NON-LINEAR ANALYSIS OF MODERATELY THICK PLATES - A NEW APPROACH.

P R E F A C E

Structural members commonly known as plates are used in machine parts, in aircraft design and also in modern structural design. The study of bending properties of such members is imperative to a design engineer. The bending properties of a plate depend greatly on its thickness as compared with its other dimensions. To study these properties we shall have to distinguish between two kinds of plates - (A) Thin plates and (B) Thick plates.

A. Thin plates :

Structural members whose one dimension is small in comparison with other two dimensions are commonly known as thin plates Within the elastic limit, the static, the thermal and the dynamic behaviours/responses of thin plates are influenced by the following factors :

(1) Material properties defined by Young's modulus E and Poisson's ratio ν. E and ν may be variable.

(2) Geometry of plate —

Geometry may be simple such as circular or complicated. Thickness of the plate may be variable.

(3) Types of loading and

(4) Nature of supports i.e. edge conditions.

It is well-known that if deflections w of a thin plate are small in comparison with its thickness h, a very satisfactory approximate theory of bending of the plate under lateral loads can be developed by making the following assumptions :

(a) There is no deformation in the middle plane of the plate and this plane remains neutral during bending.

(b) Points initially lying on a normal to the middle plane of the plate remain on the normal to the middle surface of the plate after bending and

(c) The normal stresses in the direction transverse to the plate can be disregarded.

The above assumptions constitute the simplest and most widely used classical small deflection theory developed by Lagrange [1].

The first assumption is completely satisfied only if a plate is bent into a developable surface. In other cases bending of a plate is accompanied by strain in the middle plane, but calculations show that the corresponding stresses in the middle plane are negligible if the deflections of the plate are small in comparison with its thickness. If the deflections are not small, these supplementary stresses must be taken into account in deriving the differential equations governing the deflections of the plates. In this way, we obtain non-linear equations and the solution of the problem becomes much more complicated.

With the advent of modern plate and shell constructions subjected to severe operational conditions, the classical linear theory for small deflections is no longer applicable in many cases. Methods of analysis dealing with large deflections, therefore, are of increasingly practical importance. It is well-known that the classical plate equations for studying the nonlinear behaviours of thin plates are due to Von Karman [2]. Von Karman's equations are in the coupled form and hence difficult to solve. Different numerical methods have been offered by several authors to solve them. Outstanding research workers who worked on Von Karman's equations are Chu and Herrman [3], Yamaki [4], Nowiniski [5] and Baur [6]. Other note worthy works in this field are due to Dutta [7] and Chowdhury [8], [9].

Berger [10] offered a simplified approach to study the nonlinear behaviours of thin plates. According to Berger's hypothesis the elastic energy due to the second invariant of the membrane strain may be disregarded as compared to the square of the first invariant without appreciably impairing the accuracy of the results. The Euler-Lagrange equations so derived from the variational equations turn out to be much simpler than those of Von Karman. Hence, this method gains popularity due to its simplicity, but its application is limited to the case of immovable edge conditions only [11]. Successful research workers who carried out useful investigations on this method are Nash and Modeer [12], Wah [13], Nowiniski [14], Banerjee [15]. Other interesting works on Berger's equations are due to Kamaiya [16], Karmakar [17] who carried out their investigations on sandwich plates. Later Banerjee [18] offered a modified strain-energy expression for the investigation of the nonlinear behaviours of thin plates. Banerjee's hypothesis is based on introducing directly the expression for the membrane stress into the total potential energy of the system. As a consequence, a new set of differential equations has been obtained in an uncoupled form. This hypothesis states that the radial stretching is proportional to $\left(\frac{dw}{dr}\right)^2$. This is reasonable as because the contribution of the term $\left(\frac{dw}{dr}\right)^2$ in the expression for the radial term is greater than that of $\frac{du}{dr}$ in bending. The author has carried out investigations on the nonlinear analysis of different elastic plates [19], [20] and obtained satisfactory results. Later Banerjee with Sinha Roy extended his line of thought to the large deflection of shallow shells [21] and obtained excellent results.

Another useful method to carry out the non-linear behaviours of thin plates is the finite element method. Eminent research workers in this field are Striz, Jang and Bert [22] and Chi-lung Huang [23].

B. Thick plates :

The approximate theories of thin plates discussed above, become unreliable in the case of plates of considerable thickness. In such a case, the thick plate theory should be applied. This theory considers the problem as a three dimensional problem of elasticity where the effects of transverse shear deformations and rotatory inertia are to be considered.

In recent years, a number of plate theories has been developed in an effort to extend the range of applicability of classical plate theory to that of thicker plates by including the effects of transverse shear deformation and transverse normal stress. It has been shown by Reissner [24], [25] that the inclusion of transverse shear deformation permits a return to Navier's three dimensional boundary conditions. Later on, Reissner [26] proposed a variational principle for the development of both the governing equations and the boundary conditions. Donnell [27] has given a three dimensional solution in the form of an infinite series in the loading functions for plates. Fredrick [28] investigated stresses on thick plates on elastic foundation. Donnell and Lee [29] have studied the problem of thick plates under tangential loads applied on the faces. Rectangular plates under different edge conditions have been studied in detail by many authors among which the works of Salerno and Goldberg [30], Volterra [31], Essenburg [32] and Volterra [33] need special mention. All these authors used either

Reissner's theory in their investigation or equations very simil to those obtained by Reissner, Starting with the assumptions concerning the components of displacements. Ariman [34] quite successfully investigated stresses of thick plates on elastic foundation. Lee [35] has given a three dimensional solution fo simply supported thick rectangular plates by applying the method followed by Donnell. Goldenviezer [36] has given an approxima theory of bending of a plate by the method of asymptotic integration of the governing equations. A three dimensional elasticity solution for rectangular plates has been developed by Srinivas [37]. This paper is also interesting.

The study of the nonlinear behaviours of moderately thick plates is gaining momentum day by day due to its wide application in modern structure and design. An attractive work in this field is due to Wu and Vinson [38]. The authors have used an improvec Reissner's variational theorem along with Berger's hypothesis to propose a set of governing equations including the effects of transverse shear deformation and rotatory inertia for large amplitude free vibrations of plates composed of transversely isotropic material. Another important work is due to Iyenger, Chandrashekhara and Sebastian [39] who carried out the analysis of thick rectangular plates by using a higher order theory which is an extension of Reissner's shear deformation theory. Kanaka Raju and Venkateswara Rao [40] have studied the axisymmetric vibrations of circular plates including the effects of geometric non-linearity, shear deformation and rotatory inertia by employing the finite element method to obtain their solution. Another paper can be located by Kanaka Raju, [41] where the nonlinear vibrations of beams considering shear deformation and rotatory inertia have been studied in detail. Stresses in a thick plate with a circular hole under axisymmetric loading have been quite successfully investigated by Chandrashekhara and Muthanna [42]. The authors have obtained an exact theoretical solution in terms of Fourier -Bessel series and integrals. Kanaka Raju, Venkateswara Rao and I. S. Raju [43] further studied the geometric nonlinearity on the free flexural vibrations of moderately thick rectangular plates. The authors employed finite element formulation to obtain the non-linear to linear period ratios for rectangular plates. A conformal finite element of rectangular shape, wherein the effects of shear deformation and rotatory inertia are included is developed and used for the analysis. Another paper by Kanaka Raju and Hinton, [44] needs special mention in which they quite satisfactorily analysed the non-linear vibrations of thick plates of different shapes having different boundary conditions by using Mindlin plate elements.

A discussion on various non-linear theories applicable for moderately thick plates can be found in papers by Sathyamoorthy and Chia [45] and Sathyamoorthy [46] where it has been shown that the effects of transverse shear and rotatory inertia play a significant role in the large amplitude vibrations of moderatel thick plates of various geometries. Reddy and Chao [47] have studied the finite element analysis of the equations governing the large amplitude free, flexural oscillations of laminated anisotropic rectangular plates.

Very recently Reissner [48] has generalised some formulas of the theory of moderately thick plates. The author restates formulas for stresses and stress couples for a theory of isotropic moderately thick plates (in the classical tests of Love and of Timoshenko) in a simplified form. Fuh-Gwo Yuan and Miller [49] have presented the development of a straight forward displacement type rectangular finite element for bending a flat plate with the inclusion of transverse (or lateral) shear effects. A simple higher order non-linear shear deformation plate theory has been proposed by Lee, Senthilnathan, Lim and Chow [50]. The Von Karman extension of the theory is found to be remarkably simple for obtaining the approximate solution for the non-linear bending and vibration of thick, isotropic and transversely isotropic plates.

To sum up

(i) Thick plate theory is an extension of the classical thin plate theory, where the effects of transverse shear deformation and rotatory inertia are to be included.

(ii) The analytical works so far carried out for investigation of the non-linear behaviours of thick plates are based mainly on single mode approximation and have often been done with the aid of either Von Karman type nonlinear equations or Berger type approximation, along with Reissner's variational principle.

(iii) Finite element formulation has recently been used by some authors.

It is to be noted that Berger's equation is a purely approximate method. It is meaningful only for immovable edge conditions. Von Karman equations are in the coupled form and thus difficult to solve, whereas finite element method needs much computational labour and lacks in the essence of formulation of the classical plate equations.

Aim of the present project :

The aim of the present thesis is to offer a simplified approach for the non-linear analysis of thick plates by using Reissner's variational theorem along with Banerjee's hypothesis. A set of uncoupled differential equations have been formed to study the non-linear behaviours of different elastic plates showing the effects of shear deformation and rotatory inertia. Accuracy of the results obtained from these equations has been tested for different plates and compared with other known results. The present study seems to be more advantageous than the previous investigations, because,

(a) The results can be obtained from a single differential equation both for movable as well as immovable edge conditions.

(b) The results are sufficiently accurate from the practical point of view.

(c) The proposed differential equations are in the uncoupled form and hence easy to solve. Computational labour is minimum for its simple form.

The thesis has been divided into three chapters. The first chapter is devoted to deducing the proposed differential equations governing the vibrations of thick plates with shear deformation and rotatory inertia effect. Banerjee's hypothesis suggesting a modified strain energy expression along with Reissner's variational principle has been utilised.

The second chapter deals with the application of Berger's equation on thick plate theory. Non-linear responses of thick plates having different shapes placed on elastic foundation have been studied in detail. Numerical results showing the effects of shear modulus and rotatory inertia for different values of the foundation modulus have been given in tables and compared with other known results. The study shows that Berger's approximate theory can be conveniently applied to the analysis of the thick plates. But it has been shown that for movable edge conditions Berger's theory fails miserably.

The third and the concluding chapter is devoted to the application of the new set of differential equations proposed in the present thesis. The non-linear dynamic behaviours of thick plates of square, circular and regular polygonal shapes have been studied in detail. The static behaviours of elliptical and right angled isosceles triangular plates have also been studied. Different edge conditions have been considered. For regular polygonal plates conformal mapping technique has been employed. Numerical results showing the ratio of the non-linear time periods to linear time periods for different values of transverse shear deformations have been plotted graphically in few cases and given in tabular form for other cases. It has been observed that the results obtained from the present study are in very good agreement with other known results. So, the proposed differential equations of the present project, showing the effects of shear deformation and rotatory inertia, seem to predict the non-linear behaviours of different thick elastic plates of both movable as well as immovable edges, with ease and accuracy.