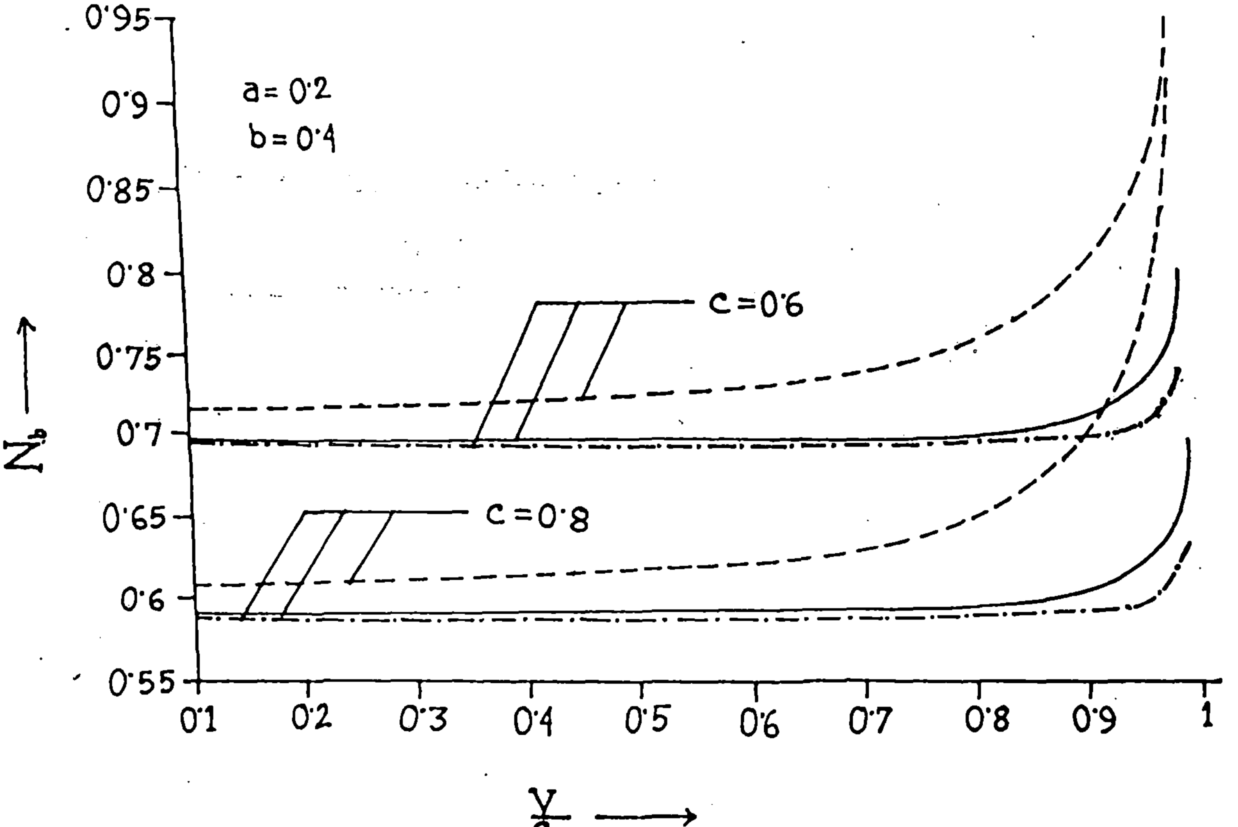

Fig.11. Stress intensity factor N_b vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

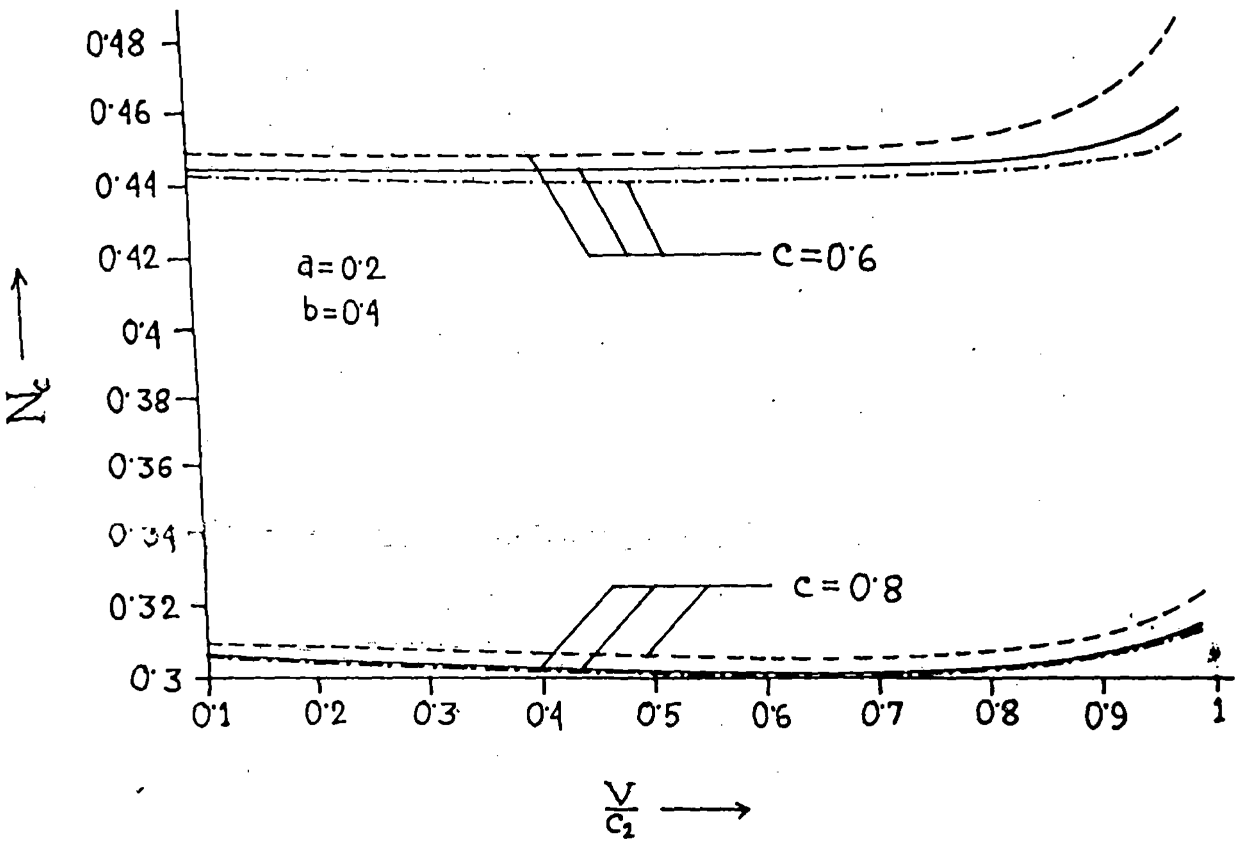

Fig.12. Stress intensity factor N_c vs. V/c_2.
(---- h=1, —— h=2, —·—·— h=5).

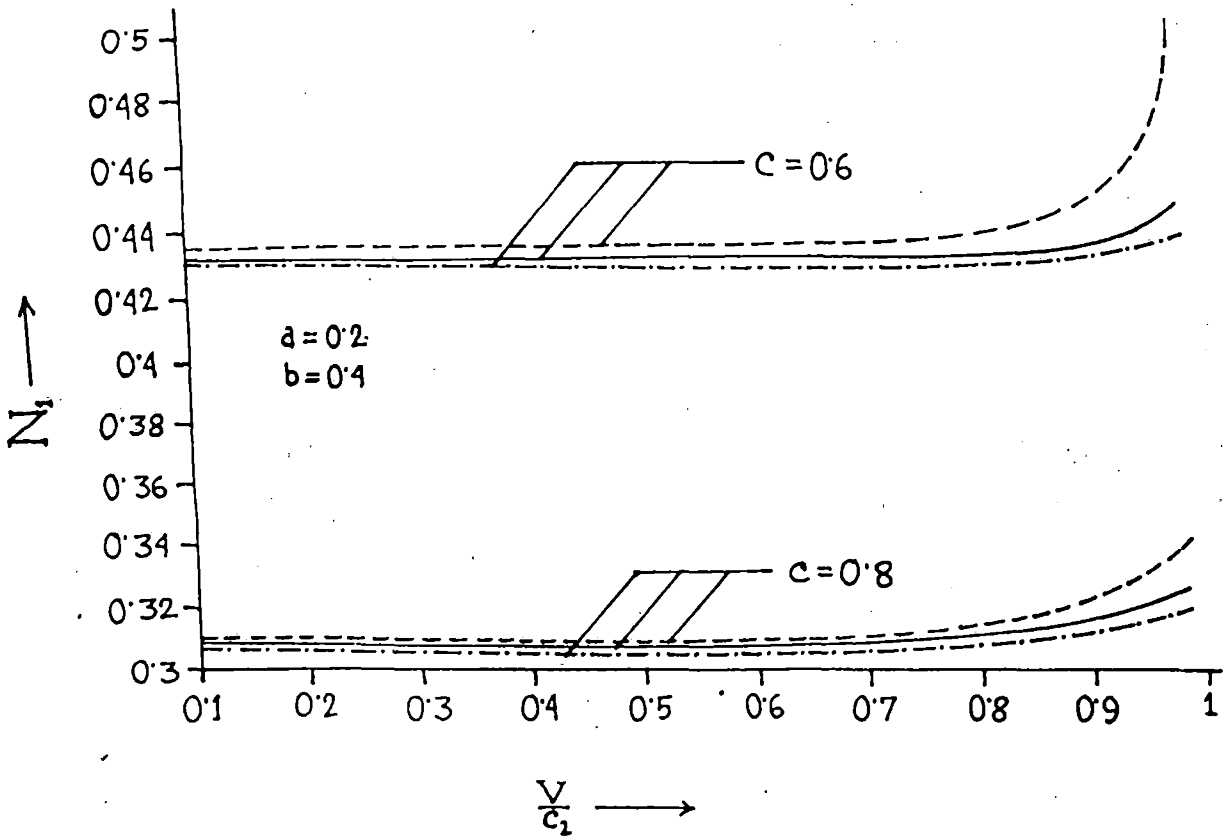

Fig.13. Stress intensity factor N_1 vs. V/c_2.
(---- h=1, —— h=2, —.—.— h=5).

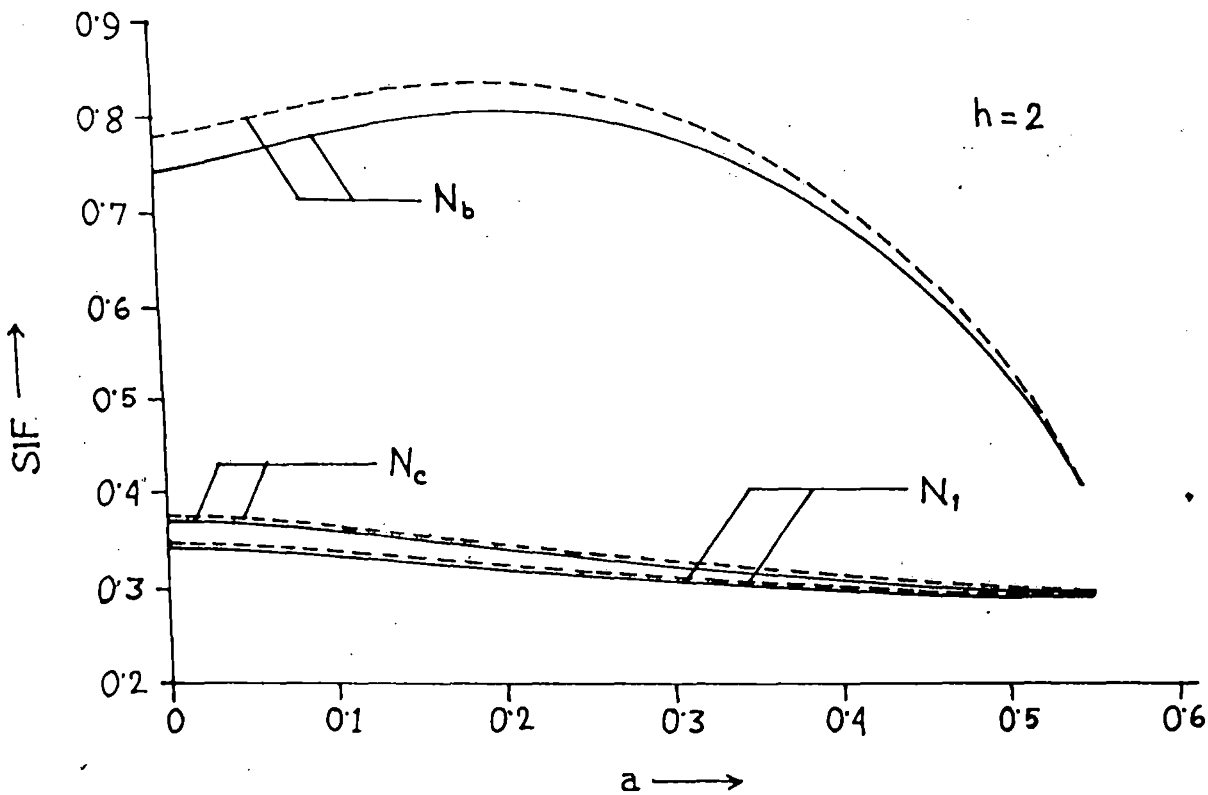

Fig. 14. Stress intensity factors vs. a.
(—— V/c_2=0.1, ----- V/c_2=0.8).

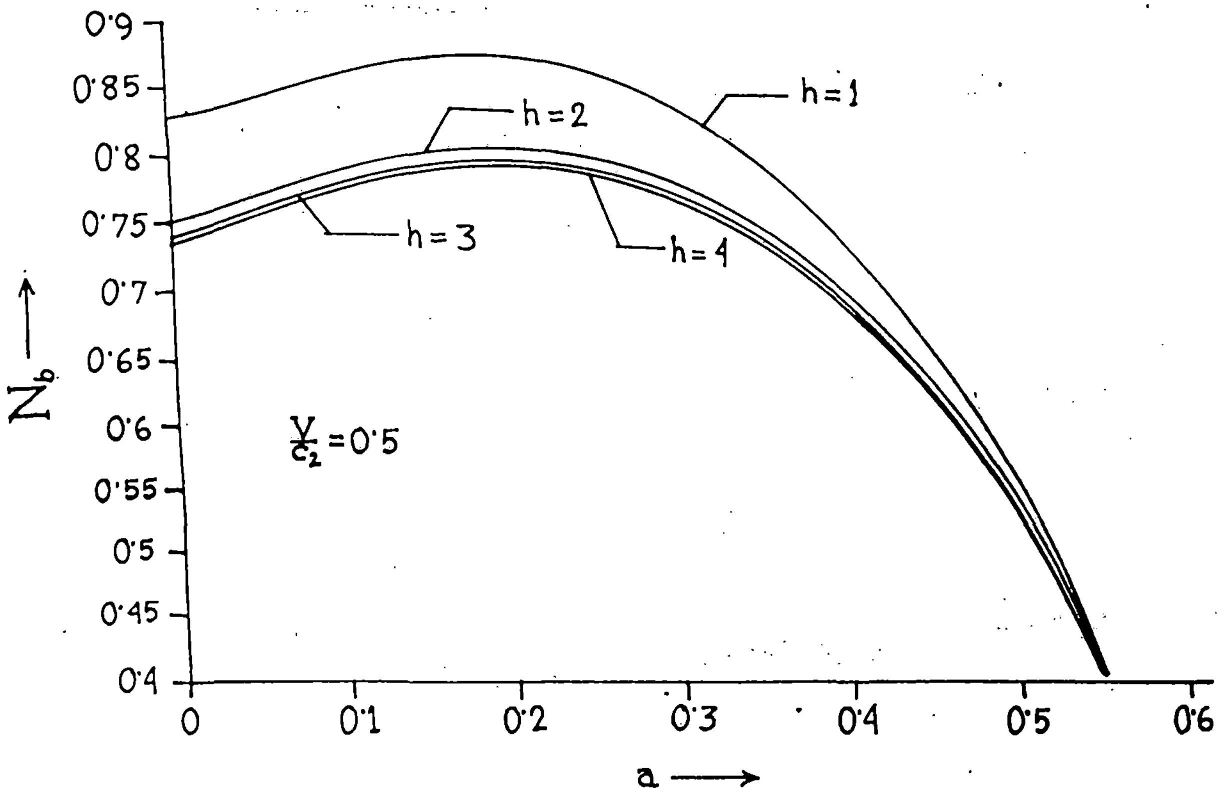

Fig.15. Stress intensity factor N_b vs. a.
($V/c_2=0.5$).

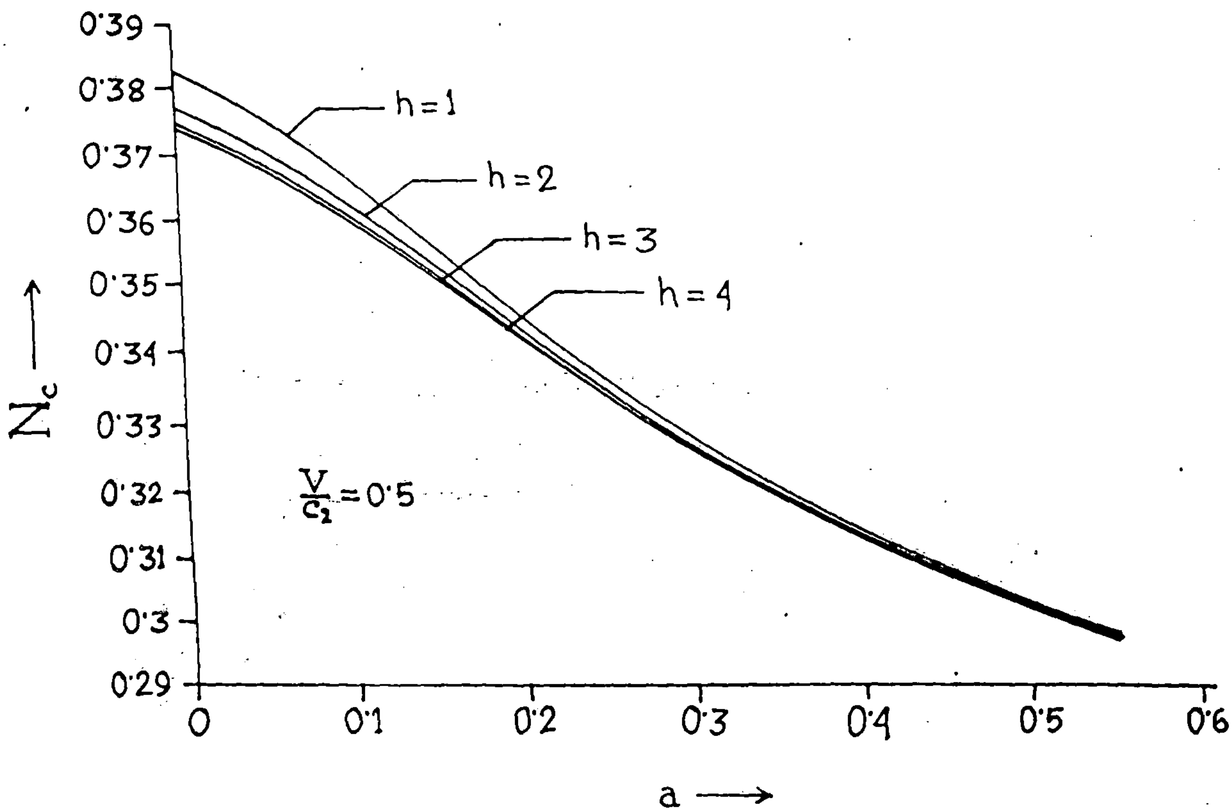

Fig.16. Stress intensity factor N_c vs. a. (V/c_2=0.5).

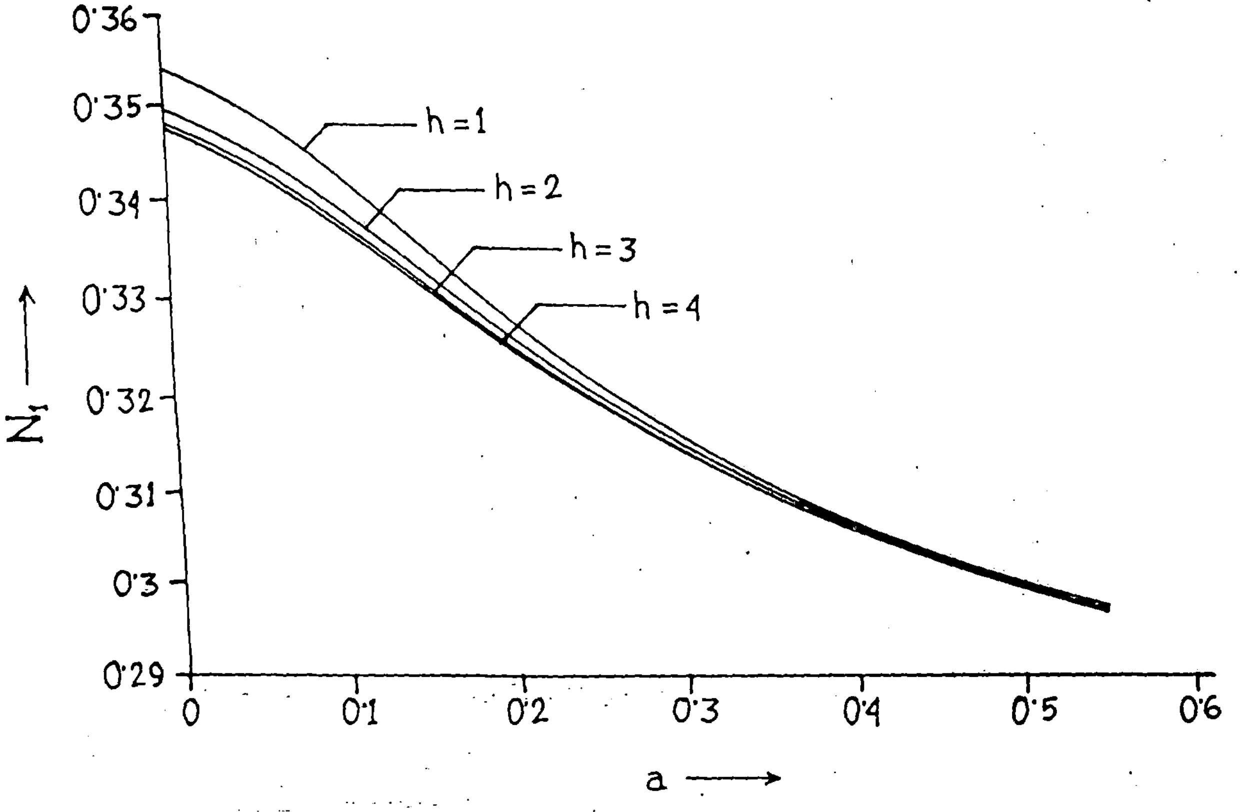

Fig.17. Stress intensity factor N_1 vs. a. (V/c_2=0.5).