P R E F A C E

Vibrational phenomena in a variety of systems constitute one of the most important fields of scientific study in general and of engineering in special. Vibrations are also of interest to astronomical studies and to medical science, the living body itself being a vast treasurehouse of complicated vibratory systems.

In engineering practice, vibrations are encountered in innumerable technical problems of significance. Such problems can be thoroughly understood only on the basis of the theory of vibrations. With the increase of size and velocity of action of various systems and machineries, the analyses of vibration problems become all the more important in design work. The working conditions of a system require that the undesirable critical condition of vibration must, as far as possible, be removed. Achievement of this demands the most favourable design proportions which can be found only by judicious enquiries into the characteristics of the vibrating system, particularly when vibrations are of large amplitudes [1].

Further, the deformational response of plates subjected to deflections of considerable magnitudes are significantly sensitive in the thickness direction. Plates are very often used in possibly all machineries to economise on weight.

Any system, to speak of, possesses the capability of vibration and most systems can vibrate subject to different conditions. Vibrations are usually complicated phenomena except possibly the linear ones which are idealised to be the simplest forms. The linear theory has been developed primarily for the purpose of giving an insight into the problems of vibrations. It was founded upon an extension of the more general cases of results which were obtained from certain exact or approximate solutions of the equations of equilibrium of elastic solids.

When, however, the extent of vibrations exceeds a certain level of amplitudes so that the middle surface strains cannot be ignored, the vibrations are called non-linear. Non-linear vibrations are liable to cause undesirable effect of upsetting the favourable working conditions of a machinery, and may even cause severe damage to the object or in the environment.

The linear vibration is but a restricted case of vibrations, the most general case being non-linear. Linear vibrations seldom occur in practice and in nature.

Non-linear vibrations may be initiated by a sudden and heavy dynamic loading such as blast waves, shocks, high speed motions and the like. Studies of non-linear vibrations have assumed of late a new significance with the advent of space industry. Non-linear problems also frequently arise in the analysis of control systems [2].

In the linear theory of thin elastic plates, the following assumptions are made [3] :

(a) Points on a normal to the middle plane before bending remain on the normal of the middle surface after bending.

(b) Normal stress across each thin layer parallel to the middle plane is small compared with the other stress components and may be neglected in the stress-strain relation.

(c) The slope of the deflected middle surface of the plate, called the neutral surface, in any direction is small compared to unity.

The two great landmarks that have followed the investigation of bending of a beam by Galileo [4] in 1638, the first mathematician, are the discovery of Hooke's Law [5] in 1660 and the general differential equations of equilibrium of vibrating solids by Navier [6] in 1821. Navier's [6] analyses, however, were subsequently disputed by Todhunter and Pearson [7]. The theoretical works of Euler [8] and of Daniell Bernoulli [9] on vibrations of bars and the experimental works of Chladni [10] in 1802 on vibrating plates are mentionable works of the period between 1638 and 1821.

During this interval, investigations were chiefly directed to theories of bars and to more special problems of plates. But the development so far may be summed up as an inadequate theory of flexure, an erroneous theory of torsion, an unproved theory of vibrations of bars and plates, and the definition of Young's modulus [1].

The theories of elasticity established by Cauchy [11] and by Poisson [12] in 1829 were applied by Clapeyron [13] and Lamé [14] to numerous vibration problems. Saint-Venant [15] is credited for bringing the problems of beams under the general theory of elasticity. The theory of Saint-Venant [15] was subsequently developed by Phillips [16], and discussed by Mindlin and Goodman [17]. Lord Kelvin [18] developed the problem of a vibrating sphere in 1890. The history of the science from the initial enquires of Galileo [4] down to the investigations of Saint-Venant [15] and Lord Kelvin [18] records a continuous progress [1].

Vibration problems of rings, bars and shafts have been elaborately investigated by various authors, reference to which may be found in the well-known treatise "Vibration Problems in Engineering" by Timoshenko [19]. The analyses done by Timoshenko [19] himself is of special interest in the sense that the author has solved vibration problems considering the effects of foundation, of the mass of the vibrating body, of rotatory inertia and of the shear due to deflection [1]. Lord Rayleigh [20] considered the energy of a vibrating system taking account of the details as far as practically significant, and devised an approximate method of which the Ritz's method [21] is a further development successfully applied to the problem of a bar of variable cross-section. The Ritz's method [21] has been extensively discussed with elaborate references by Jacobsen [2]. The general theory of the transverse vibrations of a circular plate was obtained by Kirchoff [22]. The problem of a conical bar was, however, first investigated by the same author [23]. Approximate solutions for the vibrations of hulls and akin structures [24-26] were obtained using step-by-step method of numerical [27, 28] and graphical integration [29] by some authors [19]. Step-by-step method does not lend itself to generalisations as analytical expressions do but the method unto itself is a powerful one [2].

Vibration problems of membranes have been discussed by Lord Rayleigh [20] and the Rayleigh-Ritz method [2] has been found to be very useful in calculating the frequencies of the natural modes of vibrating membranes. For such problems it is assumed that the membrane is a perfectly flexible and infinitely thin lamina and that it is uniformly stretched in all directions by a tension so large that fluctuations in this tension due to small deflections can be neglected.

The problems of bars, membranes, plates and shells have been very elegantly and extensively discussed by Lord Rayleigh [20] in his famous book "The Theory of Sound" and also by Timoshenko [19].

Analysis of linear vibrations of elastic plates since long has attracted attention resulting in a wealth of papers published by numerous authors. An extensive bibliography on the subject may be found in the handbook of Gontkevich [30].

Chulay [31] used polynomials, and Joga Rao and Lakshmi Kantam [32] used Ritz's functions [21] for the vibration of elastically restrained rectangular plates. Linear flexural vibrations of sandwich plates have been investigated in great detail by Yu [33-39]. Yu [33-38] noted that the results indicate that the transverse shear deformation of the core in general must be considered. This conclusion has been shown to be equally valid for sandwich plates with honey-comb cores [39]. Investigations of vibrations of plates with holes by Joga Rao and Pickett [40], though lack in graphical presentations, are commendable. The authors [40] used the energy method.

Some notable works were also done during the period 1965-1970 [41-44]. Elegant and extensive discussions on linear vibrations have been presented by

Leissa [45], and it appears that investigations of systems, undergoing mechanical vibrations caused by linear motions, are exhaustive.

But investigations of vibrations due to thermal shocks to which dynamic systems are often subjected appear to be rare in literature. Thermally induced vibrations are of interest in aircrafts and machine designs, in chemical and nuclear engineering and even in astronautical engineering [46]. In case of sudden heating or for temperature fields varying harmonically at high frequencies, the influence of inertia terms on deflections cannot be neglected. Boley and Barbar [47] have pointed out the importance of the role of inertia in the design of high-speed vehicles and machineries. Solutions to problems of thermally induced vibrations obtained by retaining the inertia term is referred to as dynamic solutions. Boley and Weiner [48], and Nowacki [46] independently investigated vibrations of rectangular isotropic plates and noted that the results are qualitatively similar. Sarkar [49,50] has treated some thermal deflection problems. Biswas [51,52] has solved the problem of a right isosceles and an equilateral triangular plate [51] and later, also of an orthotropic rectangular plate [52] in 1977.

In case of non-linear vibrations, the assumptions of the linear theory do not hold and the middle surface strains have to be considered whenever the magnitude of deflections of the plates is comparable to their thickness. Geometrical non-linearity creeps into the non-linear problems. But it is assumed that no physical non-linearity exists due to overstrain i.e. Hooke's Law is assumed to be valid.

Exact analytical solutions of non-linear problems due to inclusion of the middle surface strains are, in general, difficult. Thus recourse must be had, all the more, to approximate methods. Though approximate, the methods are of sufficient accuracy to satisfy nearly all the demands required by engineering applications [2,3].

Kirchoff [22] and Clebsch [53] were first to devise theories in 1862 taking account of the possibility that the displacement of the middle surface may be of any order of magnitude provided that the plate or shell is not overstrained. Kirchoff [22] adopted the multi-constant theory in the investigations of plates and thin rods. Clebsch [53] has extensively discussed the impact effect on a railway track or bridge due to a live load. The problem [53] is of great engineering importance and has been investigated by many other workers, reference to which may be found in [19]. Since the advancement of theories by Kirchoff [22] and Clebsch [53], various approximate graphical methods of solution of non-linear differential equations have been developed. The method based on time-displacement curve proposed by Lord Kelvin [18] is perhaps one of the first methods [1]. Another graphic way of solution is an adaptation of the phase-plane method of Lamoën [54] as has been applied by Jacobsen [2] to solve complicated non-linear technical problems. Rayleigh-Ritz method [2], also an approximate method originally given for solving membrane problems, can solve the non-linear problems of circular plates elegantly [1, 2, 19]. Timoshenko [19] also has extensively discussed non-linear problems including the strain of the middle plane of the plate.

In 1910, von Kármán [55] proposed a system of coupled field equations involving the deflections and the stress functions developed in a plate due to large bending.

Due to enhanced resistance of the plate to large stretching, the rigidity and hence the frequency due to non-linear vibrations increase with the pressure acting on the plate [19]. Such increase in frequency was experimentally established by Powell and Roberts [56] in 1923.

Von Kármán equations [55] have been extended to dynamic cases by Grigoliuk [57], Chu and Herrmann [58], Nowinski [59] and Nowinski and Ismail [60] and Yu [61].

Chu and Herrmann [58] solved the problem of rectangular plates with hinged immovable edges using a set of plate equations [62] considered to be dynamic analog of the von Kármán plate theory [55]. Since the vibrations of plates are principally transverse, the authors [58] neglected the inertia term inherent with the motion of the middle plane of the plate, and applied the perturbation method previously adopted by Carrier [63] and Eringen [64,65] in the solution of non-linear problems of strings [63], membranes [64] and bars [65]. Chu and Herrmann [58] observed that the aspect ratio has a relatively small effect on the magnitude of the membrane stress and that, percentage-wise, the large amplitudes affect the stress markedly less than the period of vibration. The total stress, however, was noted to be considerably larger than that predicted by the classical theory. Nowinski [59] adopted the orthogonalisation procedure in the investigation of circular built-in plates. Results obtained by Nowinski [59] agree reasonably well with those of Grigoliuk [57]. Nowinski's [59] results for orthotropy, reduced to linear case, agree with that obtained by Lekhnitsky [66]. Lekhnitsky [66] obtained results directly from Lagrange's equations of motions of the second kind. But for isotropy, Nowinski's results [59] differ by about seventeen percent from those computed by Cox and Klein [67]. Cox and Klein [67] followed a method essentially based upon the method of collocation [59]. Nowinski and Ismail [60] solved, through Galerkin procedure, the problem of anisotropic right triangular plate with built-in contour. Galerkin in 1915 introduced a method of approximate solutions of elastic rods and plates without calling attention to Ritz's original authorship [2]. Many years later, however, Klotter [68] called attention to the identity of Ritz's second method and the Galerkin method [2]. In both the above problems [59,60], the co-ordinate functions were presented in a separable form involving the deflections of the plate and the stress functions. The stress functions were not, however, determined in the investigations [59,60]. Nowinski and Ismail [60] also noted a marked effect associated with freely movable edges on the attenuation of the membrane stresses in the interior of the plate. Ramachandran [69] attempted the solution of non-linear vibrations of simply supported isotropic rectangular plate carrying concentrated mass and concluded that the time ratio is dependent on the mass.

The non-linear bending response of rectangular and circular plates subjected to various boundary conditions have been investigated by Baur [70] very elegantly.

Baur [70] transformed the dependent time functions through Airy stress functions [20] such that Lighthill's extension [71] of Poincare's perturbation method can be employed to eliminate the time and find the stress function [1]. The investigation of Baur [70] is of special interest in the sense that in addition to a step-function load, Baur [70] also treated the exponentially decaying pulse which is supposed to be adequate to describe a blast load. The investigation [70] records pronounced effects of non-linearities of immovably constrained boundaries on the response of simply supported plates.

Nayfeh [72], Kevorkian [73] and Atluri [74] used the perturbation procedure of multiple time scale to investigate non-linear vibration problems of various mechanical systems. The non-linear vibration problem of simply supported angle ply laminated plates was solved by Benette [75], and that of orthotropic triangular plates was solved by Vendhan and Dhoopar [76]. Solutions of a number of non-linear vibration problems are found in [77].

Duffing [2] made a pioneer study in 1918 on non-linear vibration problems and discussed methods of approximation and questions of stability with technical applications. Minorsky [78] gave an extensive survey of the international literature upto 1947 with a clear discussion of topological and analytical methods and non-linear resonance. Stoker [79] is credited for a clean exposition of the topological methods of solving autonomous systems undergoing non-linear vibrations with fairly complete discussion of stability. A notable book due to MClachlan [80] contains an excellent discussion of the better-known analytical procedures containing extensive bibliography to 1950. Pipes [81] in 1951 introduced the operational methods to reduce the algebraic labour of the solutions of certain classes of non-linear technical problems. Hayashi [82] gave an elegant discussion of forced oscillations of non-linear systems by transforming the equations such that topological methods can be used. Hayashi [82] also investigated the possibility of sub-harmonic solutions in non-linear systems. Clauser [83] published an excellent expository paper presenting a diversified group of non-linear phenomena. Clauser's [83] work contains a comprehensive bibliography of recent papers.

After von Kármán [55] another elegant, but still approximate, method was advanced by Berger [84] in 1955 for investigations of non-linear problems. During the span 1934-1953 exact solutions of some non-linear problems are, however, found recorded in the literature [85-91].

Berger [84] in his doctoral dissertation proposed an approximate method suggested by Prof. Williams, a pioneer in the field for the investigation of large deflections of initially flat isotropic plates. This method is essentially based on the neglect of the second invariant, in comparison to the first invariant, of the middle surface strains in the expression for the total potential energy of the system. Thus the variation of potential energy with respect to the in-plane displacements leads to the drastic simplification that the first invariant of the middle surface strain is constant. The resulting differential equations for the deflection and displacements, though approximate, are still non-linear but may be decoupled in such a manner that they may be solved readily. Berger [84] states that there appears to be no completely satisfactory simple physical justification for neglecting the second invariant of the middle surface strain and thus its justification must be based on comparison of the resulting approximate solutions with available exact solutions [85-91].

Berger's [84] well-known method has been employed to investigate large deflections of plates by a number of authors like Iwinski and Nowinski [92], Nowinski [93 - 95] and Sinha [96]. A survey of literature, however, shows that Nash and Modeer [97] were first to utilise Berger's method [84] in 1960 to solve non-linear vibration problems of isotropic rectangular plates with hinged restrained edges and of circular plates with periphery somewhat elastically restrained against rotation. Nash and Modeer [98] also solved the problems of simply supported circular and rectangular isotropic plates. The periods obtained [97] for rectangular plates are independent of the aspect ratio, whereas the corresponding results obtained by Chu and Herrmanna [58] are dependent on that ratio. Dutta [99] investigated the non-linear vibrations of irregular isotropic plates on elastic foundations by the conformal mapping technique in conjunction with Berger's method [84] and concluded that the periods will depend on the aspect ratio. Moreover, the analysis of Dutta [99] allows the solution of the eigen-values from a unified point of view since the trial functions assumed are the same for all shapes of plates. Banerjee [100, 101] has solved the problems of an elliptic plate [100] and also of an isotropic isosceles right triangular plate [101]. In solving the problem of elliptic plates with boundary elastically restrained against rotation [100] use has been made of Mathieu functions of zero order. The results [100] reduced to a circular symmetry agree with those of Nash and Modeer [97]. But the investigation [100] lacks in both graphical presentation and numerical evaluations due to possibly the induction of

Mathieu functions. Banerjee [102] analysed non-linear vibrations of rectangular and isosceles right triangular orthotropic plates by Berger's method [84], but it lacks in presentation of graphs. The treatment by Chiang and Chen [103] of the vibration of circular isotropic plates with concentric rigid mass is a restricted one in the sense that the mass is assumed at the centre of the plate. Ramachandran [104] has investigated the vibrations of isotropic rectangular plates with one pair of edges elastically restrained. The deflection equation given by Ramachandran [104] is a good approximation as can be seen by comparing the available solutions [105] of extreme cases of supports. Kamiya [106] extended Berger's method [84] in a modified way to the problem of large deflections of laterally loaded rectangular sandwich plates.

In Chapter I of the present thesis, the solution to the problem of vibrations induced in a right isosceles triangular plate on elastic foundations by thermal shocks, has been obtained in a closed form. The temperature distribution has been assumed in the form of a double trigonometric series and Laplace transform has been used for the dynamic part of the solution. The double trigonometric series is much more rapidly convergent than the series assumed by Biswas [51].

The amplitude-frequency characteristics of non-linear vibrations of various members have been investigated in Chapter II. For non-linear vibrations it is of interest to establish the influence of large amplitudes on the salient parameters of the system, e.g., the frequency of free vibrations.

In Section A, the problem of an elastically restrained rectangular plate and that of a clamped elliptic plate, each carrying a concentrated mass, have been solved applying von Kármán equations [65] in their dynamical forms. The stresses are found in each case. Galerkin procedure has been applied to the problem of the elliptic plate.

Problems solved by Berger's method [84] in Section B, are simply supported equilateral triangular plates resting on elastic foundations, right-angled isosceles orthotropic triangular plates carrying a concentrated mass and sandwich rectangular plates. The same method [84] has also been profitably utilised in the investigations of clamped circular plates with variable rigidity and of annular plates clamped along outer edge. Solution to the equilateral triangular plate is sought in conjunction with tri-linear co-ordinates [107]. Galerkin procedure has been applied to the cases of orthotropic and circular plates.

Results obtained for problems in different chapters have been compared with those of standard works and discussed. Graphical presentations are given in all the cases investigated. Some of the numerical results of non-linear solutions have also been exhibited through common parametrisation for comparative study.

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