

January 2023

Dedicated to my Parents...

Declaration

I, **Anirban Chanda**, declare that the research in the thesis is my own *bona fide* work and carried out under the supervision of **Prof. (Dr.) Bikash Chandra Paul** from June 2017 to June 2022 at the **Department of Physics**, University of North Bengal, Darjeeling, Siliguri. The research contribution in the thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credit to the research workers whenever and wherever their works have been cited in my work in the thesis. I further declare that I have not willfully copied any others' work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, thesis, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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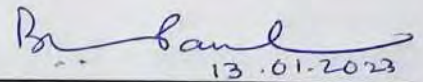
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CERTIFICATE OF SUPERVISOR

It is certified that the research work presented in the thesis entitled: "**Cosmological Models of the Universe in Different Gravitational Theories and Astronomical Observations**" by Mr. Anirban Chanda, Ph.D. Registration No.- *Ph.D./Phy.(1305)/963/R-2019*, has been carried out under my supervision. The work is original and not submitted elsewhere for a degree.

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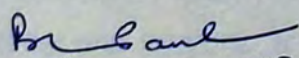
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Cosmological Models of the Universe in Different Gravitational Theories and Astronomical Observations This thesis is submitted for the degree of Doctor of Philosophy by Anirban Chanda Under the supervision of Prof. Bikash Chandra Paul DEPARTMENT OF PHYSICS NORTH BENGAL UNIVERSITY SILIGURI, Dist: DARJEELING WEST BENGAL, Pin - 734013, INDIA REGISTRATION NO. YEAR OF SUBMISSION Ph.D/Phy.(1305)/963/R-2019 2023

Chapter 1 Introduction " There are more things in Heaven and Earth, Horatio, than are dreamt of in your philosophy. " -- William Shakespeare, Hamlet. I n 1915 Einstein proposed the general theory of relativity (GR) which revolutionized the theoretical understanding of the universe [2]. In 1917, Einstein applied his theory to obtain the model of the universe which initiated the study of relativistic cosmology [3]. As the astronomical observations during that time predicted a static universe, Einstein wanted to get a static model but failed to accommodate a static universe with his theory. In order to get a static universe, Einstein introduced a cosmological constant (Λ) which provides the repulsive force to obtain a stable static universe which is called Einstein's static universe (ESU). However, in 1922 Friedmann obtained relativistic solutions which describe an expanding universe without a cosmological constant [4]. But the cosmological solution was unattended and remained of academic interest for a long time. Later Lemaître used the relativistic solution to formulate the Big Bang cosmological model [5]. However, in the year 1929 Hubble discovered that the spectral lines emitted from remote galaxies appear systematically shifted towards the red end of the spectrum indicating an expansion of the universe [6]. Hubble showed that galaxies recede in all directions and more distant ones recede more rapidly proportional to the distance between them. Hubble's discovery led Einstein to abandon the cosmological constant term (Λ), which he introduced to construct a static universe. Hubble's discovery indicates that the universe is expanding. If one extrapolates this cosmic expansion backward in time following the known laws of physics then the universe can be traced back to an extremely hot and dense state [7]. It led to a singularity, confirming initial explosion that occurred approximately 13.8 billion years ago called Big Bang. After the Big Bang, the universe began to expand and cooled sufficiently allowing nucleosynthesis, that led to the formation of stars and galaxies as we observe today. With the advent of modern science, cosmology is transforming from a


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