INTRODUCTION

The phenomenon of stress wave propagation in elastic solids offers us a rich variety of waves which was developed a century ago. Some pioneer theoretical workers in this line are Rayleigh, Love, Stokes, Kelvin and others whose contribution in the field of wave propagation in elastic solids and vibrating bodies extended the theory of elasticity. In the first three decades of this century the subject lost its interest to the research workers due to lack of sophisticated instrument, electronic devices like high-speed computer and practical methods for observing the passage of stress waves in elastic solids. But in the later stage, several theoretical and experimental worker's keen interest in this field made a large number of technical papers giving various information.

During the past two decades, seismology has made a tremendous progress, mainly because of the advent of modern computers and improvements in data acquisition systems, which are now capable of digital and analog recording of ground motion over a frequency range of five orders of magnitude. These technological developments have enabled seismologists to make measurements with far greater precision and sophistication than was previously possible. As a result, far reaching advances in our knowledge of the earth's structure and the nature of the earthquake have occurred.

We here point out the milestones of progress in elastic waves in chronological order.

1678: Robert Hooke (England) established the stress-strain relation for elastic bodies.

1760: John Michell (England) recognized that earthquakes originate within the earth and send out elastic waves through earth's interior.

1821: Louis Navier (France) derived the differential equations of the theory of elasticity.

1828: Simeo-Denis Poisson (France) predicted theoretically the existence of longitudinal and transverse elastic waves.

1849: George Gabriel Stokes (England) conceived the first mathematical model of an earthquake source.

1857: First systematic attempt to apply physical principles to earthquake effects by Robert Mallet (Ireland).

1883: Rosi-Forel scale for earthquake effects published.

1885: C.Somigliana (Italy) produced formal solutions to Navier equations for a wide class of sources and boundary conditions.

1885: Lord Rayleigh (England) predicted the existence of elastic surface waves.

1899: C.G.Knott (England) derived the general equations for the reflection and refraction of plane seismic waves at plane boundaries.

1903: A.E.H.Love (England) developed the fundamental theory of point sources in an infinite elastic space.

1904: Horace Lamb (England) solved the problem on the propagation of tremors over the surface of an elastic solid.

1907: Vito Voltera (Italy) published the theory of dislocations based on Somigliana's solution.

1909: K.Zoeppritz and L.Geiger (Germany) calculated velocities of longitudinal waves in earth's mantle.

1940: Sir Harold Jeffreys (England) and K.E.Bullen (Australia) published travel time tables for seismic waves in earth.

1949: Lapwood, E.R. first considered the distribution due to a line source in a semi- infinite elastic medium.

1959: Ari Ben-Menahem (Israel) discovered that the energy release in earthquakes takes place through a propagating rupture over the causative fault.

1967: Global seismicity patterns and earthquake generation linked to plate motions.

In recent years the problem which mostly attract the researchers both theoretical and experimental, in relation to the generation and propagation of waves in elastic medium are:

(i) diffraction of propagating waves through the medium due to an obstacle, cavity or a crack of any shape situated some where in the medium;

(ii) wave motion generated due to punch on some bounded region of the medium;

(iii) reflection and refraction of a wave at a plane surface of discontinuity;

(iv) wave motion generated in a medium when a source of disturbance is static or moving along the medium.

(v) transmission and reflection of elastic waves by topographical irregularities.

(vi) elastodynamic problems involving crack propagation, crack kinking and bifurcation.

The solution of these problems need advance level of mathematical techniques, which may roughly be grouped into the following categories:

(a) Theory of analytic function

(b) The Fredholm integral equation

(c) The singular integral equation

(d) Integral transforms and Representations

(e) Dual integral and series equations

(f) Harmonic function. Potential theory

(g) The Dirichlet and Neumann problems

(h) Green's functions

(i) The Cauchy problem

(j) Wiener- Hopf technique

(k) Riemann- Hilbert problem

(l) The method of Matched Asymptotic expansion

(m) Perturbation technique

(n) Variational method, The Ritz method

(o) The method of finite element

(p) The method of boundary element

and others.

The problem of propagation of elastic waves in the presence of topographical irregularities and also in the presence of variation of material properties in the horizontal direction have drawn the attention of many investigators of the present time, due to their applications in seismology.

The problem of transient wave propagation in a half-space composed of two elastic quarter spaces of different materials was considered by Achenbach (1969). Dutta and Mitra (1974) considered SH-wave motion in an elastic quarter space in welded contact with a uniform elastic layer of different material. The problems of SH-wave transmission across an irregularity were considered by Bose (1975),Chakroborty et. al. (1983). Wolf(1967, 1970), Sinha (1980) considered the transmission of Love waves across an topographical irregularity while scattering of Rayleigh waves by a plane barrier in a shallow ocean was considered by Mann and Deshwal (1986).

A series of problems involving the scattering of elastic waves by two dimensional and three dimensional topographical irregularities have been solved by Sanchez- Sesma (1979, 1982, 1983, 1985) by using a newly developed boundary method. The diffracted fields are constructed with linear combinations of solutions which form c- complete families for the wave equation and boundary conditions are then satisfied in a least square sense. Adopting the representation theorem due to Knopoff (1956) , Knopoff and Hudson (1964) studied the transmission of Love waves past a continental margin considering the crust to have an abrupt increase in the thickness on the continental side. Sato (1961) discussed the problem of propagation of Love waves in an elastic layer of variable thickness overlying a semi- infinite elastic medium. Approximate expressions for the transmission and reflection factors are obtained by the application of a method based on Wiener- Hopf technique.. Abubakar (1963) also studied the effect of an irregular surface with an isolated irregularity like a tough or ditch on the incident P- and SV- waves using perturbation technique.

Apart from its academic interest, the propagation of elastic waves in layered media has important applications in geophysics and seismology. Since the propagation of characteristic of earth vary with depth, the first approximation to the actual problem can be achieved by regarding earth as formed of several layers in each of which properties are constant. The problem is very cumbersome as far as the mathematics is concerned. We mention the books by Brekhovskikh (1960),Eringen and Suhubi (Vol II,1975).

Recently, the problem of propagation of waves in layered elastic medium has been solved by Zaman et.al.(1987).

The problem of fracture is the central problem of the science of resistance of materials . Fracture mechanics in the broad sense of this concept includes the part of the science of strength of its materials and structures which relates to a study of the carrying capacity of the body both with or without consideration of initial cracks and also to a study of various laws governing crack development. In general the first stage of the investigation on fracture mechanics associated with the names of Robert Hooke, C.A.de Coulomb, B. de Saint Venant, Otto Mohr is characterized by extensive studies of deformation properties of solids and by the development of various failure criteria termed strength theories.

The dynamic process of fracture is made up of two stages, crack initiation and propagation, each of these stages following its particular laws. The criterion for the initiation of crack propagation, which forms the basis of fracture mechanics, does not follow from the equations of equilibrium and motion of continuum mechanics. This is an additional boundary condition in the solution of the problem of limiting equilibrium of a cracked body. The limiting state is said to be reached if a crack-like cut can propagate. The cut then becomes a crack.

Criterion for the initiation of crack propagation can be obtained on the basis of both energy and force considerations. Historically, at first an energy fracture criterion was proposed by A.A.Griffith (1920) and G.R.Irwin (1957) formulated a force criterion.

Yoffe (1951) first investigated the propagation of a finite crack with a constant speed through a stretched isotropic elastic solid. She showed that for small crack tip velocities the maximum tensile stress acts on the line $\theta=0$. Therefore, one may reasonably anticipate that the crack extends in a straight line; but at higher crack tip velocities, starting with $0.6c_2$ the line on which the maximum tensile stress is acting begins to make an angle with the initial crack axis. The angle increases very rapidly with the crack tip velocity i.e. the crack tends to become curved at propagation velocities higher than $0.6c_2$ as shown in

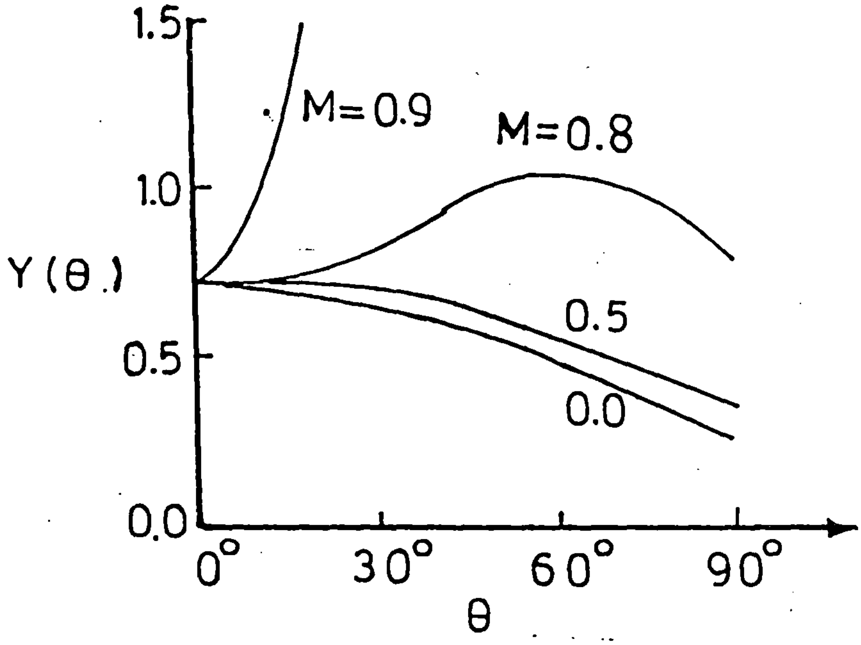

Fig. 1: Variation of Yoffe function with θ.

Fig.1. Complex variable technique to this case was latter applied by Radok (1956).

Indeed, it is now known that this variation is common to all crack tip stress distributions, as cam be verified form Irwin (1957), Williams (1957), Rice (1968). The noted variation of circumferential distribution with crack speed was proposed as an explanation for crack branching or bifurcation. While it is likely that the distribution of stress around a crack tip during acceleration may provide a mechanism by which the crack searches out alternate fracture paths, it does not explain why or how alternate paths are in fact executed.

In later stage, Yoffe's concept was utilized extensively by Singh et. al. (1981), Kassir and Tse (1983), De and Patra (1990). Three dimensional problem on moving crack was also solved by Itou (1979).

Recently, Das (1992,1993) has extended Yoffe's moving crack problems to the cases of three moving cracks.

The anti-plane problem of stress distribution around a semi-infinite crack moving with constant velocity in a strip of elastic material was solved by Sih and Chen (1972). The problem was reduced to Reimann - Hilbert problem by application of Scwartz- Christoffel transformation and the theory of complex functions.Closed form solutions were obtained for the two cases of practical importance: (i) the boundaries of the strip are clamped and displaceod in equal and opposite directions causing a tearing motion along the leading edge of the crack and (ii) the crack sheared longitudinally by a pair of concentrated forces moving with the crack while the strip boundaries are free of tractions. In both the cases, the effect of the strip width on the dynamic stress was examined.

Nilsson (1972) also solved the problem of a strip of material containing a moving semi-infinite crack using Fourier integral transform and Weiner- Hopf technique.

Here we give some techniques which are generally used in moving crack problems in elastodynamics.

1. Integral transform technique:

As the equations of motion in the theory of elasticity are partial differential equations which may be discussed with reference either to Helmholtz equation or to Laplace's equation, the method of integral transform is one of the most effective methods for solving such equations as application of this method to such equations results in the lowering of the dimension of an equation by one. There are several forms of integral transform and the choice of an integral transform depends on the structure of the equation and the geometry of the domain.

The integral transform $\overline{f}(\rho)$ of a function $f(x)$ defined on an interval (a,∞) is an expression of the form

$$\overline{f}(\rho) = \int_a^\infty f(x)\ K(x,\rho)\ dx \tag{1}$$

where a is a real number and ρ is a complex parameter varying over some region D of the complex plane. $K(x,\rho)$ is called the kernel of the transformation. The transformation (1) becomes particularly useful if it possesses inverse mapping. In that case one can express $f(x)$ in terms of its integral transform by

$$f(x) = \frac{1}{2\pi i} \int_\Gamma \overline{f}(\rho)\ M(x,\rho)\ d\rho \tag{2}$$

where $M(x,\rho)$ is a suitable function defined in $a<x<\infty$ and $\rho\in D$ and is called the kernel of the inverse transform, which is defined for all x in the interval (a,∞). The complex parameter ρ is in the region D while Γ is a suitable path of integration in D. After reducing the governing partial differential equation, the reduced problem can be solved for $\bar{f}(\rho)$. The solution of the original equation can be expressed in terms of the inverse integral, which may then be evaluated. The inversion from the the transformed space to the space of actual variables usually involved very complicated integrations. In many cases even the numerical integration can not be performed successfully because of the highly oscillatory character of the integrands [cf Eringen and Suhubi(1975), chap. 7; Achenbach (1975), chap. 7]. In particular, mixed boundary value problems like the dynamic response of a punch on an elastic half-space and the problem involving the presence of a crack or a strip inside an elastic medium may be reduced to Fredholm integral equation of first kind or to dual integral equations.

2. The factorization problem. The Wiener-Hopf technique:

Let a function $\phi(z)$ analytic in the interval $y_- < \text{Im } z < y_+$ be defined in the plane of a complex variable z. It is required to express $\phi(z)$ in the form

$$\phi(z) = \phi_+(z)\phi_-(z) \qquad (3)$$

where $\phi_+(z)$ and $\phi_-(z)$ are functions analytic in the half -plane $\text{Im } z > y_-$ and the half-plane $\text{Im } z < y_+$ respectively. The problem is called factorization problem. In a more general case, it is required to define two functions $\phi_+(z)$ and $\phi_-(z)$ which are analytic in the same half- planes respectively and which satisfy the following relation in the interval

$$A(z)\phi_+(z) + B(z)\phi_-(z) + C(z) = 0 \qquad (4)$$

where A(z),B(z) and C(z) are given analytic functions in the interval. It is obvious that if C(z) = 0, we obtain the representation (3) after the corresponding changes in the notation.

Let us assume that the function $\phi(z)$ which is to be factorised does not have any zeros in the interval $y_- < \text{Im } z < y_+$ and tends to infinity as $x \longrightarrow \infty$. In this case, neither of the functions $\phi_+(z)$ and $\phi_-(z)$ will have any zero, and we can take the logarithm of both sides of the relation (3)

$$\log \phi(z) = \log \phi_+(z) + \log \phi_-(z) \tag{5}$$

The function $F(z) = \log \phi(z)$ satisfies the condition

$$|F(x+iy)| < c|x|^{-p}, \quad (p>0 \text{ for } x \longrightarrow \infty) \tag{6}$$

and hence the relation (5) can always be solved with the help of the transformation

$$F(z) = F_+(z) + F_-(z) \tag{7}$$

Finally, we get

$$\phi(z) = e^{F_+(z)} e^{F_-(z)} \tag{8}$$

If the function $\phi(z)$ has zeros in the intervals we must consider a new function

$$\phi_1(z) = \frac{(z^2+b^2)^{N/2}\phi(z)}{\prod_{i=1}^{N_1} (z-z_i)^{\alpha_i}} \tag{9}$$

where z_i and α_i are the zeros, their multiplicity in the interval $N_1 \leq N$, where N is the total number of zeros, $b>(y_+,y_-)$. The factor in the numerator of (9) ensures that the properties of auxiliary functions are conserved at infinity.

Let us now consider the relation (4) and carry out its factorisation into L_+ and $1/L_-$ for the same interval of the ratio A/B. The relation (4) can be represented in the form

$$L_+(z)\phi_+(z) + L_-(z)\phi_-(z) + L_-(z)C(z)/B(z) = 0 \tag{10}$$

The expression $L_-(z)C(z)/B(z)$ can be represented in the following form in accordance with (7)

$$E_+(z) + E_-(z)$$

where $\phi_+(z)$ and $\phi_-(z)$ are functions analytic in the half-plane $y > y_-$ and the half-plane $y < y_+$ respectively. Taking this into account, we get

$$L_+(z)\phi_+(z) + E_+(z) = - L_-(z)\phi_-(z) - E_-(z) \qquad (11)$$

It follows from the generalized Liouville's theorem that the left as well as right hand side of (11) represents the same polynomial $P_n(z)$ of nth degree.

3. Hilbert transform technique:

If $p(y) \in L_2(a,b)$, then the equation

$$\int_a^b \frac{h(x)}{x - y}\, dx = \pi p(y), \quad y \in (a,b) \qquad (12)$$

has the solution

$$h(x) = \frac{1}{\pi}\left[\frac{x - a}{b - x}\right]^{1/2} \int_a^b \left[\frac{b - y}{y - a}\right]^{1/2} \frac{p(y)}{x - y}\, dy + \frac{C}{\sqrt{(x-a)(b-x)}} \qquad (13)$$

where C is an arbitrary constant, and the first term belongs to the class $L_2(a,b)$.

Using the above theorem, we find that the solution to the integral equation

$$\int_a^b \frac{2xh(x^2)}{x^2 - y^2}\, dx = \pi p(y), \quad y \in (a,b) \qquad (14)$$

(provided that p satisfies the conditions of the above theorem) is given by

$$h(x^2) = \frac{1}{\pi}\left[\frac{x^2 - a^2}{b^2 - x^2}\right]^{1/2} \int_a^b \left[\frac{b^2 - y^2}{y^2 - a^2}\right]^{1/2} \frac{2yp(y)}{x^2 - y^2}\, dy + \frac{C}{\sqrt{(x^2-a^2)(b^2-x^2)}}$$

where C is an arbitrary constant.

4. Schmidt method:

To solve for unknown constants $c_n(\zeta)$ occurring in

$$\sum_{n=1}^{\infty} c_n(\zeta)\; F_n(\zeta,x) = f(\zeta,x) \quad \text{for } x \in (a,b) \tag{15}$$

where $F_n(\zeta,x)$ and $f(\zeta,x)$ are known functions, we adopt Schmidt method.

Let $H_n(\zeta,x)$ be a set of orthogonal functions which satisfy

$$\int_a^b H_n(\zeta,x) H_m(\zeta,x)\; dx = N_n \delta_{nm} \tag{16}$$

where $$N_n = \int_a^b H_n^2(\zeta,x)\; dx \tag{17}$$

Then $H_n(\zeta,x)$'s can be computed from the functions $F_n(\zeta,x)$ in the following way

$$H_n(\zeta,x) = \sum_{i=1}^{\infty} \frac{c_{in}}{c_{nn}}\; F_i(\zeta,x) \tag{18}$$

with c_{in} as the cofactor of the e_{in} in D_n which is defined as

$$D_n = \begin{vmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots\cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots\cdots & e_{nn} \end{vmatrix} , \quad e_{nm} = \int_a^b F_n(\zeta,x) F_m(\zeta,x)\; dx \tag{19}$$

Now in terms of the set of orthogonal functions $H_n(\zeta,x)$, the function $f(\zeta,x)$ can be expressed as

$$f(\zeta,x) = \sum_{i=1}^{\infty} h_i\; H_i(\zeta,x) \tag{20}$$

Substituting the values of $H_n(\zeta,x)$ from (18) in (20), we obtain after some rearrangement

$$\sum_{n=1}^{\infty} c_n(\zeta)\; F_n(\zeta,x) = \sum_{n=1}^{\infty} F_n(\zeta,x) \sum_{i=n}^{\infty} \frac{c_{ni}}{c_{ii}}\; h_i \tag{21}$$

Comparing the coefficients of $F_n(\zeta, x)$ from both sides of (21) we find

$$c_n = \sum_{i=n}^{\infty} \frac{c_{ni}}{c_{ii}} h_i \tag{22}$$

where
$$h_i = \frac{1}{N_i} \int_a^b f(\zeta, x) H_i(\zeta, x)\, dx \tag{23}$$

This is in brief Schmidt method for determining the unknown coefficients c_n.

Recently extensive study on extension of crack in elastic solid has been made. Several investigations on symmetric and non-symmetric extension of crack in its own plane in an infinite elastic medium have been carried out up-till-now. Broberg (1960) first considered the problem of symmetric extension of a crack in elastic solid.

He considered the extension of a crack in a brittlelinear elastic material using Fourier transform. He assumed that the extension of crack occurs in its own plane. The plane surface is subjected

(i) to a constant pressure, acting on an infinite strip, the width of which is symmetrically increasing from zero with a constant velocity, and

(ii) to a pressure outside the strip such that the normal displacement of the surface outside the strip is zero.

The mixed boundary value problem has been treated. The stresses in the solid and the normal displacement of the surface have been solved. The result shows that the displacement of the surface is elliptic, just as the corresponding static case.

Since Broberg's investigation of the solution of a crack expanding symmetrically with constant velocity under conditions of plane stress or strain in a homogeneous elastic field of spatially and time invariant tensile stress, a number of papers have appeared analyzing different geometrical situations. Craggs (1963) later solved the same problem as that done by Broberg but he used the method of homogeneous function to obtain the solution, while Achenbach and Brock (1971) considered the corresponding anti-plane problem. Self-similarity technique, which is the most useful technique for treating extending crack problems, are used by Atkinson (1974), Brock and Achenbach (1974), and others.

Using complex variable technique Cherepanov and Afanasev(1974), Cherepanov(1979) have solved some self-similar problems of dynamic theory of elasticity. They also used the functionally invariant method of Smirnoff and Sobolev (1932). Later, this technique is used by Das (1993a) to solve the one way extension of a crack in an infinite elastic solid due to two non-parallel plane SH-waves.

Indeed, non-symmetric extension at different velocities of the crack tips is common to the fracture of geophysical settings with pre-existing rupture planes. Problems on non-symmetric extension of a small flaw into a plane crack have been studied by Brock (1975,1976), Georgiadis (1991) and Das (1993b,1993c) using self- similarity technique.

Recently, problems on extension of cracks in cruciform paths have been solved by Brock and Deng(1985), Ong and Srivastava (1985) and Georgiadis (1987).

Here we add a few lines about self-similarity technique

5. Self-similarity technique:

A self-similar solution of a physical problem can be inferred if either the data of the problem involve no characteristic length or the only characteristic length is related to a parameter to which the solution is proportional. The principal advantage of this class of solution is that the governing partial differential equations can be replaced by another set that contains one independent variable less than those in the original set.

We begin with the homogeneous solution of scalar two dimensional wave equation

$$\nabla^2\phi = c^{-2} \frac{\partial^2\phi}{\partial t^2} \qquad (24)$$

which may be expressed as

$$\phi(x/t, y/t) = \phi\ (r/t, \theta)$$

where r, θ are polar coordinates in the plane. If we define a new independent variable

$$s = r/t$$

then polar form of (24)

$$\frac{\partial^2\phi}{\partial r^2} + r^{-1}\frac{\partial\phi}{\partial r} + r^{-2}\frac{\partial^2\phi}{\partial\theta^2} = c^{-2}\frac{\partial^2\phi}{\partial t^2}$$

is transformed into

$$s^2(1-s^2/c^2)\frac{\partial^2\phi}{\partial s^2} + s(1-2s^2/c^2)\frac{\partial\phi}{\partial s} + \frac{\partial^2\phi}{\partial\theta^2} = 0 \qquad (25)$$

We point out that (25) is of mixed type, i.e., it is elliptic in $s/c<1$ and hyperbolic in $s/c>1$. The domain of elliptcity and hyperbolicity of the differential equation thus correspond to the interior and exterior of the circle $r=ct$ centered at the origin of the coordinate system. Since the coefficient of $\frac{\partial^2\phi}{\partial s^2}$ vanishes at $s=c$, the circle $r=ct$ evidently represents a singular wave front across which we may expect discontinuities in the s-derivatives of the wave function. We suppose, however, that ϕ and $\partial\phi/\partial\theta$ are continuous across the wave front. equation (25) can be reduced to the canonical form for $s/c<1$ through Chaplygin's transformation

$$\beta = -\cosh^{-1}\frac{c}{s} = \log\left[\frac{c}{s} - \left(\frac{c^2}{s^2} - 1\right)^{1/2}\right] \qquad (26)$$

which yields Laplace's equation in β-θ coordinates

$$\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial \beta^2} = 0. \tag{27}$$

Similarly if s/c>1, then the transformation

$$\sigma = \cos^{-1} \frac{c}{s} \tag{28}$$

reduced (25) to the equation

$$\frac{\partial^2 \phi}{\partial \theta^2} - \frac{\partial^2 \phi}{\partial \sigma^2} = 0. \tag{29}$$

Recently, attention are being focused to the cases of crack extension which occurs under an arbitrary angle with it's own plane (which leads that a crack may bifurcate) as shown in Fig.2 and Fig 3..Because, it is expected that once the extension of crack has started, the primary crack often bifurcates into two or more branches , each of which may propagate over a short distance, and then again split into two or more new branches.Crack bifurcation occurs in a variety of materials, and under different external conditions. The phenomenon is, however, particularly present for essentially brittle fracture, when the speed of crack propagation becomes relatively large. Experimental observations of the magnitude of the speed of crack extension at branching suggest that elastodynamic effects play a sufficient role.It has been observed that the method of self- similar solutions provide a powerful tool for the analysis of elastodynamic skew propagation and crack bifurcation.

A necessary condition for bifurcation can be determined by comparing stress prior to branching and after branching has taken place. The comparison requires expressions for the elastodynamic fields near the crack tips of the branches. For symmetric bifurcation in anti-plane strain the near tip fields were analyzed by Achenbach (1975). The propagation of a crack which emanates under an arbitrary angle from a free surface, when the surface is subjected to anti-plane mechanical disturbances was considered by Achenbach and Varatharajulu (1974). Some cases of dynamic crack propagation in elastic medium are reviewed by

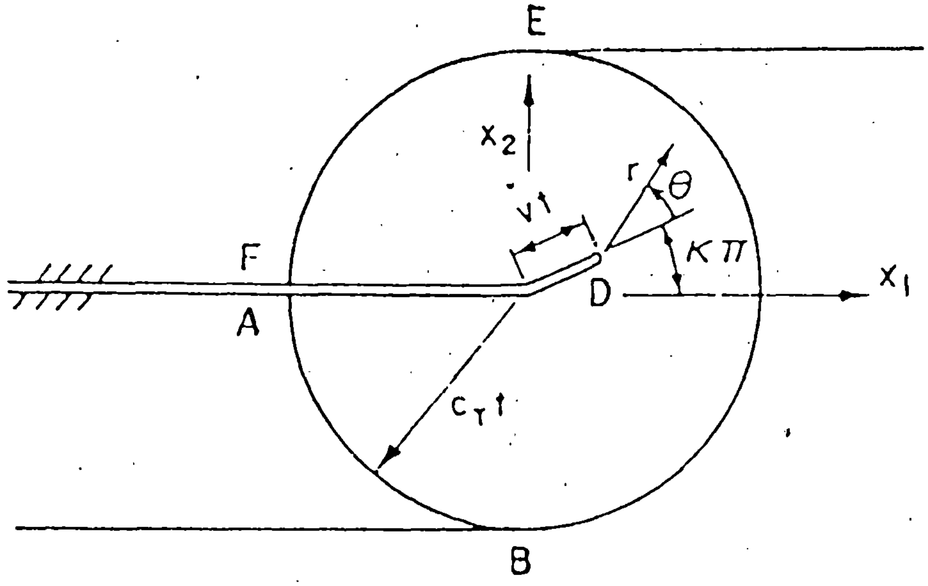

Fig. 2: Pattern of wave front and position of crack tip for skew crack propagation under the influence of a step-stress wave.

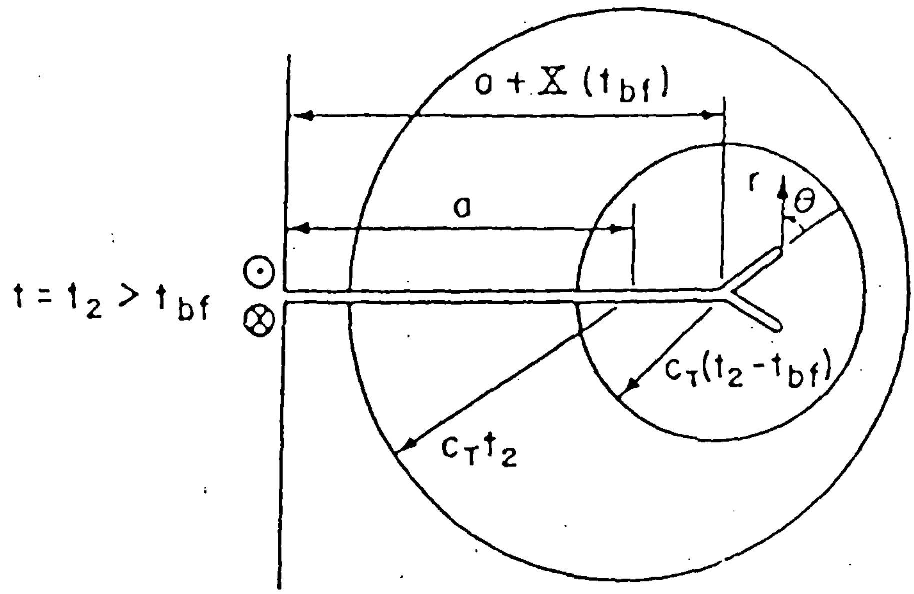

Fig. 3: Rapid propagation and bifurcation in anti-plane strain of an edge crack.

Achenbach (1972, 1976) and Freund(1976).

During the last decade, problems on Mode III crack kinking and bifurcation have been studied by several investigators. Burgers and Dempsey (1982) solved the dynamic crack bifurcation in anti-plane strain for two special cases. Corrected results for Mode III kinking of a crack under an arbitrary angle was given by Dempsey et.al. (1982). A numerical approach for the study of dynamic crack propagation of a kinked or bifurcated crack in anti-plane strain has been given by Burgers (1982). Achenbach et. al. (1984) have developed a method based on superposition principle to derive approximate expression for the elastodynamic stress intensity factors of the kinked crack. The problem of rapid tearing of a half-plane was also solved by Dempsey and Smith (1985). They considered that the surface of the half-plane is subjected to sudden anti-plane mechanical disturbance, crack initiation and subsequent crack instability are examined via two idealized problems; the first is concerned with instantaneous crack bifurcation and the second with instantaneous skew crack propagation. In either problem, crack propagation occurs at a constant subsonic velocity, under an angle $k\pi$ with the normal to the surface . For various values of the angle of crack propagation, the dependence of the elastodynamic stress intensity factors on the crack propagation velocity is investigated.

Recently, transient elastodynamic non-planar self-similar Mode III crack growth in brittle materials is examined by Dempsey et. al. (1986). The dynamic similarity and Chaplygin's transformation reduced the class of problems considered to the solution of Laplace's equation in a semi-infinite strip. The Scwartz- Christoffel transformation is subsequently employed to map the semi-infinite strip on a half-plane. The theory of analytic functions are then used. Elastodynamic influences in the vicinity of a rapidly moving tip after branching are examined in a rather general fashion.

The thesis presented here consists of some problems on cracks and wave propagation. The work has been presented in three chapters.

The first chapter deals with problems on moving cracks in infinite elastic medium and in infinitely long elastic strip.

Problems on crack extension and bifurcation have been presented in the second chapter .

The third chapter deals with the problems on wave propagation in the presence of topographical irregularities.

The summary of the thesis is presented here chapter wise.

The first problem of chapter 1 has been formulated as follows:

We have considered the problem of propagation of two coplanar Griffith cracks moving steadily in infinite long finite width strip. We consider two cracks of finite width placed on X-axis from -b to -a and a to b with reference to the rectangular coordinate system (x,y,z) which referred to a fixed coordinate system (X,Y,Z), is moving with constant velocity v along X-direction within the strip of elastic material occupying the region $-h' \leq Y \leq h'$. Employing Fourier transform and finite Hilbert transform technique, closed form solutions are obtained for two cases of practical interest. Firstly, the case when the rigidly clamped edges are pulled apart in opposite directions are considered. Secondly, we have treated the case when the lateral boundaries are subjected to shearing stresses. Exact expressions for the crack opening displacement and the stress intensity factors have been derived in both the cases.

In paper 2, we have considered the problem of two coplanar Griffith cracks moving along the interface of two dissimilar elastic media. Two cases of practical importance have been considered. Firstly, the case of two coplanar Griffith cracks moving along the interface of two semi-infinite dissimilar elastic media has been treated ; secondly, the problem of propagation of two coplanar Griffith cracks along the interface of an elastic layer overlying a semi-infinite medium of different elastic properties has been considered. Employing Fourier transform we reduced these problems to solving a set of triple integral equations with cosine kernel weight functions. These equations are solved using finite Hilbert transform technique. In the second case, analytical expressions are retained up to h^{-4} , where h is the thickness of the upper layer, for deriving the dynamic stress intensity factors and crack opening displacement.

The problem of two coplanar Griffith cracks running steadily under three dimensional loading has been considered in the third paper of chapter 1. It is assumed that equal and opposite tractions which are triaxial in nature are applied to the crack surfaces. The two dimensional Fourier transforms have been used to reduce the mixed boundary value problem to the solution of triple integral equations. In order to solve the problem , the transformed surface displacement is expanded in a series of Chebyshev polynomials which is automatically zero out side the cracks and also satisfies the edge conditions. Finally, Schmidt method has been used to determine the unknown coefficients occurring in the series. The expression for stress intensity factors at the crack tips and the crack opening displacement have also been derived for different values of the parameters. An interesting feature of this paper is that there is the possibility of curving or branching of the cracks at the outer edge at very low velocities of the cracks whereas the cracks tend to become curved at the inner edge for values of crack tip velocity about $0.6c_2$.

The dynamic in-plane problem of determining the stress and displacement due to three coplanar cracks moving steadily at a subsonic speed in fixed direction in an infinite, isotropic, homogeneous medium under normal stress and the static problem of determining the stress and displacement around three coplanar Griffith cracks in an infinite isotropic elastic medium have been considered in the fourth paper of the chapter 1. In both the cases, employing Fourier integral transform, the problems have been reduced to solving a set of four integral equations. The integral equations have been solved using finite Hilbert transform technique and Cook's result to obtain the exact form of crack opening displacement and stress intensity factors which are presented in the form of graphs.

In the fifth paper of the chapter 1 we have treated the dynamic anti-plane problem of determining stress and displacement due to three coplanar cracks moving steadily at a constant speed in an infinite elastic strip.Employing the same technique as that used in solving the problem considered in paper four, the problem when the lateral boundaries of the strip are subjected to shearing stress has been solved. Numerical results for stress intensity factors have been presented in the form of graphs.

The dynamic in-plane problem of determining the stress and displacement due four coplanar Griffith cracks moving steadily at a subsonic speed in fixed direction in an infinite, isotropic, homogeneous medium under normal stress has been treated in the sixth paper of this chapter.The static problem of determining the stress and displacement around four coplanar Griffith cracks in an infinite isotropic elastic medium have also been considered in this paper. In both the cases,employing Fourier integral transform, the problems have been reduced to solving a set of five integral equations. The integral equations have been solved using finite Hilbert transform technique to obtain the exact form of crack opening displacement and stress intensity factors which are presented in the form of graphs.

In chapter 2, the first problem deals with the non-symmetric extension of a plane crack due to plane SH- waves in a pre-stressed infinite elastic medium. We considered two identical plane waves defined by

$$\sigma_{yz} = A_o W_{\pm} H(W_{\pm}), \quad \sigma_{xz} = A_o \cot\theta_o W_{\pm} H(W_{\pm})$$

referring to coordinate system (x,y,z) where

$$W_{\pm} = c_2 t \pm y\sin\theta_o + x\cos\theta_o, \quad 0 \le \theta_o \le \pi/2$$

and H() is Heaviside's unit function, to propagate through the infinite solid which is pre-stressed such that

$$\sigma^o_{yz} = \sigma, \quad \sigma^o_{xz} = 0.$$

Fracture is assumed to initiate at a point a finite time after the waves intersect there and the crack is assumed to extend non-symmetrically along the trace of wave intersection. Following Cherepanov and Afanase'v (1974) and Cherepanov (1979) the general solution has been derived in terms of analytic function of complex variable. Numerical results have been presented to illustrate the nature of the variation of stress intensity factors and the rate of energy flux into the crack edges with the speed of the crack tips and also with the time after fracture initiation.

In the second paper of the chapter 2, we investigated the problem of non-symmetric extension of an infinitesimal flaw into a plane crack at a constant rate due to the action of two non-parallel plane SH- waves of different amplitude propagating towards each other in an infinite isotropic elastic medium which is initially in a state of uniform anti-plane shear.A finite time after the crossing of the plane wave fronts, a fracture is assumed to initiate along the line where the wave fronts crossed and the crack is then assumed to travel non-symmetrically along the trace of wave intersection. Superposition considerations allow the original problem to be separated into three self-similar problems with (0,0),(0,1)and (1,0) as the indices of self-similarity. The dynamic similarity of certain field variable in each problem suggests application of the method of homogeneous functions. Expressions for the stress intensity factors and the rate of energy flux into the extending crack edges of the crack have been derived. Finally, the nature of the variation of the stress intensity factors at the crack tips and also the rate of energy flux into the edges with velocities of the crack edges and also with the time after crack initiation have been depicted by means of graphs.

The third paper of this chapter deals with the dynamic anti-plane problem of bifurcation of a semi-infinite crack due to the incidence of two linearly varying plane SH- waves with non-parallel wave fronts in an infinite elastic medium. The semi-infinite crack is assumed to bifurcate when the plane waves intersect the crack tip. The problem has been solved using self-similarity technique which is based on the observation that certain field variables show dynamic similarities. The results include the expressions for shear stress in the planes of the cracks and the stress intensity factors at the crack tips.Finally, the variations of stress intensity factors with the angle of skew for different values of the parameters have been depicted by means of graphs.

In the first paper of chapter 3, we have studied the transmission of time step SH- wave across a step like irregularity in the surface of an elastic half- space. Considering the incident wave in the form H(T-X/c) where H() is the Heaviside's step function the problem is reduced to an integral equation by using integral transform and Green's function technique and finally using Cagniard-Dehoop method of finding inverse Laplace transform, transmitted field at any distances from the step on the free surface have been determined using iterative procedure. Numerical results have been presented in the form of graphs to illustrate the nature of transmission.

Finally, we have considered the propagation of SH- wave in a medium consisting of two welded quarter spaces of different material and having a step like change in elevation at the vertical interface. The problem is reduced to an integral equation by using the Fourier transform and Green's function technique and finally, by applying the method of steepest descent, the transmitted and reflected fields at large distances form the step have been determined. To investigate the nature of the motion, we have evaluated numerically the increment in amplitude due to the presence of the step for both the transmitted and and reflected wave s which are presented in the form of graphs.

With this brief discussion we now present the thesis chapter wise. An attempt has been made to include most of the references consistent with the problems treated in this thesis, which have come to the author's knowledge.