CHAPTER - IV

STRESSES DUE TO HEAT EXPOSURE ON THE BOUNDING SURFACE OF ELASTIC SEMI-SPACE

Paper 1 : Axisymmetric stress distribution in a semi-infinite elastic solid with constant heat flow over an elliptic area on the plane boundary.

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Paper 2 : Three dimensional thermal stresses due to periodic supply of heat on the straight edges of a semi-infinite thick plate.

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Paper 3 : Thermal stresses due to prescribed flux of heat on the surface of a thick plate.

Paper 4 : Thermal stresses due to prescribed flux of heat on the boundary of a semi-infinite elastic solid.

Axisymmetric Stress Distribution In A Semi-infinite Elastic Solid With Constant Heat-flow Over An Elliptic Area On The Plane Boundary

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1. INTRODUCTION

In this paper, axisymmetric thermal stresses in a semi-infinite elastic solid have been obtained when there is a constant supply of heat over an elliptic area on the bounding plane surface, the rest being kept at a constant temperature. Temperature and the potential of the thermo-elastic displacement are obtained in terms of Matheiu functions employing the curvilinear coordinates due to C.B.Ling [44].

2. METHOD OF SOLUTION

Let us introduce elliptical coordinates (ξ,η) connected with cartesian coordinates in the form

$$x+iy = h\cosh(\xi+i\eta) \tag{1}$$

where $0\le\eta\le2\pi$, and 2h is the distance between the foci. Let the bounding surface of the semi-infinite elastic solid be given by z=0, the axis of z being drawn into the body. The temperature field in the steady state is given by the differential equation [3]

$$\nabla^2 T = 0 \tag{2}$$

and the boundary conditions are

$$-K \frac{\partial T}{\partial z} = Q, \quad \xi < \xi_o, \quad z=0$$

$$T = 0, \quad \xi > \xi_o, \quad z=0 \tag{3}$$

$$T = 0 \quad \text{at infinity} \tag{4}$$

where K = thermal conductivity

Q = rate of flow of heat per unit area.

Equations (2) and (4) are satisfied, if we take [10]

$$T = \sum_{n=0}^{\infty} c_{2n} c_{e_{2n}}(\xi, q)\; C_{e_{2n}}(\eta, q)\; e^{-nz} \tag{5}$$

where c_{2n} is a constant to be determined from the boundary conditions (3) and

$$c_{e_{2n}}(\xi, q) = \sum_{r=0}^{\infty} A_{2r}^{(2n)} \cosh(2r\xi)$$

$$C_{e_{2n}}(\eta, q) = \sum_{r=0}^{\infty} A_{2r}^{(2n)} \cos(2r\eta)$$

are Mathieu functions of integral order [2], q being a constant obtainable from Mathieu's equation [10].

On the plane z=0, the following relations are to be satisfied

$$\sum_{n=0}^{\infty} nc_{2n} c_{e_{2n}}(\xi, q)\ C_{e_{2n}}(\eta, q) = Q/K\ ,\ \xi < \xi_{o}$$

$$\sum_{n=0}^{\infty} c_{2n} c_{e_{2n}}(\xi, q)\ C_{e_{2n}}(\eta, q) = 0\ ,\ \xi > \xi_{o} \tag{6}$$

Multiplying both sides of(6) by $C_{e_{2n}}(\eta, q)$ and integrating w.r.t. η from 0 to 2π and w.r.t. ξ from 0 to ξ_o we obtain

$$\frac{2\pi Q}{K} A_{o}^{(2n)} = \pi n\ c_{2n} \int_{0}^{\xi_o} c_{e_{2n}}(\xi, q)\ d\xi$$

where in general [10]

$$A_{m+2r}^{(m)} \cong (-1)^{r} \frac{m!}{r!(m+r)!} t^{r}, \qquad r \geq 0,\ m>0$$

and t is a function of q.

Therefore, $c_{2n} = 2QM_n/K$

where

$$M_n = \frac{A_{o}^{(2n)}}{n\int_{0}^{\xi_o} c_{e_{2n}}(\xi, q)\ d\xi} \tag{7}$$

The temperature T is ,therefore, given by

$$T = \frac{2Q}{K} \sum_{n=0}^{\infty} M_n c_{e_{2n}}(\xi, q)\, C_{e_{2n}}(\eta, q)\, e^{-nz} \tag{8}$$

To determine the stresses, the potential of thermo-elastic displacement ψ will be used. This related to the displacement components u,v,w by the equations

$$\frac{\partial\psi}{\partial x} = u, \quad \frac{\partial\psi}{\partial y} = v, \quad \frac{\partial\psi}{\partial z} = w \tag{9}$$

From the stress-strain relation in problems of thermal stresses and the equation of equilibrium [17] we have

$$\nabla^2 \psi = \beta T$$

$$= \frac{2Q}{K} \beta \sum_{n=0}^{\infty} M_n c_{e_{2n}}(\xi, q)\, C_{e_{2n}}(\eta, q)\, e^{-nz} \tag{10}$$

where $\beta = \frac{1+\nu}{1-\nu}\alpha$, ν = Poission's ratio, α is the coefficient of linear thermal expansion.

A particular integral of equation (10) is given by

$$\psi = -\frac{Q\beta z}{K} \sum_{n=0}^{\infty} n^{-1} M_n c_{e_{2n}}(\xi, q)\, C_{e_{2n}}(\eta, q)\, e^{-nz} \tag{11}$$

Now the stress components $\widehat{(\xi\xi)}_T$, $\widehat{(\eta\eta)}_T$, $\widehat{(zz)}_T$, $\widehat{(\xi z)}_T$ are calculated as

$$\frac{\widehat{(\xi\xi)}_T}{2\mu} = 2[h(\cosh(2\xi)-\cos(2\eta))]^{-2}\frac{Q\beta z}{K}\sum_{n=0}^{\infty} n^{-1}M_n\Big[\sinh 2\xi\, c'_{e_{2n}}(\xi,q)C_{e_{2n}}(\eta,q)$$

$$-\sin(2\eta)c_{e_{2n}}(\xi,q)\ C'_{e_{2n}}(\eta,q)\Big]e^{-nz} - \frac{2Q\beta}{K}\sum_{n=0}^{\infty} M_n c_{e_{2n}}(\xi,q)\ C_{e_{2n}}(\eta,q)e^{-nz} +$$

$$+2[h^2(\cosh(2\xi)-\cos(2\eta))]^{-1}\frac{\partial^2\psi}{\partial\xi^2},$$

$$\frac{\widehat{(\eta\eta)}_T}{2\mu} = 2[h(\cosh(2\xi)-\cos(2\eta))]^{-2}\frac{Q\beta z}{K}\sum_{n=0}^{\infty} n^{-1}M_n\Big[\sin 2\eta\ c_{e_{2n}}(\xi,q)C'_{e_{2n}}(\eta,q)$$

$$-\sinh(2\xi)c'_{e_{2n}}(\xi,q)\ C_{e_{2n}}(\eta,q)\Big]e^{-nz} - \frac{2Q\beta}{K}\sum_{n=0}^{\infty} M_n c_{e_{2n}}(\xi,q)C_{e_{2n}}(\eta,q)e^{-nz} +$$

$$+2[h^2(\cosh(2\xi)-\cos(2\eta))]^{-1}\frac{\partial^2\psi}{\partial\eta^2}$$

(12)

$$\frac{(\widehat{zz})_T}{2\mu} = -\frac{zQ\beta}{K} \sum_{n=0}^{\infty} nM_n c_{e_{2n}}(\xi, q) C_{e_{2n}}(\eta, q) e^{-nz} ,$$

$$\frac{(\widehat{\xi z})_T}{2\mu} = \sqrt{\cosh^2 2\xi - \cos^2 2\eta}\ \frac{Q\beta}{K} \sum_{n=0}^{\infty} n^{-1} M_n (nz-1) \Big[\sinh(2\xi) c'_{e_{2n}}(\xi, q) C_{e_{2n}}(\eta, q)$$

$$- \sin(2\eta) c_{e_{2n}}(\xi, q)\ C'_{e_{2n}}(\eta, q) \Big]\ e^{-nz} \tag{13}$$

$(\widehat{\xi\eta})_T$ and $(\widehat{\eta z})_T$ do not appear due to symmetry. Here prime stands for differentiation w.r.t. ξ and η.

It is observed that the normal stress $(\widehat{zz})_T$ vanishes for z=0. The stress $(\widehat{\xi z})_T$, however, does not vanish. In order to suppress it, the stress system $(\widehat{\xi\xi})_C$, $(\widehat{\eta\eta})_C$, $(\widehat{zz})_C$, $(\widehat{\xi z})_C$ obtained on the hypothesis that there is no temperature distribution is to be superposed.

3. Complimentary Stresses

In order to determine the complimentary stresses we use Love's function ϕ satisfying the biharmonic equation [9]

$$\nabla^4 \phi = 0 \tag{14}$$

with the boundary conditions

$$\left| \widehat{(zz)}_c \right|_{z=0} = 0$$

$$\left| \widehat{(\xi z)}_T + \widehat{(\xi z)}_c \right|_{z=0} = 0$$

and $\phi = 0$ at infinity. (15)

Let us assume the function ϕ in the form

$$\phi = \sum_{n=0}^{\infty} (C+Dnz)\, ce_{2n}(\xi,q)\; Ce_{2n}(\eta,q)\; e^{-nz} \tag{16}$$

where C and D are functions of n.

The complimentary stresses are given by [16]

$$\widehat{(\xi\xi)}_c = \frac{2\mu}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - h_1^{-1} e_{\xi\xi}\right]$$

$$\widehat{(\eta\eta)}_c = \frac{2\mu}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - h_2^{-1} e_{\eta\eta}\right]$$

$$\widehat{(zz)}_c = \frac{2\mu}{1-2\nu}\frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\phi - e_{zz}\right]$$

$$\widehat{(\xi z)}_c = \frac{2\mu}{1-2\nu}\frac{\partial}{\partial z}\left[(1-\nu)\nabla^2\phi - e_{zz}\right] \tag{17}$$

where $h_1^2 = h_2^2 = 2[h^2(\cosh(2\xi)-\cos(2\eta))]^{-1}$, $h_3 = 1.$

Using (16) in (17), we have

$$\frac{\widehat{(\xi\xi)}_c}{2\mu} = \frac{1}{1-2\nu}\Bigg\{\sum_{n=0}^{\infty} 2\nu n^3 D c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) e^{-nz} - 2[h(\cosh(2\xi)-\cos(2\eta))]^{-2}$$

$$\sum_{n=0}^{\infty} n(D-C-Dnz)\Big[-\sinh 2\xi \; c'_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) + \sin 2\eta \; c_{e_{2n}}(\xi,q) C'_{e_{2n}}(\eta,q)\Big]$$

$$\times e^{-nz} - 2[h^2(\cosh(2\xi)-\cos(2\eta))]^{-1} \frac{\partial}{\partial z}\frac{\partial^2 \phi}{\partial \xi^2}\Bigg\},$$

$$\frac{\widehat{(\eta\eta)}_c}{2\mu} = \frac{1}{1-2\nu}\Bigg\{\sum_{n=0}^{\infty} 2\nu n^3 D c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) e^{-nz} - 2[h(\cosh(2\xi)-\cos(2\eta))]^{-2}$$

$$\sum_{n=0}^{\infty} n(D-C-Dnz)\Big[\sinh 2\xi \; c'_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) - \sin 2\eta \; c_{e_{2n}}(\xi,q) C'_{e_{2n}}(\eta,q)\Big]$$

$$\times e^{-nz} - 2[h^2(\cosh(2\xi)-\cos(2\eta))]^{-1} \frac{\partial}{\partial z}\frac{\partial^2 \phi}{\partial \eta^2}\Bigg\},$$

$$\frac{\widehat{(zz)}_c}{2\mu} = \frac{1}{1-2\nu} \sum_{n=0}^{\infty} n^3 [D-2\nu D+C+Dnz] ce_{2n}(\xi, q) Ce_{2n}(\eta, q) e^{-nz}$$

$$\frac{\widehat{(\xi z)}_c}{2\mu} = [(1-2\nu)h]^{-1} \sqrt{2(\cosh^2 2\xi - \cos^2 2\eta)} \sum_{n=0}^{\infty} n^2 (2\nu D - C - Dnz) \Big[\sinh(2\xi) \times$$

$$ce'_{2n}(\xi, q) Ce_{2n}(\eta, q) - \sin(2\eta) ce_{2n}(\xi, q) \; Ce'_{2n}(\eta, q) \Big] e^{-nz} \quad (18)$$

In view of the first boundary condition

$$C = - D(1-2\nu) \quad (19)$$

The second boundary conditions gives

$$D = \frac{M_n \beta Q(1-2\nu)}{Kn^3} \quad (20)$$

With these values of C and D, the components of complimentary stresses are known.

Therefore, the resultant principal stresses are given by

$$\widehat{(\xi\xi)} = \widehat{(\xi\xi)}_T + \widehat{(\xi\xi)}_C$$

$$\widehat{(\eta\eta)} = \widehat{(\eta\eta)}_T + \widehat{(\eta\eta)}_C \quad (21)$$

$$\widehat{(\xi\xi)} = \frac{4\mu Q\beta(\nu-1)}{K} \sum_{n=0}^{\infty} M_n \Big\{ c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) - 2[nh(\cosh(2\xi)-\cos(2\eta))]^{-2} \times$$

$$\Big[\sinh(2\xi) c'_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) - \sin(2\eta) c_{e_{2n}}(\xi,q) C'_{e_{2n}}(\eta,q) - (\cosh(2\xi)-$$

$$\cos(2\eta))\Big[1 - \frac{n^2h^2}{2}\cosh(2\xi)\Big] c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q)\Big]\Big\} e^{-nz} ,$$

$$\widehat{(\eta\eta)} = \frac{4\mu Q\beta(\nu-1)}{K} \sum_{n=0}^{\infty} M_n \Big\{ c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) + 2[nh(\cosh(2\xi)-\cos(2\eta))]^{-2} \times$$

$$\Big[\sinh(2\xi) c'_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) - \sin(2\eta) c_{e_{2n}}(\xi,q) C'_{e_{2n}}(\eta,q) - (\cosh(2\xi)-$$

$$\cos(2\eta))\Big[1 - \frac{n^2h^2}{2}\cosh(2\eta)\Big] c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q)\Big]\Big\} e^{-nz}$$

(22)

On the plane $z = 0$, we have from (22)

$$[\widehat{(\xi\xi)} + \widehat{(\eta\eta)}]_{z=0} = \frac{4\mu Q\beta(\nu-1)}{K} \sum_{n=0}^{\infty} M_n \; c_{e_{2n}}(\xi,q) \; C_{e_{2n}}(\eta,q) \qquad (23)$$

$$[\widehat{(\xi\xi)} - \widehat{(\eta\eta)}]_{z=0} =$$

$$= \frac{8\mu Q\beta(\nu-1)}{K}[\cosh(2\xi)-\cos(2\eta)]^{-1}\sum_{n=0}^{\infty} n^{-2} M_n \Big\{ \Big[2 - \frac{n^2 h^2}{2}(\cosh(2\eta)+\cos(2\eta))\Big] \times$$

$$c_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) - (\cosh(2\xi)-\cos(2\eta))^{-1}\Big[\sinh(2\xi) c'_{e_{2n}}(\xi,q) C_{e_{2n}}(\eta,q) -$$

$$\sin(2\eta) c_{e_{2n}}(\xi,q)\ C'_{e_{2n}}(\eta,q)\Big]\Big\} \qquad (24)$$

So the stresses are determined and the problem is solved.

THREE DIMENSIONAL THERMAL STRESSES DUE TO PERIODIC SUPPLY OF HEAT ON THE STRAIGHT EDGES OF A SEMI INFINITE THICK PLATE

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1.INTRODUCTION

This is a thermoelastic boundary value problem of three dimensions when these thermal stresses are produced in a body by unequal distribution of temperature which may be regarded as a specified function of coordinates and time. In this paper stresses due to periodic supply of heat produced by the blow of a jet flame on the straight edges of a semi-infinite isotropic elastic thick plate distributed over a finite portion of it, have been considered.

2.SOLUTION

1. If T denotes the temperature at the point (x,y,z) and α, the coefficient of linear thermal expansion, we have the three dimensional equation of heat conduction as [3]

$$\frac{\partial T}{\partial t} = k \nabla_1^2 T \tag{1.1}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

We have the following stress-strain relation in three dimensional problems of thermal stresses [13]

$$\varepsilon_x - \alpha T = E^{-1}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y - \alpha T = E^{-1}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\varepsilon_z - \alpha T = E^{-1}\left[\sigma_z - \nu(\sigma_y + \sigma_x)\right]$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}\ ,\ \gamma_{yz} = \frac{2(1+\nu)}{E}\tau_{yz}\ ,\quad \gamma_{xz} = \frac{2(1+\nu)}{E}\tau_{xz} \qquad (1.2)$$

Solving for the stresses we find

$$\sigma_x = \frac{E}{1+\nu}\varepsilon_x + \frac{E\nu}{(1-2\nu)(1+\nu)}e - \frac{E\alpha T}{1-2\nu}$$

$$\sigma_y = \frac{E}{1+\nu}\varepsilon_y + \frac{E\nu}{(1-2\nu)(1+\nu)}e - \frac{E\alpha T}{1-2\nu}$$

$$\sigma_z = \frac{E}{1+\nu}\varepsilon_z + \frac{E\nu}{(1-2\nu)(1+\nu)}e - \frac{E\alpha T}{1-2\nu}$$

$$\tau_{xy} = \frac{1}{2}\frac{E}{1+\nu}\gamma_{xy}\ ,\ \tau_{yy} = \frac{1}{2}\frac{E}{1+\nu}\gamma_{yz}\ ,\quad \tau_{xz} = \frac{1}{2}\frac{E}{1+\nu}\gamma_{xz} \qquad (1.3)$$

where $e = \varepsilon_x + \varepsilon_y + \varepsilon_z$.

Hence from the equations of equilibrium

$$\frac{\partial}{\partial x}(\sigma_x)+\frac{\partial}{\partial y}(\tau_{xy})+\frac{\partial}{\partial z}(\tau_{xz}) = 0$$

$$\frac{\partial}{\partial x}(\tau_{xy})+\frac{\partial}{\partial y}(\sigma_y)+\frac{\partial}{\partial z}(\tau_{yz}) = 0$$

$$\frac{\partial}{\partial x}(\tau_{xz})+\frac{\partial}{\partial y}(\tau_{yz})+\frac{\partial}{\partial z}(\sigma_z) = 0 \qquad (1.4)$$

We get when expressed in terms of displacements

$$\frac{\partial e}{\partial x} +(1-2\nu)\nabla_1^2 u =2(1+\nu)\alpha \frac{\partial T}{\partial x}$$

$$\frac{\partial e}{\partial y} +(1-2\nu)\nabla_1^2 v =2(1+\nu)\alpha \frac{\partial T}{\partial y}$$

$$\frac{\partial e}{\partial z} +(1-2\nu)\nabla_1^2 w =2(1+\nu)\alpha \frac{\partial T}{\partial z} \qquad (1.5)$$

Assuming that [17]

$$u = \frac{\partial \psi}{\partial x} , \quad v = \frac{\partial \psi}{\partial y} , \quad w = \frac{\partial \psi}{\partial z} \qquad (1.6)$$

where ψ is a function of x,y,z and also of time t, the relation (1.5) reduces to

$$(1-\nu)\frac{\partial}{\partial x}\nabla_1^2\psi=(1+\nu)\alpha\frac{\partial T}{\partial x}$$

$$(1-\nu)\frac{\partial}{\partial y}\nabla_1^2\psi=(1+\nu)\alpha\frac{\partial T}{\partial y}$$

$$(1-\nu)\frac{\partial}{\partial z}\nabla_1^2\psi=(1+\nu)\alpha\frac{\partial T}{\partial z} \qquad (1.7)$$

These three equations are evidently satisfied if we take the function ψ as a solution of the equations [17]

$$\nabla_1^2\psi=\frac{1+\nu}{1-\nu}\,\alpha T \qquad (1.8)$$

Differentiating equations (1.8) with respect to t and substituting for $\partial T/\partial t$ from relation (1.1) we get

$$\nabla_1^2\frac{\partial\psi}{\partial t}=\frac{1+\nu}{1-\nu}\,\alpha K\,\nabla_1^2 T$$

We may therefore take

$$\frac{\partial\psi}{\partial t}=\frac{1+\nu}{1-\nu}\,\alpha\,KT \qquad (1.9)$$

2. Considering a semi infinite thick plate bounded by the plane with edges $y=0, z=0$, the axes of y and z being into the plate we can write the solution of (1.1) as

$$T = \int_0^\infty A(m)e^{-\alpha_m(y+z)} \cos(pt-\beta_m y) \cos(pt-\beta_m z) \cos(mx)\, dm \tag{2.1}$$

where

$$\alpha_m = \left[\frac{1}{2}m^2 + \sqrt{\frac{m^4}{4} + \frac{p^2}{4k^2}}\right]^{1/2}$$

$$\beta_m = \left[\frac{1}{2}m^2 + \sqrt{\frac{m^4}{4} + \frac{p^2}{4k}}\right]^{1/2} \tag{2.2}$$

and A(m) being an arbitrary function of m. From equation (1.9) the function ψ corresponding to this temperature becomes

$$\psi = \frac{1+\nu}{1-\nu}\frac{\alpha K}{4p} \int_0^\infty A(m)\, e^{-\alpha_m(y+z)} \sin\left[2pt-\beta_m(y+z)\right] \cos(mx)\, dm \tag{2.3}$$

The relation (2.3) represents a particular solution of the general equation (1.5). The corresponding displacements and stresses can now be calculated from relations (1.3) and (1.6)

$$u = -\frac{1+\nu}{1-\nu}\frac{\alpha K}{4p} \int_0^\infty mA(m)e^{-\alpha_m(y+z)} \sin\left[2pt-\beta_m(y+z)\right] \sin(mx)\, dm$$

$$v = w = -\frac{1+\nu}{1-\nu}\frac{\alpha K}{4p}\int_0^\infty A(m)e^{-\alpha_m(y+z)}\left[\alpha_m \sin[2pt-\beta_m(y+z)]+\beta_m \cos[2pt-\beta_m(y+z)]\right]$$

$$\times\cos(mx)\ dm \qquad (2.4)$$

$$\sigma_x = \frac{-E}{1-2\nu}\frac{\alpha K}{4p}\int_0^\infty A(m)e^{-\alpha_m(y+z)}\left[m^2 \sin[2pt-\beta_m(y+z)]+\frac{2\nu}{1-\nu}\left\{(\beta_m^2-\alpha_m^2)\sin[2pt-\beta_m\right.\right.$$

$$\left.\left.\times(y+z)]-2\alpha_m\beta_m \cos[2pt-\beta_m(y+z)]\right\}+4pK^{-1}\cos(pt-\beta_m y)\cos(pt-\beta_m z)\right]\cos(mx)dm$$

$$\sigma_y = \sigma_z = \frac{-E}{(1-2\nu)(1-\nu)}\frac{\alpha K}{4p}\int_0^\infty A(m)e^{-\alpha_m(y+z)}\left[\left\{(\beta_m^2-\alpha_m^2)\sin[2pt-\beta_m(y+z)]\right.\right.$$

$$\left.-2\alpha_m\beta_m \cos[2pt-\beta_m(y+z)]\right\} + \nu m^2 \sin[2pt-\beta_m(y+z)]+$$

$$\left.+\ 4p(1-\nu)K^{-1}\cos(pt-\beta_m y)\cos(pt-\beta_m z)\right]\cos(mx)\ dm$$

$$\tau_{xy} = \tau_{zx} = \frac{E}{1-\nu}\frac{\alpha K}{4p}\int_0^\infty mA(m)e^{-\alpha_m(y+z)}\left[\left\{\alpha_m \sin[2pt-\beta_m(y+z)]\right.\right.$$

$$+\beta_m \cos[2pt-\beta_m(y+z)]\Big\}\sin(mx)\ dm$$

$$\tau_{yz} = \frac{E}{1-\nu}\,\frac{\alpha K}{4p}\int_0^\infty A(m)e^{-\alpha_m(y+z)}\Big[\Big\{(\alpha_m^2-\beta_m^2)\sin[2pt-\beta_m(y+z)]-$$

$$+2\alpha_m\beta_m\cos[2pt-\beta_m(y+z)]\Big\}\cos(mx)dm \qquad (2.5)$$

The stresses obtained in relations (2.5) are produced by the thermal expansion. This expansion gives rise to certain stresses on the boundary of the plate. We shall therefore make the boundary free from stresses by the addition of the extra terms obtained on the hypothesis that there is no temperature distribution. In order to nullify the stresses on the boundary yz=0 we are to superimpose a complementary stress -system $(\sigma_{x_1},\sigma_{y_1},\sigma_{z_1},\tau_{xy_1},\tau_{yz_1},\tau_{zx_1})$ such that

$$(\sigma_{y_1})_0 = -(\sigma_y)_0,\quad (\sigma_{z_1})_0 = -(\sigma_z)_0,\quad (\tau_{xy_1})_0 = -(\tau_{xy})_0,\quad (\tau_{yz_1})_0 = -(\tau_{xy})_0,$$

$$(\tau_{zx_1})_0 = (\tau_{zx})_0 \qquad (2.6)$$

Considering the stress function

$$\psi = \int_0^\infty \Big[C(m)+yD(m)+zE(m)\Big]e^{-m(y+z)}\cos(mx)\ dm \qquad (2.7)$$

which satisfies the biharmonic equation, C(m),D(m) and E(m) being arbitrary functions of m, we have stresses [9]

$$\sigma_{x_1} = \left\{ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\}\psi , \quad \sigma_{y_1} = \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right\}\psi, \quad \sigma_{z_1} = \left\{ \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} \right\}\psi$$

$$\tau_{xy_1} = - \frac{\partial^2}{\partial x \partial y} \psi , \quad \tau_{yz_1} = - \frac{\partial^2}{\partial y \partial z} \psi , \quad \tau_{zx_1} = - \frac{\partial^2}{\partial x \partial z} \psi$$

So we get the complementary stresses as

$$\sigma_{x_1} = \int_0^{\infty} \left\{ 2m^2 \left[C(m) + yD(m) + zE(m) \right] - 2m \left[D(m) + E(m) \right] \right\} e^{-m(y+z)} \cos(mx) \, dm$$

$$\sigma_{y_1} = - \int_0^{\infty} 2mE(m) e^{-m(y+z)} \cos(mx) \, dm$$

$$\sigma_{z_1} = - \int_0^{\infty} 2mD(m) e^{-m(y+z)} \cos(mx) \, dm$$

$$\tau_{xy_1} = - \int_0^{\infty} \left\{ m^2 \left[C(m) + yD(m) + zE(m) \right] - mD(m) \right\} e^{-m(y+z)} \sin(mx) \, dm$$

$$\tau_{yz_1} = - \int_0^{\infty} \left\{ m^2 \left[C(m) + yD(m) + zE(m) \right] - m \left[D(m) + E(m) \right] \right\} e^{-m(y+z)} \cos(mx) \, dm$$

$$\tau_{zx_1} = -\int_0^\infty \left\{ m^2\left[C(m)+yD(m)+zE(m)\right]-mE(m)\right\} e^{-m(y+z)} \sin(mx)\ dm$$

Using relations (2.5),(2.6) and (2.8) and solving we get

$$D(m) = E(m) = \frac{-E\alpha K}{(1-\nu)(1-2\nu)8pm} A(m)\left\{(\beta_m^2-\alpha_m^2+\nu m^2)\sin(2pt)-2\alpha_m\beta_m\cos(2pt)+ \right.$$

$$\left. + \frac{1-\nu}{K} 4p \cos^2 pt \right\}$$

$$C(m)= \frac{E\alpha K}{(1-\nu)4pm^2} A(m)\left\{\left[m\alpha_m - \frac{(\beta_m^2-\alpha_m^2+\nu m^2)}{2(1-2\nu)}\right]\sin(2pt)+\left[m\beta_m + \frac{\alpha_m\beta_m}{(1-2\nu)}\right]\cos(2pt)\right.$$

$$\left. - \frac{2p(1-\nu)}{K(1-2\nu)} \cos^2 pt \right\} \qquad (2.9)$$

With the value of this constants substituted from relation (2.9)

$$\sigma_{x_1} = \int_0^\infty \left\{\frac{E\alpha Km}{(1-\nu)2p} A(m)\left[[\alpha_m\sin(2pt)+\beta_m\cos(2pt)] - \frac{(\beta_m^2-\alpha_m^2+\nu m^2)}{2(1-2\nu)m}(my+mz-1)\right.\right.$$

$$\left.\left. \sin(2pt) + \frac{\alpha_m\beta_m}{(1-2\nu)m}(my+mz-1)\cos(2pt) - \frac{4p(1-\nu)}{K(1-2\nu)}\cos^2 pt\right\} e^{-m(y+z)} \cos(mx)dm$$

$$\sigma_{y_1} = \sigma_{z_1} = \frac{E\alpha K}{(1-2\nu)(1-\nu)4p} \int_0^\infty A(m)\Big\{(\beta_m^2 - \alpha_m^2 + \nu m^2)\sin(2pt) - 2\alpha_m\beta_m\cos(2pt)$$

$$+4p(1-\nu)K^{-1}\cos^2 pt\Big\} e^{-m(y+z)} \cos(mx) \; dm$$

$$\tau_{xy_1} = \tau_{zx_1} = -\int_0^\infty \Big\{ \frac{E\alpha Km}{(1-\nu)4p} A(m)\Big[\alpha_m \sin(2pt) + \beta_m\cos(2pt) - \frac{(\beta_m^2 - \alpha_m^2 + \nu m^2)}{2(1-2\nu)m}(y+z)$$

$$\sin(2pt) + \frac{\alpha_m\beta_m}{(1-2\nu)m}(y+z)\cos(2pt) - \frac{2p(1-\nu)}{K(1-2\nu)}(y+z)\cos^2 pt\Big\} e^{-m(y+z)} \cos(mx)dm$$

$$\tau_{yz_1} = -\int_0^\infty \frac{E\alpha KA(m)}{(1-\nu)(1-2\nu)4p}\Big[m(1-2\nu)[\alpha_m \sin(2pt) + \beta_m\cos(2pt)] - \frac{(\beta_m^2 - \alpha_m^2 + \nu m^2)}{2}$$

$$(my+mz-1)\sin(2pt) + \alpha_m\beta_m(my+mz-1)\cos(2pt) - \frac{2}{K}\, p(1-\nu)(my+mz+2)\cos^2 pt\Big] \times$$

$$e^{-m(y+z)} \cos(mx)dm \qquad (2.10)$$

Thus the resultant stress components are given by

$$(\sigma_x)_R = \frac{E\alpha K}{(1-\nu)2p}\int_0^\infty\Big\{mA(m)\Big[[(\alpha_m\sin(2pt)+\beta_m\cos(2pt)] - \frac{\beta_m^2-\alpha_m^2+\nu m^2}{2(1-2\nu)m}(my+mz-1)$$

$$\sin(2pt) + \frac{\alpha_m\beta_m}{(1-2\nu)m}(my+mz-1)\cos(2pt) - \frac{4p(1-\nu)}{K(1-2\nu)}\cos^2 pt\Big\}e^{-m(y+z)}\cos(mx)dm$$

$$-\frac{E}{1-2\nu}\frac{\alpha K}{4p}\int_0^\infty A(m)\Big[m^2\sin[2pt-\beta_m(y+z)]\Big] + \frac{2\nu}{1-\nu}\Big\{(\beta_m^2-\alpha_m^2)\sin[2pt-\beta_m(y+z)] -$$

$$2\alpha_m\beta_m\cos[2pt-\beta_m(y+z)]\Big\} + \frac{4p}{K}\cos(pt-\beta_m y)\cos(pt-\beta_m z)\Big]e^{-\alpha_m(y+z)}\cos(mx)dm$$

$$(\sigma_y)_R = (\sigma_z)_R = \frac{E\alpha K}{(1-2\nu)(1-\nu)4p}\Big[\int_0^\infty A(m)\Big\{(\beta_m^2-\alpha_m^2+\nu m^2)\sin(2pt) - 2\alpha_m\beta_m\cos(2pt)$$

$$+4p(1-\nu)K^{-1}\cos^2 pt\Big\}e^{-m(y+z)}\cos(mx)dm - \int_0^\infty A(m)e^{-\alpha_m(y+z)}\Big[\Big\{(\beta_m^2-\alpha_m^2+\nu m^2)\times$$

$$\sin[2pt - \beta_m(y+z)] - 2\alpha_m\beta_m\cos[2pt - \beta_m(y+z)]\Big\} + 4p(1-\nu)K^{-1}\cos(pt-\beta_m y)$$

$$\cos(pt-\beta_m z)\Big]\cos(mx)\ dm\Bigg]$$

$$(\tau_{xy})_R=(\tau_{zx})_R=\frac{E}{1-\nu}\ \frac{\alpha K}{4p}\int_0^\infty mA(m)e^{-\alpha_m(y+z)}\Big[\Big\{\alpha_m\sin[2pt-\beta_m(y+z)]+\beta_m\cos[2pt$$

$$-\beta_m(y+z)]\Big\}\sin(mx)dm-\frac{E\alpha K}{(1-\nu)4p}\int_0^\infty\Big\{mA(m)\Big[\alpha_m\sin(2pt)+\beta_m\cos(2pt)-\sin(2pt)$$

$$\frac{(\beta_m^2-\alpha_m^2+\nu m^2)}{2(1-2\nu)}(y+z)+\frac{\alpha_m\beta_m}{(1-2\nu)}(y+z)\cos(2pt)-\frac{2p(1-\nu)}{K(1-2\nu)}(y+z)\cos^2 pt\Big\}e^{-m(y+z)}$$

$$\times\ \sin(mx)dm$$

$$(\tau_{yz})_R=-\frac{\alpha KE}{(1-\nu)(1-2\nu)4p}\int_0^\infty A(m)\Big[\ m(1-2\nu)[\alpha_m\sin(2pt)+\beta_m\cos(2pt)]\ -$$

$$\frac{\beta_m^2-\alpha_m^2+\nu m^2}{2}(my+mz-1)\sin(2pt)+\alpha_m\beta_m(my+mz-1)\cos(2pt)-\frac{2}{K}p(1-\nu)(my+mz+2)$$

$$\cos^2 pt\Big]e^{-m(y+z)}\cos(mx)dm+\frac{E}{1-\nu}\ \frac{\alpha K}{4p}\int_0^\infty A(m)e^{-\alpha_m(y+z)}\Big\{(\alpha_m^2-\beta_m^2)\sin[2pt\ -$$

$$\beta_m(y+z)] \ +2\alpha_m\beta_m\cos[2pt-\beta_m(y+z)]\Big\}\cos(mx)dm \qquad (2.11)$$

3. Suppose on the plane surface yz=0 we have

$$T = P\cos^2 pt \qquad |x|<a$$

$$= 0, \qquad |x|>a$$

From the relation (2.1) we have on the edges y=0, z=0

$$T = \int_0^\infty A(m)\ \cos^2 pt\ \ \cos(mx)\ dm$$

Hence

$$P = \int_0^\infty A(m)\ \ \cos(mx)\ dm$$

Then by Fourier's cosine transform

$$A(m) = \frac{2P\ \sin(ma)}{\pi m}$$

with this value of a(m) the complete solution is given by the relation (2.11)

Thermal Stresses Due To Prescribed Flux Of Heat On The Surface Of A Thick Plate

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1. Solution Of The Equations Of Thermoelasticity

We shall consider the temperature and displacement fields in an elastic plate of finite thickness but infinite radius which is conducting heat. It will be assumed that there is symmetry about the z- axis and any point of the solid may be expressed in terms of cylindrical coordinates (r,θ,z). For symmetrical deformation of the solid, the displacement vector will have components $(u,0,w)$ and the only non-vanishing components of the stress tensor will be rr, $\theta\theta$, zz and rz.

The temperature field is given by Laplace's equation

$$\frac{\partial^2 T}{\partial r^2} + r^{-1}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (1)$$

in the steady state and in absence of thermal sources.

Stress components are obtained by using the potential of thermo-elastic displacement ψ given by [17]

$$u_T = \frac{\partial \psi}{\partial r}, \quad w_T = \frac{\partial \psi}{\partial z} \qquad (2)$$

From the stress strain relations in problems of thermal stresses and the equations of equilibrium we have

$$\frac{\partial^2 \psi}{\partial r^2} + r^{-1}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = \beta T; \qquad \beta = \frac{1+\nu}{1-\nu}\alpha_1 \qquad (3)$$

Where T is the deviation of the absolute temperature from the temperature of the solid in a state of zero stress and strain, α_1 being the coefficient of linear thermal expansion of the solid and ν is the Poission's ratio.

A particular integral of the equation (3) is

$$\psi = \frac{\beta}{2}\int_0^\infty A \frac{J_o(\alpha r)}{\alpha}\left\{z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1} - d(1+e^{-2\alpha z})(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-2}\right\}$$

$$\times e^{\alpha(z-d)}\, d\alpha \qquad (4)$$

where A is a function of α only and 2d is the thickness of the plate.

From the relations (3) and (4) we obtain,

$$T = \int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1} J_o(\alpha r) e^{\alpha(z-d)}\, d\alpha \qquad (5)$$

which satisfies equation (1).

The stress components and the displacements can now be written as,

$$\widehat{rz}_T = 2G\frac{\partial^2\psi}{\partial r\partial z} = -G\beta\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{1+\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}\right.$$

$$\left. -\alpha d(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\} J_o(\alpha r) e^{\alpha(z-d)}\, d\alpha$$

$$\widehat{zz}_T = 2G\left(\frac{\partial^2\psi}{\partial z^2} - \nabla^2\psi\right) = G\beta\int_0^\infty A\alpha(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}\right.$$

$$\left.-d(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\}J_o(\alpha r)e^{\alpha(z-d)}\ d\alpha$$

$$\widehat{rr}_T = 2G\left(\frac{\partial^2\psi}{\partial r^2} - \nabla^2\psi\right) = G\beta\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left[\alpha\left\{z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}\right.\right.$$

$$\left.\left.-d(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\}\left\{\frac{J_1(\alpha r)}{\alpha r} - J_o(\alpha r)\right\} -2J_o(\alpha r)\right]e^{\alpha(z-d)}d\alpha$$

$$\widehat{\theta\theta}_T = 2G\left(\frac{1}{r}\frac{\partial\psi}{\partial r} - \nabla^2\psi\right) = -G\beta\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left[\alpha\left\{z(1-e^{-2\alpha z})\times\right.\right.$$

$$\left.\left.(1+e^{-2\alpha z})^{-1}-d(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\}\frac{J_1(\alpha r)}{\alpha r} +2J_o(\alpha r)\right]e^{\alpha(z-d)}d\alpha \qquad (6)$$

$$u_T = \frac{\partial\psi}{\partial r} = -\frac{\beta}{2}\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}-d(1-e^{-2\alpha d})\right.$$

$$\left.\times(1+e^{-2\alpha d})^{-1}\right\}J_1(\alpha r)e^{\alpha(z-d)}\ d\alpha$$

$$w_T = \frac{\partial\psi}{\partial z} = \frac{\beta}{2}\int_0^{\infty} A\alpha^{-1}(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{1+\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}-\alpha d \times\right.$$

$$\left.(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\}J_o(\alpha r)e^{\alpha(z-d)}\,d\alpha \qquad (7)$$

The subscript T denotes that the stresses and displacements are due to thermal expansions only, G being the modulus of elasticity in shear.

We observe that the normal stresses $\widehat{zz}_T$ vanishes at z=±d, the stress $\widehat{rz}_T$ however does not vanish. To satisfy the boundary conditions on the planes z=±d we superimpose a complimentary stress system. The components of stresses and displacements are expressed by means of Love's function ϕ by the relations [7]

$$\widehat{rr}_c = \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{\partial^2\phi}{\partial r^2}\right]$$

$$\widehat{\theta\theta}_c = \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - r^{-1}\frac{\partial\phi}{\partial r}\right]$$

$$\widehat{zz}_c = \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right]$$

$$\widehat{rz}_c = \frac{2G}{1-2\nu}\frac{\partial}{\partial r}\left[(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \qquad (8)$$

$$u_c = \frac{1}{1-2\nu}\frac{\partial^2\phi}{\partial r\partial z}$$

$$w_c = \frac{1}{1-2\nu}\left[2(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{9}$$

where ϕ satisfies the biharmonic equation [17]

$$\nabla^4\phi = 0 \tag{10}$$

A solution of equation can be assumed in the form

$$\phi = \frac{1}{2}\int_0^\infty \left\{B(1-e^{-2\alpha z})+C\alpha z(1+e^{-2\alpha z})\right\} J_o(\alpha r)e^{\alpha z}\, d\alpha \tag{11}$$

where B and C are functions of α to be determined.

The components of complementary stresses and displacements are given by

$$\widehat{zz}_c = \frac{G}{1-2\nu}\int_0^\infty \alpha^3\left[\left\{C(1-2\nu)-B\right\}(1+e^{-2\alpha z})-C\alpha z(1-e^{-2\alpha z})\right] J_o(\alpha r)e^{\alpha z}\, d\alpha$$

$$\widehat{rz}_c = \frac{G}{1-2\nu}\int_0^\infty \alpha^3\left[\left\{2C\nu+B\right\}(1-e^{-2\alpha z})-C\alpha z(1+e^{-2\alpha z})\right] J_1(\alpha r)e^{\alpha z}\, d\alpha \tag{12}$$

Now the boundary conditions to be satisfied are,

$$\left[\widehat{zz}_C\right]_{z=\pm d} = 0, \quad \left[\widehat{rz}_C + \widehat{rz}_T\right]_{z=\pm d} = 0 \tag{13}$$

Now the first relation of (13) will be satisfied, if

$$B = C\left\{1-2\nu-\alpha d(1-e^{-2\alpha d})(1+e^{-2\alpha d})^{-1}\right\} \tag{14}$$

Again, in view of the second relation of (13), we have from (6) and (12)

$$C = \frac{\beta(1-2\nu)}{\alpha^3(1+e^{-2\alpha d})} e^{-\alpha d} A \tag{15}$$

Consequently, the remaining stresses and displacements are given by

$$\widehat{rr}_C = G\beta\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left[\left\{2+\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}-\alpha d(1-e^{-2\alpha d})\right.\right.$$

$$\left.\left.\times(1+e^{-2\alpha d})^{-1}\right\}\left\{J_0(\alpha r)-\frac{J_1(\alpha r)}{\alpha r}\right\} + 2\nu\frac{J_1(\alpha r)}{\alpha r}\right]e^{\alpha(z-d)}\,d\alpha$$

$$\widehat{\theta\theta}_C = G\beta\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left[\left\{2+\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}-\alpha d(1-e^{-2\alpha d})\right.\right.$$

$$\left.\left.\times(1+e^{-2\alpha d})^{-1}-2\nu\right\}\frac{J_1(\alpha r)}{\alpha r} + 2\nu J_0(\alpha r)\right]e^{\alpha(z-d)}\,d\alpha \tag{16}$$

$$u_c = \frac{\beta}{2}\int_0^\infty A\alpha^{-1}(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\Big\{2+\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}-\alpha d(1-e^{-2\alpha d})$$

$$\times(1+e^{-2\alpha d})^{-1}-2\nu\Big\}J_1(\alpha r)e^{\alpha(z-d)}\,d\alpha$$

$$w_c = \frac{\beta}{2}\int_0^\infty A\alpha^{-1}(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\Big\{1-\alpha z(1-e^{-2\alpha z})(1+e^{-2\alpha z})^{-1}+\alpha d(1-e^{-2\alpha d})$$

$$\times(1+e^{-2\alpha d})^{-1}-2\nu\Big\}J_o(\alpha r)e^{\alpha(z-d)}\,d\alpha \qquad (17)$$

Applying (7) and (17) we have the final displacements given by,

$$u = \beta(1-\nu)\int_0^\infty A\alpha^{-1}(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}J_1(\alpha r)e^{\alpha(z-d)}\,d\alpha$$

$$w = \beta(1-\nu)\int_0^\infty A\alpha^{-1}(1-e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}J_o(\alpha r)e^{\alpha(z-d)}\,d\alpha \qquad (18a)$$

Also applying (6) and (16) we have finally,

$$\widehat{rr} = -2G\beta(1-\nu)\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\,\frac{J_1(\alpha r)}{\alpha r}\,e^{\alpha(z-d)}\,d\alpha$$

$$\widehat{\theta\theta} = 2G\beta(1-\nu)\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{\frac{J_1(\alpha r)}{\alpha r} - J_o(\alpha r)\right\}e^{\alpha(z-d)}d\alpha \quad (18b)$$

Hence we have,

$$\widehat{rr}+\widehat{\theta\theta} = -2G\beta(1-\nu)\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}J_o(\alpha r)e^{\alpha(z-d)}d\alpha$$

$$\widehat{rr}-\widehat{\theta\theta} = 2G\beta(1-\nu)\int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1}\left\{J_o(\alpha r) - 2\frac{J_1(\alpha r)}{\alpha r}\right\}e^{\alpha(z-d)}d\alpha$$

(19)

2. Temperature Distribution

We shall suppose that on the free surface z=d, there is a flux of heat within a circular region, the rest of the surface being free of any flux of heat. So the boundary conditions are, on the plane z=d,

$$\frac{\partial T}{\partial z} = f(r/a), \text{ for } 0 \le r < a$$

$$= 0, \qquad \text{for} \quad r > a \qquad (20)$$

Now from (5),

$$T = \int_0^\infty A(1+e^{-2\alpha z})(1+e^{-2\alpha d})^{-1} J_o(\alpha r) e^{\alpha(z-d)}\, d\alpha$$

Therefore

$$\frac{\partial T}{\partial z} = \int_0^\infty A\alpha(1-e^{-2\alpha z})(1+e^{-2\alpha d})^{-1} J_o(\alpha r) e^{\alpha(z-d)}\, d\alpha$$

Now we consider dimensionless coordinates ρ, η, ζ the new variable of integration being η, defined by the transformations

$$\alpha A(\alpha) = a\chi(\alpha a), \quad \eta = \alpha a, \quad \rho = r/a, \quad \zeta = z/a \tag{21}$$

Under these transformations , we have,

$$\frac{\partial T}{\partial z} = \int_0^\infty \chi(\eta)(1-e^{-2\eta\zeta})(1+e^{-2\eta d/a})^{-1} J_o(\rho\eta) e^{\eta(\zeta - d/a)}\, d\eta$$

So that on the surface z=d, we have

$$\int_0^\infty \chi(\eta)(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} J_o(\rho\eta)\, d\eta = f(\rho), \quad 0 \le \rho < 1$$
$$= 0 , \qquad \rho > 1$$

Hence by Hankel's inversion theorem [14]

$$\eta^{-1}\chi(\eta)(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} = \int_0^1 \rho f(\rho) J_o(\rho\eta)\, d\rho \tag{22}$$

Under the same set of transformations, we have on z=d,

$$u = \beta(1-\nu)a^2\int_0^\infty \frac{\chi(\eta)}{\eta^2}\, J_1(\rho\eta)\,d\eta$$

$$w = \beta(1-\nu)a^2\int_0^\infty \frac{\chi(\eta)}{\eta^2}\, (1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} J_0(\rho\eta)\,d\eta$$

$$\widehat{rr} + \widehat{\theta\theta} = -2G\beta(1-\nu)a\int_0^\infty \frac{\chi(\eta)}{\eta}\, J_0(\rho\eta)\,d\eta$$

$$\widehat{rr} - \widehat{\theta\theta} = 2G\beta(1-\nu)a\int_0^\infty \frac{\chi(\eta)}{\eta}\left\{J_0(\rho\eta) - 2\frac{J_1(\rho\eta)}{\rho\eta}\right\}d\eta \qquad (23)$$

Let us assume that $f(\rho)=k$. Then from (22), we have

$$\eta^{-1}\chi(\eta)(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} = k\int_0^1 \rho J_0(\rho\eta)\,d\rho = k\eta^{-1}J_1(\eta)$$

Hence

$$\chi(\eta) = kJ_1(\eta)(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} \qquad (24)$$

This value of $\chi(\eta)$ substituted in the relations for stresses and displacements gives the complete solution.

We find the value of $[\widehat{rr} + \widehat{\theta\theta}]_{z=d}$ with this expression for $\chi(\eta)$ as

$$[\widehat{rr}+\widehat{\theta\theta}]_{z=d} = -2G\beta(1-\nu)ak\int_0^{\infty}\eta^{-1}(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1}J_1(\eta)J_o(\rho\eta)d\eta$$

Writing,

$$(1-e^{-2\eta d/a})(1+e^{-2\eta d/a})^{-1} = 1+2\sum_{p=1}^{\infty}e^{-2p\eta d/a}$$, we obtain

$$[\widehat{rr}+\widehat{\theta\theta}]_{z=d} = \delta\int_0^{\infty}\eta^{-1}\left[1+2\sum_{p=1}^{\infty}e^{-2p\eta d/a}\right]J_1(\eta)J_o(\rho\eta)d\eta = \delta I_1+2\delta I_2 \tag{25}$$

where

$$I_1 = {}_2F_1\left(\frac{1}{2},-\frac{1}{2},1;\rho^2\right), \qquad \text{for } \rho<1$$

$$= 2/\pi, \qquad \text{for } \rho=1$$

$$= (2\rho)^{-1}{}_2F_1\left(\frac{1}{2},\frac{1}{2},2;\rho^{-2}\right), \qquad \text{for } \rho>1 \tag{26a}$$

$$I_2 = \sum_{p=1}^{\infty}\sum_{s=0}^{\infty}\frac{(-1)^s(2s)!2^{-(2s+1)}}{s!(s+1)!(p^2+4p^2d^2/a^2)^{2s+1/2}}P_{2s}\left\{\frac{2pd/a}{(p^2+4p^2d^2/a^2)^{1/2}}\right\} \tag{26b}$$

and $\delta = -2G\beta(1-\nu)ak$ (26c)

3. Paraboloidal Distribution Of Temperature

We consider a paraboloidal distribution of temperature over a circular region of exposure $0 \leq r \leq a$ while the rest of the surface is kept at zero temperature. We have therefore on the surface z=d,

$$T = T_o(1-r^2/a^2), \qquad \text{for } 0 \leq r \leq a$$

$$= 0, \qquad \text{for } r>a \tag{27}$$

By Fourier-Bessel Representation on the surface z=d

$$T = \int_0^\infty \alpha J_o(\alpha r)\,d\alpha \int_0^\infty T_o u(1-u^2/a^2) J_o(\alpha u)\,du$$

$$= 2T_o \int_0^\infty \alpha^{-1} J_2(\alpha a) J_o(\alpha r)\,d\alpha \tag{28}$$

Again, from (5), we have on z=d,

$$T = \int_0^\infty A J_o(\alpha r)\,d\alpha \tag{29}$$

Comparing (28) with (29), we get,

$$A(\alpha) = 2T_o \alpha^{-1} J_2(\alpha a)$$

Writing in the dimensionless form, we get

$$\chi(\eta) = 2T_o a^{-1} J_2(\eta)$$

Substituting in (23), we get

$$[\,\widehat{rr} + \widehat{\theta\theta}\,]_{z=d} = -4G\beta(1-\nu)T_o \int_0^\infty \eta^{-1} J_2(\eta) J_o(\rho\eta)\, d\eta$$

and

$$[\,\widehat{rr} - \widehat{\theta\theta}\,]_{z=d} = 4G\beta(1-\nu)T_o \int_0^\infty \eta^{-1} \left\{ J_2(\eta) J_o(\rho\eta) - 2(\eta\rho)^{-1} J_2(\eta) J_1(\rho\eta) \right\} d\eta$$

(31)

Thus we get finally,

$$[\,\widehat{rr} + \widehat{\theta\theta}\,]_{z=d} = -2G\beta(1-\nu)T_o \begin{cases} {}_2F_1(1,-1/2,1;\rho^2), & \rho<1 \\ 0, & \rho=1 \\ 0, & \rho>1 \end{cases} \qquad (32)$$

$$[\,\widehat{rr} - \widehat{\theta\theta}\,]_{z=d} = 2G\beta(1-\nu)T_o \begin{cases} {}_2F_1(1,-1/2,1;\rho^2) - {}_2F_1(1,-1,2;\rho^2), & \rho<1 \\ -1/2, & \rho=1 \\ -(2\rho^2)^{-1} & \rho>1 \end{cases}$$

(33)

4. NUMERICAL RESULTS

The variation of $-\left\{\left[\widehat{rr}+\widehat{\theta\theta}\right]_{z=d}\right\}/2G\beta(1-\nu)T_o$ for different values of ρ within the circle $\rho\le 1$ is given in the following table

ρ	$-\left\{\left[\widehat{rr}+\widehat{\theta\theta}\right]_{z=d}\right\}/2G\beta(1-\nu)T_o$
0.0	1.0000
0.2	0.9798
0.4	0.9185
0.6	0.8000
0.8	0.6002
1.0	0.0000

5. CONCLUSION

Thus, we note that the value of $-\left\{\left[\widehat{rr}+\widehat{\theta\theta}\right]_{z=d}\right\}/2G\beta(1-\nu)T_o$ is maximum at the origin, diminishes slowly at the initial stage, but rapidly near the edge of the circle of exposure and zero value at the edge of the exposure and outside it.

Thermal Stresses Due To Prescribed Flux Of Heat On The Boundary Of A Semi-Infinite Elastic Solid

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1. Method Of Solution

Let the bounaary surface of the semi-infinite isotropic elastic solid be given by z=0, the axis of z being drawn into the body.The temperature field in the steady state is given by Laplace's equation

$$\nabla^2 T=0 \tag{1}$$

To determine the stresses , the potential of thermo-elastic displacement ψ will be used. This is related to the displacement components u,0,w by the equation, if axially symmetrical coordinates be assumed [17],

$$u_T = \frac{\partial\psi}{\partial r}, \quad w_T = \frac{\partial\psi}{\partial z} \tag{2}$$

In this problem of axially symmetrical temperature field, the nonvanising components of displacements are u and w and since we are considering axially symmetrical coordinates the general values of u,w are not taken.

From the stress strain relations in problems of thermal stresses and the equations of equilibrium we have [17]

$$\nabla^2\psi = \beta T; \tag{3}$$

where
$$\beta = \frac{1+\nu}{1-\nu}\alpha_1 \tag{4}$$

α_1 being the coefficient of linear thermal expansion of the solid and ν is the Poission's ratio.

A particular integral of the equation (3) is

$$\psi = \frac{\beta}{2}\int_0^\infty A(\alpha)\frac{z}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}J_o(\alpha r)\,d\alpha \tag{5}$$

Substituting in (3), we get,

$$T = \int_0^\infty \alpha A(\alpha)\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}J_o(\alpha r)\,d\alpha \tag{6}$$

where $A(\alpha)$ is a function of α only to be determined.

The stress components [17] can now be written as,

$$\widehat{rz}_T = 2G\frac{\partial^2\psi}{\partial r\partial z} = -G\beta\int_0^\infty \alpha A(\alpha)(1-\alpha z)\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}J_1(\alpha r)\,d\alpha$$

$$\widehat{zz}_T = 2G\left(\frac{\partial^2\psi}{\partial z^2}-\nabla^2\psi\right) = zG\beta\int_0^\infty \alpha^2 A(\alpha)\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}J_o(\alpha r)\,d\alpha$$

$$\widehat{rr}_T = 2G\left(\frac{\partial^2\psi}{\partial r^2}-\nabla^2\psi\right) = G\beta\int_0^\infty\left[\frac{z}{r}J_1(\alpha r)+(2-\alpha z)J_o(\alpha r)\right]\alpha A(\alpha)\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}d\alpha$$

$$\widehat{\theta\theta}_T = 2G\left(\frac{1}{r}\frac{\partial\psi}{\partial r} - \nabla^2\psi\right) = G\beta\int_0^\infty\left[2J_0(\alpha r) - \frac{z}{r}J_1(\alpha r)\right]\alpha A(\alpha)\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z\,\cosh\alpha z\right\}d\alpha \qquad (7)$$

The subscript T denotes that the stresses are due to thermal expansion only, G being the modulus of elasticity in shear.

We observe that the normal stresses $\widehat{zz}_T$ vanishes at z=0, the stress $\widehat{rz}_T$ however does not vanish. In order to suppress it the stress system $(\widehat{rr}_C, \widehat{\theta\theta}_C, \widehat{zz}_C, \widehat{rz}_C)$ obtained on the hypothesis that there is no temperature distribution is to be superposed.

2. Complementary Stresses

In order to determine the complementary stresses we use Love's function satisfying biharmonic equation

$$\nabla^4\phi = 0\ ,$$

with the boundary conditions

$$\left[\widehat{zz}_C\right]_{z=0} = 0, \quad \left[\widehat{rz}_C + \widehat{rz}_T\right]_{z=0} = 0 \qquad (8)$$

and $\phi=0$ at infinity.

Let us assume the function ϕ in the form,

$$\phi = \int_0^\infty [B+C\alpha z]\frac{1}{\tanh\alpha z}\left\{\sinh\alpha z-\tanh^2\alpha z\ \cosh\alpha z\right\}J_o(\alpha r)d\alpha \tag{9}$$

where B and C are functions of α.

The complementary stresses are then given by [17]

$$\widehat{rr}_c= \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{\partial^2\phi}{\partial r^2}\right]$$

$$\widehat{\theta\theta}_c= \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[\nu\nabla^2\phi - r^{-1}\frac{\partial\phi}{\partial r}\right]$$

$$\widehat{zz}_c= \frac{2G}{1-2\nu}\frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right]$$

$$\widehat{rz}_c= \frac{2G}{1-2\nu}\frac{\partial}{\partial r}\left[(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{10}$$

In view of the first condition of (8), we have from (10)

$$B=-C(1-2\nu) \tag{11}$$

Then from (9)-(11), we get

$$\widehat{zz}_c = \frac{2Gz}{1-2\nu}\int_0^{\infty} C\alpha^4 \frac{1}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z \cosh\alpha z\right\} J_o(\alpha r)d\alpha$$

$$\widehat{rz}_c = -\frac{2G}{1-2\nu}\int_0^{\infty} C\alpha^3 \frac{1}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z \cosh\alpha z\right\}(1-\alpha z)J_o(\alpha r)d\alpha$$

$$\widehat{\theta\theta}_c = \frac{2G}{1-2\nu}\int_0^{\infty}\left[2\nu J_o(\alpha r) - (2\nu-2+\alpha z)J_1(\alpha r)/\alpha r\right]C\alpha^3 \frac{1}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z \cosh\alpha z\right\}d\alpha$$

$$\widehat{rr}_c = \frac{2G}{1-2\nu}\int_0^{\infty}\left[(2-\alpha z)J_o(\alpha r) - (2\nu-2+\alpha z)J_1(\alpha r)/\alpha r\right]C\alpha^3 \frac{1}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z \cosh\alpha z\right\}d\alpha \qquad (12)$$

The second boundary condition (8) gives

$$C = -\frac{\beta(1-2\nu)}{2\alpha^2}A(\alpha) \qquad (13)$$

We have therefore the stresses $\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{zz}$ etc. given by

$$\widehat{rr} = \widehat{rr}_C + \widehat{rr}_T = 2G\beta\int_0^\infty (1-\nu)\frac{\alpha A(\alpha)}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z\ \cosh\alpha z\right\}\frac{J_1(\alpha r)}{\alpha r}\,d\alpha$$

$$\widehat{\theta\theta} = \widehat{\theta\theta}_C + \widehat{\theta\theta}_T = 2G\beta\int_0^\infty (1-\nu)\frac{\alpha A(\alpha)}{\tanh\alpha z}\left\{\sinh\alpha z - \tanh^2\alpha z\ \cosh\alpha z\right\}\left[J_o(\alpha r) - \frac{J_1(\alpha r)}{\alpha r}\right]d\alpha \qquad (14)$$

We write the solution in a dimensionless form on putting

$$\alpha A(\alpha) = a\chi(\alpha a), \quad \eta=\alpha a, \quad \rho=r/a, \quad \zeta=z/a \qquad (15)$$

where a is some length and η a new variable of integration. We find that the solution may be written in the form

$$T = -\int_0^\infty \chi(\eta)\frac{1}{\tanh\eta\zeta}\left\{\sinh\eta\zeta - \tanh^2\eta\zeta\ \cosh\eta\zeta\right\}J_o(\rho\eta)\,d\eta \qquad (16)$$

Correspondingly, we get,

$$\widehat{rr} = 2G\beta(1-\nu)\int_0^\infty \chi(\eta)\frac{1}{\tanh\eta\zeta}\left\{\sinh\eta\zeta - \tanh^2\eta\zeta\ \cosh\eta\zeta\right\}\frac{J_1(\rho\eta)}{\rho\eta}\,d\eta$$

$$\widehat{\theta\theta} = 2G\beta(1-\nu)\int_0^\infty \chi(\eta)\frac{1}{\tanh\eta\zeta}\left\{\sinh\eta\zeta - \tanh^2\eta\zeta\ \cosh\eta\zeta\right\}\left[J_o(\rho\eta) - \frac{J_1(\rho\eta)}{\rho\eta}\right]d\eta$$

3. Prescribed Flux Of Heat On The Boundary

We shall suppose that on the free surface $z=0$, there is a flux of heat within a circular region, the rest of the surface being free of any flux of heat. So the boundary conditions are, on the surface $z=0$,

$$\frac{\partial T}{\partial z} = f(r/a), \quad \text{for} \quad 0 \le r < a$$

$$= 0, \qquad \text{for} \qquad r > a \tag{17}$$

Now from (16),

$$\frac{\partial T}{\partial z} = \int_0^{\infty} a^{-1} \eta \chi(\eta) J_o(\rho\eta) \, d\eta \tag{18}$$

Hence by Hankel's inversion theorem [14]

$$\chi(\eta) = a^{-1} \int_0^1 \rho f(\rho) J_o(\rho\eta) \, d\rho \tag{19}$$

Then we have from the relation (14)

$$\widehat{rr} + \widehat{\theta\theta} = 2G\beta(1-\nu) \int_0^{\infty} \chi(\eta) \frac{1}{\tanh\eta\zeta} \left\{ \sinh\eta\zeta - \tanh^2\eta\zeta \ \cosh\eta\zeta \right\} J_o(\rho\eta) d\eta$$

$$\widehat{rr}-\widehat{\theta\theta} = 2G\beta(1-\nu)\int_0^\infty \chi(\eta)\frac{1}{\tanh\eta\zeta}\left\{\sinh\eta\zeta - \tanh^2\eta\zeta\ \cosh\eta\zeta\right\}\left[2\frac{J_1(\rho\eta)}{\rho\eta} - J_o(\rho\eta)\right]d\eta \qquad (20)$$

When $z=0$, take the forms

$$[\ \widehat{rr}+\widehat{\theta\theta}\ ]_{z=0} = 2G\beta(1-\nu)\int_0^\infty \chi(\eta)J_o(\rho\eta)\,d\eta$$

$$[\ \widehat{rr}-\widehat{\theta\theta}\ ]_{z=0} = 2G\beta(1-\nu)\int_0^\infty \chi(\eta)\left[2\frac{J_1(\rho\eta)}{\rho\eta} - J_o(\rho\eta)\right]d\eta \qquad (21)$$

We shall obtain expressions for $[\ \widehat{rr}+\widehat{\theta\theta}\ ]_{z=0}$ and $[\ \widehat{rr}-\widehat{\theta\theta}\ ]_{z=0}$ for two particular functional value of $f(\rho)$.

Case 1.

Let us assume that $f(\rho)=k$. Then from (19), we have

$$\chi(\eta) = a^{-1}k\int_0^1 \rho J_o(\rho\eta)\ d\rho = k(a\eta)^{-1}J_1(\eta) \qquad (22)$$

Substituting this value of $\chi(\eta)$ intothe relations (21), we obtain

$$[\widehat{rr}+\widehat{\theta\theta}]_{z=0} = \frac{\delta k}{a} F\left(\frac{1}{2},-\frac{1}{2},1;\rho^2\right), \qquad \text{for } \rho<1$$

$$= \frac{2\delta k}{a\pi}, \qquad \text{for } \rho=1$$

$$= \frac{\delta k}{2a\rho} F\left(\frac{1}{2},\frac{1}{2},2;\rho^{-2}\right), \qquad \text{for } \rho>1$$

$$[\widehat{rr}-\widehat{\theta\theta}]_{z=0} = \frac{\delta k}{a}\left[F\left(\frac{1}{2},-\frac{1}{2},2;\rho^2\right)-F\left(\frac{1}{2},-\frac{1}{2},1;\rho^2\right)\right] \qquad \text{for } \rho<1$$

$$= \frac{2\delta k}{3a\pi}, \qquad \text{for } \rho=1$$

$$= \frac{\delta k}{2a\rho}\left[2F\left(\frac{1}{2},-\frac{1}{2},2;\rho^{-2}\right)-F\left(\frac{1}{2},\frac{1}{2},2;\rho^{-2}\right)\right], \quad \text{for } \rho>1 \tag{23}$$

where $\delta = 2G\beta(1-\nu)$

Case 2.

We take the flux function as a parabolic one, so that we take $f(\rho) = k(1-\rho^2)$. Then from (19)

$$\chi(\eta) = ka^{-1}\left[4\eta^{-3}J_1(\eta)-2\eta^{-2}J_0(\eta)\right] \tag{24}$$

Substituting this value of $\chi(\eta)$ into the relations (21) and integrating we get [14]

$$[\widehat{rr}+\widehat{\theta\theta}]_{z=0} = \frac{\delta k}{3a}\left[6F\left(-\frac{1}{2},-\frac{1}{2},1;\rho^2\right)-4F\left(-\frac{1}{2},-\frac{3}{2},1;\rho^2\right)\right], \qquad \text{for } \rho<1$$

$$= \frac{8\delta k}{9a\pi}, \qquad \text{for } \rho=1$$

$$= \frac{2\delta k\rho}{a}\left[F\left(-\frac{1}{2},-\frac{1}{2},1;\rho^{-2}\right)-F\left(-\frac{1}{2},-\frac{1}{2},2;\rho^{-2}\right)\right], \qquad \text{for } \rho>1$$

$$[\widehat{rr}-\widehat{\theta\theta}]_{z=0} = \frac{\delta k}{3a}\left[6F\left(-\frac{1}{2},-\frac{1}{2},2;\rho^2\right)+4F\left(-\frac{1}{2},\frac{3}{2},1;\rho^2\right)\right.$$

$$\left.-6F\left(-\frac{1}{2},-\frac{1}{2},1;\rho^2\right)-4F\left(-\frac{1}{2},\frac{3}{2},2;\rho^2\right)\right] \qquad \text{for } \rho<1$$

$$= \frac{8\delta k}{15a\pi}, \qquad \text{for } \rho=1$$

$$= \frac{\delta k\rho}{3a}\left[6F\left(-\frac{1}{2},-\frac{1}{2},-2;\rho^{-2}\right)+4F\left(-\frac{1}{2},-\frac{3}{2},1;\rho^{-2}\right)\right.$$

$$\left.-6F\left(-\frac{1}{2},-\frac{1}{2},1;\rho^{-2}\right)-4F\left(-\frac{1}{2},-\frac{3}{2},2;\rho^{-2}\right)\right] \qquad \text{for } \rho>1$$

4. Numerical Results

Taking a=1, the variation of $\left\{[\widehat{rr}-\widehat{\theta\theta}]_{z=0}\right\}/2Gk(1+\nu)\alpha_1$ for different values of ρ , in the two cases is given in the following table

Table 1.

ρ	$\left\{\left[\widehat{rr}-\widehat{\theta\theta}\right]_{z=0}\right\}/2Gk(1+\nu)\alpha_1$
0.0	0.00000
0.2	0.00505
0.4	0.02087
0.6	0.04988
0.8	0.09868
1.0	0.21212
1.5	0.27138
2.0	0.22519
3.0	0.15955
4.0	0.12203

Table 2.

ρ	$\left\{\left[\widehat{rr}-\widehat{\theta\theta}\right]_{z=0}\right\}/2Gk(1+\nu)\alpha_1$
0.0	0.00000
0.2	0.00990
0.4	0.03836
0.6	0.08140
0.8	0.13125
1.0	0.16969
1.5	0.14538
2.0	0.11686
3.0	0.08098

5. Conclusion

In the first case of distribution of temperature we note that the value of $\left\{\left[\widehat{rr}-\widehat{\theta\theta}\right]_{z=0}\right\}/2Gk(1+\nu)\alpha_t$ is zero at the origin, reaches its maximum at a point shortly beyond the edge of the region of exposure and then diminishes continuously as ρ increases further.

In the second case of distribution of temperature we observe that the nature of $\left[\widehat{rr}-\widehat{\theta\theta}\right]_{z=0}$ remains unaltered but its value diminishes rapidly outside the region of exposure.