Chapter-II

FREE CONVECTIVE FLOW AND MASS TRANSFER

Part one

UNSTEADY FREE CONVECTIVE MHD FLUID FLOW WITH MASS TRANSFER IN POROUS MEDIUM

PART ONE ➢ A

MASS TRANSFER AND FREE CONVECTIVE MHD FLOW THROUGH POROUS MEDIUM*

2.1.1 INTRODUCTION

The phenomenon of natural convective flow is not often caused entirely by the effect of temperature gradient but also by differences in concentration of dissimilar chemical species for example, in atmospheric flows there exists differences in H_2O concentration and the flow is affected by such concentration differences. Also, in a number of engineering applications, the foreign mass are injected and due to such mass transfer it has been observed that there is reduction in the wall shear stress, the mass transfer conductance or the rate of heat transfer. In such cases time dependent injection or suction velocity plays an important role. The significance of suction or injection for the boundary layer control in the field of aerodynamics and space science is well recognized. On the other hand, flows through porous medium are very much prevalent in nature and therefore, the study of flows through porous medium has become of principal interest in many scientific and engineering applications [1,2,3]. In recent years the subject of magneto-fluid dynamics has attracted many authors in view not only of its own interest but also of the applications to geophysics and engineering. When the fluid is a conductor of electricity, the free-convection and mass transfer can be influenced by an imposed magnetic field. MFD phenomena result from the mutual effect of a magnetic field and a conductivity fluid across it. Thus an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid's motion. Examination of flow models will reveal the influence of magnetic field on the velocity profile, temperature profile. Raptis and Kafousis [4] studied heat transfer in flow through a porous medium bounded by an infinite

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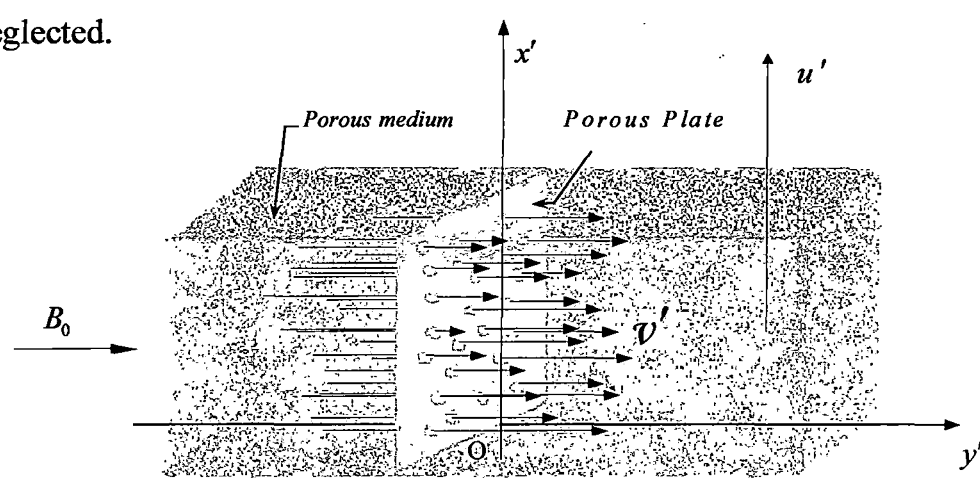
vertical plate under action of a magnetic field. Raptis and Perdikis [5] discussed magnetic effects on the flow with a great magnetic Reynolds' number by the presence of free convection and mass transfer. Later Chauhan and Jain [6] presented three dimensional MHD flow and heat transfer in the presence of a naturally permeable boundary of very small permeability. Recently Rajput *et al.* [7] studied free convection MHD flow of a stratified fluid past an oscillating porous plate with mass transfer. Hence it is of interest to make an investigation in order to analyze the effect of suction/ injection on free convective flow with mass transfer of an electrically conducting viscous fluid past an accelerated vertical infinite porous plate in a porous medium. The suction or injection velocity is taken to be time-dependent of the form $\alpha(\nu/t')^{1/2}$. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases.

2.1.2 MATHEMATICAL ANALYSIS

We consider a two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical accelerated porous plate embedded in porous medium. A magnetic field of uniform strength is applied transversely to the direction of the flow. The magnetic Reynolds' number of the flow is taken to be small enough so that the induced magnetic field can be neglected.



2.1.1 Sketch of the physical problems

The fluid is assumed to have constant properties except that the influence of thedensity variations with temperature and concentration is considered only in the body force term. At time $t'\leq 0$, the plate and the fluid are at the same temperature in a stationary condition but at time $t'>0$, the plate starts moving with velocity $U'(t')$ in its own plane and the plate temperature and concentration level is also raised to $T_w'(\neq T_\infty')$ and C_w' $(\neq C_\infty')$. In order to formulate the problem mathematically, we write down the equation of fluid motion, through a porous medium in Cartesian co-ordinates, with x'-axis along the vertical porous wall in the upward direction and y'-axis normal to it.

Under above assumptions, the physical variables except pressure P are function of y' only. Following usual Boussineseq approximation, the unsteady free convective and mass transfer flow in an electrically conducting fluid, is governed by the following equations.

$$\frac{\partial v'}{\partial y'}=0, \qquad \text{.... (2.1.1)}$$

$$\rho'\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial P'}{\partial x'}-\rho' g+\mu\frac{\partial^2 u'}{\partial y'^2}-\frac{\mu}{K'}u'-\sigma B_0^2 u', \qquad \text{.... (2.1.2)}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{\kappa}{\rho' C_p}\frac{\partial^2 T'}{\partial y'^2}, \qquad \text{.... (2.1.3)}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}. \qquad \text{.... (2.1.4)}$$

The corresponding boundary conditions are

$$\left.\begin{array}{lllll} y'=0, & u'=U'(t'), & T'=T_w', & C'=C_w' & \\ y'\to\infty, & u'\to 0, & T'\to T_\infty', & C'\to C_\infty' & t'>0 \end{array}\right\}. \qquad \text{.... (2.1.5)}$$

From (2.1.2) we have for free stream

$$0=-\frac{\partial P'}{\partial x'}-\rho'_{\infty}g. \qquad \text{.... (2.1.6)}$$

Eliminating $-\frac{\partial P'}{\partial x'}$ between (2.1.2) and (2.1.6),

$$\rho'\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=g\left(\rho'_{\infty}-\rho'\right)+\mu\frac{\partial^2 u'}{\partial y'^2}-\frac{\mu}{k'}u'-\sigma B_0^2 u', \qquad \text{.... (2.1.7)}$$

where ρ'_{∞} is the density of the flow in the free stream.

The equation of state is

$$g\left(\rho'_{\infty}-\rho'\right)=g\beta\rho'\left(T'-T'_{\infty}\right)+g\beta^*\left(C'-C'_{\infty}\right). \qquad \text{.... (2.1.8)}$$

Substituting (2.1.8) into (2.1.7), we obtain

$$\frac{\partial u'}{\partial t}+v'\frac{\partial u'}{\partial y'}=g\beta\left(T'-T'_{\infty}\right)+g\beta^*\left(C'-C'_{\infty}\right)+\nu\frac{\partial^2 u'}{\partial y'^2}-\left(\frac{\nu}{k'}+\frac{\sigma B_0^2}{\rho'}\right)u'. \qquad \text{..(2.1.9)}$$

In equation (2.1.3) the heat due to viscous dissipation is neglected, being very small in comparison with the conducting term. This is a valid assumption because of the small velocities usually encountered in free convection flows. In the same equation Joule heating term is also neglected because it is of the same order of magnitude with the viscous dissipation term.

From equation (2.1.1)

$$v'=-\alpha\left(\frac{\nu}{t'}\right)^{\frac{1}{2}}, \qquad \text{.... (2.1.10)}$$

where α represents the velocity of suction $(\alpha>0)$ or injection $(\alpha<0)$ at the plate. By introducing the following non-dimensional quantities

$$y=\frac{U_0'y'}{\nu},\ t=\frac{U_0'^2t'}{\nu},\ u=\frac{u'}{U_0'},\ U=\frac{U'}{U_0'},\ \theta=\frac{(T'-T_\infty')}{(T_w'-T_\infty')},\ C=\frac{(C'-C_\infty')}{(C_w'-C_\infty')},$$

$$Gr=\frac{g\beta\nu(T_w'-T_\infty')}{U_0'^3},\ Gm=\frac{g\beta^*\nu(C_w'-C_\infty')}{U_0'^3},\ \Pr=\frac{\nu\rho'C_p}{K'},\ Sc=\frac{\nu}{D},$$

$$K=\frac{K'U_0'^2}{\nu^2},\ M=\frac{\nu\sigma B_0^2}{\rho'U_0'^2},$$

where U_0' is a constant with dimension of velocity.

In equation (2.1.2)-(2.1.4) and taking into account (2.1.10) we get

$$\frac{\partial u}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-Lu, \qquad \text{.... (2.1.11)}$$

$$\Pr\left(\frac{\partial \theta}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial \theta}{\partial y}\right)=\frac{\partial^2 \theta}{\partial y^2}, \qquad \text{.... (2.1.12)}$$

$$Sc\left(\frac{\partial C}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial C}{\partial y}\right)=\frac{\partial^2 C}{\partial y^2}, \qquad \text{.... (2.1.13)}$$

where $L=M+\frac{1}{K}$.

The boundary conditions are as follows:

$$\left.\begin{array}{l} y=0,\ u=U(t),\ \theta=1,\ C=1 \\ y\to\infty,\ u\to 0,\ \theta\to 0,\ C\to 0 \end{array}\right\}. \qquad \text{.... (2.1.14)}$$

Assuming the parameter (L) to be small, we expand the non-dimensional velocity u as follows:

$$u(y,t)=u_0(y,t)+Lu_1(y,t)+O(L^2). \quad .... (2.1.15)$$

By substituting (2.1.15) into (2.1.11), (2.1.12) and (2.1.13) and equating the coefficients of the same powers of L, we get

$$\frac{\partial u_0}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_0}{\partial y^2}+Gr\theta+GmC, \quad .... (2.1.16)$$

$$\frac{\partial u_1}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial u_1}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}-u_0, \quad .... (2.1.17)$$

$$\Pr\left(\frac{\partial \theta}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial \theta}{\partial y}\right)=\frac{\partial^2 \theta}{\partial y^2}, \quad .... (2.1.18)$$

$$Sc\left(\frac{\partial C}{\partial t}-\alpha t^{-\frac{1}{2}}\frac{\partial C}{\partial y}\right)=\frac{\partial^2 C}{\partial y^2}. \quad .... (2.1.19)$$

The corresponding boundary conditions now become,

$$\left.\begin{array}{lllll} y=0, & u_0=U(t), & u_1=0, & \theta=1, & C=1 \\ y\to\infty, & u_0\to 0, & u_1\to 0, & \theta\to 0, & C\to 0 \end{array}\right\}. \quad .... (2.1.20)$$

Introducing the new variable

$$\eta=\frac{1}{2}yt^{-\frac{1}{2}} \quad .... (2.1.21)$$

and assuming the solutions of equations (2.1.16) and (2.1.17) is of the form

$$u_0=tf_0(\eta), \qquad u_1=t^2f_1(\eta) \quad .... (.1.22)$$

equations (2.1.16)-(2.1.19) are reduced to

$$f_0''(\eta)+2(\eta+\alpha)f_0'(\eta)-4f_0(\eta)=-4Gr\theta-4GmC, \quad .... (2.1.23)$$

$$f_1''(\eta)+2(\eta+\alpha)f_1'(\eta)-8f_1(\eta)=-4f_0(\eta), \quad \text{.... (2.1.24)}$$

$$\theta''(\eta)+2(\eta+\alpha)\theta'=0, \quad \text{.... (2.1.25)}$$

$$C''(\eta)+2(\eta+\alpha)C'=0, \quad \text{.... (2.1.26)}$$

where prime denotes differentiation with respect to η and for simplicity the Schmidt and Prandtl numbers have been taken to unity.

The boundary conditions (2.1.20) for a uniformly accelerated plate $U(t)=t$ become

$$\left.\begin{array}{ll} f_0(0)=1, & f_0(\infty)\to 0 \\ f_1(0)=0, & f_1(\infty)\to 0 \\ \theta(0)=1, & \theta(\infty)\to 0 \\ C(0)=1, & C(\infty)\to 0 \end{array}\right\}. \quad \text{.... (2.1.27)}$$

The solutions of equations (2.1.23) and (2.1.24) under the boundary conditions (2.1.27) are given as

$$u(\eta,t)=tf_0(\eta)+Lt^2 f_1(\eta), \quad \text{.... (2.1.28)}$$

where,

$$f_0(\eta)=\left[1-(Gr+Gm)\right]\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_0\left(\sqrt{2}\alpha\right)}+(Gr+Gm)\frac{Hh_0\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_0\left(\sqrt{2}\alpha\right)},$$

$$f_1(\eta)=\left[Gr+Gm-1\right]\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)}-\frac{(Gr+Gm)}{2}\frac{Hh_0\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_0\left(\sqrt{2}\alpha\right)}$$

$$+\left[1-\frac{(Gr+Gm)}{2}\right]\frac{Hh_4\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_4\left(\sqrt{2}\alpha\right)}.$$

The solution of equations (2.1.25) and (2.1.26) under the boundary condition (2.1.27) are

$$\theta = \frac{Hh_0\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_0\left(\sqrt{2}\alpha\right)}, \qquad \text{.... (2.1.29)}$$

$$C = \frac{Hh_0\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_0\left(\sqrt{2}\alpha\right)}, \qquad \text{.... (2.1.30)}$$

where the functions $Hh_n(n = 0, \pm 1, \pm 2, ...)$ are defined in Jefferys and Jefferys [8].

Finally the expression for the non-dimensional skin friction τ is given by

$$\tau = \frac{1}{2}t^{\frac{1}{2}}\left[f_0'(0) + Ltf_1'(0)\right], \qquad \text{.... (2.1.31)}$$

where

$$f_0'(0) = -\sqrt{2}\left[\left[1-(Gr+Gm)\right]\frac{Hh_1\left(\sqrt{2}\alpha\right)}{Hh_2\left(\sqrt{2}\alpha\right)} + (Gr+Gm)\frac{Hh_{-1}\left(\sqrt{2}\alpha\right)}{Hh_0\left(\sqrt{2}\alpha\right)}\right],$$

$$f_1'(0) = -\sqrt{2}\left[\left[Gr+Gm-1\right]\frac{Hh_1\left(\sqrt{2}\alpha\right)}{Hh_2\left(\sqrt{2}\alpha\right)} - \frac{(Gr+Gm)}{2}\frac{Hh_{-1}\left(\sqrt{2}\alpha\right)}{Hh_0\left(\sqrt{2}\alpha\right)}\right]$$

$$+\left[2 - \frac{(Gr+Gm)}{2}\frac{Hh_3\left(\sqrt{2}\alpha\right)}{Hh_4\left(\sqrt{2}\alpha\right)}\right].$$

2.1.3 DISCUSSION

In order to study the effects of free convection currents, mass transfer, magnetic field and suction or injection on velocity field numerical calculations are carried out for different values of *Gr*, *Gm*, *M* and α. As usual *Gr*>0 represents an externally cooled plate and $Gr<0$ corresponds to an externally heated plate. The velocity profiles for different values of Gr, Gm, α, M and K are shown in Fig. 2.1.1, Fig. 2.1.2, Fig. 2.1.3 and Fig. 2.1.4. From these figures it is observed that velocity increases as Gm increases for $\alpha > 0$ and $\alpha < 0$. In case of suction, velocity decreases steadily but in case of injection there is an increase in velocity near the plate and then decreases for $Gr>0$. For $Gr<0$, velocity decreases steadily for both suction and injection, but in case of injection, there is an increase in velocity near the plate and then decreases for $Gm=5$. Fig.2.1.1 and Fig. 2.1.2 show that the velocity decreases as M increases for $\alpha > 0$ and $\alpha < 0$. From Fig. 2.1.3 and Fig. 2.1.4, it is observed that velocity increases as K increases for both $\alpha > 0$ and $\alpha < 0$. In case of $Gr>0$, velocity at the plate wall is same for suction and then decreases steadily, but for injection, velocity increases near the plate and then decreases.

The numerical values of skin friction (τ) are given in Table 2.1.1. and Table 2.1.2 to study the effect of permeability of the medium and magnetic parameter for

Table: - 2.1.1

Value of skin friction (τ) when K=2, t=0.2

Gr	*M*	α / *Gm*	-0.5	0.5
2	0	3	0.89088	0.46880
	0.3	3	0.86637	0.44908
	0	5	1.4118	0.91777
	0.3	5	1.3815	0.89402
-2	0	3	- 0.15095	-0.42914
	0.3	3	- 0.16400	-0.44080
	0	5	0.36996	0.01982
	0.3	5	0.35118	0.00413

different values of *Gr*, *Gm* and α .Skin friction (τ) increases as *K* and *Gm* increases for all *Gr*. Skin friction (τ) decreases *as M* increases for *Gr*<0 and *Gr*>0, but increases as *Gm* increases. We have also studied the behavior of temperature and concentration distribution for different values of α (Pr=1, *Sc*=1) in Fig. 2.1.5 and Fig. 2.1.6 respectively. For suction, both temperature and concentration increases as α increases but in case of injection it exhibits opposite characteristic.

Table: - 2.1.2

Value of skin friction (τ) when M=0.5, t=.2

Gr	*K*	*Gm* \ α	-0.5	0.5
2	0.5	3	0.72748	0.33736
	2	3	0.85003	0.43539
	0.5	5	1.21020	0.75947
	2	5	1.36140	0.87819
-2	0.5	3	- 0.23794	-0.50685
	2	3	- 0.17270	-0.44857
	0.5	5	0.24476	0.08474
	2	5	0.33866	0.00631

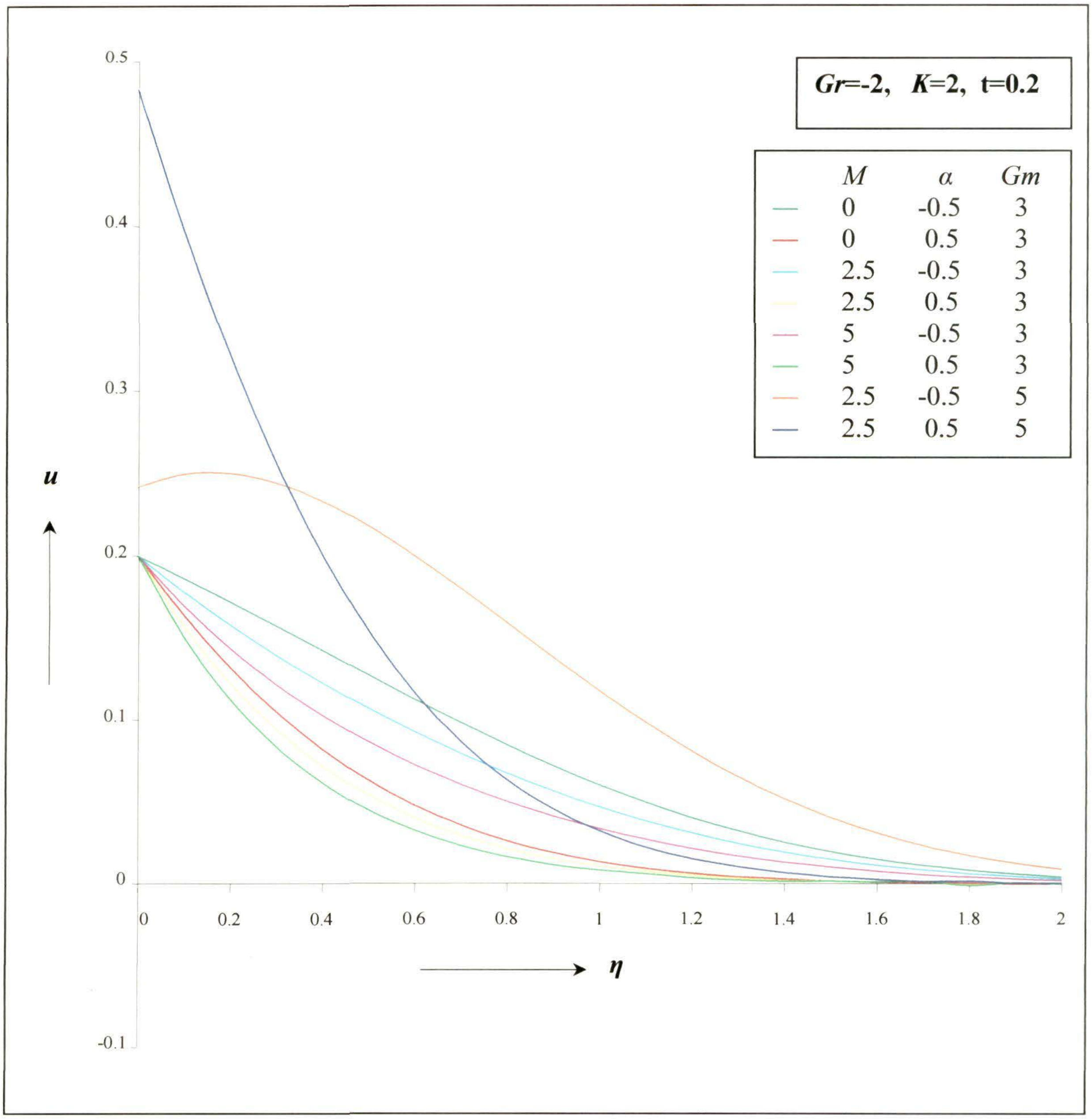


***Fig.* 2.1.1 Graph of velocity against η for different values of M, α and Gm (Gr=-2, K=2, t=0.2).**

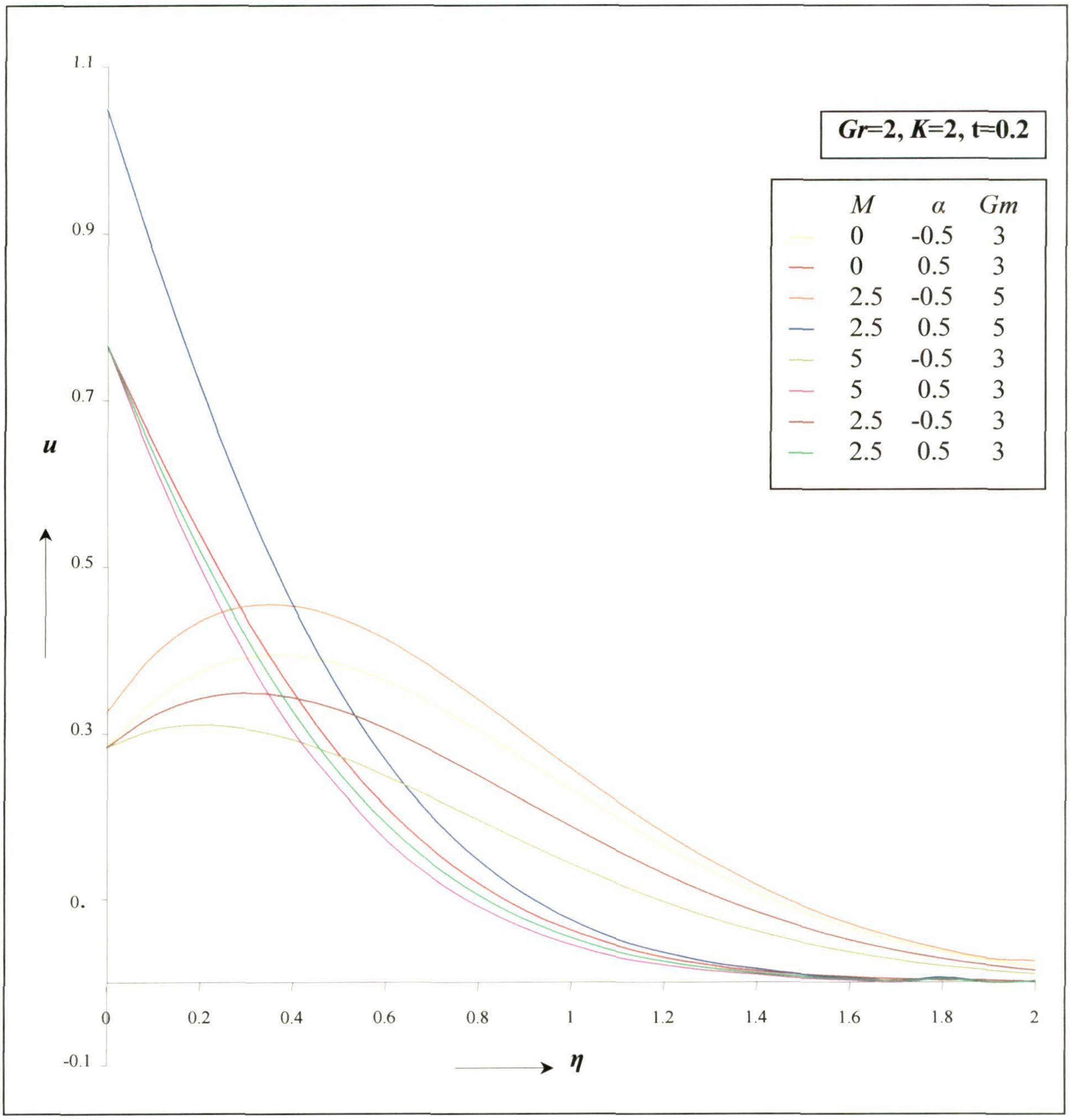


***Fig.* 2.1.2 Graph of velocity against η for different values of M, α and Gm (Gr=2, K=2, t=0.2).**

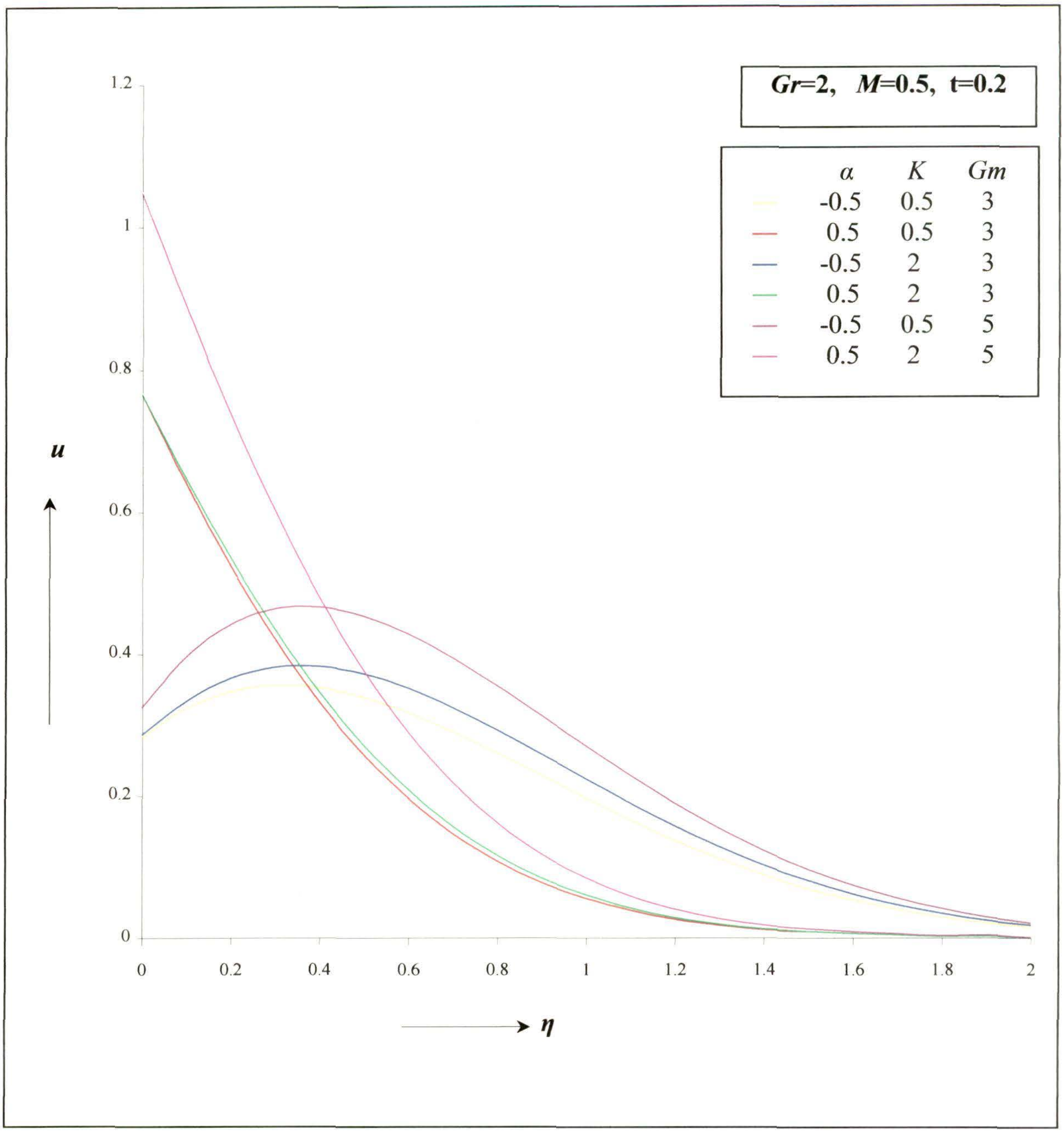


Fig. 2.1.3 **Graph of velocity against η for different values of K, α and Gm (Gr=2, M=0.5, t=0.2).**

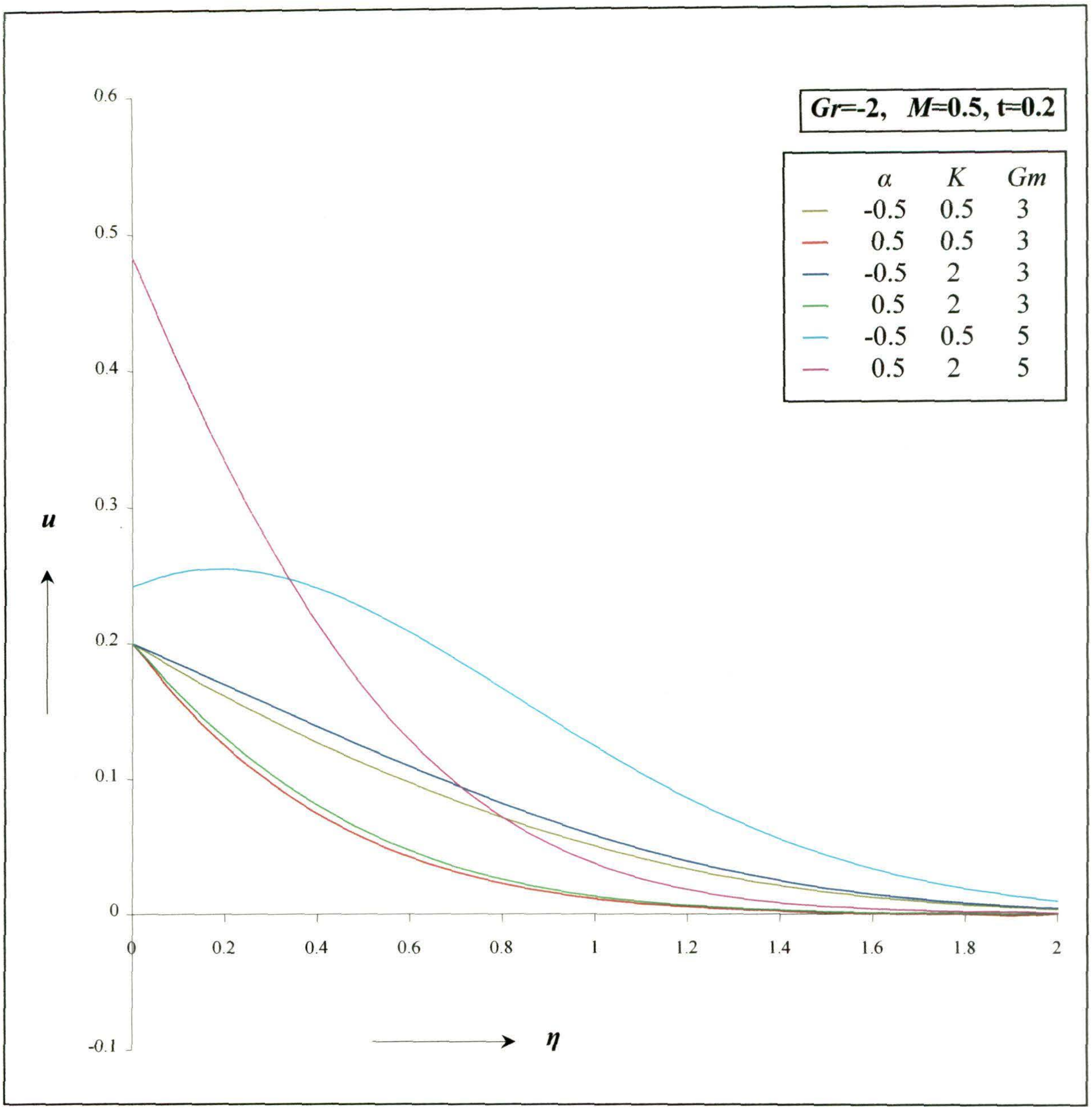


***Fig.* 2.1.4 Graph of velocity against *η* for different values of *K*, *α* and *Gm* (*Gr*=-2, *M*=0.5, t=0.2).**

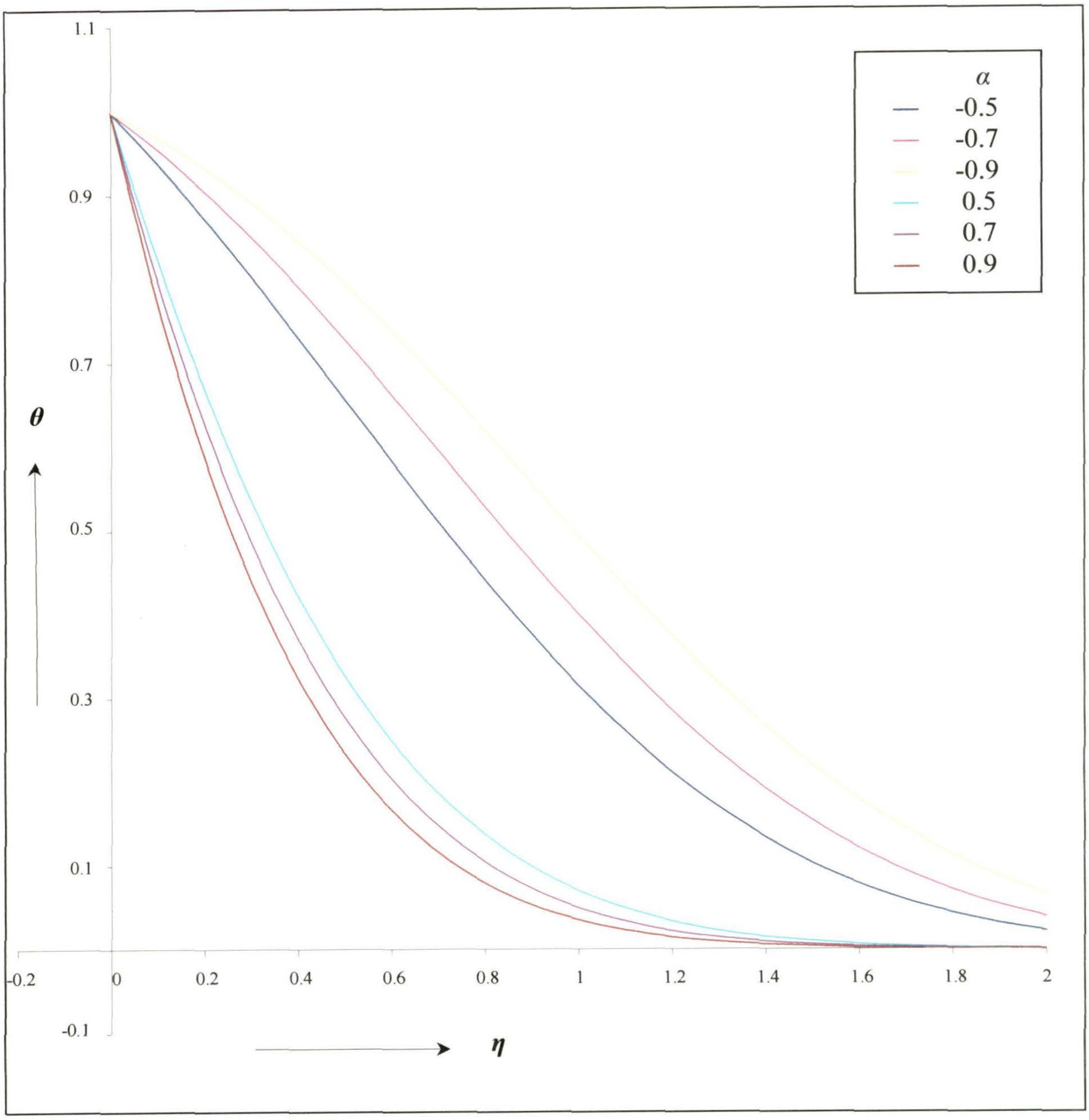


Fig. **2.1.5 Graph of temperature against η for different values of α.**

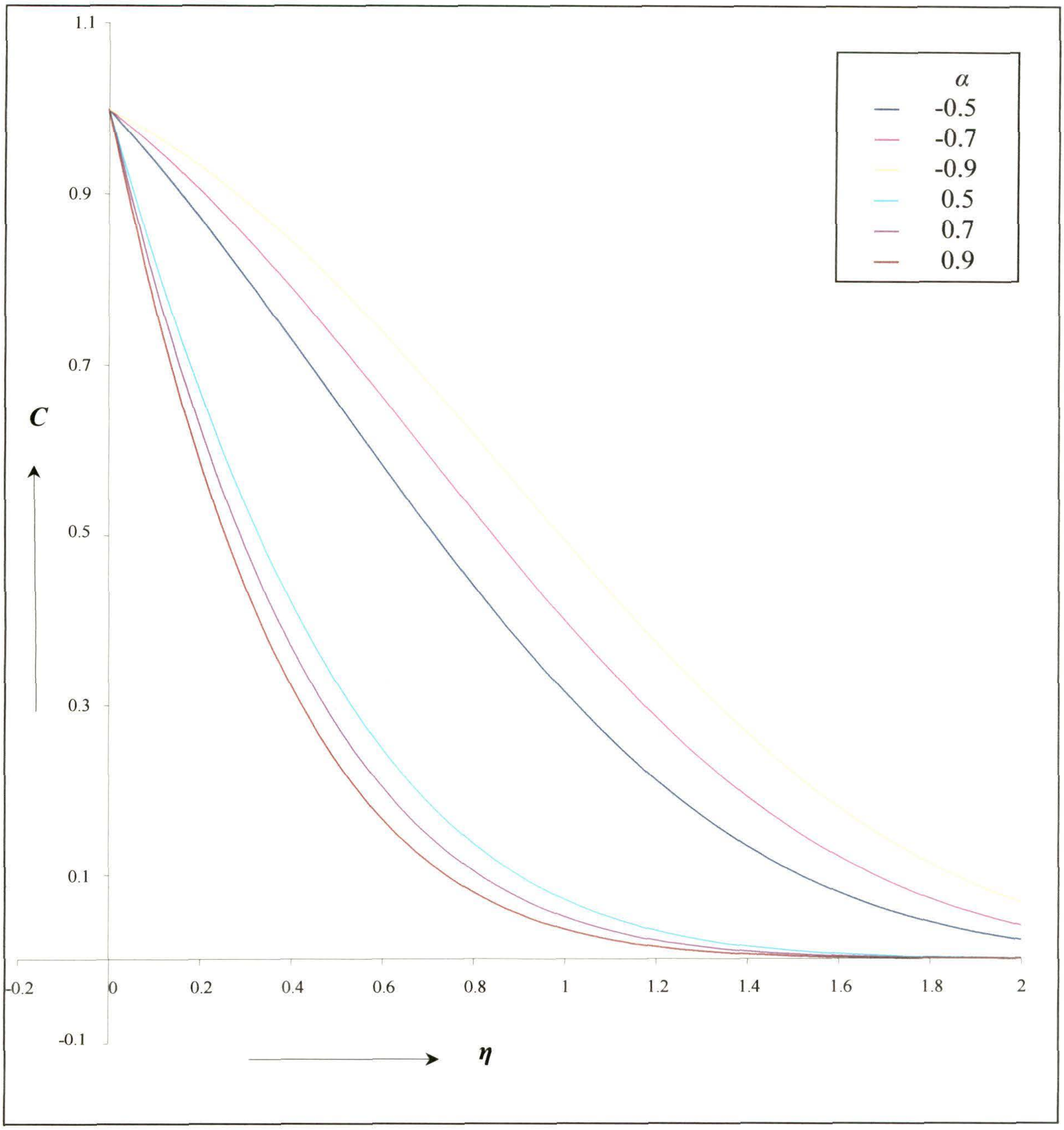


***Fig.* 2.1.6 Graph of concentration against η for different values of α.**

PART ONE ➢ B

HEAT AND MASS TRANSFER TO UNSTEADY FLOW OF MHD FLUID THROUGH A POROUS MEDIUM BOUNDED BY AN INFINITE VERTICAL HOT POROUS PLATE WITH CONSTANT SUCTION IN PRESENCE OF HEAT SOURCE

2.1.4 INTRODUCTION

Convective heat transfer in a porous medium has been the subject of intensive study for the last few decades owing to its application in different field such as chemical engineering, geothermal, petroleum and reservoir engineering, environmental protection, thermal insulation, cooling and processing of food etc. Yamamoto and Iwamura [9] investigated the flow with convective acceleration through a porous medium. Moreover, in nature, along with the free convection current caused by the temperature differences, the flow is also affected by chemical composition differences and gradients. The flow caused by density difference, which in turn caused by concentration differences is known as mass transfer flow. This phenomenon of free convection and mass transfer flow arises in a fluid when temperature and concentration differences cause density variations leading to body forces acting on the fluid's element. There are many interesting aspects of such flow, so in recent years many authors have presented analytical solutions to such problems of flow. Raptis, Tzivamidies and Kafousis [10] studied steady free convective and mass transfer flow through a porous medium bounded by an infinite vertical porous surface with constant suction. Raptis [2] studied unsteady free convective and mass transfer flow of an incompressible viscous fluid through a very porous medium past an infinite vertical porous surface with constant suction. Raptis *et al.* [3] studied the influence of free convection with unsteady flow of viscous fluid through the porous medium when there is a constant heat flux. The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc, in the presence of an electrically conducting fluid subject to a magnetic field. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical infinite plate was studied by Soundalagekar *et al.* [11]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al.* [12].The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate was studied by Soundalgekar *et al.* [13]. Further, the effect of constant heat flux on the flow of an electrically conducting fluid plate oscillating in its plate was studied by Soundalgekar *et al.* [14].Recently, Sriramulu *et al.* [15] studied the effect of applied magnetic field on transient free convection flow of an incompressible viscous fluid by taking into account of viscous dissipative heat along with the heat due to free convection currents in a vertical channel. We now proposed to study the effect of magnetic parameter and heat source on the heat transfer to unsteady flow of MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer. Solution of the equation governing the flow is obtained analytically.

2.1.5 MATHEMATICAL ANALYSIS

We consider heat transfer to unsteady flow of MHD fluid through a porous medium bounded by an infinite vertical hot porous plate in presence of heat source with mass transfer. The x'-axis is taken along the plate in the upward direction and the y'-axis is normal to it. A transverse magnetic field is applied in the direction of

y'-axis. Since the motion is two-dimensional and length of the plate is large therefore all the physical variable are independent of x'. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature and concentration is considered only in the body force term. Under these condition the problem is governed by the following system of equations

$$\frac{\partial v'}{\partial y'} = 0, \qquad .... (2.1.32)$$

$$\rho'\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} - \rho' g + \mu\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{K'}u' - \sigma B_0{}^2 u', \qquad .... (2.1.33)$$

$$\rho'\frac{\partial v'}{\partial t'} = -\frac{\partial p'}{\partial y'} - \frac{\mu}{K'}v', \qquad .... (2.1.34)$$

$$\rho' C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = \kappa\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty), \qquad .... (2.1.35)$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2}. \qquad .... (2.1.36)$$

where u', v' are the velocity components in the x' and y' directions respectively, g is the acceleration due to gravity, p' is the pressure, μ the viscosity, K' the permeability of the porous medium, C_p the specific heat of the fluid at constant pressure, k the thermal conductivity, T' the temperature, C' the concentration and D the chemical molecular diffusivity.

The boundary conditions of the problem are

$$\left.\begin{array}{llll} u' = 0, & T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\omega' t'}, & C' = C'_w & \text{at } y' = 0 \\ u' \to U_\infty, & T' \to T'_\infty, & C' \to C'_\infty & \text{as } y' \to \infty \end{array}\right\}. \qquad .... (2.1.37)$$

where T'_w, T'_∞, and C'_w, C'_∞ are the temperature and the species concentration on the porous limiting surface and in the free stream respectively. Also U'_∞ is the free-stream velocity and ε a positive constant (ε<1).

The continuity equation (2.1.32) gives

$$v' = -v_0, \qquad \text{.... (2.1.38)}$$

where v_0 (>0) is the constant suction velocity of the fluid through the porous surface.

For the free stream, equation (2.1.33) gives

$$0 = -\frac{\partial p'}{\partial x'} - \rho'_\infty g - \frac{\mu}{K'}U_\infty - \sigma B_0^2 U_\infty. \qquad \text{.... (2.1.39)}$$

Eliminating $\partial p'/\partial x'$ between (2.1.33) and (2.1.39), we have

$$\rho'\left(\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'}\right) = \rho'_\infty g + \frac{\mu}{K'}U_\infty + \sigma B_0^2 U_\infty$$

$$-\rho' g + \mu\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\mu}{K'} + \sigma B_0^2\right)u'. \qquad \text{.... (2.1.40)}$$

and taking into account the equation of state

$$g(\rho'_\infty - \rho') = g\beta\rho'(T' - T'_\infty) + g\beta^*\rho'(C' - C'_\infty). \qquad \text{.... (2.1.41)}$$

(β is the co-efficient of volume expansion and β^* is the volumetric co-efficient of expansion with concentration)

From (2.1.40) and (2.1.41), we write

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty)$$

$$+\nu\frac{\partial^2 u'}{\partial y'^2} + \left(\frac{\nu}{K'} + \frac{\sigma B_0^2}{\rho'}\right)(U_\infty - u'), \qquad \text{.... (2.1.42)}$$

where ν is the kinematics viscosity.

Again from (2.1.32) and (2.1.33) we can show that $\partial^2 p'/\partial y'^2=0$.

We introduce the following non-dimensional quantities:

$$y=\frac{y'v_0}{\nu},\quad t=\frac{t'v_0^2}{\nu},\quad u=\frac{u'}{U_\infty},\quad C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad \omega=\frac{\nu\omega'}{v_0^2},$$

$$T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ Sc=\frac{\nu}{D}(\text{Schmidt number}),\ P=\frac{\rho'\nu C_p}{\kappa}(\text{Prandtl number}),$$

$$K=\frac{v_0^2K'}{\nu^2}(\text{permeability parameter}),\qquad Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{U_\infty v_0^2},$$

$$M=\frac{\sigma\nu B_0^2}{\rho' v_0^2},\quad S=\frac{S'}{\rho' C_p v_0^2}(\text{heat source parameter}),$$

$$\text{and } Gm=\frac{\nu g\beta^*(C'_w-C'_\infty)}{U_\infty v_0^2}\ (\text{modified Grashoff number}).$$

Substituting the above non-dimensional quantities into equations (2.1.40), (2.1.35) and (2.1.36), we get

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=GrT+GmC+\frac{\partial^2 u}{\partial y^2}+\left(M+\frac{1}{K}\right)(1-u), \qquad \text{.... (2.1.43)}$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{P}\frac{\partial^2 T}{\partial y^2}+ST, \qquad \text{.... (2.1.44)}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \qquad \text{.... (2.1.45)}$$

and the boundary conditions (2.1.37) become

$$\left.\begin{array}{llll} u=0, & T=1+\varepsilon e^{i\omega t}, & C=1 & \text{at } y=0 \\ u\to 1, & T\to 0, & C\to 0 & \text{as } y\to\infty \end{array}\right\}. \qquad \text{.... (2.1.46)}$$

To solve the system of equations (2.1.42)-(2.1.44) under their boundary conditions (2.1.45), we assume that

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots\dots\dots\dots \\ T(y,t) &= T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \dots\dots\dots\dots \\ C(y,t) &= C_0(y) + \varepsilon e^{i\omega t} C_1(y) + \dots\dots\dots\dots \end{aligned}\right\}. \qquad \dots (2.1.47)$$

Substituting (2.1.47) into the system (2.1.42)-(2.1.44) we have

$$u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -GrT_0 - GmC_0 - \left(M + \frac{1}{K}\right) \qquad \dots (2.1.48)$$

and $$u_1'' + u_1' - \left\{\left(M + \frac{1}{K}\right) + i\omega\right\}u_1 = -GrT_1 - GmC_1, \qquad \dots (2.1.49)$$

$$T_0'' + PT_0' + PST_0 = 0, \qquad \dots (2.1.50)$$

$$T_1'' + PT_1' + P(S - i\omega)T_1 = 0, \qquad \dots (2.1.51)$$

$$C_0'' + ScC_0' = 0, \qquad \dots (2.1.52)$$

$$C_1'' + ScC_1' - i\omega ScC_1 = 0, \qquad \dots (2.1.53)$$

and the boundary conditions (2.1.45) now become

$$\left.\begin{aligned} &u_0(0) = 0,\ T_0(0) = 1,\ C_0(0) = 1, \\ &u_1(0) = 0,\ T_1(0) = 1,\ C_1(0) = 0, \\ &u_0(\infty) \to 1,\ T_0(\infty) \to 0,\ C_0(\infty) \to 0, \\ &u_1(\infty) \to 0,\ T_1(\infty) \to 0,\ C_1(\infty) \to 0 \end{aligned}\right\}. \qquad \dots (2.1.54)$$

Thus the solution of the problem is obtained by solving the differential equation (2.1.48)-(2.1.53) as

$$u(y,t) = 1 - L_1 e^{-R_1 y} - L_2 e^{-Scy}$$

$$+ L_3 e^{-R_3 y} + \varepsilon L_4 e^{i\omega t}\left(e^{-R_4 y} - e^{-R_2 y}\right), \qquad \dots (2.1.55)$$

$$T(y,t) = e^{-R_1 y} + \varepsilon e^{i\omega t} e^{-R_2 y}, \qquad \dots (2.1.56)$$

$$C(y) = e^{-Scy}, \quad \text{.... (2.1.57)}$$

where the quantities L_i, i =1,2,3,4 and R_j, j =1,2,3,4 are defined in the Appendix.

We can write the expression for the transient velocity profiles as

$$u(y,t) = u_0(y) + \varepsilon(N_r \cos\omega t - N_i \sin\omega t), \quad \text{.... (2.1.58)}$$

where $N_r + iN_i = u_1(y)$

and for $\omega t = \frac{\pi}{2}$,

$$u\left(y, \pi/2\omega\right) = u_0(y) - \varepsilon N_i. \quad \text{.... (2.1.59)}$$

2.1.6 DISCUSSION

In order to have a physical point of view of the problem, numerical calculations are carried out for different values of *M* (magnetic parameter), *K* (permeability parameter), *S* (heat source), *Gr* (Grashof number), *Gm* (modified Grashof number). The velocity profiles are shown in Fig. 2.1.7- Fig. 2.1.11. From this figures it is seen that velocity decreases as magnetic parameter *M* increases. But reverse character is seen in Fig. 2.1.8 and Fig. 2.1.9. Velocity increases as permeability parameter *K* and source heat parameter *S* increases. In Fig. 2.1.10, Grashof number represents the effects of the free convection currents and the case $Gr>0$ corresponds to an externally cooled plate while the case $Gr<0$corresponds to an externally heated plate. It is also observed from Fig. 2.1.11, velocity increases as modified Grashof number *Gm* increases. Here in all cases, we consider Pr=0.71 which corresponds physically to air while Sc=0.24 is chosen in such a way to represent hydrogen at $25^\circ C$ and 1 atmosphere (approximately). Table 2.1.3 gives the values of velocity for different values of the frequency ω.

Table- 2.1.3.

Variation of the velocity when Pr=0.71, S=2, K=0.5, M=0.6, Gr =5, Gm =2, Sc=0.24, ε=0.2, ωt=π/2

Y	ω=0.5	ω=1	ω=1.5
0	-1.192E-07	-1.192E-07	-1.192E-07
0.25	1.0859	1.0702	1.0572
0.5	1.5742	1.5505	1.5304
0.75	1.7169	1.6868	1.6615
1	1.6708	1.6337	1.6043
1.25	1.5344	1.4898	1.4576
1.5	1.3695	1.3178	1.2846
1.75	1.214	1.1566	1.1246
2	1.0893	1.0283	0.9996

Appendix

$$R_1 = 0.5\left[P + \left(P^2 - 4SP\right)^{\frac{1}{2}}\right]; \qquad R_2 = 0.5\left[P + \left\{P^2 - 4P\left(S - i\omega\right)\right\}^{\frac{1}{2}}\right];$$

$$R_3 = 0.5\left[1 + \left\{1 + 4\left(M + \frac{1}{K}\right)\right\}^{\frac{1}{2}}\right]; \qquad R_4 = 0.5\left[1 + \left\{1 + 4\left(M + \frac{1}{K} + i\omega\right)\right\}^{\frac{1}{2}}\right];$$

$$L_1 = \frac{Gr}{R_1{}^2 - R_1 - \left(M + \frac{1}{K}\right)}; \qquad L_2 = \frac{Gm}{Sc^2 - Sc - \left(M + \frac{1}{K}\right)};$$

$$L_3 = \left(L_1 + L_2 - 1\right); \qquad L_4 = -\frac{Gr}{R_2{}^2 - R_2 - \left(M + \frac{1}{K} + i\omega\right)}.$$

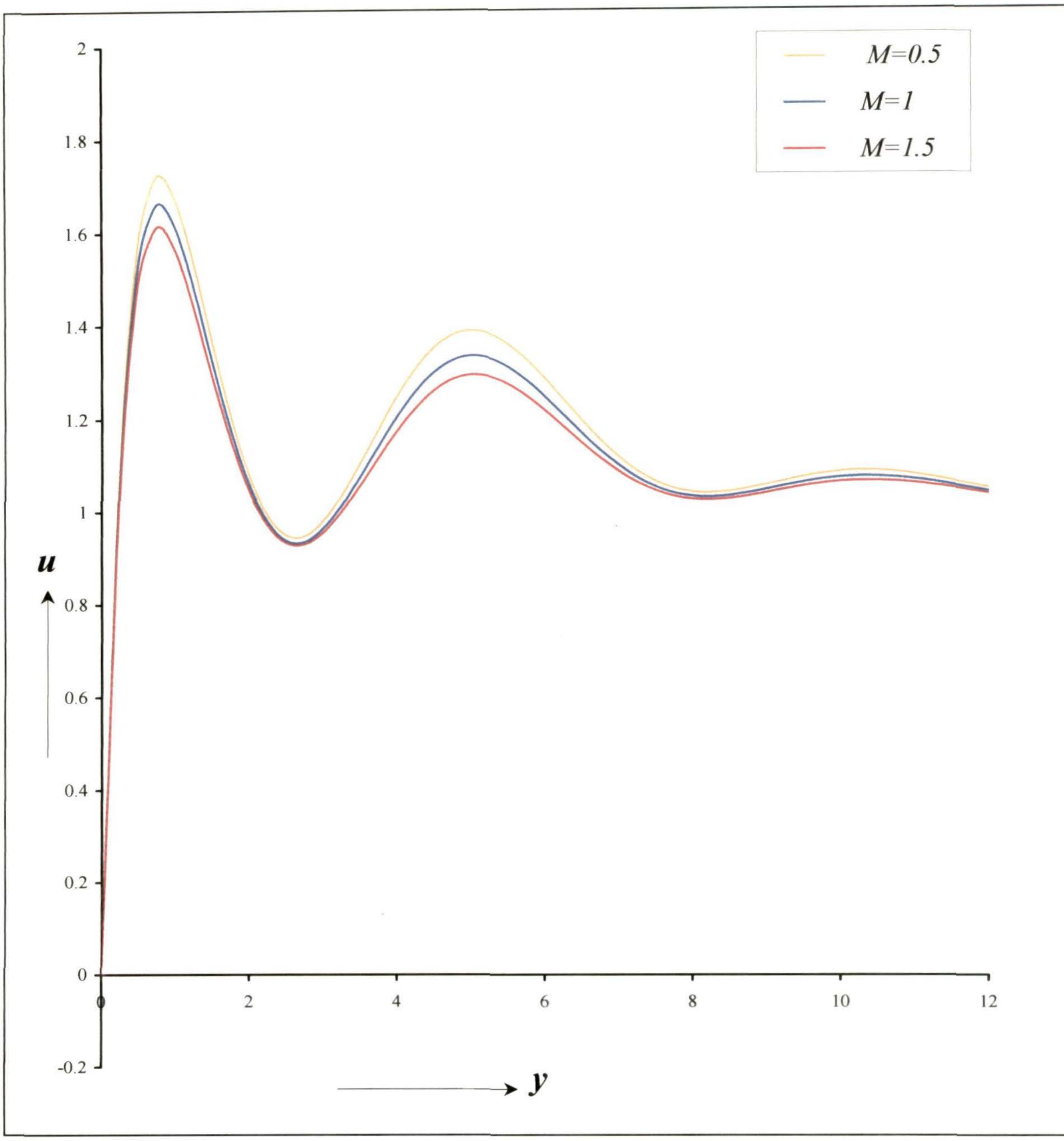


***Fig.* 2.1.7 Velocity profiles for different values of *M*: *Pr*=0.71, S=2, *K*=0.5, *ω*=0.6, *Gr*=5, *Gm*=2, *Sc*=0.24, *ε*=0.2.**

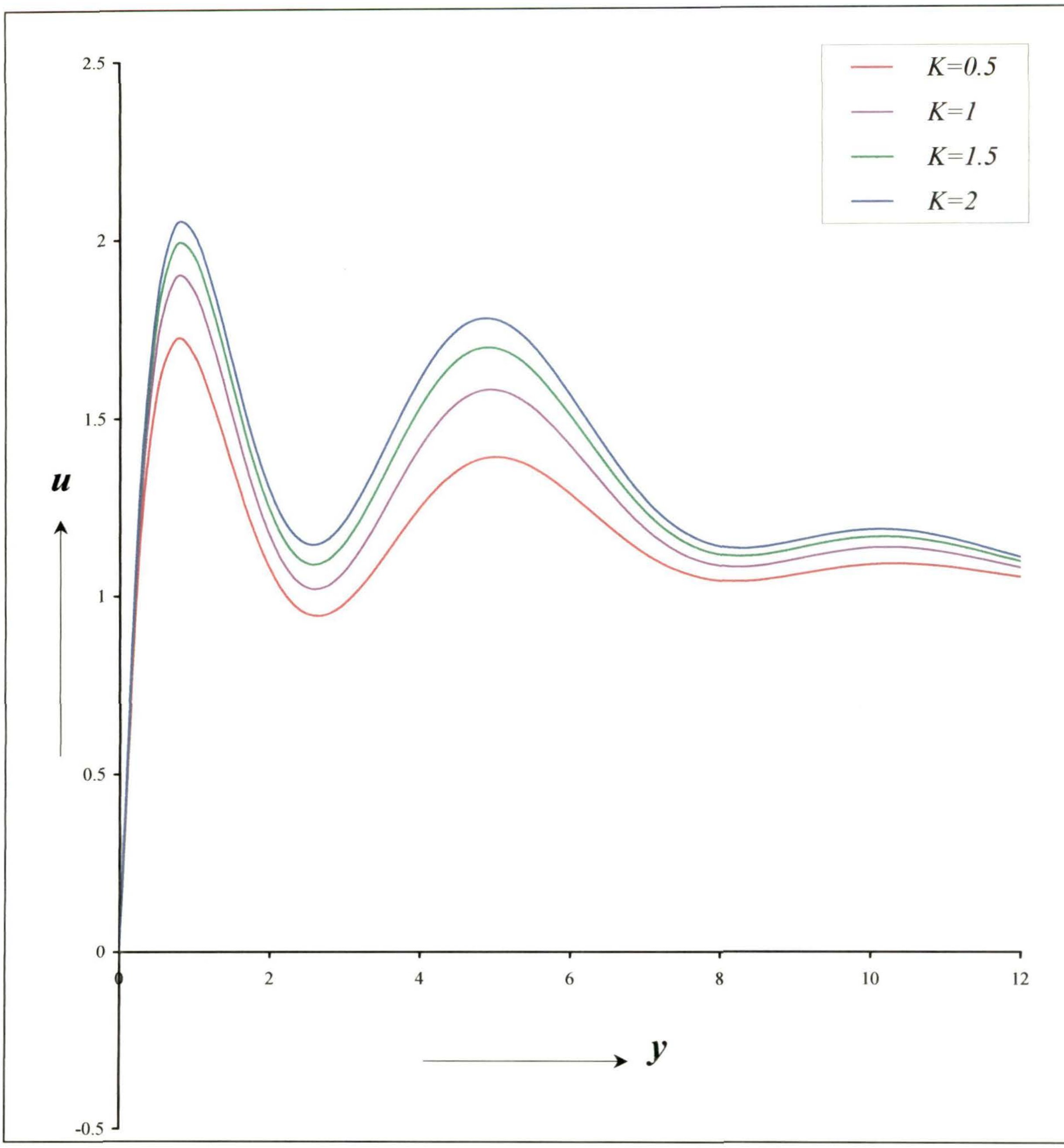


***Fig.* 2.1.8 Velocity profiles for different values of *K*: *Pr*=0.71, *S*=2, *M*=0.5, ω=0.6, *Gr*=5, *Gm*=2, *Sc*=0.24, ε=0.2.**

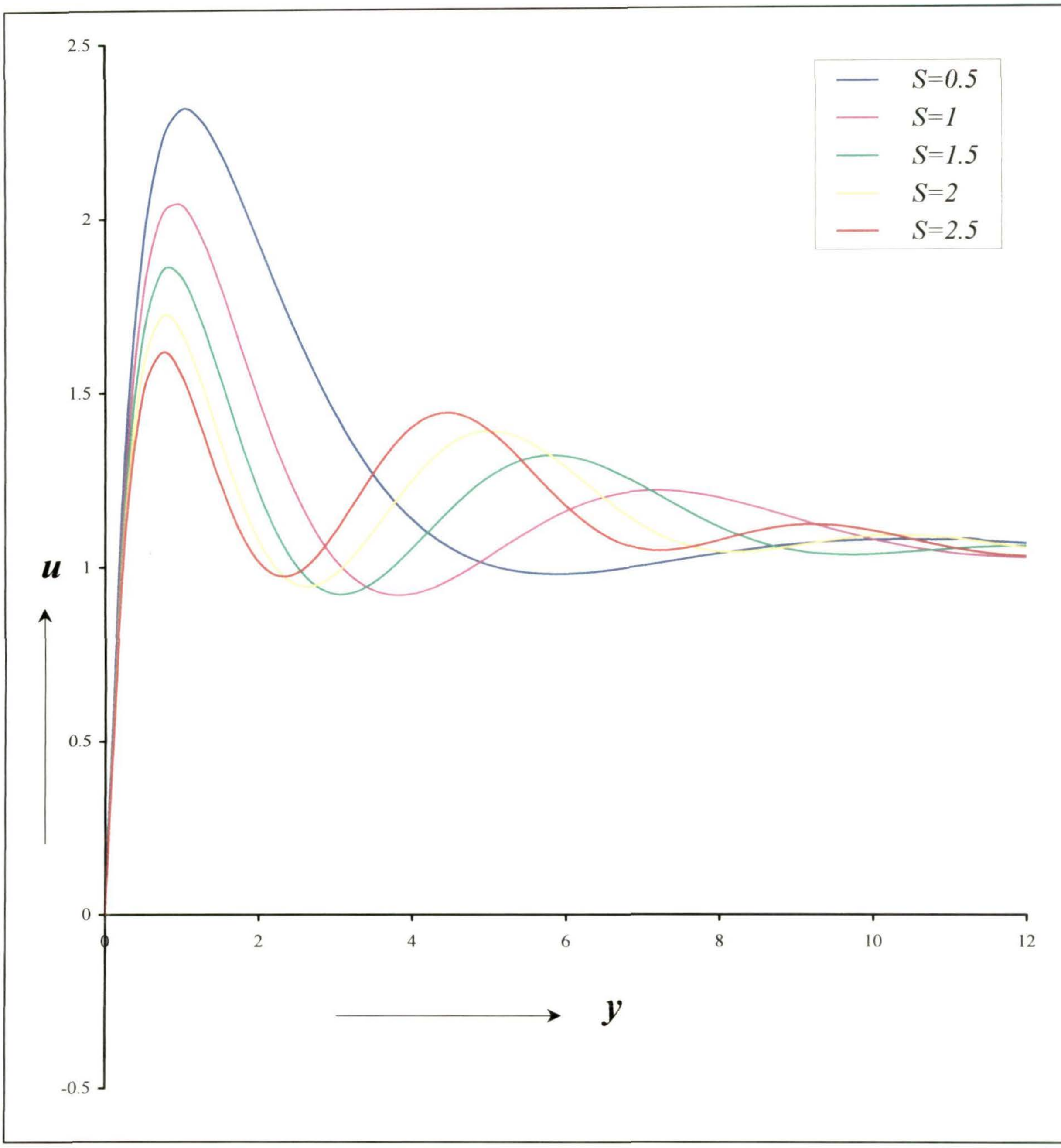


***Fig.* 2.1.9 Velocity profiles for different values of *S*: *Pr*=0.71, *Gr*=5, *M*=0.5, *ω*=0.6, K=0.5, *Gm*=2, *Sc*=0.24, *ε*=0.2.**

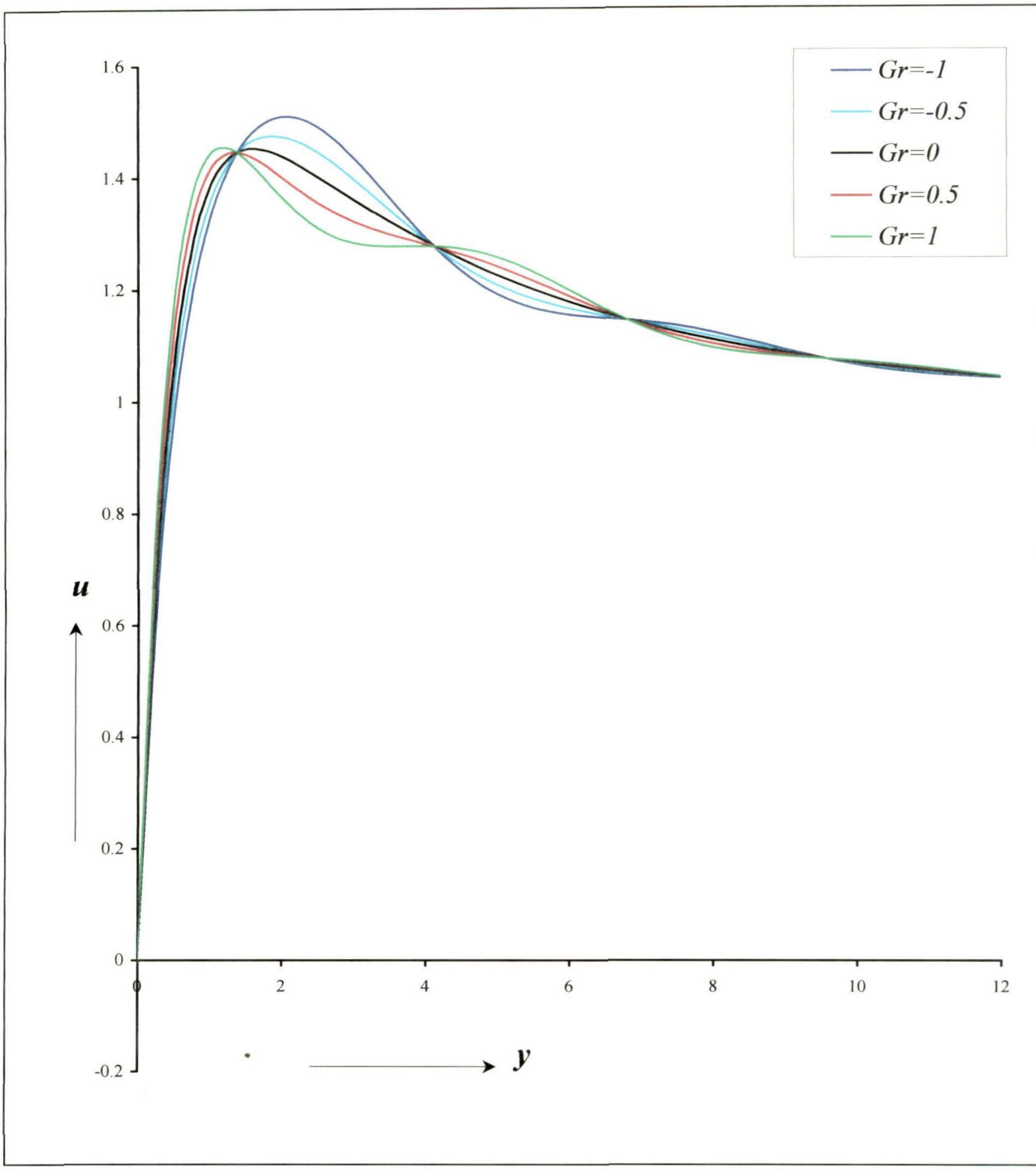


***Fig.* 2.1.10 Velocity profiles for different values of *Gr* : *P*=0.71, *S*=2, *M*=0.5, ω=0.6, *Gm*=2, *K*=0.5, *Sc*=0.24, ε=0.2.**

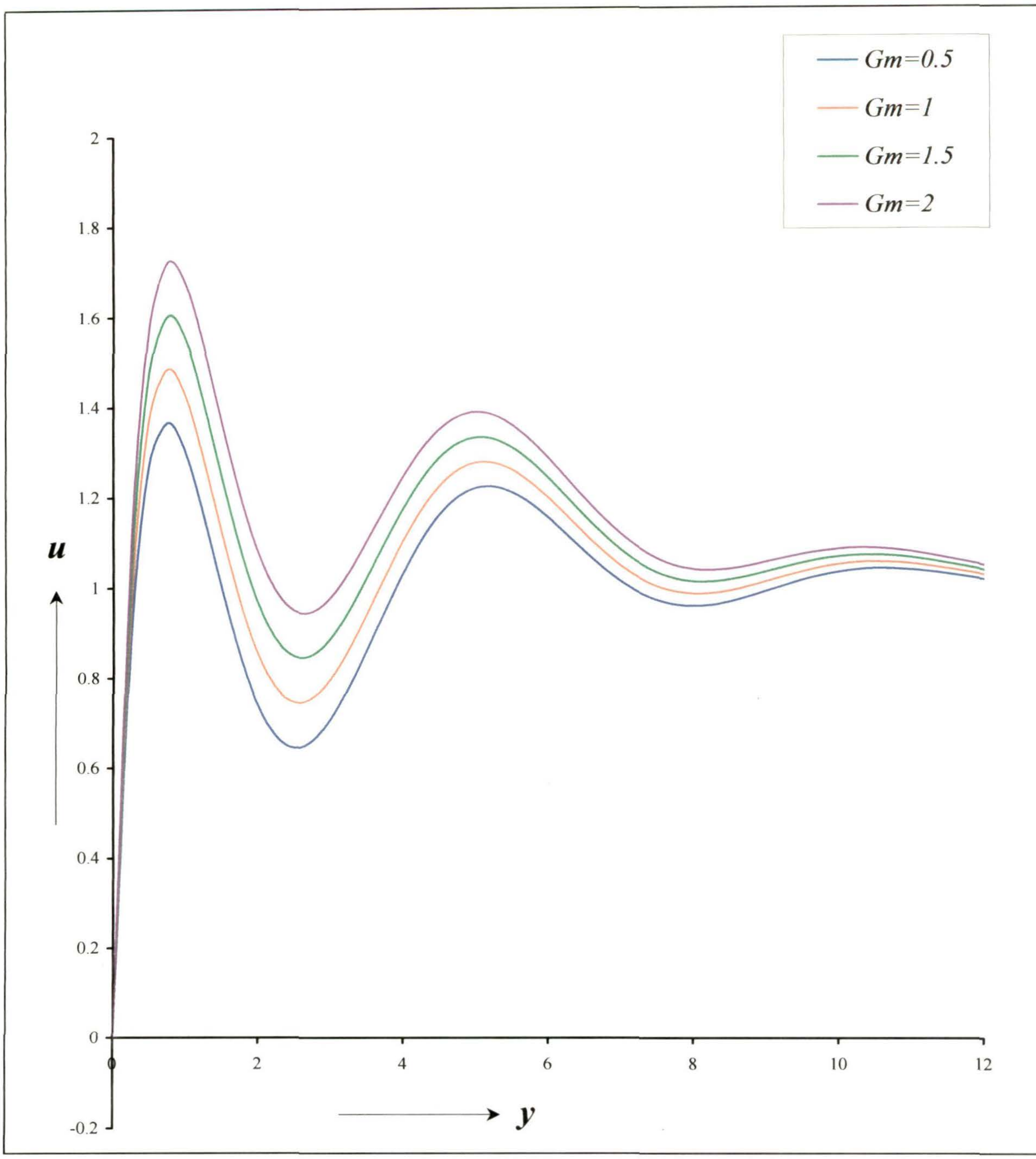


***Fig.* 2.1.11 Velocity profiles for different value of *Gm*: *Pr*=0.71, *M*=0.5, *ω*=0.6, *Gr*=5, *Sc*=0.24, *ε*=0.2, *K*=0.5.**

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Part two

UNSTEADY FREE CONVECTIVE MHD FLUID FLOW WITH MASS TRANSFER THROUGH POROUS MEDIUM IN ROTATING SYSTEM

PART TWO ➢ A

UNSTEADY FREE CONVECTIVE AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM IN ROTATING SYSTEM

2.2.1 INTRODUCTION

The flow through porous medium, under the influence of temperature differences and concentration differences, is one of the most considerable and contemporary subject, because it finds great applications in geothermy, geophysics and technology [1, 2]. Yamamoto and Iwamara [2] expressed the equations of flow through a highly porous medium. Raptis *et al.* [3,4] using the above equations studied the influences of free convection and mass transfer on the steady flow of a viscous fluid through the porous medium, which is bounded by a vertical plane surface, when the temperature and concentration on the surface is constant. Raptis *et al.* [5] also studied the influence of free convective flow on the steady flow of the viscous fluid through the porous medium, when there is a constant heat flux on the above-mentioned surface.

On the other hand, the geophysical importance of the flows in the rotating frame of reference has attracted the attention of a number of Scholars. Raptis [6] analyzed the steady free convective and mass transfer flow through porous medium in presence of a rotating fluid. Later Mahato and Maiti [7] investigated unsteady free convective flow and mass transfer in a rotating porous medium. The object of the present paper is to study the free convective and mass transfer flow of viscous fluid through a rotating porous medium bounded by a vertical porous plate subjected to a constant suction velocity in presence of constant heat flux at the plate. The temperature and concentration at the free streams are constant but the free stream velocity of the fluid vibrates about a mean constant value. The analytical expressions for velocity, temperature and concentration distribution are obtained and the results are presented graphically.

2.2.2 MATHEMATICAL ANALYSIS

We consider unsteady free convective and mass transfer flow of viscous fluid through a porous medium occupying a semi-infinite region bounded by a vertical porous plate subjected to constant suction in presence of constant heat flux at plate wall in a rotating frame of reference. The velocity of the fluid far away from the surface vibrates about a mean value with direction parallel to the plane z=0. The temperature and species concentration at the free stream are constant. A uniform magnetic field of strength B_0 is applied in vertical upward direction. The porous medium is in fact a non-homogeneous medium, which may be replaced by a homogeneous fluid having dynamical properties equal to those of a non-homogeneous continuum. We consider that the vertical infinite porous plate rotates in unison with a viscous fluid occupying the porous region with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. Cartesian co-ordinate system is chosen such that x, y-axes, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface z=0 while z-axis is normal to it. u^+, v^+, w^+ are the velocity components in x, y and z direction respectively. With the above frame of reference and assumptions, the physical variables, except the pressure p are function of z and time t only. Consequently the equation expressing the conservation of mass, momentum, energy and concentration, neglecting the heat due to viscous dissipation, which is valid for small velocities, are given by

$$\frac{\partial \omega^+}{\partial z} = 0, \qquad \text{.... (2.2.1)}$$

$$\frac{\partial u^+}{\partial t} - \omega^+ \frac{\partial u^+}{\partial z} - 2\Omega v^+ = \frac{\partial U^+}{\partial t} - 2\Omega V^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho}\right)\left(u^+ + U^+\right)$$

$$+\nu\frac{\partial^2 u^+}{\partial z^2}+g\beta\left(T^+-T_\infty^+\right)+g\beta^*\left(C^+-C_\infty^+\right), \qquad \text{.... (2.2.2)}$$

$$\frac{\partial v^+}{\partial t}-\omega^+\frac{\partial v^+}{\partial z}-2\Omega u^+=\frac{\partial V^+}{\partial t}-2\Omega U^+-$$

$$\left(\frac{\nu}{K^+}+\frac{\sigma B_0^2}{\rho}\right)\left(v^+-V^+\right)+\nu\frac{\partial^2 v^+}{\partial z^2}, \qquad \text{.... (2.2.3)}$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\nu}{K^+}\omega^+, \qquad \text{.... (2.2.4)}$$

$$\frac{\partial T^+}{\partial t}-\omega^+\frac{\partial T^+}{\partial z}=\frac{\kappa^{*+}}{\rho C_p}\frac{\partial^2 T^+}{\partial z^2}, \qquad \text{.... (2.2.5)}$$

$$\frac{\partial C^+}{\partial t}-\omega^+\frac{\partial C^+}{\partial z}=D\frac{\partial^2 C^+}{\partial z^2}, \qquad \text{.... (2.2.6)}$$

where ν is the kinematic viscosity , t is the time, ρ is the density, K^+ is the permeability of the porous medium, T^+ is the temperature and C^+ *is* the concentration.

The boundary conditions relevant to the problem are

$$\left.\begin{array}{ll} u^+=0,\ v^+=0,\ \dfrac{\partial T^+}{\partial z}=-\dfrac{s}{\kappa^{*+}},\ C^+=C_\infty^+ & at\ z=0 \\ u^+=U^+(t)=U_0\left(1+\varepsilon\cos\phi t\right), & \\ v^+=V^+(t)=0,\ T^+=T_\infty^+,\ C^+=C_\infty^+ & as\ z\to\infty \end{array}\right\}. \qquad \text{.... (2.2.7)}$$

where ϕ is the frequency of oscillation and ε is a small positive quantity.

From equation (2.2.1), we get

$$\omega^+ = -\omega_0. \qquad \text{.... (2.2.8)}$$

Let equation (2.2.2) and (2.2.3) can be combined in complex form, as

$$\frac{\partial q^+}{\partial t} - \omega_0 \frac{\partial q^+}{\partial z} + 2i\Omega q^+ = \frac{\partial Q^+}{\partial t} + 2i\Omega Q^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho}\right)\left(q^+ - Q^+\right) + \nu \frac{\partial^2 q^+}{\partial z^2}$$

$$+ g\beta\left(T^+ - T_\infty^+\right) + g\beta^*\left(C^+ - C_\infty^+\right), \qquad \text{.... (2.2.9)}$$

and equations (2.2.4) and (2.2.5), using equation (2.2.8) can be written in the form as

$$\frac{\partial T^+}{\partial t} - \omega_0 \frac{\partial T^+}{\partial z} = \frac{\kappa^{*+}}{\rho C_p} \frac{\partial^2 T^+}{\partial z^2}, \qquad \text{.... (2.2.10)}$$

$$\frac{\partial C^+}{\partial t} - \omega_0 \frac{\partial C^+}{\partial z} = D \frac{\partial^2 C^+}{\partial z^2}. \qquad \text{.... (2.2.11)}$$

We introduce the following non-dimensional quantities:

$$\eta = \frac{\omega_0}{\nu} z, \quad \tau = \frac{\omega_0^2 t}{\nu}, \quad q = \frac{q^+}{U_0}, \quad K = \frac{\omega_0^2}{\nu^2} K^+, \; Gr = \frac{\nu g\beta\left(T_w^+ - T_\infty^+\right)}{U_0 \omega_0^2},$$

$$Gm = \frac{\nu g\beta^*\left(C_w^+ - C_\infty^+\right)}{U_0 \omega_0^2}, \quad E = \frac{\Omega \nu}{\omega_0^2}, \quad C = \frac{C^+ - C_\infty^+}{C_w^+ - C_\infty^+}, \quad \Pr = \frac{\mu C_p}{\kappa^{*+}},$$

$$Sc = \frac{\nu}{D}, \quad T = \frac{T^+ - T_\infty^+}{\dfrac{\nu s}{\kappa^{*+} \omega_0}}, \quad M = \frac{\sigma \nu B_0^2}{\rho \omega_0^2}, \quad Q = \frac{Q^+}{U_0}, \quad \alpha = \frac{\phi \nu}{\omega_0^2}.$$

Using the above stated non-dimensional quantities, the equations (2.2.9), (2.2.10) and (2.2.11) reduce to

$$\frac{\partial q}{\partial \tau}-\frac{\partial q}{\partial \eta}+2iEq=\frac{\partial Q}{\partial \tau}+2iEQ$$

$$-\left(M+\frac{1}{K}\right)(q-Q)+\frac{\partial^2 q}{\partial \eta^2}+GmC+GrT, \qquad \text{.... (2.2.12)}$$

$$\frac{\partial T}{\partial \tau}-\frac{\partial T}{\partial \eta}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial \eta^2}, \qquad \text{.... (2.2.13)}$$

$$\frac{\partial C}{\partial \tau}-\frac{\partial C}{\partial \eta}=\frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2}, \qquad \text{.... (2.2.14)}$$

with boundary conditions

$$\left.\begin{aligned} & q=0,\ \frac{\partial T}{\partial \eta}=-1,\ C=1 \quad at\ \eta=0 \\ & q=1+\frac{\varepsilon}{2}\left(e^{i\alpha\tau}+e^{-i\alpha\tau}\right),\ T=0,\ C=0 \quad as\ \eta\to\infty \end{aligned}\right\}. \qquad \text{.... (2.2.15)}$$

2.2.3 SOLUTION

Let, the solutions of equations (2.2.12), (2.2.13) and (2.2.14) are assumed, respectively, as

$$q(\eta,\tau)=q_0(\eta)+\frac{\varepsilon}{2}\left\{q_1(\eta)e^{i\alpha\tau}+q_2(\eta)e^{-i\alpha\tau}\right\}, \qquad \text{.... (2.216)}$$

$$T(\eta,\tau)=T_0(\eta)+\varepsilon T_1(\eta)e^{i\alpha\tau}+\ldots\ldots\ldots\ldots, \qquad \text{.... (2.2.17)}$$

$$C(\eta,\tau)=C_0(\eta)+\varepsilon C_1(\eta)e^{i\alpha\tau}+\ldots\ldots\ldots\ldots \qquad \text{.... (2.2.18)}$$

Using equations (2.2.16), (2.2.17) and (2.2.18) in equations (2.2.12), (2.2.13) and (2.2.14), we obtain following equations

$$q_0''+q_0'-(L+2iE)q_0=-\frac{Gr}{\Pr}e^{-\Pr\eta}-Gme^{-Sc\eta}, \quad .... (2.2.19)$$

$$q_1''+q_1'-\{L+i(2E+\alpha)\}q_1=-\{L+i(2E+\alpha)\}, \quad .... (2.2.20)$$

$$q_2''+q_2'-\{L+i(2E-\alpha)\}q_1=-\{L+i(2E-\alpha)\}, \quad .... (2.2.21)$$

$$T_0''+\Pr T_0'=0, \quad .... (2.2.22)$$

$$T_1''+\Pr T_1'-i\alpha\Pr T_1=0, \quad .... (2.2.23)$$

$$C_0''+ScC_0'=0, \quad .... (2.2.24)$$

$$C_1''+ScC_1'-i\alpha ScC_1=0. \quad .... (2.2.25)$$

Using (2.2.13), (2.2.14) and (2.2.15) in (2.2.12), the boundary conditions are reduced to

$$\left.\begin{array}{l} q_0=q_1=q_2=0,\quad T_0'=-1,\quad T_1'=0, \\ C_0=1,\quad C_1=0 \qquad\qquad at\ \ \eta=0 \\ \\ q_0=q_1=q_2=1,\quad T_0=0,\quad T_1=0, \\ C_0=0,\quad C_1=0 \qquad\qquad as\ \ \eta\to\infty \end{array}\right\}. \quad .... (2.2.26)$$

The solutions of equations (2.2.19) to (2.2.25), under the boundary conditions (2.2.26) and in view of (2.2.16), (2.2.17) and (2.2.18) are given by

$$q(\eta,\tau)=1-(1-R_1-R_2)e^{-R_3\eta}-R_1e^{-\Pr\eta}$$
$$-R_2e^{-Sc\eta}+\frac{\varepsilon}{2}\left\{\left(1-e^{-R_4\eta}\right)e^{i\alpha\tau}+\left(1-e^{-R_5\eta}\right)e^{-i\alpha\tau}\right\}, \quad .... (2.2.27)$$

where

$$q_0(\eta)=1-(1-R_1-R_2)e^{-R_3\eta}-R_1e^{-\Pr\eta}-R_2e^{-Sc\eta}, \quad .... (2.2.27a)$$

$$q_1(\eta)=1-e^{-R_4\eta} \quad \text{.... (2.2.27}b)$$

and $$q_2(\eta)=1-e^{-R_5\eta}, \quad \text{....(2.2.27}c)$$

$$T(\eta)=\frac{1}{\Pr}e^{-\Pr\eta}, \quad \text{.... (2.2.28)}$$

$$C(\eta)=e^{-Sc\eta}. \quad \text{.... (2.2.29)}$$

The expressions for constant are given in appendix-I.

2.2.4 RESULTS AND DISCUSSION

Equation (2.2.27) corresponds to the velocity distribution of free convective and mass transfer flow of viscous fluid through a rotating porous medium. The expression clearly shows the existence of thin multiple Ekman boundary layer of order $O(R_3^{-1})$ super imposed with a boundary layer of thickness of order $O(\Pr^{-1})$ and $O(Sc^{-1})$. It is interesting to note that Ekman boundary layer is modified by the presence of free convection and mass transfer. We also note that this layer decrease with increase of rotation parameter and magnetic parameter and increase with increase of permeability parameter.

The solution (2.2.27a) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and phase difference due to these primary and secondary velocities for the steady flow are given by

$$|A_0|=\left(u_0^2+v_0^2\right) \qquad \text{and} \qquad \theta_0=\tan^{-1}\left(\frac{v_0}{u_0}\right),$$

where

$$u_0=1-\{(1-P_1-P_2)\cos Q_3\eta-(Q_2+Q_1)\sin Q_3\eta\}e^{-P_3\eta}-P_1e^{-\Pr\eta}-P_2e^{-Sc\eta},$$

$$v_0 = \{(Q_2 + Q_1)\cos Q_3\eta + (1 - P_1 - P_2)\sin Q_3\eta\}e^{-P_3\eta} - Q_1 e^{-\Pr\eta} - Q_2 e^{-Sc\eta}.$$

The amplitude of resultant velocity $|A_0|$ and the phase angle θ_0 for the steady part are shown graphically in Fig. 2.2.1(a,b) and Fig. 2.2.2(a,b) for various values of the rotation parameter (E) and permeability parameter (K) for fixed values of Prandtl number (Pr), Schmidt number (Sc), magnetic parameter (M), Grashof number (Gr) and modified Grashof number (Gm). It is seen from Fig. 2.2.1(a) that in case of $Gr>0$ the amplitude $|A_0|$ increase as K increases and nearly at η =2.5 these two values coincide but opposite behavior is seen for $Gr<0$ and decreases with increase in rotation. Fig. 2.2.1(b) θ_0 decreases with increase in K and increases with increasing R (both small and large) for $Gr>0$ and increases as rotation parameter increases for $Gr<0$ near the plate wall.

Variation of $|A_0|$ and θ_0 for different values of Prandtl number Pr and modified Grashof number for $Gr>0$ are shown in Fig. 2.2.3(a,b) and Fig. 2.2.4(a,b). It is clear from these figure that amplitude decreases as Pr increases and increases as Gm increases but phase difference θ_0 decreases as Gm increases and increases as Pr increases. Numerical calculation are also made for $Gr<0$ and shown in Fig. 2.2.5(a,b) and Fig. 2.2.6(a,b).It is essential to mention that equation (2.2.27b) and (2.2.27c) together give the unsteady part of the flow. This expression also exhibits boundary layer of thickness of order $O(R_4^{-1})$ and order $O(R_5^{-1})$ respectively.

The amplitude and the phase differences of shear stresses at the plate η=0 for the steady flow can be obtained as:

$$\tau_{0r} = \left(\tau_{0x}^2 + \tau_{0y}^2\right)^{\frac{1}{2}}, \qquad \theta_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right). \qquad \text{.... (2.2.30)}$$

where τ_{0x} and τ_{0y} are respectively the shear stress at the plate due to primary and secondary velocity components.

The numerical values for the resultant shear stress and the phase angle due to the shear stress are listed in Table-2.2.1.

Table-2.2.1

Sl. No.	Pr	Sc	K	E	M	Gm	Gr	τ_{0r}	θ_{0r}
1	0.71	0.3	1	1	0.5	5	10	15.7646	-0.5104
2	7	0.3	1	1	0.5	5	10	5.7257	-0.1937
3	0.71	0.66	1	1	0.5	5	10	15.1004	-0.4928
4	0.71	0.3	5	1	0.5	5	10	19.5276	-0.7104
5	0.71	0.3	1	5	0.5	5	10	9.3067	-0.2333
6	0.71	0.3	1	1	1	5	10	14.2511	-0.4069
7	0.71	0.3	1	1	0.5	10	10	20.2386	-0.5540
8	0.71	0.3	1	1	0.5	5	20	26.3319	-0.5768

These values clearly show that the shear stress τ_{0r} increases as permeability parameter K increases and decreases as rotation parameter R increases. Also the increase in permeability parameter K lead to decrease in phase difference θ_{0r} and the phase difference θ_{0r} increases as rotation parameter R increases.

Appendix-I

$$L = M + \frac{1}{K}; \qquad R_1 = P_1 + iQ_1 = \frac{Gr}{\Pr\left\{\Pr^2 - \Pr - (L + 2iE)\right\}};$$

$$R_2 = P_2 + iQ_2 = \frac{Gm}{Sc^2 - Sc - (L + 2iE)}; \qquad R_3 = P_3 + iQ_3 = \frac{1 + \sqrt{1 + 4(L + 2iE)}}{2};$$

$$R_4 = P_4 + iQ_4 = \frac{1 + \sqrt{1 + 4\{L + i(\alpha + 2E)\}}}{2}; \quad R_5 = P_5 + iQ_5 = \frac{1 + \sqrt{1 + 4\{L + i(2E - \alpha)\}}}{2};$$

$$P_1=\frac{Gr\left(\Pr^2-\Pr-L\right)}{\Pr\left\{\left(\Pr^2-\Pr-L\right)^2+4E^2\right\}};\qquad Q_1=\frac{2GrE}{\Pr\left\{\left(\Pr^2-\Pr-L\right)^2+4E^2\right\}};$$

$$P_2=\frac{Gm\left(Sc^2-Sc-L\right)}{\left\{\left(Sc^2-Sc-L\right)^2+4E^2\right\}};\qquad Q_2=\frac{2GmE}{\left\{\left(Sc^2-Sc-L\right)^2+4E^2\right\}};$$

$$P_3=\frac{\sqrt{2}+\left[(1+4L)+\sqrt{(1+4L)^2+64E^2}\right]^{\frac{1}{2}}}{2\sqrt{2}};\quad Q_3=\frac{2\sqrt{2}E}{\left[(1+4L)+\sqrt{(1+4L)^2+64E^2}\right]^{\frac{1}{2}}};$$

$$\tau_{0x}=\left\{(1-P_1-P_2)Q_3\sin Q_3\eta+(Q_2+Q_1)Q_3\cos Q_3\eta\right\}e^{-P_3\eta}$$

$$+\left\{(1-P_1-P_2)\cos Q_3\eta-(Q_2+Q_1)\sin Q_3\eta\right\}P_3e^{-P_3\eta}+P_1\Pr e^{-\Pr\eta}+P_2Sce^{-Sc\eta};$$

$$\tau_{0y}=\left\{(1-P_1-P_2)Q_3\cos Q_3\eta-(Q_2+Q_1)Q_3\sin Q_3\eta\right\}e^{-P_3\eta}$$

$$-\left\{(1-P_1-P_2)\sin Q_3\eta+(Q_2+Q_1)\cos Q_3\eta\right\}P_3e^{-P_3\eta}+Q_1\Pr e^{-\Pr\eta}+Q_2Sce^{-Sc\eta}.$$

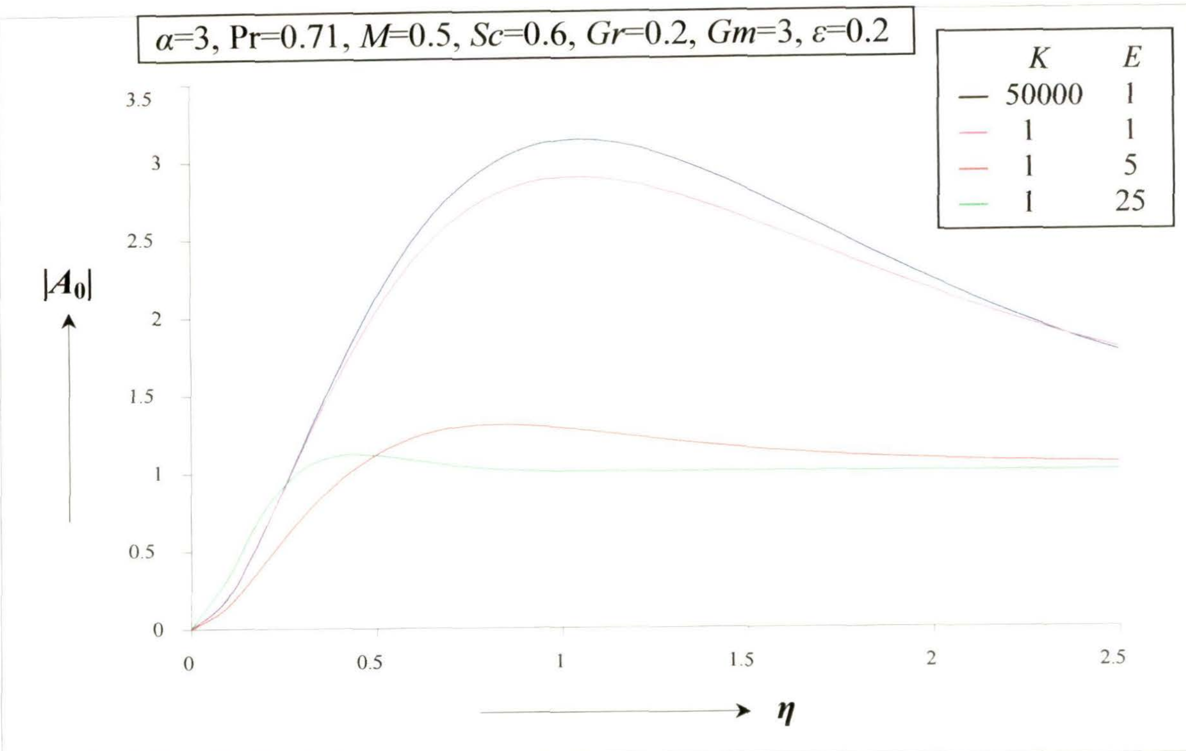

***Fig.* 2.2.1(*a*) Effects of *K* and *E* on the resultant velocity field for *Gr*>0 .**

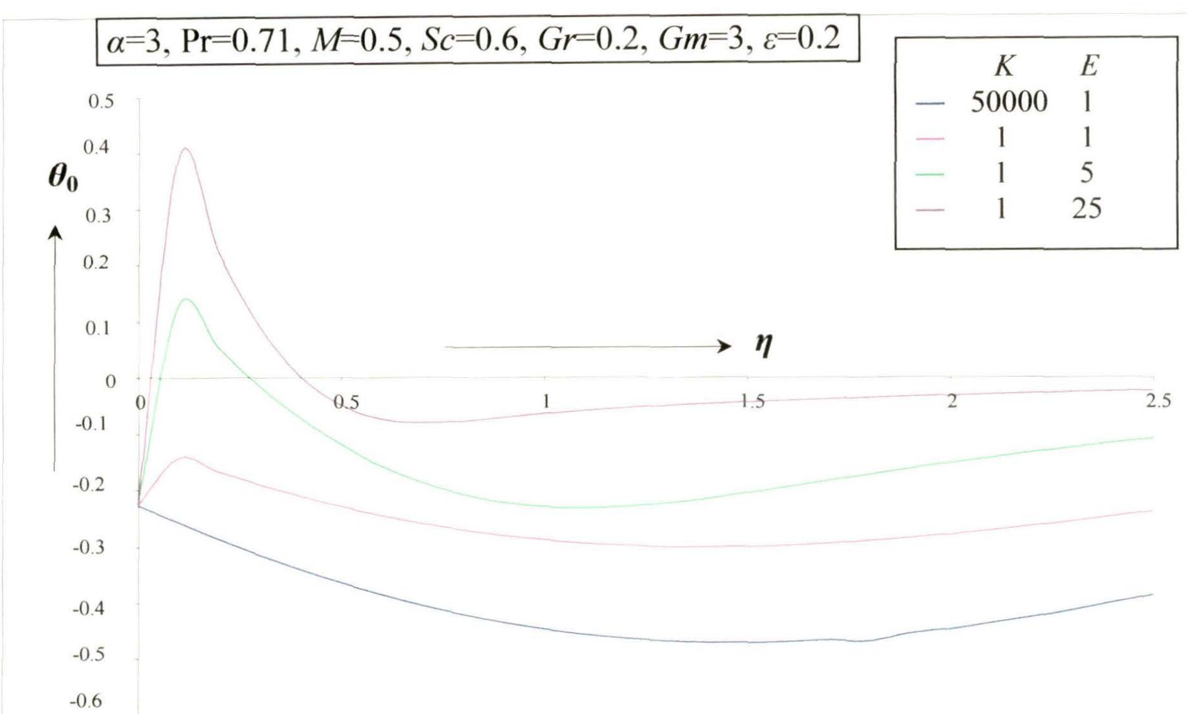

***Fig.* 2.2.1(*b*) Effects of *K* and *E* on the amplitude of the resultant velocity field for *Gr*>0.**

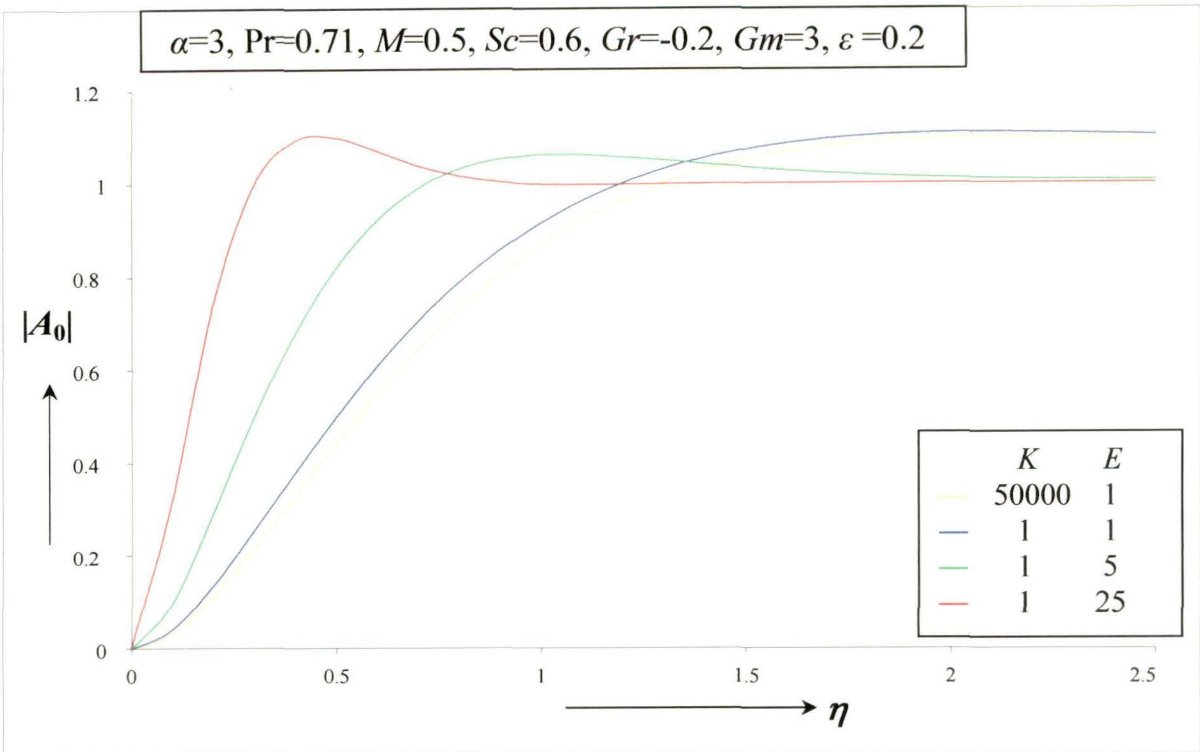

***Fig.* 2.2.2(*a*) Effects of *K* and *E* on the resultant velocity field for *Gr* < 0.**

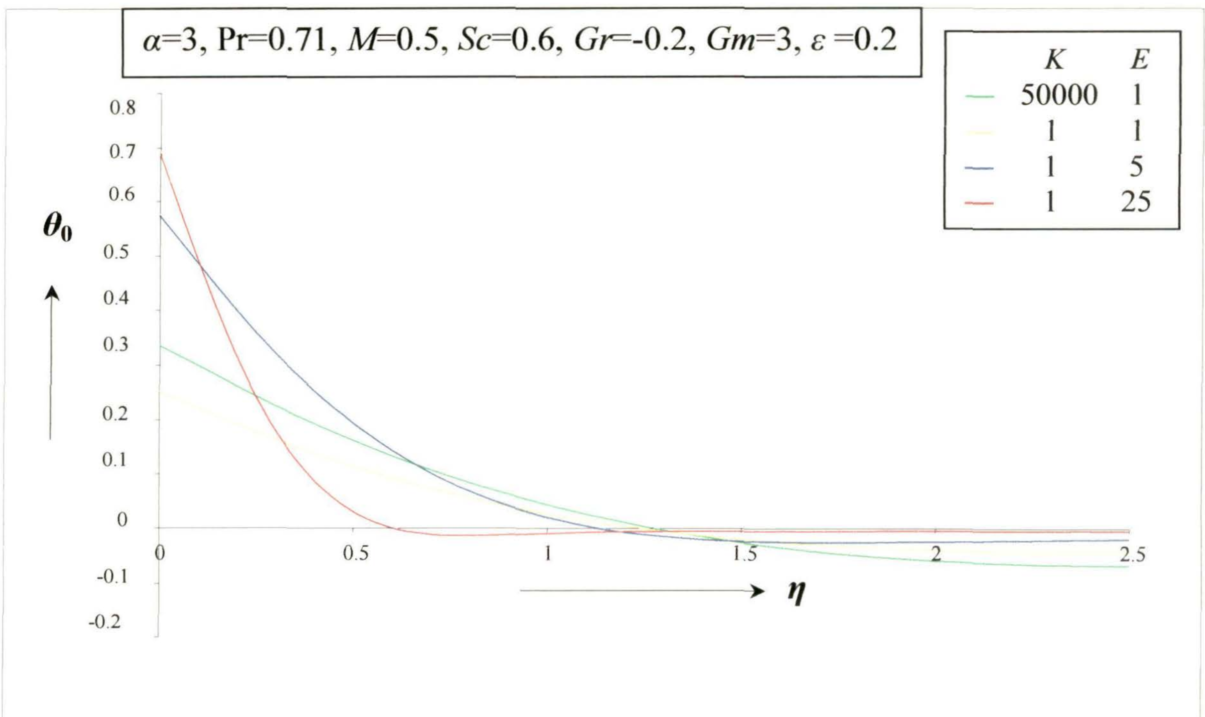

***Fig.* 2.2.2(*b*) Effects of *K* and *E* on the amplitude of the resultant velocity field for *Gr* < 0.**

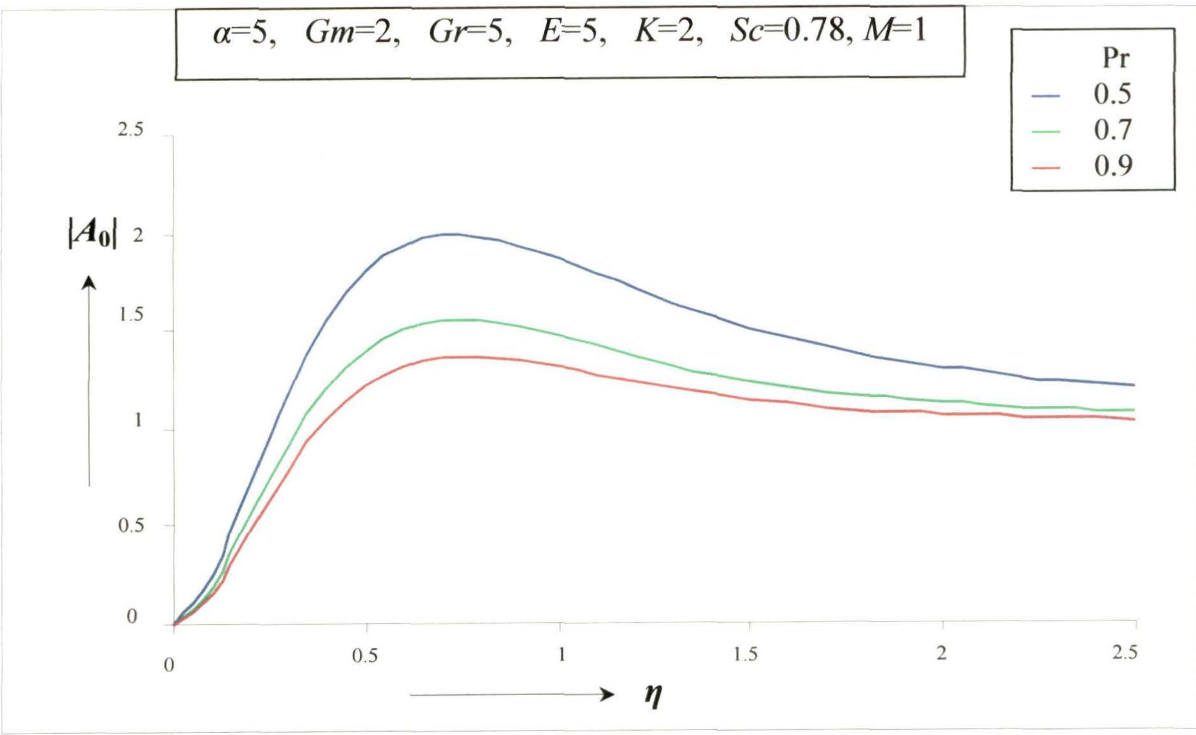

***Fig.* 2.2.3(*a*) Effects of Pr on the resultant velocity field for *Gr* > 0.**

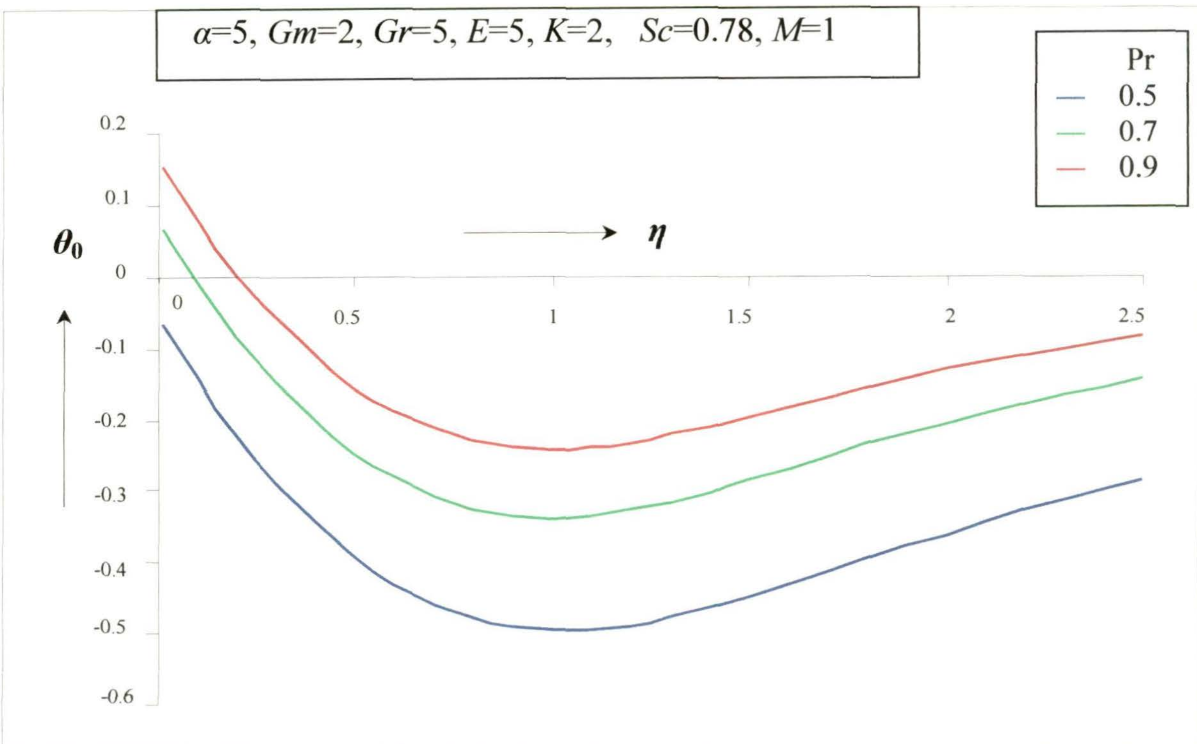

***Fig.* 2.2.3(*b*) Effects of Pr on the amplitude of the resultant velocity field for *Gr* > 0.**

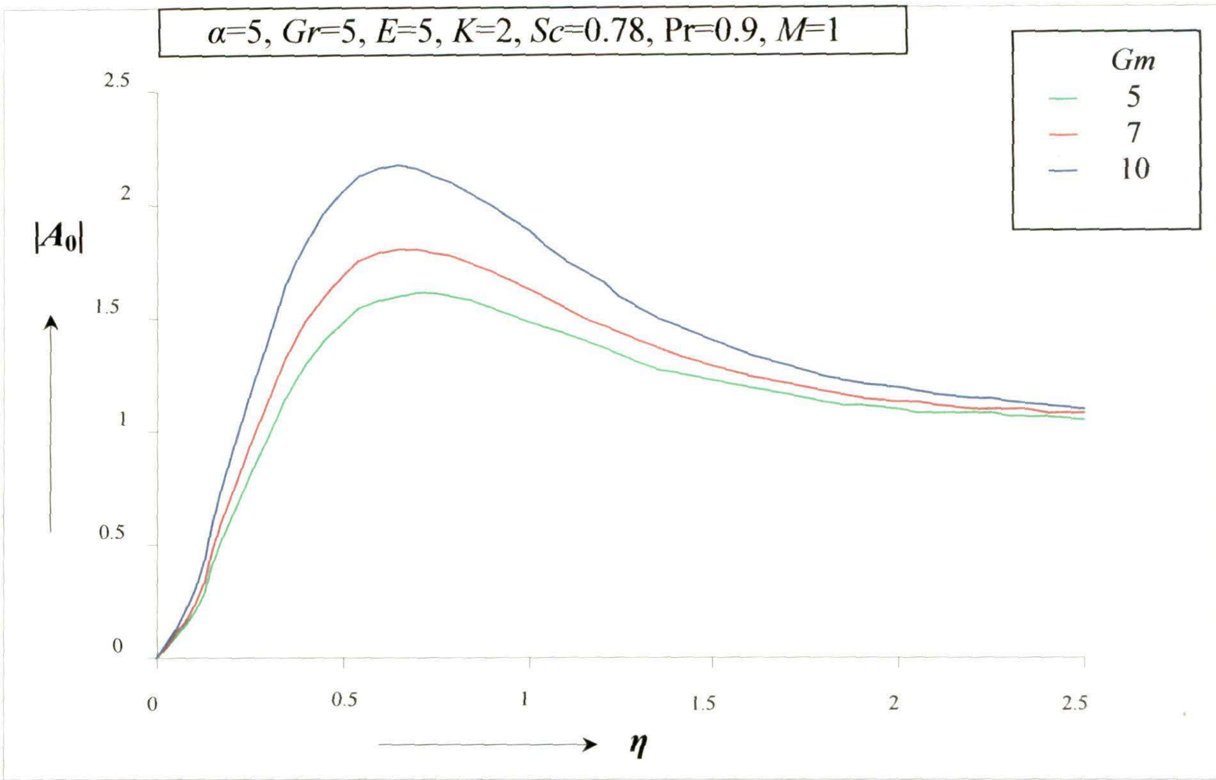

***Fig.* 2.2.4(*a*) Effects of *Gm* on the resultant velocity field for *Gr* > 0.**

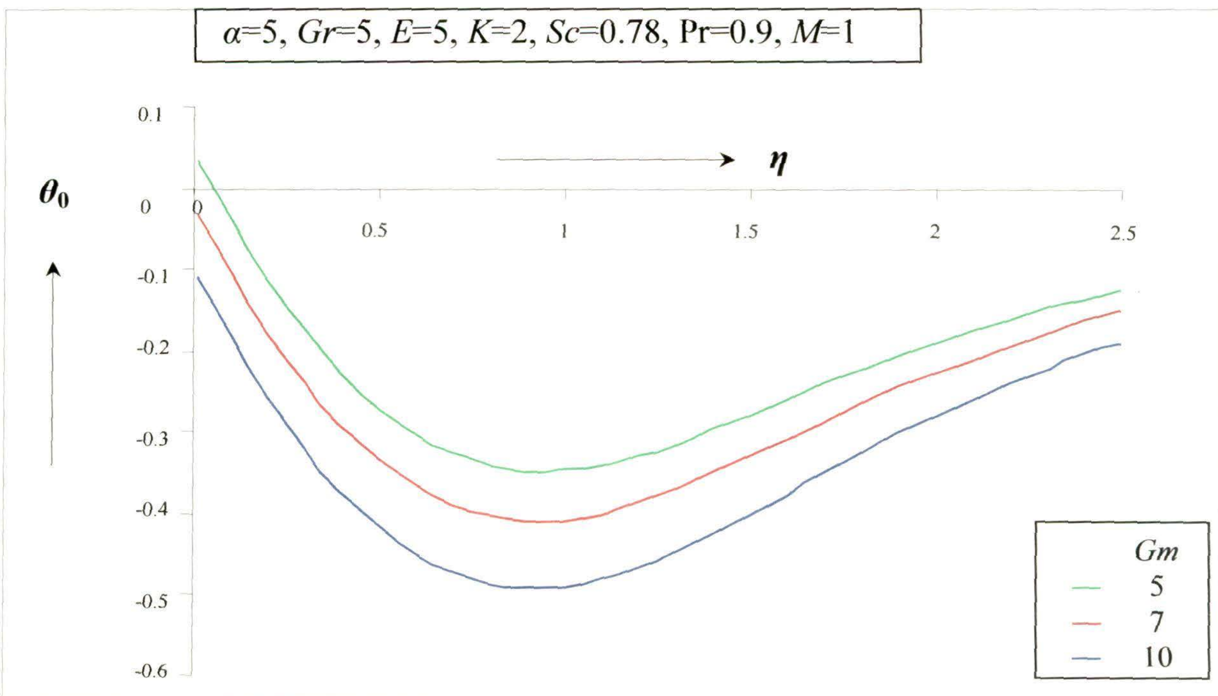

***Fig.* 2.2.4(*b*) Effects of *Gm* on the amplitude of the resultant velocity field for *Gr* > 0.**

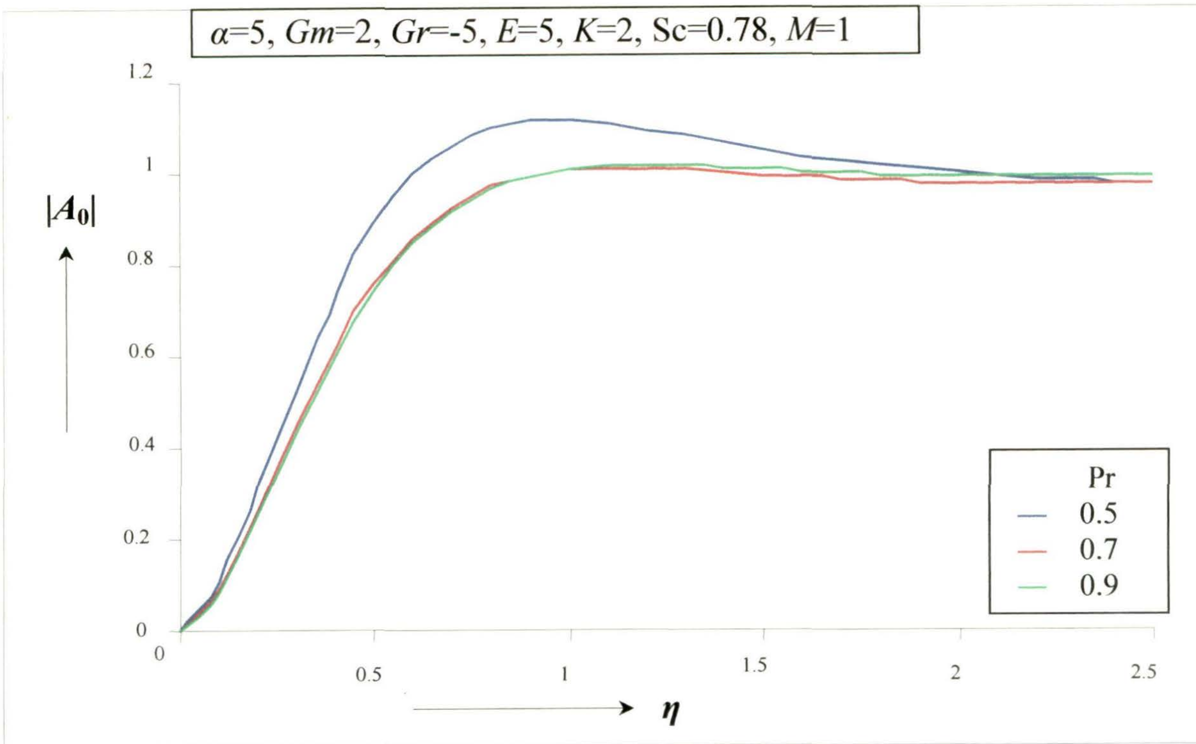

Fig. 2.2.5(a) Effects of Pr on the resultant velocity field for $Gr < 0$.

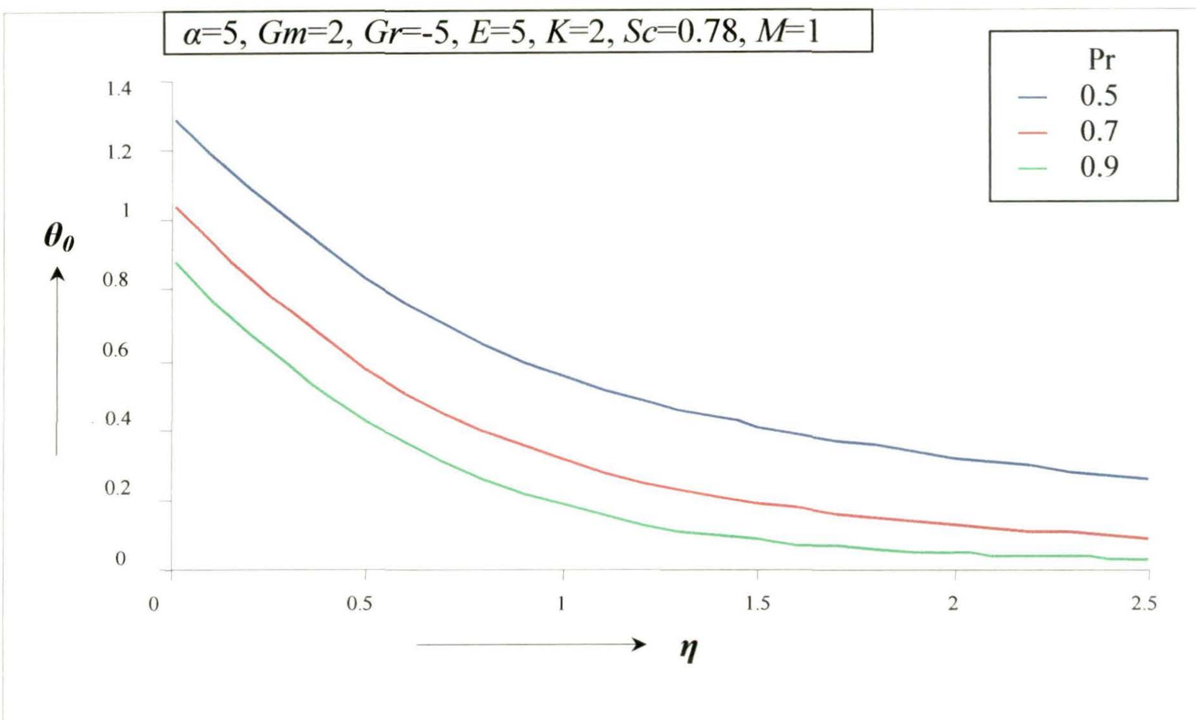

Fig. 2.2.5(b) Effects of Pr on the amplitude of the resultant velocity field for $Gr < 0$.

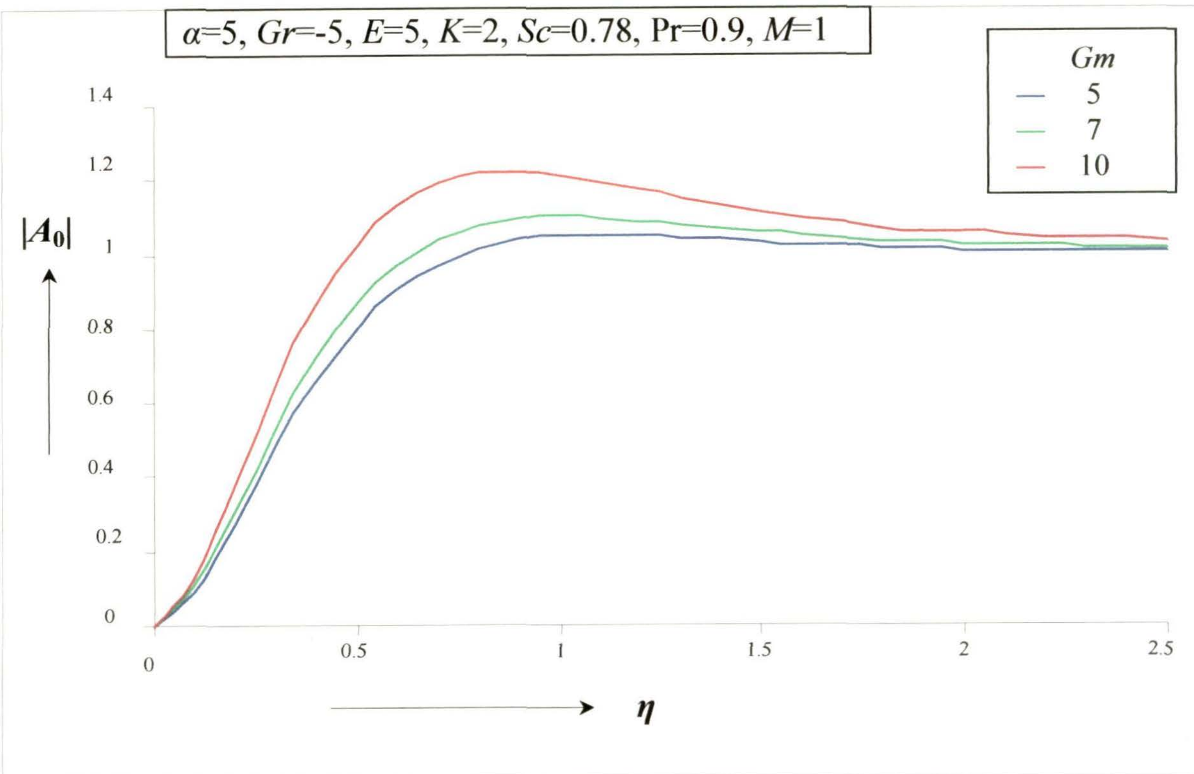

***Fig*. 2.2.6(*a*) Effects of *Gm* on the resultant velocity field for *Gr* < 0.**

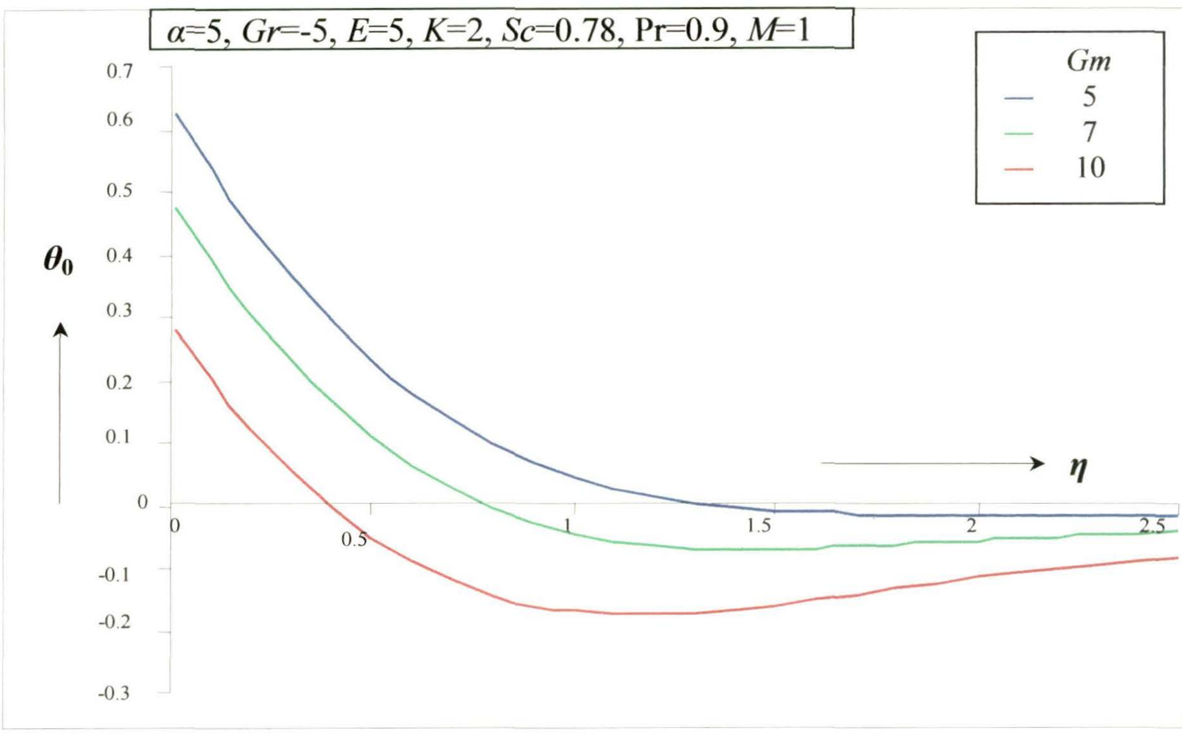

***Fig*. 2.2.6(b) Effects of *Gm* on the amplitude of the resultant velocity field for *Gr* < 0.**

PART TWO ➢ B

EFFECTS OF MASS TRANSFER AND ROTATION ON FLOW PAST A POROUS PLATE IN A POROUS MEDIUM WITH VARIABLE SUCTION IN SLIP FLOW REGIME

2.2.6 INTRODUCTION

Free convection and mass transfer flow in porous medium has received considerable attention due to it numerous application in geophysics and energy related problems. Such types of application include natural circulation in isothermal reservoirs, aquifers porous insulation in heat storage bed, grain storage, extraction of thermal energy and thermal insulation design. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical, geothermal. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Raptis *et al.* [4, 8] studied both steady and unsteady free convective flow and mass transfer of a viscous fluid through a porous medium bounded by a vertical infinite porous surface with constant suction using generalized Darcy's law. Mahato and Maiti [9] analyzed the effect of unsteady free convective flow and mass transfer during the motions of a viscous incompressible fluid in a rotating frame of references. Alam *et al.* [10] studied unsteady free convection and mass transfer flow in a rotating system with hall currents, viscous dissipation and joule heating. Later Singh *et al.* [11] studied free convection in MHD flow of a rotating viscous liquid in porous medium. Recently Singh *et al.* [12] have studied free convective MHD flow of rotating viscous fluid in a porous medium past infinite vertical porous plate. We now proposed to study of unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through porous medium bounded by a infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform magnetic field applied perpendicular to the flow of region in a rotating system is presented. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, temperature, concentration, skin friction, rate of heat transfer and rate of mass transfer are presented graphically and necessary conclusions are set out.

2.2.7 FORMULATION OF THE PROBLEM

We consider an unsteady heat and mass transfer flow of an incompressible, electrically conducting, viscous liquid flowing through a homogeneous porous medium with constant permeability K, past an infinite vertical porous plate in the presence of a constant heat source $Q=Q_0(T-T_w)$. The plate is subjected to a variable suction $\omega=-\omega_0(1+\varepsilon Ae^{i\varphi t})$ at the plate, (where ω_0 is a positive real number, A-suction parameter) under the influence of the uniform magnetic field B_0. We assume a Cartesian co-ordinate system choosing x-axis and y-axis in the plane of the porous plate and z-axis normal to the plate with velocity components u, v, w in these directions respectively. Both the liquid and the plate are in a state of rigid body rotation (with uniform angular velocity Ω) about z-axis. In this analysis buoyancy force, Hall effect, effect due to perturbation of the field, induced magnetic field and polarization effects are ignored. Following, Gebhart and Pera [13], Soret effect is taken into account. Under the above stated restrictions, the equation of motion, energy and concentration are as follows

$$\frac{\partial u}{\partial t}-\omega_0\left(1+\varepsilon Ae^{i\phi t}\right)\frac{\partial u}{\partial z}-2\Omega v=\vartheta\frac{\partial^2 u}{\partial z^2}+g\beta\left(T-T_\infty\right)$$

$$+g\beta_0\left(C-C_\infty\right)-\left(\frac{\vartheta}{K}+\frac{\sigma B_0^2}{\rho}\right)u, \qquad \ldots\ldots (2.2.31)$$

$$\frac{\partial v}{\partial t}-\omega_0\left(1+\varepsilon Ae^{i\phi t}\right)\frac{\partial v}{\partial z}+2\Omega u=\vartheta\frac{\partial^2 v}{\partial z^2}-\left(\frac{\vartheta}{K}+\frac{\sigma B_0^2}{\rho}\right)v, \qquad \ldots\ldots (2.2.32)$$

$$\frac{\partial T}{\partial t}-\omega_0\left(1+\varepsilon Ae^{i\phi t}\right)\frac{\partial T}{\partial z}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial z^2}-\frac{Q_0(T-T_\infty)}{\rho C_p}, \quad \text{.... (2.2.33)}$$

$$\frac{\partial C}{\partial t}-\omega_0\left(1+\varepsilon Ae^{i\phi t}\right)\frac{\partial C}{\partial z}=D_M\frac{\partial^2 C}{\partial z^2}+D_T\frac{\partial^2 T}{\partial z^2}. \quad \text{.... (2.2.34)}$$

where the symbols have their usual meaning.

The boundary conditions are given as

$$\left.\begin{array}{ll} u=\varepsilon U_0 e^{i\phi t}+L_1\frac{\partial u}{\partial y},\ v=0,\ T=T_w,\ C=C_w & \text{at } z=0 \\ u\to 0,\ v\to 0,\ T\to T_\infty,\ C\to C_\infty & \text{as } z\to\infty \end{array}\right\}, \quad \text{.... (2.2.35)}$$

where $L_1=\frac{(2-m_1)L}{m_1}$ and $L=\mu\left[\frac{\pi}{(2p\rho)}\right]^{\frac{1}{2}}$ is the mean free path and m_1 is the Maxwell's reflection co-efficient.

On introducing the following non-dimensional quantities

$$z^*=\frac{\omega_0 z}{\vartheta},\ t^*=\frac{\omega_0^2 t}{\vartheta},\ u^*=\frac{u}{U_0},\ \phi^*=\frac{\vartheta\phi}{\omega_0^2},\ v^*=\frac{v}{U_0},\ K^*=\frac{\omega_0^2 K}{\vartheta^2},$$

$$C^*=\frac{C-C_\infty}{C_w-C_\infty},\ T^*=\frac{T-T_\infty}{T_w-T_\infty},\ Sc=\frac{\vartheta}{D_M},\ Gm=\frac{g\beta\vartheta(C_w-C_\infty)}{U_0\omega_0^2},\ \Pr=\frac{\mu C_p}{\kappa}$$

$$Gr=\frac{g\beta_0\vartheta(T_w-T_\infty)}{U_0\omega_0^2},\quad R=\frac{\omega_0 L_1}{\vartheta}(\textit{rarefaction parameter}),$$

$$S_0=\frac{D_T}{\vartheta}\frac{(T_w-T_\infty)}{(C_w-C_\infty)}(\textit{Thermal diffusion parameter}),$$

$$M=\frac{\sigma B_0^2\vartheta}{\rho\omega_0^2}(\textit{Hartmann number}),\quad \alpha_0=\frac{Q_0\vartheta^2}{\kappa\omega_0^2}(\textit{Heat source parameter}),$$

$$E = \frac{\Omega \vartheta}{\omega_0^2} (Rotation\ parameter).$$

in equations (2.2.31)-(2.2.34) (after dropping the asterisks) can be written as

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial u}{\partial z} - 2Ev = \frac{\partial^2 u}{\partial z^2} + GrT + GmC - \left(M + \frac{1}{K}\right)u, \quad \ldots. (2.2.36)$$

$$\frac{\partial v}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial v}{\partial z} + 2Eu = \frac{\partial^2 v}{\partial z^2} - \left(M + \frac{1}{K}\right)v, \quad \ldots. (2.2.37)$$

$$\Pr\frac{\partial T}{\partial t} - \Pr\left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} - \alpha_0 T, \quad \ldots. (2.2.38)$$

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial C}{\partial z} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} + S_0\frac{\partial^2 T}{\partial z^2}, \quad \ldots. (2.2.39)$$

which leads to

$$\frac{\partial q}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial q}{\partial z} + \left[\left(M + \frac{1}{K}\right) + i2E\right]q = \frac{\partial^2 q}{\partial z^2} + GrT + GmC, \quad .. (2.2.40)$$

$$\frac{\partial^2 T}{\partial z^2} + \Pr\left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial T}{\partial z} - \Pr\frac{\partial T}{\partial t} - \alpha_0 T = 0, \quad \ldots. (2.2.41)$$

$$\frac{\partial^2 C}{\partial z^2} + Sc\left(1 + \varepsilon A e^{i\phi t}\right)\frac{\partial C}{\partial z} - Sc\frac{\partial C}{\partial t} + S_0 Sc\frac{\partial^2 T}{\partial z^2} = 0, \quad \ldots. (2.2.42)$$

where $q = u + iv$.

The boundary conditions (2.2.35) are transformed to

$$\left.\begin{aligned} &q=\varepsilon e^{i\phi t}+R\frac{\partial u}{\partial z}, \quad T=1, \quad C=1 \qquad \text{at } z=0 \\ &q\to 0, \qquad\qquad\quad T\to 0, \quad C\to 0 \qquad \text{as } z\to\infty \end{aligned}\right\}. \qquad \text{.... (2.2.43)}$$

2.2.8 SOLUTION OF THE PROBLEM

In order to solve the equations (2.2.40), (2.2.41) and (2.2.42), we assume the velocity, temperature and concentration in the neighborhood of the plate as follows

$$q(z,t)=q_0(z)+\varepsilon q_1(z)e^{i\phi t}, \qquad \text{.... (2.2.44)}$$

$$T(z,t)=T_0(z)+\varepsilon T_1(z)e^{i\phi t} \qquad \text{.... (2.2.45)}$$

and $C(z,t)=C_0(z)+\varepsilon C_1(z)e^{i\phi t}$. ....(2.2.46)

Using equations (2.2.44)-(2.2.46) in equations (2.2.40)-(2.2.42), we obtain following equations,

$$q_0''(z)+q_0'(z)-(M_1+i2E)q_0(z)=-GrT_0(z)-GmC_0(z), \qquad \text{.... (2.2.47)}$$

$$q_1''(z)+q_1'(z)-[M_1+i(2E+\phi)]q_1(z)=-Aq_0'(z)$$

$$-GrT_1(z)-GmC_1(z), \qquad \text{.... (2.2.48)}$$

$$T_0''(z)+\Pr T_0'(z)-\alpha_0 T_0(z)=0, \qquad \text{.... (2.2.49)}$$

$$T_1''(z)+\Pr T_1'(z)-(\alpha_0+i\phi\Pr)T_1(z)=-A\Pr T_0'(z), \qquad \text{.... (2.2.50)}$$

$$C_0''(z)+ScC_0'(z)=-S_0ScT_0''(z), \qquad \text{.... (2.2.51)}$$

$$C_1''(z)+ScC_1'(z)-i\phi ScC_1(z)=-AScC_0'(z)-S_0ScT_1''(z). \qquad \text{.... (2.2.52)}$$

Using (2.2.44)-(2.2.46) in (2.2.43), the boundary conditions are reduced to

$$\left.\begin{array}{ll} q_0 = R\dfrac{du_0}{dz},\ q_1 = 1 + R\dfrac{du_1}{dz},\quad T_0 = 1, & \\ T_1 = 0,\ C_0 = 1,\ C_1 = 0 & \text{at } z = 0 \\ & \\ q_0 \to 0,\ q_1 \to 0,\quad T_0 \to 0,\ T_1 \to 0, & \\ C_0 \to 0,\ C_1 \to 0 & \text{as } z \to \infty \end{array}\right\}. \qquad \text{.... (2.2.53)}$$

The solutions of equations (2.2.47) to (2.2.52), under the boundary conditions (2.2.53) are

$$T_0(z) = e^{-H_0 z}, \qquad \text{.... (2.2.54)}$$

$$T_1(z) = R_1\left(e^{-H_0 z} - e^{-H_1 z}\right), \qquad \text{.... (2.2.55)}$$

$$C_0(z) = (1 + R_0)e^{-Scz} - R_0 e^{-H_0 z}, \qquad \text{.... (2.2.56)}$$

$$C_1(z) = R_2 e^{-H_0 z} - R_3 e^{-Scz} + R_4 e^{-H_1 z} + R_5 e^{-H_2 z}, \qquad \text{.... (2.2.57)}$$

$$q_0(z) = -R_6 e^{-H_0 z} - R_7 e^{-Scz} + R_8 e^{-H_3 z}, \qquad \text{.... (2.2.58)}$$

$$q_1(z) = R_9 e^{-H_1 z} - R_{10} e^{-H_2 z}$$

$$+ R_{11} e^{-H_3 z} + R_{12} e^{-H_4 z} - R_{13} e^{-H_0 z} + R_{14} e^{-Scz}. \qquad \text{.... (2.2.59)}$$

Substituting the values of $q_0(z)$, $q_1(z)$, $T_0(z)$, $T_1(z)$, $C_0(z)$ *and* $C_1(z)$ in (2.2.44) to (2.2.46) we obtain $q(z,t)$, $T(z,t)$ and $C(z,t)$. At $\phi t = \dfrac{\pi}{2}$, we obtain,

$$u\left(z, \frac{\pi}{2\phi}\right) = u_0 - \varepsilon v_1, \qquad \text{.... (2.2.60)}$$

$$v\left(z, \frac{\pi}{2\phi}\right) = v_0 + \varepsilon u_1, \qquad \text{.... (2.2.61)}$$

where

$$u_0(z) = e^{-A_3 z}\left(P_8 \cos B_3 z + Q_8 \sin B_3 z\right) - P_6 e^{-H_0 z} - P_7 e^{-Scz},$$

$$v_0(z)=e^{-A_3 z}\left(Q_8\cos B_3 z-P_8\sin B_3 z\right)-Q_6 e^{-H_0 z}-Q_7 e^{-Scz},$$

$$u_1(z)=F_1(z)-F_2(z)+F_3(z)+F_4(z)-P_{13}e^{-H_0 z}+P_{14}e^{-Scz},$$

$$v_1(z)=f_1(z)-f_2(z)+f_3(z)+f_4(z)-Q_{13}e^{-H_0 z}+Q_{14}e^{-Scz},$$

$$F_i(z)=e^{-A_i z}\left(P_{i+8}\cos B_i z+Q_{i+8}\sin B_i z\right) \qquad i=1,2,3 \textit{ and } 4,$$

$$f_j(z)=e^{-A_j z}\left(Q_{j+8}\cos B_j z-P_{j+8}\sin B_j z\right) \qquad j=1,2,3 \textit{ and } 4.$$

The expressions for constant are given in appendix-I.

2.2.9 SKIN-FRICTION, RATE OF HEAT AND MASS TRANSFER

The non-dimensional skin-friction at the plate is

$$\tau=\left(\frac{dq_0}{dz}\right)_{z=0}+\varepsilon\left(\frac{dq_1}{dz}\right)_{z=0}e^{i\phi t}=\tau_p+i\tau_s. \qquad \text{.... (2.2.62)}$$

Hence, primary skin-friction$\left(\tau_p\right)$due to primary velocity and secondary skin friction is due to secondary velocity are given as

$$\tau_p=\tau_{op}+\varepsilon\left(J_1\cos\phi t+J_2\sin\phi t\right), \qquad \text{.... (2.2.63)}$$

$$\tau_s=\tau_{os}+\varepsilon\left(J_2\cos\phi t-J_1\sin\phi t\right). \qquad \text{.... (2.2.64)}$$

The rate of heat transfer at the plate in terms of Nusselt number $\left(Nu\right)$is

$$Nu=\left(\frac{dT_0}{dz}\right)_{z=0}+\varepsilon\left(\frac{dT_1}{dz}\right)_{z=0}e^{i\phi t}. \qquad \text{.... (2.2.65)}$$

Hence, the rate of heat transfer (considering real part only) is

$$Nu=-H_0+\varepsilon\left(G_1\cos\phi t-G_2\sin\phi t\right). \qquad \text{.... (2.2.66)}$$

which in terms of amplitude and phase, obtained as

$$Nu = -H_0 + \varepsilon |G| \cos(\phi t + \alpha).$$

The rate of mass transfer at the plate in terms of Sherwood number (S_h) is

$$S_h = \left(\frac{dC_0}{dz}\right)_{z=0} + \varepsilon \left(\frac{dC_1}{dz}\right)_{z=0} e^{i\phi t}. \qquad \text{.... (2.2.67)}$$

Hence, the rate of mass transfer is

$$S_h = H_0 R_0 - Sc(1 + R_0) + \varepsilon (N_{1r} \cos\phi t - N_{1i} \sin\phi t), \qquad \text{.... (2.2.68)}$$

which in terms of amplitude and phase, obtained as

$$S_h = H_0 R_0 - Sc(1 + R_0) + \varepsilon |N_1| \cos(\phi t + \beta).$$

The expressions for constant are given in appendix-II.

2.2.10 DISCUSSION AND CONCLUSION

In order to have physical insight into the problem we have calculated the numerical values of the primary and secondary velocity distribution, temperature distribution, concentration distribution, skin-friction, rate of heat transfer and rate of mass transfer. The effect of magnetic parameter (M),Grashof number (Gr), modified Grashof number (Gm), rotation parameter (E), rarefaction parameter (R), Prandtl number (Pr), Schmidt number (Sc), Thermal diffusion parameter (S_0), Heat source parameter (α_0), permeability parameter (K) and Suction parameter (A) on primary and secondary velocity distribution are shown graphically. In Fig. 2.2.7 and Fig. 2.2.8, the primary and secondary velocity distribution is plotted against z for Pr = 0.71, Sc = 0.3, S_0 = 1, K=10, α_0= 0.5, A= 0.2. As expected it reveals velocity jump near the plate and then decreases slowly and primary velocity decreases as M and E increases and increases as Gr, Gm and R increases but secondary velocity decreases as z increases and after attaining minimum value near the plate it increases as z increases. Secondary velocity v increases as M, E increases but near the plate behavior is opposite and v decreases as Gr, Gm and R increases. Fig. 2.2.9 and Fig. 2.2.10 show effect of Pr, Sc, S_0 , K and α_0 on primary and secondary velocity for fixed values of M, Gr , Gm , E and R. Primary velocity decreases as Pr, Sc, K and α_0 increases and decreases as S_0 and A increases. These effects are observed in opposite sense for secondary velocity. It is striking to note that in presence of He primary velocity is more than the presence of CO_2. The effects of all parameter on skin-friction are presented in tables 2.2.2 and 2.2.4. The effects of Sc, A, Pr, ϕ, t , α_0 and S_0 on rate of mass transfer is shown in table-2.2.3.The effects of A, Pr, ϕ., t and α_0 on rate of heat transfer is presented in table-2.2.5.

Table-2.2.2

Skin-friction due to primary velocity (at $\phi = 7$, $t = 2$ and $\epsilon = 0.01$)

Sl. No.	τ_p	Pr	Sc	M	K	α_0	Gr	Gm	E	S_0	A	R
1	5.41746	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
2	3.62861	7	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
3	5.30086	0.71	0.66	0.5	10	0.5	10	5	1	1	0.2	0.2
4	4.89957	0.71	0.3	1	10	0.5	10	5	1	1	0.2	0.2
5	5.50351	0.71	0.3	0.5	50	0.5	10	5	1	1	0.2	0.2
6	5.46284	0.71	0.3	0.5	10	1	10	5	1	1	0.2	0.2
7	8.60009	0.71	0.3	0.5	10	0.5	20	5	1	1	0.2	0.2
8	7.65324	0.71	0.3	0.5	10	0.5	10	10	1	1	0.2	0.2
9	3.93275	0.71	0.3	0.5	10	0.5	10	5	2	1	0.2	0.2
10	5.60372	0.71	0.3	0.5	10	0.5	10	5	1	2	0.2	0.2
11	5.41159	0.71	0.3	0.5	10	0.5	10	5	1	1	0.4	0.2
12	3.88020	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.6

Table-2.2.3

Rate of mass transfer in terms of Sherwood number (at $\epsilon = 0.01$)

Sl. No.	S_h	Sc	A	Pr	ϕ	t	α_0	S_0
1	0.043933	0.3	0.2	0.71	7	2	0.5	1
2	0.096618	0.7	0.2	0.71	7	2	0.5	1
3	0.043940	0.3	0.2	0.71	10	2	0.5	1
4	0.043784	0.3	0.2	0.71	7	5	0.5	1
5	0.124926	0.3	0.2	0.71	7	2	1	1
6	-0.084136	0.3	0.2	0.025	7	2	0.5	1
7	0.387953	0.3	0.2	0.71	7	2	0.5	2
8	0.388177	0.3	0.4	0.71	7	2	0.5	2

Table-2.2.4

Skin-friction due to secondary velocity (at $\phi = 7$, $t = 2$ and $\epsilon = 0.01$)

Sl. No.	τ_s	Pr	Sc	M	K	ϕ	Gr	Gm	E	S_0	A	R
1	-2.36531	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
2	-2.40522	7	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
3	-2.07032	0.71	0.66	0.5	10	0.5	10	5	1	1	0.2	0.2
4	-1.82312	0.71	0.3	1	10	0.5	10	5	1	1	0.2	0.2
5	-2.48214	0.71	0.3	0.5	50	0.5	10	5	1	1	0.2	0.2
6	-1.91970	0.71	0.3	0.5	10	1	10	5	1	1	0.2	0.2
7	-2.45194	0.71	0.3	0.5	10	0.5	20	5	1	1	0.2	0.2
8	-4.63711	0.71	0.3	0.5	10	0.5	10	10	1	1	0.2	0.2
9	-3.29854	0.71	0.3	0.5	10	0.5	10	5	2	1	0.2	0.2
10	-3.00924	0.71	0.3	0.5	10	0.5	10	5	1	2	0.2	0.2
11	-2.38147	0.71	0.3	0.5	10	0.5	10	5	1	1	0.4	0.2
12	-3.37970	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.6

Table-2.2.5

Rate of heat transfer in terms of Nusselt number (at $\epsilon = 0.01$)

Sl.No.	Nu	A	Pr	ϕ	t	α_0
1	-1.14666	0.2	0.71	7	2	0.5
2	-0.71972	0.2	0.025	7	2	0.5
3	-7.07928	0.4	7	7	2	0.5
4	-11.45400	0.4	11.4	7	2	0.5
5	-1.14666	0.2	0.71	10	2	0.5
6	-1.41663	0.2	0.71	7	2	1
7	-1.41712	0.4	0.71	7	2	1
8	-1.41505	0.4	0.71	7	5	1

Appendix-I

$$H_0 = \frac{\Pr + \sqrt{\Pr^2 + 4\alpha_0}}{2}; \qquad M_1 = M + \frac{1}{K};$$

$$H_1 = A_1 + iB_1 = \frac{1}{2}\left[\Pr + \sqrt{\Pr^2 + 4(\alpha_0 + i\phi \Pr)}\right];$$

$$H_2 = A_2 + iB_2 = \frac{1}{2}\left[Sc + \sqrt{Sc(Sc + i4\phi)}\right];$$

$$H_3 = A_3 + iB_3 = \frac{1}{2}\left[1 + \sqrt{1 + 4M_1 + i8E}\right];$$

$$H_4 = A_4 + iB_4 = \frac{1}{2}\left[1 + \sqrt{1 + 4M_1 + i4(2E + \phi)}\right];$$

$$R_0 = \frac{ScS_0H_0}{(H_0 - Sc)}; \qquad R_1 = P_1 + iQ_1 = \frac{A\Pr H_0}{H_0^2 + \Pr H_0 - (\alpha_0 + i\phi \Pr)};$$

$$R_2 = P_2 + iQ_2 = -\frac{H_0Sc(AR_0 + S_0R_1H_0)}{H_0^2 - Sc(H_0 + i\phi)}; \qquad R_3 = iQ_3 = -i\frac{ASc(1 + R_0)}{\phi};$$

$$R_4 = P_4 + iQ_4 = \frac{ScS_0H_1^2R_1}{H_1^2 - Sc(H_1 + i\phi)}; \qquad R_5 = P_5 + iQ_5 = R_3 - R_2 - R_4$$

$$R_6 = P_6 + iQ_6 = \frac{Gr - GmR_0}{H_0^2 - H_0 - M_1 - i2E}; \qquad R_7 = P_7 + iQ_7 = \frac{Gm(1 + R_0)}{Sc^2 - Sc - M_1 - i2E};$$

$$R_8 = P_8 + iQ_8 = \frac{R_6 + R_7 + R\{(Q_6 + Q_7)(B_3 + iA_3) + (H_0P_6 + ScP_7)\}}{(1 + A_3R)};$$

$$R_9 = P_9 + iQ_9 = \frac{GrR_1 - GmR_4}{H_1^2 - H_1 - M_1 - i(2E+\phi)};$$

$$R_{10} = P_{10} + iQ_{10} = \frac{GmR_5}{H_2^2 - H_2 - M_1 - i(2E+\phi)};$$

$$R_{11} = P_{11} + iQ_{11} = \frac{AR_8H_3}{H_3^2 - H_3 - M_1 - i2E};$$

$$R_{12} = P_{12} + iQ_{12} = \frac{1 + R_{10} + R_{13} - R_{11} - R_{14} - R_9}{(1 + A_4R)}$$

$$+\frac{R\left\{Q_{12}(B_4 + iA_4) - A_3P_{11} + B_3Q_{11} + H_0P_{13} - ScP_{14} + A_1P_9 - B_1Q_9 + A_2P_{10} - B_2Q_{10}\right\}}{1 + A_4R};$$

$$R_{13} = P_{13} + iQ_{13} = \frac{AH_0R_6 - GrR_1 - GmR_2}{H_0^2 - H_0 - M_1 - i2E};$$

$$R_{14} = P_{14} + iQ_{14} = \frac{GmR_3 - AScR_7}{Sc^2 - Sc - M_1 - i(2E+\phi)};$$

$$A_1 = \frac{\Pr}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(\Pr^2 + 4\alpha_0\right)^2 + 16\phi^2\Pr^2} + \left(\Pr^2 + 4\alpha_0\right)\right]^{\frac{1}{2}};$$

$$B_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(\Pr^2 + 4\alpha_0\right)^2 + 16\phi^2\Pr^2} - \left(\Pr^2 + 4\alpha_0\right)\right]^{\frac{1}{2}};$$

$$A_2 = \frac{Sc}{2} + \frac{1}{2\sqrt{2}}\left[Sc\left\{\sqrt{Sc^2 + 16\phi^2} + Sc\right\}\right]^{\frac{1}{2}};$$

$$B_2 = \frac{1}{2\sqrt{2}}\left[Sc\left\{\sqrt{Sc^2 + 16\phi^2} - Sc\right\}\right]^{\frac{1}{2}};$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 64E^2} + (1+4M_1)\right]^{\frac{1}{2}};$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 64E^2} - (1+4M_1)\right]^{\frac{1}{2}}.$$

$$A_4 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 16(2E+\phi)^2} + (1+4M_1)\right]^{\frac{1}{2}};$$

$$B_4 = \frac{1}{2\sqrt{2}}\left[\sqrt{(1+4M_1)^2 + 16(2E+\phi)^2} - (1+4M_1)\right]^{\frac{1}{2}};$$

$$P_1 = \frac{AH_0 \Pr a_1}{a_1^2 + b_1^2}; \qquad Q_1 = \frac{AH_0 \Pr b_1}{a_1^2 + b_1^2};$$

$$P_2 = -\frac{ScH_0\left[a_2 AR_0 + S_0 H_0 (a_2 P_1 - b_2 Q_1)\right]}{a_2^2 + b_2^2};$$

$$Q_2 = -\frac{ScH_0\left[b_2 AR_0 + S_0 H_0 (a_2 Q_1 + b_2 P_1)\right]}{a_2^2 + b_2^2}; \qquad Q_3 = -\frac{ASc(1+R_0)}{\phi};$$

$$P_4 = \frac{S_0 Sc\left[\left(A_1^2 - B_1^2\right)(a_3 P_1 - b_3 Q_1) - 2A_1 B_1 (a_3 Q_1 + b_3 P_1)\right]}{a_3^2 + b_3^2};$$

$$Q_4 = \frac{S_0 Sc\left[\left(A_1^2 - B_1^2\right)(a_3 Q_1 + b_3 P_1) + 2A_1 B_1 (a_3 P_1 - b_3 Q_1)\right]}{a_3^2 + b_3^2};$$

$$P_5 = -(P_2 + P_4); \qquad Q_5 = Q_3 - (Q_2 + Q_4);$$

$$P_6 = \frac{(Gr - GmR_0)a_4}{a_4^2 + 4E^2}; \qquad Q_6 = \frac{2E(Gr - GmR_0)}{a_4^2 + 4E^2};$$

$$P_7 = \frac{a_5 Gm(1+R_0)}{a_5^2 + 4E^2}; \qquad Q_7 = \frac{2EGm(1+R_0)}{a_5^2 + 4E^2};$$

$$P_8 = \frac{P_6 + P_7 + R\left\{Q_8 B_3 + \left(H_0 P_6 + ScP_7\right)\right\}}{\left(1 + A_3 R\right)}; \qquad Q_8 = Q_6 + Q_7;$$

$$P_9 = \frac{a_7\left(GrP_1 - GmP_4\right) + b_7\left(GrQ_1 - GmQ_4\right)}{a_7^2 + b_7^2};$$

$$Q_9 = \frac{a_7\left(GrQ_1 - GmQ_4\right) - b_7\left(GrP_1 - GmP_4\right)}{a_7^2 + b_7^2}; \qquad P_{10} = \frac{Gm\left(a_8 P_5 - b_8 Q_5\right)}{a_8^2 + b_8^2};$$

$$Q_{10} = \frac{Gm\left(a_8 Q_5 - b_8 P_5\right)}{a_8^2 + b_8^2}; \qquad P_{11} = \frac{A\left\{a_6\left(A_3 P_8 - B_3 Q_8\right) + b_6\left(A_3 Q_8 + B_3 P_8\right)\right\}}{a_6^2 + b_6^2};$$

$$Q_{11} = \frac{A\left\{a_6\left(A_3 Q_8 + B_3 P_8\right) - b_6\left(A_3 P_8 - B_3 Q_8\right)\right\}}{a_6^2 + b_6^2};$$

$$P_{12} = \left[\frac{\left(1 - P_{11} + P_{13} - P_{14} - P_9 + P_{10}\right)}{\left(1 + A_4 R\right)} + \frac{R\left(Q_{12} B_4 - A_3 P_{11} + B_3 Q_1 + H_0 P_{13} - ScP_{14} + A_1 P_9 - B_1 Q_9 + A_2 P_{10} - B_2 Q_{10}\right)}{\left(1 + A_4 R\right)}\right];$$

$$Q_{12} = Q_{10} + Q_{13} - Q_{11} - Q_{14} - Q_9;$$

$$P_{13} = \frac{\left[a_4\left(AH_0 P_6 - GrP_1 - GmP_2\right) - 2E\left(AH_0 Q_6 - GrQ_1 - GmQ_2\right)\right]}{a_4^2 + 4E^2};$$

$$Q_{13} = \frac{\left[a_4\left(AH_0 Q_6 - GrQ_1 - GmQ_2\right) + 2E\left(AH_0 P_6 - GrP_1 - GmP_2\right)\right]}{a_4^2 + 4E^2};$$

$$P_{14} = -\frac{a_5 AScP_7 + \left(2E + \phi\right)\left(GmQ_3 - AScQ_7\right)}{a_5^2 + \left(2E + \phi\right)^2};$$

$$Q_{14} = \frac{a_5\left(GmQ_3 - AScQ_7\right) - AScP_7\left(2E + \phi\right)}{a_5^2 + \left(2E + \phi\right)^2};$$

$$a_1 = H_0^2 + \Pr H_0 - \alpha_0; \qquad b_1 = \phi \Pr;$$

$$a_2 = H_0(H_0 - Sc); \qquad b_2 = \phi Sc;$$

$$a_3 = A_1^2 - B_1^2 - ScA_1; \qquad b_3 = ScB_1 + \phi Sc - 2A_1B_1;$$

$$a_4 = H_0^2 - H_0 - M_1; \qquad a_5 = Sc^2 - Sc - M_1;$$

$$a_6 = A_3^2 - B_3^2 - A_3 - M_1; \qquad b_6 = 2A_3B_3 - B_3 - 2E;$$

$$a_7 = A_1^2 - B_1^2 - A_1 - M_1; \qquad b_7 = 2A_1B_1 - B_1 - (2E + \phi);$$

$$a_8 = A_2^2 - B_2^2 - A_2 - M_1; \qquad b_8 = 2A_2B_2 - B_2 - (2E + \phi).$$

Appendix-II

$$J_1 = H_0P_{13} - P_{12} - A_3P_{11} + B_3Q_{11} - P_{14}Sc - A_1P_9 + B_1Q_9 + A_2P_{10} - B_2Q_{10};$$

$$J_2 = Q_{12} + B_3P_{11} + A_3Q_{11} - H_0Q_{13} + Q_{14}Sc + A_1Q_9 + B_1P_9 - B_2P_{10} - A_2Q_{10};$$

$$|N_1| = |N_{1r} + iN_{1i}|; \qquad \tan\beta = \frac{N_{1i}}{N_{1r}};$$

$$\tau_{op} = H_0P_6 - A_3P_8 + B_3Q_8 + ScP_7; \qquad \tau_{os} = H_0Q_6 - A_3Q_8 - B_3P_8 + ScQ_7;$$

$$G_1 = A_1P_1 - H_0P_1 - B_1Q_1; \qquad G_2 = A_1Q_1 - H_0Q_1 + B_1P_1;$$

$$G = G_1 + iG_2; \qquad |G| = |G_1 + iG_2|; \qquad \tan\alpha = \frac{G_2}{G_1};$$

$$N_{1r} = B_2Q_5 - A_2P_5 - H_0P_2 - A_1P_4 + B_1Q_4;$$

$$N_{1i} = A_2Q_5 + B_2P_5 + H_0Q_2 - ScQ_3 + A_1Q_4 + B_1P_4.$$

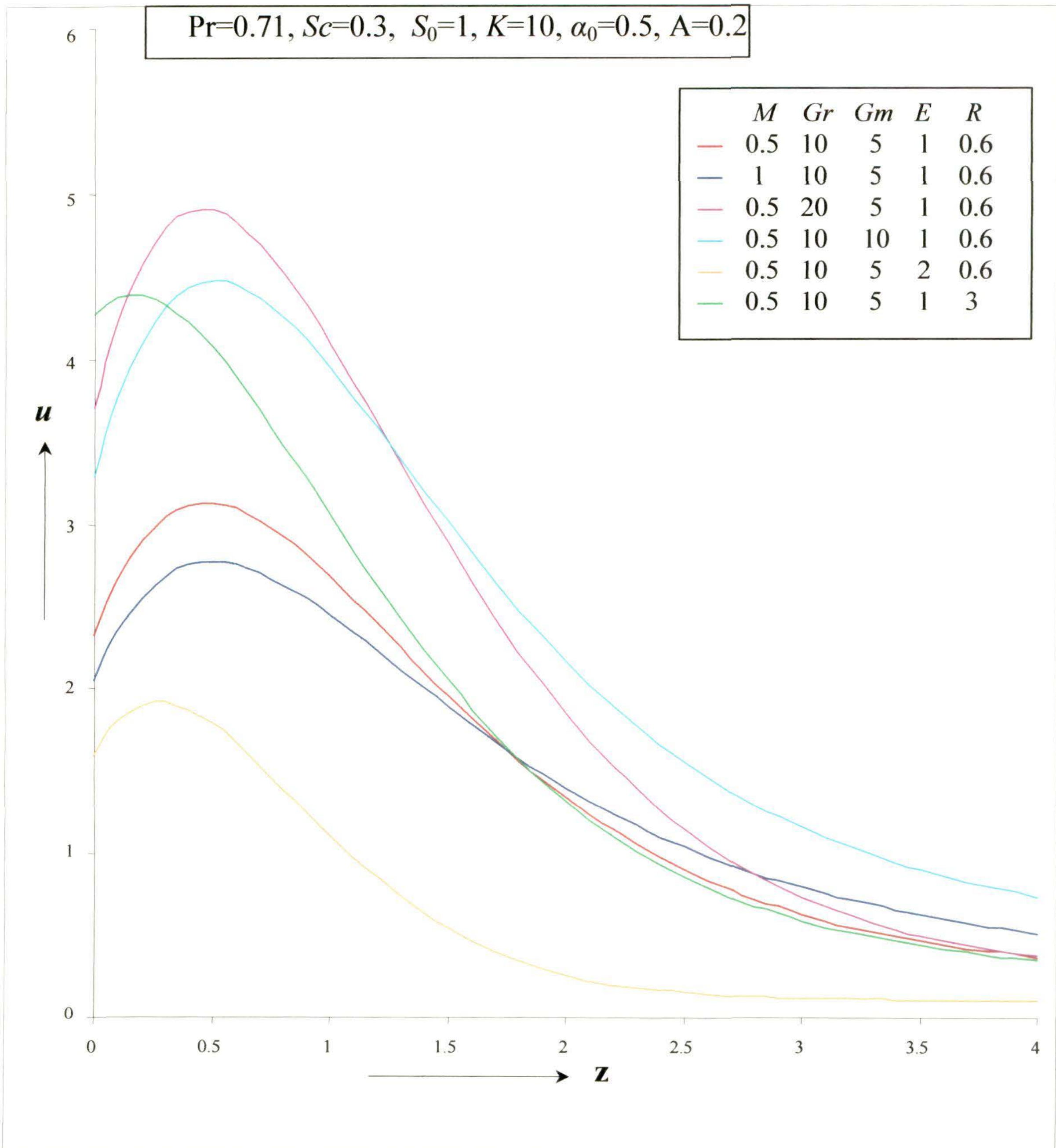

***Fig.* 2.2.7 Effects of *M*, *Gr*, *Gm*, *E* and *R* on the primary velocity field.**

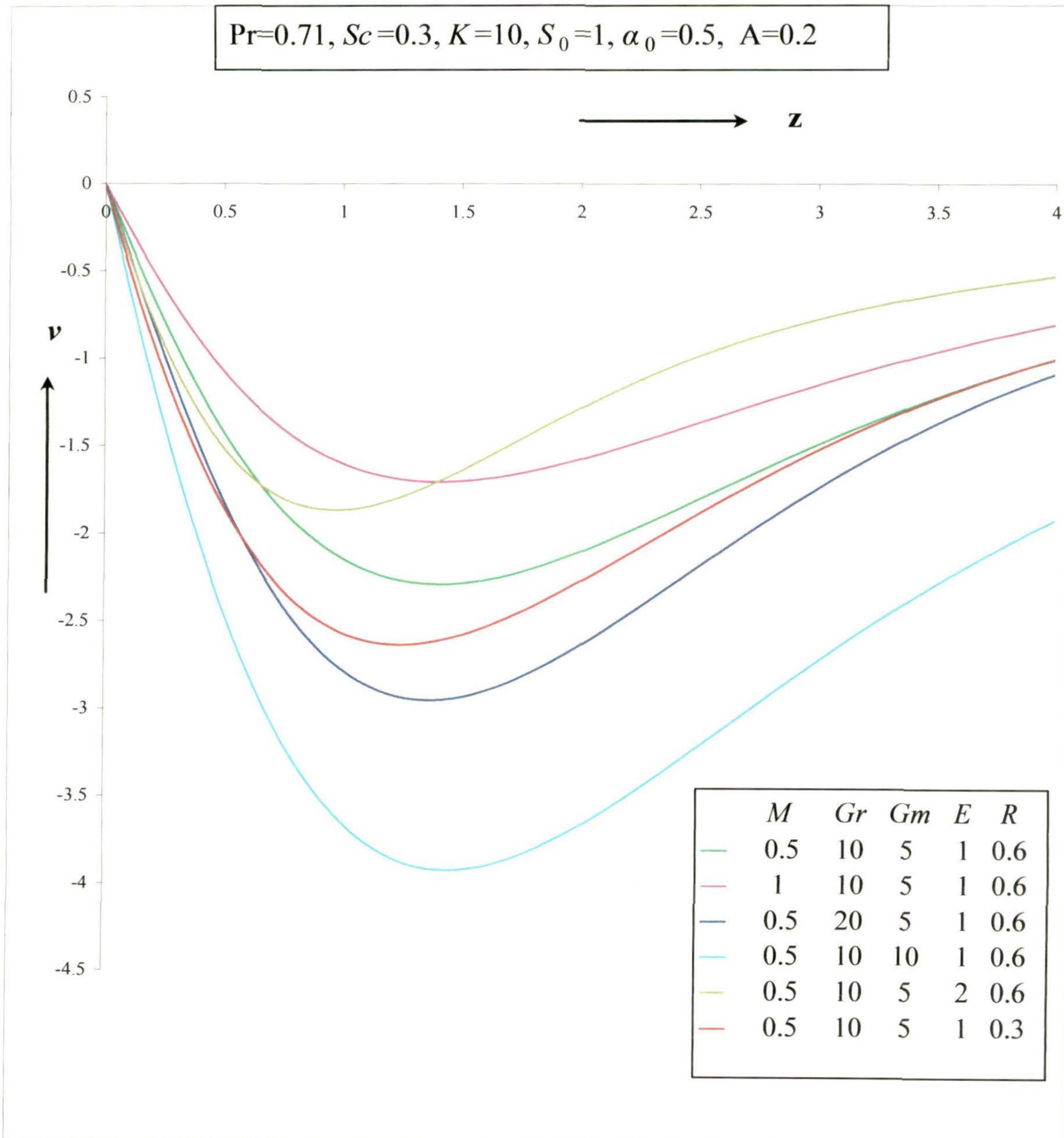

***Fig.* 2.2.8 Effects of *M*, *Gr* , *Gm* , *E* and *R* on the secondary velocity field.**

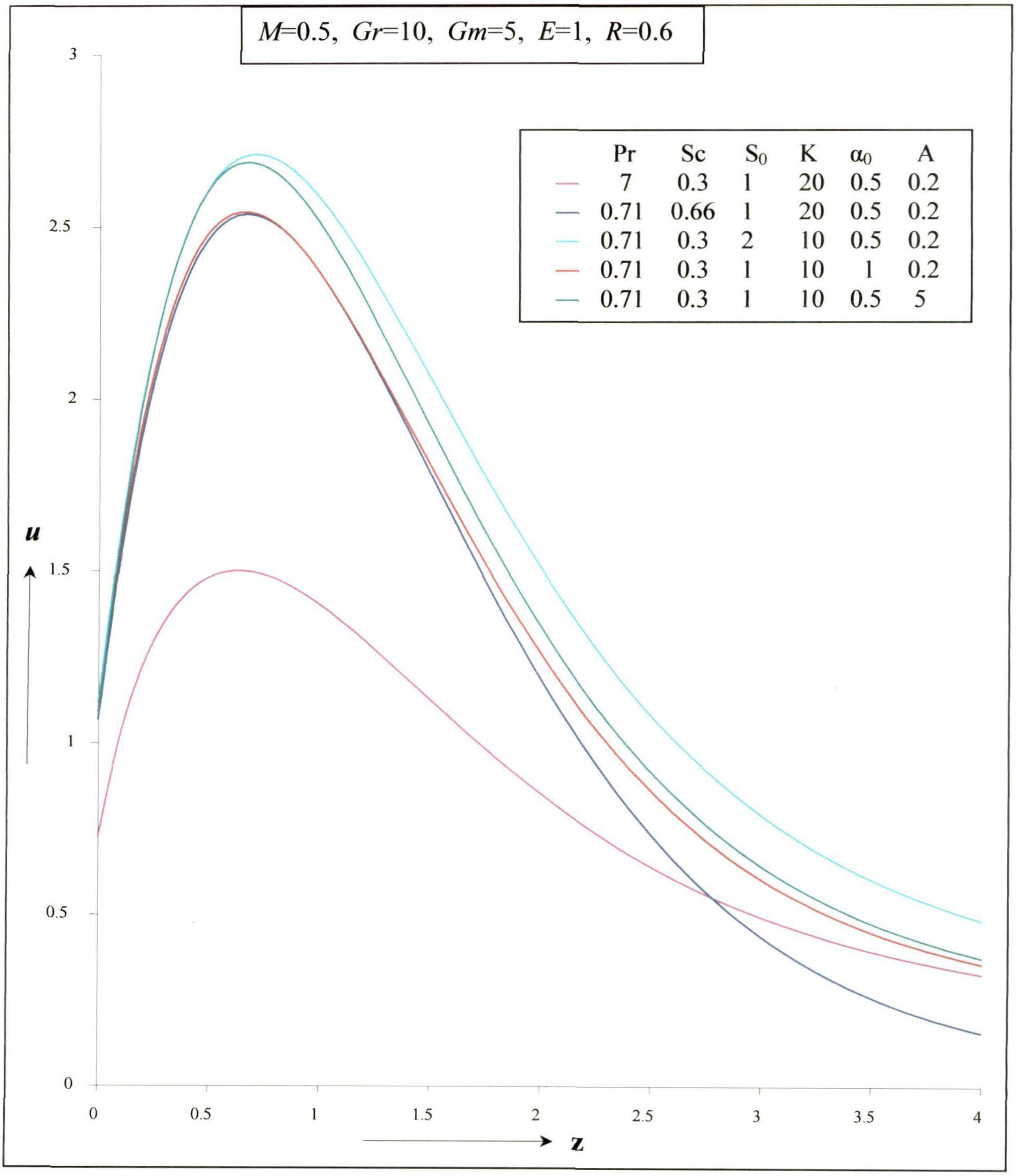

***Fig.* 2.2.9 Effects of Pr, *Sc*, *K*, S_0, α_0 and *A* on the primary velocity field**

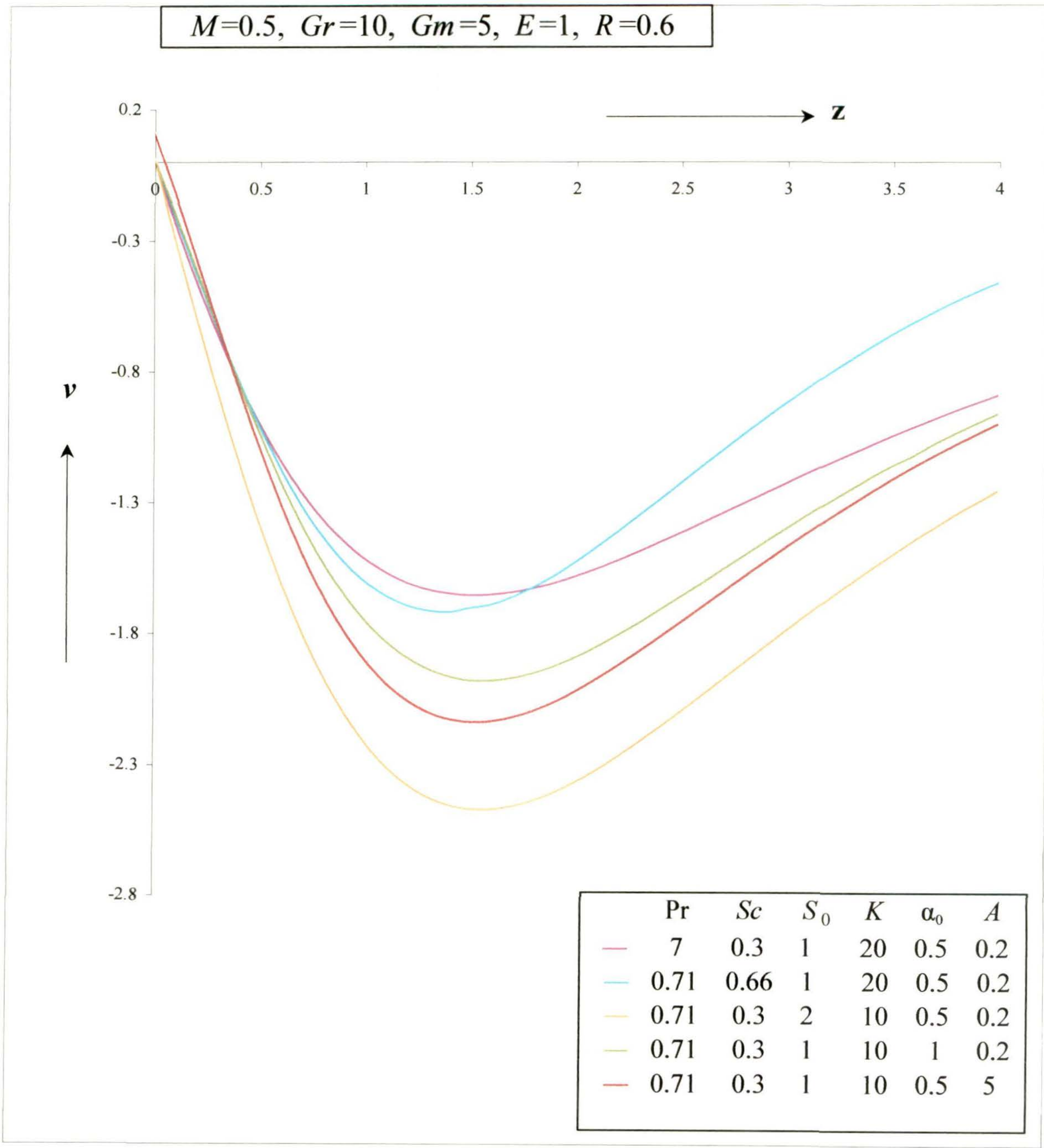

***Fig.* 2.2.10 Effects of Pr, *Sc*, *K*, S_0, α_0 and *A* on the secondary velocity field.**

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