

Chapter I

General Introduction

1.1 Introduction

The present dissertation deals with some conceptual issues in relativity theory. Some of these issues concern the pedagogical aspect of the theory while some relate a few observational puzzles in the field of cosmic ray physics and relativistic cosmology. It is interesting to note that on the one hand, the fundamental postulates of relativity is being challenged today for example, in the context of ultra high energy cosmic ray (UHECR) paradox (on the theoretical side, in the context of quantum gravity and Lorentz invariance violation (LIV) theories and the likes) on the other hand it seems that even after about hundred years since the advent of relativity theory the scientific community at large is yet to fully reconcile with the hugely counter-intuitive outcomes of the theory. The concepts and implications of the kinematics of relativity, as one may observe, have not yet fully been settled in our minds; paradoxes, resolutions and consequent debates concerning the theory still continue. For a concrete example, one may point out that nearly three hundred articles have already been written on the single issue like the twin paradox or the clock paradox and still more continues to pour in. The authors of these articles should not be accused of splitting hairs sitting on the ivory towers. They have been serving the scientific community by providing a lot of insight into various crucial questions in relativity theory.

Concerning the observational puzzles mentioned in the beginning, one often considers modification of standard relativity theories. In my opinion, to think of any possible violation of a well established theory (for one reason or the other) it is essential first to refine our understanding of the theory. I found that my humble efforts to understand the twin paradox in order also to capture newer insights, has provided me with a vantage point from where I have been able to look beyond. In

particular a newer approach to understanding the twin paradox based on the conventionality of simultaneity (CS) thesis has fascinated me. Our CS-thesis approach has also thrown some new light on the famously counter-intuitive issue. My learning of the CS-thesis in connection with the investigations on twin paradox, enabled me to obtain a novel understanding of one of the most puzzling paradoxes of physics, viz UHECR paradox. Again if our approach to deal with the UHECR problem turns out to be correct, one can show that the underlying assumptions of the novel approach may directly lead to a variable speed of light (VSL) cosmology. Indeed a simple minded approach to this effect has been a small part of the programme of my investigations. The present thesis is a compilation of the findings of the present author following investigations on the conceptual issues like the twin paradox and its ramifications, an issue in the realm of high energy physics (UHECR paradox) and an area touching VSL cosmology.

The assortment of topics of investigations has one unifying thread: They all relate to studies of space-time behavior in different inertial frames. In the treatment of twin paradox ordinarily standard relativistic transformations including ones permitted by CS-thesis have been used. In some cases however, non-relativistic transformations have been discussed for purely pedagogical purposes. In other two areas prompted by the search for new physics, a case for a deformed Lorentz transformations (LT) has seriously been considered. However predictions of a deformed LT is expected to be different from those of relativity theory, hence care has been taken so that the difference in predictions be undetectable in the domain where special relativity (SR) has been tested beyond doubt. The main text concerning the present study comprises of chapters III-VII which reports observations and results obtained by me (along with my collaborators) in the last few years. Some of these observations have been published and some have been reported in the national and

international meetings and others are under publications.

The whole volume, apart from the introductory chapters (I-II) is divided into two parts. Part-I deals with the conceptual and pedagogical issues concerning aspects of the special relativity theory in particular that of the twin paradox and its ramifications while Part-II reports investigations on UHECR paradox and VSL cosmology based on a novel LIV approach that we have developed. As has been mentioned, the CS-thesis of SR has been found to be an essential tool for the investigations. I therefore devote an entire chapter (chapter II) in order to discuss it as an essential prelude. Some novel space-time transformations (to be used later) are also discussed therein in some detail.

All the chapters of the main text are self contained. However for readers not thoroughly acquainted with the issues discussed, a brief review of the previous works has been presented in the next two sections of this introductory chapter as a background (*Background-I* and *Background-II*). Background-I pertains to topics of Part-I of the thesis which deals with the conceptual and pedagogical issues, while Background-II provides reviews of previous works concerning possible deformation of standard relativity theory in the context of UHECR paradox and VSL cosmologies. These sections will also provide the scope and objective of the present study. Finally in the last section of this chapter, I will give a topic wise summary of the main investigations. This section will provide a gateway for the main contents and the readers will find glimpse in advance of what lies ahead.

1.2 Background-I

1.2.1 Outline of the Problem

In 1905's paper on special relativity Einstein[1] predicts that of the two clocks, the one which is moved away and subsequently brought back to its original position will lag behind the stationary one. He termed the effect as "a peculiar consequence"[2]. In 1911 Einstein restates the result in a more dramatic way in the following form: "If we placed a living organism in a box ... one could arrange that the organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long since given way to new generations. For the moving organism the lengthy time of the journey was mere instant, provided the motion took place with approximately the speed of light"[3]. Historically the word "twins" first finds its appearance in the discussion when Langevin[4] has posed a thought experiment in the problematic form in which a twin leaves the earth, for a distant star at a speed closed to the speed of light and returns in the same speed to meet the stay-at-home twin on earth to discover that at the end of the trip he is younger than the earth-bound sibling. This counter-intuitive result has been given the name "twin paradox". Note that although counter-intuitive, both Einstein and Langevin did not suggest that there was any paradox in it. Indeed Einstein and Langevin recognized early that the situations for the earth-bound and the stay-at-home twins were not symmetrical and any expectation or claim of symmetrical outcome regarding their ages itself was erroneous. Hence there was no paradox (see later for more discussions). Perhaps the term "twin paradox" was coined much later. Many authors including the present one use the term "clock paradox" for the problem. We shall use in this thesis the phrases "clock paradox" and "twin paradox" inter-

changeably. However a clarification is in order. LT predicts reciprocal time dilation of moving clocks. Historically this counter-intuitive feature of SR often used to be called the “clock paradox” in the literature[5]. Whereas non-reciprocal time dilation is predicted for the round-trip journey of a clock and the “twin paradox” focuses on this asymmetrical time dilation. However the term “clock paradox” is also often substituted for “twin paradox” in the attempt to avoid the biological issue of whether a traveller’s aging is in accord with the standard clocks that he carries[5, 6]. As mentioned we will use the term “clock paradox” in the later sense.

Before we proceed let us first examine in what sense the differential aging of the twin parable due to Langevin is a paradox. There are three facets of the problem. Scott[7] has noted that “It is paradox, in the dictionary meaning of the word from the view points of (i) absolute time (ii) the special theory of relativity, and (iii) the general theory of relativity.”

(i) A common man with an intuitive notion of absolute time is puzzled by any difference between the two clocks. (Even some authors of relativity have been found to express the view that time dilation and length contraction of SR are only apparent and it arises because of distant clock synchronization alone[8]; they cannot accept true time dilation of a moving clock when it is brought back to its original position). In this sense this aspect of paradox is mainly concerned with the counter-intuitive feature of the problem.

(ii) A person aware of time dilation of SR knows that the effect depends on relative velocity and is perfectly reciprocal with respect to observers who are in relative motion hence the person gets puzzled by the contradictory claims by the twins regarding the direction of asymmetry in their ages since although there is acceleration, the effect as has been said depends on relative velocity alone. Therefore this aspect, instead of focusing on the counter-intuitive aspect (as discussed above)

concerns the logical contradiction of the issue.

(iii) There are others who take such statement of relativity theory as “the laws of physics are same in all frames of reference” too naïvely, get puzzled by the asymmetric outcome (differential aging)¹. In this context they further note that there are accelerations and since SR (as if) cannot deal with accelerated frames of reference, the problem can only be treated by general theory.

The last aspect of paradox is the most trivial one as the principle of relativity states the *equivalence of inertial frames of reference* only. (Indeed often the principle of general covariance is construed as equivalence of *all* reference frames—surely this is incorrect). The aspect of the twin paradox which is found most perplexing is the one which arises within SR. The present investigation therefore mainly concentrates on this facet of the problem. Although the chapter-III discusses the issue in the context of GR through the principle of equivalence, it attempts to focus on resolving the logical contradiction aspect rather than the counter-intuitive aspect of the issue. Coming back to Langevin’s account of the paradox², to be specific, consider two twins A and B initially on the earth. The sibling B takes a space trip to a distant star at P , a distance L from A , eventually turns around and returns to earth. If it is assumed that the periods of acceleration are negligible compared to the periods of constant velocity, the time for the round trip of B as measured by A may be

¹For example Dingle[9] in 1957 stated that Einstein made a “regrettable error” and he argued that “According to the postulates of SR, if two identical clocks separate and reunite, there is no observable phenomenon that will show in absolute sense that one rather than other moved. If the postulates of relativity is true, the clocks must be retarded equally or not at all, their readings will agree on reunion if they agreed at separation...”

²One may note here that although the standard account of the paradox considers that the traveller has a twin who stays on earth, Langevin’s original parable did not include the term “twin”. However that is a matter of history and hence besides the point.

calculated as $2L/v$. Because of the relativistic time dilation B 's clock will record $2\gamma^{-1}L/v$ time for the round trip. Since $\gamma^{-1} = (1 - v^2/c^2)^{1/2} < 1$, where c is speed of light, at the end of the trip B should be younger than A .

The beginning students of relativity immediately after getting introduced by their teachers to the paradox (which is a consequence of above result) often hears that in spite of the asymmetrical outcome, there is no paradox as such as the situations are not symmetrical for the twins: only one of the siblings experiences acceleration and hence the result is not surprising at all. As mentioned earlier Langevin and Einstein also did not see any paradox in the ‘‘peculiar consequence’’ of SR although in the later years Einstein had to fight his opponents and tried to defend his theory by giving answer to the ‘‘paradox’’ in terms of his General theory.

Authors of repute are often found to dismiss the paradox by pointing out that ‘‘The differential aging suggested by Langevin comes directly from the fact that proper time is a path dependent quantity in special relativity’’[5]. The statement may be clarified as follows. Consider the Fig.(1.1) below. One calculates the proper time τ along the two trajectories of the Minkowski space-time. It is assumed that the to and fro coordinate-speeds are the same. These trajectories are labeled as path (1) from the origin O of the earth's frame to time $t = 2T$ (say) along y -axis and path (2) from the same origin along an oblique line to the turning point P and back again to O' . These paths corresponds to the earth-bound and travelling twin respectively. The proper times are obtained by integrating along each trajectory,

$$d\tau = \gamma^{-1}dt. \quad (1.1)$$

The non-integrability of $d\tau$ follows from the fact that the results are different for the above two paths. This also follows from the elementary notion of geometry: The sum of two sides is always different from other side of triangle. However since

in SR the space-time is pseudo euclidean, $OP + PO' < OO'$. This implies that the traveller will age less. From all this it appears that as if the aspect of non-integrability of proper time is the answer to the paradox and people are often found to express surprise at the suggestion of the paradox itself.

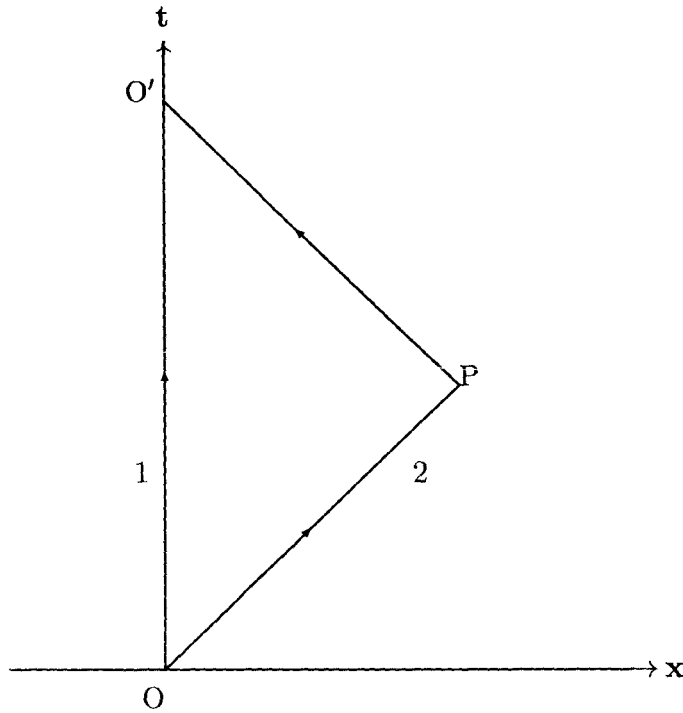


Figure 1.1: Minkowski diagram

As another example Bondi[10] compares the aging process of a human being with the mileage of a car and the journey of the traveller twin with that of a vehicle along a curvy road. Referring to the aging of the traveller Bondi has been found to remark “It will *not* therefore come as a surprise to him on his return to earth to find out that he has aged less than the people there, just as the traveller who took the curvy road cannot have been surprised that he covered a longer mileage than the traveller who followed the straight line”. “Hence there is no clock “paradox”, since it is not paradoxical for two persons with different experiences to find that the consequences

of their experiences differ". These trivializing often cavalier statements³ on the issue are made when one is concerned with the third aspect of the paradox only.

1.2.2 Resolutions

The literature is replete with discussions of twin paradox, its variations and their resolutions. That there exists a plenty of resolutions only suggest the richness of the issue. After about fifty years since Paul Langevin[4] had posed the thought experiment concerning the clock paradox, David Scott[7] identified (of the many solutions suggested at the time) three alternative approaches which to him were particularly instructive for the twin problem. We will now consider them below one by one of which the last one we have briefly mentioned in the last section.

(i) Length Contraction:

The length contraction of moving length can be used to explain the time difference. This has been discussed by Fremlin[11]. The idea is as follows: B is in a uniform motion at speed v , he can measure the distance AP (where points A and P denote the positions of stay-at-home twin and that of the distant star respectively) and will find it is $\gamma^{-1}L$ but not L because of relativistic length contraction effect. Hence B will calculate his time for round trip as $2\gamma^{-1}L/v$. In this approach, the asymmetry can be thought of as related to the fact that the distance point P is fixed relative to A and not to B .

The trouble with this explanation is that although B calculates the travel time of A 's round-trip on B 's clock correctly but the so-called resolution remains silent about the reading of A 's clock from the perspective of the traveller. Will it not be dilated?

³Often these statements are found as passing remarks by some authors who in their scholarly discourse also deal with the deeper aspects of the paradox.

(ii) Doppler Shifts:

In the literature Darwin[12] describes each of the two observers can keep a record of the other's time during the course of the trip by sending out regular time signals (see also [5]). It can be speculated that light or radio signals are sent regularly in the sender's time. In the above scenario of twins record, using the relativistic Doppler factor the frequency of a receding source is $[(c-v)/(c+v)]^{1/2}$ and for an approaching source is $[(c+v)/(c-v)]^{1/2}$. While B is on the outward trip, A will receive B 's signals at $[(c+v)/(c-v)]^{1/2}$ intervals corresponding to a red Doppler shift. Whereas when B is approaching, A will receive B 's signals at $[(c-v)/(c+v)]^{1/2}$ intervals corresponding to a violet Doppler shift.

First consider A 's record of B 's signal. Observer A will receive slow or $[(c+v)/(c-v)]^{1/2}$ signals from B for the duration of the outward trip or for the time light takes to travel from P to A . For remainder of the total time A will receive fast or $[(c-v)/(c+v)]^{1/2}$ signals from B . Hence A will record time $[(c+v)/(c-v)]^{1/2}L/(c+v) + [(c-v)/(c+v)]^{1/2}L/(c-v)$ or $2\gamma^{-1}L/v$ worth of B 's signal.

Now consider B 's record of A 's signal. B will receive slow signals from A until he reaches P and then upon reversing his motion, he will receive fast signals. If t be the total time of the trip as measured by B then for time $t/2$, B receives slow signals from A and for time $t/2$ years fast signals. Since A sends out $2L/v$ worth of signals during the trip,

$$[(c-v)/(c+v)]^{1/2}t/2 + [(c+v)/(c-v)]^{1/2}t/2 = 2L/v, \quad (1.2)$$

hence solving one obtains $t = 2\gamma^{-1}L/v$ as described before.

This treatment in terms of Doppler shifts makes it clear that the asymmetry exists because B reverses his motion and hence *immediately* observes the change in the rate of the signals. For A , he must wait for the time taken by light to traverse

the distance P to A before noting the change in the signals. However this is again merely the description of what is taking place but it does not explain the fault of the standard analysis. Besides this Doppler-shift analysis does not give the reason why there should be an abrupt change of the earth-bound clock because of B' turn around.

(iii) Word Lines:

We have mentioned this approach in the earlier section and already criticized the often made cavalier remark associated with the approach that “there is no paradox as such and the differential aging follows from the fact that the proper time is not integrable”. Although it is true that there is no paradox (indeed there cannot be a paradox in a time tested physical theory) one cannot but ignore the intricacies of the paradoxical issue. However the world line approach serves one to pose the problem in geometrical terms and provides a smart and uniform basis for discussions of the paradox as well as different variations of the clock paradox.

The time τ for the clock B , according to SR is related to the A -clock time t and displacement (x, y, z) of B as measured by A in the following way:

$$d\tau^2 = dt^2 - (dx^2 + dy^2 + dz^2)/c^2. \quad (1.3)$$

Clearly $d\tau$ is always less than dt as B departs from A . Hence for a return journey along any path (which requires integration of the above expression), the time interval on clock B will be less than that of A . One may note however that the explanation, requires that the frame attached to A is inertial. The path of observers or particles in space-time is known as world lines. Various authors describe and analyze the problem by drawing the world line of the twins (or the clocks) and sometimes for the photons they send. We have already drawn such world lines

for the twins in the earlier section. Below we draw⁴ world line diagrams for some variations of the twin paradox problem including the standard one.

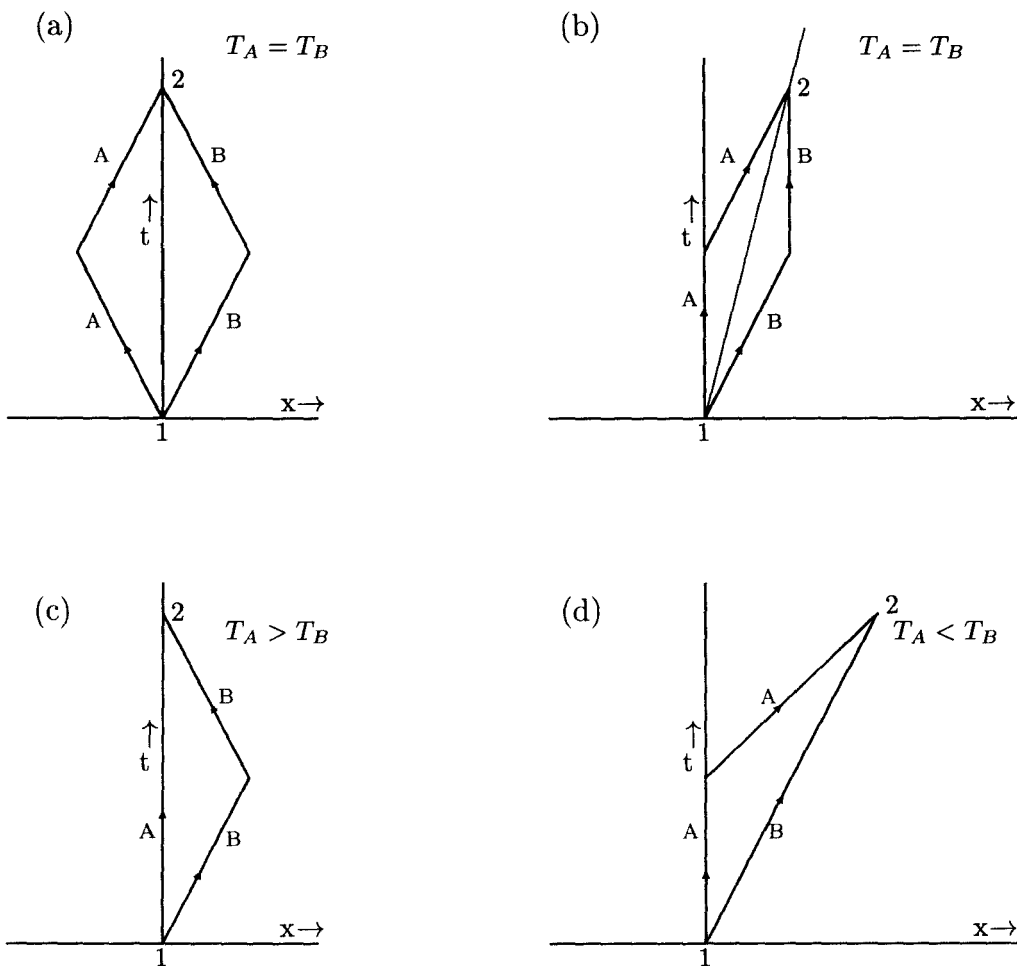


Figure 1.2: World line diagram for variations on the clock paradox problem.

In the first two diagrams there is no differential aging ($T_A = T_B$) when the twins meet at point 2, whereas the last two diagrams depict the cases where the twins' ages differ. The reader can easily reconstruct the parabolas for the twin paradoxes that the figures represent. This shows the power of these diagrams although the approach is unable to address the subtleties of the problem. Debs and Redhead in their most cited paper[5] have extensively used the pedagogical power of these diagrams in connection with their conventionality of simultaneity approach to the

⁴We owe to G. D. Scott's paper[7] for these diagrams.

problem.

1.2.3 The Genesis of the Problem at a Glance

Although the current section intends to review earlier works and ideas concerning the problem, let us briefly discuss the root of the paradox which will be elaborated later in the main chapters. Indeed the reader will then be better equipped to grasp the unsatisfactory features of some of the earlier authors' approach to the problem from the beginning.

The relativistic time dilation effect of SR relates time of two different nature. One concerns the rate of ticking of a moving clock at its positions and the corresponding time is referred to proper time of the clock. The other refers to readings of spatially separated coordinate clocks, as the concerned clock moves past these coordinate clocks. Time recorded by the coordinate clocks are known as coordinate time. Since coordinate clocks are spatially separated, the coordinate time for a given pair of events depends on the synchronization convention or the standard of simultaneity adopted to synchronize these coordinate clocks. In SR we adopt Einstein synchrony or relativity of simultaneity according to which the one-way speed (OWS) of light is stipulated to be equal to its round trip speed[13]. The proper time of a clock however is independent of any synchronization conventions. In twin paradox, the proper time of one twin (clock) is calculated from the knowledge of the coordinate time elapsed in other twin's frame of reference. We shall later see in detail that genesis of the twin paradox lies in the failure to calculate the so called proper time in the frame of reference attached to the traveller twin.

Asymmetry in Twin Parable:

We have already pointed out earlier that the paradox arises if one naïvely treats the perspectives of the twins symmetrically. Some authors seem to remain satisfied by

pointing out the asymmetrical situations for the twins. Terrell[14] elaborated this aspect by pointing out the fact that the observational data of two observers (clocks) will not be at all similar. The data of the accelerated observer obtained by means of “single” Doppler-shift and visual observation method would be peculiar and inconsistent with the data obtained by radar and “double” Doppler-shift methods. It was then remarked that the accelerated observer would be “under no temptation to consider himself in a situation equivalent with that of the unaccelerated observer ...”. In other words as Terrell pointed out, the traveller twin would then “not be surprised to discover upon returning that he had aged less than the other observer”.

However mere pointing out the asymmetry of the perspectives of the twins can hardly be called a resolution, but I consider it worthwhile here to reproduce the novel approach of the author to highlight the accelerated and unaccelerated scenarios. This will at least clear any doubt that may exist in our mind as to the non-equivalence of the situation.

(i) Unaccelerated Scenario:

Consider Lord Halsbury’s “three brothers thought experiment” [5] in which three brothers (clocks) A , B and C are moving uniformly with relative velocities along the straight line. We assume that each observer is equipped with radar, radio and optical equipment for measuring distances, relative velocity and synchronizing clocks. It is however hardly necessary to explain the detailed procedure for such measurements for the present purpose.

For definiteness one may consider that A observes twin B passes him with the velocity v at the time $t_A = 0 = t_B$. He synchronizes his clock in passing. The twin B travels a distance L at a time $t_A = L/v$ and simultaneously passes the third observer C , which moves in the travelling in the opposite direction with the same velocity v relative to A . The sibling C synchronizes his clock to that of B at the

position of passing. Finally C crosses the position A at time $t_A = 2L/v$ and A and C compare their clocks.

Now one may very well see that this is no ambiguity if one accepts the time dilation formula of SR. The observer A notes the clock of both B and C run slower by the Lorentz factor γ^{-1} as we have already discussed. Thus A observes the reading of C -clock as $2\gamma^{-1}L/v$ at the position of passing when his clock reads $t_A = 2L/v$. Since A , B and C are all assumed to be in *unaccelerated frames of reference*, according to SR then there is no abstract reason to prefer A than B or C . From the view-point of B , the clock A runs slow by the same Lorentz factor γ^{-1} as A observes for B -clock. Similarly B observes C -clock to be even slower than that of A since B measures the velocity of C to be $2v/(1 + v^2/c^2)$ which is greater than v . Hence B predicts that C -clock reads less than that of A when C passes A . Similarly according to C 's perspective, A -clock runs slower by the same Lorentz factor γ^{-1} but C observes B -clock is even slower. This clearly accounts for the fact that C -clock (which has been synchronized to that B) reads less time than A -clock when the latter passes C ⁵. However, in the present scenario, A , B and C none of has any special status; they are “equally good as observers”. Their differing observations is in accordance with SR. So far as they continue their uniform motion there is no basis for arguing that anyone’s clock is really indicating the passage of less time than another’s clock. If one says so it would imply giving preference to one of the three co-ordinate systems. This would however contradict the basic tenet of SR. Such a possibility may however become a reality if the observers are not equivalent situations.

⁵In essence, the resolution points out that during the transfer of clock information from frame of B to the frame C , the line of simultaneity has changed, with a consequent discrepancy and advance of time at A .

(ii) Accelerated Scenario:

If now one change is made in the above scenario—substituting observer B for observer C at the time of their meeting, the unaccelerated situation becomes the usual twin paradox. We then deal only with two observers i.e A and B (say). As observed by A , B synchronizes his clock in passing, travels with velocity v to distance L , then reverses direction in a time negligible with respect to L/v and returns to the position of A . The observations of A are essentially the same as in the unaccelerated case, so that B 's clock will read less than A 's upon the second meeting. The observations of B , who does not remain in a single inertial system, can be shown to be confusing and apparently internally inconsistent[14]. One may then be tempted to conclude that the acceleration which B undergoes makes a real difference in the status of two observers. At this point we feel it worthwhile to study the role of acceleration in the twin problem. Below we provide a discussion on the issue by mentioning two important works exemplifying the problem.

1.2.4 Role of Acceleration

The direct role of acceleration of travelling twin in causing the differential aging has been criticized in the literature although it is quite clear that in order to have twice intersecting trajectories of the twins (this is necessary since the clocks or ages of twins have to compare at the same space-time events) one cannot normally avoid acceleration⁶. In an interesting article Gruber and Price[15] dispel the idea of any direct connection between acceleration and asymmetric aging by presenting a variation of the paradox where although one twin is subjected to undergone an arbitrary large acceleration, no differential aging occurs. In their version, the rocket-

⁶Vide section (1.2.7) for a discussion of some novel version of the paradox, where the traveller although “unaccelerated” eventually meet the stay-at-home one.

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bound twin undergoes a periodic motion as shown in Fig.(1.3) so that

$$x = (V_{max}/\omega)\sin\omega t, \quad (1.4)$$

where x and t are the coordinates of a fixed frame on earth and V_{max} is the maximum speed achieved by the rocket-bound twin relative to the earth. The acceleration of the rocket (the rocket's 4-acceleration) has a maximum magnitude of $V_{max}\omega$, which occurs at times $\omega t = \pm\pi/2, \pm3\pi/2\dots$. These relativistic results agree with Newtonian (non-relativistic) answers i.e, the maximum velocity occurs when the particle has zero velocity relative to the fixed frame on earth.

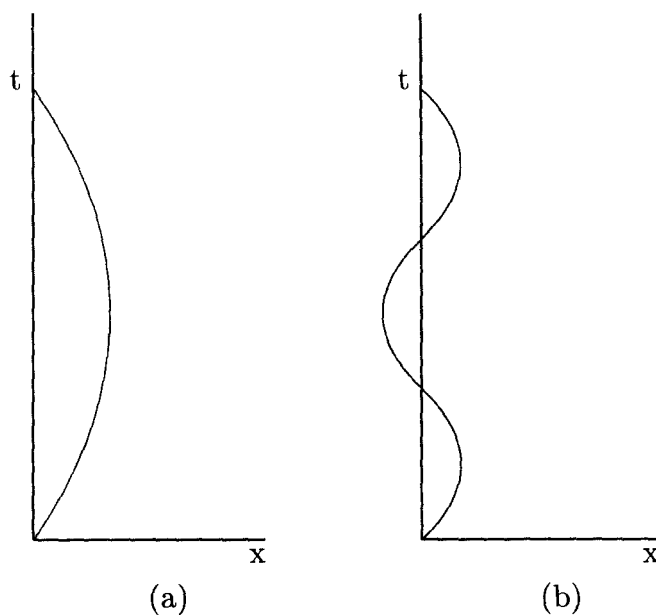


Figure 1.3: Rocket-bound twin World lines. The world line in (b) has maximum acceleration three times that for the world line in (a).

The proper time τ of rocket-bound twin relates to the coordinate time t by

$$d\tau = dt[1 - (V_{max}/c)^2\cos^2\omega t]^{1/2}. \quad (1.5)$$

A rocket trip starting and ending at the earth will take an integer number i.e, starting at $t = 0$ and lasting until $\Delta t = n\pi/\omega$. For such a trip the proper time will be

$$\Delta\tau = n \int_0^{\pi/\omega} [1 - (V_{max}/c)^2 \cos^2 \omega t]^{1/2} dt, \quad (1.6)$$

so that the ratio of elapsed rocket time to elapsed earth time is given by

$$\Delta\tau/\Delta t = (2/\pi) \int_0^{\pi/2} [1 - (V_{max}/c)^2 \cos^2 \theta]^{1/2} d\theta. \quad (1.7)$$

This integral can be evaluated numerically. The important fact on this equation is that it is independent on ω so that time dilation effect is independent on the maximum acceleration $V_{max}\omega$. These considerations eventually prove that acceleration *per se* cannot be the root of differential aging. Indeed one can show that one can have arbitrary large acceleration without any significant differential aging!

There is also a converse situation as discussed by Boughn[16] in connection with an interesting variation on the twin paradox. It is shown therein that spatially separated twins can age differently although their history of acceleration remains the same.

We shall discuss this scenario in detail in Sec.(1.2.7) where we discuss some important variations of the paradox. That the acceleration *per se* cannot play a role is evident also from the usual calculation of the age difference from the perspective of inertial frame of the stay-at-home twin if one notes that the duration of turn-around process of the rocket can be made arbitrarily small in comparison to that for the rest of the journey and hence the final age difference between the twins can then be understood in terms of usual time dilation of the travelling twin during the unaccelerated segment of her journey. In such a calculation the time dilation is also calculated during the acceleration phase (assuming the clock hypothesis to be true[5]) and is shown to contribute arbitrarily small value in the age offset if

the duration of the acceleration phase is assumed to tend to zero. Dray[17] and Barrow[18] have stated that the role of the acceleration can be eliminated by posing the problem in a closed universe setting.

Having understood all this it is important to note that although acceleration has a secondary role from the point of view of the earth-bound clock, from the stand point of the traveller the effect of acceleration is far from being trivial. The quantitative role of acceleration (from the point of view of the accelerated observer) in the asymmetry has been studied by Nekolic[19] who has estimated that the influence of acceleration on the differential aging not only depends on the value of the acceleration itself but also on the relative distance between the accelerated observer and the initial clock. The procedure predicts that acceleration has no influence if two clocks (initial and non-inertial) are at the same position. However if travelling twin moves with constant velocity and suddenly reverses the direction of motion, at this time, it will appear to him that the time of the inertial clock instantaneously jumps forward but there is no such jump of the accelerated clock from the point of view of inertial twin.

1.2.5 The Resolution in General Theory of Relativity

In spite of the fact that the clock paradox can be resolved in the context of SR, many authors feel that the introduction of the general theory of relativity (GR) and a gravitational field at the point of acceleration of the travelling twin is the right way to explain the asymmetrical aging. Although the opinion is still divided regarding the usefulness of the GR resolutions, most of the current expositions concern the intricate issues and different facets of the special relativistic resolutions. Historically among the many, Einstein[20], Tolman[21], Bohm[22] and Frisch[23] advocate the use of GR. For example, Bohm in this connection notes that two clocks running

at places of different gravitational potential will have different rates. This obvious reference of the phenomenon of gravitational red-shift, a prediction of GR was made by Bohm (and Frisch as well) around the time when the effect had just been tested for the first time by Pound and Rebka[24]. On the other hand Einstein offered his GR argument as early as in 1918[20], which indeed was his second argument. The first one was however the usual special relativistic one. An interesting history as to why Einstein had to invoke the GR argument can be seen in an interesting article by Peter Pesic[2].

In the standard version of the twin paradox the differential aging from the perspective of the stay-at-home (inertial) observer A can easily be calculated assuming that for the most parts of the journey of travelling twin B , the motion remains uniform except that there is a turn-around acceleration of the rocket so that finally the sibling are able to meet and compare their ages. The duration of the acceleration phase can be considered to be arbitrarily small compared to the time it takes during its forward and return journeys and hence the age difference occurs due to usual relativistic time dilation of a clock for its uniform motion.

$$Age\ diff = 2t_A(1 - \gamma^{-1}) \approx 2t_A v^2/c^2, \quad (1.8)$$

where $2t_A$ is the time the rocket takes for its entire journey (up and down) in uniform speed v and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor.

The paradox is resolved if one can show that B predicts the same difference in spite of the fact that the time dilation effect is reciprocal. Clearly some new considerations must offset this reciprocal time dilation and this must provide some extra aging to A from the point of view of B so that the age difference remains independent of twins perspectives. One of these considerations, as has already been discussed, is the one of the synchronization gap that B discovers due to her

change of inertial frame during the round-trip. This has been clearly demonstrated by Bondi[10] in the context of Lord Hulsbery's three brother approach to understanding the twin paradox. The other way of understanding the same thing, as remarked by Harpaz[25] and others[21, 23] is the consideration of pseudo-gravity of general relativity (GR) experienced by B because of turn-around. "Although there is no need to invoke GR in explaining the twin paradox, the student may wonder that outcome of the analysis would be if we know how to deal with accelerated reference frames. We could use space ship as our reference frame.... We would find that we must have a gravitational field in this frame to account for the accelerations... if as required in general relativity, we then compute the frequency shifts of light in this gravitational field we come to same calculation in special relativity"[26]. Harpaz[25] has argued that although special relativistic approach can correctly account for the age difference between the twins, "it does not manifest the 'physical agent' responsible for creation of a such a difference." It is held that equivalence principle (EP) provides such an agent. In order to demonstrate how EP plays the role in the analysis, Harpaz has used the gravitational red-shift formula, which can be obtained heuristically as

$$\Delta\nu = \nu_0(1 + gh/c^2), \quad (1.9)$$

where g is the acceleration due to (pseudo) gravity and $\Delta\nu$ represents the change of frequency of light observed from a distance h from the source where the frequency of the same light is seen to be ν_0 . Interpreting this red-shift effect in terms of gravitational slowing down of clocks (GSDC), the formula can be written as

$$t_1 = t_2(1 + \Delta\phi/c^2), \quad (1.10)$$

where t_1 and t_2 are times measured by clocks located at two points P_1 and P_2 (say) and $\Delta\phi = gh$, is the potential difference between these points. It has been shown

that with respect to B the acceleration plays a role by providing an extra time difference between B and A , because of GSDC during the (arbitrary) short duration of B 's acceleration. This time difference more than offsets the age difference calculated by B solely assuming the reciprocal time dilation so much so that finally B ages less by the correct amount. Indeed Einstein suggested that “according to GR, a clock works faster the higher the gravitational potential at the place where it is situated”[27], and since there are homogeneous gravitational field equivalent to the acceleration experienced by B , one should add this contribution to the calculation. Einstein asserted that “calculation shows that the consequent advancement amounts to exactly twice as much as retardation during stages of inertial motion. This completely clean up the paradox...”[27].

In an interesting paper Perin[28] has set up a specific round-trip situation from the point of view of each twin by using the gravitational field equation as its starting point. The geodesic equations of motion are solved in the travelling twin's reference frame in order to determine the time elapsed on the earth twin during the periods of acceleration. He has pointed that “The equality of the result obtained by each twin is explicitly exhibited”. Perrin's method is a generalization of Moller's[29] approach. Moller has solved the problem by transforming Lorentz metric in an arbitrary accelerated reference frame by requiring that its spatial part is to be Euclidean. Very recently Gron[30] has assumed that the travelling twin may be considered at rest in a uniformly reference frame. In such a frame the line element (in the case of time like interval) representing the proper time interval, as measured by a clock following a world line connecting two events has the form

$$-c^2 d\tau^2 = (1 + gx/c^2)c^2 dt^2. \quad (1.11)$$

For a clock at the position ($x = h$) gives

$$\Delta\tau = (1 + gh/c^2)\Delta t, \quad (1.12)$$

so that the travelling twin experiences a gravitational field each time. He has shown that at the starting and arrival points the proper time vanishes, but travelling twin can predict the aging of earth-bound twin when he experiences a gravitational field at the turning point. The earth bound twin falls freely upwards in gravitational field she experiences the gravitational field for a time $\Delta t = 2v/g$. Hence the extra time in the period of acceleration becomes $2hv/c^2$ as calculated by synchronization gap by virtue of special relativistic treatment.

1.2.6 Flaw of the GR Analysis

The essence of any general relativistic solution of the twin problem thus lies in introducing an equivalent pseudo gravitational potential to be experienced by the traveller twin at the time of her direction reversing acceleration. A consequent gravitational time offset effect then provides the extra aging of the stay-at-home twin A required to make the correct prediction by the traveller twin B . Thus acceleration is absolutely essential for the GR analysis and hence the equivalent homogeneous gravitational field as the physical cause behind the asymmetrical aging of the twins. It can be shown that the application of the equivalence principle of general theory of relativity as in the treatment by Einstein, Harpaz and others [21, 22, 25, 31, 32] is essentially trivial. We clarify this rather a strong (against the GR analysis) statement with reasons as follows:

(i) As Builder[32] has noted “ The equivalence principle states that the description of events in terms of the coordinates of an accelerated reference system is indistinguishable from the description of identical events in terms of coordinates of

reference system at rest in an equivalent gravitational field”. The principle thus allows the course of events in a gravitational field to be predicted by SR, the course of events as describes in terms of the coordinates of the equivalent accelerated reference system.

(ii) In effect the authors advocating GR resolution, have tried to answer by denying the applicability of SR, and then using instead calculations that have been obtained from SR by means of the equivalence principle. Builder[32] writes “ This tortuous procedure succeeded in evading the paradox rather than resolving it”. The procedure will be quite invalid if SR itself are indeed not properly applicable to the problem.

(iii) The clock paradox can be posed without acceleration by invoking a third inertial observer, thus a flaw of the GR analysis is that this cannot resolve the variations of the paradox which do not involve any acceleration. In addition, as pointed out by Debs and Redhead[5] and also others[33], that since in the twin problem one deals with the flat space-time (Riemann tensor $R_{\beta\mu\nu}^{\alpha} = 0$), any reference to GR in this context is quite confusing. In this context Unnikrishnan[27] has noted that “all standard resolutions of the twin paradox invoking acceleration or an equivalent pseudo-gravity as a physical effect responsible for asymmetric time dilation are flawed...”.

(iv) In an effort to find a “physical agent” responsible for the extra aging, the author of Ref.[25] and many others rely on some approximate formulas including that of the gravitational red-shift because of $v^2/c^2 \ll 1$ inherent in the analysis, and therefore, the pseudo-gravitational effect has the ability to resolve the paradox only approximately. Clearly there is no valid reason to make such small velocity approximation for the problem.

(v) The explanation based on SR relies on the fact that during the direction re-

versing acceleration, the travelling twin changes from one reference frame to another and the lack of simultaneity of one reference frame with respect to other provides the “missing time” which constitutes the reason for the differential aging. Now the lack of agreement in simultaneity is a special relativistic concept without any classical analogue, on the other hand in many standard heuristic derivations of the gravitational red-shift formula, one finds that no reference to SR is made. Indeed the well-known formula for the red-shift parameter $z = gh/c^2$ is only approximate and is derived by making use of the *classical* Doppler effect for light between the source of light and a detector placed at a distance h along the direction of acceleration g of an Einstein elevator. Thus the equivalence of gravity and acceleration in terms of gravitational red-shift or GSDC therefore turns out to be as if a purely classical (Newtonian) concept in this approximation. In chapter III we will elaborate on this issue.

1.2.7 Some Variations of the Twin Paradox

In the pages of the scientific literature, there are several variations of the clock paradox which are indeed useful additions to the pedagogy of SR in general and twin paradox in particular. In Sec.(1.2.2) we have already referred some of them indirectly through the space-time diagrams (Fig.(1.2)). In one of these diagrams one represented a scenario where neither of the twins are stay at home, instead two of them perform identical journey from a common point on earth first in opposite directions and then turn around and finally meet at the same position. Thus the asymmetry of the usual twin problem is removed in this parable but here any one of the sibling is entitled to think that the other is doing all the moving and hence must suffer time dilation indicating the contradiction! The situation is best exemplified in the so-called “circular twin paradox”. Although the paradox first appeared in

Lightman et.al[34] in 1975, a more elaborate discussion on the problem has been given by Cranor et.al[35] in the beginning of this century. The authors presented a variation of the twin paradox where “each twin leaves on one ring of a counter rotating pair of infinitesimally separated rings so that the twins travel on the same circular path but in opposite directions...”. The twin of one ring should claim the clock of the other twin be slowed down by time dilation caused by the latter’s relative velocity and other contradicting the claim. The resolution of the paradox correctly focuses attention in the connection of time dilation to clock synchronization of coordinate clocks. According to Cranor et.al’s parable the two rings have been assumed to rotate clockwise and counter clockwise with equal angular speed ω with respect to the the laboratory frame. The paper assumes that one twin Lisa with a team of observers live stationary at every point on its ring of radius R whereas the other sibling Bart lives on the other identical ring. The authors assume that Bart moves at the speed $v = \omega R$ in the counterclockwise direction through the laboratory while Lisa’s ring rotate with the same speed in the other direction. The twins will pass each other during their rotation so that they can easily compare their clocks. They are assumed to start from the same place and they notice that their clocks both read $t = 0$. Using the velocity addition formula of SR one obtains Bart’s speed with respect to the observers on Lisa’s ring as

$$v_{rel} = 2v(1 + v^2/c^2)^{-1}. \quad (1.13)$$

Hence Lisa’s team observes Bart’s clock ticking more slowly than the proper time of their clocks by a relative Lorentz factor

$$\gamma_{rel} = (1 - v_{rel}^2/c^2)^{-1/2} = (1 + v^2/c^2)(1 - v^2/c^2)^{-1}. \quad (1.14)$$

This means that his clock will lag behind the clock of next of Lisa’s team of observers that he passes. Bart’s clock should be seen to lag more and more as he passes

members of Lisa's team one by one. Finally after the half a rotation he ultimately passes his counterpart again and should find that their clock would disagree— Bart's clock lags behind Lisa's clock. Believing in the reciprocity of time dilation of SR, Bart and his team can similarly argue that Lisa's clock should lag behind his clock at future meetings indicating a contradiction.

The solution of the problem requires bringing attention to the fact that the time dilation formula of SR is applicable provided the coordinate clocks of the inertial frames are synchronized according to Einstein's method of synchronization. In this connection Cranor et.al correctly point out the difficulty in synchronizing coordinate clocks on the rotating frame in Einstein's way. In our opinion the authors are very close to the truth however some of the remarks made in the article are rather unfortunate. Let us clarify this below.

Observe that Einstein described his synchronization only for inertial frames of reference. As a rotating ring represents a set of non-inertial frames, the authors describe three other schemes of synchronization. This may briefly be reproduced here: Method 1. The ring is initially non rotating and any two infinitesimally separated coordinate clocks of the ring are synchronized according to Einstein's method (standard synchrony convention) by using the light signal following the standard convention by stipulating that light's one-way speed (OWS) is the same as its two-way speed (TWS). The ring can then be set into rotation uniformly such that all points of the ring are "treated identically". Method 2. It uses a light flash from a big laboratory clock stationed at the center of ring. The ring is again at rest to start with and the observers of the ring upon receiving the light flash can all set their clocks to read a same time, say $t = 0$. As before the rings are again uniformly put into the motion after the completion of synchronization process. Method 3. This is almost the same as method 2 in every respect except for

the fact that the coordinate clocks on the ring are synchronized when the ring is already in motion.

It is clear that the method 1, 2 and 3 synchronization schemes all suggest absolute synchrony, according to which two spatially separated events that are simultaneous with respect to the rotating ring are also simultaneous with the laboratory. In this connection the authors make the following remarks “If methods 1, 2 or 3 are used for synchronization of ring clocks, then events that are simultaneous to Lisa and Milhouse will also be simultaneous to the Lab observers. It follows that Lisa and Milhouse, and more generally the entire set of observers on Lisa’s ring, are not correctly synchronized to constitute special relativity reference frames. This explains what we already know must be true: There will be no lagging of Bart’s clock observed as it passes each of Lisa’s observers. For the relativistically inappropriately synchronized clocks of Lisa’s observers, there is no time dilation of Bart’s clock”. (In the quotation Milhouse is the closest neighbor of Lisa on the ring in the counter clockwise direction.)

From the above remark one may think that time dilation of SR is the result of the relativity of simultaneity alone. If the latter goes so does the former. This is obviously not true. It can be shown that time dilation can also be observed even if the co-ordinate clocks are inappropriately synchronized. Indeed for absolute synchrony time dilation and “time contraction” (for the inverse transformation) are both observed in the relativistic world.

The reference of “other synchronization” schemes in the context of the circular twin paradox however supports the conventionality of simultaneity thesis (conventionalist’s claim, normally denounced by relativists) in an indirect way. Much of what has been just said is a subject matter of another paper by my other colleagues and myself, however the details of which will not be included in the present thesis.

Boughn's Paradox:

In an earlier section we have discussed the relationship of time dilation and the acceleration of the traveller twin to dispel a common (students') misconception that differential aging is caused by acceleration. A case of zero time dilation in an accelerating rocket[15] has been discussed at length. It has also been briefly mentioned therein that a converse situation exists where twins experience differential aging in spite of their accelerations being the same. We now consider this variation of the twin paradox due to S. P. Boughn[16] somewhat in detail. According to Boughn's story two twins A and B on board two identical rockets (with equal amount of fuel) initially at rest a distance x_0 apart in an inertial frame S , get identical accelerations for some time in the direction AB (x -direction say), and when all their fuel has been expended they finally come to rest with respect to another inertial frame S' moving with velocity v along the positive x -direction with respect to S . By applying LT Boughn then obtains a very astonishing result that after the acceleration phases are over, B becomes older than A .

With respect to the first frame S (i.e according to Mom and Dad of the twins) the ages of the siblings (reading of the clocks) do not alter throughout their journey. Since the twins undergo identical accelerations (same velocities) the distance between twins' ships x_0 with respect to S remains same. By applying LT, the age difference and their separation can be determined from LT as,

$$\begin{aligned}x' &= \gamma(x - vt), \\t' &= \gamma(t - vx/c^2),\end{aligned}\tag{1.15}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, (x, t) and (x', t') are space-time coordinates of Mom and Dad's frame (S) and twins' final frame of reference (S') respectively.

Consider now two events marked by the birthdays of the siblings and denote the times of these events as t_A (t'_A) and t_B (t'_B) in the frame of reference S (S'). Using

the time transformation of LT, these birthday times are related as

$$\begin{aligned} t'_A &= \gamma(t_A - vx_A/c^2), \\ t'_B &= \gamma(t_B - vx_B/c^2), \end{aligned} \tag{1.16}$$

where x_A and x_B denote the spatial positions of the twins A and B respectively as measured from S . Hence

$$t'_B - t'_A = \gamma(t_B - t_A) - v(x_B - x_A)/c^2. \tag{1.17}$$

In parent's frame, as they are twins, one obviously has,

$$t_A = t_B. \tag{1.18}$$

Writing,

$$x_B - x_A = x_0, \tag{1.19}$$

the Eq.(1.17) turns out to be a time offset relation

$$\Delta t' = t'_B - t'_A = -\gamma vx_0/c^2, \tag{1.20}$$

implying that the birthday of B occurs before that of A . This means B is older than A after the trip. This is highly counter-intuitive since although both A and B have identical experiences their ages differ at the end of their journey! The paradox however can be explained by noting that for spatially separated clocks the change of relative synchronization cannot be unequivocally determined. The clocks can only be compared when they are in spatial coincidence. For example, when in S' either of the observers can slowly walk toward the other or both the observers can walk symmetrically (with respect to S') toward the other and compare their clocks (ages) when they meet[36]. However in that case one can show that they do not have identical local experiences—thus providing the resolution of the paradox[37]. While the paradoxical element of the problem goes away, the fact remains that the

result (Eq. (1.20)) is correct and this time-offset remains unchanged even if they slowly walk toward each other and compare their clocks (ages) when they meet.

This temporal offset effect of identically accelerated clocks gives an important insight into the behavior of clocks in a uniform gravitational field, for, according to EP “...all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system” [38]. This suggests, as correctly remarked by Boughn that two clocks at rest in a uniform gravitational field are in effect perpetually being accelerated into the new frames and hence the clock at the higher gravitational potential (placed forward along the direction of acceleration) runs faster. We shall see in chapter-III that the time-offset relation (Eq. (1.20)) of Boughn’s paradox can be interpreted as the accumulated time difference between two spatially separated clocks because of the pseudo-gravity experienced by the twins[37]. However the connection between gravity with this temporal offset through EP was first pointed out by Barron and Mazur [39], who derived the approximate formula for the “clock rate difference”. We shall see in chapters (III-V), the importance of the time-offset relation (Eq. (1.20)) in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin.

Twin Paradox in a Closed Universe:

Cosmology Connection

An approach to explaining the differential aging of the twins which avoids acceleration has been to put the two paths onto a closed space-time. For example one may consider a cylindrical space-time— a two-dimensional universe in the shape of an infinitely long cylinder, in which time runs up the cylinder and space runs *around* it. In the cylinder, the time axis falls parallel to the axis of rotation of

the cylinder and travelling twin departs and returns by going around the cylinder at constant velocity. The calculation of the proper times on the cylinder has been done by Dray[17] and Low[40]. In such a universe, the twin in the rocket can return to earth with constant speed without changing direction. In the standard twin paradox the “acceleration” of the traveller plays the role of an identifier as to the question who will be younger of the two. The absence of acceleration in the closed universe scenario apparently leaves no such identifier thus making the paradox more challenging.

Tevian Dray has resolved the issue by showing that there still exists an asymmetry between the two twins. He first provides a simple geometrical argument. It is believed that one should formulate the paradox in terms of “invariant concepts as opposed to observer dependent concepts”. Although we do not agree with this observation in the context of the paradox, for the sake of completeness let us briefly reproduce the argument.

According to Dray the necessary invariant notion is that of proper time which is just the Lorentzian length of the path (but not Euclidean distance). The triangle inequality in the Minkowski space-time tells that one side of the triangle is greater than sum of the other two. Knowing this, the usual twin paradox is (as if!) easily solved. This we have already discussed (and criticized) in Sec.(1.2.4). For the present paradox the things are claimed to be “just as simple”. The author remarks that a line parallel to the sides of the cylinder (time-axis) is clearly shorter than one that spirals around it. Thus the traveller twin travels along the “shorter” route and hence is found to be the older.

The author has correctly clarified that a family of observers in the closed universe “may be singled out by noting that they are the only stationary observers who cannot distinguish between “forward and backward”; e.g., by sending light

rays in both directions around the cylinder and seeing which returns first". These observers should always be the oldest in any twin paradox calculation between different stationary observers.

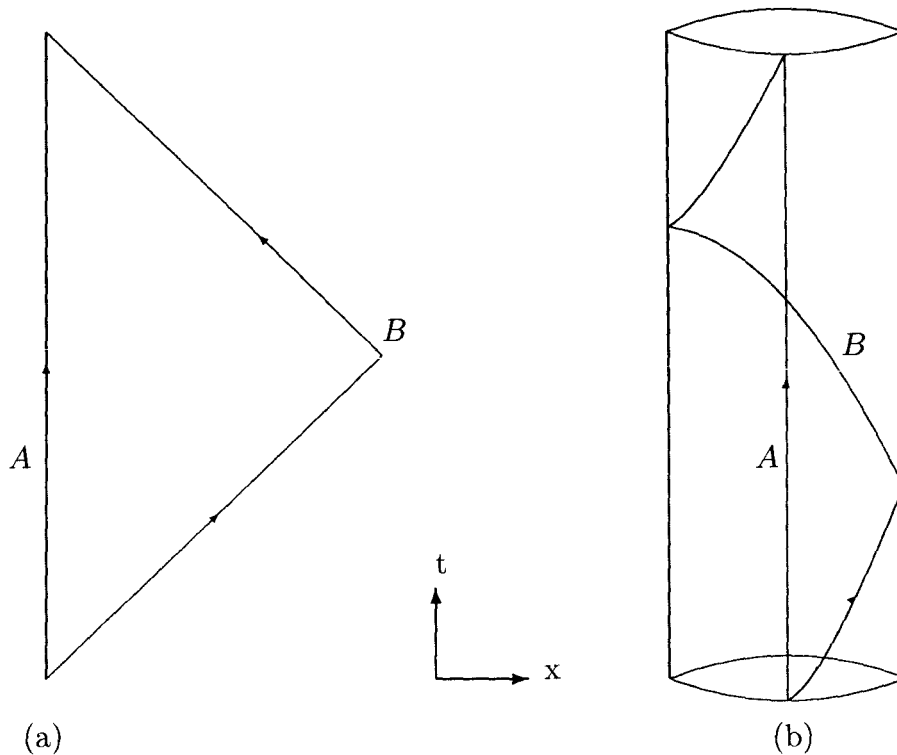


Figure 1.4: The space-time diagram of the travelling twin B and stationary twin A (a) in a usual twin paradox (b) in a cylindrical universe.

It may not be out of place to mention here that in order to observe the requirement of an asymmetry to get differential aging Hafele et.al performed one of the famous experiment called Hafele-Keating experiment[41]. In this experiment, differential aging was observed on two atomic clocks travelling on jets at the same speed around the earth in opposite directions. The rotation of earth provided the asymmetry that was necessary to produce the difference in proper times. These two paths without the rotation of the earth can be compared schematically to the two paths going

around in the opposite directions on the cylindrical space-time.

In the context of closed universe Barrow and Levin argue that “the resolution of the twin paradox in the closed universe hinges on the existence of *preferred frame* singled out by the compact topology of space”[18]. They have shown that there is only one reference frame that can be at rest and all other inertial observers in relative motion live in the universe where both space and time coordinates are known. Thus the solution of twin paradox identifies a preferred place and preferred time at the center of the universe so that observer be able to synchronize their clocks and observe the smallest volume for the universe.

The question of existence (or the role) of preferred frame in the resolution of the twin paradox has recently been discussed by Unnikrishnan[27] and Kak[42]. Referring to Einstein’s own resolution of the clock paradox in terms general relativity (gravitational slowing down of clocks) in 1918[20] and also to other standard resolutions, the former author remarks that they all suffer from a logical fallacy. Indeed Unnikrishnan has noted that “the time registered by two identical clocks that are synchronized initially can be different only if the rate of the clocks changes differently during motion, and one does not see any logical possibility of the required modification of the rate of either clock in any of the standard resolutions, including Einstein’s resolution of the twin paradox...”. According to the author (as has been discussed in Ref.[27]) the other alternative would have been to accept a ”spooky” instantaneous action-at-a-distance which is not permissible in SR. As a logically consistent possibility Unnikrishnan then suggests that one has to accept that the “rate of a clock is modified according to the standard Lorentz factor with the velocity always relative to average rest frame of the universe or the frame in which the cosmic microwave background radiation (CMBR) is isotropic...”. Obviously there does not exist a twin paradox in such a scenario. In fact in any clock com-

parison experiment (including the GPS timing and the Hafale-Keating experiment) the answer will always be unambiguous.

Clearly the foregoing analyses call for a “reassessment” of SR. In another paper the author has engaged himself to this task[43]. He holds that the novel theory has to be consistent with the “existence of the massive universe”. He further maintains that all kinematical effects of SR which depends on relative motion in flat space-time should be viewed as due to the “gravitational effects of the nearly homogeneous and isotropic universe”. According to him the correct theory should be the one with “a preferred cosmic rest frame”. This is the one with respect to which, as the author advocates in the previous paper[27], the time dilation with the usual Lorentz factor is to be considered. However, the author has to admit that the theory (termed as Cosmic Relativity by the author) should preserve Lorentz invariance.

In our opinion the last requirement emphasizes that the new theory in essence is still Lorentzian and the difference if any may at best be structural. This reminds us again of the conventionality of simultaneity thesis of SR according to which one can use a plethora of transformation equations with different synchronization parameters (for example the Reichenbäch parameter ϵ or Selleri parameter[8, 44, 45]) to describe the same relativistic physics (see chapter II for details). One of the possible convention being that of absolute synchronization[8, 13, 44, 46, 47, 48]. For absolute synchronization scheme one needs to start with a preferred frame (and usually one considers it to be the rest frame of the CMBR) and observes that time dilation takes place with respect to that frame alone. Note however that according to the true CS- thesis it is not essential to choose a particular inertial frame (CMBR frame for example) to be the preferred one and any inertial frame for that matter may be considered as the first frame[48].

More recently Kak, in consonance with Unnikrishnan believes that “the special

relativity principle formulated originally for physics in empty space is not valid in the matter filled universe.” Further, according to him physical laws are a consequence of the large scale nature of the universe so that “there will be difference in the experience of two observers in relative uniform motion if isotropy of the universe is not maintained by them equally...”. It is claimed that instead of considering the time dilation as a consequence of LT if it is viewed as a consequence of the experience of anisotropy (of the universe) by the moving observer, resolutions of the twin paradox and its variations may then become almost a trivial exercise. The basic idea of the author is much akin to the Cosmic Relativity of Unnikrishnan; however Kak in his paper[42] also suggests a means allowing one to infer (in principle) the speed of moving observer by measurements of the “distribution” of speeds of the receding distant objects.

Before we go to the next section, we would like to remark that the Cosmic Relativity theory to be truly a preferred frame one, the time dilation factor (or the length contraction factor or the both) need to differ (at least infinitesimally) from the usual relativistic expressions. In absence of this difference the theory continues to be relativistic. However there may be compelling reasons to think of a true preferred frame theory, coming from requirements elsewhere in physics. We discuss this issue in the next section in details, which will provide a background for the topics other than the twin paradox considered in this thesis.

1.3 Background-II

1.3.1 Greisen-Zatespin-Kuzim Limit and a Puzzle

After a century of successes, Einstein’s special relativity (SR) appears to be violated by certain observations on ultra high energy cosmic rays (UHECR). These cosmic

rays are high energy particles that are produced in distant galaxies and impose on the earth's atmosphere generating showers of secondary particles. By detecting these secondary particles through the particle-cascade processes, the energy of the original cosmic rays can be constructed. These observations of UHECR seem to be in contradiction with SR. The aspect of such cosmic ray observations which is in conflict with the theory of relativity concerns a theoretical limit known as the Greisen-Zatespin-Kuzim (GZK) limit [49, 50, 51]. Cosmic rays with energies above this threshold should not reach earth since they are supposed to lose energy by interactions with relic radiation of the Big Bang. The limit was calculated by K. Greisen, G. Zatespin and V. Kuzim in 1966 based on interactions predicted between the cosmic ray nucleons and photons of cosmic microwave background radiation (CMBR):

$$p + \gamma \longrightarrow p + \pi^0 \quad (1.21)$$

$$p + \gamma \longrightarrow n + \pi^+. \quad (1.22)$$

The reaction will progress when the combined center of mass energy of the proton (p) and photon (γ) equals to or greater than sum of the pion (π^0 or π^+) and proton (p) or neutron (n) masses. This can be written as

$$m_p + m_\pi + m_\pi^2 \leq q((p^2 + m_p^2)^{1/2} - p \cos \theta), \quad (1.23)$$

where q is the photon momentum along the x -axis and p is momentum of the proton hitting the photon at an angle θ in xy plane. As the pion mass is much smaller than proton or neutron mass, one may also write

$$E_p - p \cos \theta \geq m_p m_\pi / q. \quad (1.24)$$

For a thermal gas of relativistic bosons $\langle q \rangle \approx 2.7T$ and with $T_{CMBR} \approx 2.7K$ corresponds to an energy of $2.3 \times 10^{-4} eV$. On putting the pion and proton masses

a cut-off energy (E_p) of $5 \times 10^{19} eV$ is calculated. This defines the GZK limit. Thus according to this theory the cosmic rays above the threshold energy (cut-off energy) of $5 \times 10^{19} eV$ would interact with cosmic microwave background photons to produce pions.

Because of the mean path associated with the interaction (this can be calculated from the photon density of CMBR and photo-pion reaction cross-section known from laboratory studies on γ -ray and proton collisions)⁷, extragalactic cosmic rays with distances more than 50 Mpc from the earth with energies greater than this threshold energy should never be observed on earth and there are no known sources within this distance that could produce them[52]. However number of observations have been made[53, 54, 55] that appear to show cosmic ray energy spectrum from distant sources to extend well beyond GZK limit. The mechanism producing UHECR is unknown. Many models have been found in the literatures regarding the source of such high energy cosmic rays. There are different exotic origins that have been proposed, such as topological defects, active galactic nuclei and gamma-ray bursts[52]. These schemes are however ruled out by the GZK limit. Indeed the ground based detectors have detected a large number of events above 10^{20} eV, the highest energy of cosmic rays so far has been $3.2 \pm 0.9 \times 10^{20}$ eV detected by Fly's ye air shower detector in Utah[54]. In this detector the error box for the arrival direction in galactic coordinate had been centered on $b = 9.6^\circ$, $l = 163.4^\circ$ and the particle cascade reached a maximum size near a depth of $815 gm/cm^2$ in the atmosphere. However, if the sources of UHECR are believed to be extragalactic that one event at 10^{20} eV appears surprising. This is the UHECR paradox[56].

The aspect of GZK limit and its possible violation are concerned with relativity.

⁷Recall that the mean path is given by $\lambda = 1/n\sigma$ where σ is the relevant cross-section and n is the number density of the CMBR photons.

The value of the GZK threshold is purely a result of the relativistic kinematics. The observed conflict with this threshold value thus calls for a modification of special relativity or its postulates. In the literature one witness various attempts to modify the standard relativity theory in order to find an answer to the UHECR paradox which will be reviewed below. But before we do it let us note, in Amelino-Camelia's words "As with all emerging experimental paradoxes it is of course possible that the cosmic-ray paradox is the result of an incorrect analysis of the experiment, for example it is legitimate to speculate that the identification of these ultra-high-energy cosmic rays as protons produced by distant active galaxies might eventually turn out to be incorrect. But, in spite of its preliminary status, this cosmic-ray paradox provides encouragement for the study of new relativity postulates" [56].

1.3.2 Lorentz Invariance Violation: A Popular Scenario

There have been exotic proposals in the literature which try to explain trans-GZK cosmic ray events in the framework of Lorentz invariance violation (LIV) theories. For example Coleman and Glashow [57] have argued "that possible departures from strict Lorentz invariance, too small to have been detected otherwise, can affect elementary-particle kinematics so as to suppress or forbid inelastic collisions of cosmic-ray nucleons with background photons. Thereby can the GZK cutoff be relaxed or removed." The authors have argued elsewhere[58] that the velocity of light in vacuum c_0 in a moving frame relative to the rest frame of the universe can differ from the maximum attainable speed c of the material body by a small velocity parameter ϵ of the theory. The obvious consequences of this consideration is the existence of a preferred frame of reference. It is assumed that this preferred frame (the so-called ether frame) to be "the rest frame of the universe" with respect to which the CMBR is isotropic.

The precession tests for anisotropies in the velocity of light due to the CMBR frame have set a limit on this ϵ

$$|\epsilon| < 3 \times 10^{-22}. \quad (1.25)$$

However it is argued [52, 58] that stronger constraints on ϵ can be obtained from the observations on UHECR. If $c < c_0$ the photon 4-momentum becomes time like so that the energetic photon converts into an electron-positron pair. It has also been shown that the detection of primary proton energy up to 100 eV set the bound on ϵ

$$|\epsilon| < 5 \times 10^{-24}. \quad (1.26)$$

The physical basis on such a constraint on ϵ is that particle can be super luminal in vacuum. If $c_0 < c$ in such a case a proton being a charged particle will lose energy through vacuum Cerenkov radiation and will therefore fail to be detected with the super luminal speed. The limit on ϵ thus obtained does not require any postulates regarding the motion of the laboratory frame with respect to the preferred frame. According to one most popular scenarios [59], existence of different maximal speeds for different particle species is assumed and they are also assumed in general to differ from the speed of light in vacuo [see Ref.[52] and references therein]. In this way introduction of small LIV has been shown to have effects that increase rapidly with energy in such a manner that ultimately inelastic collisions with CMBR photons become kinematically forbidden.

To see briefly how the GZK cutoff is affected by Lorentz violation, consider the formation reaction yielding the first pion-nucleon resonance[57]

$$p + \gamma(CMBR) \longrightarrow \Delta(1232), \quad (1.27)$$

by which a nucleon with energy E collides inelastically with a CMBR photon of energy ω . The target photon energies are a thermal distribution with temperature

$T = 2.73$ K, or $KT \equiv \omega_0 = 2.35 \times 10^{-4}$ eV. Energy conservation provides the condition under which the reaction (1.27) can proceed:

$$2\omega = M_p^2/2E \geq (c_\Delta - c_p)E + M_\Delta^2/2E, \quad (1.28)$$

where $c_\Delta - c_p$ is the relevant Lorentz-violating parameter. If $c_\Delta = c_p$ the above equation yields the usual threshold energy for this process $E_f = (M_\Delta^2 - M_p^2)/4\omega$. Otherwise it yields a quadratic inequality in E which can be satisfied if and only if $c_{(\Delta - c_p)} < \hat{\delta}(\omega) \equiv \omega/2E_f$. As $c_\Delta - c_p$ is increased toward $\hat{\delta}$, the threshold for the formation reaction grows toward $2E_f$. However, if it exceeds its critical value,

$$c_\Delta - c_p > 2\omega^2/(M_\Delta^2 - M_p^2) \approx 1.7 \times 10^{-25}[\omega^2/\omega_0^2]. \quad (1.29)$$

The reaction (1.27) becomes kinematically forbidden for all E . They have argued that the reaction (1.27) is the dominant process leading to the GZK cutoff as originally formulated. However, if $\Delta(1232)$ formation is not possible, a weakened version of the cutoff may result from non-resonant photo-production of one or more pion:

$$p + \gamma(CMBR) \longrightarrow p + N\pi. \quad (1.30)$$

If $c_\pi = c_p$, the threshold energy for pion production is $E_p = M_\pi(2M_p + M_\pi)/4\omega$. If Lorentz invariance is violated as $c_\pi - c_p$ is increased from zero, the threshold grows. for a fixed photon energy ω

$$c_\pi - c_p < \hat{\delta}(\omega) \equiv 2\omega^2/M_\pi^2 \approx 5 \times 10^{-24}[\omega^2/\omega_0^2], \quad (1.31)$$

Reaction (1.30) and multiple pion production are kinematically forbidden at all proton energies if $c_\pi - c_p > \hat{\delta}(\omega)$.

1.3.3 Doubly Special Relativistic Theories

There are others class of theories known as doubly special relativistic (DSR) theories which consider a generalization of SR to include one more invariant scale, in addition to that of the velocity of light. The theory considers a modified LT in momentum space (DSR 1)[60, 61] and it has later been shown to be nonlinear representation of the Lorentz group[62]. Besides its primary motivation coming from experimental side, this type of theory finds encouragement from quantum gravity considerations also, where the role of Planck's scale ($E_P \approx 10^{28} eV$) might be important. It is expected then E_p would define a transition *scale* beyond which classical space-time picture will not remain valid and the description of physics will change drastically. However if the principle of relativity is to be honored one should think that the scale should be observer independent. In other words one may assume that the relativity postulates are to be revised in such a way that the description of physical phenomena changes significantly at the *observer-independent* kinematical scale. Indeed the Planck's scale has a special role in effects to unify quantum mechanics and general relativity into quantum gravity since this scale is the combination of speed of light constant, the gravitational constant and quantum mechanical Planck's constant[63]. Quantum gravity predicts a new quantum picture of space-time for particles with momentum and energy above the Planck's scale, although the classical picture remains valid below this scale. In theory of SR, the consequences of quantum gravity would be paradoxical since the theory predicts that "same particle have energy higher than the Planck's scale according to some observers and energy lower than the Planck's scale according to other observers[56]." The doubly special relativity solves this problem. In particular the variant due to Magueijo and Smolin[62] (here after called DSR 2) holds that all observers at least agree whether a particle crosses

the Planck's scale or not.

What are these modifications. According to Magueijo and Smolin theory the modification of SR is based upon four basic principles (1) the relativity of inertial frames: This predicts there is no preferred state of motion so that velocity is a purely relative quantity (2) the equivalence principle: Under the effect of gravity, freely falling observers are all equivalent to inertial observer (3) the modification: The observer independence of the Planck's energy and (4) the corresponding principle: At energy scales below Planck's energy the special relativity and general relativity both are true, i.e they will remain valid to first order in the ratio of energy scales to E_P .

Such modifications in turn deformed transformation equations which reduced to the usual ones at low energies, but keeps invariant the Planck's scale which marks the border line between classical and quantum gravity. That such DSR theories have the ability to explain high energy cosmic ray anomalies is due to the introduction of deformed dispersion relations of the form

$$E^2 = p^2 + m^2 + \lambda E^3 + \dots, \quad (1.32)$$

where λ is of the order of Planck length. Note that this is the departure from the usual relativistic dispersion relations.

$$E^2 = p^2 + m^2, \quad (1.33)$$

here we have assumed $c=1$. Following detailed analysis one can indeed show that the interaction of proton and CMBR photons leads to the corrected threshold for pion production[64, 65]

$$E_{th} = U^{-1}(E_{th0}) = U^{-1}[\left((m_p + m_\pi)^2 - m_p^2\right)/E_{CMBR}], \quad (1.34)$$

where m_p and m_π are the proton and pion rest masses, E_{CMBR} is the photon CMBR energy in the cosmological frame and E_{th0} is the GZK threshold. Note that

the U map can be chosen so as to implement various properties required from a phenomenological theory of quantum gravity.

The further development of non-linear relativity (DSR theories) is in progress. There are however many issues that are still unanswered. For example, there is the problem of how to modify the theory for composite system. With the loss of linearity, the kinematic relations valid for single particles need not be true for composite systems. It is however claimed that this is at least a desirable feature since *non-linearity builds, into the theory the concept of elementary particle*—a feature having the differentiating ability between fundamental particles and composites[65]. The solution to the problem does not come easily. For example, it can be shown that obvious straight forward covariant and composition law of energy and momentum leads to contradiction. One possible solution allows one to think that the map U use for a composite system need not be the same as that used for single particles. There are other solutions to this problem involving embedding the theory in higher dimensions however the details of which lies beyond the scope of this write up.

Another theoretical development concerns the position space picture of these theories which are usually constructed in momentum space. With the loss of linearity “duals no longer mimic one another”[62] or vectors no longer transform according to the inverse transformation of co-vectors. A number of solutions may be found either involving or avoiding non-commutative geometry[66] and quantum groups[67, 68]. One may however recover linearity by embedding the theory into a higher number of dimensions[69, 70]. The approach is elegant however alternative way to linearize the theory can also be found[71, 72]. A considerable amount of work is found in the formulating field theory[64] much of these are not relevant for what lie ahead. However one may point out that the most conceptual problem in front of DSR theory is the challenge of doing general relativity based on non-linear relativity[73].

In the context of what we are going to do in the next chapter regarding the UHECR issue let us summarize this brief review work by saying that although the UHECR paradox primarily provides encouragement for the DSR theories, the revision of dispersion relation as already stated is motivated from quantum-gravity considerations as well. These theories try to avoid the preferred frame issue prompted by the introduction of Planck's scale in the theory. The review is not exhaustive. Indeed there are many ramifications and there exist several DSR theories which try to deal with the UHECR paradox.

There are other approaches as well that may be found in the literature. Before we end the subsection we just give one such example. Booth[74] considers the reassessment of the GZK cutoff in the UHECR following his so called quasi static universe (QSU) model, according to which it is believed that the photon energy is an invariant in the cosmological reference frame so that the photon number density in the universe today is much less than (by a factor of 10^9) that of the standard model. Consequently, the mean free path of the cosmic rays (for the collision of photons and CMBR photons) will exceed the horizon distance of the universe, implying the latter to be essential transparent to the UHECRs. This QSU model therefore does not predict any cutoff for the cosmic ray spectrum. Thus it has been claimed that the reduced CMBR photon number density predicted by the model provides a natural explanation for the observed flux with energies greater than $10^{20}eV$. However the QSU model has not yet found a general acceptance and the idea although interesting can be regarded as a bit too speculative.

We follow a different approach. Referring to the most popular scenario we may recall that in an effort to look for new physics (in the theories involving LIV) one believes that the space-time is still governed by LT however other laws of physics might not remain covariant under LT. In this part of the thesis (Part II) we will deal

with the issue rather differently. As will be explained we will consider the possibility of a deformed LT (not just a deformed dispersion relation as described in the DSR theories) to relate observations performed by different inertial observers. Clearly the predictions of deformed LT will be expected to be different from those of relativity theory. However the difference in the predictions should be undetectable in the domain where SR has been tested beyond doubt. We shall see in chapter-VI that the UHECR paradox can be explained in terms of a non-preferred frame effect of the laboratory frame which is moving with velocity ≈ 300 km/sec with respect to the preferred one, assumed to be at rest with CMBR frame. Unlike some earlier efforts as discussed in Sec.(1.3.2) (the Coleman-Glashow[58] scheme for example) which consider LIV but assume that the physical kinematics is still Lorentzian, we shall propose to modify the transformation equation itself. Deformed LT are generally discussed in connection with test theories like that of Robertson[75] or Mansouri and Sexl[76] on which improved tests of SR are often based (see for example[77]). But they are not usually considered to represent a new physics that may provide a solution for the UHECR paradox.

As we have seen, some authors find it troublesome giving up the principle of relativity. In the so called DSR theories, the particle dispersion relation is modified; but the introduction of an invariant length or energy scale in addition to the invariant velocity scale of SR, the “relativity of inertial frames” is still maintained. Such theories, often motivated by quantum-gravity considerations are interesting but are unable to resolve the UHECR paradox quantitatively at the moment.

We shall attempt to deform the relativistic kinematics using heuristic means. We will do it first by identifying the objective contents of the relativity principle and then will go in for modifying these contents *minimally* to obtain a new transformation that will be able to relate space-time of an arbitrary frame of reference with

that of the universal rest frame of the cosmic substratum.

1.3.4 Variable Speed of Light

We have already pointed out the most challenging problem of the DSR proposal is that of modifying the general relativity (GR). Magueijo and Smolin[78] try to examine the question how the modification of SR proposed by the authors themselves[62, 64] can be extended to GR. The main reason of this endeavour is that there lies accessible modification of GR which is characterized by the features that the space-time geometry becomes energy dependent. For example “quanta of different energies see different classical geometries”. One outcome of this novel picture is that the speed of light (and other massless quanta) naturally becomes energy dependent. If one now turns to the investigation of cosmological models one finds naturally the so called the variable speed of light (VSL) cosmologies. Indeed it has been speculated that there may be a connection between the DSR and VSL cosmologies[79, 80, 81]. The authors of Ref.[78] have shown that this connection does follow from their proposed GR.

We shall show in the concluding chapter that this “natural” connection between DSR and VSL cosmologies also holds good for a DSR of different genre (discussed in the next chapter). In this subsection therefore it will be worthwhile to briefly review the history of VSL from both theoretical and observation points of view. Hence as the previous section gives a necessary background for the next chapter, this subsection is intended to provide the same for the concluding chapter.

VSL Theories:

Historically in 1937 Paul Dirac and others[82] started investigating the outcome of the natural constants changing with time. As an example, Dirac thought about a change of only five part of 10^{11} per year of gravitation constant G to explain

the relative meagerness of the gravitation force compared to other forces of nature. This proposal later become known as Dirac large number hypothesis.

In 1968, referring to wide speculations in cosmology by models makers without much hard facts to go on, Dirac wrote “One field of work in which there has been too much speculation is cosmology. There are very few hard facts to go on, but theoretical workers have been busy constructing various models for the universe, based on any assumptions that they fancy. These models are probably all wrong. It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular quantities which are considered to be constants of nature may be varying with cosmological time. Such variations would completely upset the model makers”[65].

Although much has changed since Dirac wrote these words, we have now many observational inputs in cosmology, the views expressed by Dirac (regarding speculations in cosmology) are still applicable. Apart from Dirac, other physicist have also entertained the possibility of varying gravitational constant G [82, 83, 84], a varying electron charge e etc.. In contrast to G and e the constant c has remains sacred for a long time. It was perhaps thought that “varying c theories are expected to cause much more structural damage to physics’ formalism than other varying constant theories”.

A VSL cosmology has independently been proposed by Petit from 1998[85, 86, 87, 88] and Moffat in 1992[89]. In Petit’s VSL model, the variation of c accompanies variation of all physical constants in such a way that all equations remain unchanged through the evolution of the universe.

One may still go to the past. Many VSL theories were considered since the advent of SR in 1905. In some context even Einstein considered such a theory long back in 1911[65]. In 1930’s VSL was used as an alternative explanation of Hubble’s

discovery of recession of galaxies[90, 91, 92]. These theories are obviously in conflict with the fine structure observations. Let us not call them the true VSL theories. The first modern VSL theory is due to J. W. Moffat[89] whose “ground breaking” paper (involving spontaneous symmetry breaking of Lorentz symmetry) gives an elegant solution to the horizon problem of the big bang cosmology.

Since then the literature on the subject continues to grow starting from the work of Albrecht and Magueijo[93] who considers the cosmological implication of light travelling faster in the earlier universe. They proposed a prescription for deriving a set of new cosmological evolution equation while the speed of light c is changing. It has been shown by them how the flatness, horizon and cosmological constant problem may be solved. Later authors try to do some thing by improving upon the theories of their earlier authors. It will be outside the scope of the present thesis to review all these efforts. Rather it will be proper to categorize these endeavours from the considerations of their *departures* from SR, since one may note that all VSL theories must be in conflict with SR in some way. The classification below will therefore be based on the nature and depth of these conflicts with SR. This classification will also help put our own VSL approach (to be discussed) in the proper perspective.

Much of what follows have been taken from a detail and excellent review work of João Magueijo[65]. We briefly reproduce them for the sake of completeness. Magueijo recalls that the main corner stone of SR are the two independent postulates. One concerns the relative nature of motion and the asserts the constancy of the velocity of light (CVL). The author holds that the VSL theories do not need to violate the principle of relativity of motion. Although it may be difficult to think of relativity of motion if the CVL is tampered with. Consequently the first criterion for differentiating the various proposals is based on whether the theories honour the

relativity of (inertial) motion.

As regards CVL there are various aspects of “constancy”. These are explained in detail in Ref.[65]. For the present purpose it is enough to say that there is a large number of combination in which these different aspects can be violated. At the outset, I would like to point out that our proposal considers violation of both the postulates in an approximate manner and hence is a preferred frame theory altogether. When both the postulates of SR are violated one may say that the Lorentz symmetry is being broken in the “Hard way”.

The main VSL mechanism proposed so far starting from Hard breaking of Lorentz symmetry are the following.

(1) Hard Breaking of Lorentz Symmetry

This model has been proposed by Albrecht and Magueijo[93] and Barrow[94]. In this model (like that of ours) both postulates of SR are violated. The authors postulate that there is a *preferred* frame in physics identified with cosmological frame. These theories describe a world where matter content of the universe as well as the laws of physics evolve in time (since the speed of light varies in time usually in the very early universe). The basic dynamical postulate is that Einstein’s field equations are valid with minimal coupling in the particular form,

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = (8\pi G/c^4)T_{\mu\nu}. \quad (1.35)$$

Note that here c is to be interpreted as a field. The metric, connection, curvature and Einstein tensors are to be evaluated in a given frame at constant c where no extra term involving gradients of c will be present. Non-covariant extra terms in gradients of c will appear only in other frames. Minimum coupling at the level of Einstein’s equations is at the heart of the model’s ability to solve the cosmological problems.

(2) Bimetric VSL Theories

This model has been initially introduced by Moffat and Clayton[95] and by Drummond[96]. In these theories the speeds of the various massless species may be different but SR is still recognized within each vicinity. The speed of the graviton is taken to be different from that of massless matter particles. By introducing two metrics (one for gravity and one for matter) the model has been further developed in scalar-tensor model[97] and vector model[98, 99]. As for example, the scalar tensor model uses a scalar field ϕ which is minimally coupled to a gravitational field (the field variables being the usual metric $g_{\mu\nu}$). However it is assumed that the matter couples to a different metric;

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi. \quad (1.36)$$

Thus one talks of gravitational metric $g_{\mu\nu}$ and matter metric $\hat{g}_{\mu\nu}$. The total action is composed of three parts;

$$S = S_g + S_\phi + \text{matter - action}, \quad (1.37)$$

where the usual gravitational action

$$S_g = (-c^4/16\pi G) \int dx^4 \sqrt{-g}(R(g) + 2\Lambda), \quad (1.38)$$

and scalar field action is

$$S_\phi = (c^4/16\pi G) \int dx^4 \sqrt{-g}[(1/2)g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)]. \quad (1.39)$$

Note that the matter action is the usual one but uses the metric $\hat{g}_{\mu\nu}$. By varying $g_{\mu\nu}$ one obtains from Eq.(1.37) the gravitational field equation with source terms containing $T_\phi^{\mu\nu}$ (the stress-energy tensor of the scalar field action), $\hat{T}_{\mu\nu}$ (of the matter action) and finally the cosmological constant term Λ . Needless to say the left hand

side of field equation is the usual Einstein tensor

$$G^{\mu\nu} = R^{\mu\nu} - (1/2)g^{\mu\nu}R. \quad (1.40)$$

The most important thing of the theory is that the speed of light is a dynamical variable predicted by a wave equation

$$\bar{g}^{\mu\nu} \hat{\Delta}_\mu \hat{\Delta}_\nu \phi + KV'[\phi] = 0, \quad (1.41)$$

where biscalar metric $\bar{g}^{\mu\nu}$ is defined in the original text [95]. This model predicts solutions with a de Sitter phase that provides sufficient inflation to solve the horizon and flatness problems. The model has also been used as an alternative explanation for dark matter and dark energy[98].

(3) Energy Dependent Speed of Light

This approach may or may not preserve the first postulate of SR (the relative nature of motion), however it violates the second one for certain. This model states that the speed of light is allowed to vary with the frequency (colour) close to Planck frequency. This is performed by deforming the photon dispersion relation ($E^2 - p^2 = m^2 = 0$). For example it was proposed that

$$E^2 = p^2 + m^2 + \lambda E^3 + \dots, \quad (1.42)$$

where λ is of the order of Planck length. If the linear Lorentz transformations are still valid and if this dispersion relation is true in one frame then it cannot be true in any other frame so that this theory also violates the principle of relativity as described in model (1). The deformed dispersion relation above implies that the group velocity c of light ($c = dE/dp$) is energy dependent. As a consequence speed of light was faster in the past suggesting a VSL cosmology.

These theories are generally used to give phenomenological description of quantum gravity[62] and as an explanation of the dark energy in terms of energy trapped

in very high momentum and low-energy quanta[65]. Finally the modified relation may lead to an explanation for the UHECR paradox[100], which is one of the main topics of this dissertation (See chapter VI).

(4) Lorentz Invariant VSL Theories

In this approach the essence of Lorentz invariance is preserved in its totality but space-time transformations relates with varying c . Moffat[89] proposed such VSL theory in which Lorentz invariance is spontaneously broken but the theory is endowed with exact local Lorentz symmetry. Typically in this scenario the speed of light undergoes *phase transition* to a value 30 order of magnitude smaller, corresponding to the currently known speed of light. In this model the entropy of the universe is reduced before the phase transition but increases afterward. This solves the enigma of the arrow of time and the second law of thermodynamics. Another model is proposed by Maguiejo[99] in which the covariant and local Lorentz invariance remain applicable when the speed of light is allowed to vary. Although elegant and interesting these theories at least suffer from the drawback that their implementation in cosmology is somewhat cumbersome.

(5) M-Theory

People often tend to consider the CVL postulate of SR as sacrosanct and try to cling to it by any means. An exotic effort to be described below is an example of this mindset. A type of VSL work has been initiated by Kiritis[101] and Alexander[102] and makes use of the brane-world picture, in which our universe is a three-dimensional brane embedded in a higher-dimensional (bulk) space-time. When a test brane is moving in the vicinity of a black-hole bulk space-time it is possible to have perfect Lorentz invariance i.e a perfect CVL in the “bulk” while one has a VSL on the “brane”. It means that the Lorentz invariance of the full theory remains intact and VSL results, so to say, from a projection effect! More

specifically, in this model the velocity of light can be found to vary depending on the distance between the brane and the black hole.

Several VSL theories in this scenario have used the Randall-Sundrum models[103] in which the extra dimensions are subject to warped compactification. The light signals in such space-time may travel faster through the extra dimensions. There are many ramifications of this sort of models and extensions, “mirage cosmology” and other exotic proposals, but the detailed description of such things are hardly necessary for the present purpose.

(6) Field Theory

This type of VSL theory describes that quantum field theory in curved space-time which predicts super luminal photon propagation. This was first proposed by Drummond[104], where one-loop vacuum polarization corrections to the photon propagation were computed in a variety of backgrounds. The phrase “super luminal light propagation” may at a first sight seem a contradiction in terms but one here typically distinguishes between c appearing in LT and the actual speed of propagation of light (which may be notified due to non-minimal coupling of gravity. A resolution of the horizon problem by means of this effect i.e in the situation where a light cone is distinct from the causal cone) was obtained in 1989, which can be found in[105]. The implications for optics and causality of “faster than light” motion are discussed by Shore[106]. The Casimir effect is an example where VSL has been discovered in fields theories. In which, vacuum quantum effects induce an anomalous speed of propagation for photons moving perpendicular to a pair of conducting plates[107].

As regards explicit breaking of Lorentz invariance one considers the possibility that LT might be a low energy phenomenon. For example Nielsen and his collaborators[108] have suggested that the “Lorentz invariance could be a stable

infrared fixed point of the renormalization group flow of a quantum field theory”. Neutrino oscillations are another example in this respect in which the endpoint of beta decay are currently being studied[109]. It is held that “high energy physics tests of CPT can also act as test of Lorentz invariance” and VSL may be studied in the framework of Lorentz violating extensions of the standard model[65].

Enough has been discussed so far although in a brief way, about the theoretical aspects of VSL motivation and scope and their different levels of departures from the basic tenets of SR but do we have enough hard facts about VSL to consider such things seriously? Perhaps yes. Below we review the observational status.

Observational status:

The most extraordinary observation with relevance for VSL is the work of Webb et.al[110], Murphy et.al[111] and Srianada et.al[112]. These authors have reported evidence for redshift dependence in the fine structure constant $\alpha = e^2/(\hbar c)$. The observations of Webb et.al[110] can be interpreted as supporting some nonstandard cosmological theories that considers VSL or the varying electron charge[113]. In the last reference it has been shown that the varying c cosmology, by transformation to standard unit can be rephrased as a varying e (electronic charge) theory. So far as the electromagnetic phenomena is concerned there has no difference if there is a c variation or e variation to account for the variation of α . If one introduces the gravitation in the discussions through the theory of black hole thermodynamics, as Paul Davies et.al[114] has shown that it is possible to test which constants might be actually varying. The authors arguments have indicated that the e variation with time are “at risk of violating both the second law of thermodynamics and the cosmic censorship hypothesis”. Although some later authors[115, 116] do not agree to this claim and the theory is not generally accepted, many other authors build up models where variation of α is considered to be as due to variation of c alone. For

the present thesis as we shall see we will assume this view point and try to match the observational results of α variation with our theory. Keeping this in mind let us continue with the observational status of varying α which is the same thing as varying c according to the stipulation.

In the beginning of this subsection we have mentioned the work of an Australian-British group that it possibly has detected a varying α (which is about one part in 10^5).

However we have a long history of null measurements. In 1956 Savedoff used the so called alkali-doublet method to estimate changes in α from the measurements of the spectra of distant quasars producing essentially a null result. Spectra of quasars at cosmological distances provide natural laboratory for studying variations in α . Dark absorption lines of radiation are produced due to intervening gas clouds which are enriched with heavy elements. The fine structure constant is known to be associated with the doublet splitting of alkali spectra. Indeed the doublet splitting is proportional to α^2 ; therefore any variation in the wavelength separation will be approximately proportional to α . Since quasars spectra contain absorption lines at different redshifts (meaning different times of the evolving universe). It is therefore possible to study in principle the time variation of α simply by looking for changes in the doublet splitting of alkaline type ions with one outer electron (for example triply ionized carbon or silicon) as a function of time (redshift). Although this seems very simple, any change in α will be very small and so the measurement accuracy is required to be extremely high. It is therefore not surprising that the early measurements which uses this technique give null results. The most accurate measurement on the fractional change prior to the work of Webb et.al give a value of $3.5(\pm 5.5) \times 10^{-6}$ [117, 118].

The study of certain isotopic abundances in the Oklo natural nuclear fission

reactor in Gabon, west Africa gives a terrestrial limit on the *alpha*-variation. The analysis of the decay product of the Oklo uranium mine (discovered in 1972) gives a range for a fractional change ($\Delta\alpha/\alpha$) which is between 0.9×10^{-7} and 1.2×10^{-7} over the period of 1.8 billion years. If one assumes a linear scale the result becomes equivalent to one part to 10^6 over the life time of the universe.

Webb et.al improved on the alkaline doublet technique to introduce a new variant whereby one compares the absorption wavelengths of magnesium and iron atoms in the same absorbing cloud. In this way the sensitivity becomes one order of magnitude more than that of the alkaline doublet method. The trend of these results following the many-multiplet heavy element transition in quasar absorption method indicate the value of α was lower in the past

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}, \quad (1.43)$$

for redshift $z \approx 0.5 - 3.5$.

This many-multiplet exploits the fact that energy of different transitions change differently for given change in the fine structure constant. The rest wavelengths of Mg II $\lambda\lambda 2797, 2803$ and Mg I $\lambda 2852$ transitions are insensitive to small changes in α whereby FeII multiplets are much sensitive to small variation in α hence the former transitions can be used as an anchor for measuring the systemic redshift. Thus measuring relative shifts between an anchor and different FeII lines can be used to measure accurately the α variation ($\Delta\alpha/\alpha$).

There are however methods other than the alkali doublet method or many multiplet method such as the one using OIII emission lines[119]. Although relatively robust, that technique is not sensitive enough to detect the small variations in $\Delta\alpha/\alpha$.

There are some investigations that rely on studies of molecular lines which have

been detected in a couple of systems[120]. These studies give $\Delta\alpha/\alpha = (-0.10 \pm 0.22) \times 10^{-5}$ and $(-0.08 \pm 0.27) \times 10^{-5}$ at z values 0.2467 and 0.6847 respectively. Studies at higher z can not be obtained due to unavailability of molecular systems.

The later works of Webb et.al group continue to conform the initial claim. For example in 2001 the authors[121] claim there are no systematic effects which can explain the positive results. However there are other authors who dispute these results on the basis of their studies at much higher sensitivities[112, 122, 123]. While many-multiplet analysis of about 143 absorption spectra obtained from the Keck/HIRES instrument used by Webb et.al group (see Ref.[124]) indicates a variation of α , the many-multiplet analysis of data obtained from ultra-violet and visual Echelle spectrograph (UVES) on the very large telescope (VLT) in Chile apparently produces a null result[112]. Murphy, Webb and Flambaum[111] however have critically reviewed the null results of the group (in particular see Ref.[122] who reports a mean relative variation of $\Delta\alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5}$). Their analysis of same fits to the absorption profiles produces the very different $\Delta\alpha/\alpha$ values with uncertainties sometimes larger by the factor of 3. They attribute the discrepancies to flawed parameter estimation techniques in the original analysis.

One may thus conclude that a reliable comparison of HIRES and UVES constraints on varying α may take place after sufficient improvements in the analysis of UVES spectra. In the concluding chapter we will develop a simple minded VSL theory on the basis of HIRES measurements and claims by Webb et.al or Murphy et.al. The null results due to Srianad et.al however we will also be discussed in the context of our proposed approach. Some more background and relevant data will be provided in that chapter in order to make it self contained.

Before we end this section I would like to reproduce a table (Tab.(1.1)) from Ref.[111] providing the summary of many-multiplet constraints on $\Delta\alpha/\alpha$ in the

literature. This is given for a ready reference. The table is self explanatory and hence no elaboration is being provided.

Table 1.1: Summary of the many-multiplet constraints on $\Delta\alpha/\alpha$ in the literature.

Instrument	N_{abs}	z_{abs}	$\Delta\alpha/\alpha[10^5]$	Reference
HIRES	30	0.5-1.6	-1.100 ± 0.400	Webb et.al (1999)[110]
HIRES	49	0.5-3.5	-0.720 ± 0.180	Murphy et.al (2001)[125]
HIRES	128	0.2-3.7	-0.543 ± 0.116	Murphy et.al (2003)[126]
HIRES	143	0.2-4.2	-0.573 ± 0.113	Murphy et.al (2004)[124]
UVES	23	0.4-2.3	-0.060 ± 0.060	Chand et.al (2004)[122]
UVES	1	1.151	-0.040 ± 0.190	Quast et.al (2004)[127]
UVES	1	1.839	$+0.240 \pm 0.380$	Levshakov et.al (2005)[128]
UVES	1	1.151	$+0.040 \pm 0.150$	Levshakov et.al (2005)[128]
UVES	1	1.151	$+0.100 \pm 0.220$	Chand et.al(2006)[129]
UVES	1	1.151	-0.007 ± 0.084	Levshakov et.al (2006)[130]
UVES	1	1.839	$+0.540 \pm 0.250$	Levshakov et.al (2007)[131]
UVES	23	0.4-2.3	-0.640 ± 0.360	Murphy et.al (2006)[111]

1.4 Topic Wise Summary

1.4.1 The Principle of Equivalence and the Twin Paradox

In chapter III the canonical twin paradox is explained by making a correct use of equivalence principle (EP) of general relativity (GR). Using EP the temporal offset effect of identically accelerated clocks is interpreted as the behavior of clocks in a uniform gravitational field. We follow an approach where such temporal offset effect is used in accounting for the asymmetrical aging of the standard twin paradox from the perspective of the traveller twin. In this chapter we have shown that the time offset effect and gravitational slowing down of clocks (GSDC) can be connected provided the world as purely relativistic in nature. To contrast this we have used Zahar transformations (following pseudo-standard synchrony in the classical world) and concluded that GSDC cannot be obtained from temporal offset effect in this world through EP. Thus in the relativistic world the temporal offset may be regarded as an integrated effect of GSDC while in the classical world (if it exists) is just an artifact of Einstein's synchrony. The experimental test of GSDC or the gravitational red shift is a test of a differential time offset effect in a way. It is worthwhile to note that the empirical verification of time offset provides empirical support for the relativity of simultaneity. Clearly because of EP the earth with its weak gravity has the ability to provide a convenient laboratory to test some special relativistic effects like relativity of simultaneity. Our approach in addition removes certain drawbacks of an earlier effort which claims to exploit EP in explaining the differential aging in the paradox.

1.4.2 Twin Paradox: A Classic Case of ‘Like Cures Like’

In chapter IV, a novel approach to understanding the ordinary twin paradox based on a variation of the paradox (called the Boughn paradox), concerning the differential aging of two identically accelerated twins is presented. This time this is done without any reference to GR or EP. The genesis of the usual twin paradox lies in the incorrect use of the relativistic time-dilation formula by the traveller twin. The current approach explains how to take into account the Boughn paradox which provides automatically a uniform standard of simultaneity of the coordinate clocks of the traveller twin’s *non-inertial* frame of reference so that the special relativistic time dilation formula can be used correctly in order to calculate the proper time of the stay-at-home sibling. There is no dearth of explanations of the twin paradox in the literature in the context of special relativity, but the present effort is a unique one where one paradox (the Boughn paradox) is used to explain another (the ordinary twin paradox), and hence may be looked upon in a lighter vein, as to justify the proverb “*like cures like*”.

1.4.3 Boughn Effect and Some Twin Paradoxes

A time offset effect between two identically accelerated twins is used to pose and resolve some interesting variations of the twin paradox in the chapter V. These novel paradoxes stem from the authors’ attempt to isolate and expose the role of the aforesaid time offset effect from that of the time dilation effect of special relativity in the usual twin paradox. The resolution of these paradoxes provides some additional insight into the century old counter-intuitive problem. The present treatment once more makes it evident why any reference to general relativity in the context of the ordinary twin paradox is redundant.

1.4.4 Relativity in “Cosmic Substratum” and the UHECR Paradox

The second part of the thesis which deals with some observational puzzles in SR, starts with chapter VI. The special theory of relativity predicts an existence of the so called Greisen-Zatsepin-Kuzmin (GZK) phenomenon according to which cosmic ray protons coming from cosmological distances with energies above $5 \times 10^{19} eV$ should not be observed on earth. The cut-off value corresponds to the threshold energy of photo-pion production by protons colliding with soft CMBR photons pervading the universe. Experimentally a number of cosmic ray events have been detected above this GZK limit which is known as the ultra high energy cosmic rays (UHECR) paradox. We suggest a resolution of this paradox through a heuristic modification of the relativistic kinematics keeping in mind that it should not lead to predictions different from those of SR in the well tested domains. It is shown that the absence of GZK limit in UHECR spectrum can be explained in terms of a non-preferred frame effect of the solar system. It is remarked that the novel theory can also be called a “doubly special relativistic”(DSR) one but now in a sense different from that of the currently known DSR theories.

1.4.5 A Simple Minded VSL Cosmology

The concluding chapter (chapter VII) concerns VSL cosmology. The deformed relativistic kinematics developed in connection with our attempt to resolve the ultra high energy cosmic rays paradox has been found to go hand in hand with a variable speed of light cosmology. Some recent observational claims concerning the cosmological evolution of the fine structure constant α , substantiates our proposal. The spectroscopic data obtained from the detailed study of absorption lines from

heavy elements in distant gas clouds along the line of sights of background quasars allow us to predict values for some parameters of the theory. A range of these values are tabulated as there is a considerable spread in the values for the observed variations of α for a given red-shift range. The present proposal although looks like a phenomenological one, has the ability to qualify itself as a “principle theory” like SR. The simple minded VSL theory is discussed in the context of theories of Albrecht and Magueijo, Barrow and Moffat and that of DSR ones.

1.5 List of Papers (Published/under Publication)

- (1) **The Principle of Equivalence and the Twin Paradox**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, *Found. Phys. Lett.* **18**(7), 603-619 (2005).
- (2) **A Twin Paradox for ‘Clever’ Students**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, *arXiv:gr-qc/0511126v1*, (2005).
- (3) **Relativity in ‘Cosmic Substratum’ and the UHECR Paradox**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,– *Proc. of Physical Interpretation of Relativity Theory*, held at Moscow, July 2-5, 2007.
- (4) **Twin Paradox: A Classic Case of ‘Like Cures Like’**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, appear in “Mathematics, Physics and Philosophy in the interpretations of Relativity theory” held in Budapest, September 7–9, 2007. [WWW.phil-inst.hu/~szekely/PIRT_BUDAPEST/ft/Ghosal-ft.pdf]
- (5) **Boughn Effect and Some Twin Paradoxes**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,(under publication).
- (6) **Twin Paradox in the Classical World**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das, (ready for communication).
- (7) **Simple Minded VSL Cosmology**– S. K. Ghosal, **Saroj Nepal** and Debarchana Das,(ready for communication).
- (8) *“Synchrony Gauge in Classical and Relativistic Sagnac Effect and Related Issues”*– S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarkar and **Saroj Nepal**, *Proc of International Conference on Gravitation and Cosmology (ICGS-04)*, held at Kochi, 5–10 January, 2004 (Abstracted).
- (9) *“Paradoxical Twins in Classical and Relativistic Worlds”*–S. K. Ghosal, **Saroj Nepal**, Debarchana Das, Biplab Raychaudhuri and Minakshi Sarkar, *Proc. of Seminar on Hundred Years of Three Seminal Papers of Albert Einstein and Con-*

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