Chapter-I

INTRODUCTION

1.1 INTRODUCTORY REMARKS

The content of this thesis are arranged in three main chapters and each chapter is subdivided into few parts. Chapter I is of review nature and deals with the introduction to the thesis and a brief discussion on the basic concepts of flow through porous medium, rotating fluid flow, unsteady free convective flow and mass transfer through porous medium, unsteady MHD free convective flow of viscous fluid with mass transfer in porous medium and unsteady convective dispersion process. Attempts are also made to give a brief survey of previous results so that the work presented in this thesis could be seen in its proper perspective.

Convective heat transfer in a porous medium is a topic of rapidly growing interest due to its application to geophysics, thermal insulation engineering, exploration of petroleum and gas field, water movement in geothermal reservoirs, underground spreading of chemical wastes, oil reservoirs engineering and packed bed storage tank etc. This transport processes occurring in nature due to temperature differences. This difference causes the density difference. This density difference is also caused by chemical composition differences and gradients or by phase constitutions. The flow caused by the density differences is known as mass transfer flow. The analysis of hydro-magnetic free convective flow in presence of mass transfer in porous medium is very important from the technological point of view. For this reason unsteady MHD free convective fluid flow with mass transfer in porous medium with or without rotation in different physical situation is considered in chapter II. Chapter II is divided into two parts and each part consists of two different problems. The first part of chapter II is associated with unsteady free convective MHD fluid flow with mass transfer through porous medium while in second part, unsteady free convective MHD fluid flows with mass transfer through porous medium in rotating system are considered. In first part of chapter II, the effect of variable suction or injection on the unsteady two dimensional free convective flow with mass transfer of an electrically conducting fluid past a vertical accelerated plate embedded in porous medium in the presence of transverse magnetic field is considered. Solutions of the equations governing the flow are obtained with the help of power series. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases. In many applications, quite often the plate temperature starts oscillating about a non-zero mean temperature. The free convection is enhanced by superimposing oscillating temperature on the mean plate temperature. In many engineering applications, transient free convective flow occurs as such a flow acts as a cooling device. Keeping this in mind, in first part of chapter II, another problem is considered where the effect of magnetic parameter and heat source on heat transfer to unsteady MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer in presence of free stream velocity is considered. The plate temperature is assumed to vary harmonically with time. Solutions to the governing equations are obtained with the help of method of perturbation. It is observed that velocity decreases as magnetic parameter increases but opposite character revealed in case of permeability parameter and heat source parameter.

Studies associated with flows through porous medium in rotating environment have some relevance in geophysical and geothermal problems. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Keeping this in mind unsteady free convective and mass transfer flow of viscous fluid through a porous medium occupying a semi-infinite region bounded by a vertical porous plate subjected to a constant suction in presence of constant heat flux at the plate wall in a rotating frame of references is studied in the second part of chapter II. The second part of chapter II, deals with the study of unsteady free convective flow and mass transfer of MHD fluid through porous medium in presence of heat source with variable suction in a rotating system. The effects of rotation on velocity, temperature and concentration field are discussed. Another problem, unsteady free convective flow and mass transfer during the motion of viscous incompressible rarefied gas through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform transverse magnetic field in a rotating system is studied in the second part of chapter II. The effects of rotation, variable suction, heat source, rarefaction parameter and magnetic parameter on velocity, temperature and concentration distribution are discussed analytically and graphically.

Chapter III is devoted to the study of dispersion of solute in MHD fluid in different geometrical conditions where generalized dispersion model proposed by Gill & Sankarasubramanian [1] is employed. The longitudinal dispersion of solute in a solvent flowing in a conduit is a phenomenon of wide application in chemical engineering, biomedical engineering, physiological fluid dynamics and environmental sciences. This motivates us to study dispersion of solute in three different realistic situations. The first problem of chapter III deals with exact analysis of the dispersion of solute in an oscillating hydro-magnetic Couette flow. Using a generalized dispersion model which is valid for all time after the injection of solute, the diffusion co-efficients $K_i(\tau)(i=1,2,3,..............)$ are determined as function of τ when the initial distribution of solute is in the form of a slug of finite extent. The second diffusion co-efficient *i.e.* $K_2(\tau)$ gives a measure of longitudinal dispersion of solute due to the combined effect of molecular diffusion and uniform or non-uniform velocity distribution. The interesting part of the analysis is that $K_2(\tau)$ consists of a steady part and a fluctuating unsteady part due to the oscillation of flow even though velocity field is independent of time. In part two of chapter III, dispersion of solute in oscillating hydro-magnetic Couette flow in a rotating system is studied. The effect of rotation and transversely applied magnetic field on the dispersion process is discussed. In part three of chapter III, unsteady dispersion of solute in two layered MHD fluid flow through parallel plates is studied. This model mainly brings out the effect of plug flow region on the overall dispersion process. It is found that initially the dispersion co-efficients decreases considerably with the increase of the value of plug flow region but becomes essentially a constant as time takes larger values. It is also seen that time required to reach the steady state depend on plug flow region. This study can be used as a starting first approximate solution for studying the dispersion in cardiovascular system.

Before we discuss various problems we present below general introduction on rotating fluid flow, flow through porous medium, free convection and mass transfer flow through porous medium, unsteady magneto-hydrodynamic free convective flow with mass transfer in porous medium, free convective MHD flow with mass transfer in rotating system, unsteady convective diffusion process in different geometries which are directly related to the concerned problem of this thesis.

1.2 ROTATING FLUID FLOWS

The study of the motion of a viscous rotating fluid has stimulated considerable interest in recent years due to its important applications. Similarly a great deal of meteorology depends upon the dynamics of a revolving fluid. The large scale and moderate motions of atmosphere are greatly affected by the vorticity of the earth's rotation. The motion in the earth's core is somehow responsible for the main geomagnetic field. It is common practice in fluid dynamics to start with simple model to investigate various effects. The problem, although idealized, retains the essential features of the investigation. It has been observed that, when the fluid is rotating near a flat plate, the pressure field of the flow far away from the plate also exists near the plate, but there is reduction in the Coriolis force near the plate owing to frictional forces. Because of this, there exists a flow in the direction in which the pressure is falling until the Coriolis forces are compensated by viscous forces. A layer in which such a flow exists is known as Ekman Layer (Prandtl [3]), was first noticed by Ekman [2] and plays a very important role in the rotating fluid flows. Thus, near the plate, the viscous and Coriolis forces are of same order of magnitude in the Ekman Layer. Rotation in fluid system produces two effects viz. the Corolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. In considering flows in rotating environment, we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the latter case it is preferable to use an inertial coordinate system fixed in space. On the other hand the flow behavior in the former case can be described in a coordinate system which rotates with the fluid and in this frame of references the fluid is at rest. The complete literature pertaining to rotating fluids is enormous and an excellent review can be found in the monograph by Greenspan [4]. The steady flow near the plate, in the Ekman layer, has been discussed by Batchelor [5]. Vidyanidhi and Nigum [6] discussed secondary flow in a rotating channel. The effects of a uniform transverse magnetic field on Ekman layer is investigated by Pop [7]. Gupta [8] obtained an exact solution of the three dimensional Navier Stokes steady state equations for the flow past a plate with uniform suction or injection (blowing) in a rotating system. Soundalgekar and Pop [9] studied on hydro-magnetic flow in a rotating fluid past an infinite porous plate. Debnath and Mukherjee [10] studied the motion of an incompressible, homogeneous, viscous fluid bounded by porous plate with uniform suction or injection. Puri [11] discussed the fluctuating flow of a viscous fluid on a porous plate in a rotating medium. Pop and Soundalgekar [12] studied the effects of constant and variable suction on the unsteady rotating flow of the fluid past an oscillating plate when both the fluid and the plate are in solid body rotation. Mazumdar *et al.* [13] investigated the effects of both Hall current and rotation on hydro-magnetic flow over a porous plate. Similar problem was studied by Jana and Datta [14], who consider the effects of Hall current and rotation on

MHD Couette flow. Mazumder [15] studied an exact solution of oscillatory Couette flow in a rotating system. Ganapathy [16] investigated an oscillatory Couette flow in a rotating system. Bhattacharjya *et al.* [17] studied the unsteady rotating flow of a compressible fluid over a finite disk. Singh [18] considered an oscillatory hydro-magnetic Couette flow in a rotating system. Kim [19] studied the unsteady two dimensional laminar flow of a viscous incompressible electrically polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of transverse magnetic field. Recently Jat and Jhankel [20] analyzed three dimensional unsteady flow of an incompressible viscous fluid in presence of transverse magnetic field through porous medium past an oscillating plate in a rotating system. Singh *et al.* [21] discussed a periodic solution of oscillatory Couette flow through porous medium in rotating system. In this thesis we consider certain problems in rotating system due to their varied applications in the field of technology.

1.3 FLOW THROUGH POROUS MEDIUM

Many materials (ex. soil, sand, packed beds) consist of a large number of particles or fibres packed closely together. In between the solid particles or fibres there is an open space, giving rise to pores through which fluid can flow. An object does not have to consist of many particles to be porous, for instance, it could simply be composed of a single continuous solid body that has many pores in it. Such is the case of certain rocks and filters. Regardless of how the porous medium is constructed, because of the irregular and tortuous nature of pores it is exceedingly difficult to model fluid flow through such materials exactly.

In recent years the flows of fluid through porous medium have attracted the attention of a number of scholars because of their possible application in many branches of science and technology. In fact a porous material containing the fluid is a non-homogeneous medium but it may be possible to treat it as a homogeneous one, for the sake of analysis, by taking its dynamical properties to be equal to the local average of original non-homogeneous continuum. Thus complicated problems of the flow through a porous medium get reduced to the flow problem of homogeneous fluid with some additional resistance. Flows of fluid through porous medium are of principal interest because these are quite prevalent in nature. Such flows are important in the field of agricultural engineering to study the underground water sources, seepage of water in river beds, in petroleum technology to study the movement to natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification processes. In view of the geophysical application of the flows through porous medium, a series of investigations have been made by Raptis *et al.* [22, 23] into the steady flow past a vertical wall.

In fluid dynamics, Darcy's law is a phenomologically derived constitutive equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth science. Although Darcy's law which is an expression of conservation of momentum was determined by Darcy; it has since been derived from the Navier-Stoke's equation via homogenization. It is analogous to Fourier's law in the field of heat conduction, Ohm's law in the field of electrical network and Fick's law in diffusion theory. One application of Darcy's law is the water flow through an acquifer. Darcy's law along with equation of conservation of mass is equivalent to the ground water flow equation, one of the basic relationships of hydrogeology. Darcy's law is also used to describe oil, water, gas flow through porous medium.

Studies associated with flows through porous medium have been based on the Darcy's empirical equation

$$\vec{q} = -\frac{\text{const.}}{\mu}\vec{\nabla} p \qquad \text{.... (1.1)}$$

Where $\vec{q}$ is the mean filter velocity, μ is the viscosity of the fluid and $\vec{\nabla} p$ is the pressure gradient. Later Muskat [24] has shown that the constant in equation (1.1) must depend on the permeability of the porous medium and showed that

$$\vec{q} = -\frac{K}{\mu}\vec{\nabla} p \qquad \text{.... (1.2)}$$

Where K is the permeability of the porous medium. Following Yamamoto and Iwamura [25], the porous medium is considered as an assemblage of small identical spherical particles fixed in space and the equation (1.2) for incompressible fluid and unsteady flow, takes the form

$$\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\vec{\nabla}\right)\vec{q} = -\frac{1}{\rho}\vec{\nabla} p - \frac{\nu}{K}\vec{q} + \nu\, \nabla^2 \vec{q} - g \qquad \text{.... (1.3)}$$

Where ν is the kinematic viscosity, t is the time and g is the acceleration of gravity.

1.4 FREE CONVECTION AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM

Fluid flow due to density differences in the external force field is generally called free convection. Such external forces are gravity forces, and the density difference, a very simple case, is the result of the temperature drop between the solid surface and the fluid. Free convection flow is not of rare occurrence in nature. In fact trade winds are due to convection currents set up in the atmosphere due to unequal heating. Also land and sea breezes arise in a similar manner. Studies on free convection have growing importance on the problem of unsteady free convection flow past an infinite vertical plate as one of the fundamental problem in heat transfer owing to its practical applications. Pop and Soundalgekar [26] have studied unsteady free convection flow past an infinite plate with constant suction and heat sources. Free convection effects on the Stoke's problem for an infinite vertical plate were investigated by Soundalgekar [27]. This problem is better known as Stokes problem for the vertical plate. Singh *et al.* [28] discussed three dimensional free convective flow and heat transfer along a porous vertical wall. Pop and Soundalgekar [29] investigated the free convection flow past an accelerated vertical infinite plate. The problem of free convective viscous flow past a vertical porous plate with periodic temperature has been solved by Acharya and Padhya [30]. Raptis [31] studied unsteady free convective flow through a porous medium. The free convection effect on the flow of an ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma [32]. Mahershi and Tak [33] studied fluctuating free convection through porous medium due to infinite vertical plate with constant heat flux.

Research on fluid flow through porous media finds great application in geothermy, geophysics and technology. Flows of fluid through porous medium are of principal interest because these are quite prevalent in nature. Such flows are important in the field of agricultural engineering to study the underground water sources, seepage of water in river beds, in petroleum technology to study the movement to natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification processes. Yamamoto and Iwamura [25] considered the flow with convective acceleration through a porous medium as assuming the porous medium as an assemblage of small identical spherical particles fixed in space. Raptis *et al.* [34, 35] studied the influence of the free convective flow on the steady flow of the viscous fluid through the porous medium when there is a constant heat flux. Raptis [36] analyzed the influence of free convection on the unsteady flow of a viscous fluid through a porous medium considering the fluctuation of the surface temperature in time about a constant non-zero mean value. The study of two dimensional flows through porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma [37]. Three dimensional free convective flow and heat transfer through a porous medium was discussed by Ahmed and Sharma [38]. Sattar *et al.* [39] studied free convection flow and heat transfer through a porous vertical flat plate immersed in a porous medium with variable suction. Singh [40] investigated three dimensional free convective flow of a viscous fluid through porous medium with time dependent suction velocity. Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability was studied by Singh *et al.* [41]. Free convection flow of viscous fluid in porous medium in presence of heat source was investigated by Singh *et al.* [42].

However, in nature, along with the free convection currents caused by the temperature differences, the flow is also affected by chemical composition differences and gradients or by material or phase constitutions. This can be seen in our everyday life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by the temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as the mass transfer flow. When a mixture of gasses or liquid is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from a region of high concentration to regions of low concentration. There is also a mass transfer associated with convection in which mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level. Due to applications in various technological problems and in agricultural science, effects of mass transfer on the unsteady free convective flow past an infinite porous plate with constant or variable suction were studied by Soundalgekar [43], Soundalgekar and Wavre

[44,45], Soundalgekar [46] and Raptis *et al.*[47]. Raptis *et al.* [48] examined free convection and mass transfer flow through a porous medium bounded by an infinite vertical limiting surface with constant suction. Raptis *et al.* [49] studied the steady free convective flow and mass transfer of a viscous fluid through a porous medium bounded by a vertical infinite porous surface with constant suction by using generalized Darcy's law. In a subsequent paper, under the same geometrical and physical considerations, Raptis [50] studied the influences of both free convective flow and mass transfer through a porous medium. Raptis [51] studied the free convection and mass transfer flow through porous medium bounded by a plate with free stream velocity. Raptis and Perdikis [52] also analyzed the steady free convective and mass transfer flow when a viscous incompressible fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite porous plate. Singh [53] studied three dimensional unsteady free convection and mass transfer flow through a porous medium. Chitti Babu *et al.* [54] investigated three dimensional free convective flows of heat and mass transfer through a porous medium with periodic permeability.

1.5 UNSTEADY MAGNETO-HYDRODYNAMIC FREE CONVECTIVE FLOW WITH MASS TRANSFER IN POROUS MEDIUM

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many processes, industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc, in the presence of an electrically conducting fluid subjected to a magnetic field. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical infinite plate was studied by Soundalagekar *et al.*[55]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al.* [56].The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate was studied by Soundalgekar [57]. Raptis and Kafousias [58] investigated heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Further, the effect of constant heat flux on the flow of an electrically conducting fluid plate oscillating in its plane was studied by Soundalgekar *et al.* [59]. Acharya *et al.*[60] have analyzed free convection and mass transfer in steady flow through porous medium with constant suction in the presence of magnetic field. Singh and Chand [61] discussed unsteady free convective MHD flow past a vertical porous plate with variable temperature. Recently, Sriramulu *et al.* [62] studied the effect of applied magnetic field on transient free convection flow of an incompressible viscous fluid by taking into account of viscous dissipative heat along with the heat due to free convection currents in a vertical channel. Samman *et al.* [63] studied transient free convection flow of a viscous dissipative fluid with mass transfer past a semi-infinite vertical plate. Singh *et al.* [64] studied hydro-magnetic heat and mass transfer in MHD flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate embedded with porous medium of time-dependent permeability under oscillatory suction velocity normal to the plate. More recently Singh [65] investigated MHD free convection and mass transfer flow with Hall effect, viscous dissipation, Joule heating and thermal diffusion. Mukherjee *et al.* [66] studied MHD free convective flow and mass transfer through an inclined open rectangular channel. Unsteady Magneto-hydrodynamic free convection flow past an infinite vertical plate with time dependent suction and heat sink was studied by Kumar *et al.* [67]. Sarangi and Jose [68] analyzed unsteady MHD free convective flow and mass transfer through porous medium with constant suction and constant heat flux. Mishra [69] investigated heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction. Jain *et al.* [70] discussed MHD free convection flow of water at 4° C in presence of a heat under slip boundary conditions. Effects of permeability on three dimensional oscillatory free convective MHD flow and heat transfer along an infinite vertical porous plate was studied by Shrivastava *et al.* [71]. Recently effect of mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink was studied by Sharma and Kaanodia [72]. We now proposed to study the effect of variable suction or injection on the unsteady two dimensional free convective flows with mass transfer of an electrically conducting fluid past a vertical accelerated plate embedded in the presence of transverse magnetic field in the first part of chapter II. Solutions of the equations governing the flow are obtained with the help of power series. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases. In the first part of chapter II, another problem on the heat transfer to unsteady flow of MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer in presence of free stream velocity is considered. The plate temperature is assumed to vary harmonically with time considering ε to be very small, we have practically considered the plate temperature to vary only slightly from mean value. Solutions of the equations governing the velocity field, temperature field and concentration field are obtained analytically and effects of magnetic parameter, heat source and permeability parameter on the velocity field are also studied.

1.6 FREE CONVECTIVE MHD FLOW WITH MASS TRANSFER IN ROTATING SYSTEM

The study of fluid flow in a rotating system which was initiated by Greenspan [4] recently has received considerable interest due to its applications in practical situations. In particular the hydro-magnetic flow in a rotating system has numerous engineering applications e.g., in generation of MHD power in a small scale for space applications, design of heat exchangers, flow meters, etc. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical, geothermal problems. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Gupta [8] investigated the effect of hydro-magnetic flow past a rotating porous flat plate. Mohan [73] examined the free and forced convection effects in a rotating, steady hydro-magnetic viscous fluid between two parallel plates maintaining the boundaries at constant temperature gradient and taking the plates to be finite conductivity. Prasada Rao and Krishna [74] investigated the influence of Hall currents on the free and forced convective flow of a viscous conducting fluid in a rotating channel maintained at constant temperature gradient along the channel walls under the influence of transverse magnetic field. Agarwal *et al.* [75] investigated the effects of Hall current on a steady hydro-magnetic free convection flow past an infinite porous plate in a rotating viscous fluid system. Mahato and Maiti [76, 77] analyzed the effect of unsteady free convective flow and mass transfer during the motions of a viscous incompressible fluid in a rotating frame of references. The effect of magnetic field on free convective flow of electrically conducting fluids past a semi-infinite flat plate is analyzed by Sacheti *et al.* [78]. Satter and Alam [79] studied MHD free convective flow with Hall current in a porous medium for electrolytic solution. Later Alam *et al.* [80] studied unsteady free convection and mass transfer flow in a rotating system with Hall currents, viscous dissipation and joule heating. Tak and Gehlot [81, 82] studied the effects of suction on skin-friction and heat transfer in the free convection boundary layer flow along a porous vertical semi-infinite plate in presence of transverse magnetic field with or without frictional heat. Singh *et al.* [83] studied free convection in MHD flow of a rotating viscous liquid in porous medium. Recently Singh *et al.* [84] have studied free convective MHD flow of rotating viscous fluid in a porous medium past infinite vertical porous plate. Soundalgekar *et al.* [85] obtained an exact solution of the transient free convection flow past an infinite vertical plate in presence of periodic heat flux. Recently

Sahoo *et al.* [86] studied the unsteady hydro-magnetic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in presence of constant suction and heat absorbing sinks. In the second part of chapter II, we now proposed to study unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform magnetic field applied perpendicular to the flow region in rotating system. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, temperature, concentration, skin-friction, rate of heat transfer and rate of mass transfer is presented graphically and necessary conclusions are set out.

In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle of the surface has a finite tangential velocity, it slips along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Using these assumptions Gupta and Babu [87] investigated the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in the slip flow regime. Debangana [88] investigated MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime. Jain and Taneja [89] examined unsteady MHD flow with radiation through porous medium in slip flow regime. Sharma and Choudhury [90] studied effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. Singh *et al.* [91] studied magnetic field effects on free convection and mass transfer flow through porous medium with constant suction and constant heat and mass flux in slip flow regime. Sharma and Sharma [92] investigated influence of variable suction on unsteady free convective flow from a vertical flat plate and heat transfer in slip-flow regime. Varshney *et al.* [93] investigated effect of heat source on free convection and mass transfer flow through porous medium with constant heat and mass flux in slip flow regime. Sharma [94] has studied the effect of periodic heat and mass transfer on unsteady free conduction flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time about a non-zero constant mean. Singh and Gupta [95] analyzed MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Jain *et al.* [96] discussed three dimensional free convection heat transfer flow with periodic permeability and periodic temperature in a slip flow regime. Recently Emmanuel Osalusi [97] investigated effects of thermal radiation on MHD and slip flow over a porous rotating disk with variable properties. We now proposed to study another problem on free convective MHD flow with mass transfer past a porous plate in porous medium with variable suction in a slip flow regime in the second part of chapter II. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, skin friction, rate of heat transfer and rate of mass transfer is presented analytically and graphically.

1.7 UNSTEADY CONVECTIVE DIFFUSION PROCESS

The longitudinal dispersion of a solute in a solvent flowing in a conduit (pipe/ channel) is a phenomenon of wide application in chemical engineering, biomedical engineering, physiological fluid dynamics and environmental sciences. The basic principle under the dispersion theory is the spreading of a passive species in a flowing fluid due to the combined action of molecular diffusion and non-uniform velocity distribution. The first fundamental study of dispersion was that of Taylor [98] who showed that, if a solute is injected in a solvent flowing steadily in a straight tube the combined action of the lateral molecular diffusion and the variation of the velocity over the cross section would cause the solute ultimately to spread diffusively with the effective molecular diffusivity Deff given by Deff $=D_m+a^2\omega_m^2/48D_m$, where D_m is the molecular diffusivity, ω_m is the mean axial velocity and a is the radius of tube. The analysis showed that the spreading of solute is symmetrical about a point moving with average velocity ω_m of the fluid. Aris [99] using the method of moments, showed that the effective molecular diffusivity would be Deff=$D_m+a^2\omega_m^2/48D_m$, when the molecular diffusivity is also taken into account. The analysis showed that the Taylor's dispersion theory is valid for Deff>>D_m.

The time development of dispersion has most commonly been studied by calculating the evolution of axial moments of the solute concentration following its injection into the flow. Anathkrishanan *et al.* [100] obtained the exact numerical solution for the complete convective diffusion equation which takes into account both the radial and axial molecular diffusion. Their results showed that the Taylor-Aris dispersion theory gives a good description of the dispersion process if and only if the time after injection of the solid exceed about $0.5a^2/D_m$. The effect of inlet boundary conditions on the transient approach to the asymptotic Taylor-Aris dispersion theory was studied in a subsequent paper by Gill and Anathkrishanan [101] and was validated by the experimental work of Reejhsigngani *et al.* [102]. Gill [103] generalized Taylor-Aris work by proposing a series expansion about mean concentration to describe the local concentration distribution. Gill and Anathkrishanan [104] extended this theory to include the effect of finite slug inputs on the dispersion process. In a subsequent analysis Gill and Sankarasubramanian [1] showed that the method of series solution mentioned earlier provided an exact solution for the unsteady convective diffusion problem for laminar flow in a circular tube if the co-efficients in the dispersion model are obtained as suitable function of time. This model is widely referred to as the generalized dispersion model. By truncating the generalized dispersion model to two terms, Gill and Sankarasubramanian [1] showed that for all time, the mean concentration profile of the solute was symmetric about a point moving with the average velocity of the fluid. Their results also validated the findings of Anathkrishanan *et al.* that the Taylor-Aris dispersion theory is applicable for the exceeding $0.5a^2/D_m$. Gill and Sankarasubramanian [105,106] extended the scope of their model to study dispersion of solute in a time-dependent laminar flow which in principle, valid for all values of time, they confined their analysis only to the case of dispersion in a fully developed flow. They [107] extended the theory of miscible dispersion to inter phase transport system. They studied the dispersion of solute in a laminar flow through a tube with first order irreversible heterogeneous chemical reaction. Later Krishnamurthy and Subramanian [108] formulated convective diffusion theory for predictive modeling of field-flow fractionation columns used for the separation of colloidal mixture. Jayaraj and Subramanian [109] used the truncated version of generalized dispersion theory to study the relaxation phenomena in field-flow-fractionation. Annapurna and Gupta [110] and Gupta [111] analyzed the unsteady magneto hydrodynamic convective diffusion in electrically conducting fluid flowing in a parallel plate channel. Subsequently Annapurna and Gupta [112] studied the dispersion of matter in flow of a Bingham plastic in a tube using the generalized dispersion model. Later Mukherjee and Maiti [113] studied the dispersion of solute in blood stream flowing through a tube treating blood as a casson fluid model. Mandal *et al.* [114] investigated the dispersion of solute in an incompressible electrically conducting viscous fluid in a porous-walled parallel plate channel permitted by transverse magnetic field. Layek *et al.* [115] presents an exact analysis of the dispersion of a passive contaminant in a viscous fluid flowing in a parallel plate channel driven by uniform pressure gradient. The channel rotates about an axis perpendicular to its wall with uniform angular velocity resulting in a secondary flow. Later Hazra *et al.* [116] studied the dispersion of a solute in oscillating flow through a channel. Siddheshwar *et al.* [117] studied the effect of interphase mass transfer on it. Jayaram *et al.* [118] studied dispersion of solute in a fluid flowing through a curve tube with absorbing wall. Siddheshwar and Manjunath [119] have recently studied dispersion of solute in a plane poiseuille flow of a micro polar fluid. Recently Siddheshwar and Markande [120] considered unsteady convective diffusion of solute in a micro polar fluid flow through a cylindrical tube. Later Dash *et al.* [121] studied a shear augmented dispersion of solute in a casson fluid flowing in a conduit. Hossain *et* *al.* [122] studied exact analysis of dispersion of solutes in free and forced convective flow through a channel. Hossain *et al.* [123] examined the effect of radiation on unsteady convective diffusion of solute in an MHD flow through a vertical channel. Recently Nagarani *et al.* [124] discussed exact analysis of unsteady convective diffusion in casson fluid in an annulus. The first problem of chapter III deals with exact analysis of the dispersion of solute in an oscillating hydro-magnetic Couette flow. Using a generalized dispersion model which is valid for all time after the injection of solute, the diffusion co-efficients $K_i(\tau)$ (i=1,2,3,...........) are determined as function of τ when the initial distribution of solute is in the form of a slug of finite extent. The second diffusion co-efficient *i.e.* $K_2(\tau)$ gives a measure of longitudinal dispersion of solute due to the combined effect of molecular diffusion and uniform or non-uniform velocity distribution. The interesting part of the analysis is that $K_2(\tau)$ consists of a steady part and a fluctuating unsteady part due to the oscillation of flow even though velocity field is independent of time. In second problem of chapter III, dispersion of solute in oscillating hydro-magnetic Couette flow in a rotating system is studied. The effect of rotation and transversely applied magnetic field on the dispersion process is discussed. In the third problem of chapter III, unsteady dispersion of solute in two layered MHD fluid flow through parallel plates is studied. This model mainly brings out the effect of plug flow region on the overall dispersion process. It is found that initially the dispersion co-efficients decreases considerably with the increase of the value of plug flow region but becomes essentially a constant as time takes larger values. It is also seen that time required to reach the steady state depend on plug flow region. This study can be used as a starting first approximate solution for studying the dispersion in cardiovascular system.

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