C H A P T E R - V.

BREAKDOWN OF GASES BY RADIO FREQUENCY ELECTRIC FIELD IN THE PRESENCE OF A SUPERIMPOSED PARALLEL D.C. ELECTRIC FIELD.

I N T R O D U C T I O N.

Varela (1947) observed that the breakdown potential in a discharge excited by a radiofrequency source increased when a d.c. potential less than the radiofrequency breakdown voltage was applied across the discharge tube. The discharge tube had aluminium electrodes in an atmosphere of 5 cm. hydrogen with an admixture of 20% argon. It was expected however, that the presence of the d.c. field would hasten ionisation with a given radiofrequency potential by virtue of higher peak electron velocities and that a residual potential following the discharge would hasten deionisation; but the results were contrary to what was expected. Almost the same results were obtained earlier by Kirchner (1925, 1947) while studying the breakdown in gases by an r.f. field in the presence of a d.c. potential.

Varnerin and Brown (1950) calculated theoretically the distribution function of electrons in an ionised gas in the presence of both radiofrequency and d.c. fields. It is generally known that the gas in the cavity will breakdown when the losses of electrons to the walls of the cavity are replaced by ionisation in the body of the gas. When an a.c. field is applied, electrons are lost by diffusion; when, in addition, a d.c. field is applied, electrons are lost both by diffusion and mobility. It has been shown by Varnerin and Brown that the new effective diffusion length $L_{d.c.}$ of the vessel in the presence of the d.c. field is shorter than its undisturbed diffusion length L according to the equation

$$1/L_{d.c.}^2 = 1/L^2 + \left\{ \bar{K} E_{d.c.} / 2D \right\}^2 \qquad \ldots(5.1)$$

where $\bar{K}$ denotes the mobility of the electron and D the diffusion coefficient. They thus concluded that the only difference between the breakdown condition

in the a.c. - d.c. case and the pure a.c. case is the substitution of a modified diffusion length $L_{d.c.}$ for the characteristic diffusion length L. That a greater breakdown field is necessary when the d.c. voltage is superimposed on the radiofrequency field was shown by Brown (1956) in the case of air at a pressure of 38 mm. of mercury where a d.c. field upto 200 volts/cm was applied.

No, systematic study of the breakdown of gases under the simultaneous action of the a.c. and d.c. fields has so far been reported. It is expected that this study will throw more light on the mechanism of breakdown.

The object of the present investigation is to determine the breakdown voltage in some gases in a superimposed radiofrequency and d.c. field and to present a theory capable of explaining the observed results. The whole investigation is divided in to two parts. In the first part the breakdown voltage under the simultaneous action of radiofrequency and d.c. field is measured for some rare gases (He, Ne, A) and oxygen at a constant pressure of 10 m.m. mercury. The applied d.c. voltage is varied from zero to 70 volts/cm . It has been found that the breakdown voltage is higher when both the fields are present than when the gas is excited by a radiofrequency field alone and the breakdown voltage gradually increases with the increase of the d.c. field. A theoretical expression for the breakdown voltage in the presence of d.c. and radiofrequency field was deduced from the theory of electrical discharge by Kihara (1952) utilising the expression for equivalent length deduced by Varnerin and Brown. The expression thus deduced was utilised to explain the experimental results. However this treatment did not include the contribution to ionisation made by the d.c. field because the field applied was small compared to the radiofrequency breakdown field. It was evident however from the experimental results obtained that the rate of rise of breakdown field with the applied d.c. field gradually becomes smaller as the value of the d.c. field is

increased. From this it was evident that the contribution to ionisation by the d.c. field was becoming dominant with higher d.c. fields and naturally if the d.c. field be sufficiently increased then the contribution to ionisation by the d.c. field will be considerable and the radiofrequency voltage to cause breakdown will gradually decrease. At this point a second set of experiments is performed on a.c. - d.c. combined breakdown phenomena to verify the above conclusion. The gases used were oxygen, air, carbon-di-oxide and hydrogen and results were obtained for different pressures in each gas (a few m.m. mercury) by gradually increasing the d.c. voltage at regular steps. The maximum d.c. voltage used is 240 volts per cm. The variable voltage radiofrequency field had the frequency 10.3 Mc/sec. In all cases studied, the radiofrequency voltage initially increases with applied d.c. voltage attains a maximum and then with the continous increase of d.c. field, the r.f. breakdown voltage gradually decreases.

Obviously the theoretical expression deduced to explain the results of first set of experiments was a partial success. It was pointed out that the discrepancy might be due (a) to uncertainty in the values of the molecular constants introduced by Kihara and (b) the effect of d.c. ionisation was not taken into consideration. Varnerin and Brown in finding the effect of d.c. field only considered the effects of increased diffusion and mobility but at high values of d.c. field the contribution due to d.c. ionisation should also be taken into consideration. Consequently a modified theoretical expression is deduced to explain the experimental results of the second set.

EXPERIMENTAL SETUP.

In the first part of the experiment, the discharge tubes of helium, neon, argon and oxygen have been supplied by the manufacturers in sealed condition at a certain pressure and each discharge tube is fitted with aluminium electrodes. The r.f. voltage and d.c. voltage are simultaneously applied to the discharge

tubes through series connected condensers and series connected r.f. chokes respectively as explained in chapter III. The d.c. field is varied from 5 volts/cm to 70 volts/cm. The length of the discharge tubes are as follows : helium-13 cm., neon -15 cm., argon - 13.2 cm., oxygen- 13 cm.

The second part of the experiment has got the identical setup as the previous one. The discharge tube is of length 3cm. and diameter 1.25 cm and fitted with two platinum electrodes with arrangement of stopcock to control the pressure of the gas enclosed. The gases studied in this investigation are oxygen, air, hydrogen and carbon-di-oxide. Air, oxygen and carbondioxide have been chosen in order to study the effect of electron attachment and the pressure is of the order of a few mms of mercury which has been measured with the help of a mercury manometer. The radiofrequency voltage has the frequency 10.3 Mc/s and the d.c. voltage applied is from few volts/cm to 240 volts/cm. Hydrogen and oxygen have been prepared by electrolysis of barium hydroxide solution. Carbon-dioxide has been obtained by the action of dilute sulphuric acid on sodium carbonate.

RESULTS AND DISCUSSION.

1st Part :- The variation of the ratio E_1/E_o (where E_1 is the breakdown voltage per cm in the presence of both the radiofrequency and d.c. fields and E_o is the breakdown voltage per cm in the presence of the radiofrequency field only) against $E_{d.c.}$ the applied d.c. voltage per cm is shown in figs.20-a, b, c, d. There is no detectable change in radiofrequency breakdown voltage for small d.c. voltages, which was less than 5 v/cm for all the gases studied; it is clear that with the application of d.c. field, the radiofrequency breakdown voltage increases and the nature of variation of E_1/E_o with $E_{d.c.}$ is practically the same for all the gases studied. The nature of the variation is the same as that observed by the previous workers.

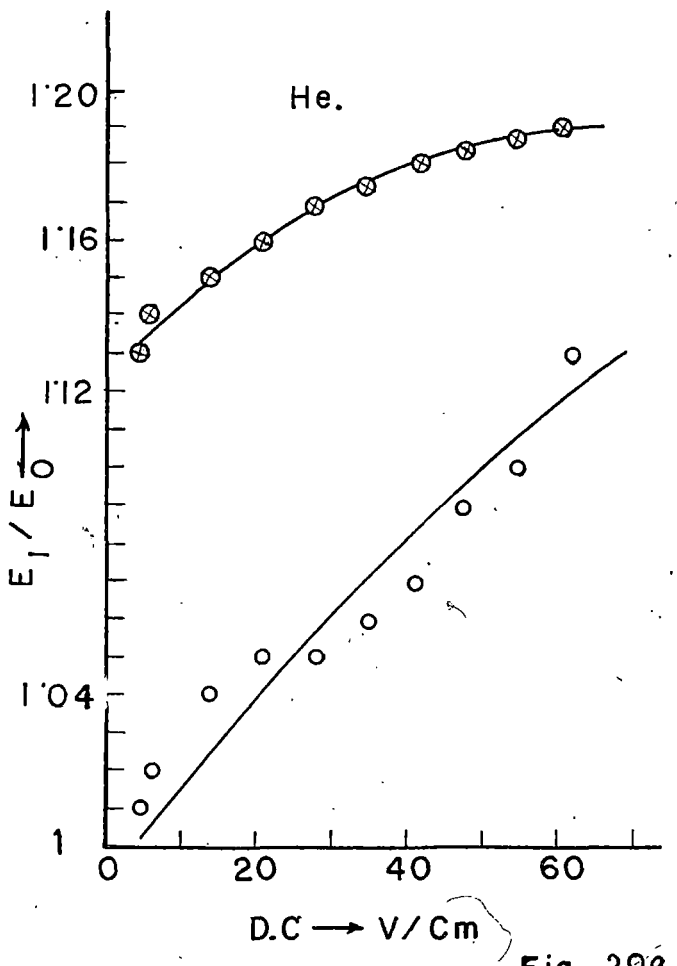

Fig-20a

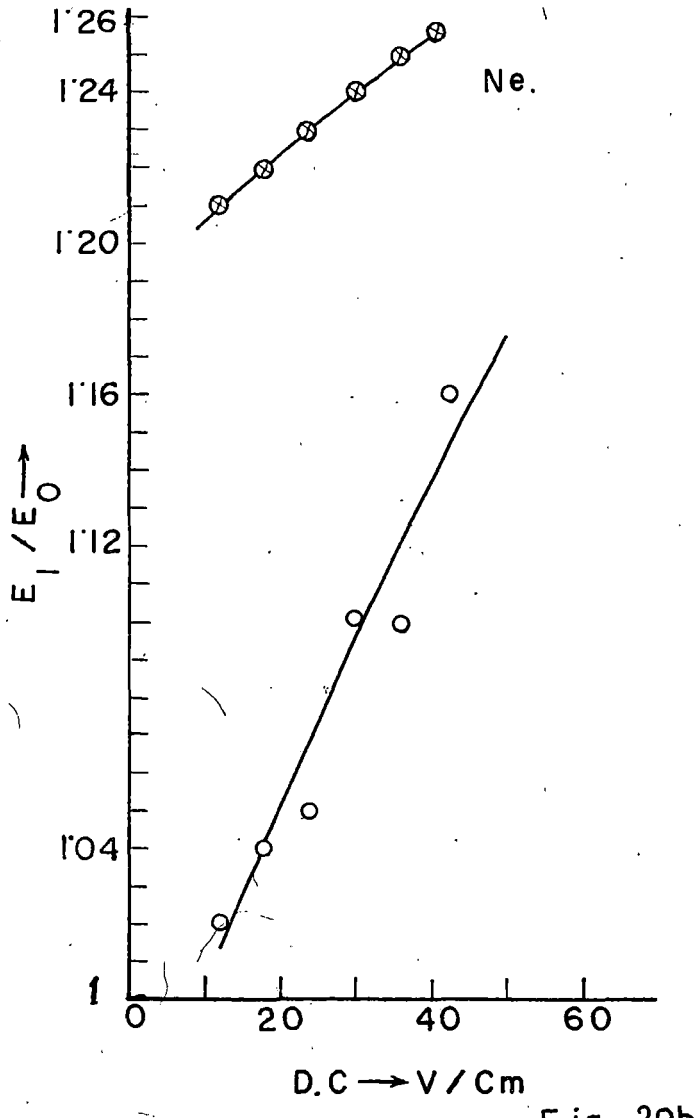

Fig-20b

[O : EXPERIMENTAL POINTS.]

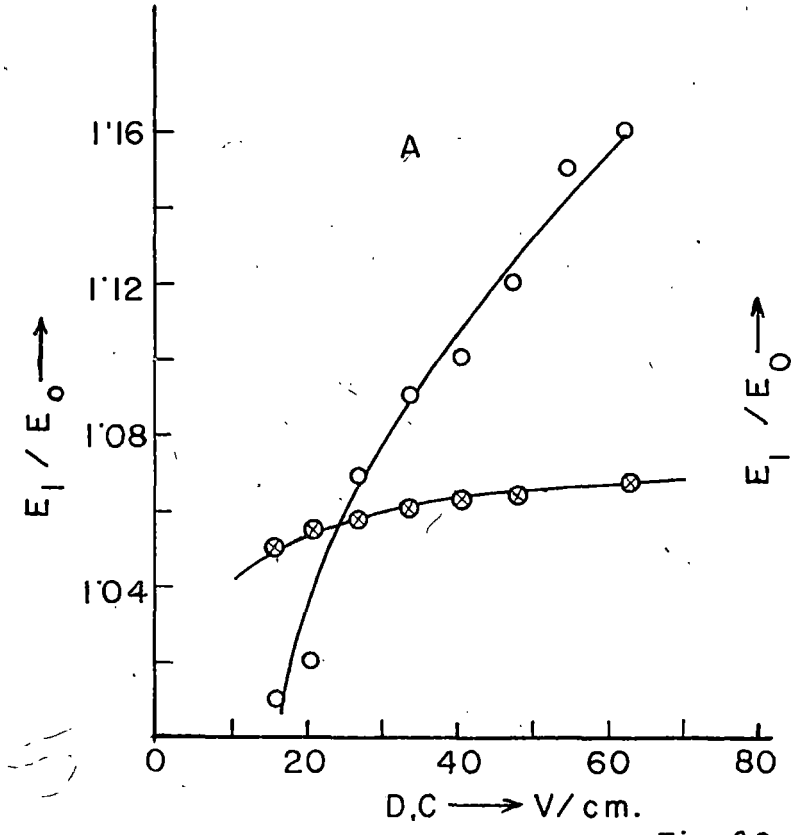

Fig-20c

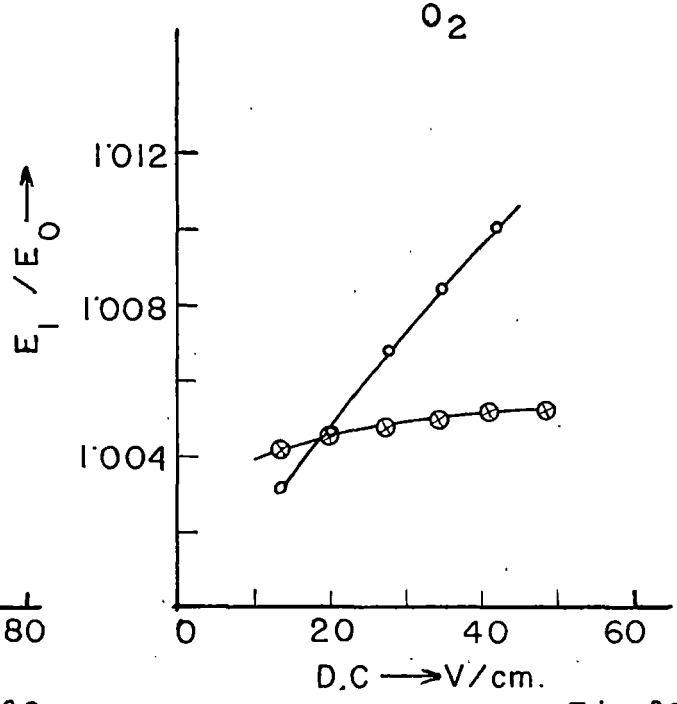

Fig-20d

[O : EXPERIMENTAL POINTS]

The physical significance of these results is at once apparent. In a r.f. field, breakdown occurs when the loss of electrons by diffusion and mobility is compensated by the generation of electrons by ionisation. The application of the d.c. field further increases the loss due to diffusion and mobility and hence a larger electric field must be applied to produce more ionisation to compensate for this increased loss. In the analysis which follows use has been made of the equations deduced by Kihara (1952) with the new diffusion length $L_{d.c.}$ introduced by Varnerin and Brown (1950).

If L_o denotes the equivalent length with the radiofrequency field alone, then according to Kihara (1952)

$$1/L_o^2 = (1/\pi^2)(\nu_o/D_o) \qquad \ldots (5.2)$$

where ν_o is the ionisation coefficient and D_o is the diffusion coefficient. When both the r.f. and d.c. fields are present, the breakdown condition is given by

$$1/L_{d.c.}^2 = (1/\pi^2)(\nu/D) \qquad \ldots (5.3)$$

where $$1/L_{d.c.}^2 = 1/L_o^2 + \left\{\bar{K} E_{d.c.}/2D\right\}^2 \qquad \ldots (5.4)$$

then $$1/L_o^2 + \left\{\bar{K} E_{d.c.}/2D\right\}^2 = (1/\pi^2)(\nu/D) \qquad \ldots (5.5)$$

where ν and D represent the quantities for the new breakdown field. According to Kihara (1952)

$$\nu_o/D_o = (3.\sigma.N^2\lambda/c_i)\exp\left[-mc_i^2/2K(T_e)_o\right]$$

$$\nu/D = (3.\sigma.N^2\lambda/c_i)\exp\left[-mc_i^2/2KT_e\right]$$

where σ, λ and c_i are the molecular constants introduced by Kihara and N is the number of molecules per c.c. at the pressure of the gas, K is the Boltzman constant, "m" is the mass of the electron and (T_e) is the electron temperature, which will be different for two breakdown voltages. Consequently, from equations (5.3) and (5.4) we get

$$\frac{1/L_o^2}{(1/L_o^2) + (\bar{K} E_{d.c.}/2D)^2} = \exp\left[(mc_i^2/2K)\left\{\frac{1}{T_e} - \frac{1}{(T_e)_o}\right\}\right] \quad \ldots(5.6)$$

Assuming that $T_e \approx (T_e)_o$ we get

$$\log\left[1 + \left(\bar{K} E_{d.c.} L_o/2D\right)^2\right] = -(mc_i^2/2K)\left\{\frac{(T_e)_o - (T_e)}{(T_e)_o^2}\right\} \quad \ldots(5.7)$$

But according to Kihara (1952)

$$\bar{K}/2D = e/K(T_e)_o$$

and $$K(T_e)_o = eE_o/N(6\lambda\rho)^{1/2}$$

where E_o = Peak r.f. breakdown voltage = (r.m.s. voltage). $2^{\frac{1}{2}}$

and similarly $$KT_e = eE_1/N(6\lambda\rho)^{1/2}$$

where E_1 = Peak r.f. breakdown voltage = (r.m.s. voltage) . $2^{\frac{1}{2}}$

Putting these values into equation (5.6) we get

$$\log\left[1 + \left\{eL_o E_{d.c.}/2K(T_e)_o\right\}^2\right] = N(6\lambda\rho)^{1/2}\left\{(E/E_o) - 1\right\} \quad \ldots(5.8)$$

According to Kihara $$L_o = L' - (2\bar{K}/\omega)E_o \quad \ldots(5.9)$$

where L′ is the length of the discharge tube and " ω " is the angular frequency of the applied r.f. field. Hence

$$\log\left[1+\left\{\frac{e\left[L'-(2\bar{K}/\omega)E_0\right]E_{d.c.}}{2K(T_e)_0}\right\}^2\right] = N(6\lambda\rho)^{1/2}\left\{(E_1/E_0)-1\right\}$$

$$\therefore\ E_1/E_0 = 1 + \frac{\log\left[1+\left\{\frac{e\left[L'-(2\bar{K}/\omega)E_0\right]E_{d.c.}}{2K(T_e)_0}\right\}^2\right]}{N(6\lambda\rho)^{1/2}}$$

.....(5.10)

Since the radiofrequency breakdown voltage is known from experimental results, $(T_e)_0$ can be calculated from the relation (Kihara 1952).

$$K(T_e)_0 = eE_0/N(6\lambda\rho)^{1/2}$$

The values of the mobility coefficient $\bar{K}$ for the value E_0/p have been obtained from the results reported by Brown (1959). In order to calculate the right hand side of equation (5.10), the following values have been calculated from the data of molecular constants given by Kihara (1952).

T A B L E - I.

$N = 3.55 \times 10^{17}$ P = 10mm. Hg. f = 10Mc/sec $\omega = 6.28 \times 10^7$ radians.

Gas	$\lambda\times10^8$ Cm^3/sec	$\rho\times10^{24}$ Cm.Sec.	$N(6\lambda\rho)^{1/2}$	E_0 Volts/cm	$K(T_e)_0\times10^{12}$	L' cm.	$\bar{K}\times10^{-6}$	$\frac{e[L'-(2\bar{K}/\omega)E_0]}{2K(T_e)_0}$	$(\lambda\rho)\times10^{32}$ cm^4
Argon	7.8	1.35	282.1	16.28	0.0953	13.3	0.0568	114.1	
Helium	4.4	0.20	81.58	14.61	0.2902	13	0.0821	36.17	
Neon			52.96	17.3	0.517	15	0.1329	22.86	0.3710
Oxygen			726.6	23	0.0512	13	0.1304	203.8	8.356

The right hand side of equation (5.10) can thus be calculated for different values of $E_{d.c.}$. The results are plotted side by side with the experimental curves. The quantitative agreement is not satisfactory except for argon and oxygen and also for values of $E_{d.c.}$ near about 20 v/cm. There is no agreement at all for helium and neon. The quantitative disagreement can be attributed partly to constants introduced by Kihara. The nature of the theoretical curve resembles the experimental curve to a certain extent, but the rate of rise as predicted from the theoretical deduction is smaller than that obtained experimentally. It should be noted however, that in the above deduction two effects have not been considered viz. (a) increased loss due to diffusion because of the presence of positive ions and (b) the possible ionisation contributed by the d.c. field itself. The contribution by the second factor to the theoretical computation of breakdown voltage would, however, decrease the actual value, because if there is any ionisation caused by the d.c. field, it would tend to lower the value of breakdown field. Whereas this factor may lower the values of (E_1/E_0) somewhat in the case of helium and neon, it will reduce still further the values of (E_1/E_0) in the case of argon and oxygen where the theoretical values are actually lower than the experimental values. Further more in the case of oxygen the contribution due to the attachment of electrons have not been taken into consideration, and this actually modifies the expression for breakdown voltage. The increased loss due to diffusion in the presence of positive ions may be responsible for the higher values obtained experimentally.

The theoretical and experimental results point, however, to the fact that in a discharge excited by the simultaneous action of radiofrequency and d.c. voltage, the loss of electrons is due both to diffusion and to mobility and by employing the deductions of Kihara with the expression for the equivalent length as deduced by Varnerin and Brown, it has been possible to deduce results which can be compared with the experimentally observed values. Though the evobe

treatment is incomplete in many sense, yet its success with so many uncertainities could not be ignored. So in the second part of the experiment, attention was paid to different drawbacks in this theoretical expression and the theory has been modified in the light of knowledge gathered by this result.

2nd Part :- In continuation of work done in the first part the variation of radiofrequency breakdown voltages with the applied d.c. field has been plotted in Figs. 21, 22, 23, 24, in case of air, hydrogen, oxygen and carbon-di-oxide respectively. The ordinate represents the ratio E_2 / E_o where E_2 is the radiofrequency breakdown voltage when both the radiofrequency and d.c. fields are present and E_o is the breakdown voltage when only the radiofrequency voltage is present. It is seen from the nature of the curves that the breakdown voltage gradually increases with the increase of d.c. field, then attains a maximum at a certain d.c. field which is different for different gases, and then falls as the d.c. field is further increased. A well defined maximum is obtained in each case and the variation of breakdown voltage with the d.c. field is quite general as it has been observed for all the four gases studied. It is thus quite apparent that whereas the theory of Varnerin and Brown (1950) can explain the increase of breakdown voltage when the d.c. field is small as due to increased loss due to mobility in the presence of d.c. field, the occurance of maxima and the consequent fall in breakdown voltage show that other mechanisms are also operating.

To explain the observed results it is therefore suggested that, whereas for small d.c. fields, d.c. ionisation can be neglected, the effect of d.c. ionisation has to be taken into consideration when the d.c. voltage is of the order employed in the present investigations. The effect of d.c. voltage is also to increase the mobility and thereby cause an increased loss of electrons. In the calculation which follows, it will first be considered that the ionisation

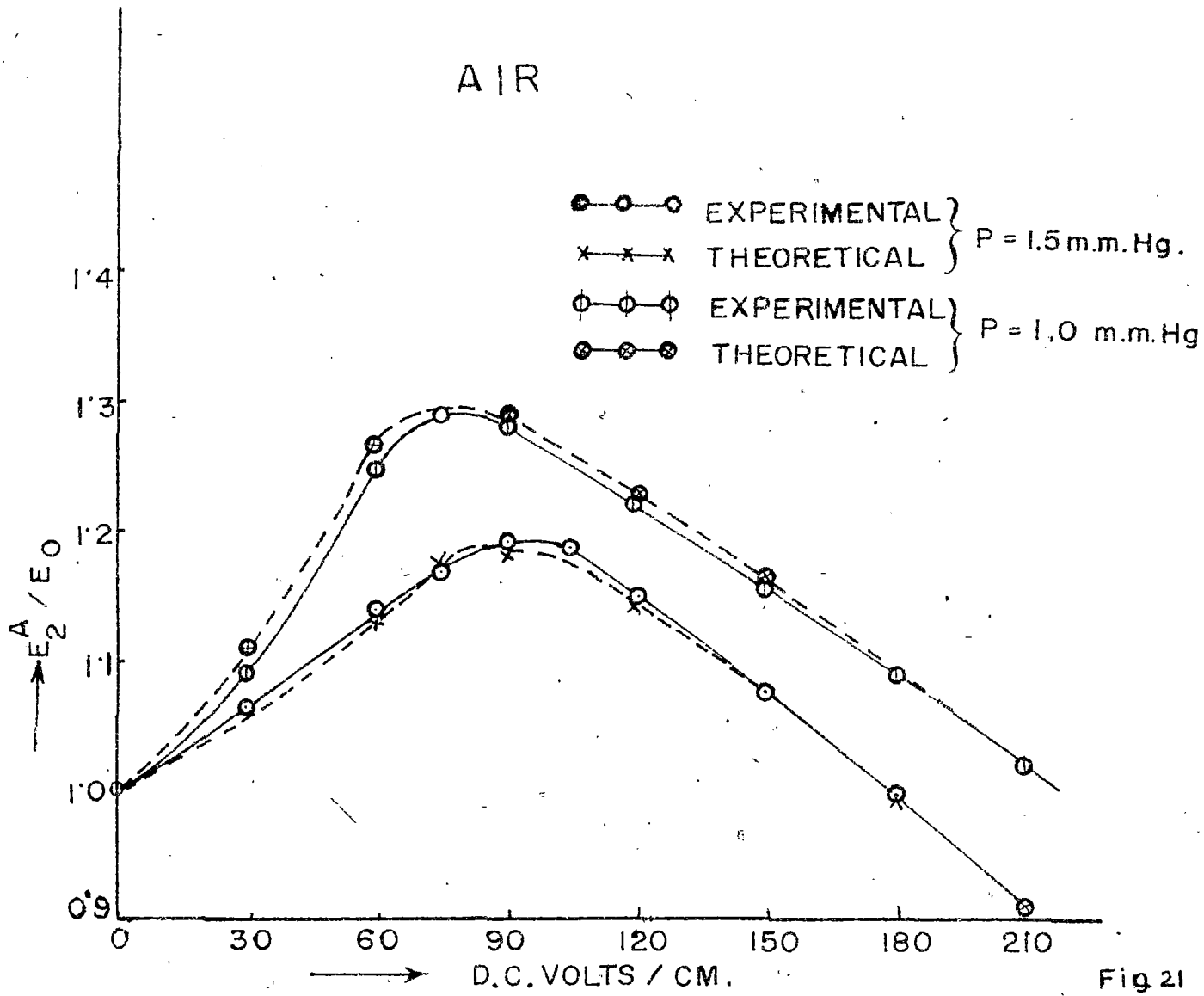
AIR
EXPERIMENTAL
THEORETICAL
P = 1.5 m.m.Hg.
EXPERIMENTAL
THEORETICAL
P = 1.0 m.m.Hg.
1·4
1·3
1·2
1·1
1·0
0·9
E_2^A / E_0
0
30
60
90
120
150
180
210
D.C. VOLTS / CM.

Fig. 21

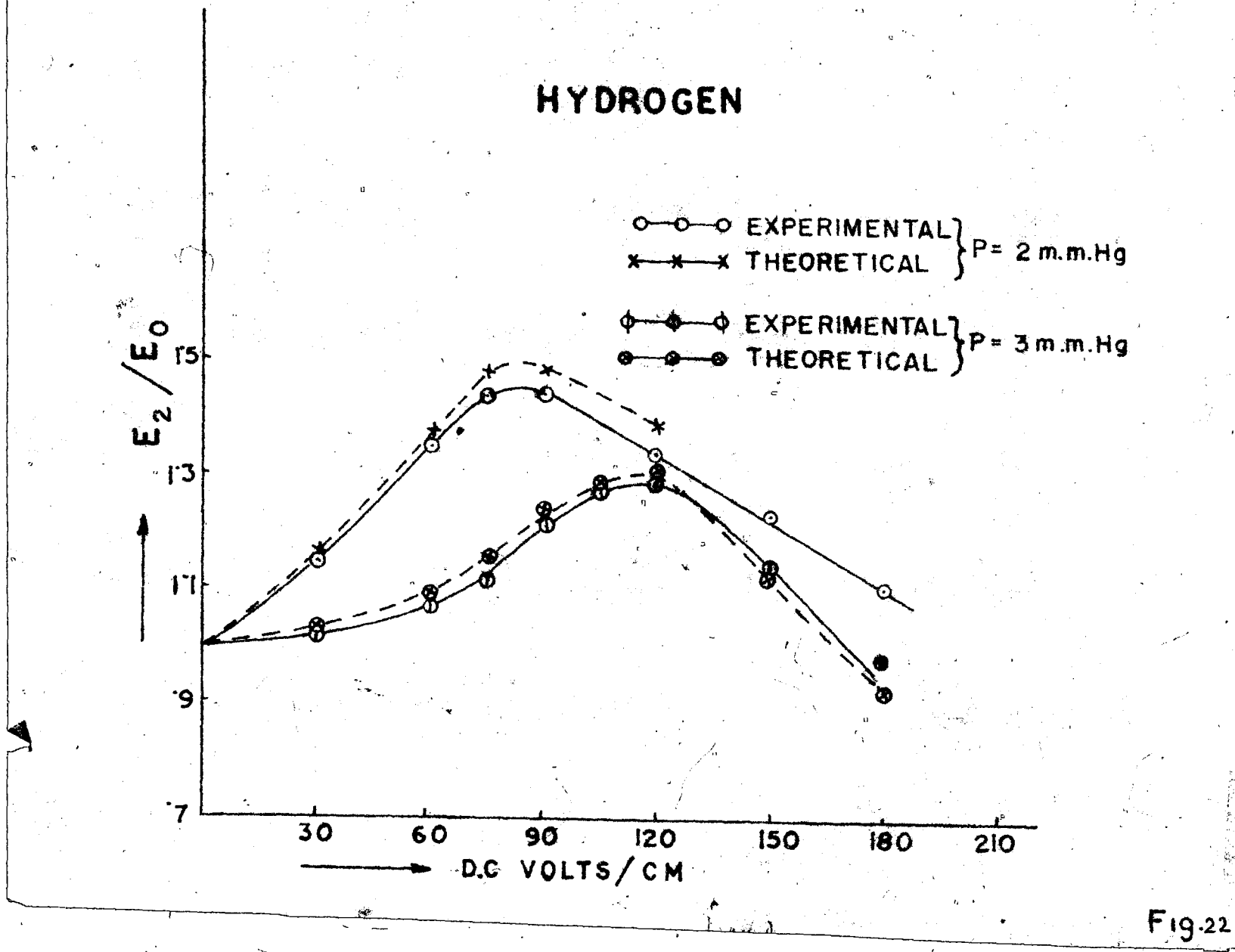
HYDROGEN
EXPERIMENTAL
THEORETICAL
P= 2 m.m.Hg
EXPERIMENTAL
THEORETICAL
P= 3 m.m.Hg
E2 / E0
1·5
1·3
1·1
·9
·7
30
60
90
120
150
180
210
D.C VOLTS / CM

Fig 22

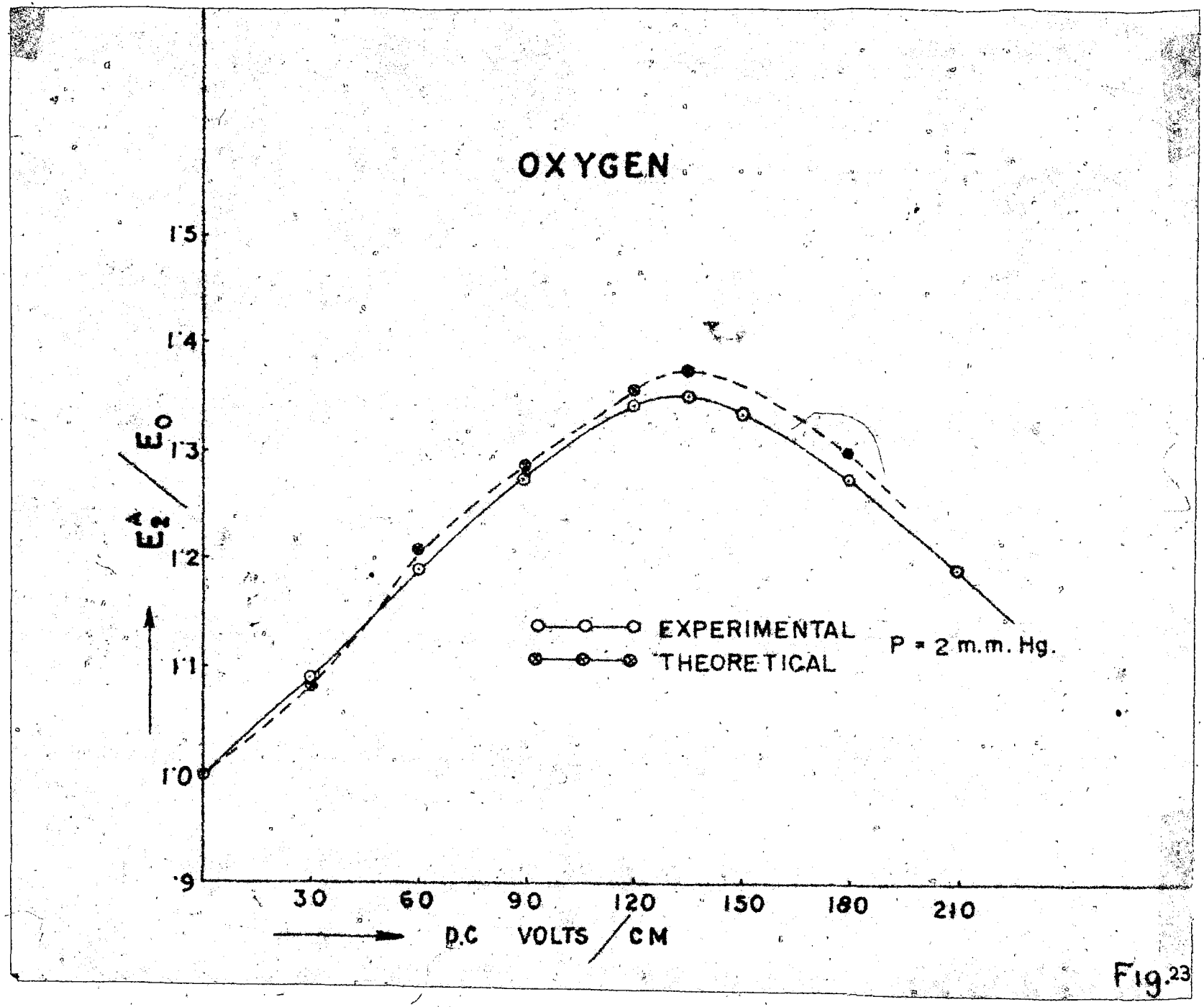
OXYGEN
1·5
1·4
1·3
1·2
1·1
1·0
·9
E^A / E^0
EXPERIMENTAL
THEORETICAL
P = 2 m.m. Hg.
30
60
90
120
150
180
210
D.C VOLTS / CM

Fig.23

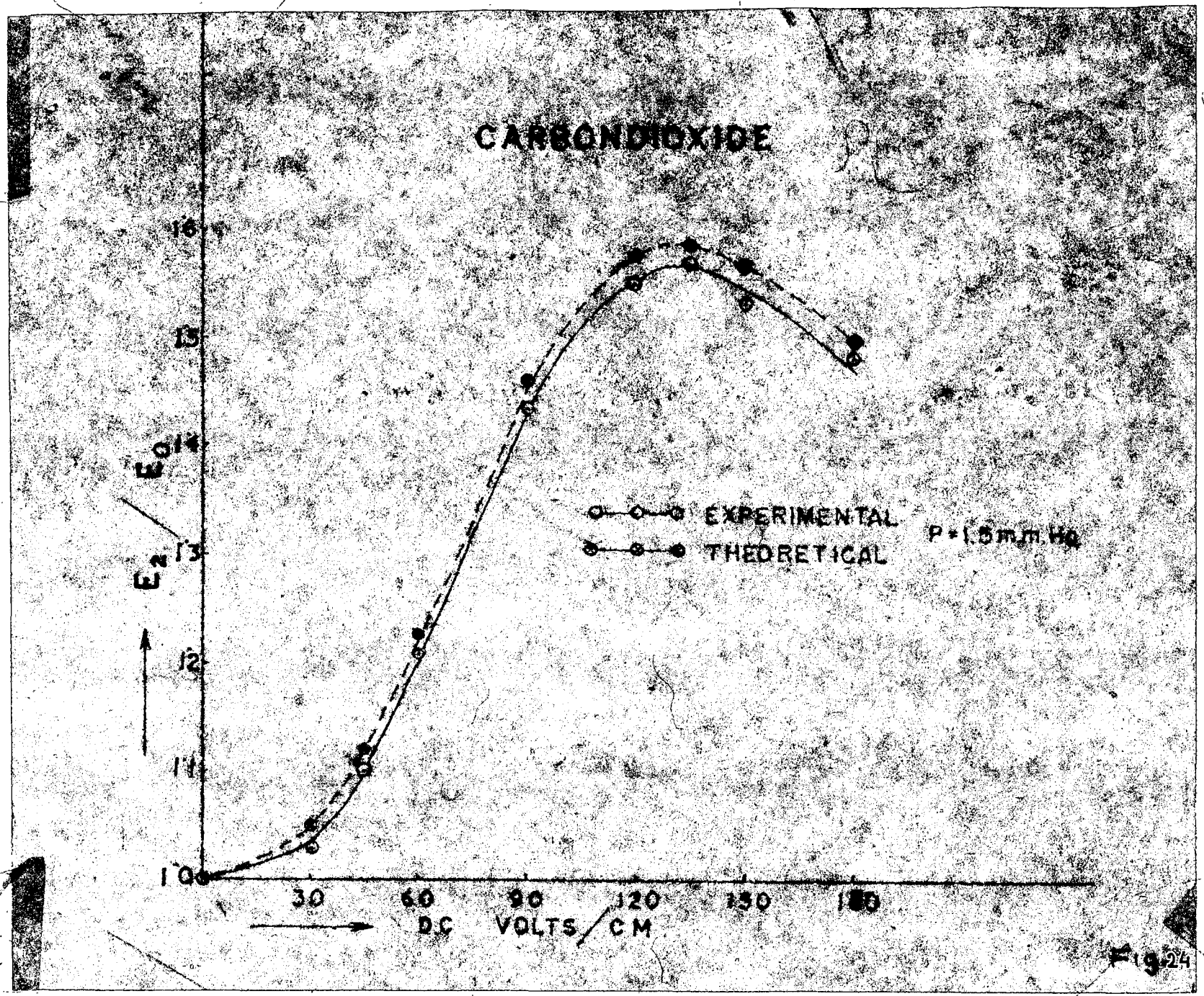
CARBONDIOXIDE
16
15
14
13
12
11
10
E_{CO_2}
E_{N_2}
EXPERIMENTAL
THEORETICAL
P = 1.5 m.m. Hg
30
60
90
120
150
180
D.C VOLTS / C M

Fig 24

due to d.c. field is absent and only the loss due to mobility to be effective. It will be then found out how this breakdown field changes when the d.c. ionisation is taken into consideration. In the presence of radiofrequency field the breakdown condition is given by

$$\nu_0/D_0 = 1/\Lambda^2 = \left(2{\cdot}405/a\right)^2 + \left(\pi/L_0\right)^2 \qquad \ldots(5.11)$$

where Λ is the diffusion length

where

$$L_0 = \left(L - 2\bar{K} E_0/\omega\right) \qquad \ldots(5.12)$$

Here ν_0 is the ionisation coefficient and D_0 is the diffusion coefficient. E_0 is the breakdown field in absence of d.c. field, L the length of the discharge tube, " a " is the radius of the tube, and $\bar{K}$ the mobility and ω the angular frequency of the applied field. When the d.c. field is also present, it has been shown by Varnerin and Brown (1950) that the breakdown condition is given by

$$\nu_1/D_1 = \left(2{\cdot}405/a\right)^2 + \left(\pi/L_0\right)^2 + \left(\bar{K} E_{d.c.}/2D\right)^2 \qquad \ldots(5.13)$$

From equations (5.11), (5.12) and (5.13)

$$\frac{\nu_0/D_0}{\nu_1/D_1} = 1/\left(1 + \beta E_{d.c.}^2\right) \qquad \ldots(5.14)$$

where $\beta = \dfrac{\left(\bar{K}/2D\right)^2}{\left(2{\cdot}405/a\right)^2 + \left(\pi/L_0\right)^2} = \dfrac{N^2(3\lambda\rho)}{4E_0^2\left[\left(2{\cdot}405/a\right)^2 + \pi^2/\left(L - 2\bar{K}E_0/\omega\right)^2\right]} = \dfrac{N^2(3\lambda\rho)}{4E_0^2/\Lambda^2}$

by putting the values of $\bar{K}$ and D from Kihara's theory (1952).

From Kihara's theory (1952)

$$\nu_0/D_0 = \left(3.\sigma.N^2.\lambda/c_i\right) \exp\left\{-mc_i^2/2K(T_e)_0\right\} \qquad \ldots(5.15)$$

$$\nu_1/D_1 = \left(3.\sigma.N^2.\lambda/c_i\right) \exp\left\{-mc_i^2/2K(T_e)_1\right\} \quad \ldots (5.16)$$

Where σ, λ, β and c_i are molecular constants introduced by Kihara, N denotes the number of the molecules per unit volume and T_e is the electron temperature. Putting the values ν_0/D_0 and ν_1/D_1 in the equation (5.14) and remembering that

$$KT_e = eE/N(3.\lambda.\beta)^{1/2}$$

and $$mc_i^2.N.(3.\lambda.\beta)^{1/2}/2eE_1 = B_0P/E_1$$

where E =r.m.s. breakdown voltage = (peak r.f. voltage) / $2^{\frac{1}{2}}$ ...Kihara (1952) we get

$$-B_0P(E_0 - E_1)/E_0E_1 = \log\left(1 + \beta E_{d.c.}^2\right) \quad \ldots (5.17)$$

where E_1 is the breakdown voltage without taking into consideration of d.c. ionisation, and B_0 is the constant introduced by Townsend. The constant β can be calculated for different gases from the values of molecular constants given by Kihara; and as the value of E_0 is known that of E_1 can can be calculated for various values of $E_{d.c.}$

To take account of the effect of d.c. ionisation let us assume that if ν_2 is the ionisation coefficient for the radiofrequency field only and $\nu_{d.c.}$ is the d.c. ionisation coefficient,

$$\nu_1 = \nu_2 + \nu_{d.c.} \quad \ldots (5.18)$$

To calculate $\nu_{d.c.}$ both the primary and secondary ionisations have to be taken into consideration; when the primary ionisation is considered,

$$i = i_0 . \exp(\alpha_1 L) \qquad \ldots(5.19)$$

when both the primary and secondary effects are considered

$$i = i_0 . \exp\left[\frac{\alpha_2 L}{1 - \gamma . \exp(\alpha_2 L)}\right] \qquad \ldots(5.20)$$

where "α" and "γ" are Townsend's first and second ionisation coefficients respectively.

If α_1 has to account for both the α and γ effects then

$$\exp(\alpha_1 L) = \frac{\exp(\alpha_2 L)}{1 - \gamma \exp(\alpha_2 L)}$$

or

$$\alpha_1 = \alpha_2 - \frac{\log\left[1 - \gamma \exp(\alpha_2 L)\right]}{L} \qquad \ldots(5.21)$$

$$\therefore \nu_{d.c.} = \alpha \bar{K} E_{d.c.} - \frac{\bar{K} E_{d.c.} \log\left[1 - \gamma . \exp(\alpha L)\right]}{L} \qquad \ldots(5.22)$$

but since ν_1 and ν_2 correspond to breakdown for radiofrequency discharge

$$\nu_1 / D_1 = 1/\Lambda^2 \quad \text{and} \quad \nu_2 / D_2 = 1/\Lambda^2$$

and we get from equation (5.18)

$$D_1/\Lambda^2 = D_2/\Lambda^2 + \alpha \bar{K} E_{d.c.} - \frac{\left[\bar{K} E_{d.c.} \log\left\{1 - \gamma . \exp(\alpha L)\right\}\right]}{L}$$

Putting the values of D_1 and D_2 from Kihara's theory

$$\frac{E_1 \bar{K}}{N (3.\lambda.\rho)^{1/2}} = \frac{E_2 \bar{K}}{N.(3.\lambda.\rho)^{1/2}} + \alpha \bar{K} \Lambda^2 E_{d.c.} - \frac{\bar{K} E_{d.c.} \Lambda^2 \log\left\{1 - \gamma . \exp(\alpha L)\right\}}{L}$$

$$\therefore E_2 = E_1 - N (3.\lambda.\rho)^{1/2} . \Lambda^2 . \alpha . E_{d.c.} + \frac{N.(3.\lambda.\rho)^{1/2} \Lambda^2 E_{d.c.} \log\left\{1 - \gamma . \exp(\alpha L)\right\}}{L}$$

$$\ldots(5.23)$$

where E_2 is the radiofrequency breakdown voltage when both the radiofrequency and d.c. fields are present.

The numerical values of the third term have been calculated in case of air and H_2 though there is insufficient data for γ in the literature. However the values calculated are so small in comparison to first and second terms on the right hand side that its contribution can be neglected. Then

$$E_2 = E_1 - N . (3.\lambda.\rho)^{1/2} . \Lambda^2 . (\alpha/p) (E_{d.c.}/p) . p^2 \qquad \ldots(5.24)$$

Consequently equation (5.24) can be utilised for calculating E_2. To calculate E_1 the value of β has been obtained for each gas from the numerical constants introduced by Kihara.

TABLE - II.

Gas	Pressure m.m. Hg.	E_o Volts/cm.	$(\lambda p).10^{32}$	$1/\lambda^2$	β	B_o Volt per cm. mm Hg	$N.(3.\lambda.p)^{1/2}.\lambda^2$
Air	1.5	106.7	14.55	16.565	1.658×10^{-3}	365	2.121
Air	1.0	90.0	14.55	16.854	1.007×10^{-3}	365	1.414
Hydrogen	2.0	56.8	4.455	16.885	3.073×10^{-3}	130	1.538
Hydrogen	3.0	68.3	4.455	16.378	5.00×10^{-3}	130	2.379
Oxygen	2.0	86.6	8.189	159.5	2.564×10^{-4}	138	0.221
Carbondioxide	1.5	93.3	21.78	61.38	8.66×10^{-4}	250	0.6742

The values of E_1 can thus be calculated from equation (5.17) and this gradually increases with the increase of d.c. field. It is however to be noted that upto a value of ($E_{d.c.}/P$) of the order of 150 volts/cm. m.m. Hg. the rise of E_1 with $E_{d.c.}$ is almost linear but when $E_{d.c.}$ is increased still further, E_1 assumes extremely high values. This fact suggests that at high values of ($E_{d.c.}/P$) greater than 150 volt/cm. m.m. Hg. the drift velocity is no longer a linear function of E/P. After calculating the values of E_1 it is possible to calculate E_2 from the equation (5.24). The values of (α/P) for various values of E/P have been taken from the data given by Brown (1959) and the values of (E_2/E_o) have been plotted in the figs. 21, 22, 23, 24 (curves indicated by dotted lines). In case of air,

Carbondioxide and oxygen the equation has further been modified because attachment will influence the value of the breakdown voltage. The effect of attachment has been calculated in the same manner as has been done by Sen and Ghosh (1963) where it was shown that

$$E_2 = (E_2^A).b + L_o\,\omega(1-b)/\bar{K} \qquad \ldots(5.25)$$

where E_2 is the breakdown voltage calculated without considering attachment,

E_2^A = breakdown voltage when attachment is taken into consideration and

$$b = \left[\frac{\alpha/P - \eta/P}{\alpha/P}\right]^{1/2} \qquad \ldots(5.26)$$

where η/P is the attachment coefficient.

The values of η/P for air and oxygen for various ($E_{d.c.}/P$) values have also been taken from Brown (1959), due to lack of adequate data, the value could not be calculated inpase of carbondioxide. In the figs. 21 and 23 the ratio E_2^A/E_o have been plotted whereas in the figs. 22 and 24 the ratio E_2/E_o have been plotted.

By comparing theoretical and experimental data it is seen that there is quite good agreement in case of air both for pressure of 1.5 mm and 1 mm. The discrepancy observed may partly be ascribed to uncertainity in the values of the molecular constants. There is reasonable agreement in case of hydrogen for a pressure of 3 mm. However, for the pressure of 2 mm, though the agreement is good for lower pressure, wide divergence is noticed for $E_{d.c.} > 90$ volt/cm. In fact, the value of E_1 the breakdown voltage without considering d.c. ionisation becomes so large that wide divergence is noticed even if d.c. ionisation is considered. This fact suggests as has been mentioned (Sen and Bhattacharjee 1965) that perhaps the drift velocity does not remain a linear function of (E/P) and the loss of electron due to mobility is actually smaller than what is predicted by Warnerin and Brown's theory. In case of oxygen also

divergence becomes dominant at high d.c. fields even though attachment correction has been applied. Results would have improved slightly if attachment data were available in case of carbondioxide.

It can thus be concluded that for small d.c. voltage the theory proposed by Varnerin and Brown that loss of electrons increases due to mobility can explain the increase of breakdown voltage, but with the increase of d.c. voltage, ionisation due to d.c. field also has to be taken into consideration and this factor gradually increases with the applied d.c. field and becomes considerable at high d.c. voltage and thereby reduces the radiofrequency voltages necessary for breakdown. The observed discrepency can be ascribed partly to the uncertainity in the values of molecular constants introduced by Kihara in his theory of electrical discharge. In case of d.c. ionisation, the primary ionisation is much more predominant than the secondary effect which depends to a major extent upon the nature of the electrode and also upon the pressure of the gas in the discharge tube.

It should however be noted that the theory which has been advanced here is based upon the theory of electrical discharge by Kihara which assumes that electron velocity distribution function is Maxwellian. But in fact it actually does not hold good when radiofrequency voltage sufficient to cause breakdown is applied and departure from the Maxwellian distribution becomes more when d.c. ionisation effects which involve large transit of electrons are considered. Consequently the above theory suffers from the same limitations as are applicable to Kihara's theory. But it is noted that even with those limitations it can explain the experimental results at least upto a certain limit of d.c. voltage per cm.

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BREAKDOWN OF A RADIO-FREQUENCY DISCHARGE IN THE PRESENCE OF A SUPERIMPOSED d-c. FIELD

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ABSTRACT

Breakdown voltages have been determined in the case of some rare gases (He, Ne, A) and oxygen at a constant pressure (10 mm Hg) when excited simultaneously by a radio-frequency field (frequency 10 Mc/sec) and a variable d-c. field. It is found in all cases that the breakdown voltage is higher when both the fields are present than when the gases are excited by the radio-frequency field alone and the breakdown voltage gradually increases with the increase of the applied d-c. field. The variation of breakdown field with d-c. field is of the same nature in all of the gases studied. A theoretical expression for the breakdown voltage in the presence of both the r-f. and d-c. fields has been deduced from the theory of electrical discharge by Kihara (1952) together with the expression of equivalent length as deduced by Varnerin and Brown (1950). The theoretical expression cannot explain satisfactorily the experimental results, and the rate of rise of breakdown voltage in the d-c. field as obtained from theory is smaller than that obtained from experimental results. The discrepancy has been ascribed partly to the uncertainty in the values of the numerical constants introduced by Kihara and also to the increase of diffusion caused by the presence of positive ions—a factor which has not been taken into consideration in the present treatment.

INTRODUCTION

Varela (1947) observed that the breakdown potential in a discharge excited by a radio-frequency source increased when a d-c. potential less than the radio-frequency breakdown voltage was applied across the discharge tube. The discharge tube had aluminium electrodes in an atmosphere of 5 cm hydrogen and 20% argon. It was expected, however, that the presence of the d-c. field would hasten ionization with a given radio-frequency potential by virtue of higher peak electron velocities and that a residual potential following the discharge would hasten deionization; but the results were contrary to what was expected. Almost the same results were obtained earlier by Kirchner (1925, 1947) while he was studying the breakdown in gases by an r-f. field in the presence of a d-c. potential.

Varnerin and Brown (1950) calculated theoretically the distribution function of electrons in an ionized gas in the presence of both radio-frequency and d-c. fields. It is generally known that the gas in the cavity will break down when the losses of electrons to the walls of the cavity are replaced by ionization in the body of the gas. When an a-c. field is applied, electrons are lost by diffusion; when, in addition, a d-c. field is applied, electrons are lost both by diffusion and mobility. It has been shown by Varnerin and Brown that the new effective diffusion length L_{dc} of the vessel in the presence of the d-c. field is shorter than its undisturbed diffusion length L according to the equation

$$\frac{1}{L_{dc}^2} = \frac{1}{L^2} + \left\{\frac{\bar{K}E_{dc}}{2D}\right\}^2, \tag{1}$$

where $\bar{K}$ denotes the mobility of the electron and D the diffusion coefficient. They thus concluded that the only difference between the breakdown condition in the a-c.–d-c. case and the pure a-c. case is the substitution of a modified diffusion length L_{dc} for the characteristic diffusion length L.

That a greater breakdown field is necessary when the d-c. voltage is superimposed on the radio-frequency field was further shown by Brown (1956) in the case of air at 38 mm Hg, where a d-c. field of up to 200 volts/cm was applied.

No systematic study of the breakdown of gases under the simultaneous action of the a-c. and d-c. fields has so far been reported. It is expected that this study will throw more light on the mechanism of breakdown. The object of the present investigation is to determine the breakdown voltage in some rare gases (such as argon, neon, helium) and oxygen in a superimposed radio-frequency and d-c. field and to present a theory capable of explaining the observed results.

EXPERIMENTAL SETUP

Discharge tubes (Geissler tubes) filled with different gases, such as argon, helium, neon, and oxygen, as supplied by the manufacturers were used. Each of the discharge tubes is fitted with two aluminium electrodes. The method of determining the breakdown voltage under radio-frequency excitation has been described in an earlier paper by Sen and Ghosh (1963). The frequency of the exciting radio-frequency field is 10 Mc/sec. The method of applying the d-c. field obtained from dry batteries is shown in Fig. 1. The pressure of the enclosed gas has been taken to be 10 mm Hg as given by the manufacturers. The method consists in measuring the breakdown potential in the presence of the radio-frequency field only and then in repeating the measurement with the application of the d-c. field superimposed upon the radio-frequency field. The d-c. field is varied from 5 volts/cm to 70 volts/cm. The gases investigated are argon, helium, neon, and oxygen. Hydrogen could not be excited with the applied radio-frequency voltage. The lengths of the discharge tubes are as follows: helium—13.0 cm, neon—15 cm, argon—13.2 cm, oxygen—13 cm.

RESULTS AND DISCUSSION

The variation of the ratio E/E_0 (where E is the breakdown voltage per cm in the presence of both the radio-frequency and d-c. fields and E_0 is the breakdown voltage per cm in the presence of the radio-frequency field only) against E_{dc}, the applied d-c. voltage per cm, is shown in Figs. 2, 3, 4, 5. There is no detectable change in breakdown voltage other than that in the radio-frequency case for small d-c. voltages, which was less than 5 V/cm for all the gases studied; it is clear that with the application of the d-c. field, the radio-frequency breakdown voltage increases and the nature of the variation of E/E_0 with E_{dc} is practically the same for all the gases studied. The nature of the variation is the same as that observed by previous workers.

The physical significance of these results is at once apparent. In a radio-frequency field, breakdown occurs when the loss of electrons by diffusion and mobility is compensated by the generation of electrons by ionization. The application of the d-c. field further increases the loss due to diffusion and mobility

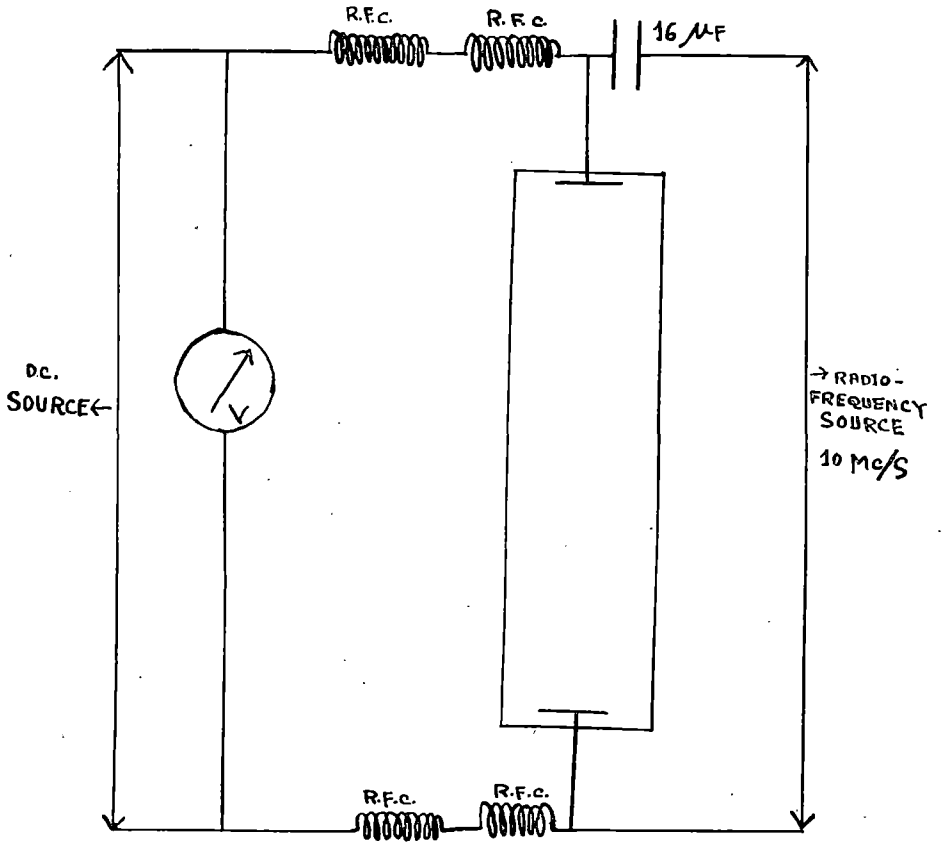

FIG. 1. Arrangement for applying d-c. voltage to the discharge tube.

and hence a larger electric field must be applied to produce more ionization to compensate for this increased loss. In the analysis which follows, we have employed the equations deduced by Kihara (1952) with the new diffusion length L_{dc} introduced by Varnerin and Brown (1950).

If L_0 denotes the equivalent length with the radio-frequency field alone, then according to Kihara (1952).

$$\frac{1}{L_0^2} = \frac{1}{\pi^2}\cdot\frac{\nu_0}{D_0}, \tag{2}$$

where ν_0 is the ionization coefficient and D_0 is the diffusion coefficient. When both the r-f. and d-c. fields are present, the breakdown condition is given by

$$\frac{1}{L_{dc}^2} = \frac{1}{\pi^2}\cdot\frac{\nu}{D}, \tag{3}$$

where

$$\frac{1}{L_{dc}^2} = \frac{1}{L_0^2} + \left\{\frac{\bar{K}E_{dc}}{2D}\right\}^2;$$

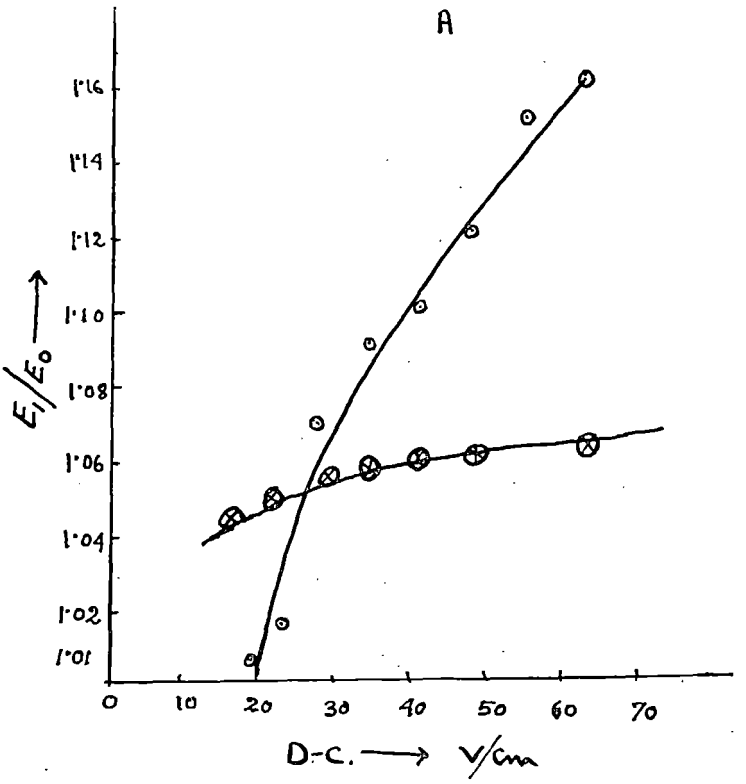

FIG. 2. Variation of E/E_0 with E_{dc} in argon (theoretical and experimental). ⊙—experimental, ⊗—from equation (6).

then

$$\frac{1}{L_0^2} + \left\{\frac{\bar{K}E_{dc}}{2D}\right\}^2 = \frac{1}{\pi^2}\cdot\frac{\nu}{D},$$

where ν and D represent the quantities for the new breakdown field. According to Kihara (1952),

$$\frac{\nu_0}{D_0} = \frac{3\sigma N^2\lambda}{C_1}\exp\left[-\frac{mC_1^2}{2K(T_e)_0}\right],$$

$$\frac{\nu}{D} = \frac{3\sigma N^2\lambda}{C_1}\exp\left[-\frac{mC_1^2}{2K(T_e)}\right],$$

where λ, σ, and C_1 are the molecular constants introduced by Kihara and N is the number of molecules per cc at the pressure of the gas, K is the Boltzmann constant, m the mass of the electron, and (T_e) is the electron temperature, which will be different for the two breakdown voltages. Consequently, from equations (2) and (3) we get

$$\frac{1/L_0^2}{1/L_0^2 + (\bar{K}E_{dc}/2D)^2} = \exp\frac{mC_1^2}{2K}\left[\frac{1}{T_e} - \frac{1}{(T_e)_0}\right]. \tag{4}$$

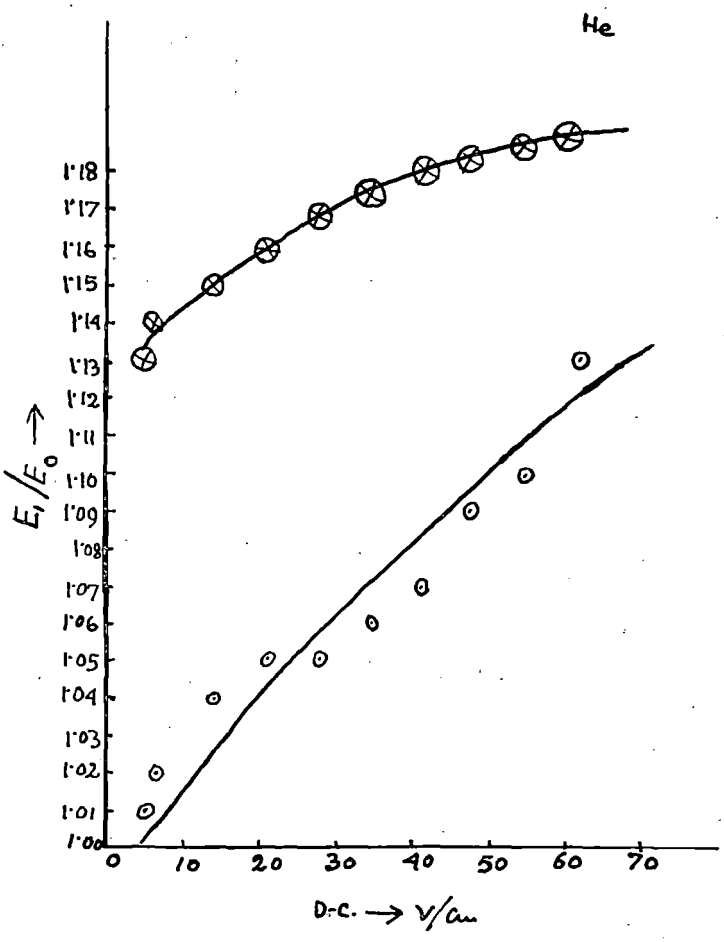

FIG. 3. Variation of E/E_0 with E_{dc} in helium (theoretical and experimental). ⊙—experimental, ⊗—from equation (6).

Assuming that $T_e = (T_e)_0$, we get

$$\log\left[1+\left(\frac{\bar{K}E_{dc}L_0}{2D}\right)^2\right] = -\frac{mC_1^2}{2K}\frac{(T_e)_0-(T_e)}{(T_e)_0^2}.$$

But according to Kihara (1952),

$$\frac{\bar{K}}{2D} = \frac{e}{K(T_e)_0}$$

and

$$K(T_e)_0 = \frac{eE_0}{N(6\lambda\rho)^{\frac{1}{2}}}$$

and similarly

$$KT_e = \frac{eE_1}{N(6\lambda\rho)^{\frac{1}{2}}}.$$

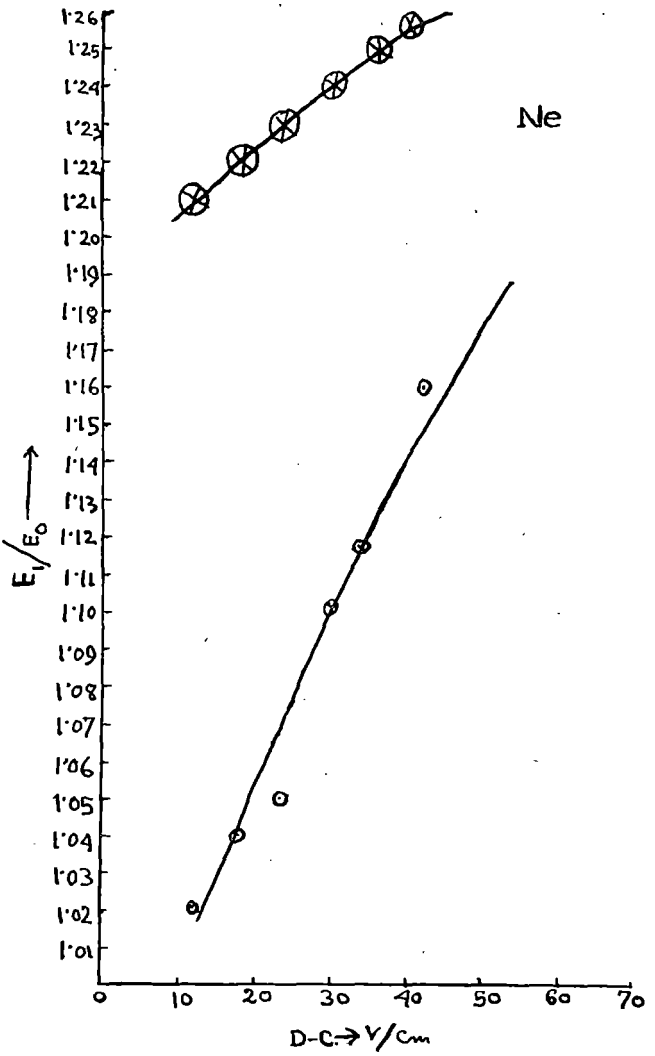

FIG. 4. Variation of E/E_0 with E_{dc} in neon (theoretical and experimental). ⊙—experimental, ⊗—from equation (6).

Putting these values into equation (5), we get

$$\log\left\{1+\left[\frac{eL_0E_{dc}}{2K(T_e)_0}\right]^2\right\}=N(6\lambda\rho)^{\frac{1}{2}}\left[\frac{E_t}{E_0}-1\right]. \tag{5}$$

According to Kihara, $L_0 = L' - (2\bar{K}/\omega)E_0$, where L' is the length of the discharge tube and ω is the angular frequency of the applied radio-frequency field. Hence

$$\log\left\{1+\left[\frac{e[L'-(2\bar{K}/\omega)E_0]E_{dc}}{2K(T_e)_0}\right]^2\right\}=N(6\lambda\rho)^{\frac{1}{2}}\left[\frac{E_t}{E_0}-1\right] \tag{6}$$

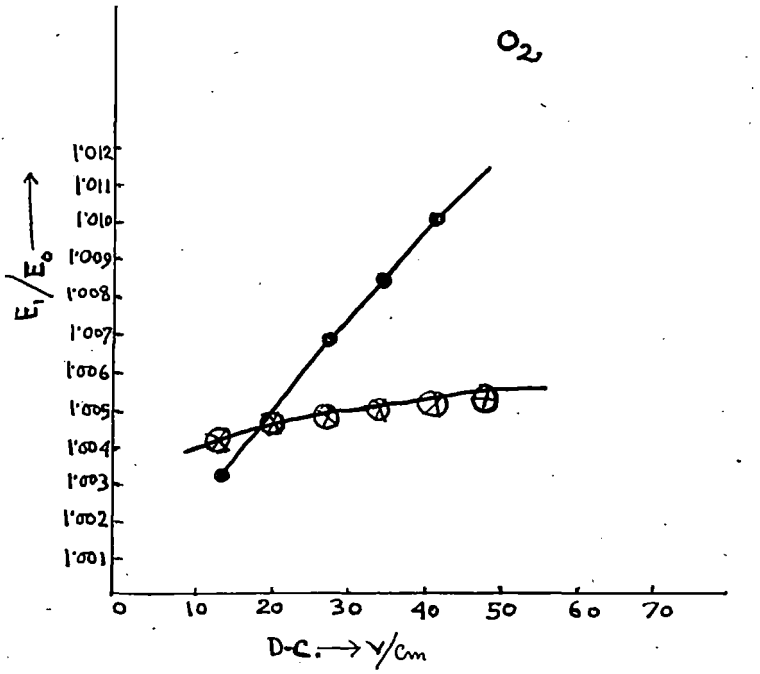

FIG. 5. Variation of E/E_0 with E_{dc} in oxygen (theoretical and experimental). ●—experimental, ⊗—from equation (6).

or
$$\frac{E_1}{E_0} = 1 + \frac{\log\{1 + [e(L' - (2\bar{K}/\omega)E_0)E_{dc}/2K(T_e)_0\}^2}{N(6\lambda\rho)^{\frac{1}{2}}}.$$

Since the radio-frequency breakdown voltage is known from experimental results, $(T_e)_0$ can be calculated from the relation (Kihara 1952)

$$K(T_e)_0 = \frac{eE_0}{N(6\lambda\rho)^{\frac{1}{2}}}.$$

The values of the mobility coefficient $\bar{K}$ for the value E_0/P have been obtained from the results reported by Brown (1959). The right-hand side of equation (6) has been calculated in this way, and the results are plotted side by side with the experimental values. In order to calculate the right-hand side of equation (6), the following values have been calculated from the data of molecular constants given by Kihara (1952).

The right-hand side of equation (6) can thus be calculated for different values of E_{dc}. The results are plotted side by side with the experimental curves. The quantitative agreement is not satisfactory except for argon and oxygen and also for values of E_{dc} near about 20 volts/cm. There is no agreement at all for helium and neon. The quantitative disagreement can be attributed partly to constants introduced by Kihara. The nature of the theoretical curve resembles the experimental curve to a certain extent, but the rate of rise as predicted from theoretical deduction is smaller than that obtained experimentally. It should be noted,

TABLE I

Gas	$\lambda \times 10^8$, cm³/sec	$\rho \times 10^{24}$, cm sec	$N(6\lambda\rho)^{\frac{1}{2}}$	E_0, volts/cm	$K(T_e)_0 \times 10^{12}$	L'_β, cm	$\bar{K} \times 10^{-6}$	$\frac{e[L-(2\bar{K}/\omega)E_0]}{2K(T_e)_0}$
Argon	7.8	1.35	282.1	16.28	0.0953	13.2	0.0368	114.1
Helium	4.4	0.20	81.58	14.61	0.2902	13.0	0.0821	36.17
Neon	$(\lambda\rho) \times 10^{32}$ 0.3710		52.96	17.3	0.517	15.0	0.1329	22.86
Oxygen	$(\lambda\rho) \times 10^{32}$ 8.356		726.6	23.0	0.0512	13.0	0.1304	203.8

$N = 3.55 \times 10^{17}$, $P = 10$ mm Hg, $f = 10$ Mc/sec, $\omega = 6.28 \times 10^7$ radians.

however, that in the above deduction we have not considered two effects: (*a*) increased loss due to diffusion because of the presence of positive ions and (*b*) the possible ionization contributed by the d-c. field itself. The contribution by the second factor to the theoretical computation of breakdown voltage would, however, decrease the actual value, because if there is any ionization caused by the d-c. field, it would tend to lower the value expected. Whereas this factor may lower the values of (E_1/E_0) somewhat in the case of helium and neon, it will reduce still further the values of (E_1/E_0) in the case of argon and oxygen, where the theoretical values are actually lower than the experimental values. Furthermore, in the case of oxygen the contribution due to the attachment of electrons has not been taken into consideration, and this actually modifies the expression for breakdown voltage. The increased loss due to diffusion in the presence of positive ions may be responsible for the higher values obtained experimentally, and in this calculation we have not taken this effect into consideration.

The theoretical and experimental results point, however, to the fact that in a discharge excited by the simultaneous action of radio-frequency and d-c. voltage, the loss of electrons is due both to diffusion and to mobility, and by employing the deductions of Kihara with the expression for the equivalent length as deduced by Varnerin and Brown, it has been possible to deduce results which can be compared with the experimentally observed values. The discrepancy can be ascribed to the uncertainty in the values of the numerical constants introduced by Kihara and to the fact that the presence of positive ions increases the diffusion—a factor which has not been taken into consideration here.

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RADIO-FREQUENCY BREAKDOWN IN A SUPERIMPOSED D-C. FIELD

S. N. Sen and B. Bhattacharjee

RADIO-FREQUENCY BREAKDOWN IN A SUPERIMPOSED D-C. FIELD

S. N. SEN AND B. BHATTACHARJEE

Following the work done by the authors (Sen and Bhattacharjee 1965), the breakdown potentials of air, hydrogen, and oxygen excited by a radio-frequency field of frequency 10.3 Mc/sec in the presence of a superimposed d-c. field varying from 0 to 240 V/cm have been determined for various values of the d-c. field. The object of undertaking this work is to see how the radio-frequency breakdown voltage changes when the applied d-c. field is large in contrast to the small d-c. fields used in the previous investigations. The discharge tube is a cylindrical glass tube fitted with two electrodes, and the distance between the two electrodes is 3 cm and the diameter 1.25 cm. The method is the same

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as adopted in the previous paper. The variation of E_1/E_0, where E_1 is the radio-frequency breakdown voltage in the presence of a d-c. field and E_0 is the radio-frequency breakdown voltage in the absence of a d-c. field, has been plotted against d-c. voltage per cm. Measurements were taken for various values of pressure for the three gases but only three representative curves for the three gases have been plotted, each for a pressure of 2 mm Hg, in Fig. 1. The results show that the breakdown voltage in the presence of a d-c. field increases for small values of d-c. field, attains a maximum at a certain value

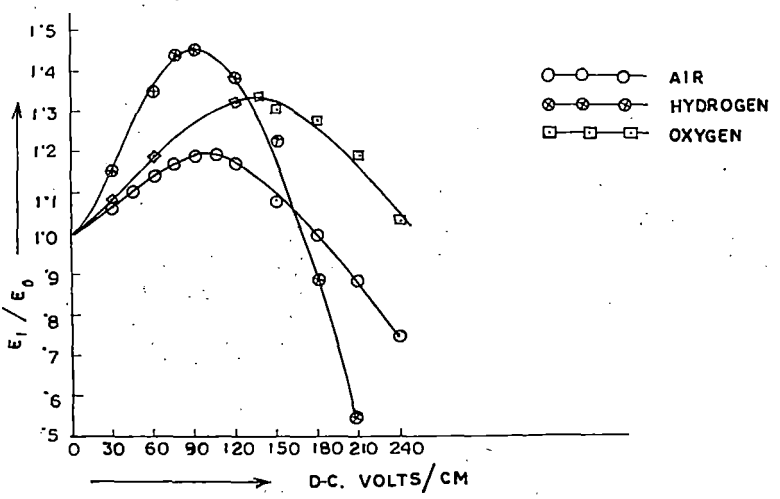

FIG. 1. Variation of radio-frequency voltage with superimposed d-c. field.

of d-c. field which is different for different gases, and then falls as the d-c. field is further increased. The variation is quite general, as has been observed in other gases also.

It is quite clear that whereas the increase of radio-frequency breakdown voltage can be explained by the theory put forward by Varnerin and Brown (1950) as due to increased loss due to mobility in the presence of a d-c. field, the occurrence of maxima and the consequent fall in the breakdown voltage show that other mechanisms are also operating. For small d-c. fields, the incorporation of Varnanin and Brown's increased diffusion length into Kihara's theory could at least give qualitative agreement with experimental results, as has been shown for rare gases by the present authors (Sen and Bhattacharjee 1965).

To explain the present results therefore the following mechanism is suggested. (*a*) For small values of d-c. field the contribution due to d-c. ionization has been neglected but when the d-c. field becomes of the order of that employed in the present investigation, there is a definite contribution to ionization due to the d-c. field in addition to ionization produced by the radio-frequency field, and this factor increases as the magnitude of the field increases. (*b*) Provided that the loss mechanism due to diffusion and mobility remains

unchanged, it is obvious that breakdown will take place when the combined loss due to diffusion and mobility is compensated by the combined ionization due to radio-frequency and d-c. field. Smaller values of the radio-frequency field will be necessary to cause breakdown because the d-c. field is also contributing to ionization. This can explain the fall in the radio-frequency breakdown voltage when the d-c. field is increased. With the increase of the d-c. field, the radio-frequency voltage necessary to cause breakdown will decrease gradually because d-c. ionization will be more dominant. (*c*) For breakdown at any d-c. voltage the rate of ionization due to the r-f. field plus the rate of ionization due to the d-c. field must equal the rate of loss due to mobility for the d-c. field plus the rate of loss due to diffusion. And the maximum in the curve will occur when the total rate of loss due to diffusion and mobility minus the rate of ionization due to the d-c. field becomes a maximum.

A generalized quantitative theory regarding the breakdown voltage in the presence of a superimposed d-c. field is being worked out, and a paper describing the results will be sent for publication soon.

In conclusion, it may be noted that data are available in the existing literature for the drift velocity of electrons for small E/P values where it is shown that for most of the gases, the drift velocity is linearly proportional to E/P, but it is doubtful whether the drift velocity is a simple linear function of E/P when E/P is of the order used in the present experiment. Consequently experiments are in progress in this laboratory for the measurement of the drift velocity of electrons when E/P varies from 15 to 200 V/cm mm of Hg. and the results obtained will be utilized in deducing the theory of r-f. breakdown in the presence of a d-c. field.

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Breakdown of a Radiofrequency Discharge in a Superimposed D. C. Field

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Breakdown of a Radiofrequency Discharge in a Superimposed D.C. Field

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(Received January 9, 1967)

Breakdown voltages have been measured in case of air, hydrogen, oxygen and carbon dioxide at a pressure of a few millimeters of mercury when excited simultaneously by a radiofrequency field (frequency 10.3 Mc/sec) and a variable d.c. field which varies from 0 to 240 volt/cm. It is found that the breakown voltage increases when the d.c. field is small, and when the field is further increased it shows a maximum and then gradually falls for all the gases studied, the maximum occuring at a d.c. voltage which is different for different gases. It has been shown that, when the d.c. field is small, the dominant factor is the loss of electrons by diffusion as well as by mobility which causes the breakdown voltage to increase, but when the d.c. field is increased, contribution due to d.c. ionization has also to be taken into consideration. This d.c. ionization causes a decrease in the radiofrequency voltage necessary for breakdown. A mathematical expression has been deduced which explains satisfactorily the observed experimental results. In the expression deduced the effect of secondary ionization as well as that of electron attachment have been taken into consideration.

§ 1. Introduction.

In a previous paper (Sen and Bhattacharjee[1]) the breakdown voltages for some rare gases (He, Ne, A) and oxygen at a constant pressure (10mm Hg) were determined when excited simultaneously by a radiofrequency field (frequency 10 Mc/sec) and a variable d.c. field. It was found that the breakdown voltage is higher when both the fields are present than when the gas is excited by a radiofrequency field alone and the breakdown voltage gradually increases with the increase of d.c. field. A theoretical expression for the breakdown voltage in the presence of d.c. and radiofrequency field was deduced from the theory of electrical discharge by Kihara[2] utilizing the expression for equivalent length deduced by Varnerin and Brown.[3] In the expression thus deduced the contribution to ionization made by the d.c. field was not taken into consideration because the field applied was small compared to the breakdown radiofrequency field. It was evident however from the experimental results obtained in the previous paper that the rate of rise of breakdown field with the applied d.c. field gradually becomes smaller as the value of the d.c. field is increased. From this it was evident that the contribution to ionization by the d.c. field was gradually becoming dominant with higher d.c. fields and naturally if the d.c. field be sufficiently increased then the contribution to ionization by the d.c. field will be considerable and the radiofrequency voltage to cause breakdown will gradually decrease. The theoretical expression deduced in the previous paper could not explain satisfactorily the experimental results, it was pointed out that the discrepancy might be due (a) to uncertainty in the values of the molecular constants introduced by Kihara and (b) the effect of d.c. ionization was not taken into consideration. Vernerin and Brown[3] in finding the effect of d.c. field only considered the effects of increased diffusion and mobility but at high values of d.c. field the contribution due to ionization should also be taken into consideration. To find the effect of d.c. ionization, d.c. fields higher than those used in the previous investigation have to be applied. The purpose of the present investigation is to determine experimentally the values of breakdown voltage in the simultaneous presence of radiofrequency voltage and a d.c. voltage where the values of d. c. voltages are much greater than the radiofrequency voltages applied, and to present a consistent theory capable of explaining the experimental results. A research note covering the preliminary investigation has been published. (Sen and Bhattacharjee.[4])

§ 2. Experimental Arrangement

The method of measuring the breakdown voltage in the presence of radiofrequency and d.c. field is the same as in the previous paper (Sen and Bhattacharjee.[1]) The discharge tubes are of length 3 cm and diameter 1.25 cm and fitted with two platinum electrodes. The gases studied in the investigation are air, hydrogen, oxygen and carbon dioxide. Air, oxygen and carbon dioxide have been chosen in order to study the effect of electron attachment and the pressure is of the order of a few millimeters which has been measured with the help of a mercury manometer. The frequency of the radiofrequency voltage is 10.3 Mc/sec which has been accurately measured with a communication receiver; and the applied d.c. voltage, which has been provided with a series of dry batteries, has been varied from a few volts per cm to 240 volts per cm. Hydrogen and oxygen have been prepard by the electrolysis of a very dilute solution of barium Hydroxide in water and then allowing the gas to pass through traps of phosphorus pentoxide; Carbon dioxide has been prepared by the action of dilute sulphuric acid on a pure sample of sodium carbonate. The gas has been passed through a trap of distilled water to absorb traces of acid and finally dried by phosphorus pentoxide and collected in an evacuated chamber.

§ 3. Results and Discussion

The variation of radiofrequency breakdown voltage with the applied d. c. field has been plotted in Figs. 1,2,3 and 4 in case of air, hydrogen, oxygen and carbon dioxide respectively. The ordinate represents the ratio E_2/E_0 where E_2 is the radiofrequency breakdown voltage when both the radiofrequency and d.c. fields are present and E_0 is the breakdown voltage when only the radiofrequency voltage is present. It is seen from the nature of the curves that the breakdown voltage gradually increases with the increase of the d.c. field, then attains a maximum at a certain d.c. field which is different for different gases, and then falls as the d.c. field is further increased. A well defined maximum is obtained in each case and the variation of breakdown voltage with the d.c. field is quite general as it has been observed for all the four gases studied. It is thus quite apparent that whereas the theory of Varnerin and Brown[3] can explain the increase of breakdown voltage when the d. c. field is small as due to increased loss due to mobility in the presence of d.c. field, the occurance of maxima and the consequent fall in voltage shows that other mechanisms are also operating.

To explain the observed results it is therefore suggested that, whereas for small d.c. fields d.c. ionization can be neglected, the effect of d.c. ionization has to be taken into consideration when the d.c. voltage is of the order employed in the present investigation. The effect of d.c. voltage is also to increase the mobility and thereby cause an increased loss of electrons. In the calculation which follows we shall first consider the ionization due to d.c. field to be absent and only the loss due to mobility to be effective. We shall then find out how this breakdown field changes when the d.c. ionization is taken into consideration. In the presence of rediofrequency field the breakdown condition is given by,

$$\frac{\nu_0}{D_0}=\left(\frac{2{\cdot}405}{a}\right)^2+\frac{\pi^2}{L_0^2}=\frac{1}{\Lambda^2}$$

where $$L_0=\left(L-\frac{2\bar{K}E_0}{\omega}\right) \qquad (1)$$

Here ν_0 is the ionization coefficient and D_0 is the diffusion coefficient. E_0 the breakdown field in absence of d.c. field, L the length of the discharge tube, a is the radius of the tube, and $\bar{K}$ the mobility and ω the angular frequency of the applied field. When the d.c. field is also present, it has been shown by Varnerin and Brown[3] that the breakdown condition is given by

$$\frac{\nu_1}{D_1}=\left(\frac{2{\cdot}405}{a}\right)^2+\frac{\pi^2}{L_0^2}+\left(\frac{\bar{K}E_{DC}}{2D}\right)^2 \qquad (2)$$

From eqs. (1) and (2)

$$\frac{\frac{\nu_0}{D_0}}{\frac{\nu_1}{D_1}}=\frac{1}{1+\beta E_{DC}^2} \qquad (3)$$

where $$\beta=\frac{(\bar{K}/2D)^2}{\left(\frac{2{\cdot}405}{a}\right)^2+\frac{\pi^2}{L_0^2}}=\frac{N^2(3\lambda\rho)}{4E_0^2\left[\left(\frac{2{\cdot}405}{a}\right)^2+\frac{\pi^2}{\left(L-\frac{2\bar{K}E_0}{\omega}\right)^2}\right]}=\frac{N^2(3\lambda\rho)}{4E_0^2\frac{1}{\Lambda^2}}$$

From Kihara's paper[2]

$$\frac{\nu_0}{D_0}=\frac{3\sigma N^2\lambda}{c_i}\exp\left\{\frac{-mc_i^2}{2k(T_e)_0}\right\}$$

$$\frac{\nu_1}{D_1}=\frac{3\sigma N^2\lambda}{c_i}\exp\left\{\frac{-mc_i^2}{2k(T_e)_1}\right\}$$

where σ, λ, ρ and C_i are molecular constants introduced by Kihara, N denotes the number of molecules per unit volume and T_e is the electron temperature. Putting the values of ν_0/D_0 and ν_1/D_1 in the equation (3) and remembering (Kihara[2])

$$KT_e=\frac{eE}{N(3\lambda\rho)^{1/2}} \text{ and } \frac{mc_i^2N(3\lambda\rho)^{1/2}}{2eE_1}=\frac{B_0P}{E_1}$$

we get,

$$\frac{-B_0P(E_0-E_1)}{E_0E_1}=\log[1+\beta E_{DC}^2]\,, \tag{4}$$

where E_1 is the breakdown voltage without d.c. ionization, and B_0 is the constant introduced by Townsend. The constant β can be calculated for different gases from the values of molecular constants given by Kihara; and as the value of E_0 is known that of E_1 can be calculated for various values of E_{DC}.

To take account of the effect of d.c. ionization let us assume that, if ν_2 is the ionization coefficient for the radiofrequency field only and ν_{DC} is the d.c. ionization coefficient,

$$\nu_1=\nu_2+\nu_{DC} \tag{5}$$

To calculate ν_{DC} both the primary and secondary ionization have to be taken into consideration; when the primary ionization is considered,

$$i=i_0\exp\alpha_1L$$

when both the primary and secondary effects are considered,

$$i=i_0\exp\alpha_2L/1-\gamma\exp\alpha_2L\,.$$

If α_1 has to account for both the α and γ effects

then
$$\exp\alpha_1L=\frac{\exp\alpha_2L}{1-\gamma\exp\alpha_2L}\,.$$

$$\text{or } \alpha_1=\alpha_2-\frac{\log(1-\gamma\exp\alpha_2L)}{L}\,.$$

$$\therefore\quad \nu_{DC}=\alpha\bar{K}E_{DC}-\frac{\bar{K}E_{DC}\log(1-\gamma\exp\alpha L)\,.}{L}$$

but since ν_1 and ν_2 correspond to breakdown for radiofrequency discharge

$$\frac{\nu_1}{D_1}=\frac{1}{A^2} \text{ and } \frac{\nu_2}{D_2}=\frac{1}{A^2}$$

and we get from equation (5)

$$\frac{D_1}{A^2}=\frac{D_2}{A^2}+\alpha\bar{K}E_{DC}-\frac{\bar{K}E_{DC}\log(1-\gamma\exp\alpha L)\,.}{L}$$

Putting the values of D_1 and D_2 from Kihara's theory

$$\frac{E_1\bar{K}}{N(3\lambda\rho)^{1/2}}=\frac{E_2\bar{K}}{N(3\lambda\rho)^{1/2}}+\alpha\bar{K}A^2E_{DC}-\frac{\bar{K}E_{DC}A^2\log(1-\gamma\exp\alpha L)\,.}{L}$$

$$\text{or } E_2=E_1-N(3\lambda\rho)^{1/2}A^2\alpha E_{DC}+\frac{N(3\lambda\rho)^{1/2}A^2E_{DC}\log(1-\gamma\exp\alpha L)\,.}{L}$$

Table I.

Gas	Pressure mmHg	E_0 volt/cm	$(\lambda\rho)\times10^{32}$	$1/A^2$	β	B_0 volt/cm mmHg	$N(3\lambda\rho)^{1/2}A^2$.
Air	1.5	106.7	14.55	16.565	1.658×10^{-3}	365	2.121
〃	1.0	90.0	〃	16.854	1.007×10^{-3}	〃	1.414
Hydrogen	2.0	56.8	4.455	16.885	3.073×10^{-3}	130	1.538
〃	3.0	68.3	〃	16.378	5.00×10^{-3}	〃	2.379
Oxygen	2.0	86.6	8.189	159.5	2.564×10^{-4}	138	.221
Carbon dioxide.	1.5	93.3	21.78	61.38	8.66×10^{-4}	250	.6742

Where E_2 is the radiofrequency breakdown voltage when both the radiofrequency and d.c. fields are present.

The numerical values of the third term have been calculated in case of air and H_2 though there is insufficient data for γ in the literature. However the values calculated are so small in comparison to first and second terms on the right-hand side that its contribution can be neglected. Then

$$E_2=E_1-N(3\lambda\rho)^{1/2}A^2\left(\frac{\alpha}{P}\right)\left(\frac{E_{DC}}{P}\right)P^2 . \qquad (6)$$

Consequently equation (6) can be utilised for calculating E_2. To calculate E_1, the value of β has been obtained for each gas from the numerical constants introduced by Kihara.

The values of E_1 can thus be calculated from equation (4), and this gradually increases with the increase of d.c. field. It is however to be noted that up to a value of (E_{DC}/P) of the order of 150 volt/cm mmHg the rise of E_1 with E_{DC} is almost linear but when E_{DC} is increased still further, E_1 assumes extremely high values. This fact suggests that at high values of (E_{DC}/P) greater than 150 volt/cm mmHg the drift velocity is no longer a linear function of (E/P). There is no available experimental data in literature; hence experiments are in progress in this laboratory to measure mobility of electrons in various gases when (E/P) is of the same order of magnitude as has been used in this experiment. After calculating the values of E_1, it is possible to calculated E_2 from the equation (6). The values of (α/P) for various values of (E/P) have been taken from the data given by Brown[5] and the values of (E_2/E_0) have been plotted in the Figs. 1 to 4 (curves indicated by dotted lines). In case of air, carbon dioxide and oxygen the equation has further been modified because attachment will influence the value of the breakdown voltage. The effect of attachment has been calculated in the same manner as has been done by Sen and Ghosh,[6] where it was shown that

$$E_2=E_2{}^Ab+\frac{L_0\omega(1-b)}{\bar{K}}$$

where E_2 is the breakdown voltage calculated without considering attachment

$E_2{}^A$=breakdown voltage when attachment is taken into consideration

and $b=\left[\dfrac{\alpha/P-h/P}{\alpha/P}\right]^{1/2}$

where h/P is the attachment coefficient.

The values of h/P for air and oxygen for various (E_{DC}/P) values have also been taken from Brown[5]; due to lack of adequate data, the value could not be calculated in case of carbon dioxide. In the Figs. 1 and 3, the ratio $E_2{}^A/E_0$ have been plotted whereas in the Figs. 2 and 4, the ratio E_2/E_0 have been plotted.

By comparing theoretical and experimental data it is seen that there is quite good agreement in case of air both for pressure of 1.5mm and 1 mm. The discripency observed may partly be ascribed to uncertainty in the values of the molecular cohstants. There is reasonable agreement in case of hydrogen for a pressure of 3mm However, for the pressure of 2mm, though the agreement is good for lower pressure, wide divergence is noticed for $E_{DC}>90$ volt/cm. In fact,

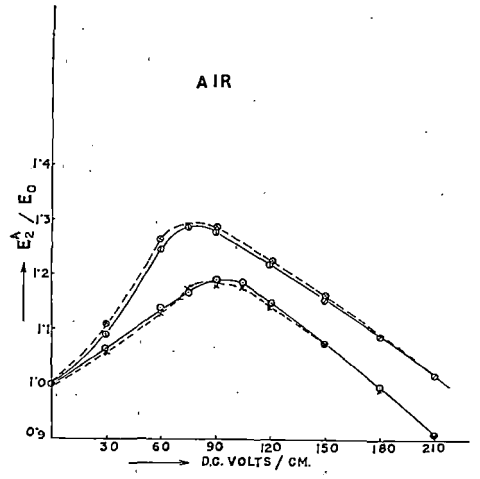

Fig. 1. Variation of E_2^A/E_0 with E_{DC} in air (theoretical and experimental)

○—○—○ Experimental
×—×—× Theoretical — P=1.5 mmHg
①—①—① Experimental
⊗—⊗—⊗ Theoretical — P=1.0 m.m.

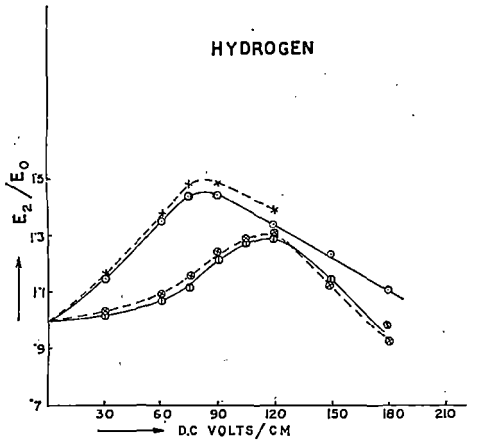

Fig. 2. Variation of E_2/E_0 with E_{DC} in hydrogen (theoretical and experimental)

○—○—○ Experimental
×—×—× Theoretical — P=2 mmHg
①—①—① Experimental
⊗—⊗—⊗ Theoretical — P=3 m.m.

the value of E_1 the breakdown voltage without considering d.c. ionization becomes so large that wide divergence is noticed even if d.c. ionization is considered. This fact suggests as has been mentioned before that perhaps the drift velocity does not remain a linear funear function of (E/P) and the loss of electrons due to mobility is actually smaller than what is predicted by Varnerin and Brown's theory. In case of oxygen also divergence becomes dominant at high d.c. fields even though attachment correction has been

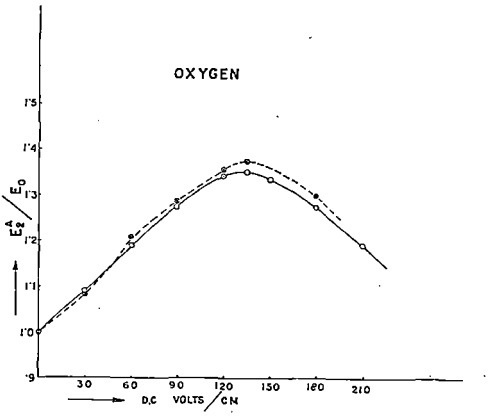

Fig. 3. Variation of E_2^A/E_0 with E_{DC} in oxygen (theoretical and experimental)

○—○—○ Experimental
⊗—⊗—⊗ Theoretical — P=2 mmHg

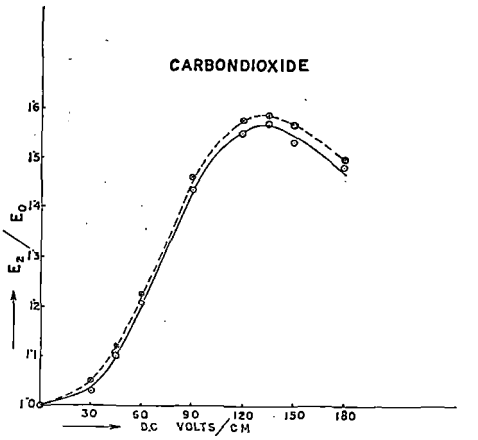

Fig. 4. Variation of E_2/E_0 with E_{DC} in CO_2 (theoretical and experimental)

○—○—○ Experimental
⊗—⊗—⊗ Theoretical — P=1.5 mmHg

applied. Results would have improved slightly if attachment data were available in case of carbon dioxide.

It can thus be concluded that for small d.c. voltage the theory proposed by Varnerin and Brown that loss of electrons increases due to mobility can explain the increase of breakdown voltage, but, with the increase of d.c. voltage, ionization due to d.c. field also has to be taken into consideration, and this factor gradually increases with the applied d.c. field and becomes considerable at high d.c. voltage and thereby reduces the radiofrequency voltages necessary for breakdown. The observed discripency can be ascribed partly to the uncertainty in the values of molecular constants introduced by

Kihara in his theory of electrical discharge. In case of d.c. ionization, the primary ionization is much more predominant than the secondary effect which depends to a major extent upon the nature of the electrode and also upon the pressure of the gas in the discharge tube.

It should however be noted that the theory which has been advanced here is based upon the theory of electrical discharge by Kihara which assumes that electron velocity distribution function is Maxwellian. But in fact it actually does not hold good when radiofrequency voltage sufficient to cause breakdown is applied and departure from the Maxwellian distribution becomes more when d.c. ionization effects which involve large transit of electrons are considered. Consequently the above theory suffers from the same limitations as are applicable to Kihara's theory.

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