3. EXTENDED SYNOPSIS

In this chapter the Author wishes to report in details the work carried out by him during the period of investigation with an introduction to the present work, objectives and scope of the subject after having a literature survey on the topics of the thesis. The theory and formulation of the subject under consideration will be presented as per need. A summary report on the findings of the results of the present investigation has been presented. There are every possibility of exploring some avenues for further investigations in future on the present topics and as such some recommendations has been made for the said purpose.

3.1 INTRODUCTION TO THE PRESENT WORK.

In engineering and building designs, especially in several of the present day high-technology industries, high speed spacecrafts, nuclear power plants, offshore and ship building mechanics, storage and high-rise structures including applications in many branches of engineering mechanics and aeronautics, the widespread use of plates and shells are made for which there arises the need for reliance upon different methods of analysis.

Analytical techniques have serious limitations because of difficulties in closed-form solutions of nonlinear differential equations. Therefore despite the simplified nature of plates and shell theory and the effort that has been extended in this area, relatively few solutions are known, particularly when the structures behave geometrical nonlinearity.

Thin plates and shells of regular polygonal and irregular shapes made of isotropic, orthotropic and sandwich materials are often subjected to different kinds of mechanical and thermal loading and as a result such structural components are prone to deformations, bucking and vibrations for which proper analysis are required to be made and of great interest to designers, engineers, scientists and researchers.

Analysis of deformations, buckling and vibrations of different kinds of plates and shells could be made by the applications of linear and nonlinear theory. In the linear theory the components of strains $\varepsilon_{11}, \varepsilon_{22}$ and ε_{12} of the deflected middle surface have negligible magnitude but for nonlinear theory these

components cannot be neglected and are to be taken into consideration for deriving the basic governing nonlinear partial differential equations.
Attempts have been made during the course of the present investigation so as to fill in the gaps where some more emphasis has to be made or to investigate further the elastic behavior of structures often used in modern technology.

3.2. OBJECTIVES AND SCOPE

As already been stated that a closed form solution of governing differential equation, to study the nonlinear and sometimes even the linear static or dynamic analysis of structures, is at most difficult if not impossible.

The main objective of the present work would be to find ways and means so as to overcome the difficulties with the help of existing methodologies as far as possible.

Also the other aim would be to identify problems, which have either been overlooked, or to have a new search for the development of the existing ones. With the advancement of modern technologies and subsequent applications of them in practice creates problems anew. The present investigator hopes to add something new, which may be of little magnitude, yet for which an honest attempt will be made.

However, the main sphere of investigation will be restricted to justify the Title of the present thesis.

3.3 SUMMERY OF LITERATURE SURVEY

Use of analytical techniques in solving such problems has considerable limitations because of difficulties in having closed-form solutions of nonlinear differential equations involved therein. Therefore despite the simplified nature of plates and shells theory and the efforts that have been extended to this area, relatively few solutions are known, particularly when the structures have large deformations.

Moreover, thin isotropic, orthotropic and sandwich plate or shell structures of regular polygonal and irregular shapes are often subjected to different kinds of mechanical and thermal loading and as a result such structural components are prone to small or large deformations, bucking and vibrations, for which

proper analyses are of great interest to designers, engineers, scientists and researchers.
In deformation, buckling and vibration analysis of different kinds of plates and shells may be made by using both the linear and nonlinear theory. In the linear theory the components of strains $\varepsilon_{11}, \varepsilon_{22}$ and ε_{12} of the deflected middle surface have little significance but for nonlinear theory these components cannot be neglected and rather they are to be taken into consideration for deriving the basic governing equations leading to nonlinear partial differential equations of higher orders.

Worth mentioning research works on static and dynamic analysis of thin plates and many researchers using different boundary conditions have carried out researches on shell structures under mechanical, thermal and other types of loadings. Unfortunately, most of them are restricted to linear analysis only. Extensive references are cited in the works of S.Timoshenko and S.Woinowsky Krieger [1], N.J.Hoff [2], B.E.Gatewood [3],Witold Nowacki [4], D.J.Jones [5],B.A.Boley and J.H.Weiner [6] J.L.Nowinski [7], H.Parkus [8], E. A.Thornton [9], L.H.Donnel [182] and P.Biswas [10].

Elaborate discussions on the temperature and membrane stress distribution in an elastic plate with an insulated central elliptical hole have been made by K.S.Rao, M.N.Bapu Rao and T.Ariman[11] using linear theory. Also mention may be made of some other major research works in analysing the thermal stress distribution and vibrations of different structures [12- 18].G.Fanonneau and R.D.Marangoni [19] considered the effect of a thermal gradient on the natural frequencies of rectangular plates. In such cases under elevated temperature, the elastic coefficients of homogeneous materials are no longer constants but become functions of space variables [2] and so application of non-homogeneous theory becomes a necessity.

Based on this non-homogeneous theory several other papers may be cited of which mention may be made of the work of N.Ganesan [20] who considered linear vibration analysis of a rectangular plate subject to a thermal gradient and J.S.Tomar and A.K.Gupta [21,22] who considered such an analysis for orthotropic rectangular and elliptic plates of linearly varying thickness and of non-uniform thickness and temperature, respectively.

Some other papers [23 - 25] deal with finite .deformations, post buckling behavior of heated rectangular plates with temperature-dependent material properties and thermo-elastic analysis in orthotropic elastic semi-space and

finite orthotropic slab [24-25]. T.R.Tauchert [26] presented an analysis on thermal shock of orthotropic rectangular plates.
The literature has also been enriched by the works on thermal post-buckling behavior of skew plates [27-28], thick elastic rectangular plate on an elastic foundation subject to a steady temperature distribution [29] and its extension to, a thick right-angled isosceles triangular plate [30]. Some other relevant works related with different structures or different technical methods may be found in the literature cited in references [31-43].

The above bulk of classical approach in Applied Mechanics rests on the assumption that the linear mathematical model has described the phenomenon involved. However, with the advent of modern technology and systems exposed to oppressive operational conditions induce large deflections, i.e., deflections that are of the same order as the plate and shell thickness and small compared to in-plane dimensions of the structures. Thus, when the deflections are no longer small in comparison with the thickness but small compared to the in-plane dimensions, the middle surface strains must be considered in deriving the differential equations of thin plates and shell structures. In this way one gets the nonlinear differential equations in the classical nonlinear theory.

For the analysis of large deflections of plates von Kármán's [44] coupled nonlinear partial differential equations have extensively been employed by many a earlier researcher. These equations which are of the fourth order with respect to the unknown deflection W and stress function F enables one to determine W and F. These equations can also be expressed in terms of displacement components u,v and W and conveniently been used for the nonlinear analysis of different kind of plates and shells. von Kármán's equations are generally difficult to deal with because of its coupled nonlinearity and as yet no general solutions of these equations are known. However, approximate and different numerical and computational methods have been adopted for the solution of such large deflections analysis of plates and shells.

There is a galaxy of outstanding research workers who employed von Kármán equations to the analysis nonlinear behavior of thin plates, both isotropic and orthotropic under mechanical and other kinds of loading and subsequently extended to shallow shells with the inclusion of curvature.

Several other authors extended von Kármán's equations plates and shells with the inclusion of thermal loading, both stationary (steady) and non-stationary (time-dependent) and further extension were made to sandwich plates and shells under mechanical and thermal loading [46-78].

Meanwhile C.Y.Chia [79] has published an excellent book entitled "Non-Linear Analysis of Plates" in which problems on orthotropic and laminated plates have been analyzed in addition to other problems with an Kármán extensive bibliography of other related works. Subsequently von Karman's equations have been extended with the inclusion of thermal loading in the static case for both plates and shells as have been nicely presented by W.Nowacki in his famous monograph [4]. Further studies by different authors using type field Kármán equations for different structures can be fond in Refs.[80-82].

As von Kármán's equations are in the coupled form and very difficult to deal with and yet having no closed-form solutions, H.M.Berger [83] proposed a pair of quasi-linear partial differential equations for the analysis of large deflection of isotropic plates. These equations being in the decoupled form have obvious advantages for getting solution of large deflection problems of elastic plates with much ease and computational effort. In Berger's method, the second strain invariant in the middle surface in the expression for total potential energy has been neglected. An application of the variational techniques of the Calculus of Variations to this simplified energy expression ultimately gives rise to the Berger's equations. Although no physical explanation of this method has been provided, yet results obtained by him and other authors agree well with those obtained from more precise analysis. However it has been shown by J.L.Nowinski and H.Ohnabe [84] and G.Prathap [85] that this method miserably fails i.e. this method gives absurd results for plates with movable edge boundary conditions. Considering the obvious advantages of Berger's method due to its quasi-linearity and de-coupled forms this method has been employed by many authors and further extended to the dynamic cases with and without thermal loading by William A. Nash and J.Modeer [86] followed by others [87-107]

Berger's method has further been extended with the inclusion of thermal loading in a good number of papers in the static and dynamic cases by many authors in the literature . Noteworthy mention may be made of the works of S.Basuli [108] who first extended Berger's method with the inclusion of thermal loading and investigated large deflection of some elastic plates under uniform load and heating. Intensive use of this method has been continued for a long period [108-143]

Over the years investigations on finite deformations and vibrations of Sandwich plates and shells under mechanical and thermal loading have been gaining importance due to wide applications in aerospace industry, high-speed aircrafts, missiles and in different components of structural mechanics.

However,Berger's equations have certain limitations and inaccuracies as discussed by several authors [84-85, 104] . These are most accurate for the immovable clamped edge conditions and fairly accurate for immovable simply supported edge conditions. Berger's assumptions yield absurd results for movable edge conditions. This is due to the fact that neglect of e_2 [the second strain invariant] for movable edge fails to imply freedom of rotation in the meridian planes where membrane stress exists. For movable edges the in-plane displacement u is never zero and thus Berger's equations lead to absurd results. On the other hand, for clamped edge conditions $u = 0$ and $dw/dr = 0$ at the boundary and Berger's equations are most accurate here. But for simply-supported edge conditions u is zero but $dw/dr \neq 0$ at the boundary and thus Berger's equations yield fairly accurate results. It is also interesting to note that under many loading conditions, especially uniform and under relatively smooth and regular boundary conditions–the distortional energy and its variation should be substantially smaller than the dilatation. Hence the Berger's assumption which too simplistically has been translated into assuming a Poisson's ratio of unity and has patently absurd . For this reason this assumption has always yielded reasonably good practical results for the uniform or smoothly varying loading. The circular is the best geometry, but as any in-plane large distortional changes even in rectangular plates is usually confined to the corners, reasonable results should also be expected there. On the other hand disparities such as movable boundary suggest large energy changes and the basic hypothesis becomes questionable. Consequently due to severe criticism of the application of Berger's equations B.Banerjee *et.al* [144,145] offered a new set of decoupled differential equations to investigate the nonlinear behaviors of elastic plates under different types of loading. The new set of differential equations are formed under a modified energy expression containing the expression for the in-plane stress σ_{rr}.

The differential equations as proposed in [144, 145] are decoupled and hence can be solved without any difficulty for any type of loading and are valid for both movable and immovable edge conditions with ease and with relative accuracy and further applied by them for spherical and cylindrical shells in the static and dynamic cases [146-147]. Following the same approach B.Banerjee alone [148] presented large deflection analysis of circular plates of variable thickness. Exponential variation of thickness useful in design and discussed fully by S.Timoshenko and S.Woinowsky-Krieger [1] was considered in this paper. Further S .Datta and B .Banerjee [149] and P.Bhattacharya and B.Banerjee [150] extended this modified Berger's method to large deflection analysis of sandwich plates. Subsequently, using this modified Berger's method D.N.Paliwal et.al. [151-157] considered problems on nonlinear deformations and vibrations of elastic plates and shells under mechanical and thermal loading and resting on Pasternak and Kerr type of elastic. However, using this method M.M.Banerjee *et al.*[158] investigated large amplitude free vibration of shallow spherical shell subjected to thermal gradient including effects of temperature dependent modulus of elasticity of material and expressed some reservations on the use of this modified Berger's method. This modified Berger's has also been further extended to heated sandwich plates [159-160]

In addition to the voluminous works of plate and shell problems using von Kármán's method, Berger's method and modified Berger's method, a great deal of remarkable progress has been made for the analysis of nonlinear static and dynamic behavior of plate and shell structures subjected to different types of loading-mechanical and thermal, by using other analytical methods. Such analytical methods are not always suitable to deal with problems on plates and shells having complex geometrical and boundary conditions. For such cases, numerical methods like finite element method, boundary element method, complex variables method and conformal mapping technique have been used by many researchers and scientists some of which have been cited in this thesis.

During the course of revision of this Thesis, the Author has made further literature survey and discovered some recent published works on topics related with those of the Thesis. These current works have helped the Author in many ways for the revision of this Thesis. These References [201-208] have been included in the Reference Section of the Thesis.

3.4 SUMMARY OF FORMULATIONS.

With usual notations, the total strain energy is given by

$$U=\frac{1}{2}\iiint(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_{xy}\varepsilon_{xy})\,dzdxdy$$

whereas the kinetic energy is

$$T_e=(\rho h/2)\iint(\dot{u}^2+\dot{v}^2+\dot{w}^2)dxdy$$

and the work done is

$$W_k=\iint pw\,dxdy$$

Formulating the Lagrangian with the help of the above expressions and applying Hamilton's principle, a straightforward application of the variational calculus will yield the following equations of motion [192]

$$D\nabla^4 w=hS(F,w)-h\left(\frac{F,_{yy}}{R_x}+\frac{F,_{xx}}{R_y}-2\frac{F,_{xy}}{R_{xy}}\right)+q-\rho hw,_{tt}$$

and

$$D\nabla^4 F=-\frac{E}{2}hS(w,w)+E\left(\frac{w,_{yy}}{R_x}+\frac{w,_{xx}}{R_y}-2\frac{w,_{xy}}{R_{xy}}\right)$$

where the operator *S(w,F)* stands for

$$S(w,F)\equiv\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}$$

Here 'F' denotes the Airy-Stress function as found in the literature

$$\int_{-h/2}^{h/2}\sigma_{xx}dz=N_x=h\frac{\partial^2 F}{\partial y^2},\quad \int_{-h/2}^{h/2}\sigma_{yy}dz=N_y=h\frac{\partial^2 F}{\partial x^2},\quad \int_{-h/2}^{h/2}\sigma_{xy}dz=N_{xy}=-h\frac{\partial^2 F}{\partial x\partial y},$$

whereas

$$M_x=\int_{-h/2}^{h/2}\sigma_x zdz=-D[w,_{xx}+\nu w,_{yy}]\quad,\quad M_y=\int_{-h/2}^{h/2}\sigma\ zdz=-D[w,_{yy}+\nu w,_{xx}]$$

$$M_{xy}=\int_{-h/2}^{h/2}\sigma_{xy}zdz=-D(1-\nu)w,_{xy}$$

and the (,) notation signifies partial derivative with respect to the suffix

If we are concerned with the doubly-curved shells we may put $1/R_{xy} = 0$, and R_x *and* R_y are suitably chosen with proper signs for Gaussian curvature and one may get

$$D\nabla^4 w = hS(F,w) - h\left(\frac{F,_{yy}}{R_x} + \frac{F,_{xx}}{R_y} - 2\frac{F,_{xy}}{R_{xy}}\right) + q - \rho\, hw,_{tt}$$

and

$$D\nabla^4 F = -\frac{E}{2} hS(w,w) + E\left(\frac{w,_{yy}}{R_x} + \frac{w,_{xx}}{R_y} - 2\frac{w,_{xy}}{R_{xy}}\right)$$

with $1/R_{xy} = 0$.

But when plate problems will be considered we also put all $1/R_x$ *and* $1/R_y$ to be zero, i.e with $1/R_x$ *and* $1/R_y$ equal to zero i.e.,

$$D\nabla^4 w = hS(\phi, w) + q - \rho h w_{tt}$$
$$\nabla^4 \phi = -(E/2)S(w,w),$$

When thermal contribution is added to the problem the strain components will take the form

Median surface stress-strain relations are given by

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y) - \frac{\alpha_t ET}{(1-\nu)}$$
$$\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x) - \frac{\alpha_t ET}{(1-\nu)}$$
$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

For a cylindrical shell Panel the von-Karman strain displacement equations are given by Donnell[182]

$$\varepsilon_x = u,_x + \frac{1}{2}(W,_x)^2 - zW,_{xx}$$

$$\varepsilon_y = u,_y + \frac{1}{2}(W,_y)^2 - zW,_{yy} - \frac{W}{R}$$

$$\gamma_{xy} = u,_x + v,_y + W,_x W,_y - 2zW,_{xy}$$

The forces N_{xx}, N_{yy}, N_{xy} and the moments M_{xx}, M_{yy}, M_{xy} can be expressed by the matrix equation

$$\begin{bmatrix} N_{xx}, N_{yy}, N_{xy} \\ M_{xx}, M_{yy}, M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x, \sigma_y, \tau_{xy} \\ z\sigma_x, z\sigma_y, z\tau_{xy} \end{bmatrix} dz$$

The in-plane equations of equilibrium in the X and Y directions are

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$

These equations are identically satisfied by introducing the Airy stress function defined by the relations

$$N_{xx} = F,_{yy}$$

$$N_{yy} = F,_{xx}$$

$$N_{xy} = -F,_{xy}$$

Considering preceding equations one gets

$$\varepsilon_x = \frac{1}{Eh}(F,_{yy} - \nu F,_{xx}) - zW,_{xx} + \frac{\alpha_t N_T}{h}$$

$$\varepsilon_y = \frac{1}{Eh}(F,_{xx} - \nu F,_{yy}) - zW,_{yy} + \frac{\alpha_t N_T}{h}$$

$$\gamma_{xy} = -\frac{2(1+\nu)}{Eh} F_{,xy} - 2zW,_{xy}$$

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} - \frac{1}{R}\frac{\partial^2 W}{\partial x^2}$$

By virtue of the above equations one gets the following differential equation for the stress function in terms of deflection function

$$\nabla^4 F = Eh\left[\left(W_{,xy}^2\right) - W_{,xx} W_{,yy}\right] - \alpha_t E\left(\nabla^2 N_T\right) - \frac{Eh}{R} W_{,xx}$$

where

$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} T(x,y,z)dz = \text{Thermal Stress Couple}$$

Considering the expressions for the moments from the equations as stated above and considering the following equation of equilibrium [1,page-379]

$$\frac{\partial^2}{\partial x^2}(M_{xx}) - 2\frac{\partial^2}{\partial x \partial y}(M_{xy}) + \frac{\partial^2}{\partial y^2}(M_{yy}) = -\left[N_{xx}W_{,xx} + 2N_{xy}W_{,xy} + N_{yy}W_{,yy}\right]$$

one gets

$$D\nabla^4 W + \frac{\alpha_t E}{(1-\nu)}\left(\nabla^2 M_T\right) = \left[F_{,xx} W_{,yy} - 2F_{,xy} W_{,xy} + F_{,yy} W_{,xx}\right]$$

where

$$M_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} zT(x,y,z)dz = \text{Thermal Moment}$$

The above two equations constitute coupled nonlinear partial differential equations in the von Karman sense for determining the large thermal deflections of a shallow cylindrical shell panel.

If the plate is assumed to be comparatively thin and normal to the plane

$$\bar{\varepsilon}_x = \frac{1}{E}(\sigma_{xm} - \nu\sigma_{ym}) = \frac{1}{E}(\phi_{yy} - \nu\phi_{xx}) = u_x + \tfrac{1}{2}w_x^2$$

$$\bar{\varepsilon}_y = \frac{1}{E}(\sigma_{ym} - \nu\sigma_{xm}) = \frac{1}{E}(\phi_{xx} - \nu\phi_{yy}) = v_y + \tfrac{1}{2}w_y^2$$

$$\bar{\varepsilon}_{xy} = \frac{2(1+\nu)}{E}\sigma_{xym} = -\frac{2(1+\nu)}{E}\phi_{xy} = u_y + v_x + w_x w_y$$

Performing necessary integrations, the total strain energy can be expressed in terms of the first and second invariants of the middle surface of the plate

$$\xi = \frac{D}{2}\iint \left\{ \left(\nabla^2 w\right)^2 + \frac{12}{h^2} e_1^2 - 2(1-\nu)\left[\frac{12}{h^2} e_2 + w_{xx}\, w_{yy} - w_{xy}^2\right] \right\} dx\, dy - \iint p\, dx\, dy$$

where $e_1 = \bar{\varepsilon}_x = +\bar{\varepsilon}_y$, $e_2 = \bar{\varepsilon}_x \bar{\varepsilon}_y - \frac{1}{4}\bar{\varepsilon}_{xy}^2$

The kinetic energy of the plate is

$$T = \frac{\rho h}{2}\iint (u_t^2 + v_t^2 + w_t^2)^2\, dx\, dy$$

Using the Lagrangian L = (T − ξ^*) and applying Hamilton's principle we get the dynamic analogue of the von Karman equations (in the absence of time derivative), as:

$$u_{xx} + w_x w_{xx} + \nu(v_{xy} + w_y w_{xy}) + \tfrac{1}{2}(1-\nu)\,(u_{yy} + v_{xy} + w_x w_{yy} + w_y\, w_{xy}) = \frac{\rho(1-\nu^2)}{E} u_{tt}$$

$$v_{yy} + w_y w_{yy} + \nu(u_{xy} + w_x\, w_{xy}) + \tfrac{1}{2}(1-\nu)\,(u_{xy} + v_{xx} + w_x\, w_{xy} + w_y\, w_{xx}) = \frac{\rho(1-\nu^2)}{E} v_{tt}$$

$$\frac{h^2}{12}\nabla^4 w = u_x\, w_{xx} + \tfrac{1}{2} w_x^2\, w_{xx} + v_y\, w_{yy} + \tfrac{1}{2} w_y^2\, w_{yy} + \nu(v_y\, w_{xx} + \tfrac{1}{2} w_y^2\, w_{xx} + v_y\, w_{yy} + \tfrac{1}{2} w_x^2\, w_{yy}) +$$

$$(1-\nu)(u_y\, w_{xy} + v_x\, w_{xy} + w_x\, w_y\, w_{xy}) + \frac{\rho(1-\nu^2)}{E}(w_x\, u_{tt} + w_y\, v_{tt} + w_{tt}) + q$$

The above equations may be re-written in terms of stress resultants and moments for a plate with moderately large amplitude as

$$\frac{\partial}{\partial x}(\sigma_{xm}) + \frac{\partial}{\partial y}(\sigma_{xym}) = \rho h u_{tt} \quad , \quad \frac{\partial}{\partial y}(\sigma_{ym}) + \frac{\partial}{\partial x}(\sigma_{xym}) = \rho h v_{tt}$$

and

$$\xi^* = \iint \Big\{ -\frac{1}{2E}\big[(\phi_{xx}+\phi_{yy})^2 + 2(1+\nu)(\phi_{xy}^2 - \phi_{xx}\phi_{yy})\big] + \frac{D}{2}\big[(w_{xx}+w_{yy})^2 + 2(1-\nu)(w_{xy}^2 - w_{xx}w_{yy})\big]$$

$$+\frac{h}{2}\big[\phi_{yy}w_{xx}^2 + \phi_{xx}w_{yy}^2 - 2\phi_{xy}w_x w_y\big] - p\,w \Big\} dx\,dy$$

$$\frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial}{\partial x}(h\phi_{yy}w_x) + \frac{\partial}{\partial y}(h\phi_{xx}w_y) + \frac{\partial}{\partial x}(-h\phi_{xy}w_y) + \frac{\partial}{\partial y}(-h\phi_{xy}w_x) = \rho h w_{tt}$$

In general the preceding equations may represent the motion of the plate. Yet for practical purpose let us simplify the basic equations with the assumption that the effect of both the longitudinal and rotatory inertia forces can be neglected. The basic equations governing the nonlinear vibration of plates subjected to a normal load 'p' may be reduced to

$$\frac{\partial}{\partial x}(\sigma_{xm}) + \frac{\partial}{\partial y}(\sigma_{xym}) = 0 \quad , \quad \frac{\partial}{\partial y}(\sigma_{ym}) + \frac{\partial}{\partial x}(\sigma_{xym}) = 0$$

$$\nabla^4 \phi = E(w_{xy}^2 - w_{xx}\, w_{yy})$$

$$L(w,\phi) \equiv D\nabla^4 w - h(\phi_{yy}\, w_{xx} + \phi_{xx}\, w_{yy} - 2\,\phi_{xy}\, w_{xy}) + \rho h w_{tt} - p = 0$$

Besides the expressions for total energy, the required basic governing equations can be derived.

METHODOLOGIES AVAILABLE IN THE LITERATURE.

A. METHODS OF APPROXIMATE SOLUTION.

Once the basic equations have been established it is now the time for investigation of plate problems. The linear approach may sometime look easier than that of nonlinear ones, but sometimes linear problems also involve other geometric nonlinearities or other factors that make the investigation a little harder when it becomes necessary to explore possibilities for an approximate solution. Thus our next purpose will be to identify such methods which are broadly in use.

B. RAYLEIGH,RITZ OR RAYLEIGH-RITZ METHOD.

The Ritz Method is simple and convenient for determining solutions to plate problems. It involves choosing the deflection function in advance in the form of a series as,

$$w = \sum_{i=1}^{n} C_i \ \varphi_i$$

where C_i's are undetermined parameters obtained by minimizing the total potential (V) satisfying the condition

$$\frac{\partial V}{\partial C_k} = 0, \qquad k = 1,2,3,\ldots\ldots n$$

We are interested here only the important observations made on the use of this method rather than illustrating any particular problem here.

The problem of rectangular plates with all possible mixed boundary conditions has been included in an excellent paper by Warburton [193] which gave formulas for finding the natural frequencies of all twenty-one possible distinct combinations of simple boundary conditions, using the Rayleigh method with single-term deflection modes composed of products of "beam functions"

The concept of using beam functions with the Rayleigh-Ritz method to obtain highly accurate frequencies and mode shapes was set forth in the classic work of Young [194] Young used superposition of beam functions and determined the eigenvectors of the amplitude coefficients by the minimizing scheme of Ritz [195].

An important component in the application of R-R method is the selection of appropriate admissible functions for use in the series representing the deflection of the plate in concern. Different sets of functions have been proposed by different authors. Gram-Schmidt process [196] is the important tool to generate sets of Orthogonal Polynomial functions, the first member of each set satisfying the geometric and natural boundary conditions of an equivalent beam, the remainder of the set satisfying, automatically, only the geometric boundary conditions of the beam.
For example, if the deflection function is set as

$X_m(x)$ *aswellas* $Y_n(y)$ may be generated by using Gram-Schimdt process.

$$(w(x,y) = \sum_{m=1}^{p} \sum_{n=1}^{q} A_{mn} W_{mn}(x,y),\ W_{mn}(x,y) = X_m(x) Y_n(y)$$

Gram-Schimdt considered the starting function as $X_0(x)$ and generated the subsequent functions as

$X_1(x) = (x - B_1)X_0(x),\ \ X_k(x) = (x - B_k)X_{k-1}(x) - C_k\, X_{k-2}(x),\ \ k > 1$

with

$$B_k = \int_a^b x\, w(x) X_{k-1}^2(x)\,dx \Big/ \int_a^b X_{k-1}^2(x)\,dx$$

$w(x)$being the weighted unction and the polynomials $X_k(x)$ satisfy the orthogonality condition

$$\int_a^b w(x) X_k(x) X_l(x) dx = \begin{cases} 0 & \text{if } k \neq l \\ a_{kl} & \text{if } k = l \end{cases}$$

Bhat [197] opted to choose the weighted function as unity, the interval as 0 to 1 and the coefficients of the polynomials are so chosen as to make the polynomials orthonormal,

$$\int_0^1 X_k^2(x)\,dx = 1$$

Since the orthogonal polynomials satisfy only the geometric boundary conditions, except for the first member, they do not over restrain the structure, unlike the beam functions. Hence Bhut observes that the functions are able to closely approximate the true boundary conditions of the plate with the application of R-R[Rayleigh-Ritz] procedure.

Thus with a first polynomial $X_1(x)$ satisfying the geometric and natural boundary conditions of the equivalent beam function the subsequent terms may be obtained from $X_2(x) = (x - B_2)X_1(x)$ and $X_k(x) = (x - B_k)X_{k-1}(x) - C_k\, X_{k-2}(x),\ \ k > 2$, where

$$B_k = \int_0^1 x X_{k-1}^2(x)\,dx \Big/ \int_0^1 X_{k-1}^2(x)\,dx,$$

and

$$C_k = \int_0^1 x X_{k-1}(x)\, X_{k-2}(x) dx \Big/ \int_0^1 X_{k-2}^2(x) dx.$$

In conclusion he adds that the characteristic orthogonal polynomials as proposed by him yield superior results for lower modes, particularly when the plate has free edges and are simple to construct and possess the orthogonal property which simplifies the analysis as in the case of beam functions[198] and simply-supported plate functions . Another important observation has been made by Liew and Lam[199] by using characteristic orthogonal polynomials in Rayleigh-Ritz method for flexural vibration, first introduced by Bhat.

Numerical results for skew plates depict that the convergence pattern for different modes of vibration for skew angles 15° and 45° stable convergence is reached with twenty-five terms used in the series for the expression of the transverse deflection in terms of two-dimensional orthogonal plate function. For fundamental mode, the convergence may be reached with lesser terms but for higher modes use of more than fifteen terms seems to be essential. The convergence study made by the authors [199] shows that for m=n=6 in the expression for *W(x,y)* the convergence is very rapid for all classes where free edges exist and that the use of a single term starting function (suggested by the authors) yields more satisfactory results.

The results presented by Dickinson and Blasio [200] confirm that the Gram-Schmidt generated polynomial functions proposed by Bhat are very satisfactory for use in Rayleigh-Ritz method for the study of variety of plate problems.

C.GALERKIN METHOD

In finding the solution for the equation $L[w(x,y)] = p(x,y)/D$, if somehow we can find an exact solution

$$w(x,y) = w_o(x,y) = \sum_{j=1}^{n} a_j\, w_j(x,y) ,$$

then

$$L[w_o(x,y)] - p(x,y)/D = 0$$

But if it is not so then will yield some error given by

$$E_r(x,y) = \sum_{j=1}^{n} a_j L[w_j(x,y)] - p(x,y)/D$$

known as the error function. The Galerkin procedure requires that the error function be orthogonal to with all the approximate functions ϕ_j., i.e.,

$$\iint_R E_r(x,y)\,\phi_j(x,y)\,dx\,dy = 0$$

which in turn yields *'n'* simultaneous equations for determination of the unknown coefficients in equation

The Galerkin Method has some advantages over the Ritz Method and thus its application extends over a broader range.. However, there are many other methods in the literature which are omitted here for the sake of brevity