Abstract

In this thesis, a scale invariant analysis on the set of real number system R is developed using the concept of relative infinitesimals , scale free infinitesimals and their corresponding non-archimedean valuation (absolute value). With this valuation we first extend the real number system R to an infinite dimensional non-archimedean system $\mathbf{R}$ accommodating infinitesimally small and infinitely large numbers. Next we determine the corresponding inversion mediated metric space $\mathcal{R}$ and interpret a directed variation of a real variable in a dynamical sense. Then applying the scale free analysis on $\mathcal{R}$, we present a new elementary proof of the well known Prime Number Theorem. Next this analysis is applied on a class of differential equations. We report in particular, some simple but nontrivial applications of this nonlinear formalism leading to emergence of complex nonlinear structures even from a linear differential system. These emergent nonlinear phenomena from a linear system is argued to offer, a new non-perturbative method for computing solutions and estimate amplitude, frequency etc. for a specific nonlinear system, viz, the Van der Pol equation. It is also shown that anomalous mean square fluctuations can arise naturally from the ordinary diffusion equation interpreted scale invariantly in the present formalism endowing real numbers with a non-archimedean multiplicative structure.