

P R E F A C E

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Structural members are frequently used in various machine parts and the scope of study of bending properties of such members is sufficiently broad. Within elastic limit, various plate problems have been dealt with by numerous eminent scientists and research workers. All these problems may be classified as

- i) Static Problems,
- ii) Dynamic Problems and
- iii) Thermal Problems.

Any elastic behaviour (whether 'Static' or 'Dynamic' or 'Thermal') of plates is influenced by the following factors -

i) Material properties defined by Young's Modulus 'E' and Poisson's Ratio ' ν '. Both of 'E' and ' ν ' may be constant or variable.

ii) Geometry of the plate. Geometry of the plate may be simple such as Circular, Rectangular, Triangular, Elliptic etc., or may be complicated like Parabolic, Polygonal or Skewed one.

iii) Thickness of the plate 'h' which may be constant or varying as well.

iv) Types of loading such as Transverse (or Lateral) Loading, In-plane Loading, Concentrated Loading, Edge Loading, Combined Loading etc.

and

v) Nature of support i.e. edge conditions of the plate such as 'Simply-Supported' or 'Clamped' having 'Movable' or 'Immovable' states.

Whenever the deflections 'W' of a thin plate are small in comparison with its thickness 'h' a very approximate but satisfactory theory of bending of the plate by lateral loads can be developed by

making the following assumptions -

- i) there is no deformation in the middle plane of the plate and hence this plane remains 'Neutral' during bending,
- ii) points of the plate lying initially on a normal-to-the middle plane of the plate remain on the normal-to-the-middle surface of the plate after bending,
- iii) the stresses normal-to-the-middle plane of the plate, arising from the applied loading, can be disregarded.

These assumptions constitute the simplest and widely used "Classical Small Deflection Theory" or "Linear Theory", developed by Joseph Louis Lagrange in 1811. According to this theory all the stress components can be expressed by the deflection 'W' of the plate, which is a function of the two co-ordinates in the plane of the plate. This function has to satisfy a linear partial differential equation which together with the boundary conditions, completely defines 'W'. Thus the solution of this equation gives all the necessary information for calculating stresses at any point of the plate. Following the above-mentioned linear theory, different works on the bending of thin plates have been carried out by numerous research workers and the Bibliography of these works has been nicely incorporated in the book, "Theory of plates and Shells" by S. Timoshenko and S. Woinowski - Krieger (1962).

In most of these cases, bending of a plate is accompanied by strain in the middle plane, but, careful calculations show that the corresponding stresses in the middle plane are negligible if the deflections of the plate are small in comparison with its thickness. If the deflections are not small in comparison with the plate-thickness, these supplementary stresses in the middle plane of the plate must be taken into account in deriving the governing differential equations of the

plate. The differential equations so obtained become non-linear in character and their solutions are much more complicated.

With the advent of modern technology and systems exposed to oppressive operation conditions, the linear hypothesis could no longer be 'retained'. Whenever Forces, Deformations, Velocities, Temperatures and other factors become excessive, non-linear effects come into play and their influences can no longer be ignored. This situation occurs also in the particular field of Applied mechanics involving plates and shallow shells. These elements, when used in modern structures, such as "High-Speed Aeroplanes", "Missiles" and "Space-Vehicles", are often subject to large transverse deflections and reveal a clearly non linear response.

METHODS OF ATTACK OF THE NON-LINEAR PROBLEMS OF THIN PLATES :

So far there has been a wide application of the three types of differential equations for the non-linear analyses of thin plates as well as thick plates. They are

- i) Von-Kármán's Equations,
- ii) Berger's Equations and
- iii) Banerjee's Equations.

i) The first formulation of the theory of plates with a stretching of the middle plane and moderately large deflections was developed by Gehring (1860) and was improved upon by G. Kirchhoff (1883). The potential energy per unit area of the plates was written as a sum of a quadratic function of the quantities defining the Extension of the middle plane and a quadratic function of the quantities defining Bending. The equations of motion with finite displacements were deduced by the Principle of Virtual work. The stress function satisfying the in-plane

force equilibrium equations was introduced by A. Föppl (1907). The currently popular form of the two governing equations, in the rectangular cartesian co-ordinates, was given by T. Von-Kármán (1910). The equations of Von-Kármán are in the coupled form and involve Transverse Deflection and the Membrane Stress Function as unknown functions. These are difficult to solve. Several numerical methods have been employed by different investigators for solutions of Von-Kármán's equations.

Interesting Works On Von-Kármán's Equations

Among the authors who initially treated the non-linear analysis of plates, the works of S. Timoshenko (1937) and S. Way (1934, 1938) need special mention. Since then many authors investigated the different plate problems using Von-Karman's Equations. Useful works in this field are due to S. Levy (1942), S. Levy and S. Greenman (1942) W.Z.Chien(1947) Chi-Teh Wang (1948), H. N. Chu and G. Herrmann (1955, 1956), N.A.Weil and N. M. Newmark (1956), S. J. Medwadowski (1958), W. A. Nash (1959), W. A. Nash and I. D. Cooley (1959), N. Yamaki (1964), J. L. Nowinski (1962, 1963, 1964), A. M. Alwan (1964), J. L. Nowinski and I. A. Ismail (1965), G. Z. Harris and E. H. Mansfield (1967), J. B. Kennedy and Simon Ng. (1967), Robert Schmidt (1968), H. F. Bauer (1968), J. M. Whitney and A. W. Leissa (1969), M. Sathyamoorthy and K. A. V.Pandalai (1972), Richard Bolton (1972), J. L. Nowinski and H. Ohnabe (1973), K. Kanakeraju and C. R. V. Rao (1976), S. Dutta (1976, 1978), M.A.Sayed and R. Schmidt (1977), B. M. Karmakar (1978, 1979), M. Sathyamoorthy (1978, 1979), M. Sathyamoorthy and C. Y. Chia (1979, 1980), B.Banerjee and S.Dutta (1980), J. N. Reddy and W. C. Chao (1981), J.N.Reddy and C. L. Huang (1981), B. Banerjee (1982, 1983, 1984), S.K.Chaudhuri(1982, 1984), S. Das (1984), K. Kanakaraju and C.R.V.Rao (1986), P.C.Dumir(1988),

G. L. Ostiguy and N. Nguyen (1988), Yeh Kai-Yuan, Zheng Xiao-Jing and Zhou You-he (1989), M. Gorji (1989), H. Kobayashi and K. Sonoda (1989), K. M. Liew and K. Y. Lam (1990), J. W. Zhang (1991), B. Singh and S. Chakravorty (1991), H. Kobayashi and K. Sonoda (1991), U. S. Gupta R. Lal and S. K. Jain (1991), M. Ganapathi, T. K. Varadan and B. S. Sarma (1991), G. B. Chai (1991), and D. J. Gorman (1991).

All these workers solved the coupled form of Van-Kármán's equations by different numerical methods which are elegant but laborious

ii) H. M. Berger (1955) offered an approximate method for non-linear analyses of thin elastic plates by neglecting the second strain invariant in the expression for the total potential energy of the system. He investigated the large deflections of circular and rectangular plates both for clamped immovable and simply-supported immovable edges. Although no physical justification of his assumption was given in the paper, the results obtained in both the cases are in very good agreement with other known results. The speciality of Berger's equations is that the equations have been decoupled so that the solutions of the differential equations have been simplified and obtained in the closed form. Actually Berger's equations are linear in character. The essential non-linearity depends on the coupling parameter ' α '. These specialities have made Berger's equations very popular and many useful works on plates of immovable edges have been done following Berger's equations.

Useful Works On Berger's Equations

The comprehensive works on Berger's equations which need special mention are due to T. Iwinski and J. L. Nowinski (1957), M. L. Williams (1958), N. A. Nash and J. R. Modeer (1959, 1960), S. Basuli (1961), J. E. Hassert and J. L. Nowinski (1962), T. Wah (1963), S. N. Sinha (1963),

J. L. Nowinski and T. A. Ismail (1965), M. C. Pal (1967), B. Banerjee (1967, 1968, 1969), Cheng-Ih Wu and T. R. Vinson (1968, 1969), M. C. Pal (1969, 1970, 1973), M. Sathyamoorthy and K. A. V. Pandalai (1970, 1973, 1974), J. Mazumdar and R. Jones (1974), P. Biswas (1975), S. Dutta (1975, 1976), M. M. Banerjee (1976), N. Kamiya (1976), B. M. Karmakar (1977), J. Mazumdar and R. Jones (1977), S. Dutta (1976, 1977), M. Sathyamoorthy (1977, 1978), N. Kamiya (1978), B. Banerjee and S. Dutta (1979), B. Banerjee (1982), S. Das (1984) and T. Das (1986).

It is to be noted that Berger's equations are meaningful for immovable edge conditions only. It leads to meaningless results for movable edge conditions. This has been pointed out by J. L. Nowinski and H. Ohnabe (1972) in an excellent research work.

iii) In 1981, B. Banerjee and S. Dutta offered a modified strain energy expression to investigate non-linear problems of thin elastic plates. A set of decoupled differential equations are obtained under this modified strain energy expression. The authors have tested the accuracy of their equations by solving different non-linear plate problems. They have obtained sufficiently accurate results both for movable and immovable edge conditions. The equivalent hypothesis of Banerjee's equations is that the radial stretching of the plate is proportional to $(dw/dr)^2$. This is reasonable because under any type of loading and under any boundary condition the extra strain imposed by bending is represented by the term $(dw/dr)^2$. It is believed that Banerjee's equations are more welcome from the practical point of view because unlike Von-Kármán's equations, they are uncoupled and unlike Berger's equations they give reasonable results both for movable and immovable edge conditions.

Important Works on Banerjee's Equations

So far many useful works have been carried out on Banerjee's equations. Important works are due to B. Banerjee and S. Dutta (1981), B. Banerjee (1984), B. Banerjee and G. Sinharoy (1985, 1986) and S. K. Ghosh (1991) who have utilised Banerjee's equations in different non-linear thin plate problems under different types of loading and achieved satisfactory results.

Employing Banerjee's hypothesis on non-linear thick plates, R. Bhattacharjee and B. Banerjee (1988) have achieved satisfactory results under different types of loading on thick plates.

Utilising Banerjee's equations on skewed plates, satisfactory results were also obtained by A. Roy, B. Banerjee and B. Bhattacharjee (1991, 1992, 1993, 1994) for rhombic plates under different types of loading. Numerical results obtained in the above cases are interesting and useful from the practical point of view.

Works On Sandwich Plates

Investigations on finite deformations of sandwich plates are gaining importance day by day due to its wide applications in modern designs. Outstanding research workers who carried out interesting investigations in this field are Reissner (1948), Alwan (1964) and Nowinski and Ohnabe (1973)

Reissner proposed the basic differential equations for finite transverse deflections of sandwich plates under the assumptions that the plate consists of a core layer and two face layers of such construction that the face parallel stresses in the core and the variation of the face stresses over the thickness of the face layers are negligible.

The resultant equations permit the analysis of the effect of transverse shear stress deformation and transverse normal stress deformation in the core on the overall behaviour of the plate. It is shown that, in general, the effect of the transverse normal stresses in the core is negligibly small compared with the effect of the transverse shear stresses. The equations, when simplified by the omission of the transverse normal stress terms, are brought into a form suitable for the solution of rectangular-plate problems. It was further shown that the range of deflections for which the linear theory is adequate decreases in accordance with a simple explicit formula as the core is made softer relative to the faces. He used the finite deflection equations to obtain plate - buckling equations that include the effect of transverse shear stress deformation on buckling loads.

Alwan proposed differential equations for large deflection of sandwich plates with orthotropic core in terms of the Airy stress function. This work is very interesting and useful for modern design.

Nowinski and Ohnabe presented a generalization of Reissner equations for the isotropic plates to the case when the material of both the core and the faces is isotropic and of differing elastic characteristics. They derived a system of four governing equations involving the deflection W , stress function ϕ , and two functions, α and β , characterizing the shearings of the core. A transformation enables one to reduce the system to two equations involving two functions, W and ϕ only. For the isotropic faces the system reduces to two equations obtained by Alwan for large deflections of plates and to the equations of Cheng (1968) for small deflections. The integral analysis based on the application of Reissner-Hellinger variational principles is simpler than the five equations differential approach of Wempner and Baylor (1965). The authors obtained an explicit solution, for a simply supported rectangular

plate prevented from inplane motions on the contour, by means of Galerkin's procedure. Numerical calculations using UNIVAC 1108 computer was performed for various aspect ratios and the thickness of the plate as well as for several contrasting sets of elastic constants of orthotropic and isotropic materials. This paper is an attractive work on orthotropic sandwich plates. All these investigations involve non-linear partial differential equations in the coupled form. These equations are difficult to solve. Different numerical techniques have been employed to get desired solutions.

To get over the difficulties in the solutions of coupled differential equations for sandwich plates, N. Kamiya (1976) has employed Berger's well known techniques to solve non-linear problems of sandwich plates. The author has offered a new set of governing equations by using Berger's approximation to study the non-linear static behaviours of sandwich plates. He has analysed in detail the case of rectangular sandwich plates. The accuracy of his method depends on a correction factor $F(b/a)$. Thus although Kamiya has been able to simplify the coupled differential equations of sandwich plates proposed by earlier authors, his attempt has been restricted to a particular plate geometry due to introduction of this correction factor. More over Berger's method fails completely for movable edge conditions.

Later B. Karmakar (1978) also employed Berger's line of thought to investigate free vibrations of rectangular sandwich plates. The numerical results obtained in this study are interesting.

Large deflection analysis of heated sandwich plates is very important in modern design. Unfortunately work in this area has not received much attention so far. An attractive work in this field is due to Kamiya (1978) who has quite elegantly analysed the large deflections of

rectangular sandwich plates under thermal loading by using Berger-type equations. The author's analysis is useful. He has compared the results with those obtained from conventional governing equations solved by the Ritz-Galerkin method. These results are approximate due to Berger's approximation which is fairly accurate for immovable simply supported edges and fails for movable edges. Ritz-Galerkin's method also involves approximations.

Aim of The Present Project

The aim of the present project is to propose a new set of uncoupled differential equations in rectangular cartesian co-ordinate system to analyse the non-linear behaviours of sandwich plates. Banerjee's hypothesis (1981) has been employed. Accuracy of the results of different sandwich plates both for movable as well as immovable edge conditions has been tested under different types of loading and compared with other known results.

Later applying the hypothesis of the present study, A. Roy, S. Dutta & B. Banerjee (1993) have offered a new set of uncoupled differential equations to solve non-linear thermal behaviours of sandwich plates. The case of a rectangular plate has been carried out in detail. Numerical results of a simply-supported rectangular plate with movable as well as immovable edges have been given in graphs. The results obtained by the authors are interesting and useful for modern design.

The present study seems to be more advantageous because

- i) Results of the immovable as well as movable edge condition can be obtained from the same cubic equations with ease and accuracy.
- ii) The proposed differential equations are in decoupled form and hence results can be obtained employing minimum computational labour.

iii) The decoupled differential equations yield sufficiently accurate and useful results acceptable for the practical purpose, and (iv) previous investigations by Kamiya (1976) show that accuracy of the results depend on a correction factor, which is a function of the ratio of the sides of plate geometry, and hence this factor will vary according to the plate geometry. But the present study does not depend on any correction factor. This is an advantage of the present study.

The thesis is divided into four chapters.

The first chapter contains formulations of the governing equations for non-linear analyses of sandwich plates.

The second chapter contains the application of the governing equations to the sandwich plates under dynamic and static loading. The case of a rectangular plate has been discussed in detail.

The third chapter contains two problems on sandwich plates of orthotropic materials. The first paper of this chapter deals with the problem of large deflections of sandwich plates with orthotropic cores where the faces are isotropic. The second paper of this chapter is devoted to the large deflections of sandwich plates with orthotropic core and faces. The analysis of rectangular plates under uniform load has been carried out in both cases.

The fourth and the last chapter of this thesis contains two problems on polygonal sandwich plates. The first paper of this chapter contains the problem of large deflections of polygonal sandwich plates under uniform load. The second paper of this chapter deals with the problem of non-linear free vibrations of polygonal plates.

Numerical results for all the problems in different chapters of the present study have been computed, given in tables and compared with other known results. It has been observed that the results of the present study offer sufficiently accurate results for practical purpose.