PREFACE

This thesis systematically devolops the problems connected with nonhomogeneity of mechanical properties of materials in plates and shells for isotropic or anisotropic cases. Such problems, besides being of great scientific importance, play a very significant role in many technical applications and in engineering practice.

Increasing use of composite materials in aerospace applications or wide utilization of concrete to change the surface of the earth or timber-technology draws the attention to solve the problems of plane-elasticity, thermoelasticity and visco-elasticity in static as well as dynamic conditions where the nonhomogeneous material is isotropic or anisotropic. Experiments in recent times show that in a medium with definite type of temperature, the homogeneous character breaks down giving rise to nonhomogeneity to elastic modulii as well as the coefficients of thermal expansion. In practical situations these inhomogeneities are also produced by temperature gradients.

The investigators satisfaction lies in designing a structure with minimum material but it must be of maximum strength. This objective can be fulfilled with structures made of nonhomogeneous materials or structures formed by layered media having continuously increasing or decreasing stiffness. Therefore it is worthy to develop the expression of displacements and stresses in plates and shells taking into account the nonhomogeneous character of the material.

The opening chapter of this thesis is devoted to non-linear problems on plates of isotropic material. The second chapter deals with an anisotropic disk problem. The remaining four chapters are on shells. The third chapter covers the problems on twisted spheres and spherical caps of isotropic nature. Fourth, fifth and sixth chapters are on spherical and cylindrical shells made of anisotropic materials in static or dynamic state of course, nonhomogeneity is an essential feature in every problem.

The first chapter consists of two large deflection problems. It may be noted that in solving the large deflection plate problems one has to encounter the non-linear differential equations which cannot be exactly solved. However, the relevant differential equations may be obtained as a fourth order differential equation coupled with a non-linear second order equation when the same is deduced from the total strain-energy of the system in which the strain-energy due to second strain invariant is neglected. This method was adopted first by Berger (1955) [4] who has successfully solved the equations for circular and rectangular plates under uniform load and suitable boundary conditions. Though no justification has yet been given for such type of approximation, the results obtained in this process tally with those whenever available from more exact theories or from experiments. This approximate method has been extended to various problems of plates and shells by many authors e.g. Iwinski and Nowinski (1957) [23] on ortho-tropic plates, Nash and Modeer (1960) [31] on flexural vibrations of plates, Basuli (1968) [3] on heated plates, Das (1972) [15], (1973) [17] on shallow spherical shell under variable load, heated circular plate under variable load and others. The first paper of this chapter considers the large deflection of a square plate of uniform thickness made of nonhomogeneous material where the plate is under normal pressure and three dimensional temperature distribution. The case of an infinite strip of nonhomogeneous material with uniform thickness subjected to normal pressure and heating is also dealt with. The nonhomogeneity of the material is characterized by the linear variation of the Young's modulus. The normal deflections which are vitally important to expose the behavior of the plate or strip are obtained. The normal deflection of homogeneous strip is found approximately four times that of nonhomogeneous strip. The second paper of this chapter is intended to find the axisymmetric large deflection of a thermo-elastic annular circular plate subjected to lateral load and heating, in which the elastic property varies as any power of the distance from its centre. The load function is any function of the radius of the plate and the temperature varies in the directions of the thickness and the radius of the plate. The normal displacements of the plate under different boundary conditions are presented in simple forms.

The second chapter contains only one paper on anisotropic nonhomogeneous spinning disk. Stresses in a symmetrical rotating annular disk of variable thickness under variable thermal loading are calculated. A particular type of anisotropy is considered here so that the principal axes of anisotropy coincide with the principal axes of stress at each point of the solid which, in addition, is continuously inhomogeneous with mechanical properties varying along the radius. The results found by Leopold (1948) [26], Sengupta (1949) [38], Mollah (1976) [29] or Gururaja and Srinath (1973) [21] are deducible from the results presented here.

The third chapter has two papers. The first one is devoted to determine the stresses and displacement due to two equal and opposite couples applied on the surface of a composite inhomogeneous elastic sphere having homogeneous concentric inclusion. General solutions are presented in terms of Whittaker functions and associated Legendre functions. Chakraborty's (1970) [12] results for the same problem with homogeneous outer shell can be derived from the general results. The torsional vibration of a truncated cone with spherical caps is discussed in the second paper. Equations of motion are solved under two sets of boundary conditions. The frequencies of vibration have been calculated. The rigidity and the density of the material of the structure are proptional to any power of the distance from the vertex.

The chapter four is entirely on static problems of shells of considerable thickness. The materials concerned are nonhomogeneous and anisotropic. Anisotropic nonhomogeneous elastic media are the subjects of many authors like Greif and Chou (1971) [20], Brodeau (1959) [9], Bors (1959) [8], Kaliski (1959) [24] and others. Spherically isotropic materials have their physical properties transversely isotropic about the radius vector drawn from some particular point, fixed with respect to the material. In other words, in these materials, the physical properties are the same for all directions tangent to any sphere with its origin at a certain point. The elastic properties of such a material is described by five elastic parameters. An isotropic material with two independent elastic parameters is a special case of spherical isotropy. This definition of "spherical isotropy" is sometimes followed to mean "spherical anisotropy". But in this thesis the term 'spherical isotropy' is used everywhere. The first paper of this chapter deals with a spherically isotropic elastic medium bounded by two concentric spherical surfaces subjected to normal pressures. The material of the structure is spherically isotropic and, in addition, is continuously inhomogeneous with mechanical properties varying exponentially as the radius and as the square of the radius. Exact solutions for both the variations are presented in terms of Whittaker functions. The St.Venant's solution in the case of homogeneous material and Lamé's solutions in the case of homogeneous isotropic material are derived here from the general solutions. The problem of a solid sphere of the same medium under external pressure is also solved as a particular case of the above problem. The displacements and stresses of a composite sphere consisting of a solid spherical body made of homogeneous material and a nonhomogeneous concentric spherical shell covering the inclusion, both of them being spherically isotropic, are obtained when the sphere is under uniform compression. Such structures with the above mentioned material property are in extensive use where the radiation sheilding is needed. The above analysis is also of much help for layered media having exponentially increasing or decreasing stiffness. The second paper of this chapter is still more important. Here the elasticity problem of a cylindrically anisotropic elastic medium bounded by two axisymmetric cylindrical surfaces subjected to normal pressures (plane-strain) is studied. The material of the structure is orthotropic with cylindrical anisotropy and, in addition, is continuously inhomogeneous with mechanical properties varying exponentially along the radius. Stresses and displacements are found in terms of Whittaker functions. The results obtained by St.Venant for homogeneous cylindrically anisotropic medium follow from the general solutions. The problem of a solid cylinder of the same medium under the external pressure is also solved as a particular case of the above problem. Problems of the type covered in this paper are encountered in nuclear reactor design. The last paper of this chapter is no less important. The object of this paper is to determine the radial displacement and relevant stresses in cylindrical shell (plane-strain) on the basis of three dimensional linear theory of elasticity. The nature of the material property is same as stated for the second article of this chapter.

The fifth chapter of this thesis contains one paper on time-hardening and time-softening anisotropic nonhomogeneous thick elastic shells. The elastic parameter is assumed to be a function of time of the asymptotic type - another new idea in elasticity introduced by

Paria (1966) [34] and Bychawski and Piszczek (1959) [10]. Das (1972) [14] has developed this idea to obtain transient displacements and stresses that exist in thick elastic shells under internally and externally applied time-depedent loads in radial motion of an infinitely long circular cylindrical shell and in radially symmetric motion of a spherical shell. Das (1973) [16] has further extended the idea to nonhomogeneous body and found out the displacement and stresses in a thin circular plate with a central hole under a state of extensional vibration due to dynamic loading on the boundary of the hole and on the edge of the plate. The transient displacements and stresses in thick elastic spheres and cylinders have been obtained here when the surfaces are subjected to dynamic loads internally and externally for the following problems —

(A) radially symmetric motion of a spherical shell,

(B) pure radial motion of an infinitely long circular cylinder.

The material of the structures is continuously inhomogeneous with mechanical properties varying along the radius and depending on time. Moreover, it is spherically isotropic for (A) and orthotropic with cylindrical anisotropy for (B).

The concluding chapter has only one paper on wave propagation in visco-elastic medium from a spherical cavity. The medium is spherically isotropic and nonhomogeneous. Radial displacement in such a medium is found when the surface of the spherical cavity in the medium is loaded.