CHAPTER TWO
SOME METHODOLOGICAL ISSUES

The main thrust of the present study is to measure current state of market volatility in historical perspective and provide a plausible explanation of this price movement. To investigate these different issues, seperate methodologies have been adopted in the present work, mention about which has been made independently in the following two sections.

Section I : Measurement of Stock Price Volatility

Any such study, at the outset, requires to develop a clear concept of "what volatility is ?", "how to measure it?", "which data sources to be used ?", and "what span of time to be considered ?". In a sense, this section deals with different methodological issues relevant to the measurement of share price fluctuation.

Volatility Defined

Volatility means changeability or randomness of asset prices. Theoretically a change in the volatility of either future cash flows or discount rates causes a change in the volatility of share prices. "Fads" or "Bubbles" introduce additional sources of volatility (Schwert, 1989).

From the finance literature, it appears that the operating definition of volatility is the relative dispersion of changes in stock prices relative to some average for a period (Jones and Wilson, 1989). Hence it is usually measured by the standard deviation or variance of stock prices or the rates of return.

Perception of both public and press about stock market volatility, in fact, is largely based on point changes. Finance academicians, however, widely agree that volatility should be measured in terms of percentage change in prices or rates of return, thus discard the use of absolute 'amount' of changes in asset prices. Point changes usually exceed percentage changes because the index levels from where the prices move are often greater than 100 (Jones and Wilson, 1989). Thus the point changes mostly favoured by press, overestimate and create a false impression regarding magnitude of volatility among investors. For obvious reason, present study also relies on percentage not on point changes of prices, the approach that is rightly followed by finance economists.

The Measurement

This section aims to examine the problem of estimating stock price volatility parameters from the most available forms of public data. We shall consider below only those volatility estimators that rely on data which are truly universal in their accessibility to investors, namely, the historical opening, closing, high and low prices appearing in the financial pages of different dailies. After evaluating all the parameters, we shall use the volatility estimate, which is most suitable for the nature of data readily available in our country.

In the literature of financial economics, there is the mention of a number of volatility estimators to measure share price fluctuation. All the measures (See Appendix) may, however, be conveniently clubbed into two broad groups : close-to-close standard deviation method and High-low estimator. Two widely used models representing these groups are discussed below.

1. Close-to-Close Standard Deviation Method

The most commonly used measure of stock return volatility is standard deviation based on close-to-close price data (Jones and Wilson, 1989; Schwert, 1990; Cho and Frees, 1988).

Let $(P_1, P_2, \ldots , P_{n+1})$ be a set of share prices quoted at the close of each trading day. Then the standard deviation of the rate of return (r) on share price, is traditionally estimated as follows : Letting $r_t = I_n(P_{t+1}/P_t)$, $t = 1, 2, \ldots, n$ = rate of return over the daily closing prices, then

$$\sigma = \left[1/n \sum_{t=1}^{n} (r_t - \bar{r})^2 \right]^{1/2}$$

2. High-Low Estimator

Market volatility can also be examined by using alternative estimators of volatility, in addition to the standard close-to-close daily price changes. As it is alleged that much of the recent volatility has been in intraday price movements rather than in day-to-day price changes (Edwards, 1988), intuition would tell us that high/low prices contain more information regarding volatility than do close-to-close prices. This intuition is correct. Parkinson (1980) had shown that simply using the daily high (H_t) and low (L_t) prices, the variance estimator $\hat{\sigma}_2^2 = [I_n(H_t) - I_n(L_t)]^2/4\, I_n\, 2$, had an efficiency of 5.2. It implies about 80% less data (and thus, an 80% smaller time interval) is needed for the extreme value method than for the traditional method (Parkinson, 1980). Thus the use of these extreme values (the high and low prices) provides a far superior estimate to the traditional close-to-close variance.

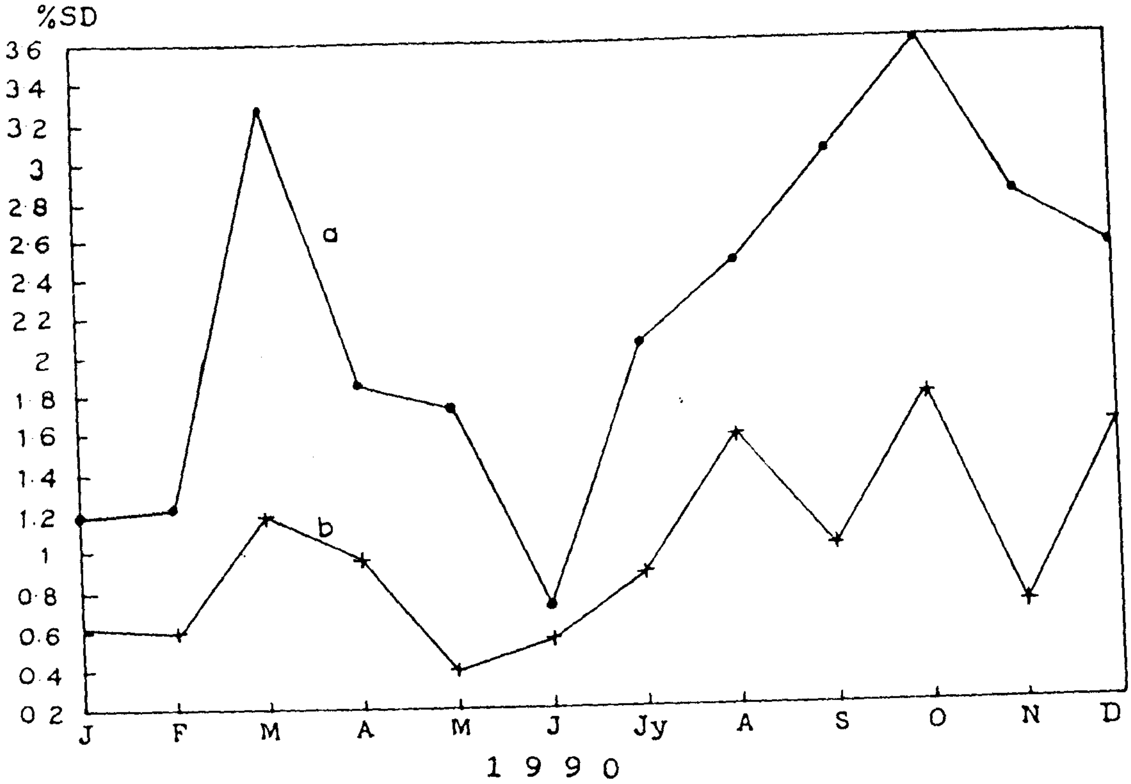
The principal advantage of the traditional close-to-close standard deviation is its simplicity of use. Closing prices are measured in a consistent fashion from period to period, and there is little question about the time interval being spanned by the estimator (Garman and Klass, 1980). The main disadvantage is, it ignores other readily available information that may be conveniently used to measure fluctuation more efficiently.

The High-low estimator, on the other hand, measures the intraday price movements and thereby reflects much more information. Hence this model may be considered as more efficient than and far superior to the traditional method. It is also very easy to apply in practice provided daily high and low prices are available. The following limitations may, however, blemish otherwise theoretically sound high-low estimator.

First, if the stock does not trade sufficiently there would be no significant difference between high and low prices. Secondly, one may expect bad data points show up more easily in the high and low price quotations. Finally, high and low prices may reflect trades by disadvantaged buyers or sellers and could therefore be less reliable indicators of the true value of the underlying security. Though in general the daily range should give us a better indication of the variability of the price, the aforementioned weaknesses could annihilate this advantage (Backers, 1983).

Whatever might be the strength and weakness of the two estimators, ultimate selection of a particular technique to be used truly depends on the availability of data. The relevant

Fig.2.1 Alternative Estimators of Volatility, Average Daily Share Price of Top 20 Companies by Turnover (January-December 1990).



a. Day-to-day price changes measured by standard deviation of $r_t = I_n (P_{t+1}/P_t)$

b. Intraday price movements measured by standard deviation of I_n (H/L). This intraday 'price range' estimator is closely related to Parkinson's high-low estimator and provides a more intuitive measure of volatility.

data on intraday high and low share price indices are not available. Hence in the present study close-to-close standard deviation method has been used which is mostly suitable for the data readily available in our country over decades. However for an illustrative purpose, comparative volatility measurement for the year 1990, using two alternative methods is displayed in Figure 2.1.

Two estimates have been measured from the daily high-low and closing prices of top 20 companies by turnover. The above figure suggests that intraday price range estimator is a better measure having lower variance than that of other (Beckers, 1983).

Data Sources

To measure stock market volatility, one commonly used series of data is the index number of share prices (Jones and Wilson, 1989; Harris, 1989; Schwert, 1989; 1990). Hence, the data used in the study are based on different share price indices reported by financial papers.

The sample data consist of mainly two sets. The first set comprises of the " 'Capital' Stock and Share Index", compiled and published by the CAPITAL on monthly basis for a period January 1935 to December 1960. The second one is the series of index number of share prices, namely, "The Economic Times Index Numbers of Ordinary Share Prices", compiled and published by The Economic Times on daily basis for a period 1961 to 1992. The data for the indices have been collected from the various issues of the CAPITAL and the Economic Times for the respective period.

Though share price index is the most commonly used series in any such study, it suffers from some limitations to be reckoned before using it. First, an index may give a completely false impression for the extent of price fluctuations (Kempt and Reid, 1971). This is because the process of averaging the price changes of different shares may wash out the individualistic portions of the price changes, leaving only the movement that is common to all shares. Secondly, Kendall (1953) has suggested, "Aggregative index numbers behave more systematically than their components. This might be due to the reduction of the random elements by averaging and the consequent emergence of systematic constituents; but it could equally well be due to chance".

Inspite of these limitations we have no alternative of the index number, specially for the measurement of aggregate share price movement. Hence we have used it as the means to have an end. One should, therefore, keep the above limitations in mind, while interpreting empirical results of our study.

Period Covered

Studies of similar nature in developed countries often consider a longer period of over 100 years in measuring volatility (Jones and Wilson, 1989; Harris, 1989; Schwert, 1989; 1990). But the present study of volatility measurement is confined to a period of only 58 years spanning over 1935-92. Selection of the specific period is primarily guided by some practical considerations, the most important of which is the availability of share price index. To the best of our knowledge, no reliable share price index was readily available before 1935. The CAPITAL started publishing price series on monthly basis since 1935, while The Economic Times daily share price index is available only from 1961. Since none of the price indices was solely available for the entire period, the above two series have been considered successively (1935-60 and 1961-92) to cover a relatively long period of time of nearly 6 decades. The period witnessed many big and small events directly or indirectly related with stock market activities.

Simply to measure volatility for a longer period, the validity of using two indices comprising of different portfolios may be questioned. At least there are two points to defend the approach of using two indices in our study.

1. No index maintain the same portfolio for a longer period of time. The portfolio is often reshuffled replacing active by inactive shares. Hence even the use of a single index gives no guarantee that the volatility estimate would be based on same set of shares for the entire period.

2. There is no evidence of significant variation in volatility estimates even when several alternative indices comprising of different portfolios have been used simultaneously to measure it (Schwert, 1989).

In view of the above facts, two indices that represent the most highly traded securities of the respective time period, may be conveniently used to cover a relatively longer period.

Section II : Volatility and Market Efficiency

Since in an efficient market, price changes at any time represent the efficient discounting of new information, movement of share prices should be reconciled with the variation in the expectation of future dividends. In this section we discuss the methodology to investigate whether stock price movement in our market can be attributed to subsequent changes in dividends. In the present study we have applied the simple efficient market model and variance inequality tests developed by Robert J.Shiller (1981) to examine the above research question[1].

The Efficient Market Model

According to the simple efficient market model, the real price P_t of a share at the beginning of the time period t is given by

$$P_t = \sum_{k=0}^{\infty} y^{k+1} \; E_t \; D_{t+k} \qquad 0 < y < 1 \qquad \ldots (1)$$

where D_t is the real dividend paid at (let us say, the end of) time t, E_t denotes mathematical expectation conditional on information available at time t, and y is the constant real discount factor = $1/(1 + r)$, where r is defined to be the constant real interest rate.

The model (1) can be restated in terms of series as a proportion of the long-run growth factor : $p_t = P_t \cdot \lambda^{T-t}$, $d_t = D_t \cdot \lambda^{T-t-1}$ where the growth factor is $\lambda^{T-t} = (1 + g)^{T-t}$, g is the rate of growth, and T is the base year. The growth factor λ^{T-t} has the effect of eliminating heteroscedasticity due to the gradually increasing size of the market. Multiplying (1) by λ^{T-t} and substituting we find

$$p_t = \sum_{k=0}^{\infty} (\lambda y)^{k+1} E_t d_{t+k}$$

$$= \sum_{k=0}^{\infty} \bar{y}^{k+1} E_t d_{t+k} \qquad \ldots \quad (2)$$

The growth rate g must be less than the discount rate r if (1) is to give a finite price, and hence $\bar{y} \equiv \lambda y < 1$, and defining $\bar{r}$ by $\bar{y} \equiv 1/(1 + \bar{r})$, the discount rate appropriate for the p_t and d_t series is $\bar{r} > 0$. The discount rate $\bar{r} = E(d)/E(p)$.[2]

The above model may be written in terms of the ex post rational price series p_t^* as follows -

$$p_t = E_t\ (p_t^*) \qquad \ldots (3)$$

Where $$p_t^* = \sum_{k=0}^{\infty} \bar{y}^{\,k+1} d_{t+k} = \bar{y}\,(p_{t+1}^* + d_t)$$

Since the summation extends to infinity, we never observe p_t^* without some error. However, with a long enough dividend series we may observe an approximate p_t^*. If we select an arbitrary value for the terminal value of p_t^* then we may determine p_t^* recursively by $p_t^* = \bar{y}\ (p_{t+1}^* + d_t)$ working backward from the terminal date. If we choose a different terminal condition, the result would be to add or substract an exponential trend from the p^*. As we move back from the terminal date, the importance of the terminal value chosen, however, declines (See Shiller, 1981).

In equation (3), p_t is the mathematical expectation conditional on all information available at time t of p_t^*. In other words, p_t is the optimal forecast of p_t^*. We can define the forecast error as $u_t = p_t^* - p_t$. It can be shown from the theory of conditional expectations that u_t must be uncorrelated with p_t. If u_t is correlated to p_t ,then the forecast can be improved.

Since the variance of the sum of two uncorrelated variables is the sum of their variances, we can write

$$\text{var}(p^*) = \text{var}(u) + \text{var}(p)$$

$$\text{or, } \text{var}(p) \leq \text{var}(p^*) \qquad \text{(since var}(u) \geq 0)$$

$$\text{or, } \sigma(p) \leq \sigma(p^*) \qquad \ldots (4)$$

The above inequality puts a limit on the standard deviation of p_t in terms of the standard deviation of p_t^*. The efficient market model may be restated in innovation form which allows better understanding of the limits on stock price volatility imposed by the model.

For this purpose, it is convenient to adopt notation for the innovation in a variable. Let us define the innovation operator $\delta_t \equiv E_t - E_{t-1}$ where E_t is the conditional expectations operator. Then for any variable x_t the term $\delta_t x_{t+k}$ $= E_t x_{t+k} - E_{t-1} x_{t+k}$ which is the change in the conditional expectation of x_{t+k} that is made in response to new information arriving between t-1 and t. Since conditional expectations operators satisfy $E_j E_k = E_{\min(j,k)}$ it follows that $E_{t-m} \delta_t x_{t+k} = E_{t-m}(E_t x_{t+k} - E_{t-1} x_{t+k}) = E_{t-m} x_{t+k} - E_{t-m} x_{t+k} = 0$, $m \geq 0$. This means that $\delta_t x_{t+k}$ must be uncorrelated for all k with all information known at time t-1 and must, since lagged innovations are information at time t, be uncorrelated with $\delta_{t'} x_{t+j}$, $t' < t$, all j, i.e., innovations in variables are serially uncorrelated.

The model implies that the innovation in price $\delta_t p_t$ is observable. Since (2) can be written $p_t = \bar{y}(d_t + E_t p_{t+1})$, we know, solving, that $E_t p_{t+1} = p_t / \bar{y} - d_t$. Hence

$\delta_t p_t \equiv E_t p_t - E_{t-1} p_t = p_t + d_{t-1} - p_{t-1}/\bar{y} = \Delta p_t + d_{t-1}$
$- \bar{r}\, p_{t-1}$.

The model also implies that the innovation in price is related to the innovations in dividends by

$$\delta_t p_t = \sum_{k=0}^{\infty} \bar{y}^{k+1} \delta_t d_{t+k} \qquad \ldots (5)$$

This expression is identical to (2) except that δ_t replaces E_t. Unfortunately, while $\delta_t p_t$ is observable in this model, the $\delta_t d_{t+k}$ terms are not directly observable, that is, we do not know when the public gets information about a particular dividend. Thus, in deriving inequalities below, one is obliged to assume the "worst possible" pattern of information accrual.

To find a limit on the standard deviation of δp for a given standard deviation of d_t, first note that d_t equals its unconditional expectation plus the sum of its innovations :

$$d_t = E(d) + \sum_{k=0}^{\infty} \delta_{t-k} d_t \qquad \ldots (6)$$

If we regard $E(d)$ as $E_{-\infty}(d_t)$, then this expression is just a tautology. It tells us, though, that d_t $t = 0,1,2, \ldots$ are just different linear combinations of the same innovations in dividends that enter into the linear combination in (5) which determine $\delta_t p_t$ $t = 0,1,2, \ldots$ Thus it can be asked how large $var(\delta p)$ might be for given $var(d)$. Since innovations are serially uncorrelated, we know from (6) that the variance of the sum is the sum of the variances :

$$\text{var}(d) = \sum_{k=0}^{\infty} \text{var}(\delta d_k) = \sum_{k=0}^{\infty} \sigma_k^2 \qquad \cdots (7)$$

The assumption of stationarity for d_t implies that $\text{var}(\delta_{t-k}\, d_t) \equiv \text{var}(\delta d_k) \equiv \sigma_k^2$ is independent of t.

In expression (5) we have no information that the variance of the sum is the sum of the variances since all the innovations are time t innovations, which may be correlated. In fact, for given σ_0^2 , σ_1^2 , ... , the maximum variance of the sum in (5) occurs when the elements in the sum are perfectly positively correlated. This means then that so long as $\text{var}(\delta d) \neq 0$, $\delta_t\, d_{t+k} = a_k \delta_t\, d_t$, where $a_k = \sigma_k / \sigma_0$. Substituting this into (6) implies

$$\hat{d}_t = \sum_{k=0}^{\infty} a_k \, \varepsilon_{t-k} \qquad \cdots (8)$$

Where $\hat{d}_t \equiv d_t - E(d)$ and $\varepsilon_t \equiv \delta_t\, d_t$. Thus, if $\text{var}(\delta p)$ is to be maximised for given σ_0^2, σ_1^2, ... , the dividend process must be a moving average process in terms of its own innovations. It can thus be shown, rather than assumed, that if the variance of δp is to be maximised, the forecast of d_{t+k} will have the usual ARIMA form as in the forecast popularized by Box and Jenkins (1970).

We can now find the maximum possible variance for δp for given variance of d. Since the innovations in (5) are perfectly positively correlated, var$(\delta p) = (\sum_{k=0}^{\infty} \bar{y}^{k+1} \sigma_k)^2$. To maximise this subject to the constraint var(d) $= \sum_{k=0}^{\infty} \sigma_k^2$ with respect to $\sigma_0, \sigma_1, \ldots$, one may use the Lagrangean :

$$L = \left(\sum_{k=0}^{\infty} \bar{y}^{k+1} \sigma_k\right)^2 + \lambda\left(\text{var}(d) - \sum_{k=0}^{\infty} \sigma_k^2\right) \cdots (9)$$

where λ is the Lagrangean multiplier. The first-order conditions for σ_j , $j = 0, \ldots \infty$ are

$$\frac{\delta L}{\delta \sigma_j} = 2\left(\sum_{k=0}^{\infty} \bar{y}^{k+1} \sigma_k\right) \bar{y}^{j+1} - 2\lambda\sigma_j = 0 \qquad \cdots \ (10)$$

Which in turn means that σ_j is proportional to $\bar{y}^j$. The second-order conditions for a maximum are satisfied, and the maximum can be viewed as a tangency of an isoquant for var(δp), which is a hyperplane in $\sigma_0, \sigma_1, \sigma_2, \ldots$ space, with the hypersphere represented by the constraint. At the maximum $\sigma_k^2 = (1-\bar{y}^2)$ var(d) $\bar{y}^{2k}$ and var$(\delta p) = \bar{y}^2$var(d)$/(1-\bar{y}^2)$ and so, converting to standard deviations for ease of interpretation, we have

$$\sigma(\delta p) \leq \sigma(d)/\sqrt{\bar{r}_2} \qquad \cdots \ (11)$$

Where $\bar{r}_2 = (1 + \bar{r})^2 - 1$

The maximum occurs, then, when d_t is a first order auto-regressive process, $\hat{d}_t = \bar{y}\, \hat{d}_{t-1} + \varepsilon_t$, and $E_t\, \hat{d}_{t+1} = \bar{y}^k\, \hat{d}_t$, where $\hat{d} \equiv d - E(d)$ as before.

Other inequality analogous to (11) can also be derived in the same way. We can put an upper bound to the standard deviation of the change in price (rather than the innovation in price) for given standard deviation in dividend. The only difference induced in the above procedure is that Δp_t is a different linear combination of innovations in dividends. Using $\Delta p_t = \delta_t p_t + \bar{r}p_{t-1} - d_{t-1}$ we find

$$\Delta p_t = \sum_{k=0}^{\infty} \bar{y}^{k+1} \delta_t d_{t+k} + \bar{r} \sum_{j=1}^{\infty} \delta_{t-j} \sum_{k=0}^{\infty} \bar{y}^{k+1} d_{t+k-1}$$

$$- \sum_{j=1}^{\infty} \delta_{t-j} \quad d_{t-1} \qquad \ldots (12)$$

As above, the maximisation of the variance of δp for given variance of d requires that the time t innovations in d be perfectly correlated (innovations at different times are necessarily uncorrelated) so that again the dividend process must be forecasted as an ARIMA process. However, the parameters of the ARIMA process for d which maximise the variance of Δp will be different. After maximising the Lagrangean expression [analogous to (9)] an inequality slightly different from (11), is given by

$$\sigma(\Delta p) \leq \sigma(d)/\sqrt{2\bar{r}} \qquad \ldots (13)$$

The upper bound is attained if the optimal dividend forecast is first-order autoregressive, but with an autoregressive coefficient slightly different from that which induced the upper bound to (11). The upper bound to (13) is attained if $\hat{d}_t = (1-\bar{r})\hat{d}_{t-1} + \varepsilon_t$ and $E_t\, d_{t+k} = (1-\bar{r})^k\, \hat{d}_t$, where, as before, $\hat{d}_t \equiv d_t - E(d)$.

The definitions and terminologies are illustrated in Table 2.1.

Table 2.1 : Definition of Principal Symbols

γ	= real discount factor for series before detrending; $\gamma = 1/(1+r)$
$\bar{\gamma}$	= real discount factor for detrended series : $\bar{\gamma} \equiv \lambda\gamma$
D_t	= real dividend accruing to stock price index (before detrending)
d_t	= real detrended dividend; $d_t \equiv D_t \cdot \lambda^{T-t-1}$
Δ	= first difference operator $\Delta x_t \equiv x_t - x_{t-1}$
δ_t	= innovation operator; $\delta_t\, x_{t+k} \equiv E_t x_{t+k} - E_{t-1}\, x_{t+k}$; $\delta x \equiv \delta_t x_t$
E	= unconditional mathematical expectation operator. $E(x)$ is the true (population) mean of x.
E_t	= mathematical expectations operator conditional on information at time t; $E_t x_t \equiv E(x_t/I_t)$ where I_t is the information set known at time t.
λ	= trend factor for price and dividend series; $\lambda \equiv 1+g$ where g is the long-run growth rate of price and dividends
P_t	= real stock price index (before detrending)
p_t	= real detrended stock price index; $p_t = P_t \cdot \lambda^{T-t}$
p_t^*	= ex post rational stock price index (expression 3)
r	= one-period real discount rate for series before detrending
$\bar{r}$	= real discount rate for detrended series; $\bar{r} = (1-\bar{\gamma})/\bar{\gamma}$
$\bar{r}_2$	= two-period real discount rate for detrended series; $\bar{r}_2 = (1+\bar{r})^2-1$
t	= time (year)
T	= base year for detrending and for wholesale price index.

As refered earlier, we apply the variance inequalities (4), (11) and (13) to test the stock price volatility of our market, using the data set given below.

Data Set and Time Period

Price series P_t used here is the average price of the portfolio representing 30 selected shares[3], for the period 1968 to 1991, divided by the average Wholesale Price Index of the respective year scaled to 1 in the base year 1991. The dividend series D_t is the average dividends for the calender year ending March, accruing to the portfolio represented by the shares considered in the price series, divided by the same yearly Wholesale Price Index scaled to 1 in the base year 1991. Both price and dividend series have been collected from various issues of The Stock Exchange Official Directory, Bombay.

The variable p_t for 1968-91 is the P_t times a scale factor λ^{T-t} which has the effect of eliminating heteroscedasticity due to the gradually increasing size of the market. The variable d_t for the corresponding period is the real total dividends D_t times λ^{T-t-1}.

Period of the study is confined to only 24 years spanning over 1968 to 1991. Admittedly, it is a short period for a study of this nature. However, there is no readily available data package required for this type of analysis, that restrains us for considering a longer period of time.

To maintain the data series P_t and D_t for 24 years, we have changed the portfolio time to time. Of course the volatility comparisons that will be made in the study have the advantage that the tests would not be affected, even if portfolios are occassionaly reshuffled, so long the volatility of the series is not misstated (Shiller, 1981).

Notes

1. In developing the variance inequality tests from Efficient Market Model, we have grossly used the writings of Robert J.Shiller (1981).

2. Taking unconditional expectations of both sides of equation (2) we find

$$E(p) = \frac{\bar{\gamma}}{1-\bar{\gamma}} E(d)$$

using $\bar{\gamma} = 1/(1+\bar{r})$ and solving we find $\bar{r} = E(d)/E(p)$.

3. Shares have been selected from those companies having financial year ending March.

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