

**HEAT AND MASS TRANSFER
TO FLOW OF FLUID THROUGH POROUS MEDIUM
AND
DISPERSION OF SOLUTE**

**Thesis
Submitted for the Degree of
Doctor of Philosophy in Science**

**By
Dhritiman Sarkar**

**Under the guidance of
Dr. Swapna Mukherjee**



**University of North Bengal
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Dedicated to my Beloved Parents

CERTIFICATE

This is to certify that the thesis entitled “ Heat and Mass transfer to flow of fluid through porous medium and Dispersion of solute” being submitted by Dhritiman Sarkar for the award of the Degree of Doctor of Philosophy in Science to the University of North Bengal, Raja-Rammohunpur, Darjeeling, is a record of bonafide research work carried out by him under my supervision and guidance. Thesis has reached the standard fulfilling the requirements of the regulations to the Degree. The results embodied in this thesis have not been submitted to any other Institute or University for the award of any Degree or Diploma.

Date: 20.12.07.

Swapna Mukherjee
(Dr. Swapna Mukherjee)
Principal,
Dr. Meghnad Saha College,
Itahar, Uttar Dinajpur
West Bengal

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Date: 20.12.07

Dhritiman Sarkar

(Dhritiman Sarkar)

Assistant Teacher

S. D. P. U. Vidyachakra

P.O. Raiganj

Dist.- Uttar Dinajpur

West Bengal, India

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Chapter-I

INTRODUCTION

1.1 INTRODUCTORY REMARKS

The content of this thesis are arranged in three main chapters and each chapter is subdivided into few parts. Chapter I is of review nature and deals with the introduction to the thesis and a brief discussion on the basic concepts of flow through porous medium, rotating fluid flow, unsteady free convective flow and mass transfer through porous medium, unsteady MHD free convective flow of viscous fluid with mass transfer in porous medium and unsteady convective dispersion process. Attempts are also made to give a brief survey of previous results so that the work presented in this thesis could be seen in its proper perspective.

Convective heat transfer in a porous medium is a topic of rapidly growing interest due to its application to geophysics, thermal insulation engineering, exploration of petroleum and gas field, water movement in geothermal reservoirs, underground spreading of chemical wastes, oil reservoirs engineering and packed bed storage tank etc. This transport processes occurring in nature due to temperature differences. This difference causes the density difference. This density difference is also caused by chemical composition differences and gradients or by phase constitutions. The flow caused by the density differences is known as mass transfer flow. The analysis of hydro-magnetic free convective flow in presence of mass transfer in porous medium is very important from the technological point of view. For this reason unsteady MHD free convective fluid flow with mass transfer in porous medium with or without rotation in different physical situation is considered in chapter II. Chapter II is divided into two parts and each part consists of two different problems. The first part of chapter II is associated with unsteady free convective MHD fluid flow with mass transfer through porous medium while in second part, unsteady free convective MHD fluid flows with mass transfer through porous medium in rotating system are considered. In first part of chapter II, the effect of variable suction or injection on the unsteady two dimensional free

convective flow with mass transfer of an electrically conducting fluid past a vertical accelerated plate embedded in porous medium in the presence of transverse magnetic field is considered. Solutions of the equations governing the flow are obtained with the help of power series. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases. In many applications, quite often the plate temperature starts oscillating about a non-zero mean temperature. The free convection is enhanced by superimposing oscillating temperature on the mean plate temperature. In many engineering applications, transient free convective flow occurs as such a flow acts as a cooling device. Keeping this in mind, in first part of chapter II, another problem is considered where the effect of magnetic parameter and heat source on heat transfer to unsteady MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer in presence of free stream velocity is considered. The plate temperature is assumed to vary harmonically with time. Solutions to the governing equations are obtained with the help of method of perturbation. It is observed that velocity decreases as magnetic parameter increases but opposite character revealed in case of permeability parameter and heat source parameter.

Studies associated with flows through porous medium in rotating environment have some relevance in geophysical and geothermal problems. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Keeping this in mind unsteady free convective and mass transfer flow of viscous fluid through a porous medium occupying a semi-infinite region bounded by a vertical porous plate subjected to a constant suction in presence of constant heat flux at the plate wall in a rotating frame of references is studied in the second part of chapter II. The second part of chapter II, deals with the study of unsteady free convective flow and mass transfer of MHD fluid through porous medium in presence of heat

source with variable suction in a rotating system. The effects of rotation on velocity, temperature and concentration field are discussed. Another problem, unsteady free convective flow and mass transfer during the motion of viscous incompressible rarefied gas through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform transverse magnetic field in a rotating system is studied in the second part of chapter II. The effects of rotation, variable suction, heat source, rarefaction parameter and magnetic parameter on velocity, temperature and concentration distribution are discussed analytically and graphically.

Chapter III is devoted to the study of dispersion of solute in MHD fluid in different geometrical conditions where generalized dispersion model proposed by Gill & Sankarasubramanian [1] is employed. The longitudinal dispersion of solute in a solvent flowing in a conduit is a phenomenon of wide application in chemical engineering, biomedical engineering, physiological fluid dynamics and environmental sciences. This motivates us to study dispersion of solute in three different realistic situations. The first problem of chapter III deals with exact analysis of the dispersion of solute in an oscillating hydro-magnetic Couette flow. Using a generalized dispersion model which is valid for all time after the injection of solute, the diffusion co-efficients $K_i(\tau)$ ($i=1,2,3,\dots$) are determined as function of τ when the initial distribution of solute is in the form of a slug of finite extent. The second diffusion co-efficient *i.e.* $K_2(\tau)$ gives a measure of longitudinal dispersion of solute due to the combined effect of molecular diffusion and uniform or non-uniform velocity distribution. The interesting part of the analysis is that $K_2(\tau)$ consists of a steady part and a fluctuating unsteady part due to the oscillation of flow even though velocity field is independent of time. In part two of chapter III, dispersion of solute in oscillating hydro-magnetic Couette flow in a rotating system is studied. The effect of rotation and transversely applied magnetic field on the dispersion process is discussed. In part three of chapter III,

unsteady dispersion of solute in two layered MHD fluid flow through parallel plates is studied. This model mainly brings out the effect of plug flow region on the overall dispersion process. It is found that initially the dispersion co-efficients decreases considerably with the increase of the value of plug flow region but becomes essentially a constant as time takes larger values. It is also seen that time required to reach the steady state depend on plug flow region. This study can be used as a starting first approximate solution for studying the dispersion in cardiovascular system.

Before we discuss various problems we present below general introduction on rotating fluid flow, flow through porous medium, free convection and mass transfer flow through porous medium, unsteady magneto-hydrodynamic free convective flow with mass transfer in porous medium, free convective MHD flow with mass transfer in rotating system, unsteady convective diffusion process in different geometries which are directly related to the concerned problem of this thesis.

1.2 ROTATING FLUID FLOWS

The study of the motion of a viscous rotating fluid has stimulated considerable interest in recent years due to its important applications. Similarly a great deal of meteorology depends upon the dynamics of a revolving fluid. The large scale and moderate motions of atmosphere are greatly affected by the vorticity of the earth's rotation. The motion in the earth's core is somehow responsible for the main geomagnetic field. It is common practice in fluid dynamics to start with simple model to investigate various effects. The problem, although idealized, retains the essential features of the investigation. It has been observed that, when the fluid is rotating near a flat plate, the pressure field of the flow far away from the plate also exists near the plate, but there is reduction in the Coriolis force near the plate owing to frictional forces. Because of this, there exists

a flow in the direction in which the pressure is falling until the Coriolis forces are compensated by viscous forces. A layer in which such a flow exists is known as Ekman Layer (Prandtl [3]), was first noticed by Ekman [2] and plays a very important role in the rotating fluid flows. Thus, near the plate, the viscous and Coriolis forces are of same order of magnitude in the Ekman Layer. Rotation in fluid system produces two effects viz. the Coriolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. In considering flows in rotating environment, we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the latter case it is preferable to use an inertial coordinate system fixed in space. On the other hand the flow behavior in the former case can be described in a coordinate system which rotates with the fluid and in this frame of references the fluid is at rest. The complete literature pertaining to rotating fluids is enormous and an excellent review can be found in the monograph by Greenspan [4]. The steady flow near the plate, in the Ekman layer, has been discussed by Batchelor [5]. Vidyanidhi and Nigum [6] discussed secondary flow in a rotating channel. The effects of a uniform transverse magnetic field on Ekman layer is investigated by Pop [7]. Gupta [8] obtained an exact solution of the three dimensional Navier Stokes steady state equations for the flow past a plate with uniform suction or injection (blowing) in a rotating system. Soundalgekar and Pop [9] studied on hydro-magnetic flow in a rotating fluid past an infinite porous plate. Debnath and Mukherjee [10] studied the motion of an incompressible, homogeneous, viscous fluid bounded by porous plate with uniform suction or injection. Puri [11] discussed the fluctuating flow of a viscous fluid on a porous plate in a rotating medium. Pop and Soundalgekar [12] studied the effects of constant and variable suction on the unsteady rotating flow of the fluid past an oscillating plate when both the fluid and the plate are in solid body rotation. Mazumdar *et al.* [13] investigated the effects of both Hall current and rotation on hydro-magnetic flow over a porous plate. Similar problem was studied by Jana and Datta [14], who consider the effects of Hall current and rotation on

MHD Couette flow. Mazumder [15] studied an exact solution of oscillatory Couette flow in a rotating system. Ganapathy [16] investigated an oscillatory Couette flow in a rotating system. Bhattacharjya *et al.* [17] studied the unsteady rotating flow of a compressible fluid over a finite disk. Singh [18] considered an oscillatory hydro-magnetic Couette flow in a rotating system. Kim [19] studied the unsteady two dimensional laminar flow of a viscous incompressible electrically polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of transverse magnetic field. Recently Jat and Jhankel [20] analyzed three dimensional unsteady flow of an incompressible viscous fluid in presence of transverse magnetic field through porous medium past an oscillating plate in a rotating system. Singh *et al.* [21] discussed a periodic solution of oscillatory Couette flow through porous medium in rotating system. In this thesis we consider certain problems in rotating system due to their varied applications in the field of technology.

1.3 FLOW THROUGH POROUS MEDIUM

Many materials (ex. soil, sand, packed beds) consist of a large number of particles or fibres packed closely together. In between the solid particles or fibres there is an open space, giving rise to pores through which fluid can flow. An object does not have to consist of many particles to be porous, for instance, it could simply be composed of a single continuous solid body that has many pores in it. Such is the case of certain rocks and filters. Regardless of how the porous medium is constructed, because of the irregular and tortuous nature of pores it is exceedingly difficult to model fluid flow through such materials exactly.

In recent years the flows of fluid through porous medium have attracted the attention of a number of scholars because of their possible application in many branches of science and technology. In fact a porous material containing the fluid is a non-homogeneous medium but it may be possible to treat it as a homogeneous

one, for the sake of analysis, by taking its dynamical properties to be equal to the local average of original non-homogeneous continuum. Thus complicated problems of the flow through a porous medium get reduced to the flow problem of homogeneous fluid with some additional resistance. Flows of fluid through porous medium are of principal interest because these are quite prevalent in nature. Such flows are important in the field of agricultural engineering to study the underground water sources, seepage of water in river beds, in petroleum technology to study the movement to natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification processes. In view of the geophysical application of the flows through porous medium, a series of investigations have been made by Raptis *et al.* [22, 23] into the steady flow past a vertical wall.

In fluid dynamics, Darcy's law is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth science. Although Darcy's law which is an expression of conservation of momentum was determined by Darcy; it has since been derived from the Navier-Stoke's equation via homogenization. It is analogous to Fourier's law in the field of heat conduction, Ohm's law in the field of electrical network and Fick's law in diffusion theory. One application of Darcy's law is the water flow through an aquifer. Darcy's law along with equation of conservation of mass is equivalent to the ground water flow equation, one of the basic relationships of hydrogeology. Darcy's law is also used to describe oil, water, gas flow through porous medium.

Studies associated with flows through porous medium have been based on the Darcy's empirical equation

$$\vec{q} = -\frac{\text{const.} \vec{\nabla} p}{\mu} \quad \dots (1.1)$$

Where \vec{q} is the mean filter velocity, μ is the viscosity of the fluid and $\vec{\nabla} p$ is the pressure gradient. Later Muskat [24] has shown that the constant in equation (1.1) must depend on the permeability of the porous medium and showed that

$$\vec{q} = -\frac{K \vec{\nabla} p}{\mu} \quad \dots (1.2)$$

Where K is the permeability of the porous medium. Following Yamamoto and Iwamura [25], the porous medium is considered as an assemblage of small identical spherical particles fixed in space and the equation (1.2) for incompressible fluid and unsteady flow, takes the form

$$\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \vec{\nabla} \right) \vec{q} = -\frac{1}{\rho} \vec{\nabla} p - \frac{\nu}{K} \vec{q} + \nu \nabla^2 \vec{q} - g \quad \dots (1.3)$$

Where ν is the kinematic viscosity, t is the time and g is the acceleration of gravity.

1.4 FREE CONVECTION AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM

Fluid flow due to density differences in the external force field is generally called free convection. Such external forces are gravity forces, and the density difference, a very simple case, is the result of the temperature drop between the solid surface and the fluid. Free convection flow is not of rare occurrence in nature. In fact trade winds are due to convection currents set up in the atmosphere due to unequal heating. Also land and sea breezes arise in a similar manner. Studies on free convection have growing importance on the problem of unsteady free convection flow past an infinite vertical plate as one of the fundamental problem in heat transfer owing to its practical applications. Pop and Soundalgekar [26] have

studied unsteady free convection flow past an infinite plate with constant suction and heat sources. Free convection effects on the Stoke's problem for an infinite vertical plate were investigated by Soundalgekar [27]. This problem is better known as Stokes problem for the vertical plate. Singh *et al.* [28] discussed three dimensional free convective flow and heat transfer along a porous vertical wall. Pop and Soundalgekar [29] investigated the free convection flow past an accelerated vertical infinite plate. The problem of free convective viscous flow past a vertical porous plate with periodic temperature has been solved by Acharya and Padhya [30]. Raptis [31] studied unsteady free convective flow through a porous medium. The free convection effect on the flow of an ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma [32]. Mahershi and Tak [33] studied fluctuating free convection through porous medium due to infinite vertical plate with constant heat flux.

Research on fluid flow through porous media finds great application in geothermy, geophysics and technology. Flows of fluid through porous medium are of principal interest because these are quite prevalent in nature. Such flows are important in the field of agricultural engineering to study the underground water sources, seepage of water in river beds, in petroleum technology to study the movement to natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification processes. Yamamoto and Iwamura [25] considered the flow with convective acceleration through a porous medium as assuming the porous medium as an assemblage of small identical spherical particles fixed in space. Raptis *et al.* [34, 35] studied the influence of the free convective flow on the steady flow of the viscous fluid through the porous medium when there is a constant heat flux. Raptis [36] analyzed the influence of free convection on the unsteady flow of a viscous fluid through a porous medium considering the fluctuation of the surface temperature in time about a constant non-zero mean value. The study of two dimensional flows through porous medium

bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma [37]. Three dimensional free convective flow and heat transfer through a porous medium was discussed by Ahmed and Sharma [38]. Sattar *et al.* [39] studied free convection flow and heat transfer through a porous vertical flat plate immersed in a porous medium with variable suction. Singh [40] investigated three dimensional free convective flow of a viscous fluid through porous medium with time dependent suction velocity. Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability was studied by Singh *et al.* [41]. Free convection flow of viscous fluid in porous medium in presence of heat source was investigated by Singh *et al.* [42].

However, in nature, along with the free convection currents caused by the temperature differences, the flow is also affected by chemical composition differences and gradients or by material or phase constitutions. This can be seen in our everyday life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by the temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as the mass transfer flow. When a mixture of gasses or liquid is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from a region of high concentration to regions of low concentration. There is also a mass transfer associated with convection in which mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level. Due to applications in various technological problems and in agricultural science, effects of mass transfer on the unsteady free convective flow past an infinite porous plate with constant or variable suction were studied by Soundalgekar [43], Soundalgekar and Wavre

[44,45], Soundalgekar [46] and Raptis *et al.*[47]. Raptis *et al.* [48] examined free convection and mass transfer flow through a porous medium bounded by an infinite vertical limiting surface with constant suction. Raptis *et al.* [49] studied the steady free convective flow and mass transfer of a viscous fluid through a porous medium bounded by a vertical infinite porous surface with constant suction by using generalized Darcy's law. In a subsequent paper, under the same geometrical and physical considerations, Raptis [50] studied the influences of both free convective flow and mass transfer through a porous medium. Raptis [51] studied the free convection and mass transfer flow through porous medium bounded by a plate with free stream velocity. Raptis and Perdikis [52] also analyzed the steady free convective and mass transfer flow when a viscous incompressible fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite porous plate. Singh [53] studied three dimensional unsteady free convection and mass transfer flow through a porous medium. Chitti Babu *et al.* [54] investigated three dimensional free convective flows of heat and mass transfer through a porous medium with periodic permeability.

1.5 UNSTEADY MAGNETO-HYDRODYNAMIC FREE CONVECTIVE FLOW WITH MASS TRANSFER IN POROUS MEDIUM

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many processes, industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc, in the presence of an electrically conducting fluid subjected to a magnetic field. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal

vertical infinite plate was studied by Soundalagekar *et al.*[55]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalagekar *et al.* [56].The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate was studied by Soundalagekar [57]. Raptis and Kafousias [58] investigated heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Further, the effect of constant heat flux on the flow of an electrically conducting fluid plate oscillating in its plane was studied by Soundalagekar *et al.* [59]. Acharya *et al.*[60] have analyzed free convection and mass transfer in steady flow through porous medium with constant suction in the presence of magnetic field. Singh and Chand [61] discussed unsteady free convective MHD flow past a vertical porous plate with variable temperature. Recently, Sriramulu *et al.* [62] studied the effect of applied magnetic field on transient free convection flow of an incompressible viscous fluid by taking into account of viscous dissipative heat along with the heat due to free convection currents in a vertical channel. Samman *et al.* [63] studied transient free convection flow of a viscous dissipative fluid with mass transfer past a semi-infinite vertical plate. Singh *et al.* [64] studied hydro-magnetic heat and mass transfer in MHD flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate embedded with porous medium of time-dependent permeability under oscillatory suction velocity normal to the plate. More recently Singh [65] investigated MHD free convection and mass transfer flow with Hall effect, viscous dissipation, Joule heating and thermal diffusion. Mukherjee *et al.* [66] studied MHD free convective flow and mass transfer through an inclined open rectangular channel. Unsteady Magneto-hydrodynamic free convection flow past an infinite vertical plate with time dependent suction and heat sink was studied by Kumar *et al.* [67]. Sarangi and Jose [68] analyzed unsteady MHD free convective flow and mass transfer through porous medium with constant suction and constant heat flux. Mishra [69] investigated heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent

suction. Jain *et al.* [70] discussed MHD free convection flow of water at 4°C in presence of a heat under slip boundary conditions. Effects of permeability on three dimensional oscillatory free convective MHD flow and heat transfer along an infinite vertical porous plate was studied by Shrivastava *et al.* [71]. Recently effect of mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink was studied by Sharma and Kaanodia [72]. We now proposed to study the effect of variable suction or injection on the unsteady two dimensional free convective flows with mass transfer of an electrically conducting fluid past a vertical accelerated plate embedded in the presence of transverse magnetic field in the first part of chapter II. Solutions of the equations governing the flow are obtained with the help of power series. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases. In the first part of chapter II, another problem on the heat transfer to unsteady flow of MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer in presence of free stream velocity is considered. The plate temperature is assumed to vary harmonically with time considering ε to be very small, we have practically considered the plate temperature to vary only slightly from mean value. Solutions of the equations governing the velocity field, temperature field and concentration field are obtained analytically and effects of magnetic parameter, heat source and permeability parameter on the velocity field are also studied.

1.6 FREE CONVECTIVE MHD FLOW WITH MASS TRANSFER IN ROTATING SYSTEM

The study of fluid flow in a rotating system which was initiated by Greenspan [4] recently has received considerable interest due to its applications in practical situations. In particular the hydro-magnetic flow in a rotating system has numerous engineering applications e.g., in generation of MHD power in a small scale for

space applications, design of heat exchangers, flow meters, etc. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical, geothermal problems. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Gupta [8] investigated the effect of hydro-magnetic flow past a rotating porous flat plate. Mohan [73] examined the free and forced convection effects in a rotating, steady hydro-magnetic viscous fluid between two parallel plates maintaining the boundaries at constant temperature gradient and taking the plates to be finite conductivity. Prasada Rao and Krishna [74] investigated the influence of Hall currents on the free and forced convective flow of a viscous conducting fluid in a rotating channel maintained at constant temperature gradient along the channel walls under the influence of transverse magnetic field. Agarwal *et al.* [75] investigated the effects of Hall current on a steady hydro-magnetic free convection flow past an infinite porous plate in a rotating viscous fluid system. Mahato and Maiti [76, 77] analyzed the effect of unsteady free convective flow and mass transfer during the motions of a viscous incompressible fluid in a rotating frame of references. The effect of magnetic field on free convective flow of electrically conducting fluids past a semi-infinite flat plate is analyzed by Sacheti *et al.* [78]. Satter and Alam [79] studied MHD free convective flow with Hall current in a porous medium for electrolytic solution. Later Alam *et al.* [80] studied unsteady free convection and mass transfer flow in a rotating system with Hall currents, viscous dissipation and joule heating. Tak and Gehlot [81, 82] studied the effects of suction on skin-friction and heat transfer in the free convection boundary layer flow along a porous vertical semi-infinite plate in presence of transverse magnetic field with or without frictional heat. Singh *et al.* [83] studied free convection in MHD flow of a rotating viscous liquid in porous medium. Recently Singh *et al.* [84] have studied free convective MHD flow of rotating viscous fluid in a porous medium past infinite vertical porous plate. Soundalgekar *et al.* [85] obtained an exact solution of the transient free convection flow past an infinite vertical plate in presence of periodic heat flux. Recently

Sahoo *et al.* [86] studied the unsteady hydro-magnetic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in presence of constant suction and heat absorbing sinks. In the second part of chapter II, we now proposed to study unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of uniform magnetic field applied perpendicular to the flow region in rotating system. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, temperature, concentration, skin-friction, rate of heat transfer and rate of mass transfer is presented graphically and necessary conclusions are set out.

In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle of the surface has a finite tangential velocity, it slips along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Using these assumptions Gupta and Babu [87] investigated the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in the slip flow regime. Debangana [88] investigated MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime. Jain and Taneja [89] examined unsteady MHD flow with radiation through porous medium in slip flow regime. Sharma and Choudhury [90] studied effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. Singh *et al.* [91] studied magnetic field effects on free convection and mass transfer flow through porous medium with constant suction and constant heat and mass flux in slip flow regime. Sharma and Sharma [92] investigated influence of variable suction on unsteady free convective flow from a vertical flat plate and heat transfer in slip-flow regime. Varshney *et al.* [93] investigated effect of heat source on free convection and mass transfer flow through porous medium with constant heat and

mass flux in slip flow regime. Sharma [94] has studied the effect of periodic heat and mass transfer on unsteady free conduction flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time about a non-zero constant mean. Singh and Gupta [95] analyzed MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Jain *et al.* [96] discussed three dimensional free convection heat transfer flow with periodic permeability and periodic temperature in a slip flow regime. Recently Emmanuel Osalusi [97] investigated effects of thermal radiation on MHD and slip flow over a porous rotating disk with variable properties. We now proposed to study another problem on free convective MHD flow with mass transfer past a porous plate in porous medium with variable suction in a slip flow regime in the second part of chapter II. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, skin friction, rate of heat transfer and rate of mass transfer is presented analytically and graphically.

1.7 UNSTEADY CONVECTIVE DIFFUSION PROCESS

The longitudinal dispersion of a solute in a solvent flowing in a conduit (pipe/channel) is a phenomenon of wide application in chemical engineering, biomedical engineering, physiological fluid dynamics and environmental sciences. The basic principle under the dispersion theory is the spreading of a passive species in a flowing fluid due to the combined action of molecular diffusion and non-uniform velocity distribution. The first fundamental study of dispersion was that of Taylor [98] who showed that, if a solute is injected in a solvent flowing steadily in a straight tube the combined action of the lateral molecular diffusion and the variation of the velocity over the cross section would cause the solute ultimately to spread diffusively with the effective molecular diffusivity D_{eff} given by $D_{eff} = D_m + \frac{a^2 \omega_m^2}{48 D_m}$, where D_m is the molecular diffusivity, ω_m is the mean axial velocity and a is the radius of tube. The analysis showed that the spreading of

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solute is symmetrical about a point moving with average velocity ω_m of the fluid. Aris [99] using the method of moments, showed that the effective molecular diffusivity would be $D_{eff} = D_m + \alpha^2 \omega_m^2 / 48 D_m$, when the molecular diffusivity is also taken into account. The analysis showed that the Taylor's dispersion theory is valid for $D_{eff} \gg D_m$.

The time development of dispersion has most commonly been studied by calculating the evolution of axial moments of the solute concentration following its injection into the flow. Anathkrishanan *et al.* [100] obtained the exact numerical solution for the complete convective diffusion equation which takes into account both the radial and axial molecular diffusion. Their results showed that the Taylor-Aris dispersion theory gives a good description of the dispersion process if and only if the time after injection of the solid exceed about $0.5\alpha^2/D_m$. The effect of inlet boundary conditions on the transient approach to the asymptotic Taylor-Aris dispersion theory was studied in a subsequent paper by Gill and Anathkrishanan [101] and was validated by the experimental work of Reejhsingani *et al.* [102]. Gill [103] generalized Taylor-Aris work by proposing a series expansion about mean concentration to describe the local concentration distribution. Gill and Anathkrishanan [104] extended this theory to include the effect of finite slug inputs on the dispersion process. In a subsequent analysis Gill and Sankarasubramanian [1] showed that the method of series solution mentioned earlier provided an exact solution for the unsteady convective diffusion problem for laminar flow in a circular tube if the co-efficients in the dispersion model are obtained as suitable function of time. This model is widely referred to as the generalized dispersion model. By truncating the generalized dispersion model to two terms, Gill and Sankarasubramanian [1] showed that for all time, the mean concentration profile of the solute was symmetric about a point moving with the average velocity of the fluid. Their results also validated the findings of Anathkrishanan *et al.* that the Taylor-Aris dispersion theory is applicable for the exceeding $0.5\alpha^2/D_m$. Gill and Sankarasubramanian [105,106] extended the scope of

their model to study dispersion of solute in a time-dependent laminar flow which in principle, valid for all values of time, they confined their analysis only to the case of dispersion in a fully developed flow. They [107] extended the theory of miscible dispersion to inter phase transport system. They studied the dispersion of solute in a laminar flow through a tube with first order irreversible heterogeneous chemical reaction. Later Krishnamurthy and Subramanian [108] formulated convective diffusion theory for predictive modeling of field-flow fractionation columns used for the separation of colloidal mixture. Jayaraj and Subramanian [109] used the truncated version of generalized dispersion theory to study the relaxation phenomena in field-flow-fractionation. Annapurna and Gupta [110] and Gupta [111] analyzed the unsteady magneto hydrodynamic convective diffusion in electrically conducting fluid flowing in a parallel plate channel. Subsequently Annapurna and Gupta [112] studied the dispersion of matter in flow of a Bingham plastic in a tube using the generalized dispersion model. Later Mukherjee and Maiti [113] studied the dispersion of solute in blood stream flowing through a tube treating blood as a casson fluid model. Mandal *et al.* [114] investigated the dispersion of solute in an incompressible electrically conducting viscous fluid in a porous-walled parallel plate channel permitted by transverse magnetic field. Layek *et al.* [115] presents an exact analysis of the dispersion of a passive contaminant in a viscous fluid flowing in a parallel plate channel driven by uniform pressure gradient. The channel rotates about an axis perpendicular to its wall with uniform angular velocity resulting in a secondary flow. Later Hazra *et al.* [116] studied the dispersion of a solute in oscillating flow through a channel. Siddheshwar *et al.* [117] studied the effect of interphase mass transfer on it. Jayaram *et al.* [118] studied dispersion of solute in a fluid flowing through a curve tube with absorbing wall. Siddheshwar and Manjunath [119] have recently studied dispersion of solute in a plane poiseuille flow of a micro polar fluid. Recently Siddheshwar and Markande [120] considered unsteady convective diffusion of solute in a micro polar fluid flow through a cylindrical tube. Later Dash *et al.* [121] studied a shear augmented dispersion of solute in a casson fluid flowing in a conduit. Hossain *et*

al. [122] studied exact analysis of dispersion of solutes in free and forced convective flow through a channel. Hossain *et al.* [123] examined the effect of radiation on unsteady convective diffusion of solute in an MHD flow through a vertical channel. Recently Nagarani *et al.* [124] discussed exact analysis of unsteady convective diffusion in casson fluid in an annulus. The first problem of chapter III deals with exact analysis of the dispersion of solute in an oscillating hydro-magnetic Couette flow. Using a generalized dispersion model which is valid for all time after the injection of solute, the diffusion co-efficients $K_i(\tau)$ ($i=1,2,3,\dots$) are determined as function of τ when the initial distribution of solute is in the form of a slug of finite extent. The second diffusion co-efficient *i.e.* $K_2(\tau)$ gives a measure of longitudinal dispersion of solute due to the combined effect of molecular diffusion and uniform or non-uniform velocity distribution. The interesting part of the analysis is that $K_2(\tau)$ consists of a steady part and a fluctuating unsteady part due to the oscillation of flow even though velocity field is independent of time. In second problem of chapter III, dispersion of solute in oscillating hydro-magnetic Couette flow in a rotating system is studied. The effect of rotation and transversely applied magnetic field on the dispersion process is discussed. In the third problem of chapter III, unsteady dispersion of solute in two layered MHD fluid flow through parallel plates is studied. This model mainly brings out the effect of plug flow region on the overall dispersion process. It is found that initially the dispersion co-efficients decreases considerably with the increase of the value of plug flow region but becomes essentially a constant as time takes larger values. It is also seen that time required to reach the steady state depend on plug flow region. This study can be used as a starting first approximate solution for studying the dispersion in cardiovascular system.

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Chapter-II

FREE CONVECTIVE FLOW AND MASS TRANSFER

Part one

UNSTEADY FREE CONVECTIVE MHD
FLUID FLOW WITH MASS TRANSFER
IN POROUS MEDIUM

*PART ONE > A***MASS TRANSFER AND FREE CONVECTIVE MHD
FLOW THROUGH POROUS MEDIUM*****2.1.1 INTRODUCTION**

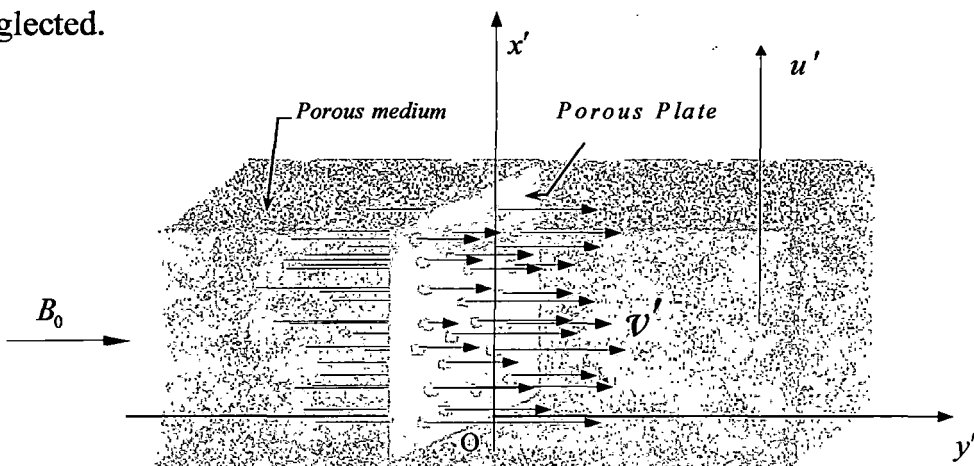
The phenomenon of natural convective flow is not often caused entirely by the effect of temperature gradient but also by differences in concentration of dissimilar chemical species for example, in atmospheric flows there exists differences in H_2O concentration and the flow is affected by such concentration differences. Also, in a number of engineering applications, the foreign mass are injected and due to such mass transfer it has been observed that there is reduction in the wall shear stress, the mass transfer conductance or the rate of heat transfer. In such cases time dependent injection or suction velocity plays an important role. The significance of suction or injection for the boundary layer control in the field of aerodynamics and space science is well recognized. On the other hand, flows through porous medium are very much prevalent in nature and therefore, the study of flows through porous medium has become of principal interest in many scientific and engineering applications [1,2,3]. In recent years the subject of magneto-fluid dynamics has attracted many authors in view not only of its own interest but also of the applications to geophysics and engineering. When the fluid is a conductor of electricity, the free-convection and mass transfer can be influenced by an imposed magnetic field. MFD phenomena result from the mutual effect of a magnetic field and a conductivity fluid across it. Thus an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid's motion. Examination of flow models will reveal the influence of magnetic field on the velocity profile, temperature profile. Raptis and Kafousis [4] studied heat transfer in flow through a porous medium bounded by an infinite

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vertical plate under action of a magnetic field. Raptis and Perdakis [5] discussed magnetic effects on the flow with a great magnetic Reynolds' number by the presence of free convection and mass transfer. Later Chauhan and Jain [6] presented three dimensional MHD flow and heat transfer in the presence of a naturally permeable boundary of very small permeability. Recently Rajput *et al.* [7] studied free convection MHD flow of a stratified fluid past an oscillating porous plate with mass transfer. Hence it is of interest to make an investigation in order to analyze the effect of suction/ injection on free convective flow with mass transfer of an electrically conducting viscous fluid past an accelerated vertical infinite porous plate in a porous medium. The suction or injection velocity is taken to be time-dependent of the form $\alpha(vt')^{1/2}$. The behavior of velocity distribution, temperature distribution and concentration distribution is discussed for different parameters. It is observed that velocity decreases as magnetic parameter increases.

2.1.2 MATHEMATICAL ANALYSIS

We consider a two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical accelerated porous plate embedded in porous medium. A magnetic field of uniform strength is applied transversely to the direction of the flow. The magnetic Reynolds' number of the flow is taken to be small enough so that the induced magnetic field can be neglected.



2.1.1 Sketch of the physical problems

The fluid is assumed to have constant properties except that the influence of the density variations with temperature and concentration is considered only in the body force term. At time $t' \leq 0$, the plate and the fluid are at the same temperature in a stationary condition but at time $t' > 0$, the plate starts moving with velocity $U'(t')$ in its own plane and the plate temperature and concentration level is also raised to $T'_w (\neq T'_\infty)$ and $C'_w (\neq C'_\infty)$. In order to formulate the problem mathematically, we write down the equation of fluid motion, through a porous medium in Cartesian coordinates, with x' -axis along the vertical porous wall in the upward direction and y' -axis normal to it.

Under above assumptions, the physical variables except pressure P are function of y' only. Following usual Boussinesq approximation, the unsteady free convective and mass transfer flow in an electrically conducting fluid, is governed by the following equations.

$$\frac{\partial v'}{\partial y'} = 0, \quad \dots (2.1.1)$$

$$\rho' \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = -\frac{\partial P'}{\partial x'} - \rho' g + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{K'} u' - \sigma B_0^2 u', \quad \dots (2.1.2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho' C_p} \frac{\partial^2 T'}{\partial y'^2}, \quad \dots (2.1.3)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}. \quad \dots (2.1.4)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} y' = 0, \quad u' = U'(t'), \quad T' = T'_w, \quad C' = C'_w \\ y' \rightarrow \infty, \quad u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad t' > 0 \end{aligned} \right\} \dots (2.1.5)$$

From (2.1.2) we have for free stream

$$0 = -\frac{\partial P'}{\partial x'} - \rho'_\infty g. \quad \dots (2.1.6)$$

Eliminating $-\frac{\partial P'}{\partial x'}$ between (2.1.2) and (2.1.6),

$$\rho' \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = g(\rho'_\infty - \rho') + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{k'} u' - \sigma B_0^2 u', \quad \dots (2.1.7)$$

where ρ'_∞ is the density of the flow in the free stream.

The equation of state is

$$g(\rho'_\infty - \rho') = g\beta\rho'(T' - T'_\infty) + g\beta^*(C' - C'_\infty). \quad \dots (2.1.8)$$

Substituting (2.1.8) into (2.1.7), we obtain

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + v' \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{v}{k'} + \frac{\sigma B_0^2}{\rho'} \right) u'. \quad \dots (2.1.9)$$

In equation (2.1.3) the heat due to viscous dissipation is neglected, being very small in comparison with the conducting term. This is a valid assumption because of the small velocities usually encountered in free convection flows. In the same equation Joule heating term is also neglected because it is of the same order of magnitude with the viscous dissipation term.

From equation (2.1.1)

$$v' = -\alpha \left(\frac{v}{l'} \right)^{\frac{1}{2}}, \quad \dots (2.1.10)$$

where α represents the velocity of suction ($\alpha > 0$) or injection ($\alpha < 0$) at the plate.

By introducing the following non-dimensional quantities

$$y = \frac{U'_0 y'}{\nu}, \quad t = \frac{U'_0{}^2 t'}{\nu}, \quad u = \frac{u'}{U'_0}, \quad U = \frac{U'}{U'_0}, \quad \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \quad C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)},$$

$$Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U'_0{}^3}, \quad Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U'_0{}^3}, \quad Pr = \frac{\nu\rho' C_p}{K'}, \quad Sc = \frac{\nu}{D},$$

$$K = \frac{K'U'_0{}^2}{\nu^2}, \quad M = \frac{\nu\sigma B_0^2}{\rho'U'_0{}^2},$$

where U'_0 is a constant with dimension of velocity.

In equation (2.1.2)-(2.1.4) and taking into account (2.1.10) we get

$$\frac{\partial u}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - Lu, \quad \dots (2.1.11)$$

$$Pr \left(\frac{\partial \theta}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2}, \quad \dots (2.1.12)$$

$$Sc \left(\frac{\partial C}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial C}{\partial y} \right) = \frac{\partial^2 C}{\partial y^2}, \quad \dots (2.1.13)$$

where $L = M + \frac{1}{K}$.

The boundary conditions are as follows:

$$\left. \begin{aligned} y = 0, \quad u = U(t), \quad \theta = 1, \quad C = 1 \\ y \rightarrow \infty, \quad u \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \end{aligned} \right\} \dots (2.1.14)$$

Assuming the parameter (L) to be small, we expand the non-dimensional velocity u as follows:

$$u(y,t) = u_0(y,t) + Lu_1(y,t) + O(L^2). \quad \dots (2.1.15)$$

By substituting (2.1.15) into (2.1.11), (2.1.12) and (2.1.13) and equating the coefficients of the same powers of L , we get

$$\frac{\partial u_0}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} + Gr\theta + GmC, \quad \dots (2.1.16)$$

$$\frac{\partial u_1}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} - u_0, \quad \dots (2.1.17)$$

$$Pr \left(\frac{\partial \theta}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2}, \quad \dots (2.1.18)$$

$$Sc \left(\frac{\partial C}{\partial t} - \alpha t^{-\frac{1}{2}} \frac{\partial C}{\partial y} \right) = \frac{\partial^2 C}{\partial y^2}. \quad \dots (2.1.19)$$

The corresponding boundary conditions now become,

$$\left. \begin{array}{l} y=0, \quad u_0 = U(t), \quad u_1 = 0, \quad \theta = 1, \quad C = 1 \\ y \rightarrow \infty, \quad u_0 \rightarrow 0, \quad u_1 \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \end{array} \right\}. \quad \dots (2.1.20)$$

Introducing the new variable

$$\eta = \frac{1}{2} y t^{-\frac{1}{2}} \quad \dots (2.1.21)$$

and assuming the solutions of equations (2.1.16) and (2.1.17) is of the form

$$u_0 = t f_0(\eta), \quad u_1 = t^2 f_1(\eta) \quad \dots (2.1.22)$$

equations (2.1.16)-(2.1.19) are reduced to

$$f_0''(\eta) + 2(\eta + \alpha) f_0'(\eta) - 4f_0(\eta) = -4Gr\theta - 4GmC, \quad \dots (2.1.23)$$

$$f_1''(\eta) + 2(\eta + \alpha)f_1'(\eta) - 8f_1(\eta) = -4f_0(\eta), \quad \dots (2.1.24)$$

$$\theta''(\eta) + 2(\eta + \alpha)\theta' = 0, \quad \dots (2.1.25)$$

$$C''(\eta) + 2(\eta + \alpha)C' = 0, \quad \dots (2.1.26)$$

where prime denotes differentiation with respect to η and for simplicity the Schmidt and Prandtl numbers have been taken to unity.

The boundary conditions (2.1.20) for a uniformly accelerated plate $U(t) = t$ become

$$\left. \begin{aligned} f_0(0) = 1, & \quad f_0(\infty) \rightarrow 0 \\ f_1(0) = 0, & \quad f_1(\infty) \rightarrow 0 \\ \theta(0) = 1, & \quad \theta(\infty) \rightarrow 0 \\ C(0) = 1, & \quad C(\infty) \rightarrow 0 \end{aligned} \right\} \dots (2.1.27)$$

The solutions of equations (2.1.23) and (2.1.24) under the boundary conditions (2.1.27) are given as

$$u(\eta, t) = tf_0(\eta) + Lt^2 f_1(\eta), \quad \dots (2.1.28)$$

where,

$$f_0(\eta) = \left[1 - (Gr + Gm) \right] \frac{Hh_2(\sqrt{2}(\eta + \alpha))}{Hh_0(\sqrt{2}\alpha)} + (Gr + Gm) \frac{Hh_0(\sqrt{2}(\eta + \alpha))}{Hh_0(\sqrt{2}\alpha)},$$

$$f_1(\eta) = \left[Gr + Gm - 1 \right] \frac{Hh_2(\sqrt{2}(\eta + \alpha))}{Hh_2(\sqrt{2}\alpha)} - \frac{(Gr + Gm) Hh_0(\sqrt{2}(\eta + \alpha))}{2 Hh_0(\sqrt{2}\alpha)}$$

$$+ \left[1 - \frac{(Gr + Gm)}{2} \right] \frac{Hh_4(\sqrt{2}(\eta + \alpha))}{Hh_4(\sqrt{2}\alpha)}.$$

The solution of equations (2.1.25) and (2.1.26) under the boundary condition (2.1.27) are

$$\theta = \frac{Hh_0(\sqrt{2}(\eta + \alpha))}{Hh_0(\sqrt{2}\alpha)}, \quad \dots (2.1.29)$$

$$C = \frac{Hh_0(\sqrt{2}(\eta + \alpha))}{Hh_0(\sqrt{2}\alpha)}, \quad \dots (2.1.30)$$

where the functions Hh_n ($n = 0, \pm 1, \pm 2, \dots$) are defined in Jefferys and Jefferys [8].

Finally the expression for the non-dimensional skin friction τ is given by

$$\tau = \frac{1}{2}t^{\frac{1}{2}}[f'_0(0) + Ltf'_1(0)], \quad \dots (2.1.31)$$

where

$$f'_0(0) = -\sqrt{2} \left[\left[1 - (Gr + Gm) \right] \frac{Hh_1(\sqrt{2}\alpha)}{Hh_2(\sqrt{2}\alpha)} + (Gr + Gm) \frac{Hh_{-1}(\sqrt{2}\alpha)}{Hh_0(\sqrt{2}\alpha)} \right],$$

$$f'_1(0) = -\sqrt{2} \left[\left[Gr + Gm - 1 \right] \frac{Hh_1(\sqrt{2}\alpha)}{Hh_2(\sqrt{2}\alpha)} - \frac{(Gr + Gm)}{2} \frac{Hh_{-1}(\sqrt{2}\alpha)}{Hh_0(\sqrt{2}\alpha)} \right]$$

$$+ \left[2 - \frac{(Gr + Gm)}{2} \frac{Hh_3(\sqrt{2}\alpha)}{Hh_4(\sqrt{2}\alpha)} \right].$$

2.1.3 DISCUSSION

In order to study the effects of free convection currents, mass transfer, magnetic field and suction or injection on velocity field numerical calculations are carried out for different values of Gr , Gm , M and α . As usual $Gr > 0$ represents an

externally cooled plate and $Gr < 0$ corresponds to an externally heated plate. The velocity profiles for different values of Gr , Gm , α , M and K are shown in Fig. 2.1.1, Fig. 2.1.2, Fig. 2.1.3 and Fig. 2.1.4. From these figures it is observed that velocity increases as Gm increases for $\alpha > 0$ and $\alpha < 0$. In case of suction, velocity decreases steadily but in case of injection there is an increase in velocity near the plate and then decreases for $Gr > 0$. For $Gr < 0$, velocity decreases steadily for both suction and injection, but in case of injection, there is an increase in velocity near the plate and then decreases for $Gm = 5$. Fig. 2.1.1 and Fig. 2.1.2 show that the velocity decreases as M increases for $\alpha > 0$ and $\alpha < 0$. From Fig. 2.1.3 and Fig. 2.1.4, it is observed that velocity increases as K increases for both $\alpha > 0$ and $\alpha < 0$. In case of $Gr > 0$, velocity at the plate wall is same for suction and then decreases steadily, but for injection, velocity increases near the plate and then decreases.

The numerical values of skin friction (τ) are given in Table 2.1.1. and Table 2.1.2 to study the effect of permeability of the medium and magnetic parameter for

Table: - 2.1.1

Value of skin friction (τ) when $K=2$, $t=0.2$

Gr	α		-0.5	0.5
	M	Gm		
2	0	3	0.89088	0.46880
	0.3	3	0.86637	0.44908
	0	5	1.4118	0.91777
	0.3	5	1.3815	0.89402
-2	0	3	-0.15095	-0.42914
	0.3	3	-0.16400	-0.44080
	0	5	0.36996	0.01982
	0.3	5	0.35118	0.00413

different values of Gr , Gm and α . Skin friction (τ) increases as K and Gm increases for all Gr . Skin friction (τ) decreases as M increases for $Gr < 0$ and $Gr > 0$, but increases as Gm increases. We have also studied the behavior of temperature and concentration distribution for different values of α ($Pr=1, Sc=1$) in Fig. 2.1.5 and Fig. 2.1.6 respectively. For suction, both temperature and concentration increases as α increases but in case of injection it exhibits opposite characteristic.

Table: - 2.1.2

Value of skin friction (τ) when $M=0.5, t=.2$

Gr	α		-0.5	0.5
	K	Gm		
2	0.5	3	0.72748	0.33736
	2	3	0.85003	0.43539
	0.5	5	1.21020	0.75947
	2	5	1.36140	0.87819
-2	0.5	3	-0.23794	-0.50685
	2	3	-0.17270	-0.44857
	0.5	5	0.24476	0.08474
	2	5	0.33866	0.00631

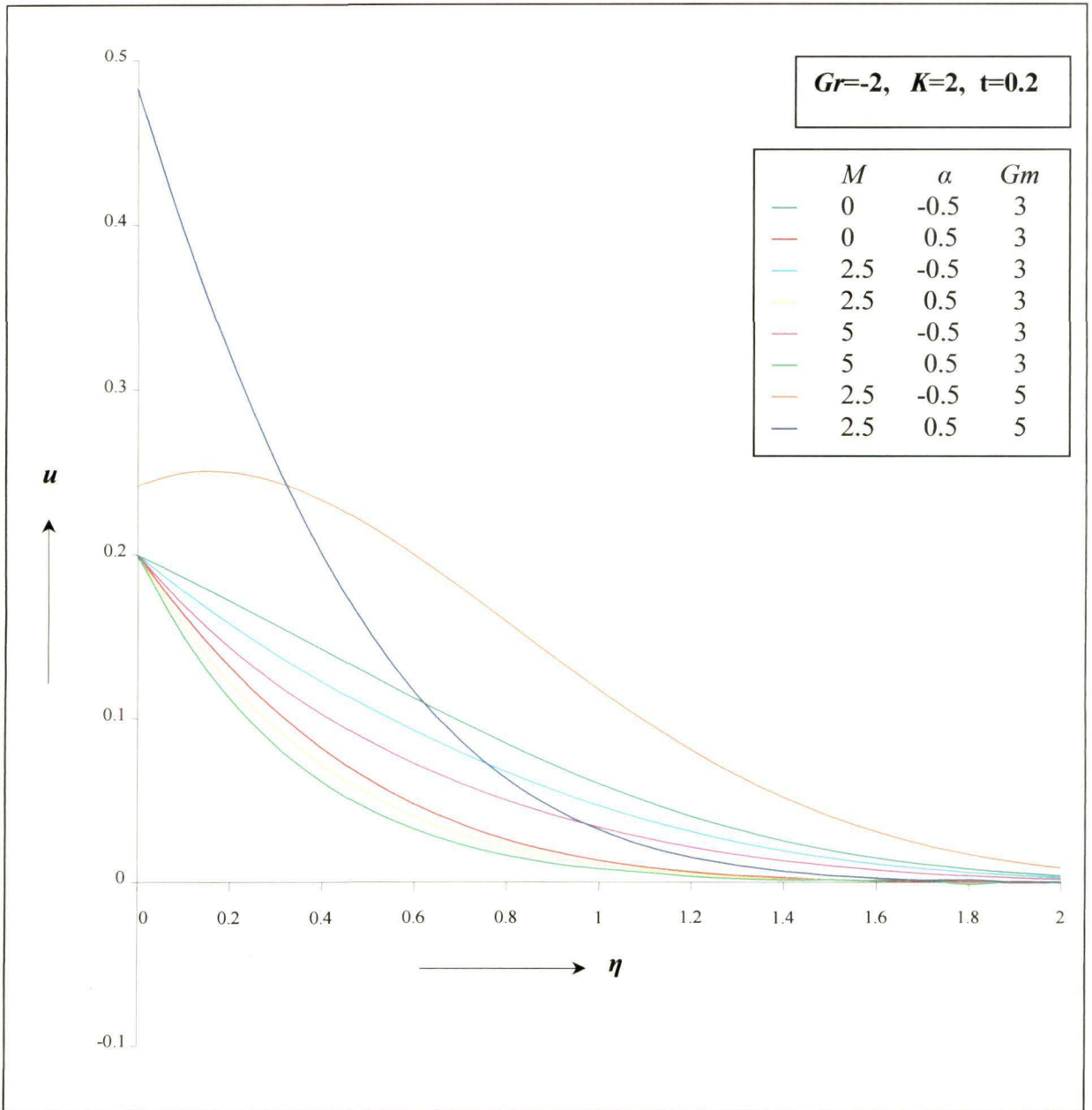


Fig. 2.1.1 Graph of velocity against η for different values of M , α and Gm ($Gr=-2, K=2, t=0.2$).

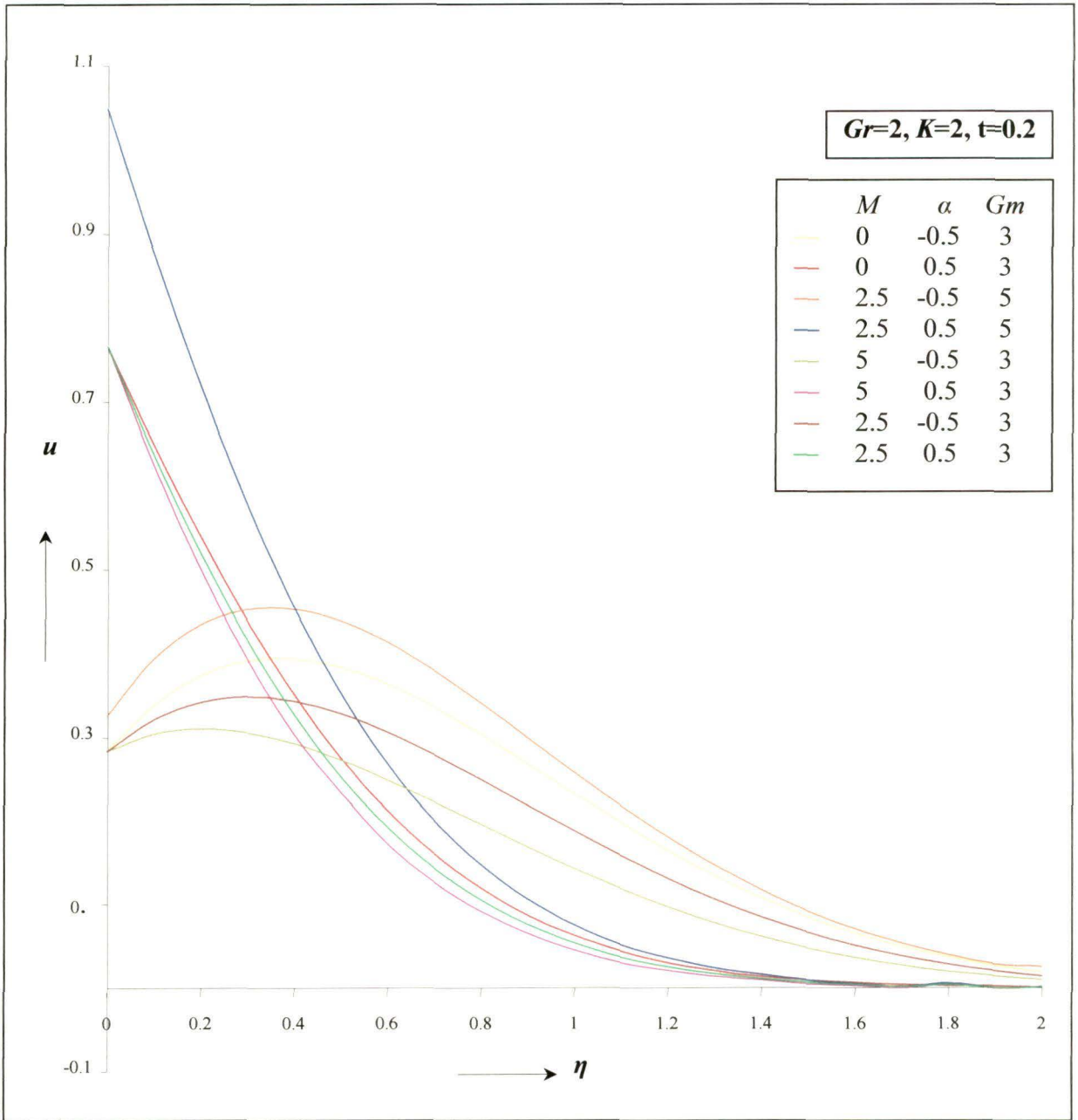


Fig. 2.1.2 Graph of velocity against η for different values of M , α and Gm ($Gr=2, K=2, t=0.2$).

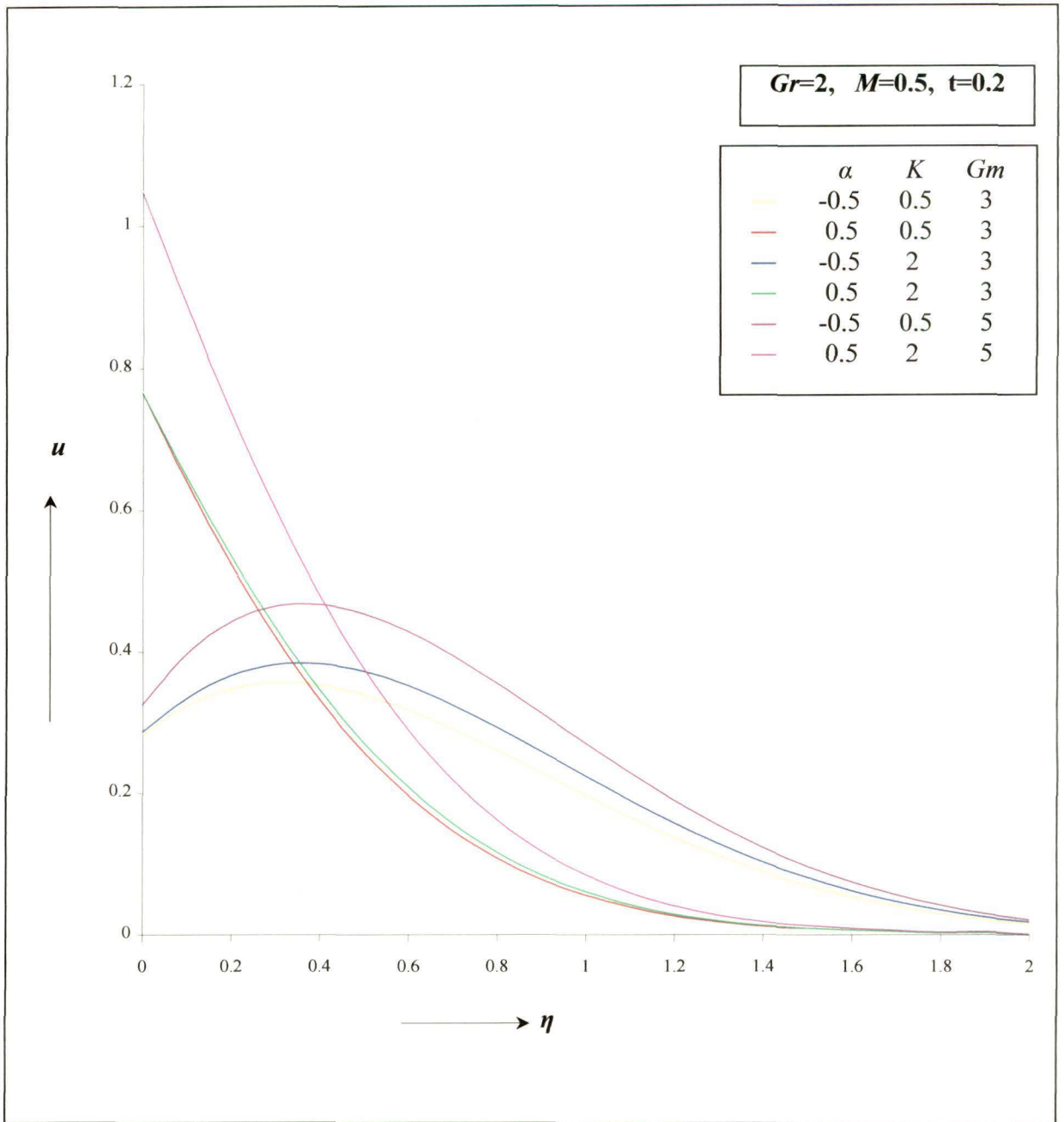


Fig. 2.1.3 Graph of velocity against η for different values of K , α and Gm ($Gr=2, M=0.5, t=0.2$).

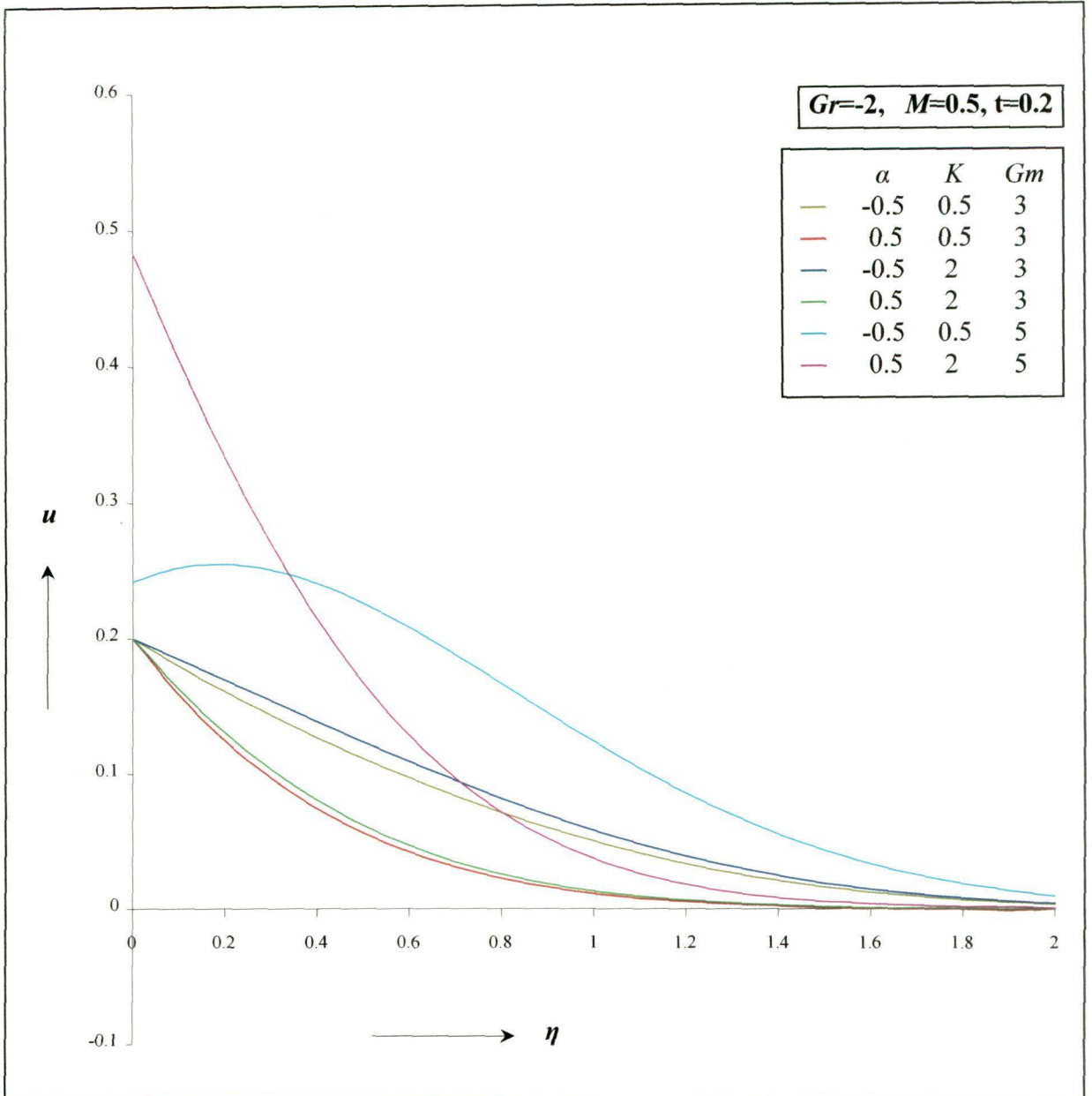


Fig. 2.1.4 Graph of velocity against η for different values of K , α and Gm ($Gr=-2, M=0.5, t=0.2$).

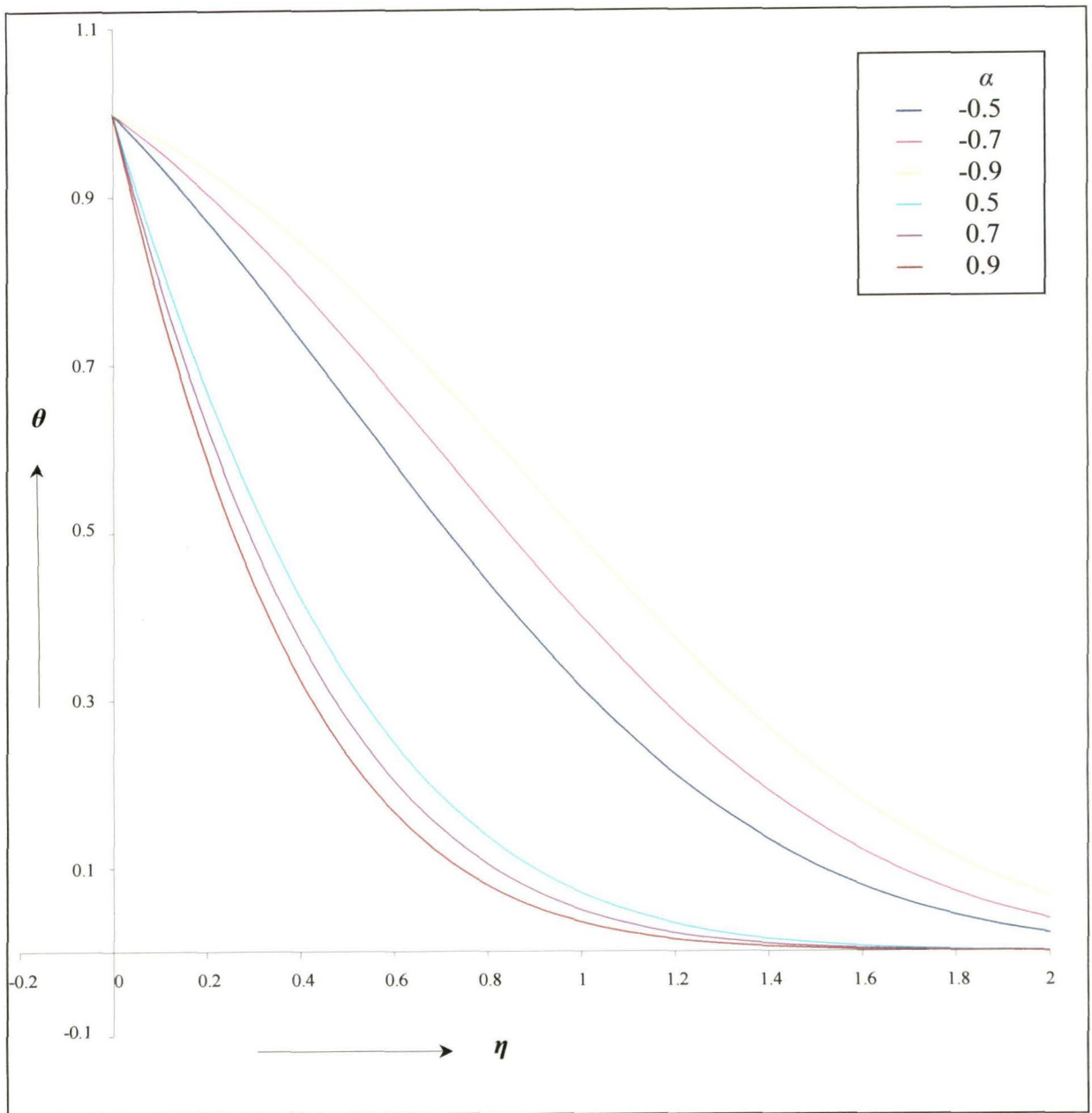


Fig. 2.1.5 Graph of temperature against η for different values of α .

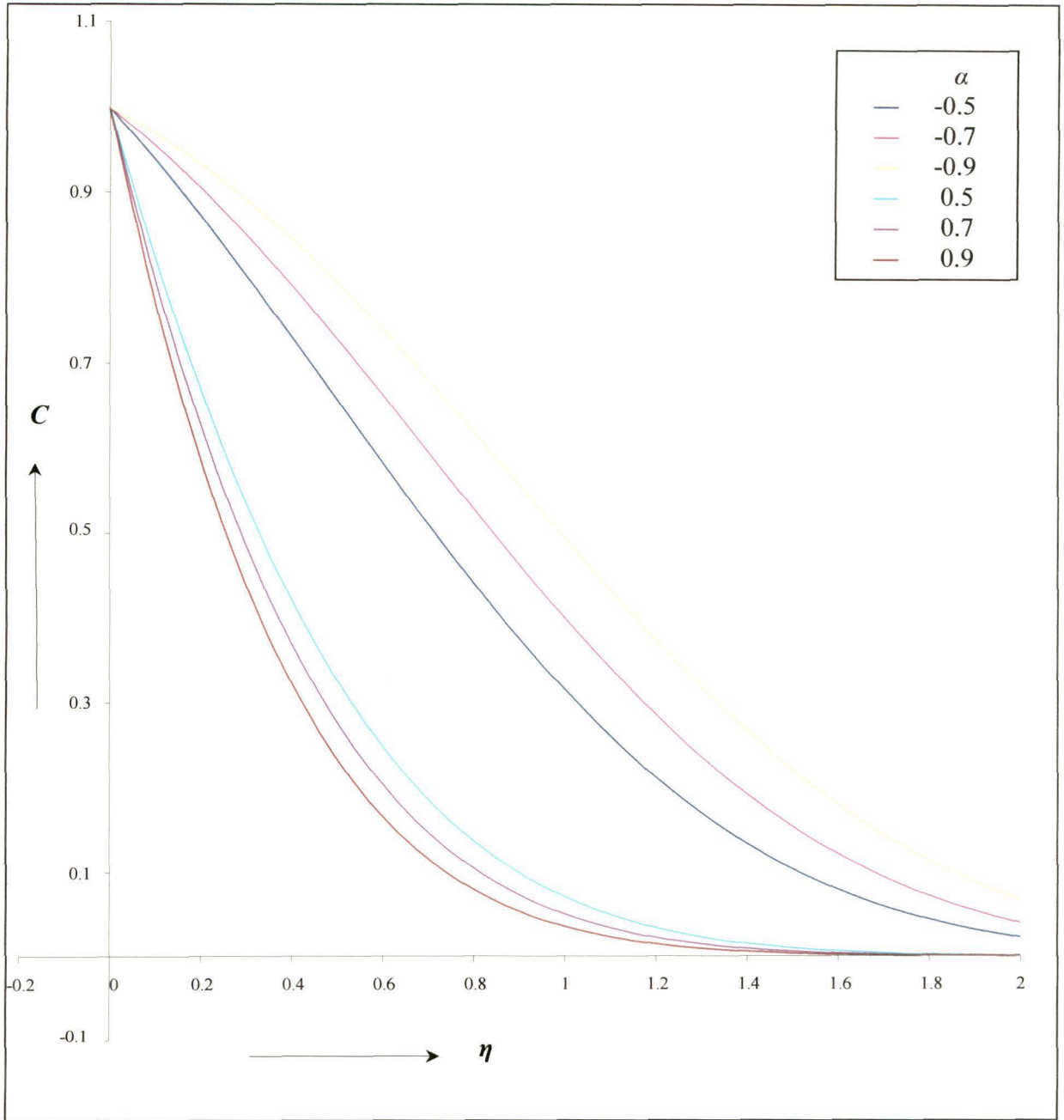


Fig. 2.1.6 Graph of concentration against η for different values of α .

PART ONE > B**HEAT AND MASS TRANSFER TO UNSTEADY FLOW OF MHD FLUID THROUGH A POROUS MEDIUM BOUNDED BY AN INFINITE VERTICAL HOT POROUS PLATE WITH CONSTANT SUCTION IN PRESENCE OF HEAT SOURCE****2.1.4 INTRODUCTION**

Convective heat transfer in a porous medium has been the subject of intensive study for the last few decades owing to its application in different field such as chemical engineering, geothermal, petroleum and reservoir engineering, environmental protection, thermal insulation, cooling and processing of food etc. Yamamoto and Iwamura [9] investigated the flow with convective acceleration through a porous medium. Moreover, in nature, along with the free convection current caused by the temperature differences, the flow is also affected by chemical composition differences and gradients. The flow caused by density difference, which in turn caused by concentration differences is known as mass transfer flow. This phenomenon of free convection and mass transfer flow arises in a fluid when temperature and concentration differences cause density variations leading to body forces acting on the fluid's element. There are many interesting aspects of such flow, so in recent years many authors have presented analytical solutions to such problems of flow. Raptis, Tzivamidies and Kafousis [10] studied steady free convective and mass transfer flow through a porous medium bounded by an infinite vertical porous surface with constant suction. Raptis [2] studied unsteady free convective and mass transfer flow of an incompressible viscous fluid through a very porous medium past an infinite vertical porous surface with constant suction. Raptis *et al.* [3] studied the influence of free convection with unsteady flow of viscous fluid through the porous medium when there is a constant heat flux. The influence of magnetic field on viscous incompressible flow

of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads etc, in the presence of an electrically conducting fluid subject to a magnetic field. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical infinite plate was studied by Soundalagekar *et al.* [11]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalagekar *et al.* [12]. The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate was studied by Soundalagekar *et al.* [13]. Further, the effect of constant heat flux on the flow of an electrically conducting fluid plate oscillating in its plate was studied by Soundalagekar *et al.* [14]. Recently, Sriramulu *et al.* [15] studied the effect of applied magnetic field on transient free convection flow of an incompressible viscous fluid by taking into account of viscous dissipative heat along with the heat due to free convection currents in a vertical channel. We now proposed to study the effect of magnetic parameter and heat source on the heat transfer to unsteady flow of MHD fluid through porous medium bounded by infinite vertical porous plate with mass transfer. Solution of the equation governing the flow is obtained analytically.

2.1.5 MATHEMATICAL ANALYSIS

We consider heat transfer to unsteady flow of MHD fluid through a porous medium bounded by an infinite vertical hot porous plate in presence of heat source with mass transfer. The x' -axis is taken along the plate in the upward direction and the y' -axis is normal to it. A transverse magnetic field is applied in the direction of

y' -axis. Since the motion is two-dimensional and length of the plate is large therefore all the physical variable are independent of x' . All the fluid properties are assumed to be constant except that the influence of the density variation with temperature and concentration is considered only in the body force term. Under these condition the problem is governed by the following system of equations

$$\frac{\partial v'}{\partial y'} = 0, \quad \dots (2.1.32)$$

$$\rho' \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = -\frac{\partial p'}{\partial x'} - \rho' g + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{K'} u' - \sigma B_0^2 u', \quad \dots (2.1.33)$$

$$\rho' \frac{\partial v'}{\partial t'} = -\frac{\partial p'}{\partial y'} - \frac{\mu}{K'} v', \quad \dots (2.1.34)$$

$$\rho' C_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = \kappa \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty), \quad \dots (2.1.35)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}. \quad \dots (2.1.36)$$

where u' , v' are the velocity components in the x' and y' directions respectively, g is the acceleration due to gravity, p' is the pressure, μ the viscosity, K' the permeability of the porous medium, C_p the specific heat of the fluid at constant pressure, k the thermal conductivity, T' the temperature, C' the concentration and D the chemical molecular diffusivity.

The boundary conditions of the problem are

$$\left. \begin{aligned} u' = 0, \quad T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\omega t'}, \quad C' = C'_w \quad \text{at } y' = 0 \\ u' \rightarrow U_\infty, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \end{aligned} \right\} \dots (2.1.37)$$

where T'_w , T'_∞ , and C'_w , C'_∞ are the temperature and the species concentration on the porous limiting surface and in the free stream respectively. Also U'_∞ is the free-stream velocity and ε a positive constant ($\varepsilon < 1$).

The continuity equation (2.1.32) gives

$$v' = -v_0, \quad \dots (2.1.38)$$

where v_0 (> 0) is the constant suction velocity of the fluid through the porous surface.

For the free stream, equation (2.1.33) gives

$$0 = -\frac{\partial p'}{\partial x'} - \rho'_\infty g - \frac{\mu}{K'} U_\infty - \sigma B_0^2 U_\infty. \quad \dots (2.1.39)$$

Eliminating $\partial p'/\partial x'$ between (2.1.33) and (2.1.39), we have

$$\begin{aligned} \rho' \left(\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} \right) &= \rho'_\infty g + \frac{\mu}{K'} U_\infty + \sigma B_0^2 U_\infty \\ &\quad - \rho' g + \mu \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\mu}{K'} + \sigma B_0^2 \right) u'. \end{aligned} \quad \dots (2.1.40)$$

and taking into account the equation of state

$$g(\rho'_\infty - \rho') = g\beta\rho'(T' - T'_\infty) + g\beta^*\rho'(C' - C'_\infty). \quad \dots (2.1.41)$$

(β is the co-efficient of volume expansion and β^* is the volumetric co-efficient of expansion with concentration)

From (2.1.40) and (2.1.41), we write

$$\begin{aligned} \frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} &= g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \\ &\quad + \nu \frac{\partial^2 u'}{\partial y'^2} + \left(\frac{\nu}{K'} + \frac{\sigma B_0^2}{\rho'} \right) (U_\infty - u'), \end{aligned} \quad \dots (2.1.42)$$

where ν is the kinematics viscosity.

Again from (2.1.32) and (2.1.33) we can show that $\partial^2 p' / \partial y^2 = 0$.

We introduce the following non-dimensional quantities:

$$y = \frac{y'v_0}{\nu}, \quad t = \frac{t'v_0^2}{\nu}, \quad u = \frac{u'}{U_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \omega = \frac{\nu\omega'}{v_0^2},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Sc = \frac{\nu}{D} \text{ (Schmidt number)}, \quad P = \frac{\rho' \nu C_p}{\kappa} \text{ (Prandtl number)},$$

$$K = \frac{v_0^2 K'}{\nu^2} \text{ (permeability parameter)}, \quad Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{U_\infty v_0^2},$$

$$M = \frac{\sigma \nu B_0^2}{\rho' v_0^2}, \quad S = \frac{S'}{\rho' C_p v_0^2} \text{ (heat source parameter)},$$

and $Gm = \frac{\nu g \beta^* (C'_w - C'_\infty)}{U_\infty v_0^2}$ (modified Grashoff number).

Substituting the above non-dimensional quantities into equations (2.1.40), (2.1.35) and (2.1.36), we get

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = GrT + GmC + \frac{\partial^2 u}{\partial y^2} + \left(M + \frac{1}{K} \right) (1 - u), \quad \dots (2.1.43)$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P} \frac{\partial^2 T}{\partial y^2} + ST, \quad \dots (2.1.44)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2}, \quad \dots (2.1.45)$$

and the boundary conditions (2.1.37) become

$$\left. \begin{aligned} u = 0, \quad T = 1 + \varepsilon e^{i\omega t}, \quad C = 1 \quad \text{at } y = 0 \\ u \rightarrow 1, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \dots (2.1.46)$$

To solve the system of equations (2.1.42)-(2.1.44) under their boundary conditions (2.1.45), we assume that

$$\left. \begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots \\ T(y,t) &= T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \dots \\ C(y,t) &= C_0(y) + \varepsilon e^{i\omega t} C_1(y) + \dots \end{aligned} \right\} \dots (2.1.47)$$

Substituting (2.1.47) into the system (2.1.42)-(2.1.44) we have

$$u_0'' + u_0' - \left(M + \frac{1}{K} \right) u_0 = -GrT_0 - GmC_0 - \left(M + \frac{1}{K} \right) \dots (2.1.48)$$

$$\text{and } u_1'' + u_1' - \left\{ \left(M + \frac{1}{K} \right) + i\omega \right\} u_1 = -GrT_1 - GmC_1, \dots (2.1.49)$$

$$T_0'' + PT_0' + PST_0 = 0, \dots (2.1.50)$$

$$T_1'' + PT_1' + P(S - i\omega)T_1 = 0, \dots (2.1.51)$$

$$C_0'' + ScC_0' = 0, \dots (2.1.52)$$

$$C_1'' + ScC_1' - i\omega ScC_1 = 0, \dots (2.1.53)$$

and the boundary conditions (2.1.45) now become

$$\left. \begin{aligned} u_0(0) &= 0, T_0(0) = 1, C_0(0) = 1, \\ u_1(0) &= 0, T_1(0) = 1, C_1(0) = 0, \\ u_0(\infty) &\rightarrow 1, T_0(\infty) \rightarrow 0, C_0(\infty) \rightarrow 0, \\ u_1(\infty) &\rightarrow 0, T_1(\infty) \rightarrow 0, C_1(\infty) \rightarrow 0 \end{aligned} \right\} \dots (2.1.54)$$

Thus the solution of the problem is obtained by solving the differential equation (2.1.48)-(2.1.53) as

$$\begin{aligned} u(y,t) &= 1 - L_1 e^{-R_1 y} - L_2 e^{-Scy} \\ &\quad + L_3 e^{-R_3 y} + \varepsilon L_4 e^{i\omega t} \left(e^{-R_4 y} - e^{-R_2 y} \right), \end{aligned} \dots (2.1.55)$$

$$T(y,t) = e^{-R_1 y} + \varepsilon e^{i\omega t} e^{-R_2 y}, \dots (2.1.56)$$

$$C(y) = e^{-Scy}, \quad \dots (2.1.57)$$

where the quantities $L_i, i=1,2,3,4$ and $R_j, j=1,2,3,4$ are defined in the Appendix.

We can write the expression for the transient velocity profiles as

$$u(y,t) = u_0(y) + \varepsilon(N_r \cos \omega t - N_i \sin \omega t), \quad \dots (2.1.58)$$

where $N_r + iN_i = u_1(y)$

and for $\omega t = \frac{\pi}{2}$,

$$u\left(y, \frac{\pi}{2\omega}\right) = u_0(y) - \varepsilon N_i. \quad \dots (2.1.59)$$

2.1.6 DISCUSSION

In order to have a physical point of view of the problem, numerical calculations are carried out for different values of M (magnetic parameter), K (permeability parameter), S (heat source), Gr (Grashof number), Gm (modified Grashof number). The velocity profiles are shown in Fig. 2.1.7- Fig. 2.1.11. From this figures it is seen that velocity decreases as magnetic parameter M increases. But reverse character is seen in Fig. 2.1.8 and Fig. 2.1.9. Velocity increases as permeability parameter K and source heat parameter S increases. In Fig. 2.1.10, Grashof number represents the effects of the free convection currents and the case $Gr > 0$ corresponds to an externally cooled plate while the case $Gr < 0$ corresponds to an externally heated plate. It is also observed from Fig. 2.1.11, velocity increases as modified Grashof number Gm increases. Here in all cases, we consider $Pr = 0.71$ which corresponds physically to air while $Sc = 0.24$ is chosen in such a way to represent hydrogen at $25^\circ C$ and 1 atmosphere (approximately). Table 2.1.3 gives the values of velocity for different values of the frequency ω .

Table- 2.1.3.

Variation of the velocity when $Pr=0.71, S=2, K=0.5, M=0.6, Gr =5, Gm =2,$
 $Sc=0.24, \varepsilon=0.2, \omega t=\pi/2$

Y	$\omega=0.5$	$\omega=1$	$\omega=1.5$
0	-1.192E-07	-1.192E-07	-1.192E-07
0.25	1.0859	1.0702	1.0572
0.5	1.5742	1.5505	1.5304
0.75	1.7169	1.6868	1.6615
1	1.6708	1.6337	1.6043
1.25	1.5344	1.4898	1.4576
1.5	1.3695	1.3178	1.2846
1.75	1.214	1.1566	1.1246
2	1.0893	1.0283	0.9996

Appendix

$$R_1 = 0.5 \left[P + \left(P^2 - 4SP \right)^{\frac{1}{2}} \right];$$

$$R_2 = 0.5 \left[P + \left\{ P^2 - 4P(S - i\omega) \right\}^{\frac{1}{2}} \right];$$

$$R_3 = 0.5 \left[1 + \left\{ 1 + 4 \left(M + \frac{1}{K} \right) \right\}^{\frac{1}{2}} \right];$$

$$R_4 = 0.5 \left[1 + \left\{ 1 + 4 \left(M + \frac{1}{K} + i\omega \right) \right\}^{\frac{1}{2}} \right];$$

$$L_1 = \frac{Gr}{R_1^2 - R_1 - \left(M + \frac{1}{K} \right)};$$

$$L_2 = \frac{Gm}{Sc^2 - Sc - \left(M + \frac{1}{K} \right)};$$

$$L_3 = (L_1 + L_2 - 1);$$

$$L_4 = - \frac{Gr}{R_2^2 - R_2 - \left(M + \frac{1}{K} + i\omega \right)}.$$

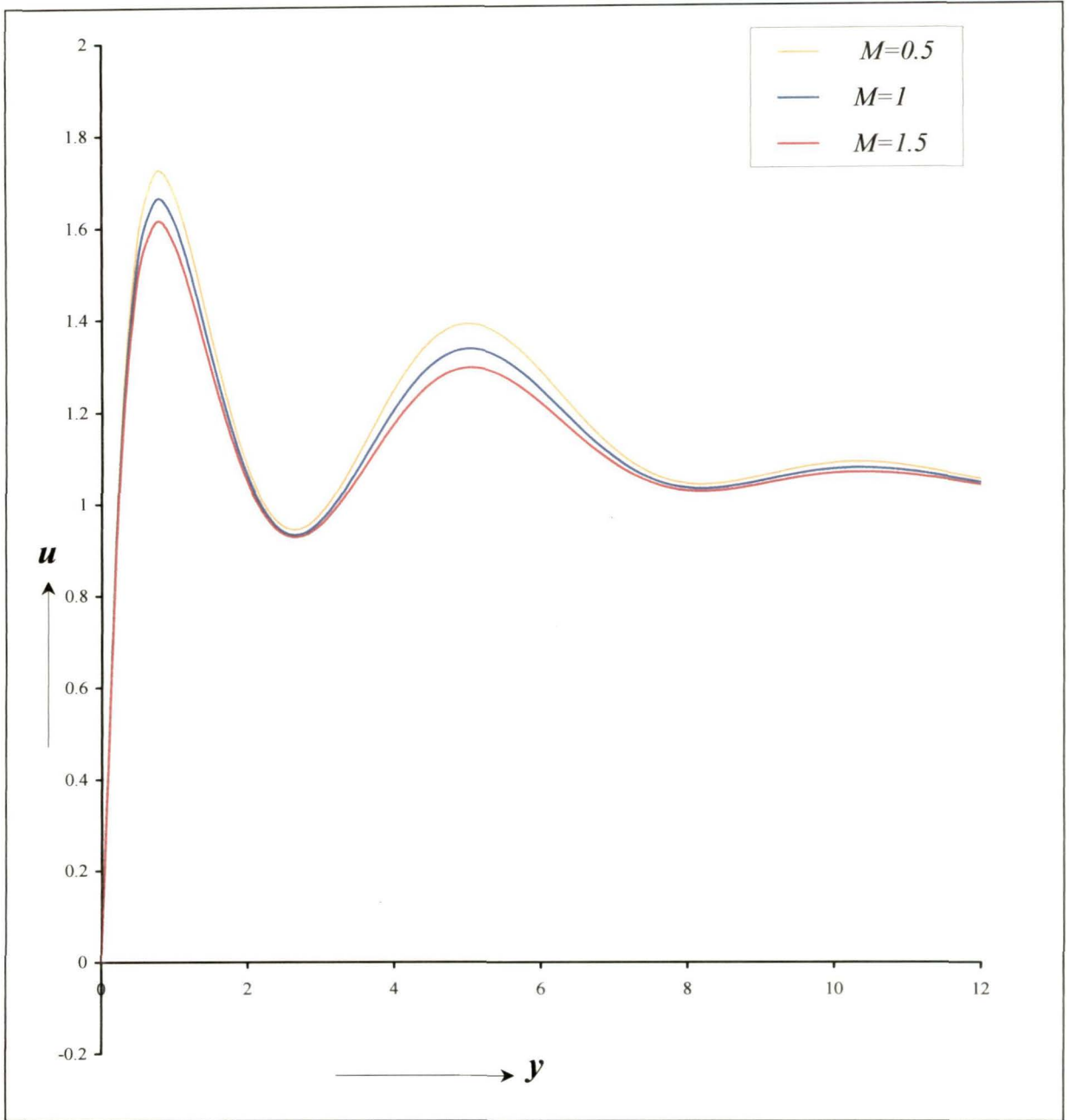


Fig. 2.1.7 Velocity profiles for different values of M : $Pr=0.71$, $S=2$, $K=0.5$, $\omega=0.6$, $Gr=5$, $Gm=2$, $Sc=0.24$, $\varepsilon=0.2$.

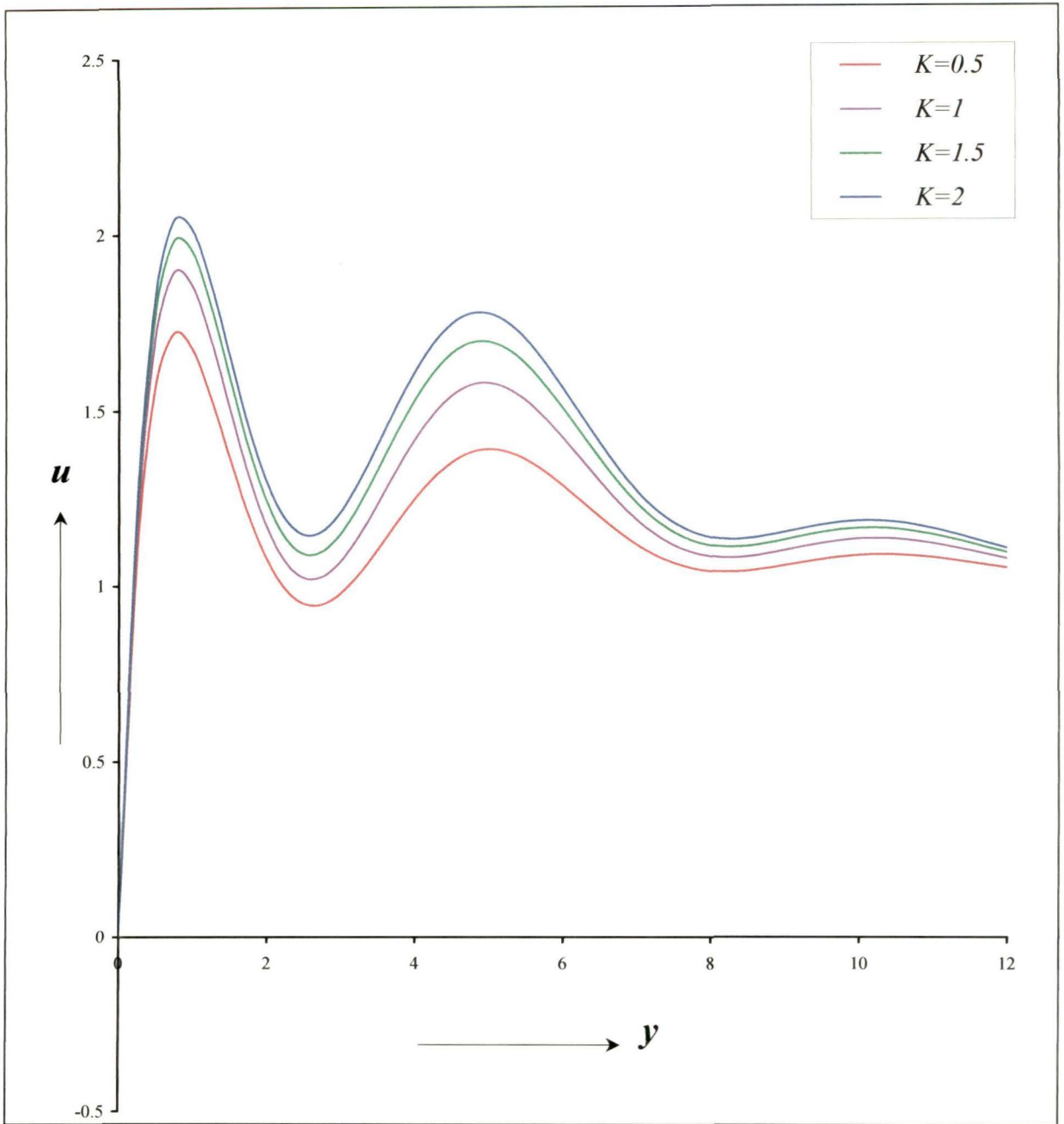


Fig. 2.1.8 Velocity profiles for different values of K : $Pr=0.71$, $S=2$, $M=0.5$, $\omega=0.6$, $Gr=5$, $Gm=2$, $Sc=0.24$, $\varepsilon=0.2$.

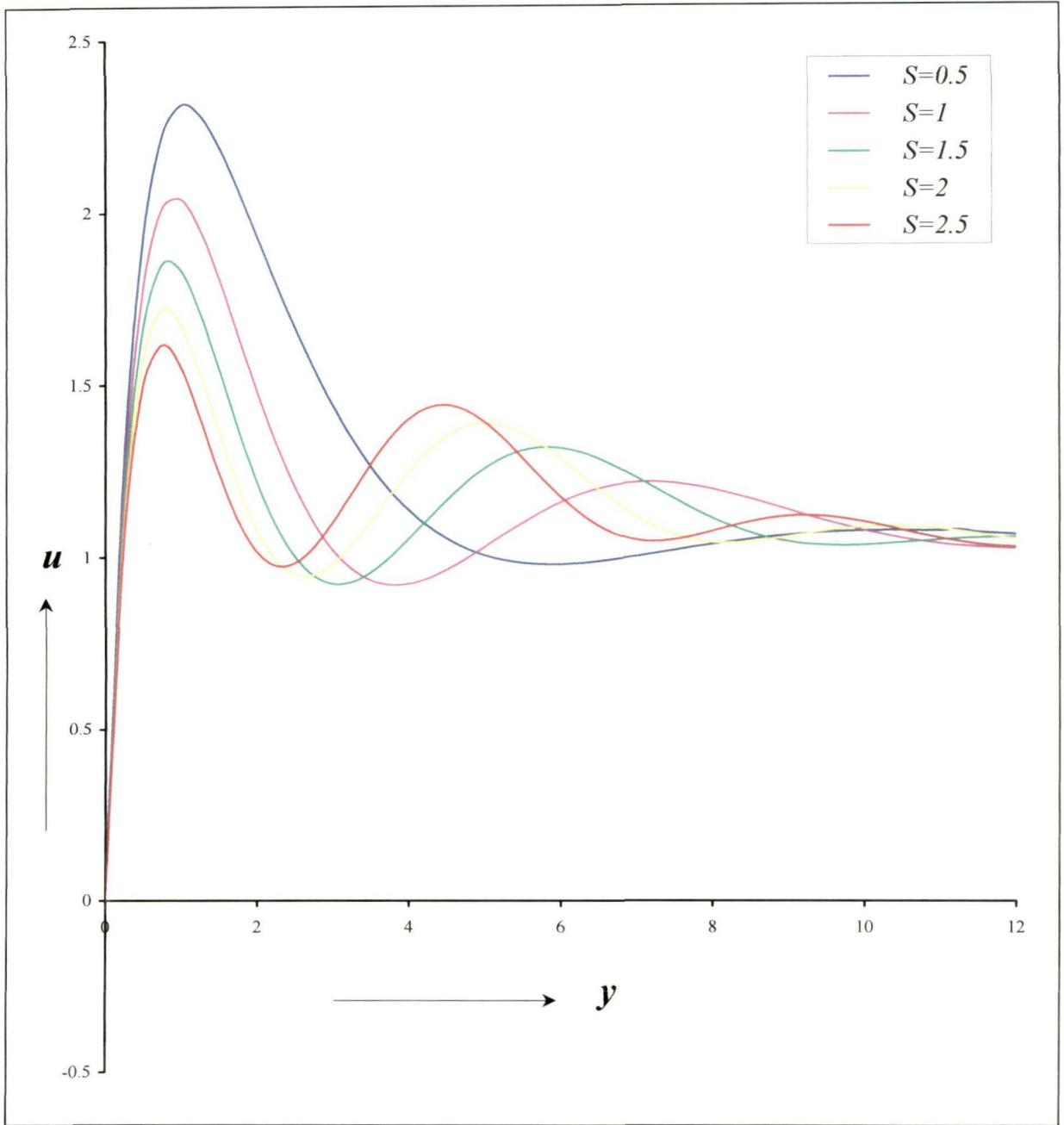


Fig. 2.1.9 Velocity profiles for different values of S : $Pr=0.71$, $Gr=5$, $M=0.5$, $\omega=0.6$, $K=0.5$, $Gm=2$, $Sc=0.24$, $\varepsilon=0.2$.

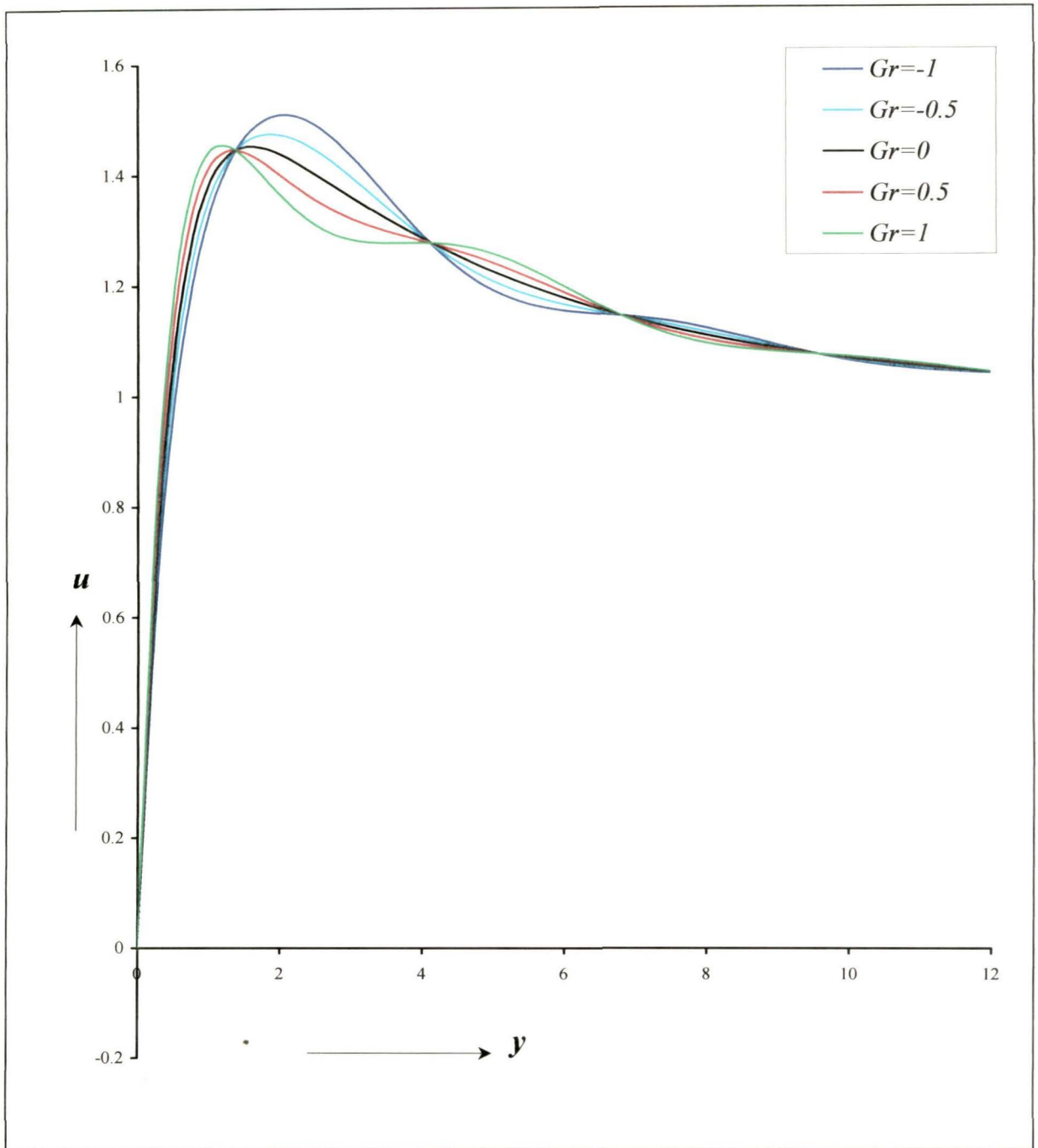


Fig. 2.1.10 Velocity profiles for different values of Gr : $P=0.71$, $S=2$, $M=0.5$, $\omega=0.6$, $Gm=2$, $K=0.5$, $Sc=0.24$, $\varepsilon=0.2$.

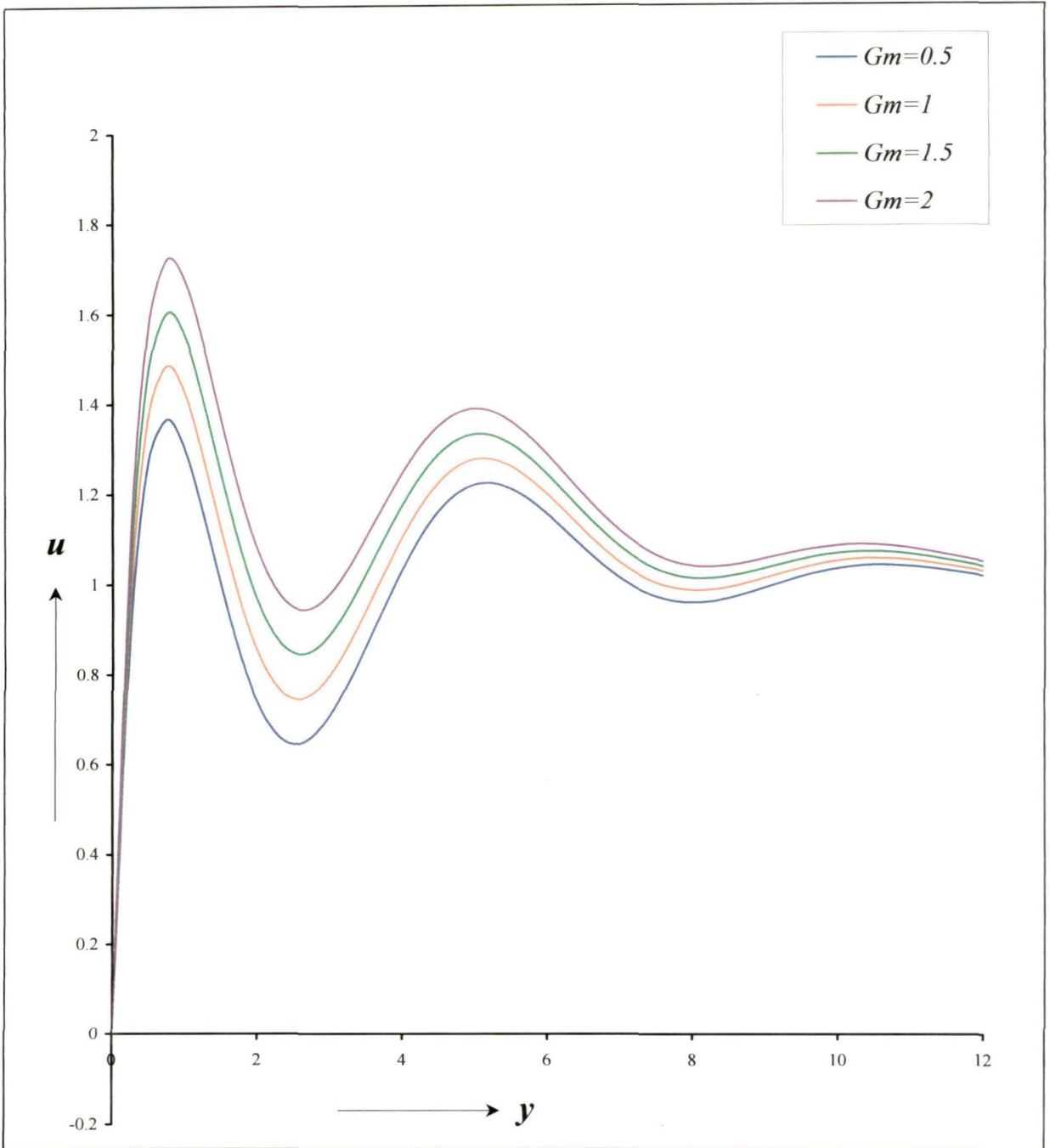


Fig. 2.1.11 Velocity profiles for different value of Gm : $Pr=0.71$, $M=0.5$, $\omega=0.6$, $Gr=5$, $Sc=0.24$, $\varepsilon=0.2$, $K=0.5$.

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Part two

UNSTEADY FREE CONVECTIVE
MHD FLUID FLOW WITH MASS
TRANSFER THROUGH POROUS
MEDIUM IN ROTATING SYSTEM

PART TWO > A**UNSTEADY FREE CONVECTIVE AND MASS
TRANSFER FLOW THROUGH POROUS MEDIUM
IN ROTATING SYSTEM****2.2.1 INTRODUCTION**

The flow through porous medium, under the influence of temperature differences and concentration differences, is one of the most considerable and contemporary subject, because it finds great applications in geothermy, geophysics and technology [1, 2]. Yamamoto and Iwamara [2] expressed the equations of flow through a highly porous medium. Raptis *et al.* [3,4] using the above equations studied the influences of free convection and mass transfer on the steady flow of a viscous fluid through the porous medium, which is bounded by a vertical plane surface, when the temperature and concentration on the surface is constant. Raptis *et al.* [5] also studied the influence of free convective flow on the steady flow of the viscous fluid through the porous medium, when there is a constant heat flux on the above-mentioned surface.

On the other hand, the geophysical importance of the flows in the rotating frame of reference has attracted the attention of a number of Scholars. Raptis [6] analyzed the steady free convective and mass transfer flow through porous medium in presence of a rotating fluid. Later Mahato and Maiti [7] investigated unsteady free convective flow and mass transfer in a rotating porous medium. The object of the present paper is to study the free convective and mass transfer flow of viscous fluid through a rotating porous medium bounded by a vertical porous plate subjected to a constant suction velocity in presence of constant heat flux at the plate. The temperature and concentration at the free streams are constant but the free stream velocity of the fluid vibrates about a mean constant value. The

analytical expressions for velocity, temperature and concentration distribution are obtained and the results are presented graphically.

2.2.2 MATHEMATICAL ANALYSIS

We consider unsteady free convective and mass transfer flow of viscous fluid through a porous medium occupying a semi-infinite region bounded by a vertical porous plate subjected to constant suction in presence of constant heat flux at plate wall in a rotating frame of reference. The velocity of the fluid far away from the surface vibrates about a mean value with direction parallel to the plane $z=0$. The temperature and species concentration at the free stream are constant. A uniform magnetic field of strength B_0 is applied in vertical upward direction. The porous medium is in fact a non-homogeneous medium, which may be replaced by a homogeneous fluid having dynamical properties equal to those of a non-homogeneous continuum. We consider that the vertical infinite porous plate rotates in unison with a viscous fluid occupying the porous region with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. Cartesian co-ordinate system is chosen such that x, y -axes, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z=0$ while z -axis is normal to it. u^+, v^+, w^+ are the velocity components in x, y and z direction respectively. With the above frame of reference and assumptions, the physical variables, except the pressure p are function of z and time t only. Consequently the equation expressing the conservation of mass, momentum, energy and concentration, neglecting the heat due to viscous dissipation, which is valid for small velocities, are given by

$$\frac{\partial \omega^+}{\partial z} = 0, \quad \dots (2.2.1)$$

$$\frac{\partial u^+}{\partial t} - \omega^+ \frac{\partial u^+}{\partial z} - 2\Omega v^+ = \frac{\partial U^+}{\partial t} - 2\Omega V^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho} \right) (u^+ + U^+)$$

$$+v \frac{\partial^2 u^+}{\partial z^2} + g\beta(T^+ - T_\infty^+) + g\beta^*(C^+ - C_\infty^+), \quad \dots (2.2.2)$$

$$\frac{\partial v^+}{\partial t} - \omega^+ \frac{\partial v^+}{\partial z} - 2\Omega u^+ = \frac{\partial V^+}{\partial t} - 2\Omega U^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho} \right) (v^+ - V^+) + \nu \frac{\partial^2 v^+}{\partial z^2}, \quad \dots (2.2.3)$$

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\nu}{K^+} \omega^+, \quad \dots (2.2.4)$$

$$\frac{\partial T^+}{\partial t} - \omega^+ \frac{\partial T^+}{\partial z} = \frac{\kappa^{*+}}{\rho C_p} \frac{\partial^2 T^+}{\partial z^2}, \quad \dots (2.2.5)$$

$$\frac{\partial C^+}{\partial t} - \omega^+ \frac{\partial C^+}{\partial z} = D \frac{\partial^2 C^+}{\partial z^2}, \quad \dots (2.2.6)$$

where ν is the kinematic viscosity, t is the time, ρ is the density, K^+ is the permeability of the porous medium, T^+ is the temperature and C^+ is the concentration.

The boundary conditions relevant to the problem are

$$\left. \begin{aligned} u^+ = 0, v^+ = 0, \frac{\partial T^+}{\partial z} = -\frac{s}{\kappa^{*+}}, C^+ = C_\infty^+ \quad \text{at } z = 0 \\ u^+ = U^+(t) = U_0(1 + \varepsilon \cos \phi t), \\ v^+ = V^+(t) = 0, T^+ = T_\infty^+, C^+ = C_\infty^+ \quad \text{as } z \rightarrow \infty \end{aligned} \right\} \dots (2.2.7)$$

where ϕ is the frequency of oscillation and ε is a small positive quantity.

From equation (2.2.1), we get

$$\omega^+ = -\omega_0. \quad \dots (2.2.8)$$

Let equation (2.2.2) and (2.2.3) can be combined in complex form, as

$$\begin{aligned} \frac{\partial q^+}{\partial t} - \omega_0 \frac{\partial q^+}{\partial z} + 2i\Omega q^+ = \frac{\partial Q^+}{\partial t} + 2i\Omega Q^+ - \left(\frac{\nu}{K^+} + \frac{\sigma B_0^2}{\rho} \right) (q^+ - Q^+) + \nu \frac{\partial^2 q^+}{\partial z^2} \\ + g\beta (T^+ - T_\infty^+) + g\beta^* (C^+ - C_\infty^+), \quad \dots (2.2.9) \end{aligned}$$

and equations (2.2.4) and (2.2.5), using equation (2.2.8) can be written in the form as

$$\frac{\partial T^+}{\partial t} - \omega_0 \frac{\partial T^+}{\partial z} = \frac{\kappa^{*+}}{\rho C_p} \frac{\partial^2 T^+}{\partial z^2}, \quad \dots (2.2.10)$$

$$\frac{\partial C^+}{\partial t} - \omega_0 \frac{\partial C^+}{\partial z} = D \frac{\partial^2 C^+}{\partial z^2}. \quad \dots (2.2.11)$$

We introduce the following non-dimensional quantities:

$$\eta = \frac{\omega_0}{\nu} z, \quad \tau = \frac{\omega_0^2 t}{\nu}, \quad q = \frac{q^+}{U_0}, \quad K = \frac{\omega_0^2}{\nu^2} K^+, \quad Gr = \frac{\nu g\beta (T_w^+ - T_\infty^+)}{U_0 \omega_0^2},$$

$$Gm = \frac{\nu g\beta^* (C_w^+ - C_\infty^+)}{U_0 \omega_0^2}, \quad E = \frac{\Omega \nu}{\omega_0^2}, \quad C = \frac{C^+ - C_\infty^+}{C_w^+ - C_\infty^+}, \quad Pr = \frac{\mu C_p}{\kappa^{*+}},$$

$$Sc = \frac{\nu}{D}, \quad T = \frac{T^+ - T_\infty^+}{\frac{\nu s}{\kappa^{*+} \omega_0}}, \quad M = \frac{\sigma \nu B_0^2}{\rho \omega_0^2}, \quad Q = \frac{Q^+}{U_0}, \quad \alpha = \frac{\phi \nu}{\omega_0^2}.$$

Using the above stated non-dimensional quantities, the equations (2.2.9), (2.2.10) and (2.2.11) reduce to

$$\frac{\partial q}{\partial \tau} - \frac{\partial q}{\partial \eta} + 2iEq = \frac{\partial Q}{\partial \tau} + 2iEQ$$

$$-\left(M + \frac{1}{K}\right)(q - Q) + \frac{\partial^2 q}{\partial \eta^2} + GmC + GrT, \quad \dots (2.2.12)$$

$$\frac{\partial T}{\partial \tau} - \frac{\partial T}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 T}{\partial \eta^2}, \quad \dots (2.2.13)$$

$$\frac{\partial C}{\partial \tau} - \frac{\partial C}{\partial \eta} = \frac{1}{Sc} \frac{\partial^2 C}{\partial \eta^2}, \quad \dots (2.2.14)$$

with boundary conditions

$$\left. \begin{aligned} q = 0, \quad \frac{\partial T}{\partial \eta} = -1, \quad C = 1 \quad \text{at } \eta = 0 \\ q = 1 + \frac{\varepsilon}{2} \left(e^{i\alpha\tau} + e^{-i\alpha\tau} \right), \quad T = 0, \quad C = 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \right\} \dots (2.2.15)$$

2.2.3 SOLUTION

Let, the solutions of equations (2.2.12), (2.2.13) and (2.2.14) are assumed, respectively, as

$$q(\eta, \tau) = q_0(\eta) + \frac{\varepsilon}{2} \left\{ q_1(\eta) e^{i\alpha\tau} + q_2(\eta) e^{-i\alpha\tau} \right\}, \quad \dots (2.2.16)$$

$$T(\eta, \tau) = T_0(\eta) + \varepsilon T_1(\eta) e^{i\alpha\tau} + \dots, \quad \dots (2.2.17)$$

$$C(\eta, \tau) = C_0(\eta) + \varepsilon C_1(\eta) e^{i\alpha\tau} + \dots \quad \dots (2.2.18)$$

Using equations (2.2.16), (2.2.17) and (2.2.18) in equations (2.2.12), (2.2.13) and (2.2.14), we obtain following equations

$$q_1(\eta) = 1 - e^{-R_4\eta} \quad \dots (2.2.27b)$$

$$\text{and } q_2(\eta) = 1 - e^{-R_3\eta}, \quad \dots(2.2.27c)$$

$$T(\eta) = \frac{1}{Pr} e^{-Pr\eta}, \quad \dots (2.2.28)$$

$$C(\eta) = e^{-Sc\eta}. \quad \dots (2.2.29)$$

The expressions for constant are given in appendix-I.

2.2.4 RESULTS AND DISCUSSION

Equation (2.2.27) corresponds to the velocity distribution of free convective and mass transfer flow of viscous fluid through a rotating porous medium. The expression clearly shows the existence of thin multiple Ekman boundary layer of order $O(R_3^{-1})$ super imposed with a boundary layer of thickness of order $O(Pr^{-1})$ and $O(Sc^{-1})$. It is interesting to note that Ekman boundary layer is modified by the presence of free convection and mass transfer. We also note that this layer decrease with increase of rotation parameter and magnetic parameter and increase with increase of permeability parameter.

The solution (2.2.27a) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and phase difference due to these primary and secondary velocities for the steady flow are given by

$$|A_0| = \left(u_0^2 + v_0^2 \right) \quad \text{and} \quad \theta_0 = \tan^{-1} \left(\frac{v_0}{u_0} \right),$$

where

$$u_0 = 1 - \left\{ (1 - P_1 - P_2) \cos Q_3\eta - (Q_2 + Q_1) \sin Q_3\eta \right\} e^{-P_3\eta} - P_1 e^{-Pr\eta} - P_2 e^{-Sc\eta},$$

$$v_0 = \{(Q_2 + Q_1) \cos Q_3 \eta + (1 - P_1 - P_2) \sin Q_3 \eta\} e^{-P_3 \eta} - Q_1 e^{-Pr \eta} - Q_2 e^{-Sc \eta}.$$

The amplitude of resultant velocity $|A_0|$ and the phase angle θ_0 for the steady part are shown graphically in Fig. 2.2.1(a,b) and Fig. 2.2.2(a,b) for various values of the rotation parameter (E) and permeability parameter (K) for fixed values of Prandtl number (Pr), Schmidt number (Sc), magnetic parameter (M), Grashof number (Gr) and modified Grashof number (Gm). It is seen from Fig. 2.2.1(a) that in case of $Gr > 0$ the amplitude $|A_0|$ increase as K increases and nearly at $\eta = 2.5$ these two values coincide but opposite behavior is seen for $Gr < 0$ and decreases with increase in rotation. Fig. 2.2.1(b) θ_0 decreases with increase in K and increases with increasing R (both small and large) for $Gr > 0$ and increases as rotation parameter increases for $Gr < 0$ near the plate wall.

Variation of $|A_0|$ and θ_0 for different values of Prandtl number Pr and modified Grashof number for $Gr > 0$ are shown in Fig. 2.2.3(a,b) and Fig. 2.2.4(a,b). It is clear from these figure that amplitude decreases as Pr increases and increases as Gm increases but phase difference θ_0 decreases as Gm increases and increases as Pr increases. Numerical calculation are also made for $Gr < 0$ and shown in Fig. 2.2.5(a,b) and Fig. 2.2.6(a,b). It is essential to mention that equation (2.2.27b) and (2.2.27c) together give the unsteady part of the flow. This expression also exhibits boundary layer of thickness of order $O(R_4^{-1/2})$ and order $O(R_5^{-1/2})$ respectively.

The amplitude and the phase differences of shear stresses at the plate $\eta=0$ for the steady flow can be obtained as:

$$\tau_{0r} = \left(\tau_{0x}^2 + \tau_{0y}^2 \right)^{1/2}, \quad \theta_{0r} = \tan^{-1} \left(\frac{\tau_{0y}}{\tau_{0x}} \right). \quad \dots (2.2.30)$$

where τ_{0x} and τ_{0y} are respectively the shear stress at the plate due to primary and secondary velocity components.

The numerical values for the resultant shear stress and the phase angle due to the shear stress are listed in Table-2.2.1.

Table-2.2.1

Sl. No.	Pr	Sc	K	E	M	Gm	Gr	τ_{0r}	θ_{0r}
1	0.71	0.3	1	1	0.5	5	10	15.7646	-0.5104
2	7	0.3	1	1	0.5	5	10	5.7257	-0.1937
3	0.71	0.66	1	1	0.5	5	10	15.1004	-0.4928
4	0.71	0.3	5	1	0.5	5	10	19.5276	-0.7104
5	0.71	0.3	1	5	0.5	5	10	9.3067	-0.2333
6	0.71	0.3	1	1	1	5	10	14.2511	-0.4069
7	0.71	0.3	1	1	0.5	10	10	20.2386	-0.5540
8	0.71	0.3	1	1	0.5	5	20	26.3319	-0.5768

These values clearly show that the shear stress τ_{0r} increases as permeability parameter K increases and decreases as rotation parameter R increases. Also the increase in permeability parameter K lead to decrease in phase difference θ_{0r} and the phase difference θ_{0r} increases as rotation parameter R increases.

Appendix-I

$$L = M + \frac{1}{K};$$

$$R_1 = P_1 + iQ_1 = \frac{Gr}{Pr \left\{ Pr^2 - Pr - (L + 2iE) \right\}};$$

$$R_2 = P_2 + iQ_2 = \frac{Gm}{Sc^2 - Sc - (L + 2iE)};$$

$$R_3 = P_3 + iQ_3 = \frac{1 + \sqrt{1 + 4(L + 2iE)}}{2};$$

$$R_4 = P_4 + iQ_4 = \frac{1 + \sqrt{1 + 4\{L + i(\alpha + 2E)\}}}{2}; \quad R_5 = P_5 + iQ_5 = \frac{1 + \sqrt{1 + 4\{L + i(2E - \alpha)\}}}{2};$$

$$P_1 = \frac{Gr(\text{Pr}^2 - \text{Pr} - L)}{\text{Pr} \left\{ (\text{Pr}^2 - \text{Pr} - L)^2 + 4E^2 \right\}};$$

$$Q_1 = \frac{2GrE}{\text{Pr} \left\{ (\text{Pr}^2 - \text{Pr} - L)^2 + 4E^2 \right\}};$$

$$P_2 = \frac{Gm(\text{Sc}^2 - \text{Sc} - L)}{\left\{ (\text{Sc}^2 - \text{Sc} - L)^2 + 4E^2 \right\}};$$

$$Q_2 = \frac{2GmE}{\left\{ (\text{Sc}^2 - \text{Sc} - L)^2 + 4E^2 \right\}};$$

$$P_3 = \frac{\sqrt{2} + \left[(1+4L) + \sqrt{(1+4L)^2 + 64E^2} \right]^{\frac{1}{2}}}{2\sqrt{2}}; \quad Q_3 = \frac{2\sqrt{2}E}{\left[(1+4L) + \sqrt{(1+4L)^2 + 64E^2} \right]^{\frac{1}{2}}};$$

$$\tau_{0x} = \left\{ (1 - P_1 - P_2)Q_3 \sin Q_3\eta + (Q_2 + Q_1)Q_3 \cos Q_3\eta \right\} e^{-P_3\eta}$$

$$+ \left\{ (1 - P_1 - P_2) \cos Q_3\eta - (Q_2 + Q_1) \sin Q_3\eta \right\} P_3 e^{-P_3\eta} + P_1 \text{Pr} e^{-\text{Pr}\eta} + P_2 \text{Sc} e^{-\text{Sc}\eta};$$

$$\tau_{0y} = \left\{ (1 - P_1 - P_2)Q_3 \cos Q_3\eta - (Q_2 + Q_1)Q_3 \sin Q_3\eta \right\} e^{-P_3\eta}$$

$$- \left\{ (1 - P_1 - P_2) \sin Q_3\eta + (Q_2 + Q_1) \cos Q_3\eta \right\} P_3 e^{-P_3\eta} + Q_1 \text{Pr} e^{-\text{Pr}\eta} + Q_2 \text{Sc} e^{-\text{Sc}\eta}.$$

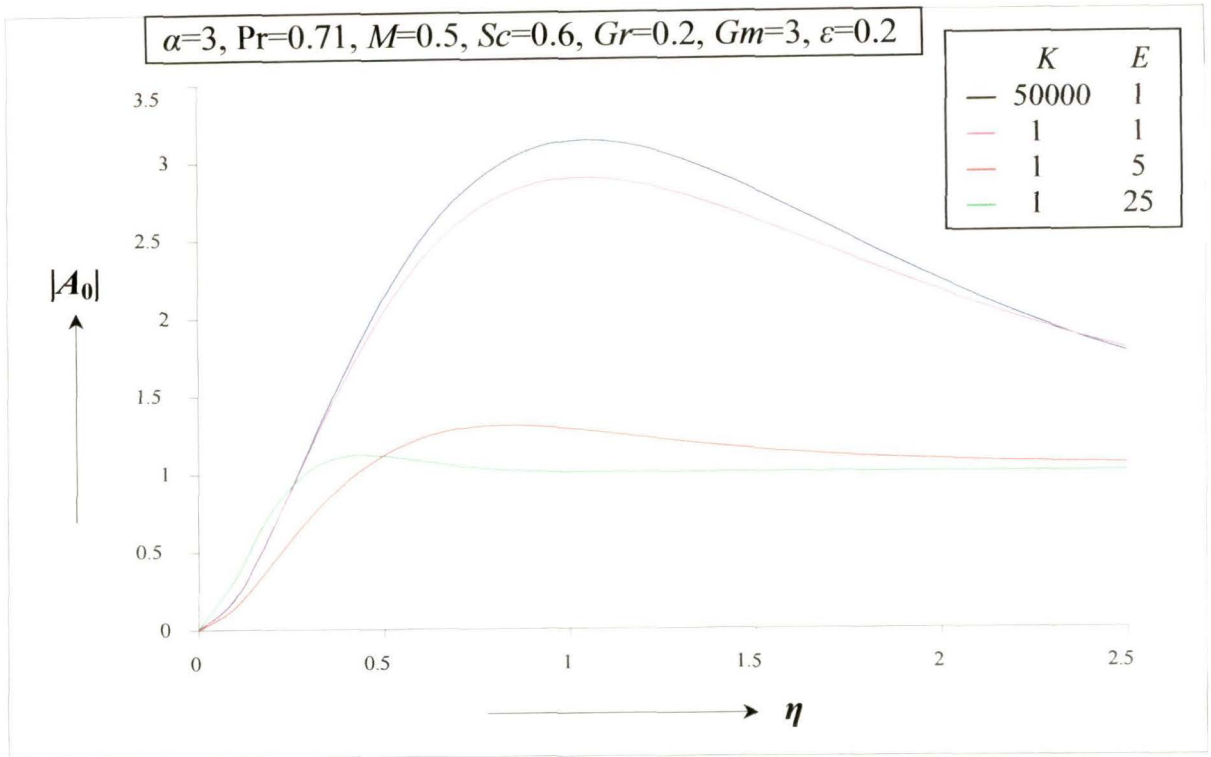


Fig. 2.2.1(a) Effects of K and E on the resultant velocity field for $Gr > 0$.

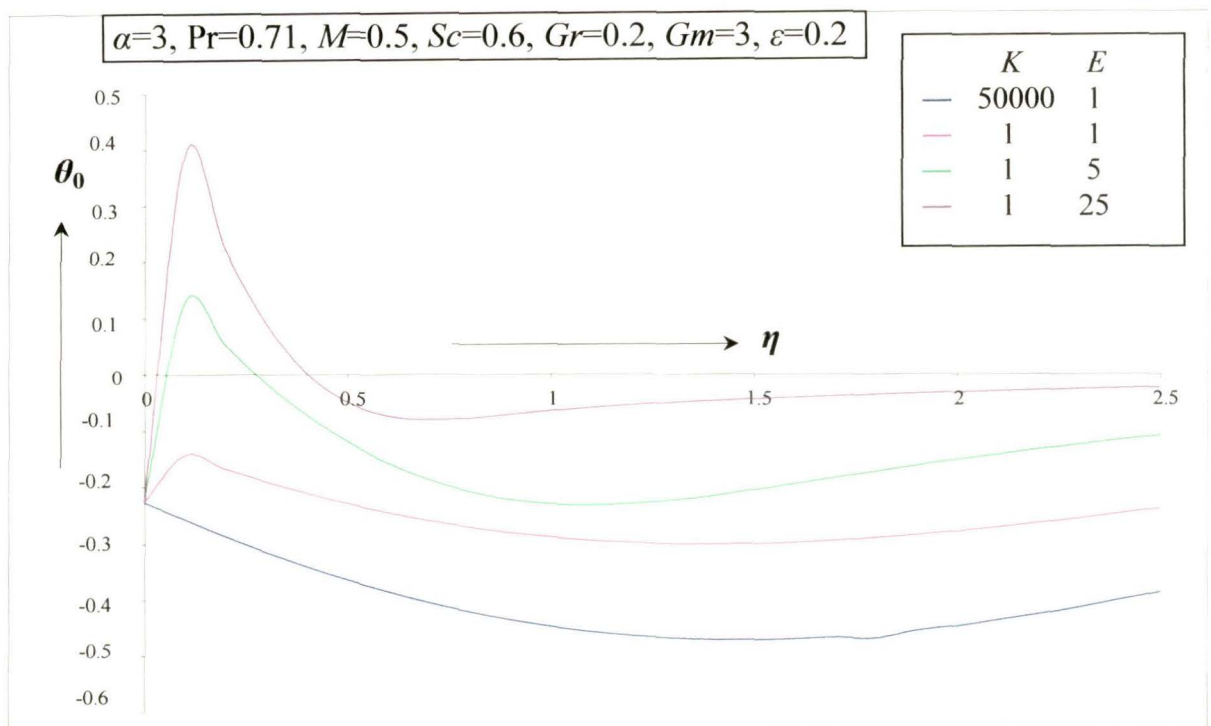


Fig. 2.2.1(b) Effects of K and E on the amplitude of the resultant velocity field for $Gr > 0$.

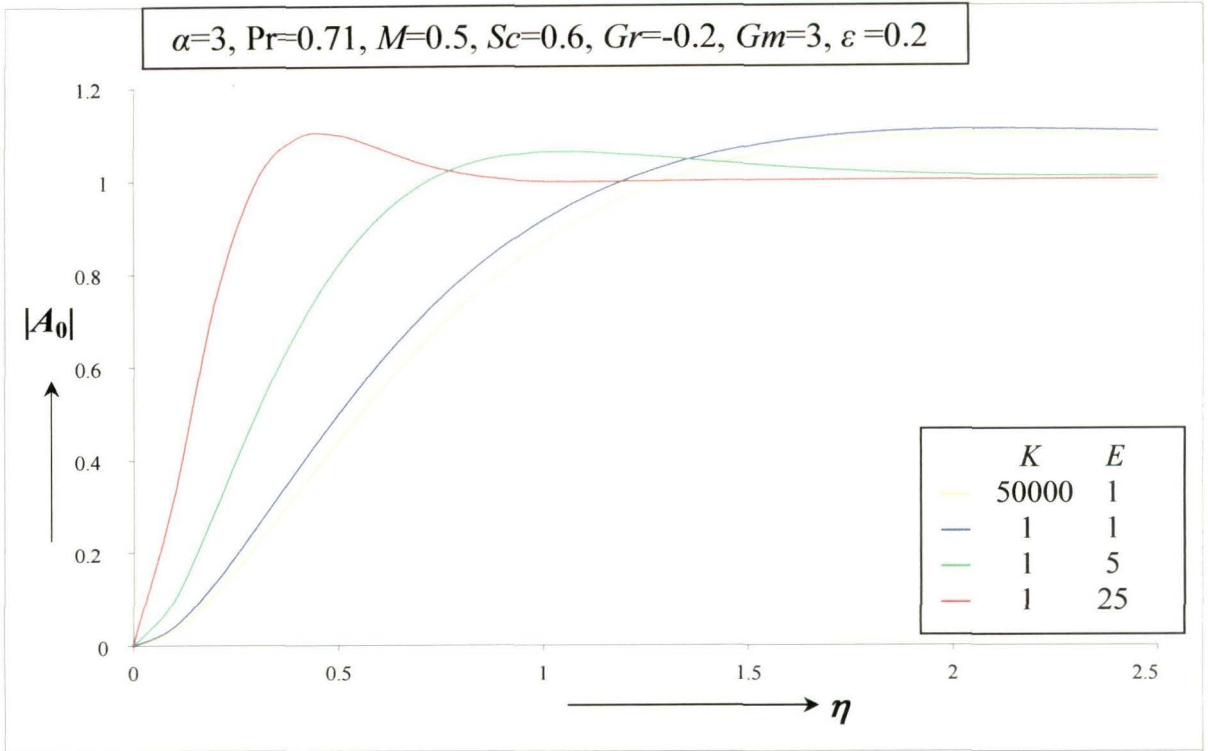


Fig. 2.2.2(a) Effects of K and E on the resultant velocity field for $Gr < 0$.

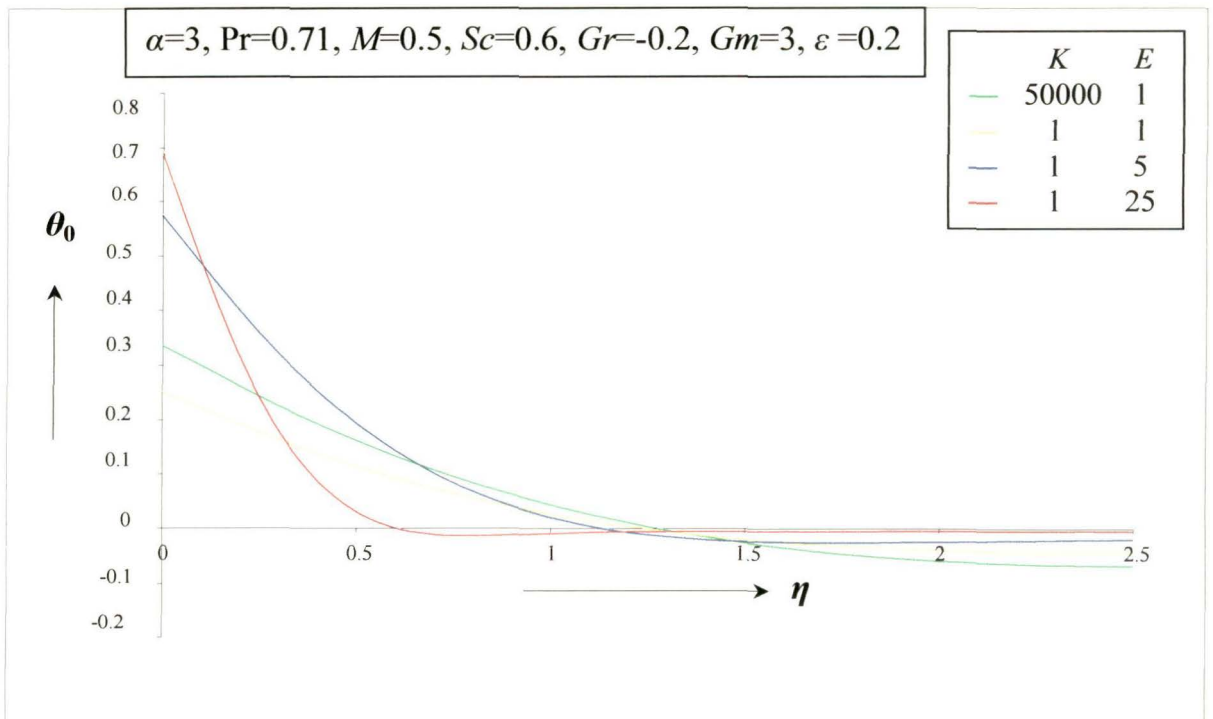


Fig. 2.2.2(b) Effects of K and E on the amplitude of the resultant velocity field for $Gr < 0$.

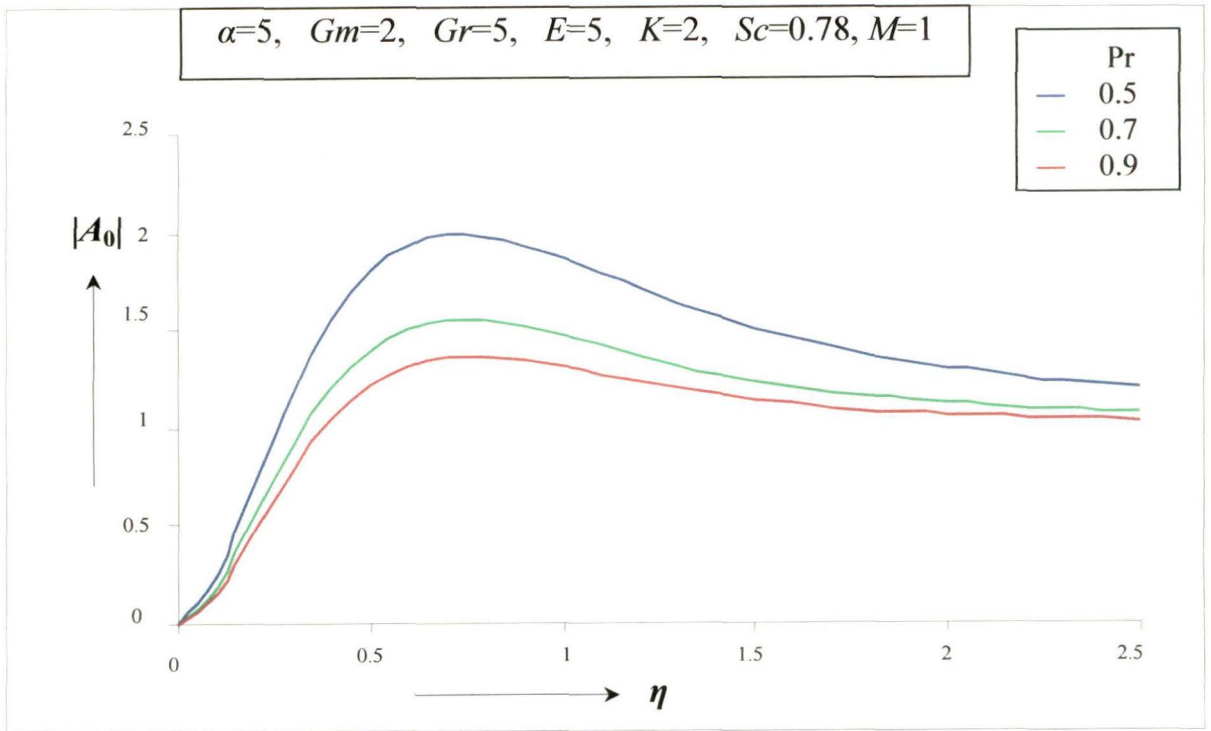


Fig. 2.2.3(a) Effects of Pr on the resultant velocity field for $Gr > 0$.

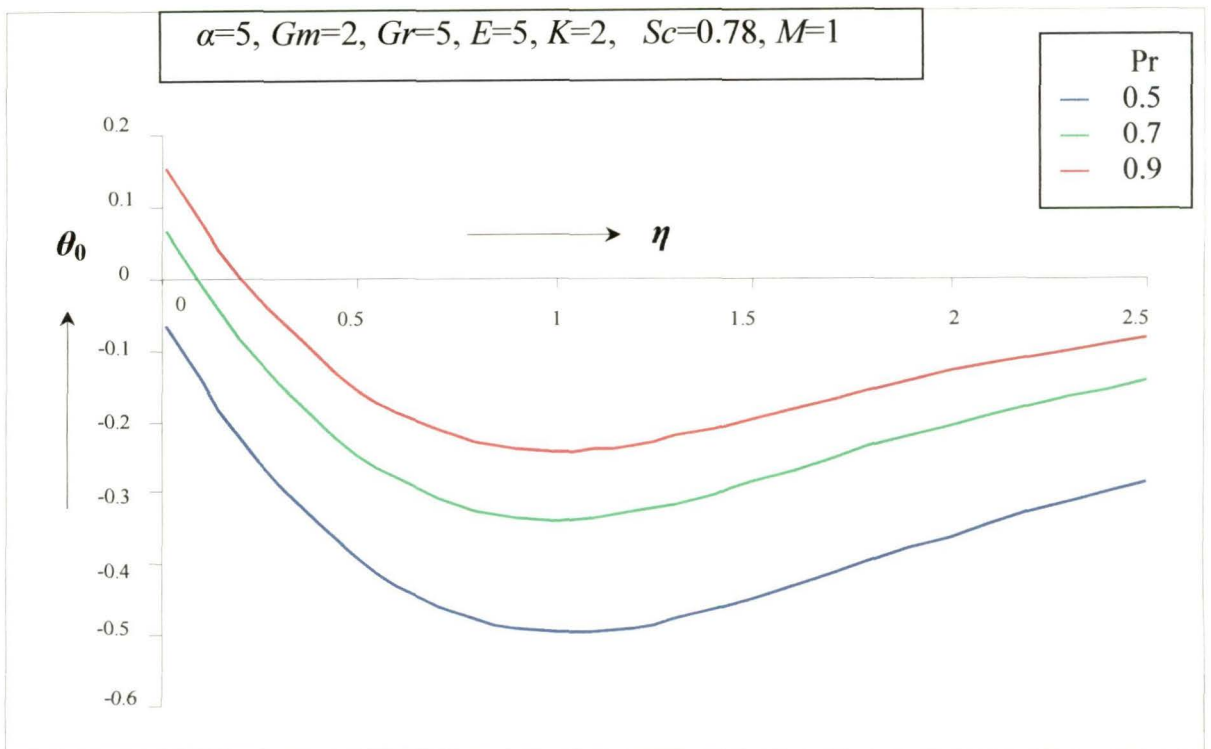


Fig. 2.2.3(b) Effects of Pr on the amplitude of the resultant velocity field for $Gr > 0$.

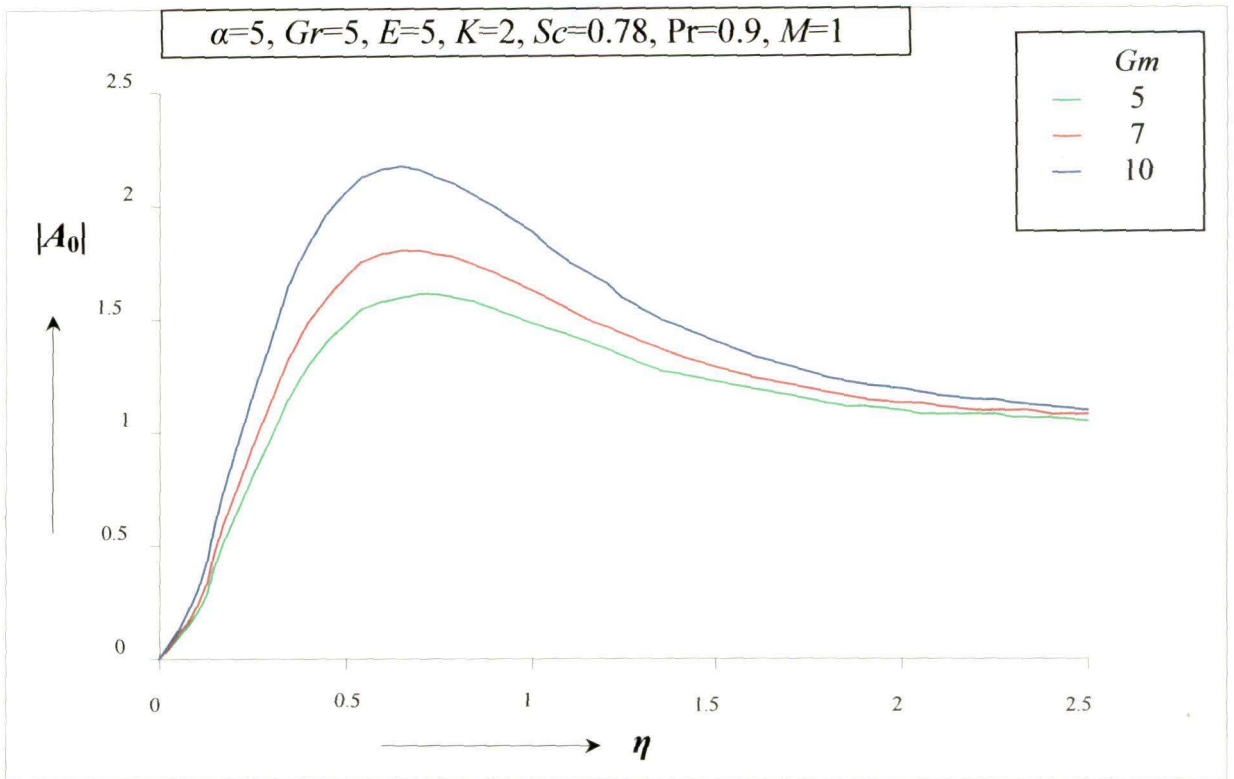


Fig. 2.2.4(a) Effects of Gm on the resultant velocity field for $Gr > 0$.

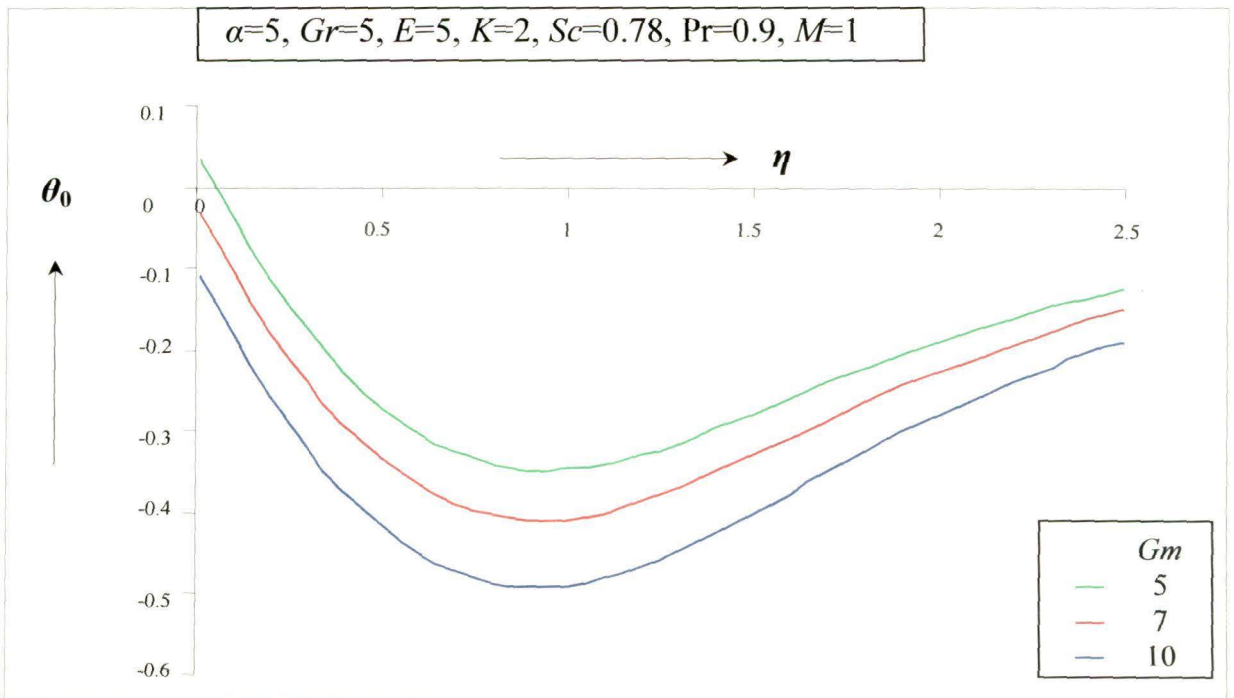


Fig. 2.2.4(b) Effects of Gm on the amplitude of the resultant velocity field for $Gr > 0$.

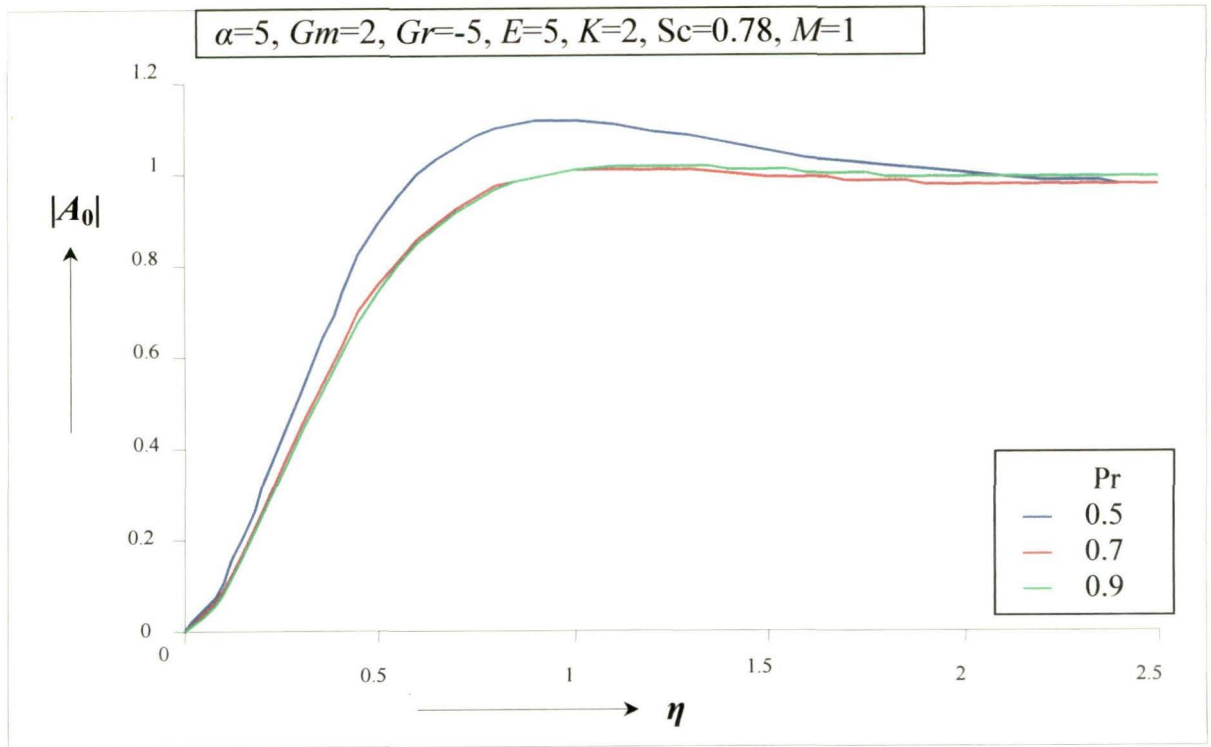


Fig. 2.2.5(a) Effects of Pr on the resultant velocity field for $Gr < 0$.

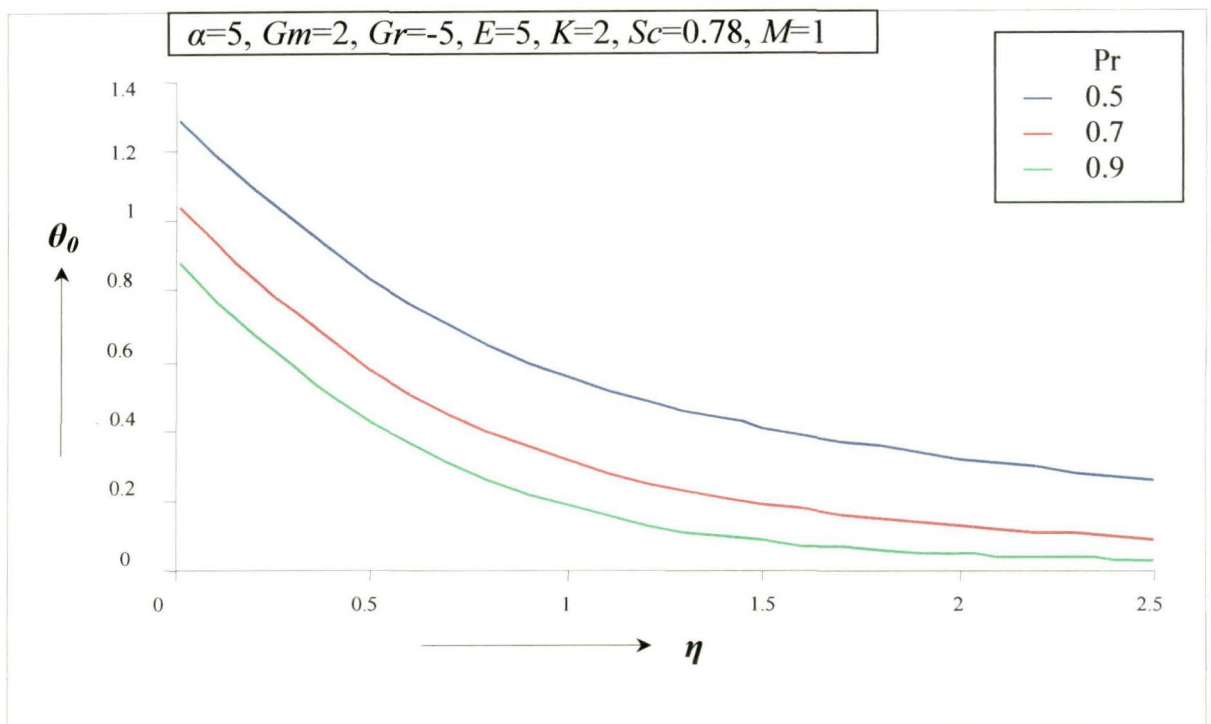


Fig. 2.2.5(b) Effects of Pr on the amplitude of the resultant velocity field for $Gr < 0$.

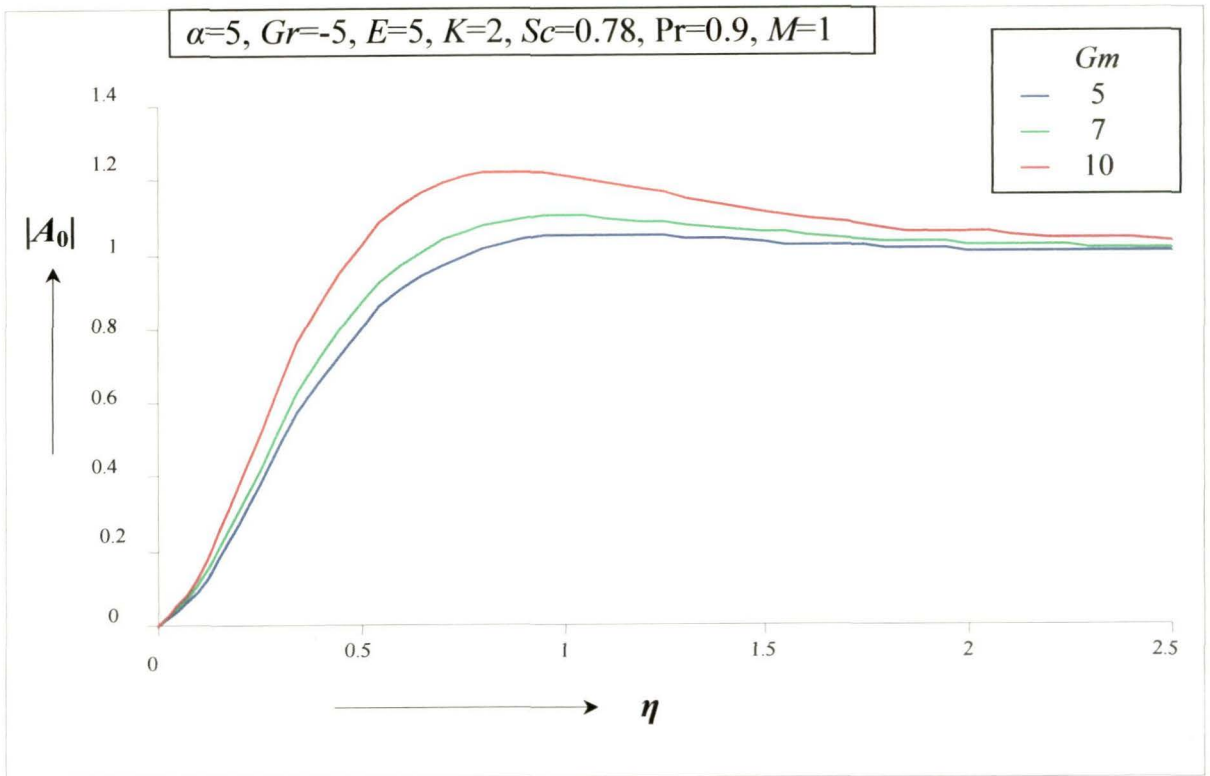


Fig. 2.2.6(a) Effects of Gm on the resultant velocity field for $Gr < 0$.

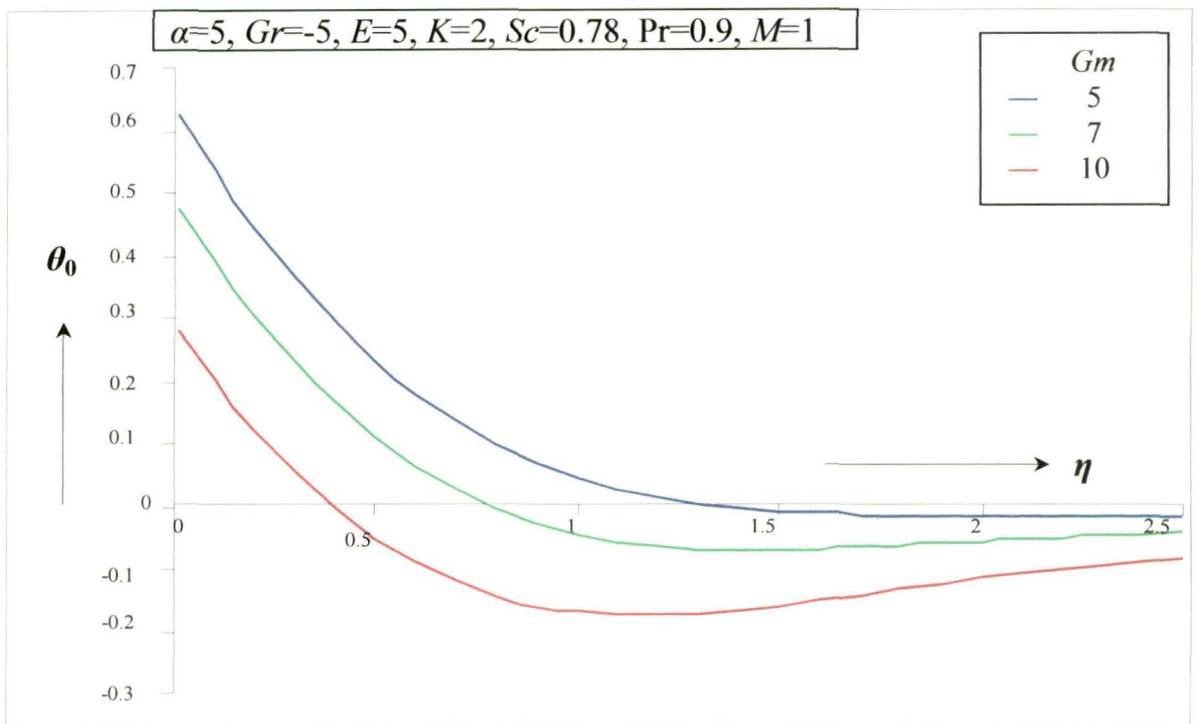


Fig. 2.2.6(b) Effects of Gm on the amplitude of the resultant velocity field for $Gr < 0$.

PART TWO > B**EFFECTS OF MASS TRANSFER AND ROTATION ON FLOW PAST A POROUS PLATE IN A POROUS MEDIUM WITH VARIABLE SUCTION IN SLIP FLOW REGIME****2.2.6 INTRODUCTION**

Free convection and mass transfer flow in porous medium has received considerable attention due to its numerous applications in geophysics and energy related problems. Such types of application include natural circulation in isothermal reservoirs, aquifers porous insulation in heat storage bed, grain storage, extraction of thermal energy and thermal insulation design. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical, geothermal. Many aspects of the motion in a rotating frame of references of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Raptis *et al.* [4, 8] studied both steady and unsteady free convective flow and mass transfer of a viscous fluid through a porous medium bounded by a vertical infinite porous surface with constant suction using generalized Darcy's law. Mahato and Maiti [9] analyzed the effect of unsteady free convective flow and mass transfer during the motions of a viscous incompressible fluid in a rotating frame of references. Alam *et al.* [10] studied unsteady free convection and mass transfer flow in a rotating system with hall currents, viscous dissipation and joule heating. Later Singh *et al.* [11] studied free convection in MHD flow of a rotating viscous liquid in porous medium. Recently Singh *et al.* [12] have studied free convective MHD flow of rotating viscous fluid in a porous medium past infinite vertical porous plate. We now propose to study of unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through porous medium bounded by an infinite vertical porous plate in presence of heat source with variable suction under the influence of

uniform magnetic field applied perpendicular to the flow of region in a rotating system is presented. The porous plate and the porous medium are assumed to rotate in a solid body rotation. The study of velocity, temperature, concentration, skin friction, rate of heat transfer and rate of mass transfer are presented graphically and necessary conclusions are set out.

2.2.7 FORMULATION OF THE PROBLEM

We consider an unsteady heat and mass transfer flow of an incompressible, electrically conducting, viscous liquid flowing through a homogeneous porous medium with constant permeability K , past an infinite vertical porous plate in the presence of a constant heat source $Q=Q_0(T-T_w)$. The plate is subjected to a variable suction $\omega=-\omega_0(1+\varepsilon Ae^{i\phi t})$ at the plate, (where ω_0 is a positive real number, A -suction parameter) under the influence of the uniform magnetic field B_0 . We assume a Cartesian co-ordinate system choosing x -axis and y -axis in the plane of the porous plate and z -axis normal to the plate with velocity components u , v , w in these directions respectively. Both the liquid and the plate are in a state of rigid body rotation (with uniform angular velocity Ω) about z -axis. In this analysis buoyancy force, Hall effect, effect due to perturbation of the field, induced magnetic field and polarization effects are ignored. Following, Gebhart and Pera [13], Soret effect is taken into account. Under the above stated restrictions, the equation of motion, energy and concentration are as follows

$$\frac{\partial u}{\partial t} - \omega_0 \left(1 + \varepsilon A e^{i\phi t} \right) \frac{\partial u}{\partial z} - 2\Omega v = g \frac{\partial^2 u}{\partial z^2} + g\beta(T - T_\infty) + g\beta_0(C - C_\infty) - \left(\frac{g}{K} + \frac{\sigma B_0^2}{\rho} \right) u, \quad \dots (2.2.31)$$

$$\frac{\partial v}{\partial t} - \omega_0 \left(1 + \varepsilon A e^{i\phi t} \right) \frac{\partial v}{\partial z} + 2\Omega u = g \frac{\partial^2 v}{\partial z^2} - \left(\frac{g}{K} + \frac{\sigma B_0^2}{\rho} \right) v, \quad \dots (2.2.32)$$

$$\frac{\partial T}{\partial t} - \omega_0 \left(1 + \varepsilon A e^{i\phi t} \right) \frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial z^2} - \frac{Q_0 (T - T_\infty)}{\rho C_p}, \quad \dots (2.2.33)$$

$$\frac{\partial C}{\partial t} - \omega_0 \left(1 + \varepsilon A e^{i\phi t} \right) \frac{\partial C}{\partial z} = D_M \frac{\partial^2 C}{\partial z^2} + D_T \frac{\partial^2 T}{\partial z^2}. \quad \dots (2.2.34)$$

where the symbols have their usual meaning.

The boundary conditions are given as

$$\left. \begin{aligned} u &= \varepsilon U_0 e^{i\phi t} + L_1 \frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } z = 0 \\ u &\rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } z \rightarrow \infty \end{aligned} \right\}, \quad \dots (2.2.35)$$

where $L_1 = \frac{(2 - m_1)L}{m_1}$ and $L = \mu \left[\frac{\pi}{(2p\rho)} \right]^{\frac{1}{2}}$ is the mean free path and m_1 is the Maxwell's reflection co-efficient.

On introducing the following non-dimensional quantities

$$z^* = \frac{\omega_0 z}{g}, \quad t^* = \frac{\omega_0^2 t}{g}, \quad u^* = \frac{u}{U_0}, \quad \phi^* = \frac{g\phi}{\omega_0^2}, \quad v^* = \frac{v}{U_0}, \quad K^* = \frac{\omega_0^2 K}{g^2},$$

$$C^* = \frac{C - C_\infty}{C_w - C_\infty}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad Sc = \frac{g}{D_M}, \quad Gm = \frac{g\beta g(C_w - C_\infty)}{U_0 \omega_0^2}, \quad Pr = \frac{\mu C_p}{\kappa}$$

$$Gr = \frac{g\beta_0 g(T_w - T_\infty)}{U_0 \omega_0^2}, \quad R = \frac{\omega_0 L_1}{g} \text{ (rarefaction parameter),}$$

$$S_0 = \frac{D_T (T_w - T_\infty)}{g (C_w - C_\infty)} \text{ (Thermal diffusion parameter),}$$

$$M = \frac{\sigma B_0^2 g}{\rho \omega_0^2} \text{ (Hartmann number),} \quad \alpha_0 = \frac{Q_0 g^2}{\kappa \omega_0^2} \text{ (Heat source parameter),}$$

$$E = \frac{\Omega g}{\omega_0^2} (\text{Rotation parameter}).$$

in equations (2.2.31)-(2.2.34) (after dropping the asterisks) can be written as

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial u}{\partial z} - 2Ev = \frac{\partial^2 u}{\partial z^2} + GrT + GmC - \left(M + \frac{1}{K}\right)u, \quad \dots (2.2.36)$$

$$\frac{\partial v}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial v}{\partial z} + 2Eu = \frac{\partial^2 v}{\partial z^2} - \left(M + \frac{1}{K}\right)v, \quad \dots (2.2.37)$$

$$\text{Pr} \frac{\partial T}{\partial t} - \text{Pr} \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} - \alpha_0 T, \quad \dots (2.2.38)$$

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial C}{\partial z} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2} + S_0 \frac{\partial^2 T}{\partial z^2}, \quad \dots (2.2.39)$$

which leads to

$$\frac{\partial q}{\partial t} - \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial q}{\partial z} + \left[\left(M + \frac{1}{K}\right) + i2E\right]q = \frac{\partial^2 q}{\partial z^2} + GrT + GmC, \quad \dots (2.2.40)$$

$$\frac{\partial^2 T}{\partial z^2} + \text{Pr} \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial T}{\partial z} - \text{Pr} \frac{\partial T}{\partial t} - \alpha_0 T = 0, \quad \dots (2.2.41)$$

$$\frac{\partial^2 C}{\partial z^2} + Sc \left(1 + \varepsilon A e^{i\phi t}\right) \frac{\partial C}{\partial z} - Sc \frac{\partial C}{\partial t} + S_0 Sc \frac{\partial^2 T}{\partial z^2} = 0, \quad \dots (2.2.42)$$

where $q = u + iv$.

The boundary conditions (2.2.35) are transformed to

$$\left. \begin{aligned} q &= \varepsilon e^{i\phi t} + R \frac{\partial u}{\partial z}, & T &= 1, & C &= 1 & \text{at } z = 0 \\ q &\rightarrow 0, & T &\rightarrow 0, & C &\rightarrow 0 & \text{as } z \rightarrow \infty \end{aligned} \right\} \dots (2.2.43)$$

2.2.8 SOLUTION OF THE PROBLEM

In order to solve the equations (2.2.40), (2.2.41) and (2.2.42), we assume the velocity, temperature and concentration in the neighborhood of the plate as follows

$$q(z, t) = q_0(z) + \varepsilon q_1(z) e^{i\phi t}, \quad \dots (2.2.44)$$

$$T(z, t) = T_0(z) + \varepsilon T_1(z) e^{i\phi t} \quad \dots (2.2.45)$$

$$\text{and } C(z, t) = C_0(z) + \varepsilon C_1(z) e^{i\phi t}. \quad \dots (2.2.46)$$

Using equations (2.2.44)-(2.2.46) in equations (2.2.40)-(2.2.42), we obtain following equations,

$$q_0''(z) + q_0'(z) - (M_1 + i2E)q_0(z) = -GrT_0(z) - GmC_0(z), \quad \dots (2.2.47)$$

$$\begin{aligned} q_1''(z) + q_1'(z) - [M_1 + i(2E + \phi)]q_1(z) &= -Aq_0'(z) \\ &\quad -GrT_1(z) - GmC_1(z), \quad \dots (2.2.48) \end{aligned}$$

$$T_0''(z) + PrT_0'(z) - \alpha_0 T_0(z) = 0, \quad \dots (2.2.49)$$

$$T_1''(z) + PrT_1'(z) - (\alpha_0 + i\phi Pr)T_1(z) = -APrT_0'(z), \quad \dots (2.2.50)$$

$$C_0''(z) + ScC_0'(z) = -S_0 ScT_0''(z), \quad \dots (2.2.51)$$

$$C_1''(z) + ScC_1'(z) - i\phi ScC_1(z) = -AScC_0'(z) - S_0 ScT_1''(z). \quad \dots (2.2.52)$$

Using (2.2.44)-(2.2.46) in (2.2.43), the boundary conditions are reduced to

$$\left. \begin{aligned}
 q_0 &= R \frac{du_0}{dz}, \quad q_1 = 1 + R \frac{du_1}{dz}, \quad T_0 = 1, \\
 T_1 &= 0, \quad C_0 = 1, \quad C_1 = 0 && \text{at } z = 0 \\
 q_0 &\rightarrow 0, \quad q_1 \rightarrow 0, \quad T_0 \rightarrow 0, \quad T_1 \rightarrow 0, \\
 C_0 &\rightarrow 0, \quad C_1 \rightarrow 0 && \text{as } z \rightarrow \infty
 \end{aligned} \right\} \dots (2.2.53)$$

The solutions of equations (2.2.47) to (2.2.52), under the boundary conditions (2.2.53) are

$$T_0(z) = e^{-H_0 z}, \quad \dots (2.2.54)$$

$$T_1(z) = R_1 \left(e^{-H_0 z} - e^{-H_1 z} \right), \quad \dots (2.2.55)$$

$$C_0(z) = (1 + R_0) e^{-Scz} - R_0 e^{-H_0 z}, \quad \dots (2.2.56)$$

$$C_1(z) = R_2 e^{-H_0 z} - R_3 e^{-Scz} + R_4 e^{-H_1 z} + R_5 e^{-H_2 z}, \quad \dots (2.2.57)$$

$$q_0(z) = -R_6 e^{-H_0 z} - R_7 e^{-Scz} + R_8 e^{-H_3 z}, \quad \dots (2.2.58)$$

$$\begin{aligned}
 q_1(z) &= R_9 e^{-H_1 z} - R_{10} e^{-H_2 z} \\
 &\quad + R_{11} e^{-H_3 z} + R_{12} e^{-H_4 z} - R_{13} e^{-H_0 z} + R_{14} e^{-Scz}.
 \end{aligned} \quad \dots (2.2.59)$$

Substituting the values of $q_0(z)$, $q_1(z)$, $T_0(z)$, $T_1(z)$, $C_0(z)$ and $C_1(z)$ in (2.2.44) to (2.2.46) we obtain $q(z, t)$, $T(z, t)$ and $C(z, t)$. At $\phi t = \frac{\pi}{2}$, we obtain,

$$u\left(z, \frac{\pi}{2\phi}\right) = u_0 - \varepsilon v_1, \quad \dots (2.2.60)$$

$$v\left(z, \frac{\pi}{2\phi}\right) = v_0 + \varepsilon u_1, \quad \dots (2.2.61)$$

where

$$u_0(z) = e^{-A_3 z} (P_8 \cos B_3 z + Q_8 \sin B_3 z) - P_6 e^{-H_0 z} - P_7 e^{-Scz},$$

$$v_0(z) = e^{-A_3 z} (Q_8 \cos B_3 z - P_8 \sin B_3 z) - Q_6 e^{-H_0 z} - Q_7 e^{-Scz},$$

$$u_1(z) = F_1(z) - F_2(z) + F_3(z) + F_4(z) - P_{13} e^{-H_0 z} + P_{14} e^{-Scz},$$

$$v_1(z) = f_1(z) - f_2(z) + f_3(z) + f_4(z) - Q_{13} e^{-H_0 z} + Q_{14} e^{-Scz},$$

$$F_i(z) = e^{-A_i z} (P_{i+8} \cos B_i z + Q_{i+8} \sin B_i z) \quad i = 1, 2, 3 \text{ and } 4,$$

$$f_j(z) = e^{-A_j z} (Q_{j+8} \cos B_j z - P_{j+8} \sin B_j z) \quad j = 1, 2, 3 \text{ and } 4.$$

The expressions for constant are given in appendix-I.

2.2.9 SKIN-FRICTION, RATE OF HEAT AND MASS TRANSFER

The non-dimensional skin-friction at the plate is

$$\tau = \left(\frac{dq_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dq_1}{dz} \right)_{z=0} e^{i\phi t} = \tau_p + i\tau_s. \quad \dots (2.2.62)$$

Hence, primary skin-friction (τ_p) due to primary velocity and secondary skin friction is due to secondary velocity are given as

$$\tau_p = \tau_{op} + \varepsilon (J_1 \cos \phi t + J_2 \sin \phi t), \quad \dots (2.2.63)$$

$$\tau_s = \tau_{os} + \varepsilon (J_2 \cos \phi t - J_1 \sin \phi t). \quad \dots (2.2.64)$$

The rate of heat transfer at the plate in terms of Nusselt number (Nu) is

$$Nu = \left(\frac{dT_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dT_1}{dz} \right)_{z=0} e^{i\phi t}. \quad \dots (2.2.65)$$

Hence, the rate of heat transfer (considering real part only) is

$$Nu = -H_0 + \varepsilon (G_1 \cos \phi t - G_2 \sin \phi t). \quad \dots (2.2.66)$$

which in terms of amplitude and phase, obtained as

$$Nu = -H_0 + \varepsilon |G| \cos(\phi t + \alpha).$$

The rate of mass transfer at the plate in terms of Sherwood number (S_h) is

$$S_h = \left(\frac{dC_0}{dz} \right)_{z=0} + \varepsilon \left(\frac{dC_1}{dz} \right)_{z=0} e^{i\phi t}. \quad \dots (2.2.67)$$

Hence, the rate of mass transfer is

$$S_h = H_0 R_0 - Sc(1 + R_0) + \varepsilon (N_{1r} \cos \phi t - N_{1i} \sin \phi t), \quad \dots (2.2.68)$$

which in terms of amplitude and phase, obtained as

$$S_h = H_0 R_0 - Sc(1 + R_0) + \varepsilon |N_1| \cos(\phi t + \beta).$$

The expressions for constant are given in appendix-II.

2.2.10 DISCUSSION AND CONCLUSION

In order to have physical insight into the problem we have calculated the numerical values of the primary and secondary velocity distribution, temperature distribution, concentration distribution, skin-friction, rate of heat transfer and rate of mass transfer. The effect of magnetic parameter (M), Grashof number (Gr), modified Grashof number (Gm), rotation parameter (E), rarefaction parameter (R), Prandtl number (Pr), Schmidt number (Sc), Thermal diffusion parameter (S_0), Heat source parameter (α_0), permeability parameter (K) and Suction parameter (A) on primary and secondary velocity distribution are shown graphically. In Fig. 2.2.7 and Fig. 2.2.8, the primary and secondary velocity distribution is plotted against z for $Pr = 0.71$, $Sc = 0.3$, $S_0 = 1$, $K=10$, $\alpha_0 = 0.5$, $A = 0.2$. As expected it reveals velocity jump near the plate and then decreases slowly and primary velocity decreases as M and E increases and increases as Gr , Gm and R increases but secondary velocity decreases as z increases and after attaining minimum value

near the plate it increases as z increases. Secondary velocity v increases as M , E increases but near the plate behavior is opposite and v decreases as Gr , Gm and R increases. Fig. 2.2.9 and Fig. 2.2.10 show effect of Pr , Sc , S_0 , K and α_0 on primary and secondary velocity for fixed values of M , Gr , Gm , E and R . Primary velocity decreases as Pr , Sc , K and α_0 increases and decreases as S_0 and A increases. These effects are observed in opposite sense for secondary velocity. It is striking to note that in presence of He primary velocity is more than the presence of CO_2 . The effects of all parameter on skin-friction are presented in tables 2.2.2 and 2.2.4. The effects of Sc , A , Pr , ϕ , t , α_0 and S_0 on rate of mass transfer is shown in table-2.2.3. The effects of A , Pr , ϕ , t and α_0 on rate of heat transfer is presented in table-2.2.5.

Table-2.2.2

Skin-friction due to primary velocity (at $\phi = 7$, $t = 2$ and $\epsilon = 0.01$)

Sl. No.	τ_p	Pr	Sc	M	K	α_0	Gr	Gm	E	S_0	A	R
1	5.41746	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
2	3.62861	7	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
3	5.30086	0.71	0.66	0.5	10	0.5	10	5	1	1	0.2	0.2
4	4.89957	0.71	0.3	1	10	0.5	10	5	1	1	0.2	0.2
5	5.50351	0.71	0.3	0.5	50	0.5	10	5	1	1	0.2	0.2
6	5.46284	0.71	0.3	0.5	10	1	10	5	1	1	0.2	0.2
7	8.60009	0.71	0.3	0.5	10	0.5	20	5	1	1	0.2	0.2
8	7.65324	0.71	0.3	0.5	10	0.5	10	10	1	1	0.2	0.2
9	3.93275	0.71	0.3	0.5	10	0.5	10	5	2	1	0.2	0.2
10	5.60372	0.71	0.3	0.5	10	0.5	10	5	1	2	0.2	0.2
11	5.41159	0.71	0.3	0.5	10	0.5	10	5	1	1	0.4	0.2
12	3.88020	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.6

Table-2.2.3

Rate of mass transfer in terms of Sherwood number (at $\epsilon = 0.01$)

Sl. No.	S_h	Sc	A	Pr	ϕ	t	α_0	S_0
1	0.043933	0.3	0.2	0.71	7	2	0.5	1
2	0.096618	0.7	0.2	0.71	7	2	0.5	1
3	0.043940	0.3	0.2	0.71	10	2	0.5	1
4	0.043784	0.3	0.2	0.71	7	5	0.5	1
5	0.124926	0.3	0.2	0.71	7	2	1	1
6	-0.084136	0.3	0.2	0.025	7	2	0.5	1
7	0.387953	0.3	0.2	0.71	7	2	0.5	2
8	0.388177	0.3	0.4	0.71	7	2	0.5	2

Table-2.2.4Skin-friction due to secondary velocity (at $\phi = 7$, $t = 2$ and $e = 0.01$)

Sl. No.	τ_s	Pr	Sc	M	K	ϕ	Gr	Gm	E	S_0	A	R
1	-2.36531	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
2	-2.40522	7	0.3	0.5	10	0.5	10	5	1	1	0.2	0.2
3	-2.07032	0.71	0.66	0.5	10	0.5	10	5	1	1	0.2	0.2
4	-1.82312	0.71	0.3	1	10	0.5	10	5	1	1	0.2	0.2
5	-2.48214	0.71	0.3	0.5	50	0.5	10	5	1	1	0.2	0.2
6	-1.91970	0.71	0.3	0.5	10	1	10	5	1	1	0.2	0.2
7	-2.45194	0.71	0.3	0.5	10	0.5	20	5	1	1	0.2	0.2
8	-4.63711	0.71	0.3	0.5	10	0.5	10	10	1	1	0.2	0.2
9	-3.29854	0.71	0.3	0.5	10	0.5	10	5	2	1	0.2	0.2
10	-3.00924	0.71	0.3	0.5	10	0.5	10	5	1	2	0.2	0.2
11	-2.38147	0.71	0.3	0.5	10	0.5	10	5	1	1	0.4	0.2
12	-3.37970	0.71	0.3	0.5	10	0.5	10	5	1	1	0.2	0.6

Table-2.2.5Rate of heat transfer in terms of Nusselt number (at $e = 0.01$)

Sl.No.	Nu	A	Pr	ϕ	t	α_0
1	-1.14666	0.2	0.71	7	2	0.5
2	-0.71972	0.2	0.025	7	2	0.5
3	-7.07928	0.4	7	7	2	0.5
4	-11.45400	0.4	11.4	7	2	0.5
5	-1.14666	0.2	0.71	10	2	0.5
6	-1.41663	0.2	0.71	7	2	1
7	-1.41712	0.4	0.71	7	2	1
8	-1.41505	0.4	0.71	7	5	1

Appendix-I

$$H_0 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 + 4\alpha_0}}{2}; \quad M_1 = M + \frac{1}{K};$$

$$H_1 = A_1 + iB_1 = \frac{1}{2} \left[\text{Pr} + \sqrt{\text{Pr}^2 + 4(\alpha_0 + i\phi \text{Pr})} \right];$$

$$H_2 = A_2 + iB_2 = \frac{1}{2} \left[\text{Sc} + \sqrt{\text{Sc}(\text{Sc} + i4\phi)} \right];$$

$$H_3 = A_3 + iB_3 = \frac{1}{2} \left[1 + \sqrt{1 + 4M_1 + i8E} \right];$$

$$H_4 = A_4 + iB_4 = \frac{1}{2} \left[1 + \sqrt{1 + 4M_1 + i4(2E + \phi)} \right];$$

$$R_0 = \frac{\text{Sc}S_0H_0}{(H_0 - \text{Sc})}; \quad R_1 = P_1 + iQ_1 = \frac{A\text{Pr}H_0}{H_0^2 + \text{Pr}H_0 - (\alpha_0 + i\phi \text{Pr})};$$

$$R_2 = P_2 + iQ_2 = -\frac{H_0\text{Sc}(AR_0 + S_0R_1H_0)}{H_0^2 - \text{Sc}(H_0 + i\phi)}; \quad R_3 = iQ_3 = -i\frac{A\text{Sc}(1 + R_0)}{\phi};$$

$$R_4 = P_4 + iQ_4 = \frac{\text{Sc}S_0H_1^2R_1}{H_1^2 - \text{Sc}(H_1 + i\phi)}; \quad R_5 = P_5 + iQ_5 = R_3 - R_2 - R_4$$

$$R_6 = P_6 + iQ_6 = \frac{Gr - GmR_0}{H_0^2 - H_0 - M_1 - i2E}; \quad R_7 = P_7 + iQ_7 = \frac{Gm(1 + R_0)}{\text{Sc}^2 - \text{Sc} - M_1 - i2E};$$

$$R_8 = P_8 + iQ_8 = \frac{R_6 + R_7 + R \left\{ (Q_6 + Q_7)(B_3 + iA_3) + (H_0P_6 + \text{Sc}P_7) \right\}}{(1 + A_3R)};$$

$$R_9 = P_9 + iQ_9 = \frac{GrR_1 - GmR_4}{H_1^2 - H_1 - M_1 - i(2E + \phi)};$$

$$R_{10} = P_{10} + iQ_{10} = \frac{GmR_5}{H_2^2 - H_2 - M_1 - i(2E + \phi)};$$

$$R_{11} = P_{11} + iQ_{11} = \frac{AR_8H_3}{H_3^2 - H_3 - M_1 - i2E};$$

$$R_{12} = P_{12} + iQ_{12} = \frac{1 + R_{10} + R_{13} - R_{11} - R_{14} - R_9}{(1 + A_4R)}$$

$$+ \frac{R\{Q_{12}(B_4 + iA_4) - A_3P_{11} + B_3Q_{11} + H_0P_{13} - ScP_{14} + A_1P_9 - B_1Q_9 + A_2P_{10} - B_2Q_{10}\}}{1 + A_4R};$$

$$R_{13} = P_{13} + iQ_{13} = \frac{AH_0R_6 - GrR_1 - GmR_2}{H_0^2 - H_0 - M_1 - i2E};$$

$$R_{14} = P_{14} + iQ_{14} = \frac{GmR_3 - AScR_7}{Sc^2 - Sc - M_1 - i(2E + \phi)};$$

$$A_1 = \frac{Pr}{2} + \frac{1}{2\sqrt{2}} \left[\sqrt{\left(\text{Pr}^2 + 4\alpha_0\right)^2 + 16\phi^2 \text{Pr}^2 + \left(\text{Pr}^2 + 4\alpha_0\right)} \right]^{\frac{1}{2}};$$

$$B_1 = \frac{1}{2\sqrt{2}} \left[\sqrt{\left(\text{Pr}^2 + 4\alpha_0\right)^2 + 16\phi^2 \text{Pr}^2 - \left(\text{Pr}^2 + 4\alpha_0\right)} \right]^{\frac{1}{2}};$$

$$A_2 = \frac{Sc}{2} + \frac{1}{2\sqrt{2}} \left[Sc \left\{ \sqrt{Sc^2 + 16\phi^2} + Sc \right\} \right]^{\frac{1}{2}};$$

$$B_2 = \frac{1}{2\sqrt{2}} \left[Sc \left\{ \sqrt{Sc^2 + 16\phi^2} - Sc \right\} \right]^{\frac{1}{2}};$$

$$A_3 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[\sqrt{(1+4M_1)^2 + 64E^2} + (1+4M_1) \right]^{\frac{1}{2}};$$

$$B_3 = \frac{1}{2\sqrt{2}} \left[\sqrt{(1+4M_1)^2 + 64E^2} - (1+4M_1) \right]^{\frac{1}{2}}.$$

$$A_4 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \left[\sqrt{(1+4M_1)^2 + 16(2E+\phi)^2} + (1+4M_1) \right]^{\frac{1}{2}};$$

$$B_4 = \frac{1}{2\sqrt{2}} \left[\sqrt{(1+4M_1)^2 + 16(2E+\phi)^2} - (1+4M_1) \right]^{\frac{1}{2}};$$

$$P_1 = \frac{AH_0 \text{Pr} a_1}{a_1^2 + b_1^2}; \quad Q_1 = \frac{AH_0 \text{Pr} b_1}{a_1^2 + b_1^2};$$

$$P_2 = -\frac{ScH_0 [a_2 AR_0 + S_0 H_0 (a_2 P_1 - b_2 Q_1)]}{a_2^2 + b_2^2};$$

$$Q_2 = -\frac{ScH_0 [b_2 AR_0 + S_0 H_0 (a_2 Q_1 + b_2 P_1)]}{a_2^2 + b_2^2};$$

$$Q_3 = -\frac{ASc(1+R_0)}{\phi};$$

$$P_4 = \frac{S_0 Sc \left[(A_1^2 - B_1^2)(a_3 P_1 - b_3 Q_1) - 2A_1 B_1 (a_3 Q_1 + b_3 P_1) \right]}{a_3^2 + b_3^2};$$

$$Q_4 = \frac{S_0 Sc \left[(A_1^2 - B_1^2)(a_3 Q_1 + b_3 P_1) + 2A_1 B_1 (a_3 P_1 - b_3 Q_1) \right]}{a_3^2 + b_3^2};$$

$$P_5 = -(P_2 + P_4);$$

$$Q_5 = Q_3 - (Q_2 + Q_4);$$

$$P_6 = \frac{(Gr - GmR_0)a_4}{a_4^2 + 4E^2};$$

$$Q_6 = \frac{2E(Gr - GmR_0)}{a_4^2 + 4E^2};$$

$$P_7 = \frac{a_5 Gm(1+R_0)}{a_5^2 + 4E^2};$$

$$Q_7 = \frac{2EGm(1+R_0)}{a_5^2 + 4E^2};$$

$$P_8 = \frac{P_6 + P_7 + R\{Q_8 B_3 + (H_0 P_6 + Sc P_7)\}}{(1 + A_3 R)}; \quad Q_8 = Q_6 + Q_7;$$

$$P_9 = \frac{a_7(Gr P_1 - Gm P_4) + b_7(Gr Q_1 - Gm Q_4)}{a_7^2 + b_7^2};$$

$$Q_9 = \frac{a_7(Gr Q_1 - Gm Q_4) - b_7(Gr P_1 - Gm P_4)}{a_7^2 + b_7^2}; \quad P_{10} = \frac{Gm(a_8 P_5 - b_8 Q_5)}{a_8^2 + b_8^2};$$

$$Q_{10} = \frac{Gm(a_8 Q_5 - b_8 P_5)}{a_8^2 + b_8^2}; \quad P_{11} = \frac{A\{a_6(A_3 P_8 - B_3 Q_8) + b_6(A_3 Q_8 + B_3 P_8)\}}{a_6^2 + b_6^2};$$

$$Q_{11} = \frac{A\{a_6(A_3 Q_8 + B_3 P_8) - b_6(A_3 P_8 - B_3 Q_8)\}}{a_6^2 + b_6^2};$$

$$P_{12} = \left[\frac{(1 - P_{11} + P_{13} - P_{14} - P_9 + P_{10})}{(1 + A_4 R)} + \frac{R(Q_{12} B_4 - A_3 P_{11} + B_3 Q_1 + H_0 P_{13} - Sc P_{14} + A_1 P_9 - B_1 Q_9 + A_2 P_{10} - B_2 Q_{10})}{(1 + A_4 R)} \right];$$

$$Q_{12} = Q_{10} + Q_{13} - Q_{11} - Q_{14} - Q_9;$$

$$P_{13} = \frac{[a_4(AH_0 P_6 - Gr P_1 - Gm P_2) - 2E(AH_0 Q_6 - Gr Q_1 - Gm Q_2)]}{a_4^2 + 4E^2};$$

$$Q_{13} = \frac{[a_4(AH_0 Q_6 - Gr Q_1 - Gm Q_2) + 2E(AH_0 P_6 - Gr P_1 - Gm P_2)]}{a_4^2 + 4E^2};$$

$$P_{14} = -\frac{a_5 A Sc P_7 + (2E + \phi)(Gm Q_3 - A Sc Q_7)}{a_5^2 + (2E + \phi)^2};$$

$$Q_{14} = \frac{a_5(Gm Q_3 - A Sc Q_7) - A Sc P_7(2E + \phi)}{a_5^2 + (2E + \phi)^2};$$

$$a_1 = H_0^2 + Pr H_0 - \alpha_0;$$

$$b_1 = \phi Pr;$$

$$a_2 = H_0(H_0 - Sc);$$

$$b_2 = \phi Sc;$$

$$a_3 = A_1^2 - B_1^2 - ScA_1;$$

$$b_3 = ScB_1 + \phi Sc - 2A_1B_1;$$

$$a_4 = H_0^2 - H_0 - M_1;$$

$$a_5 = Sc^2 - Sc - M_1;$$

$$a_6 = A_3^2 - B_3^2 - A_3 - M_1;$$

$$b_6 = 2A_3B_3 - B_3 - 2E;$$

$$a_7 = A_1^2 - B_1^2 - A_1 - M_1;$$

$$b_7 = 2A_1B_1 - B_1 - (2E + \phi);$$

$$a_8 = A_2^2 - B_2^2 - A_2 - M_1;$$

$$b_8 = 2A_2B_2 - B_2 - (2E + \phi).$$

Appendix-II

$$J_1 = H_0P_{13} - P_{12} - A_3P_{11} + B_3Q_{11} - P_{14}Sc - A_1P_9 + B_1Q_9 + A_2P_{10} - B_2Q_{10};$$

$$J_2 = Q_{12} + B_3P_{11} + A_3Q_{11} - H_0Q_{13} + Q_{14}Sc + A_1Q_9 + B_1P_9 - B_2P_{10} - A_2Q_{10};$$

$$|N_1| = |N_{1r} + iN_{1i}|;$$

$$\tan \beta = \frac{N_{1i}}{N_{1r}};$$

$$\tau_{op} = H_0P_6 - A_3P_8 + B_3Q_8 + ScP_7;$$

$$\tau_{os} = H_0Q_6 - A_3Q_8 - B_3P_8 + ScQ_7;$$

$$G_1 = A_1P_1 - H_0P_1 - B_1Q_1;$$

$$G_2 = A_1Q_1 - H_0Q_1 + B_1P_1;$$

$$G = G_1 + iG_2; \quad |G| = |G_1 + iG_2|; \quad \tan \alpha = \frac{G_2}{G_1};$$

$$N_{1r} = B_2Q_5 - A_2P_5 - H_0P_2 - A_1P_4 + B_1Q_4;$$

$$N_{1i} = A_2Q_5 + B_2P_5 + H_0Q_2 - ScQ_3 + A_1Q_4 + B_1P_4.$$

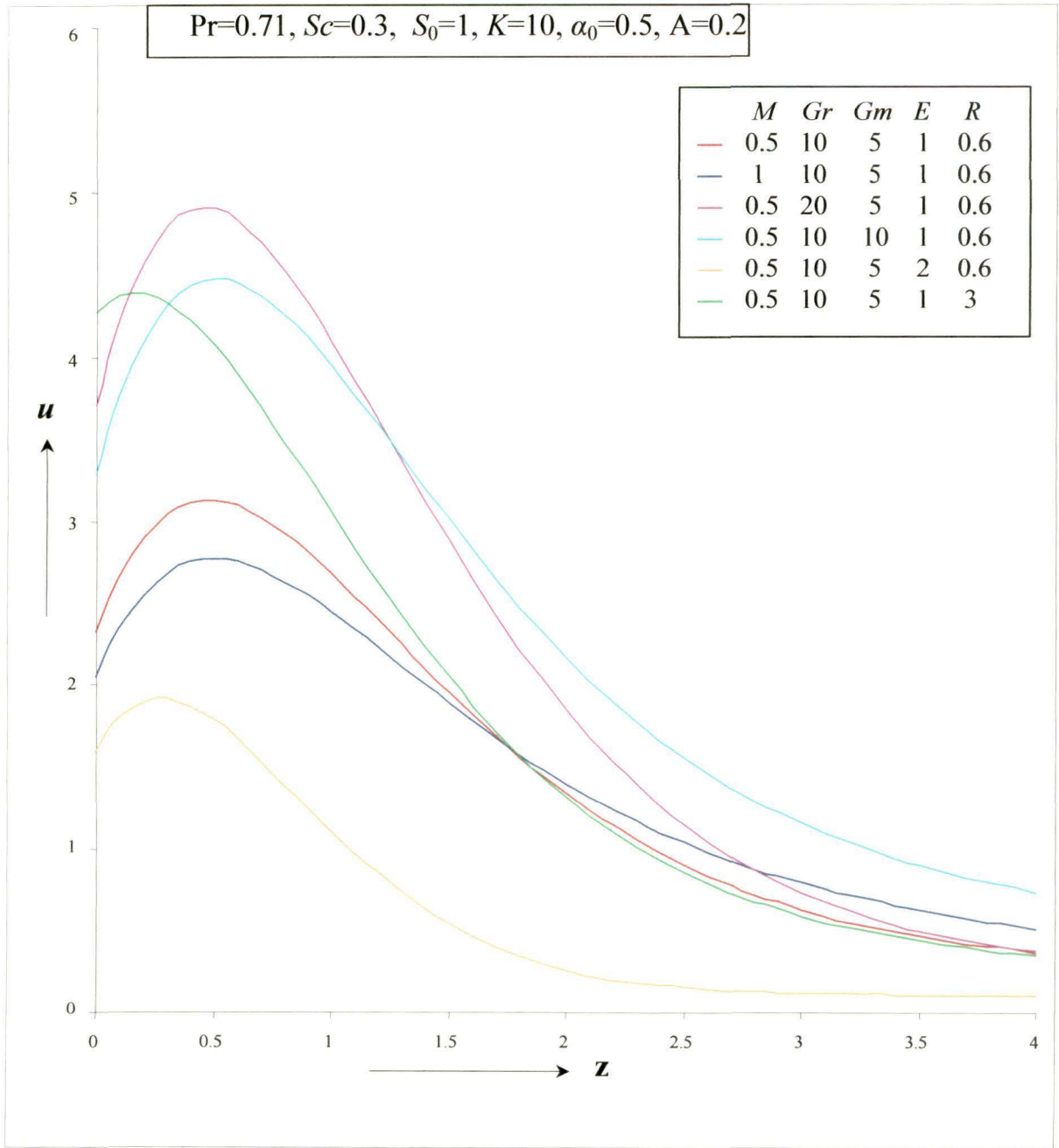


Fig. 2.2.7 Effects of M , Gr , Gm , E and R on the primary velocity field.

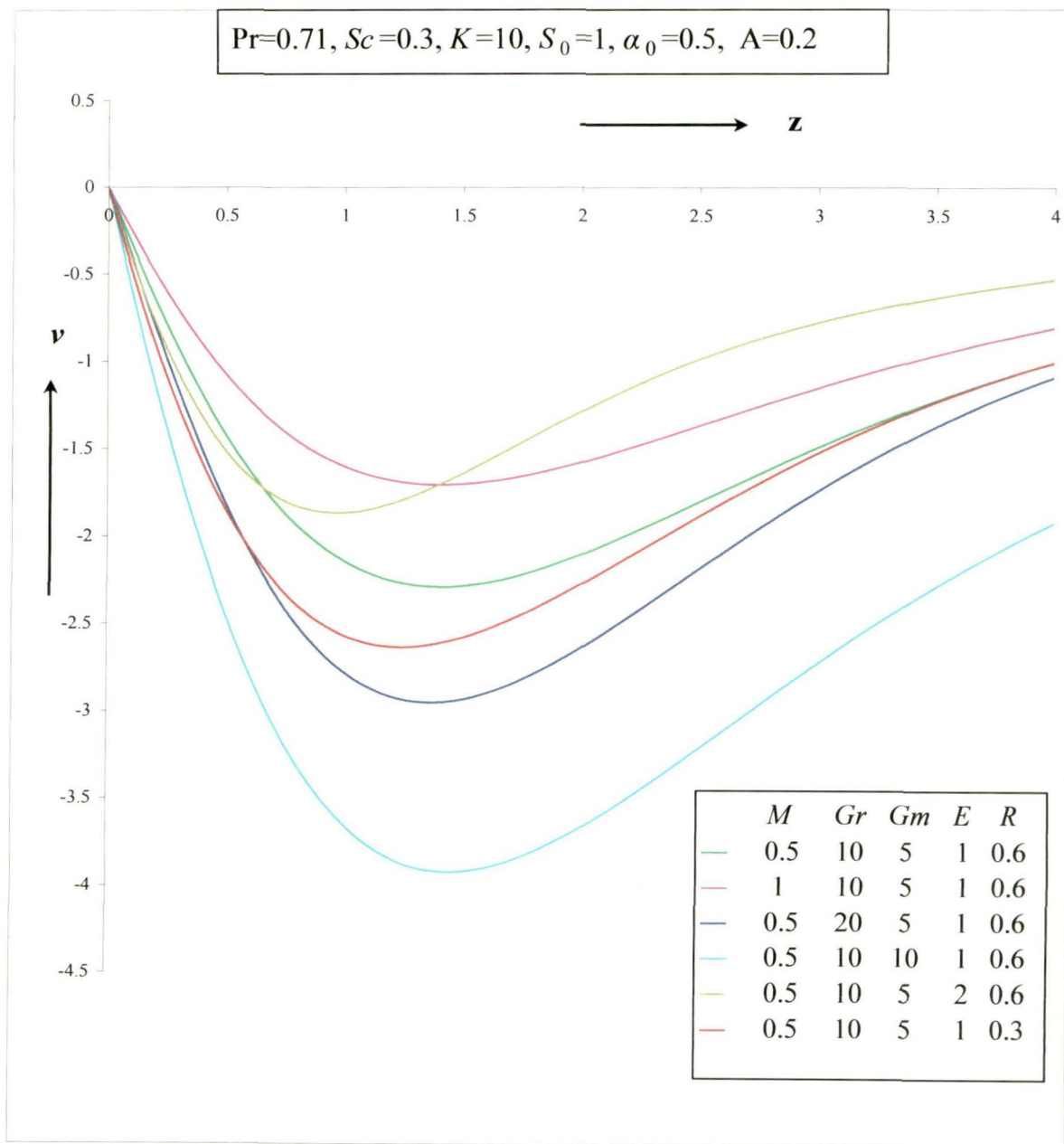


Fig. 2.2.8 Effects of M , Gr , Gm , E and R on the secondary velocity field.

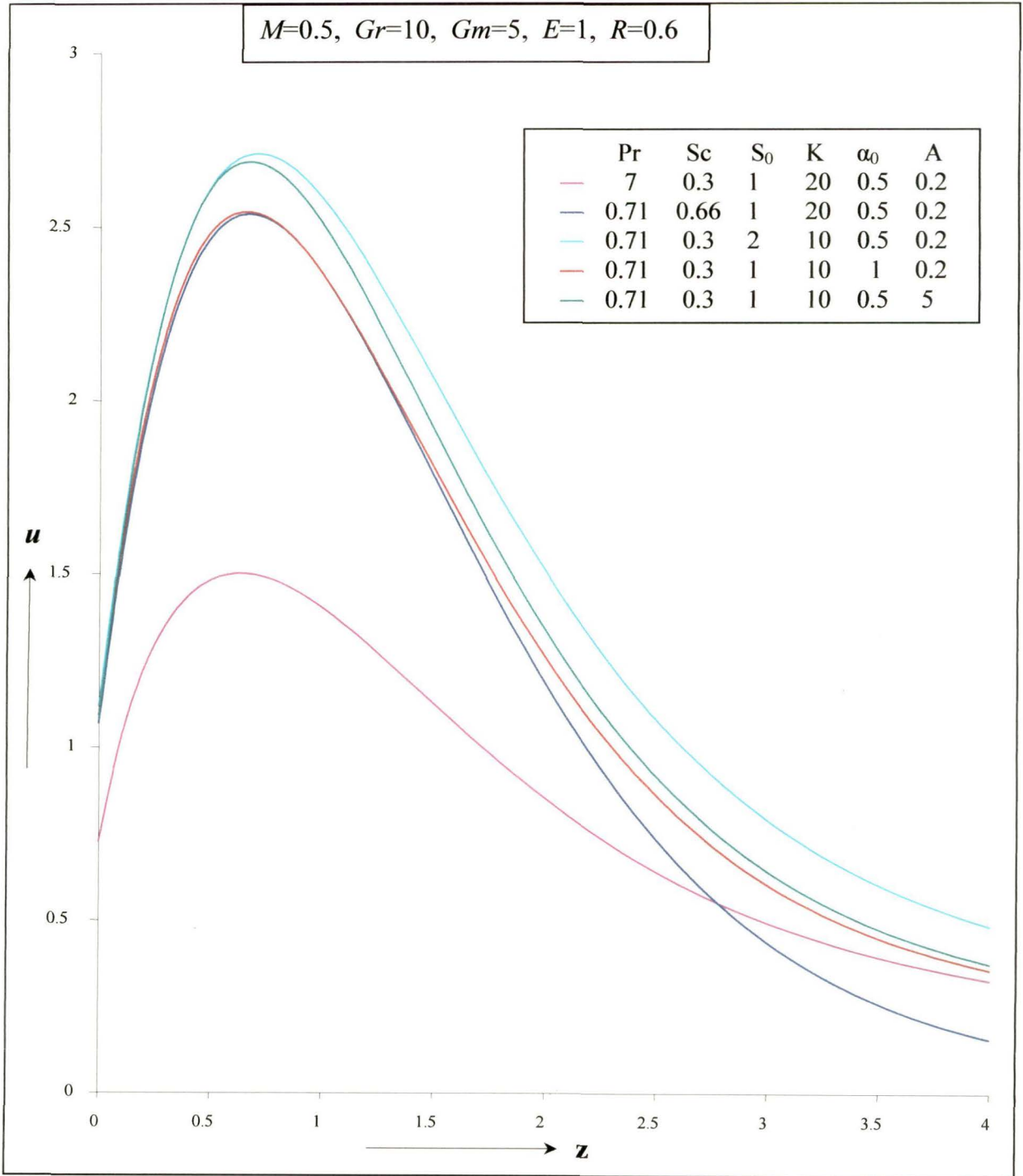


Fig. 2.2.9 Effects of Pr, Sc, K, S_0, α_0 and A on the primary velocity field

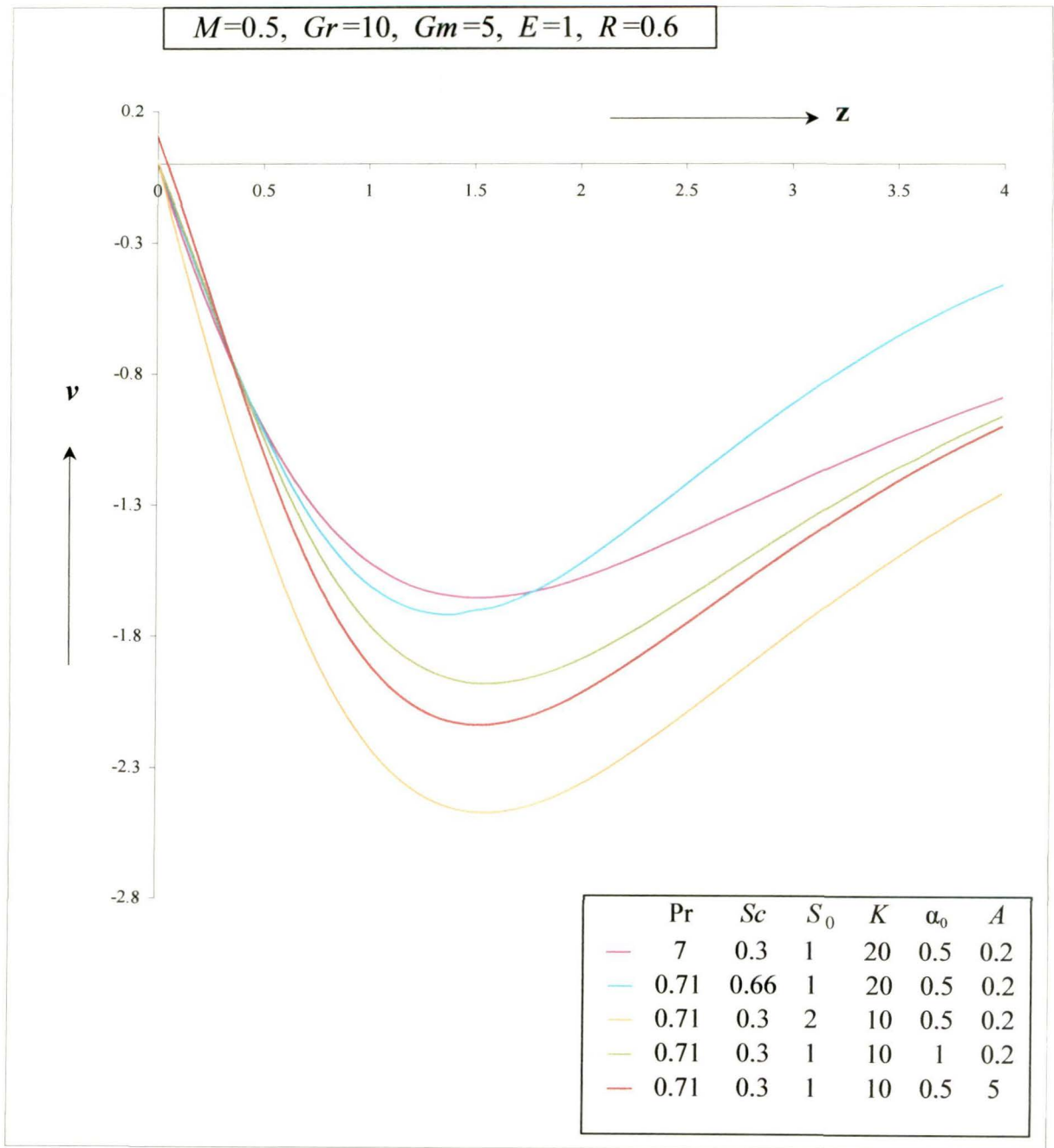


Fig. 2.2.10 Effects of Pr, Sc, K, S_0 , α_0 and A on the secondary velocity field.

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Chapter-III

DISPERSION OF SOLUTE

*PART ONE***DISPERSION OF SOLUTE IN OSCILLATING
HYDROMAGNETIC COUETTE FLOW*****3.1 INTRODUCTION**

Taylor [1] showed that if a solute is injected into a solvent flowing steadily in a tube, the combined action of the lateral molecular diffusion and the variation of velocity over the cross-section would cause the solute ultimately to spread diffusively with the effective molecular diffusivity given by $D_{eff.} = a^2 \omega_m^2 / 48 D_m$, where D_m is the molecular diffusivity, ω_m is the mean velocity and a is the radius of the tube. Some restrictions in this model of Taylor [1] was overcome partially by Aris [2] using a statistical approach. Gill and Sankarasubramanian [3] constructed a dispersion model for the steady flow of fluid in a tube which is valid for all time after the injection of solute by allowing the diffusion coefficients to vary with time. Although authors [4] extended the scope of their model to study dispersion of solute in a time-dependent laminar flow which in principle, valid for all values of time, they confined their analysis only to the case of dispersion in a fully developed steady flow. Recently Hazra *et al.* [5] studied dispersion of solute in pulsatile flow of viscous fluid in a parallel plate channel.

The aim of the present paper is to study the effects of transversely applied uniform magnetic field on the dispersion of solute in oscillatory Couette flow. The analysis is based on the generalized dispersion model introduced by Gill and Sankarasubramanian. The effect of magnetic field and oscillation of plate on diffusion coefficients are investigated. The interesting part of the analysis is that $K_2(\tau)$, second dispersion coefficient, consists of a steady part S and a fluctuating part $D_2(\tau)$ due to the oscillation of plate. Dispersion of solute in oscillatory hydro-magnetic Couette flow is of interest in the field of chemical engineering, biomedical engineering and environmental sciences and in such areas as chemical

reaction design and studies on flow transients using probes based on diffusion-controlled electrode reaction.

3.2 MATHEMATICAL ANALYSIS

The dispersion of passive solute in the form of a slug of finite extent (in the region $-x_s/2 \leq x \leq x_s/2$) in a fully developed oscillatory flow between two parallel flat plates $z = \pm\delta$ is considered. The lower plate is stationary and the upper one is oscillating in its own plane with a velocity $U(t)$ about a non zero constant mean velocity U_0 . The z -axis is taken perpendicular to the plates and a transverse magnetic field of uniform strength B_0 is applied along z -axis. Since the plates are infinite in length, all physical quantities except pressure depend on z and t only.

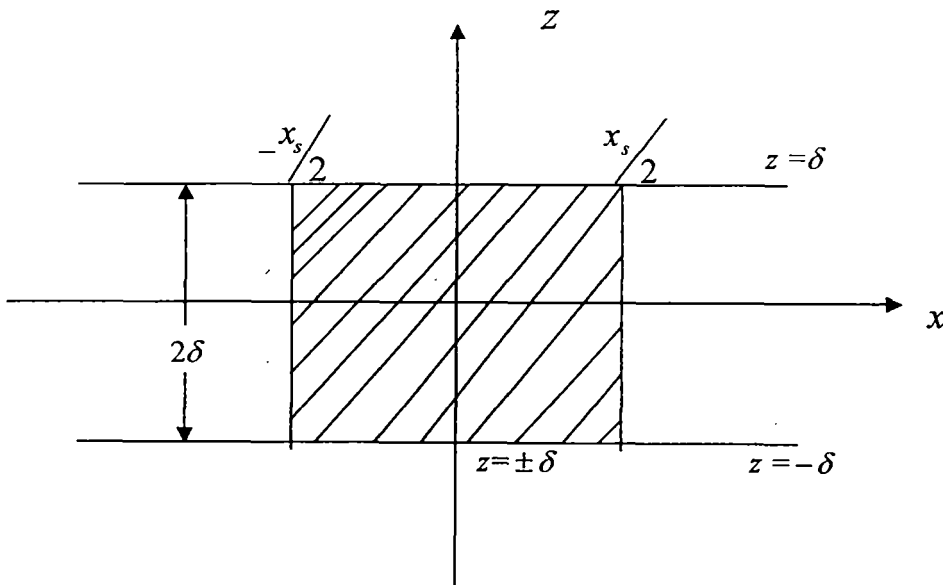


Fig. 3.1. Physical sketch of the problem

The resulting velocity field $u(z,t)$, which is along x -axis, satisfies the Navier–Stokes' equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u, \quad \dots (3.1)$$

with boundary conditions

$$\left. \begin{aligned} u=0 & \quad \text{at } z = -\delta \\ u = U(t) = U_0(1 + \varepsilon \cos \beta t) & \quad \text{at } z = \delta \end{aligned} \right\}, \quad \dots (3.2)$$

where β is the frequency of oscillation.

The solution of (3.1) satisfying (3.2) is given as

$$u(z,t) = U_0 \left[u_0(z) + \frac{\varepsilon}{2} \left\{ u_1(z) e^{i\beta t} + u_2(z) e^{-i\beta t} \right\} \right], \quad \dots (3.3)$$

where,

$$u_0(z) = 1 - \frac{\sinh(\delta - z)M}{\sinh 2M\delta}, \quad \dots (3.4)$$

$$u_1(z) = 1 - \frac{\sinh(\delta - z)L}{\sinh 2L\delta}, \quad \dots (3.5)$$

$$u_2(z) = 1 - \frac{\sinh(\delta - z)L'}{\sinh 2L'\delta} \quad \dots (3.6)$$

$$\text{and } L = \sqrt{M^2 + i\omega}, \quad L' = \sqrt{M^2 - i\omega}, \quad \dots (3.7)$$

$$\text{where } M = \frac{\sigma B_0^2}{\rho}, \quad \omega = \frac{\beta \delta^2}{\nu}. \quad \dots (2.8)$$

If a solute diffuses in the above fully developed flow, then the concentration $C(t,x,z)$ of solute satisfies

$$\frac{\partial c}{\partial t} + u(z,t) \frac{\partial c}{\partial x} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial z^2} \right), \quad \dots (3.9)$$

where D is the molecular diffusivity.

The initial and boundary conditions are

$$\left. \begin{aligned} c(0,x,z) &= c_0 & \text{for } |x| \leq \frac{x_s}{2} \\ &= 0 & \text{for } |x| > \frac{x_s}{2} \end{aligned} \right\}, \quad \dots (3.10a)$$

$$\frac{\partial c}{\partial z} = 0 \quad \text{at } z = \pm \delta, \quad \dots (3.10b)$$

$$c(t, \infty, z) = 0, \quad \dots (3.10c)$$

where (3.10c) expresses the condition of zero mass flux at the plate walls.

We introduce the dimensionless quantities as

$$\theta = \frac{c}{c_0}, U(\eta, \tau) = \frac{u(z, t)}{\bar{u}}, X = \frac{x_s}{\delta^2 \bar{u}}, \tau = \frac{Dt}{\delta^2}, Pe = \frac{\bar{u}\delta}{D}, \eta = \frac{z}{\delta}, \quad \dots (3.11)$$

where \bar{u} is the time-averaged axial velocity on the central line $z = 0$ given by

$$\bar{u} = \frac{2\pi}{\beta} \int_0^{\beta/2\pi} u(t, 0) dt \quad \dots (3.12)$$

using (3.11) in (3.9) and (3.10) we get

$$\frac{\partial \theta}{\partial \tau} + U(\eta, \tau) \frac{\partial \theta}{\partial X} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial \eta^2} \quad \dots (3.13)$$

with the initial and boundary conditions are as follows

$$\left. \begin{aligned} \theta(0, X, \eta) &= 1 && \text{for } |X| \leq \frac{1}{2} X_s \\ &= 0 && \text{for } |X| > \frac{1}{2} X_s \end{aligned} \right\}, \quad \dots (3.14a)$$

$$\theta(\tau, \infty, \eta) = 0, \quad \dots (3.14b)$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, \pm 1) = 0. \quad \dots (3.14c)$$

Following Gill and Sankarasubramanian [3], the solution of (3.13) subject to (3.14) is formulated as

$$\theta(\tau, X, \eta) = \sum_{k=0}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial X^k}, \quad \dots (3.15)$$

where

$$\theta_m = \frac{1}{2} \int_{-1}^1 \theta d\eta. \tag{3.16}$$

Substituting (3.15) in (3.13) we get

$$\begin{aligned} \frac{\partial \theta_m}{\partial \tau} + U(\eta, \tau) \frac{\partial \theta_m}{\partial X} - \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} + \sum_{k=0}^{\infty} \left[\left(\frac{\partial f_k}{\partial \tau} - \frac{\partial^2 f_k}{\partial \eta^2} \right) \frac{\partial^k \theta_m}{\partial X^k} \right. \\ \left. + U(\tau, \eta) f_k \frac{\partial^{k+1} \theta_m}{\partial X^{k+1}} - \frac{1}{Pe^2} f_k \frac{\partial^{k+2} \theta_m}{\partial X^{k+2}} + f_k \frac{\partial^{k+1} \theta_m}{\partial \tau \partial X^k} \right] = 0. \end{aligned} \tag{3.17}$$

Integration of (3.13) gives upon using (3.16),

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} - \frac{1}{2} \int_{-1}^1 U(\tau, \eta) \frac{\partial \theta}{\partial X} d\eta. \tag{3.18}$$

We assume that the process of distribution of θ_m is diffusive in nature right from time zero, then following Gill and Sankarasubramanian's approach. They generalized dispersion model with time dependent dispersion coefficients can be written as

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial X^i}, \tag{3.19}$$

where

$$K_1(\tau) = -\frac{1}{2} \int_{-1}^1 U(\tau, \eta) f_0(\tau, \eta) d\eta, \tag{3.20a}$$

$$K_2(\tau) = \frac{1}{Pe^2} - \frac{1}{2} \int_{-1}^1 U(\tau, \eta) f_1(\tau, \eta) d\eta, \tag{3.20b}$$

⋮

$$K_{i+2}(\tau) = -\frac{1}{2} \int_{-1}^1 U(\tau, \eta) f_{i+1} d\eta \quad (i=1,2,3,\dots). \tag{3.20c}$$

Substituting (3.19) in (3.17) and equating the coefficients $\partial^k \theta_m / \partial X^k$, we obtain the following equations for $f_k(\tau, \eta)$:

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial \eta^2}, \quad \dots (3.21a)$$

$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial \eta^2} - [U(\tau, \eta) + K_1(\tau)] f_0. \quad \dots (3.21b)$$

$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial \eta^2} + \left[\frac{1}{Pe^2} - K_2(\tau) \right] f_0 - [U(\tau, \eta) + K_1(\tau)] f_1. \quad \dots (3.21c)$$

⋮

$$\begin{aligned} \frac{\partial f_k}{\partial \tau} = & \frac{\partial^2 f_k}{\partial \eta^2} - [U(\tau, \eta) + K_1(\tau)] f_{k-1} \\ & + \left[\frac{1}{Pe^2} - K_2(\tau) \right] f_{k-2} - \sum_{i=3}^k K_i f_{k-i} \quad (k=3,4,5,\dots). \quad \dots (3.21d) \end{aligned}$$

Equations (3.14a) and (3.16) give

$$\left. \begin{aligned} \theta_m(0, X) = 1 & \quad \text{for} \quad |X| \leq \frac{X_s}{2} \\ \theta_m(0, X) = 0 & \quad \text{for} \quad |X| > \frac{X_s}{2} \end{aligned} \right\}, \quad \dots (3.22a)$$

$$\theta_m(\tau, \infty) = 0. \quad \dots (3.22b)$$

Now from (3.15), the initial conditions for f_k can be taken as

$$f_0(0, \eta) = 1, \quad f_k(0, \eta) = 0 \quad \text{for } k=1,2,3,\dots \quad \dots (3.23)$$

Similarly the boundary conditions for f_k are derived from (3.14) and (3.15) as

$$\frac{\partial f_k}{\partial \eta} = 0 \quad \text{at } \eta = \pm 1 \quad (k=0,1,2,\dots). \quad \dots (3.24)$$

$$A = \frac{A' \frac{\alpha}{\sqrt{2}} + B' \frac{\omega}{\sqrt{2\alpha}}}{\frac{\alpha^2}{2} + \frac{\omega^2}{2\alpha^2}}; \quad B = \frac{B' \frac{\alpha}{\sqrt{2}} - A' \frac{\omega}{\sqrt{2\alpha}}}{\frac{\alpha^2}{2} + \frac{\omega^2}{2\alpha^2}};$$

$$A' = \frac{\sinh \sqrt{2\alpha\delta} \cos \frac{\sqrt{2\omega\delta}}{\alpha} - \sinh \sqrt{2\alpha\delta} \cosh \sqrt{2\alpha\delta}}{\sinh^2 \sqrt{2\alpha\delta} \cos^2 \frac{\sqrt{2\omega\delta}}{\alpha} + \cosh^2 \sqrt{2\alpha\delta} \sin^2 \frac{\sqrt{2\omega\delta}}{\alpha}}; \quad \dots (3.30)$$

$$B' = \frac{\sin \frac{\sqrt{2\omega\delta}}{\alpha} \cos \frac{\sqrt{2\omega\delta}}{\alpha} - \cosh \sqrt{2\alpha\delta} \sin \frac{\sqrt{2\omega\delta}}{\alpha}}{\sinh^2 \sqrt{2\alpha\delta} \cos^2 \frac{\sqrt{2\omega\delta}}{\alpha} + \cosh^2 \sqrt{2\alpha\delta} \sin^2 \frac{\sqrt{2\omega\delta}}{\alpha}};$$

$$\alpha = \left\{ M^2 + \sqrt{M^4 + \omega^2} \right\}^{\frac{1}{2}}.$$

To determine $f_1(\tau, \eta)$, one has to solve (3.21b) subject to the initial and boundary conditions (3.23) – (3.25) with $f_0=1$. From Duhamel's theorem, it follows that

$$f_1(\tau, \eta) = \frac{\partial}{\partial \tau} \int_0^\tau F(\tau - \xi, \eta, \xi) d\xi, \quad \dots (3.31)$$

where $F(\tau, \eta, \xi)$ satisfies

$$\frac{\partial F}{\partial \tau} = \frac{\partial^2 F}{\partial \eta^2} - [U(\xi, \eta) + K_1(\xi)]. \quad \dots (3.32)$$

Subject to

$$\left. \begin{aligned} F(0, \eta, \tau) &= 0 \\ \left(\frac{\partial F}{\partial \eta} \right)_{\eta = \pm 1} &= 0, \quad \int_{-1}^1 F d\eta = 0 \end{aligned} \right\}. \quad \dots (3.33)$$

Note that ξ behaves like a parameter while solving (3.32). Substituting for U and K_I from (3.26) and (3.29) in (3.32) and solving the resulting equation subject to (3.33), we get

$$\begin{aligned}
 F(\tau, \eta, \xi) = & \frac{1}{\delta P M \sinh 2\delta M} \left[\frac{\sinh(1-\eta)\delta M}{-\delta M} - \frac{(1 - \cosh 2\delta M)}{4} \eta^2 \right] \\
 & + \frac{\varepsilon}{2P\delta} \left[\frac{\sinh(1-\eta)\delta L}{\delta L^2 \sinh 2\delta L} e^{i\zeta\xi} + \frac{\sinh(1-\eta)\delta L'}{\delta L'^2 \sinh 2\delta L'} e^{-i\zeta\xi} - (A \cos \zeta\xi - B \sinh \zeta\xi) \frac{\eta^2}{2} \right] \\
 & - \left[\frac{(1 + \cosh 2\delta M)}{2\delta P M \sinh 2\delta M} + \frac{\varepsilon}{2P\delta} \{ (2C + A) \cos \zeta\xi + (2\bar{D} - B) \sin \zeta\xi \} \right] \eta \\
 & + \frac{1}{2\delta P M \sinh 2\delta M} \left[\frac{(\cosh 2\delta M - 1)}{\delta^2 M^2} + \frac{1}{6} (1 - \cosh 2\delta M) \right] + \\
 & \frac{\varepsilon}{2P\delta} \left[\frac{(\cosh 2\delta L - 1)}{\delta^2 L^3 \sinh 2\delta L} e^{i\zeta\xi} + \frac{(\cosh 2\delta L' - 1)}{\delta^2 L'^3 \sinh 2\delta L'} e^{-i\zeta\xi} + \frac{(A \cos \zeta\xi - B \sin \zeta\xi)}{3} \right] \\
 & + \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 \tau} \cos \lambda_n \eta, \quad \dots (3.34)
 \end{aligned}$$

where

$$\begin{aligned}
 \lambda_n = n\pi; \quad C = \frac{\frac{\alpha}{\sqrt{2}} C' - \frac{\omega}{\sqrt{2\alpha}} D'}{\frac{\alpha^2}{2} + \frac{\omega^2}{2\alpha^2}}; \quad \bar{D} = \frac{\frac{\omega}{\sqrt{2\alpha}} C' + \frac{\alpha}{\sqrt{2}} D'}{\frac{\alpha^2}{2} + \frac{\omega^2}{2\alpha^2}}; \\
 C = \frac{\sinh \sqrt{2\delta\alpha} \cosh \sqrt{2\delta\alpha}}{\sinh^2 \sqrt{2\delta\alpha} \cos^2 \frac{\sqrt{2\omega\delta}}{\alpha} + \cosh^2 \sqrt{2\delta\alpha} \sin^2 \frac{\sqrt{2\omega\delta}}{\alpha}};
 \end{aligned}$$

$$D' = \frac{\sin \frac{\sqrt{2}\omega\delta}{\alpha} \cos \frac{\sqrt{2}\omega\delta}{\alpha}}{\sinh^2 \sqrt{2}\delta\alpha \cos^2 \frac{\sqrt{2}\omega\delta}{\alpha} + \cosh^2 \sqrt{2}\delta\alpha \sin^2 \frac{\sqrt{2}\omega\delta}{\alpha}};$$

$$d_n = \frac{\cos \lambda_n (1 - \cosh 2\delta M)}{\delta P M \sinh 2\delta M} \left[\frac{\delta M}{\lambda_n^2 + \delta^2 M^2} + \frac{1}{\lambda_n^2} \right] - \frac{\varepsilon \cos \lambda_n}{2 P}$$

$$\times \left[\frac{(\cosh 2\delta L - 1)}{L^2 (\lambda_n^2 + \delta^2 L^2) \sinh 2\delta L} e^{i\zeta\xi} + \frac{(\cosh 2\delta L' - 1)}{L'^2 (\lambda_n^2 + \delta^2 L'^2) \sinh 2\delta L'} e^{-i\zeta\xi} \right.$$

$$\left. - 2 \frac{(A \cos \zeta\xi - B \sin \zeta\xi)}{\lambda_n^2} \right]. \quad (n=1,2,3,\dots) \quad \dots (3.35)$$

Equations (3.31) and (3.34) now give

$$f_1(\tau, \eta) = \frac{1}{\delta P M \sinh 2\delta M} \left[\frac{\sinh(1-\eta)\delta M}{-\delta M} - \frac{(1 - \cosh 2\delta M)}{4} \eta^2 \right]$$

$$+ \frac{\varepsilon}{2P\delta} \left[\frac{\sinh(1-\eta)\delta L}{\delta L^2 \sinh 2\delta L} e^{i\zeta\tau} + \frac{\sinh(1-\eta)\delta L'}{\delta L'^2 \sinh 2\delta L'} e^{-i\zeta\tau} - (A \cos \zeta\tau - B \sin \zeta\tau) \frac{\eta^2}{2} \right]$$

$$- \frac{\varepsilon}{2P\delta} \{ (2C + A) \cos \zeta\tau + (2\bar{D} - B) \sin \zeta\tau \} \eta +$$

$$\frac{1}{2\delta P M \sinh 2\delta M} \left[\frac{(\cosh 2\delta M - 1)}{\delta^2 M^2} + \frac{1}{6} (1 - \cosh 2\delta M) \right] - \frac{(1 + \cosh 2\delta M)}{2\delta P M \sinh 2\delta M} \eta$$

$$+ \frac{\varepsilon}{2P\delta} \left[\frac{(\cosh 2\delta L - 1)}{\delta^3 L^3 \sinh 2\delta L} e^{i\zeta\tau} + \frac{(\cosh 2\delta L' - 1)}{\delta^2 L'^3 \sinh 2\delta L'} e^{-i\zeta\tau} + \frac{1}{3} (A \cos \zeta\tau - B \sin \zeta\tau) \right]$$

$$+ \sum_{n=1}^{\infty} \left[\frac{(1 - \cosh 2\delta M) \cos \lambda_n}{\delta P M \sinh 2\delta M} \left(\frac{\delta M}{\lambda_n^2 + \delta^2 M^2} + \frac{1}{\lambda_n^2} \right) e^{-\lambda_n^2 \tau} - \frac{\varepsilon \cos \lambda_n}{2 P \delta} \times \right.$$

$$\left\{ \frac{(\cosh 2\delta L - 1) (i\zeta e^{i\zeta\tau} + \lambda_n^2 e^{-\lambda_n^2\tau})}{L^2 \sinh 2\delta L (\lambda_n^2 + \delta^2 L^2)(\lambda_n^2 + i\zeta)} + \frac{(\cosh 2\delta L' - 1)}{L'^2 \sinh 2\delta L'} \right. \\ \times \frac{(\lambda_n^2 e^{-\lambda_n^2\tau} - i\zeta e^{-i\zeta\tau})}{(\lambda_n^2 + \delta^2 L'^2)(\lambda_n^2 - i\zeta)} - \frac{2}{\lambda_n^2} \left(\frac{A(\zeta^2 \cos \zeta\tau - \lambda_n^2 \zeta \sin \zeta\tau + \lambda_n^4 e^{-\lambda_n^2\tau})}{\lambda_n^4 + \zeta^2} \right. \\ \left. \left. \frac{B(\lambda_n^2 \zeta \cos \zeta\tau + \zeta^2 \sin \zeta\tau - \lambda_n^2 \zeta e^{-\lambda_n^2\tau})}{\lambda_n^4 + \zeta^2} \right) \right\} \dots (3.36)$$

Substituting (3.26) and (3.36) in (3.20b), we obtain the diffusion coefficient $K_2(\tau)$ as follows:

$$K_2(\tau) = \frac{1}{Pe^2} + S + D_2(\tau), \dots (3.37)$$

where S and $D_2(\tau)$ represent the steady and time dependent part of the diffusion coefficient, respectively.

The expressions for S and $D_2(\tau)$ are given as

$$S = \frac{1}{P\delta \{ \sinh 2\delta M - \sinh \delta M \}} \left[\frac{2 \sinh^2 \delta M}{\delta^2 M^3} - \frac{(1 - \cosh 2\delta M)}{2M} \left\{ \frac{1}{3} - \frac{1}{2 \sinh 2\delta M} \left(\frac{2 \sinh^2 \delta M}{\delta M} + \frac{4 \sinh^2 \delta M}{M^3 \delta^3} - \frac{4 \cosh \delta M \sinh \delta M}{M^2 \delta^2} \right) \right\} - \frac{\varepsilon^2 \sinh 2\delta M}{8 \delta^2 \sinh 2\delta L \sinh 2\delta L'} \left(\frac{\sinh 2(L+L')\delta}{L+L'} - \frac{\sinh 2(L-L')\delta}{L-L'} \right) \left(\frac{1}{L^2} + \frac{1}{L'^2} \right) + \frac{(1 + \cosh 2\delta M) \cosh \delta M}{\delta M^2 \sinh 2\delta M} \left(\frac{\sinh \delta M}{M\delta} - \cosh \delta M \right) + \frac{(\cosh 2\delta M - 1)}{M} \right]$$

$$\times \left(\frac{1}{\delta^2 M^2} - \frac{1}{6} \right) \left(1 - \frac{\sinh^2 \delta M}{\delta M \sinh 2\delta M} \right) - \frac{\varepsilon^2 \sinh 2\delta M}{2\delta^3 \sinh 2\delta L \sinh 2\delta L'}$$

$$\times \left\{ \frac{(\cosh 2\delta L - 1) \sinh^2 \delta L'}{L'L^3} + \frac{(\cosh 2\delta L' - 1) \sinh^2 \delta L}{LL^3} \right\}.$$

$$D_2(\tau) = -\frac{1}{\delta P^2 M \sinh 2\delta M} \left[\frac{\varepsilon e^{i\zeta\tau}}{2\delta M \sinh \delta L} \left(\frac{\sinh 2(L+M)\delta}{2(L+M)\delta} - \frac{\sinh 2(L-M)\delta}{2(L-M)\delta} \right) \right.$$

$$+ \frac{\varepsilon e^{-i\zeta\tau}}{2\delta M \sinh \delta L'} \left(\frac{\sinh 2(M+L')\delta}{2(M+L')\delta} - \frac{\sinh 2(M-L')\delta}{2(M-L')\delta} \right) + \varepsilon \cos \zeta\tau$$

$$\times \left\{ \frac{(1 - \cosh 2\delta M)}{6} - \frac{2 \sinh^2 \delta M}{\delta^2 M^2} \right\} + \frac{\varepsilon e^{i\zeta\tau} (1 - \cosh 2\delta M)}{8 \sinh \delta L} \left\{ \frac{\sinh^2 \delta L}{\delta L} \right.$$

$$+ \frac{4 \sinh^2 \delta L}{\delta^3 L^3} - \frac{2 \sinh 2\delta L}{L^2 \delta^2} \left. \right\} + \frac{\varepsilon e^{-i\zeta\tau} (1 - \cosh 2\delta M)}{8 \sinh \delta L'} \left\{ \frac{4 \sinh^2 \delta L'}{\delta^3 L'^3} \right.$$

$$\left. - \frac{2 \sinh 2\delta L'}{\delta^2 L'^2} + \frac{\sinh^2 \delta L'}{\delta L'} \right\} - \frac{\varepsilon}{2\delta P^2} \left[\frac{2e^{i\zeta\tau} \sinh^2 \delta L}{\delta^2 L^3 \sinh 2\delta L} + \frac{2e^{-i\zeta\tau} \sinh^2 \delta L'}{\delta^2 L'^3 \sinh 2\delta L'} \right.$$

$$- \frac{e^{i\zeta\tau}}{\delta L^2 \sinh 2\delta M \sinh 2\delta L} \left\{ \frac{\sinh 2(L+M)\delta}{2(L+M)\delta} - \frac{\sinh 2(L-M)\delta}{2(L-M)\delta} \right\}$$

$$- \frac{e^{-i\zeta\tau}}{\delta L'^2 \sinh 2\delta M \sinh 2\delta L'} \left\{ \frac{\sinh 2(L'+M)\delta}{2(L'+M)\delta} - \frac{\sinh 2(M-L')\delta}{2(M-L')\delta} \right\}$$

$$\left. - \frac{\varepsilon e^{2i\zeta\tau}}{2\delta L^2 \sinh^2 2\delta L} \left\{ \frac{\sinh 4\delta L}{4\delta L} - 1 \right\} - \frac{\varepsilon e^{-2i\zeta\tau}}{2\delta L'^2 \sinh^2 2\delta L'} \left\{ \frac{\sinh 4\delta L'}{4\delta L'} - 1 \right\} \right]$$

$$\begin{aligned}
& + \frac{2\varepsilon \cos \zeta \tau \sinh^2 \delta L}{\delta^2 L^3 \sinh 2\delta L} e^{i\zeta \tau} + \frac{2\varepsilon \cos \zeta \tau \sinh^2 \delta L'}{\delta^2 L'^3 \sinh 2\delta L'} e^{-i\zeta \tau} - (A \cos \zeta \tau - B \sin \zeta \tau) \\
& \times \left\{ \frac{2}{3} - \frac{1}{\sinh 2\delta M} \left(\frac{\sinh^2 \delta M}{\delta M} + \frac{2 \sinh^2 \delta M}{M^3 \delta^3} - \frac{\sinh 2\delta M}{M^2 \delta^2} \right) \right\} - \frac{\varepsilon e^{i\zeta \tau}}{2 \sinh 2\delta L} \\
& \times \left\{ \frac{2 \sinh^2 \delta L}{\delta L} + \frac{4 \sinh^2 \delta L}{L^3 \delta^3} - \frac{2 \sinh 2\delta L}{L^2 \delta^2} \right\} - \frac{\varepsilon e^{-i\zeta \tau}}{2 \sinh 2\delta L'} \left\{ \frac{2 \sinh^2 \delta L'}{\delta L'} \right. \\
& \left. + \frac{4 \sinh^2 \delta L'}{L'^3 \delta^3} - \frac{2 \sinh 2\delta L'}{L'^2 \delta^2} \right\} + \frac{2}{3} \varepsilon \cos \zeta \tau \left[-\frac{1}{P} \left[\frac{(1 + \cosh 2\delta M)}{2P\delta M \sinh 2\delta M} \right. \right. \\
& \left. \left. \times \left\{ \frac{\varepsilon e^{i\zeta \tau}}{2 \sinh 2\delta L} \left(\frac{\sinh 2\delta L}{L^2 \delta^2} - \frac{2 \cosh^2 \delta L}{\delta L} \right) + \frac{\varepsilon e^{-i\zeta \tau}}{2 \sinh 2\delta L'} \left(\frac{\sinh 2\delta L'}{L'^2 \delta^2} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{2 \cosh^2 \delta L'}{\delta L'} \right) \right\} + \frac{\varepsilon}{2P\delta \sinh 2\delta M} \left(\frac{\sinh 2\delta M}{M^2 \delta^2} - \frac{2 \cosh^2 \delta M}{M\delta} \right) \right. \\
& \left. \times \{ (2C + A) \cos \zeta \tau + (2\bar{D} - B) \sin \zeta \tau \} + \frac{\varepsilon^2}{2P\delta} \{ (2C + A) \cos \zeta \tau \right. \\
& \left. + (2\bar{D} - B) \sin \zeta \tau \} \left\{ \frac{e^{i\zeta \tau}}{2 \sinh 2\delta L} \left(\frac{\sinh 2\delta L}{L^2 \delta^2} - \frac{2 \cosh^2 \delta L}{\delta L} \right) + \frac{e^{-i\zeta \tau}}{2 \sinh 2\delta L'} \right. \right. \\
& \left. \left. \times \left(\frac{\sinh 2\delta L'}{L'^2 \delta^2} - \frac{2 \cosh^2 \delta L'}{\delta L'} \right) \right\} \right] + \frac{1}{P} \left[\frac{\varepsilon (\cosh 2\delta M - 1)}{2\delta P M \sinh 2\delta M} \left(\frac{1}{\delta^2 M^2} - \frac{1}{6} \right) \right. \\
& \left. \times \left\{ \frac{e^{i\zeta \tau} \sinh^2 \delta L}{\delta L \sinh 2\delta L} + \frac{e^{-i\zeta \tau} \sinh^2 \delta L'}{\delta L' \sinh 2\delta L'} - 2 \cos \zeta \tau \right\} - \frac{\varepsilon}{P\delta} \left(1 - \frac{\sinh^2 \delta M}{\delta M \sinh 2\delta M} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \times \left\{ \frac{(\cosh 2\delta L - 1) e^{i\zeta\tau}}{\delta^2 L^3 \sinh 2\delta L} + \frac{(\cosh 2\delta L' - 1) e^{-i\zeta\tau}}{\delta^2 L'^3 \sinh 2\delta L'} + \frac{1}{3} (A \cos \zeta\tau - B \sin \zeta\tau) \right\} \\
& + \frac{\varepsilon^2}{2P\delta} \left\{ \frac{(\cosh 2\delta L - 1) e^{i\zeta\tau}}{\delta^2 L^3 \sinh 2\delta L} + \frac{(\cosh 2\delta L' - 1) e^{-i\zeta\tau}}{\delta^2 L'^3 \sinh 2\delta L'} + \frac{1}{3} (A \cos \zeta\tau - B \sin \zeta\tau) \right\} \\
& \times \left\{ \frac{\sinh^2 \delta L}{\delta L \sinh 2\delta L} e^{i\zeta\tau} + \frac{\sinh^2 \delta L'}{\delta L' \sinh 2\delta L'} e^{-i\zeta\tau} - 2 \cos \zeta\tau \right\} + \sum_{n=1}^{\infty} \frac{1}{P} \left[\frac{\cos \lambda_n}{\delta P M \sinh 2\delta M} \right. \\
& \left. - \frac{\cos \lambda_n \cosh 2\delta M}{\delta P M \sinh 2\delta M} \left(\frac{\delta M}{\lambda_n^2 + \delta^2 M^2} + \frac{1}{\lambda_n^2} \right) e^{-\lambda_n^2 \tau} - \frac{\varepsilon \cos \lambda_n}{2P\delta} \right. \\
& \times \left\{ \frac{(\cosh 2\delta L - 1)(i\zeta e^{i\zeta\tau} + \lambda_n^2 e^{-\lambda_n^2 \tau})}{L^2(\lambda_n^2 + \delta^2 L^2)(\lambda_n^2 + i\zeta) \sinh 2\delta L} + \frac{(\cosh 2\delta L' - 1)(\lambda_n^2 e^{-\lambda_n^2 \tau} - i\zeta e^{-i\zeta\tau})}{L'^2(\lambda_n^2 + \delta^2 L'^2)(\lambda_n^2 - i\zeta) \sinh 2\delta L'} \right. \\
& \left. - \frac{2}{\lambda_n^2} \left(A \frac{(\zeta^2 \cos \zeta\tau - \lambda_n^2 \zeta \sin \zeta\tau + \lambda_n^4 e^{-\lambda_n^2 \tau})}{\lambda_n^4 + \zeta^2} - \right. \right. \\
& \left. \left. B \frac{(\lambda_n^2 \zeta \cos \zeta\tau + \zeta^2 \sin \zeta\tau - \lambda_n^2 \zeta e^{-\lambda_n^2 \tau})}{\lambda_n^4 + \zeta^2} \right) \right\} \times \left[\frac{M\delta \cos \lambda_n (\cosh 2\delta M - 1)}{(\lambda_n^2 + M^2 \delta^2) \sinh 2\delta M} \right. \\
& \left. + \frac{\varepsilon \delta L \cos \lambda_n (\cosh 2\delta L - 1) e^{i\zeta\tau}}{2(\lambda_n^2 + L^2 \delta^2) \sinh 2\delta L} + \frac{\varepsilon \delta L' \cos \lambda_n (\cosh 2\delta L' - 1) e^{-i\zeta\tau}}{2(\lambda_n^2 + L'^2 \delta^2) \sinh 2\delta L'} \right].
\end{aligned}$$

The foregoing procedure may be repeated and higher order dispersion coefficients K_3, K_4, \dots , may be determined with the help of (3.20c). It is also noted from previous work [3, 4] that higher order dispersion coefficients decrease rapidly in magnitude. Neglecting K_3 and higher order dispersion coefficients equation (3.19) can be written as

$$\frac{\partial \theta_m}{\partial \tau} = K_1(\tau) \frac{\partial \theta_m}{\partial X} + K_2(\tau) \frac{\partial^2 \theta_m}{\partial X^2} \quad \dots (3.38)$$

The solution of equation (3.38) subject to (3.22a) and (3.22b) is obtained as

$$\theta_m = \frac{1}{2} \left[\operatorname{erf} \frac{\frac{1}{2} X_s + X_1}{2T_0^{\frac{1}{2}}} + \operatorname{erf} \frac{\frac{1}{2} X_s - X_1}{2T_0^{\frac{1}{2}}} \right],$$

where

$$X_1 = X + \int_0^\tau K_1(\eta) d\eta ; \quad T_0 = \int_0^\tau K_2(\eta) d\eta .$$

3.3 RESULTS AND DISCUSSION

The most striking result of this analysis is that the dispersion of solute in the unsteady flow arising out of oscillation of upper plate gives rise to dispersion coefficient which consists of both steady and unsteady part. We have computed S , the steady part of dispersion coefficient, for various values of ω (oscillation parameter) and M (magnetic parameter) with $Sc=1000$ (for liquid Sc is very large) and plotted in Fig.-3.2. and Fig.-3.3. It can be seen that from Fig.-3.2 that for fixed M , S decreases with increases in ω and becomes more or less constant for $\omega \geq 2$. It may be remarked that Fig.-3.2 shows no variation for higher values of oscillation parameter, which is quite natural. In Fig.-3.3 it is noted that S decreases slowly with increase in M for fixed ω but decreases rapidly for $M \geq 0.5$ and this steepness is prominent for $\omega=1$. The time-dependent part $D_2(\tau)$ of dispersion coefficient $K_2(\tau)$ is evaluated for different values of τ , ω and M for $Sc=1000$. Fig.-3.4 shows the plot of $D_2(\tau)$ with τ for various values of ω with fixed M . Fig.-3.4 shows irregular oscillation for higher values of ω . Similarly Fig.-3.5 depicts the graph of $D_2(\tau)$ with τ for different values of M with fixed ω . It is interesting to note that no. of oscillation is more for large ω but amplitude of oscillation decreases as M increases. Fig.-3.6 shows the variation of $D_2(\tau)$ with ω for different values of M .

The amplitude of oscillation is less for higher values of ω and the nature of oscillation becomes regular as ω increases. Thus we can conclude that frequency of oscillation has great influence on steady and unsteady part of dispersion coefficient.

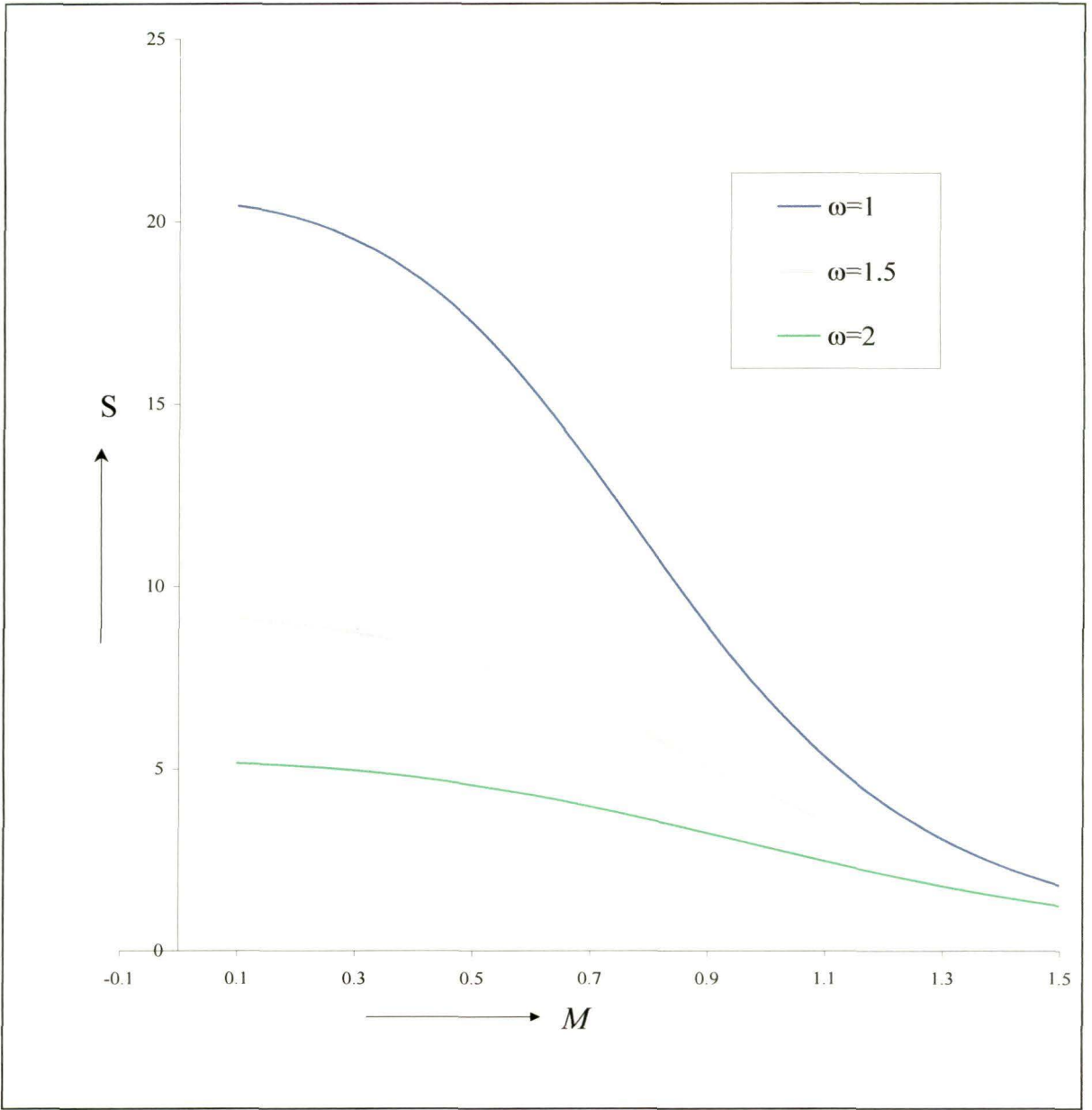


Fig. 3.2 S decreases with increase in ω for fixed M.

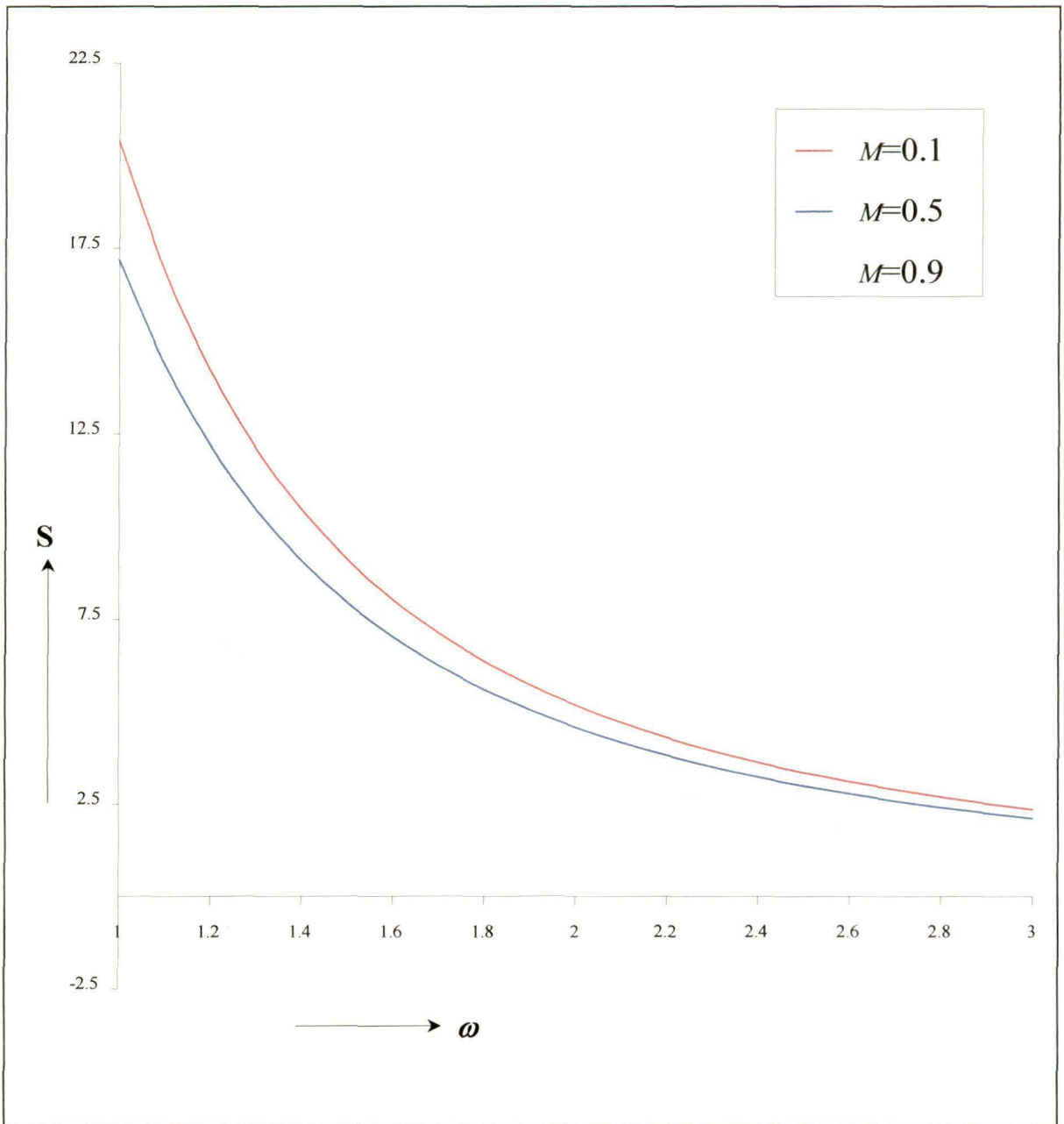


Fig. 3.3 S decreases slowly with increase in M for ω , but decreases rapidly for $M > 0.5$.

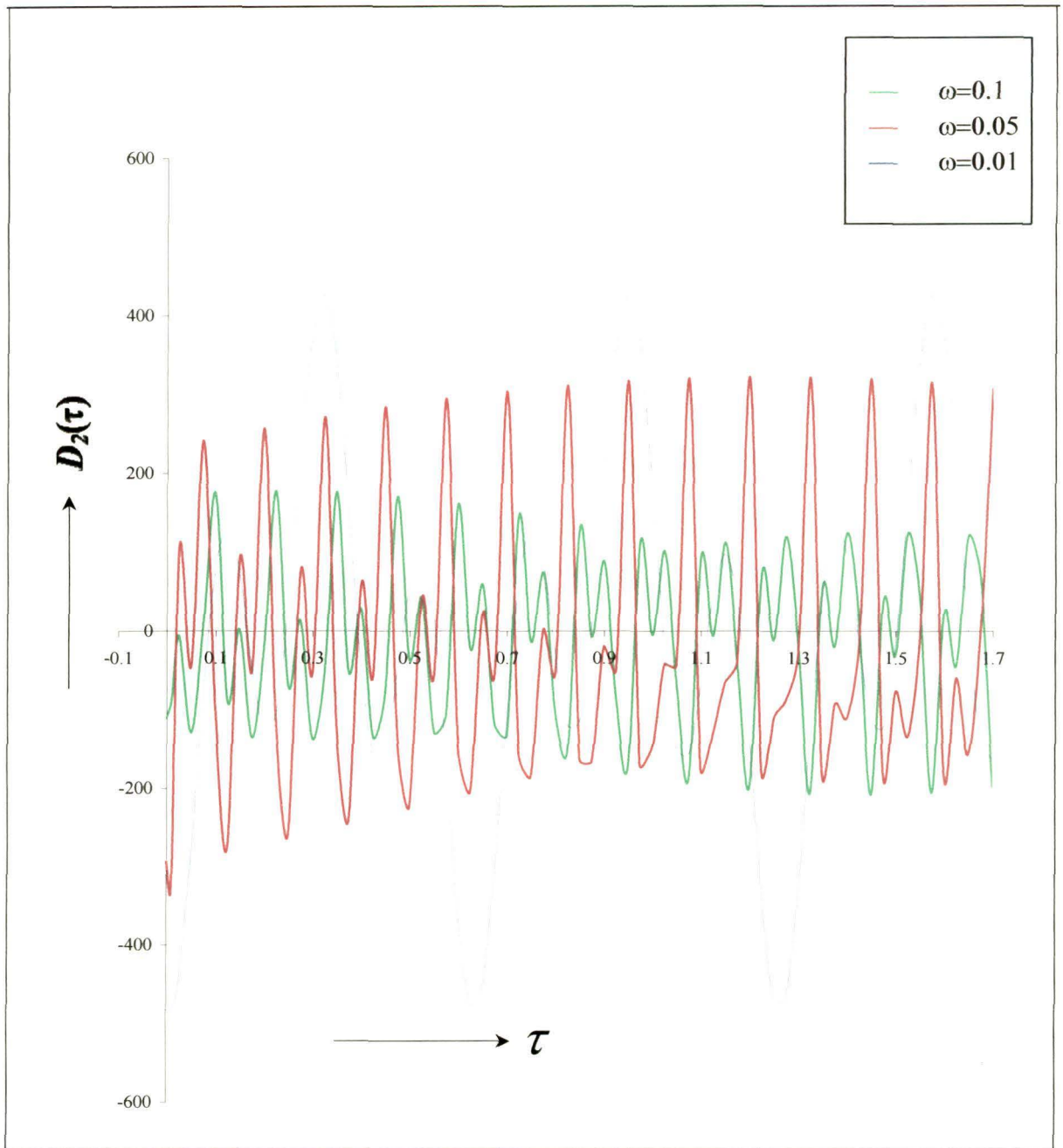


Fig. 3.4 Variation of $D_2(\tau)$ vs. τ for different values of ω with $M=0.3$, $Sc=1000$.

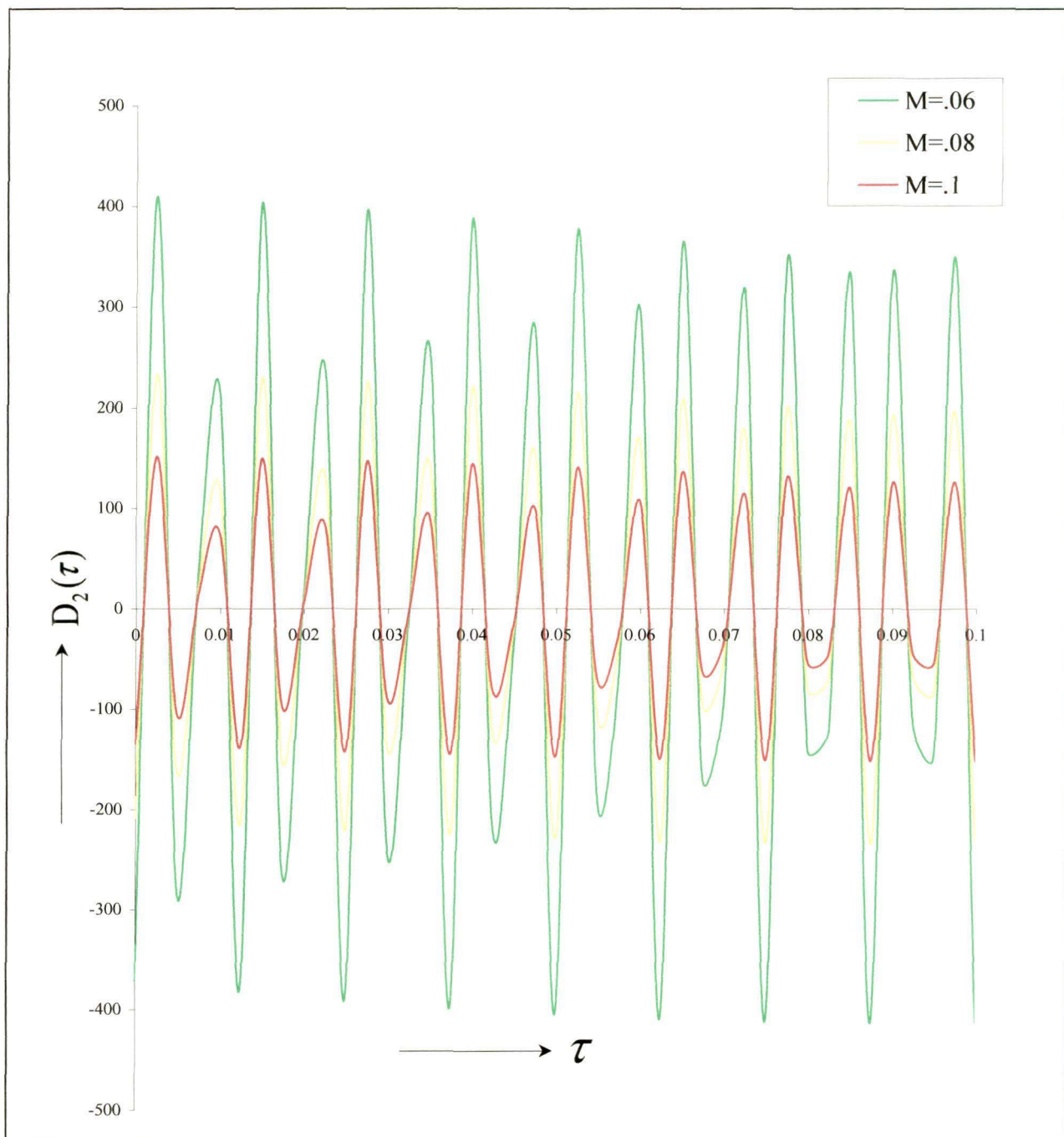


Fig. 3.5 Variation of $D_2(\tau)$ vs. τ for different values of M with $\omega=1$, $Sc=1000$

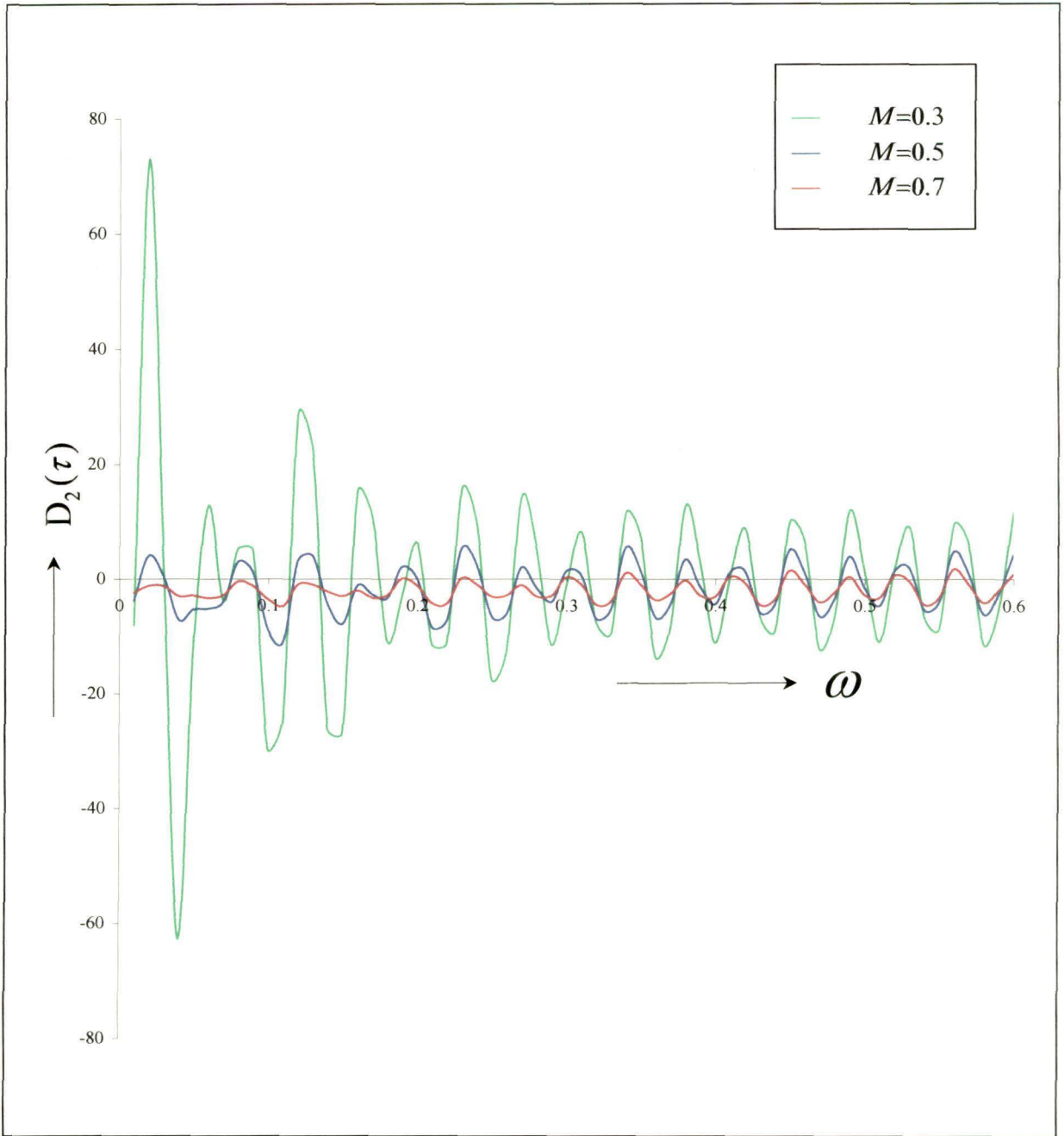


Fig. 3.6 Variation of $D_2(\tau)$ vs. ω for different values of M with $\tau=0.8$, $Sc=1000$

PART TWO

**EXACT ANALYSIS OF UNSTEADY CONVECTIVE
DIFFUSION OF SOLUTE IN TWO-LAYERED MHD
FLOW THROUGH PARALLEL PLATES***

3.4 INTRODUCTION

The longitudinal dispersion of solute in a solvent flowing in a conduit is a phenomenon of wide application in chemical engineering, biomedical engineering, physiological fluid dynamics and environmental science. The basic principle underlying the dispersion theory is the spreading of a passive species in a flowing fluid due to combined effect of molecular diffusion and non-uniform velocity distribution.

The first fundamental study on dispersion was that of Taylor [1] who showed that if a solute is injected into a solvent flowing steadily in a straight tube, the combined effect of lateral molecular diffusion and variation of velocity over the cross-section would cause the solute ultimately to spread diffusively with the effective molecular diffusivity $Deff.$ [$Deff. = \alpha^2 \omega_m^2 / 48 D_m$], where D_m is molecular diffusivity, ω_m is the mean axial velocity and α is the radius of the tube. Aris [2] using the method of moments, showed that the effective molecular diffusivity would be $Deff. = D_m + \alpha^2 \omega_m^2 / 48 D_m$ when the contribution of axial molecular diffusion is also taken into account. Gill [3] generalized Taylor-Aris's work by proposing a series expansion about the mean concentration to describe the local concentration distribution. In a subsequent analysis, Gill and Sankarasubramanian showed that the method of series solution mentioned earlier provides an exact solution for the unsteady convective diffusion problem for laminar flow in a circular tube if the coefficients in the dispersion model are obtained as suitable function of time t . This model is widely referred to as the generalized dispersion

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The steady state all the physical variables except pressure will be function of y only. Further, blood is assumed to be Newtonian fluid. Since blood is a the suspension of red cells in plasma, the cells have a tendency to move away from the walls and this forms a peripheral plasma layer (PPL) near the wall and a core-region consisting of cells and plasma. Since the plasma is an electrically non-conducting fluid, the flow in PPL is not affected by the magnetic field; however, due to electric charges on red cells, the flow in the core region is influenced by the magnetic field.

The velocity field as obtained by Chaturani and Saxena [8] is given as

$$\begin{aligned} u_x &= u_c && \text{when } -(h-\delta) \leq y \leq (h-\delta), \\ &= u_p && \text{" } -h \leq y \leq -(h-\delta), (h-\delta) \leq y \leq h, \end{aligned}$$

where $\eta = \frac{y}{h}$, $M = B_0 h \sqrt{\frac{\sigma}{\eta_c}}$ (M is a non-dimensional number, called Hartmann number);

$$\begin{aligned} u_c &= \frac{P_0}{\sigma B_0^2} - \left\{ \frac{P_0}{\sigma B_0^2} + \frac{P_0 h^2}{2\eta_p} \left(\frac{\delta}{h} \right) \left(\frac{\delta}{h} - 2 \right) \right\} \frac{\cosh M\eta}{\cosh M \left(1 - \frac{\delta}{h} \right)}, \\ u_p &= \frac{P_0 h^2}{2\eta_p} (\eta^2 - 1), \end{aligned}$$

where u_c and u_p are velocities of fluid in the core-region and PPL respectively, σ is electrical conductivity of blood, B_0 is applied constant uniform magnetic field (Fig. 3.7), η_c and η_p are viscosities of the fluid in the core-region and PPL respectively, $P_0 (= -\partial p / \partial x)$ is pressure gradient in the x -direction and δ is the thickness of PPL.

If a solute diffuses in the above fully developed flow, the concentration $c(t, x, y)$ of the solute satisfies

$$\frac{\partial c}{\partial t} = D\nabla^2 c - u_x \frac{\partial c}{\partial x}, \quad \dots (3.39)$$

where D is the molecular diffusivity.

We introduce the dimensionless variables as

$$\left. \begin{aligned} \theta &= \frac{c}{c_0}, & X &= \frac{Dx}{h^2 \bar{u}}, & \tau &= \frac{Dt}{h^2}, \\ \eta &= \frac{y}{h}, & X_s &= \frac{Dx_s}{h^2 \bar{u}}, & Pe &= \frac{\bar{u}h}{D} \end{aligned} \right\} \dots (3.40)$$

where c_0 is the concentration of initial slag input and $\bar{u} = \frac{1}{2} \int_{-1}^1 u_x d\eta$ is the average velocity.

We introduce a new axial co-ordinate moving with the average velocity \bar{u} as $x_1 = x - \bar{u}t$ which in dimensionless form is given as

$$\xi = X - \tau, \quad \text{where } \xi = \frac{Dx_1}{h^2 \bar{u}}. \quad \dots (3.41)$$

Using (3.40) and (3.41) in (3.39), we get

$$\frac{\partial \theta}{\partial \tau} + \Psi(\eta) \frac{\partial \theta}{\partial \xi} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}, \quad \dots (3.42)$$

where $\Psi(\eta) = \frac{u_x}{\bar{u}} - 1$.

$$\Psi(\eta) = \frac{LL_1 - L^2(1 - \frac{L}{3}) + \frac{L_2}{M} \tanh M(1-L) - L_2 \frac{\cosh M\eta}{\cosh M(1-L)}}{Q},$$

$$\text{when } -(1-L) \leq \eta \leq (1-L). \quad \dots (3.43a)$$

$$= \frac{(1-\eta^2) - L^2\left(1 - \frac{L}{3}\right) - L_1(1-L) + \frac{L_2}{M} \tanh M(1-L)}{Q},$$

$$\text{when } -1 \leq \eta \leq -(1-L), (1-L) \leq \eta \leq 1. \quad \dots (3.43b)$$

Velocity field in non-dimensional form is

$$\left. \begin{aligned} U(\eta) &= L_1 - L_2 \frac{\cosh M\eta}{\cosh M(1-L)} \quad \text{when } -(1-L) \leq \eta \leq (1-L) \\ &= 1 - \eta^2 \quad \text{when } -1 \leq \eta \leq -(1-L), (1-L) \leq \eta \leq 1 \end{aligned} \right\} \dots (3.44)$$

$$\text{where } L = \frac{\delta}{h}; \quad L_1 = \frac{2\eta_p}{\sigma B_0^2 h^2}; \quad L_2 = L_1 + L(L-2);$$

$$Q = L^2 \left(1 - \frac{L}{3}\right) + L_1(1-L) - \frac{L_2}{M} \tanh M(1-L).$$

The initial and boundary conditions for (3.42) are

$$\left. \begin{aligned} \theta(0, \xi, \eta) &= 1 \quad \text{for } |\xi| \leq \frac{1}{2} \xi_s, \\ \theta(0, \xi, \eta) &= 0 \quad \text{" } |\xi| > \frac{1}{2} \xi_s, \\ \theta(0, \infty, \eta) &= 0 \\ \text{and } \frac{\partial \theta}{\partial \eta}(\tau, \xi, -1) &= \frac{\partial \theta}{\partial \eta}(\tau, \xi, 1) = 0 \end{aligned} \right\} \dots (3.45)$$

where the last two conditions are consistent with the no mass at the channel walls.

3.6 SOLUTION

We now assume that the solution of equation (3.42) is formulated as a series

$$\text{expansion in } \frac{\partial^k \theta_m}{\partial \xi^k}$$

such that

$$\theta = \theta_m(\tau, \xi) + \sum_{k=1}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial \xi^k}, \quad \dots (3.46)$$

where $\theta_m = \frac{1}{2} \int \theta d\eta$.

Substitution of (3.46) in (3.42) gives

$$\begin{aligned} \frac{\partial \theta_m}{\partial \tau} + \Psi(\eta) \frac{\partial \theta_m}{\partial \xi} - \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial \xi^2} + \sum_{k=1}^{\infty} \left[\left(\frac{\partial f_k}{\partial \tau} - \frac{\partial^2 f_k}{\partial \eta^2} \right) \frac{\partial^k \theta_m}{\partial \xi^k} + \psi(\eta) \frac{\partial^{k+1} \theta_m}{\partial \xi^{k+1}} \right. \\ \left. - \frac{1}{Pe^2} f_k \frac{\partial^{k+2} \theta_m}{\partial \xi^{k+2}} + f_k \frac{\partial^{k+1} \theta_m}{\partial \tau \partial \xi^k} \right] = 0. \quad \dots (3.47) \end{aligned}$$

Following Gill and Sankarasubramanian, we can introduce the generalized dispersion model with time dependent dispersion coefficients as

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial \xi^i}. \quad \dots (3.48)$$

This gives

$$\frac{\partial^{k+1} \theta_m}{\partial \tau \partial \xi^k} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^{i+k} \theta_m}{\partial \xi^{i+k}}. \quad \dots (3.49)$$

Introduction of (3.48) and (3.49) into (3.47) and rearrangement of terms give

$$\begin{aligned} \left[\frac{\partial f_1}{\partial \tau} - \frac{\partial^2 f_1}{\partial \eta^2} + \Psi(\eta) + K_1(\tau) \right] \frac{\partial \theta_m}{\partial \xi} + \left[\frac{\partial f_2}{\partial \tau} - \frac{\partial^2 f_2}{\partial \eta^2} + \psi(\eta) f_1 + f_1 K_1 + K_2 \right. \\ \left. - \frac{1}{Pe^2} \right] \frac{\partial^2 \theta_m}{\partial \xi^2} + \sum_{k=1}^{\infty} \left[\frac{\partial f_{k+2}}{\partial \tau} - \frac{\partial^2 f_{k+2}}{\partial \eta^2} + \psi(\eta) f_{k+1} + f_{k+1} K_1 + \left(K_2 - \frac{1}{Pe^2} \right) f_k \right. \end{aligned}$$

$$-\sum_{i=3}^{k+2} K_i f_{k+2-i} \left] \frac{\partial^{k+2} \theta_m}{\partial \xi^{k+2}} = 0 \quad \dots (3.50)$$

with the understanding that $f_0 = 1$.

Equating the coefficients of $\partial^k \theta_m / \partial \xi^k$ ($k=1,2,3,\dots$) to zero, an infinite set of differential equations are obtained as

$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial \eta^2} - \psi(\eta) - K_1(\tau), \quad \dots (3.51)$$

$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial \eta^2} - \psi(\eta) f_1 - f_1 K_1(\tau) + K_2(\tau) - \frac{1}{Pe^2}, \quad \dots (3.52)$$

⋮

and
$$\frac{\partial f_{k+2}}{\partial \tau} = \frac{\partial^2 f_{k+2}}{\partial \eta^2} - \psi(\eta) f_{k+1} - K_1(\tau) - f_{k+1} - \left\{ K_2(\tau) - \frac{1}{Pe^2} \right\} f_k - \sum_{i=3}^{k+2} K_i(\tau) f_{k+2-i}, \quad \text{where } k = 1, 2, 3, \dots \dots (3.53)$$

Now θ_m will be chosen to satisfy the initial conditions on θ given by (3.45).

The conditions on $f_k(\tau, \eta)$ are given as

$$f_k(0, \eta) = 0, \quad \frac{\partial f_k}{\partial \eta} \Big|_{\tau, \pm 1} = 0, \quad \text{where } k = 1, 2, 3, 4, \dots \dots (3.54)$$

Further equations (3.46) and (3.47) require that

$$\int_{-1}^1 f_k d\eta = 0, \quad k = 1, 2, 3, \dots \dots (3.55)$$

Integrating (3.51) w. r. to η from -1 to 1 and using (3.54) and (3.55) we get

$$K_1(\tau) = 0. \tag{3.56}$$

Following the same procedure, we find from (3.52), (3.54) and (3.55), the expression for $K_2(\tau)$ as

$$K_2(\tau) = \frac{1}{Pe^2} - \frac{1}{2} \int_{-1}^1 \psi(\eta) f_1 d\eta. \tag{3.57}$$

From (3.53), using $f_0 = 1$ we obtain in a similar manner

$$K_{k+2}(\tau) = -\frac{1}{2} \int \psi(\eta) f_{k+1}(\eta) d\eta, \quad k = 1, 2, 3, \dots \tag{3.58}$$

with $K_1(\tau)=0$, we then solve (3.51) subject to (3.54) and (3.55) by using Duhamel’s theorem and get the expression for f_1 as

$$\left. \begin{aligned} f_1(\tau, \eta) &= \frac{1}{Q} \left\{ L_1 \frac{\eta^2}{2} - \frac{L_2 \cosh M\eta}{M^2 \cosh M(1-L)} \right\} - \frac{\eta^2}{2} + C_1 \eta \\ &+ C_2 + \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 \tau} \cos \lambda_n \eta, \quad \text{when } -(1-L) \leq \eta \leq (1-L). \\ &= \frac{1}{Q} \left\{ \frac{\eta^2}{2} - \frac{\eta^4}{12} \right\} - \frac{\eta^2}{2} + C_3 + \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 \tau} \cos \lambda_n \eta, \\ &\quad \text{when } -1 \leq \eta \leq -(1-L), (1-L) \leq \eta \leq 1. \end{aligned} \right\} \tag{3.59}$$

where

$$C_1 = \frac{1}{Q} \left[\frac{2}{3} - L^2 + \frac{L^3}{3} - L_1(1-L) + \frac{L_2}{M} \tanh M(1-L) \right],$$

$$C_2 = \frac{1}{Q} \left[L \left\{ 1 - \frac{(1-L)^2}{6} - L_1 \right\} \frac{(1-L)^2}{2} + \frac{LL_2}{M^2} - \frac{3}{40} - \frac{L_1(1-L)^3}{12} \left\{ \frac{(1-L)^2}{10} - 1 \right\} \right. \\ \left. - \frac{L_1(1-L)^3}{6} + \frac{L_2}{M^3} \tanh M(1-L) \right] + \frac{1}{6} - C_1(1-L)L,$$

$$\begin{aligned}
C_3 &= C_2 - \frac{1}{Q} \left[\left\{ 1 - \frac{(1-L)^2}{6} - L_1 \right\} \frac{(1-L)^2}{2} + \frac{L_2}{M^2} \right] + C_1(1-L), \\
d_n &= \frac{1}{Q} \left[\left\{ \frac{(1-L)^2}{\lambda_n} - \frac{2}{\lambda_n^3} - \frac{(1-L)^4}{6\lambda_n} + \frac{2(1-L)^2}{\lambda_n^3} + \frac{4}{\lambda_n^5} - \frac{L_1(1-L)^2}{\lambda_n} + \frac{2L_1}{\lambda_n^3} \right\} \right. \\
&\quad \times \sin \lambda_n(1-L) - \left. \left\{ \frac{2(1-L)^3}{3\lambda_n^2} - \frac{2(1-L)}{\lambda_n^2} - \frac{4(1-L)}{\lambda_n^4} + \frac{2L_1(1-L)}{\lambda_n^2} \right\} \cos \lambda_n(1-L) \right. \\
&\quad + \frac{L_2 \{ \lambda_n \sin \lambda_n(1-L) \cosh M(1-L) + M \sinh M(1-L) \cos \lambda_n(1-L) \}}{M^2(\lambda_n^2 + M^2) \cosh M(1-L)} \\
&\quad \left. - \left(\frac{4}{3\lambda_n^2} + \frac{4}{\lambda_n^4} \right) \cos \lambda_n \right] - \frac{2(C_2 - C_3)}{\lambda_n} \sin \lambda_n(1-L).
\end{aligned}$$

Substituting (3.59) in (3.57), the expression for $K_2(\tau)$ is obtained as

$$\begin{aligned}
K_2(\tau) - \frac{1}{Pe^2} &= -\frac{1}{2Q} \left[\frac{2}{Q} \left\{ \frac{13}{210} - \frac{(1-L)^3}{6} + \frac{7(1-L)^5}{60} - \frac{(1-L)^7}{84} \right\} + \frac{(1-L)^3}{3} \right. \\
&\quad - \frac{(1-L)^5}{5} + 2C_3L + \frac{2C_3}{3}(1-L)^3 - \frac{2C_3}{3} - \frac{2}{15} + \frac{2}{Q} \left\{ \frac{L_1(1-L)^3}{6} \right. \\
&\quad \left. - \frac{L_1L_2}{M^3} \tanh M(1-L) \right\} - \frac{L_1(1-L)^3}{3} + 2C_2L_1(1-L) - \frac{2L_2}{Q} \\
&\quad \times \left[\frac{L_1(1-L)^2 \tanh M(1-L)}{2M} - \frac{L_1(1-L)}{M^2} + \frac{L_1}{M^3 \tanh M(1-L)} \right. \\
&\quad \left. - \frac{L_2(1-L) \sec^2 M(1-L)}{2M^2} - \frac{L_2}{4M^3} \tanh M(1-L) \sec hM(1-L) \right] \left. \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{Q} \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 \tau} \left[\frac{2(1-L)}{\lambda_n^2} \cos \lambda_n (1-L) - \frac{2}{\lambda_n^2} \cos \lambda_n - \left\{ \frac{1}{\lambda_n} - \frac{(1-L)^2}{\lambda_n} + \frac{2}{\lambda_n^3} \right\} \right. \\
& \times \sin \lambda_n (1-L) + \sin \frac{\lambda_n (1-L)}{\lambda_n} - L_2 \frac{M \lambda_n \cos \lambda_n (1-L) \tanh M(1-L)}{M^2 + \lambda_n^2} \\
& \left. + \frac{\lambda_n^2 \sin \lambda_n (1-L)}{M^2 + \lambda_n^2} \right]. \quad \dots (3.60)
\end{aligned}$$

For large τ , the equation (3.60) reduces to the form

$$\begin{aligned}
K_2^*(\infty) &= K_2(\infty) - \frac{1}{Pe^2} \\
&= -\frac{1}{2Q} \left[\frac{2}{Q} \left\{ \frac{13}{210} - \frac{(1-L)^3}{6} + \frac{7(1-L)^5}{60} - \frac{(1-L)^7}{84} \right\} + \frac{(1-L)^3}{3} - \frac{(1-L)^5}{5} \right. \\
&+ \frac{4C_3 L}{3} + \frac{2C_3(1-L)^3}{3} - \frac{2}{15} + \frac{2}{Q} \left\{ \frac{L_1(1-L)^3}{6} - \frac{L_1 L_2}{M^3} \tanh M(1-L) \right\} \\
&- \frac{L_1(1-L)^3}{3} + 2C_2 L_1(1-L) - \frac{2L_2}{Q} \left\{ \frac{L_1(1-L)^2}{2M} \tanh M(1-L) - \frac{L_1(1-L)}{M^2} \right. \\
&+ \frac{L_1}{M^3} \tanh M(1-L) - \frac{L_2}{2M^2} (1-L) \operatorname{sech}^2 M(1-L) \\
&- \left. \frac{L_2}{4M^3} \tanh M(1-L) \operatorname{sech} M(1-L) \right\} + 2L_2 \left\{ \frac{\tanh M(1-L)}{M^3} - \frac{(1-L)}{M^2} \right\} \\
&\left. - \frac{2C_2 L_2 \tanh M(1-L)}{M} \right]. \quad \dots (3.61)
\end{aligned}$$

The asymptotic form for $f_2(\tau, \eta)$ as $\tau \rightarrow \infty$ is obtained from (3.52) as the solution of

$$\frac{d^2 f_2}{d\eta^2} - \psi(\eta) f_1 - K_2(\tau) + \frac{1}{Pe^2} = 0, \quad \dots (3.62)$$

where $K_1(\tau) = 0$.

Substituting the limiting form of $f_1(\tau, \eta)$ and $K_2(\tau)$ as $\tau \rightarrow \infty$ from (3.59) and (3.60) respectively and using the conditions (3.54) and (3.55) for $k=2$, we get the solution of (3.62) as

$$f_2(\eta) = \left(\frac{1}{Q} - 1\right) \left[\frac{1}{Q} \left\{ \frac{\eta^4}{24} - \frac{\eta^6}{360} \right\} - \frac{\eta^4}{24} + C_3 \frac{\eta^2}{2} \right] - \frac{1}{Q} \left[\frac{1}{Q} \left\{ \frac{\eta^6}{60} - \frac{\eta^8}{672} \right\} - \frac{\eta^6}{60} + \frac{C_3 \eta^4}{12} \right] + K_2^*(\infty) + \alpha_1, \quad \text{when } -1 \leq \eta \leq -(1-L), (1-L) \leq \eta \leq 1. \quad \dots (3.63)$$

$$= \frac{1}{Q} \left[\frac{L_1}{Q} \left\{ \frac{L_1 \eta^4}{24} - \frac{L_2 \cosh M\eta}{M^4 \cosh M(1-L)} \right\} - \frac{L_1}{24} \eta^4 + \frac{L_1 C_1}{6} \eta^3 + \frac{L_1 C_2}{2} \eta^2 \right] - \frac{L_2}{Q \cosh M(1-L)} \left[\frac{1}{Q} \left\{ L_1 \left(\frac{\eta^2}{2M^2} \cosh M\eta - \frac{2\eta}{M^3} \sinh M\eta + \frac{3 \cosh M\eta}{M^4} \right) - \frac{L_2}{2M^2 \cosh M(1-L)} \left(\frac{\eta^2}{2} + \frac{\cosh 2M\eta}{4M^2} \right) \right\} - \left(\frac{\eta^2}{2M^2} \cosh M\eta - \frac{2\eta}{M^3} \sinh M\eta + \frac{3 \cosh M\eta}{M^4} \right) + C_1 \left(\frac{\eta \cosh M\eta}{M^2} - \frac{2 \sinh M\eta}{M^3} \right) + \frac{C_2}{M^2} \cosh M\eta \right] - \left[\frac{1}{Q} \left\{ \frac{L_1 \eta^4}{24} - \frac{L_2 \cosh M\eta}{M^4 \cosh M(1-L)} \right\} - \frac{\eta^4}{24} + \frac{C_1}{6} \eta^3 + \frac{C_2}{2} \eta^2 \right]$$

$$+K_2^*(\infty)\frac{\eta^2}{2} + C''\eta + \alpha_2, \quad \text{when } -(1-L) \leq \eta \leq (1-L). \quad \dots (3.64)$$

where

$$\begin{aligned} \alpha_2 = & \frac{C_2(1-L)^3}{6} + \frac{1}{Q} \left\{ \frac{L_1(1-L)^5}{120} - \frac{L_2}{M^5} \tanh M(1-L) \right\} + \frac{L_2}{Q} \left[\left(\frac{L_1}{Q} - 1 \right) \right. \\ & \times \left. \left\{ \frac{(1-L)^2}{2M^3} \tanh M(1-L) - \frac{3(1-L)}{M^4} + \frac{6}{M^5} \tanh M(1-L) \right\} \right. \\ & \left. - \frac{L_2}{2M^2Q \cosh^2 M(1-L)} \left\{ \frac{(1-L)^3}{6} + \frac{\sinh 2M(1-L)}{8M^3} \right\} + \frac{C_2}{M^3} \tanh M(1-L) \right] \\ & + \frac{1}{Q} \left[\frac{67}{30240Q} - \frac{1}{420} + \frac{C_3}{60} \right] - \frac{(1-L)^5}{120} - \frac{1}{Q} \left[\left(\frac{L_1}{Q} - 1 \right) \frac{L_1(1-L)^5}{120} \right. \\ & \left. - \frac{L_1L_2}{M^5Q} \tanh M(1-L) + \frac{L_1C_2(1-L)^3}{3} \right] - \frac{K_2^*(\infty)}{6} - \left(\frac{1}{Q} - 1 \right) \\ & \times \left[\frac{209}{2520Q} - \frac{1}{120} + \frac{C_3}{6} \right] - \frac{1}{Q} \left[\left(\frac{1}{Q} - 1 \right) \frac{(1-L)^7}{420} - \frac{(1-L)^9}{6048Q} + \frac{C_3(1-L)^5}{60} \right] \\ & + \left(\frac{1}{Q} - 1 \right) \left[\left(\frac{1}{Q} - 1 \right) \frac{(1-L)^5}{120} - \frac{(1-L)^7}{2520Q} + \frac{C_3(1-L)^3}{6} \right] - LA, \\ A = & \frac{1}{Q} \left[\left(\frac{L_1}{Q} - 1 \right) \frac{L_1(1-L)^4}{24} - \frac{L_1L_2}{M^4Q} + \frac{L_1C_1(1-L)^3}{6} + \frac{L_1C_2(1-L)^2}{2} \right] \\ & - \frac{L_2}{Q} \left[\left(\frac{L_1}{Q} - 1 \right) \left\{ \frac{(1-L)^2}{2M^2} - \frac{2(1-L)}{M^3} \tanh M(1-L) + \frac{3}{M^4} \right\} \right. \\ & \left. - \frac{L_2}{2M^2Q \cosh^2 M(1-L)} \left\{ \frac{(1-L)^2}{2} + \frac{\cosh 2M(1-L)}{4M^2} \right\} + C_1 \left\{ \frac{(1-L)}{M^2} \right. \right. \end{aligned}$$

$$\begin{aligned}
& -\frac{2}{M^3} \tanh M(1-L) \left. \right\} + \frac{C_2}{M^2} \left. \right] - \left[\left(\frac{L_1}{Q} - 1 \right) \frac{(1-L)^4}{24} - \frac{L_2}{M^4 Q} + C_1 \frac{(1-L)^3}{6} \right. \\
& \left. + \frac{C_2(1-L)^2}{2} \right] + \frac{K_2^*(\infty)(1-L)^2}{2} + C''(1-L) - \left(\frac{1}{Q} - 1 \right) \left[\left(\frac{1}{Q} - 1 \right) \frac{(1-L)^4}{24} \right. \\
& \left. - \frac{(1-L)^6}{360Q} + \frac{C_3(1-L)^2}{2} \right] + \frac{1}{Q} \left[\left(\frac{1}{Q} - 1 \right) \frac{(1-L)^6}{60} - \frac{(1-L)^8}{672Q} + \frac{C_3(1-L)^4}{12} \right] \\
& - K_2^*(\infty) \frac{(1-L)^2}{2}
\end{aligned}$$

$$\alpha_1 = \alpha_2 + A,$$

$$\begin{aligned}
C'' &= \left(\frac{1}{Q} - 1 \right) \left[\frac{1}{Q} \left\{ \frac{(1-L)^3}{6} - \frac{(1-L)^5}{60} \right\} - \frac{(1-L)^3}{6} + C_3(1-L) \right] - \frac{1}{Q} \left[\frac{1}{Q} \left\{ \frac{(1-L)^5}{10} \right. \right. \\
& \left. \left. - \frac{(1-L)^7}{84} \right\} - \frac{(1-L)^5}{10} + \frac{C_3(1-L)^3}{3} \right] - \frac{1}{Q} \left[\frac{L_1}{Q} \left\{ \frac{L_1(1-L)^3}{6} - \frac{L_2}{M^3} \tanh M(1-L) \right\} \right. \\
& \left. - \frac{L_1(1-L)^3}{6} + \frac{L_1 C_1(1-L)^2}{2} + L_1 C_2(1-L) \right] + \frac{L_2}{Q} \left[\frac{1}{Q} \left\{ \frac{L_1(1-L)^2}{2M} \tanh M(1-L) - \frac{L_1(1-L)}{M^2} \right. \right. \\
& \left. \left. + \frac{L_1}{M^3} \tanh M(1-L) - \frac{L_2}{2M^2} (1-L) \operatorname{sech} M(1-L) - \frac{L_2}{4M^3} \tanh M(1-L) \right\} \right. \\
& \left. - \frac{(1-L)^2}{2M} \sinh M(1-L) + \frac{(1-L)}{M^2} \cosh M(1-L) - \frac{\sinh M(1-L)}{M^3} \right. \\
& \left. + C_1 \left\{ \frac{(1-L)}{M} \sinh M(1-L) - \frac{\cosh M(1-L)}{M^2} \right\} + \frac{C_2}{M} \sinh M(1-L) \right]
\end{aligned}$$

$$+ \left[\frac{1}{Q} \left\{ \frac{L_1(1-L)^3}{6} - \frac{L_2}{M^3} \tanh M(1-L) \right\} - \frac{(1-L)^3}{6} + \frac{C_1(1-L)^2}{2} + C_2(1-L) \right].$$

Substituting the value of $f_2(\eta)$ in (3.58) with $k=1$, the asymptotic form of $K_3(\tau)$ as $\tau \rightarrow \infty$ is given by

$$\begin{aligned} K_3(\infty) = & \left(\frac{1}{Q} - 1 \right) \left[\left(\frac{1}{Q} - 1 \right) \left\{ \left(\frac{1}{Q} - 1 \right) \frac{(1-L)^5}{120} - \frac{(1-L)^7}{2520Q} + \frac{C_3(1-L)^3}{6} - \frac{1}{126Q} \right. \right. \\ & + \left. \frac{1}{120} - \frac{C_3}{6} \right\} - \frac{1}{Q} \left\{ \left(\frac{1}{Q} - 1 \right) \frac{(1-L)^7}{420} - \frac{(1-L)^9}{6048Q} + \frac{C_3(1-L)^5}{60} + \frac{67}{30240Q} \right. \\ & \left. \left. - \frac{1}{420} + \frac{C_3}{60} \right\} + K_2^*(\infty) \left\{ \frac{(1-L)^3}{6} - \frac{1}{6} \right\} - \alpha_1 L \right] - \left(\frac{L_1}{Q} - 1 \right) \left[\frac{1}{Q} \left\{ \left(\frac{L_1}{Q} - 1 \right) \right. \right. \\ & \times \left. \frac{L_1(1-L)^5}{120} - \frac{L_1 L_2}{M^5 Q} \tanh M(1-L) + \frac{L_1 C_2(1-L)^3}{6} \right\} - \frac{L_2}{Q} \left\{ \left(\frac{1}{Q} - 1 \right) \left(\frac{(1-L)^2}{2M^3} \right. \right. \\ & \times \left. \left. \tanh M(1-L) - \frac{3(1-L)}{M^4} + \frac{6}{M^5} \tanh M(1-L) \right) - \frac{L_2}{2M^2 Q \cosh^2 M(1-L)} \right. \\ & \times \left. \left(\frac{(1-L)^3}{6} + \frac{\sinh 2M(1-L)}{8M^3} \right) + \frac{C_2}{M^3} \tanh M(1-L) \right\} - \left[\left(\frac{1}{Q} - 1 \right) \frac{(1-L)^5}{120} \right. \\ & \left. \left. - \frac{L_2}{M^5 Q} \tanh M(1-L) + \frac{C_2(1-L)^3}{6} \right] + \frac{K_2^*(\infty)(1-L)^3}{6} + \alpha_2(1-L) \right] \\ & + \frac{L_2}{Q} \left[\left(\frac{1}{Q} - 1 \right) \frac{L_1}{24Q} \left\{ \frac{(1-L)^4}{M} \tanh M(1-L) - \frac{4}{M^2} (1-L)^3 + \frac{12(1-L)^2}{M^3} \right. \right. \\ & \times \left. \left. \tanh M(1-L) - \frac{24}{M^4} (1-L) + \frac{24}{M^5} \tanh M(1-L) \right\} - \frac{L_1 L_2}{2M^4 Q^2} \right] \end{aligned}$$

$$\begin{aligned}
& \times \left\{ (1-L) \operatorname{sech}^2 M(1-L) + \frac{\sinh 2M(1-L)}{2M \cosh^2 M(1-L)} \right\} + \frac{L_1 C_2}{Q} \left\{ \frac{(1-L)^2}{2M} \right. \\
& \times \left. \tanh M(1-L) - \frac{(1-L)}{M^2} + \frac{\tanh M(1-L)}{M^3} \right\} - \frac{L_2}{Q \cosh^2 M(1-L)} \\
& \times \left(\frac{L_1}{Q} - 1 \right) \left\{ \frac{(1-L)^3}{12M^2} + \frac{(1-L)^2 \sinh 2M(1-L)}{8M^3} - \frac{5(1-L)}{8M^4} \cosh 2M(1-L) \right. \\
& \left. + \frac{17}{16M^5} \sinh 2M(1-L) + \frac{3(1-L)}{2M^4} \right\} + \frac{L_2^2}{2M^2 Q^2 \cosh^2 M(1-L)} \\
& \times \left\{ \frac{(1-L)^2}{2M} \tanh M(1-L) - \frac{(1-L)}{M^2} + \frac{9}{8M^3} \tanh M(1-L) \right. \\
& \left. + \frac{\sinh 3M(1-L)}{24M^3 \cosh M(1-L)} \right\} - \frac{L_2 C_2}{Q \cosh^2 M(1-L)} \left\{ \frac{(1-L)}{2M^2} + \frac{\sinh 2M(1-L)}{4M^3} \right\} \\
& - \frac{1}{24} \left(\frac{L_1}{Q} - 1 \right) \left\{ \frac{(1-L)^4}{M} \tanh M(1-L) - \frac{4(1-L)^3}{M^2} + \frac{12(1-L)^2}{M^3} \tanh M(1-L) \right. \\
& \left. - \frac{24}{M^4} (1-L) + \frac{24}{M^5} \tanh M(1-L) \right\} + \frac{L_2}{2M^4 Q \cosh^2 M(1-L)} \left\{ 1-L \right. \\
& \left. + \frac{\sinh 2M(1-L)}{2M} \right\} - C_2 \left\{ \frac{(1-L)^2}{2M} \tanh M(1-L) - \frac{(1-L)}{M^2} + \frac{\tanh M(1-L)}{M^3} \right\} \\
& + K_2^*(\infty) \left\{ \frac{(1-L)^2}{2M} \tanh M(1-L) - \frac{(1-L)}{M^2} + \frac{\tanh M(1-L)}{M^3} \right\} \\
& + \frac{\alpha_2 \tanh M(1-L)}{M} \left] + \frac{1}{Q} \left[\left(\frac{1}{Q} - 1 \right) \left\{ \frac{16}{2385Q} - \frac{1}{168} + \frac{C_3}{10} \right\} - \frac{1}{Q} \right.
\end{aligned}$$

$$\times \left\{ \frac{571}{332640Q} - \frac{1}{540} + \frac{C_3}{84} \right\} + \frac{K_2^*(\infty)}{10} + \frac{\alpha_1}{3} \Big]. \quad \dots (3.65)$$

The foregoing procedure may be repeated and higher order dispersion coefficients may be determined to any order although computations will be more and more difficult. It is also noted that higher order dispersion coefficients decrease rapidly in magnitude. Neglecting K_3 and higher order dispersion coefficients, equation (3.48) reduces to

$$\frac{\partial \theta_m}{\partial \tau} = K_2(\tau) \frac{\partial^2 \theta_m}{\partial \xi^2}. \quad \dots (3.66)$$

Since a slug is being considered, θ_m will have to satisfy

$$\left. \begin{aligned} \theta_m(0, \xi) &= 1 && \text{when } |\xi| \leq \frac{1}{2} \xi_s \\ &= 0 && \text{" } |\xi| > \frac{1}{2} \xi_s \end{aligned} \right\}, \quad \dots (3.67)$$

$$\theta_m(\tau, \eta) = 0. \quad \dots (3.68)$$

The solution of equation (3.66) subject to (3.51), (3.52), (3.53) is obtained as

$$\theta_m = \frac{1}{2} \left[\operatorname{erf} \frac{\frac{1}{2} X_s - \xi}{2T_0^{\frac{1}{2}}} + \operatorname{erf} \frac{\frac{1}{2} X_s + \xi}{2T_0^{\frac{1}{2}}} \right],$$

where $T_0 = \int_0^{\tau} K_2(\eta) d\eta$.

3.7 RESULTS AND DISCUSSIONS

The time dependent nature of the dispersion coefficient K_2 is illustrated in Fig.-3.8. and Fig.-3.9. The plot of $[K_2(\tau) - Pe^{-2}]$ versus time τ for dispersion in a pipe with different values of L (plug flow parameter) is shown in Fig.-3.8. whereas

the variation of $[K_2(\tau) - Pe^{-2}]$ with τ for different values of magnetic parameter M is shown in Fig.-3.9. It is observed from Fig.-3.8. that initially ($\tau = 0.1$) the values of dispersion coefficient $[K_2(\tau) - Pe^{-2}]$ increase considerably with the increasing values of L [for fixed value of M]. It is also observed that the amount of dispersion becomes slower with the increasing values of L for a certain period of time $\tau < 0.2$ approx. and after that as the time increases ($\tau > 0.6$ approx.), the nature of dispersion is nearly same for all values L and attains a constant value after $\tau \geq 0.6$. The time τ required to reach the steady state is seen to depend on plug flow parameter L . It is observed from the Fig.-3.9 that initially the values of dispersion coefficient $[K_2(\tau) - Pe^{-2}]$ decrease with the increasing values of M . It is further observed that the nature of the graph is nearly same for all values of M and for the increasing values of $\tau > 0.35$ the rate of dispersion becomes steady. In Fig. - 3.10. It is observed that the dispersion coefficient $K_2^*(\infty)$ increases upto a certain value of $L < 0.3$ and then for increasing values of L the dispersion coefficient decreases. It is further observed that there is no effect of magnetic parameter M on the dispersion coefficient for $L=1$. In Fig. - 3.11. It is observed that the dispersion coefficient increases upto a certain value of $L < 0.48$ approx. and then for increasing values of L the dispersion coefficient $K_3(\infty)$ decreases. It is also observed that there is no effect of magnetic parameter M on the dispersion coefficient for $L=1$.

The objective of the present investigations is to study the unsteady dispersion of a solute in the two-layered MHD flow through parallel plates due to its wide practical application. This study may be relevant in understanding many physiological processes, which involve injecting a quantity of solute into the concentration at some downstream point. This analysis can also be applied to artificial blood handling devices such as blood oxy-generators.

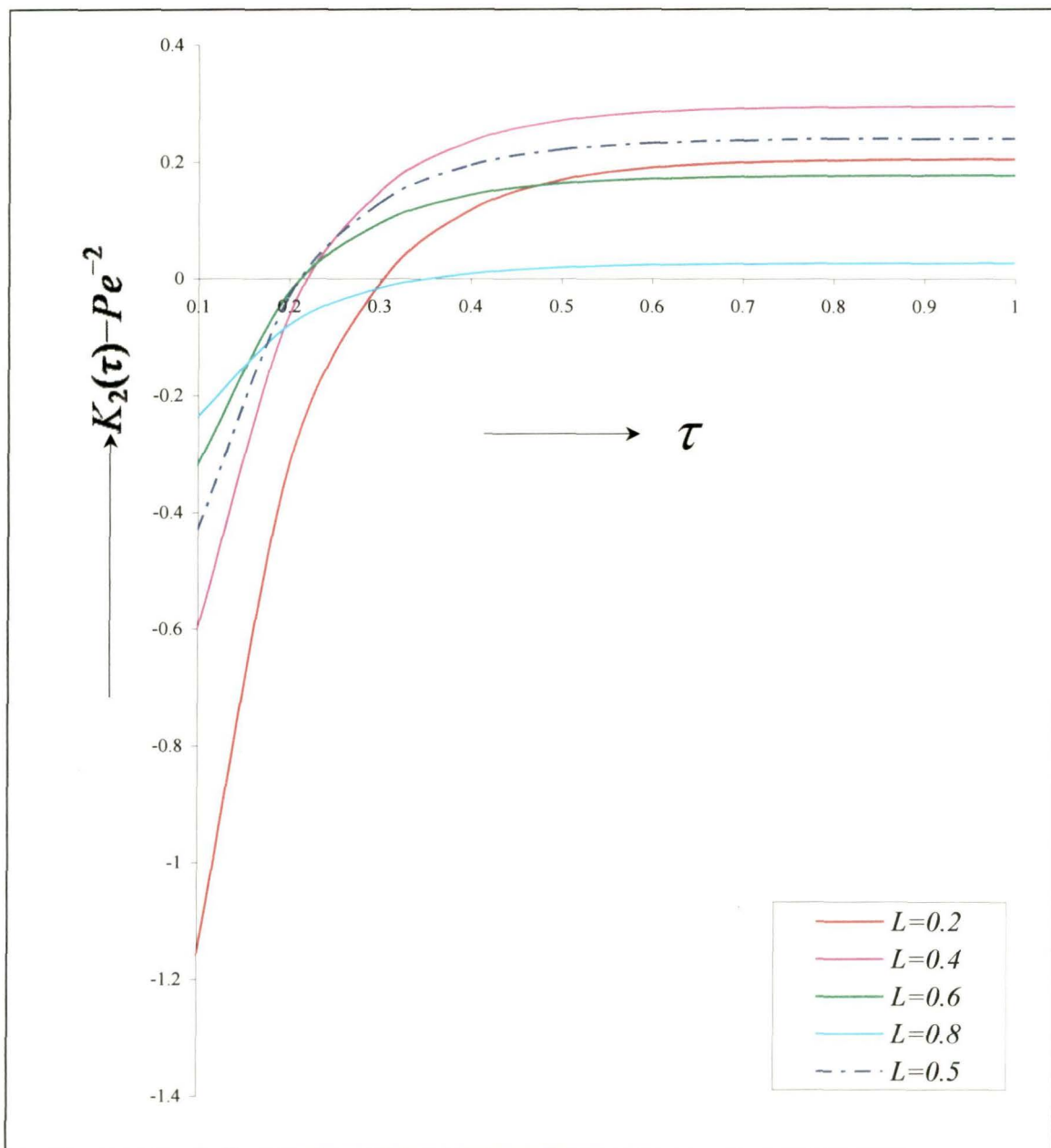


Fig. 3.8 Plot of $[K_2(\tau) - Pe^{-2}]$ against τ for different values of L .

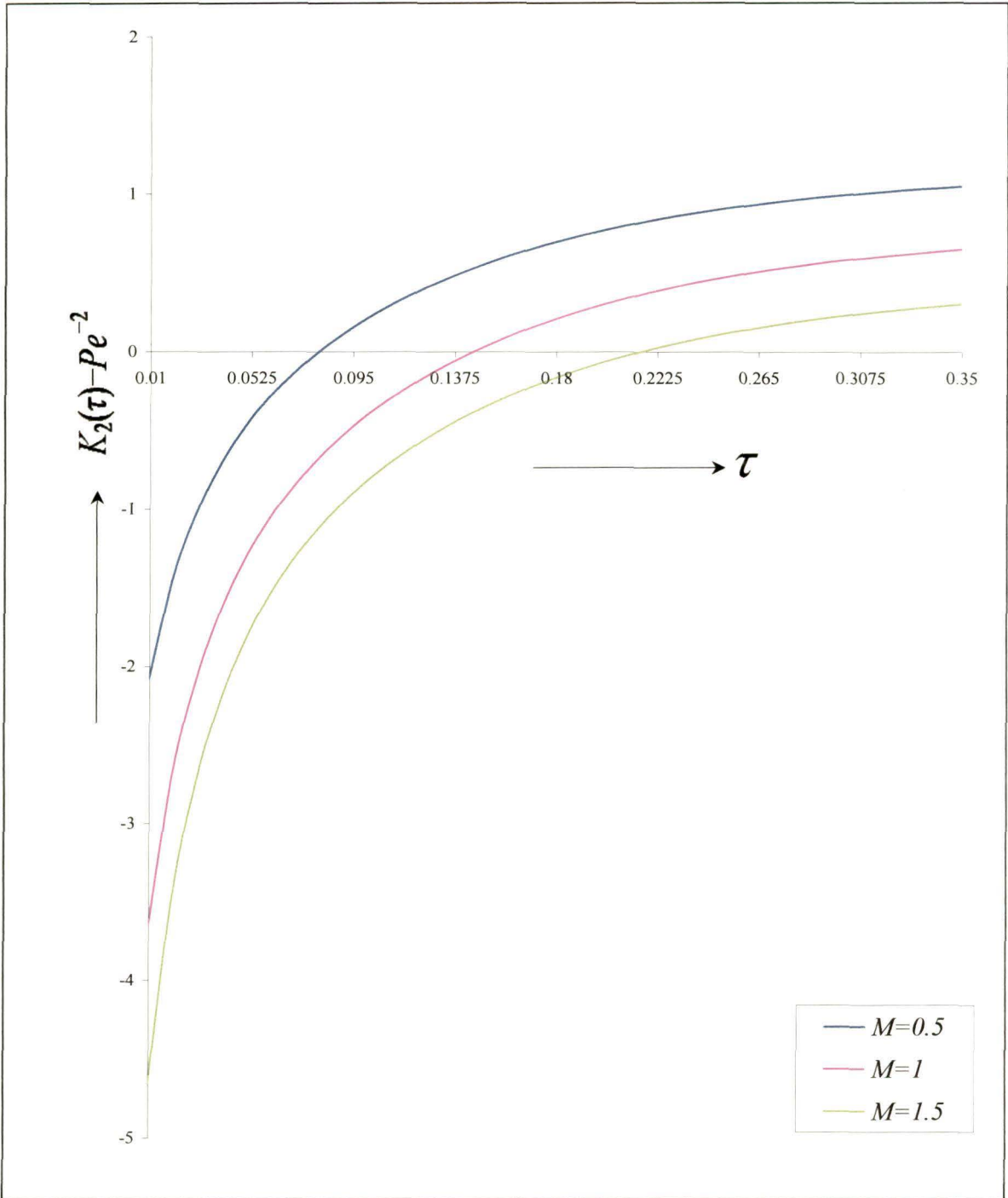


Fig. 3.9 Plot of $[K_2(\tau) - Pe^{-2}]$ against τ for different values of M .

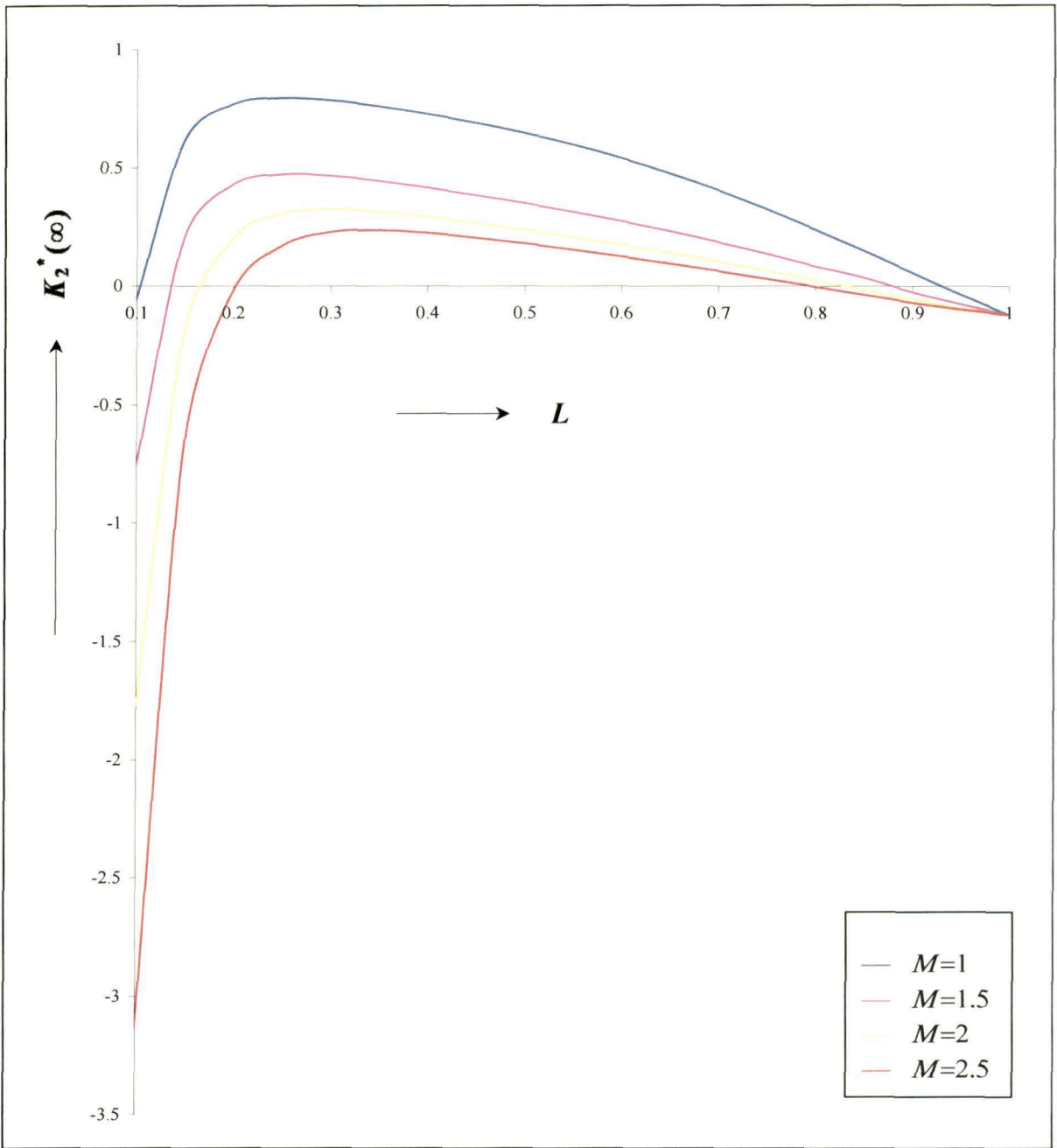


Fig. 3.10 Plot of $K_2^*(\infty)$ against L for different values of M .

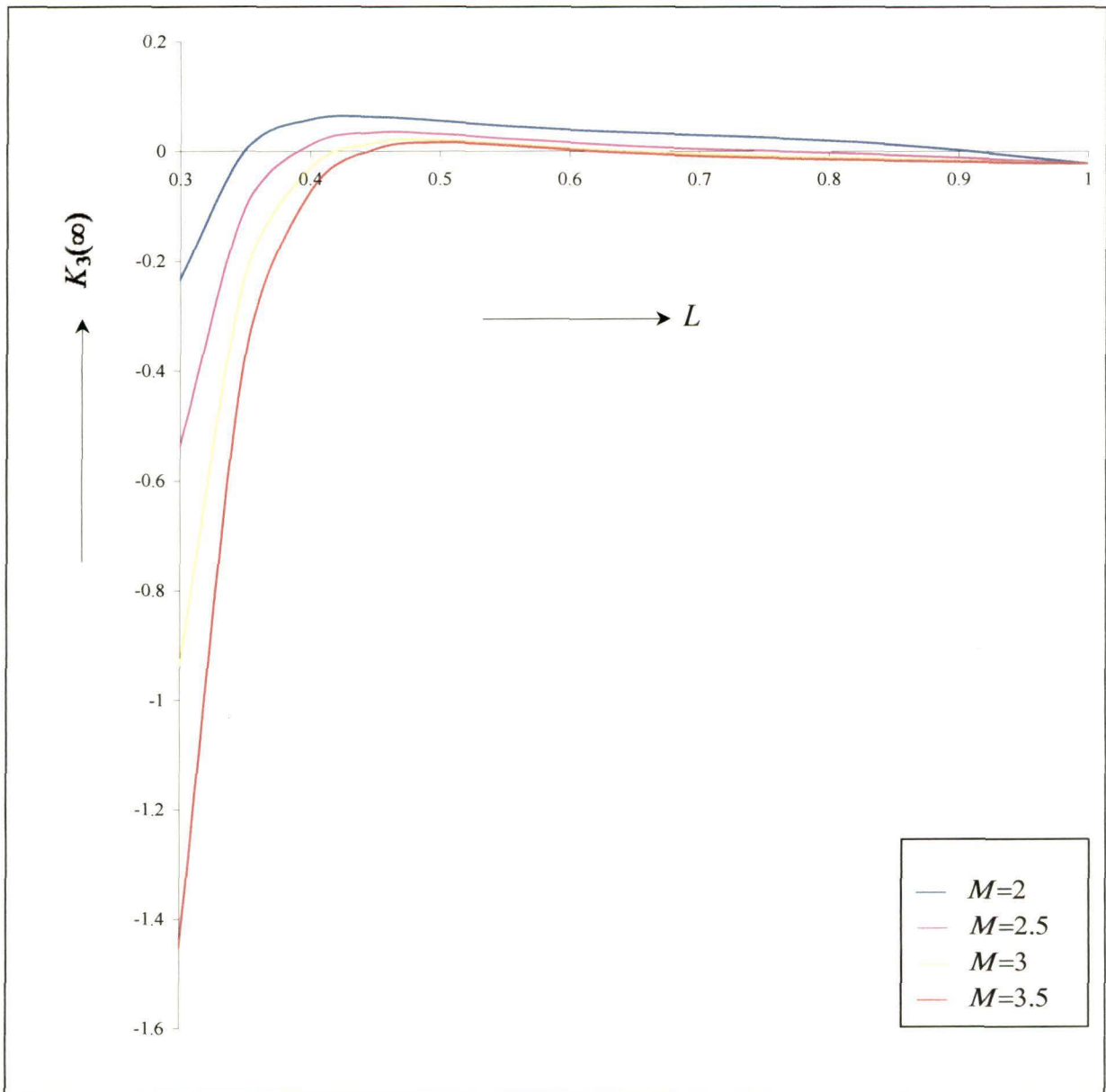


Fig. 3.11 Plot of $K_3(\infty)$ against L for different values of M .

PART THREE

**DISPERSION OF SOLUTE IN OSCILLATING
HYDROMAGNETIC COUETTE FLOW IN A
ROTATING SYSTEM***

3.8 INTRODUCTION

The dispersion of passive impurities or solute in an incompressible viscous fluid flowing in a circular pipe under laminar conditions was investigated by Taylor[1]. He showed that relative to a plane moving with the mean speed of the flow, the solute is dispersed with an effective longitudinal dispersion coefficient $Deff. = \alpha^2 \omega^2 / 48 D_m$ Where D_m is the molecular diffusivity, ω_m is the mean velocity and a is the radius of the tube. However his conceptual model is asymptotically valid for large time. Gill and Sankarasubramanian [3] constructed a dispersion model for problem of convective diffusion, which is valid for all time, by allowing the dispersion coefficients to vary with time. This dependence on time might account for the considerable amount of scatter, which generally found in experimental data on dispersion. Following this generalized dispersion model, Gill and Sankarasubramanian [4], Annapurna and Gupta [9], Mukherjee *et al.* [10] and Layek *et al.* [11] studied the dispersion of solute in different geometrical conditions. Although the generalized dispersion of solute in time-dependent laminar flow which in principle valid for all value of time, they confined their analysis only to cases of dispersion in a fully developed steady flow. Moreover, to the best of our knowledge, the effect of rotation on the dispersion of solute in a time-dependent flowing fluid has not been studied in the literature. But it is well known that earth's rotation plays an important role in the dynamics of thin sheets of fluid. Any dispersion problem involving say, **damping of waste materials in thin sheets of liquids (say river or lakes)**, one would expect that earth's rotation would have significant effect.

The above consideration provides the motivation for our present study. Here we have studied the effect of rotation and transversely applied magnetic field on the dispersion of solute in oscillating hydromagnetic Couette flow. The interesting part of the analysis is that $K_2(\tau)$, second dispersion coefficient consists of a steady part S and a fluctuating part $D_2(\tau)$ due to rotation and oscillation of plate.

3.9 MATHEMATICAL ANALYSIS

An unsteady Couette flow of an electrically conducting, viscous, incompressible fluid is considered between two parallel plates of distance ' δ ' apart. The lower plate is stationary and the upper one is oscillating in its own plane with a velocity $U(t)$ about a non-zero constant mean velocity U_0 . Choose the origin on the lower plate and x -axis parallel to the direction of the upper plate. The z -axis taken perpendicular to the plate, which is the axis of rotation about which the entire system is rotating with constant angular velocity Ω . A transverse magnetic field of uniform strength B_0 is applied along the axis of rotation. Since the plates are infinite in extent, all physical quantities, except the pressure, depend on z and t only. If a solute diffuses in the above fully developed flow, then the concentration $c(t, x, z)$ of solute satisfies

$$\frac{\partial c}{\partial t} + u(z, t) \frac{\partial c}{\partial x} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial z^2} \right), \quad \dots (3.70)$$

where D is the molecular diffusivity.

The initial and boundary conditions are

$$\left. \begin{aligned} c(0, x, z) &= c_0 & \text{for } |x| \leq \frac{x_s}{2} \\ &= 0 & \text{for } |x| > \frac{x_s}{2} \end{aligned} \right\}, \quad \dots (3.70a)$$

$$\frac{\partial c}{\partial z} = 0 \quad \text{at } z = \pm \delta, \quad \dots (3.70b)$$

$$c(t, \infty, z) = 0, \quad \dots (3.70c)$$

where (3.70c) expresses the condition of zero mass flux at the plate walls.

We introduce the dimensionless quantities as

$$\theta = \frac{c}{c_0}, \Psi(\eta, \tau) = \frac{u(z, t)}{\bar{u}}, X = \frac{x_s}{\delta^2 \bar{u}}, \tau = \frac{Dt}{\delta^2}, Pe = \frac{\bar{u} \delta}{D}, \eta = \frac{z}{\delta}, \quad \dots (3.71)$$

where \bar{u} is the time-averaged axial velocity on the central line $z = 0$ given by

$$\bar{u} = \frac{2\pi}{\omega} \int_0^{\omega/2\pi} u(t, 0) dt. \quad \dots (3.72)$$

Using (3.71) in (3.69) and (3.70) we get

$$\frac{\partial \theta}{\partial \tau} + \Psi(\eta, \tau) \frac{\partial \theta}{\partial X} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial \eta^2}. \quad \dots (3.73)$$

The complete velocity field in this flow was determined by Singh [12] which is given as follows

$$\Psi(\eta, \tau) = \frac{U_0}{\bar{u}} \left[q_0(\eta) + \frac{\varepsilon}{2} \left\{ q_1(\eta) e^{i\lambda\tau} + q_2(\eta) e^{-i\lambda\tau} \right\} \right], \quad \dots (3.74)$$

where $q_0(\eta)$, $q_1(\eta)$ and $q_2(\eta)$ are the combination of primary and secondary velocity distribution and

$$q_0(\eta) = 1 - \frac{\sinh(1-\eta)L_1}{\sinh L_1}, \quad \dots (3.75)$$

$$q_1(\eta) = 1 - \frac{\sinh(1-\eta)L_2}{\sinh L_2}, \quad \dots (3.76)$$

$$q_2(\eta) = 1 - \frac{\sinh(1-\eta)L_3}{\sinh L_3}, \quad \dots (3.77)$$

$$\left. \begin{aligned}
 L_1 &= \left(M^2 + 2iK \right)^{\frac{1}{2}}, \quad L_2 = \left(M^2 + 2iK + i\lambda \right)^{\frac{1}{2}}, \\
 L_3 &= \left(M^2 + 2iK - i\lambda \right)^{\frac{1}{2}}, \\
 K &= \frac{\Omega d^2}{\nu} \text{ is the rotation parameter,} \\
 \lambda &= \frac{\omega d^2}{\nu} \text{ is frequency parameter,} \\
 M &= B_0 d \left(\frac{\sigma}{\mu} \right)^{\frac{1}{2}} \text{ is the Hartman number.}
 \end{aligned} \right\} \dots (3.78)$$

The initial and boundary condition are as follows

$$\left. \begin{aligned}
 \theta(0, X, \eta) &= 1 && \text{for } |X| \leq \frac{1}{2} X_s \\
 &= 0 && \text{for } |X| > \frac{1}{2} X_s
 \end{aligned} \right\}, \dots (3.79)$$

$$\theta(\tau, \infty, \eta) = 0, \dots (3.80a)$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, \pm 1) = 0. \dots (3.80b)$$

3.10 SOLUTION

Following Gill and Sankarasubramanian [3], the solution of (3.73) subject to (3.79) and (3.80) is formulated as

$$\theta(\tau, X, \eta) = \sum_{k=0}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial X^k}, \dots (3.81)$$

where

$$\theta_m = \frac{1}{2} \int_{-1}^1 \theta d\eta. \dots (3.82)$$

Substituting (3.81) in (3.73) we get

$$\frac{\partial \theta_m}{\partial \tau} + \Psi_1(\eta, \tau) \frac{\partial \theta_m}{\partial X} - \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} + \sum_{k=0}^{\infty} \left[\left(\frac{\partial f_k}{\partial \tau} - \frac{\partial^2 f_k}{\partial \eta^2} \right) \frac{\partial^k \theta_m}{\partial X^k} + \Psi(\tau, \eta) f_k \frac{\partial^{k+1} \theta_m}{\partial X^{k+1}} - \frac{1}{Pe^2} f_k \frac{\partial^{k+2} \theta_m}{\partial X^{k+2}} + f_k \frac{\partial^{k+1} \theta_m}{\partial \tau \partial X^k} \right] = 0. \quad \dots (3.83)$$

Integration of (3.73) gives upon using (3.82)

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} - \frac{1}{2} \int_{-1}^1 \Psi(\tau, \eta) \frac{\partial \theta}{\partial X} d\eta. \quad \dots (3.84)$$

We assume that the process of distribution of θ_m is diffusive in nature right from time zero. Following Gill and Sankarasubramanian's approach, the generalized dispersion model with time dependent dispersion coefficients can be written as

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial X^i}, \quad \dots (3.85)$$

where

$$K_1(\tau) = -\frac{1}{2} \int_{-1}^1 \Psi(\tau, \eta) f_0(\tau, \eta) d\eta, \quad \dots (3.86a)$$

$$K_2(\tau) = \frac{1}{Pe^2} - \frac{1}{2} \int_{-1}^1 \Psi(\tau, \eta) f_1(\tau, \eta) d\eta, \quad \dots (3.86b)$$

⋮

$$K_{i+2}(\tau) = -\frac{1}{2} \int_{-1}^1 \Psi(\tau, \eta) f_{i+1} d\eta \quad (i=1,2,3,\dots). \quad \dots (3.86c)$$

Substituting (2.85) in (2.83) and equating the coefficients $\partial^k \theta_m / \partial X^k$, we obtain the following equations for $f_k(\tau, \eta)$:

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial \eta^2}, \tag{3.87a}$$

$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial \eta^2} - [\Psi(\tau, \eta) + K_1(\tau)] f_0, \tag{3.87b}$$

$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial \eta^2} + \left[\frac{1}{Pe^2} - K_2(\tau) \right] f_0 - [\Psi(\tau, \eta) + K_1(\tau)] f_1, \tag{3.87c}$$

⋮

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial \eta^2} - [\Psi(\tau, \eta) + K_1(\tau)] f_{k-1} + \left[\frac{1}{Pe^2} - K_2(\tau) \right] f_{k-2} - \sum_{i=3}^k K_i f_{k-i} \quad (k=3,4,5,\dots). \tag{3.87d}$$

Equations (3.79) and (3.82) give

$$\left. \begin{aligned} \theta_m(0, X) &= 1 & \text{for } |X| &\leq \frac{X_s}{2} \\ \theta_m(0, X) &= 0 & \text{for } |X| &> \frac{X_s}{2} \end{aligned} \right\}, \tag{3.88a}$$

$$\theta_m(\tau, \infty) = 0. \tag{3.88b}$$

Now from (3.81), the initial conditions for f_k can be taken as

$$f_0(0, \eta) = 1, \quad f_k(0, \eta) = 0 \quad \text{for } k=1, 2, 3, \dots \tag{3.89}$$

Similarly the boundary conditions for f_k are derived from (3.79), (3.80) and (3.81) as

$$\frac{\partial f_k}{\partial \eta} = 0 \quad \text{at } \eta = \pm 1 \quad (k=0, 1, 2, \dots). \tag{3.90}$$

Further Equations (3.81) and (3.82) are consistent if

$$\int_{-1}^1 f_0 d\eta = 2 \quad \text{and} \quad \int_{-1}^1 f_k d\eta = 0 \quad \text{for } k = 1, 2, 3, \dots \dots (3.91)$$

With the initial and boundary conditions given by (3.89)–(3.91), we determine the functions $f_k(\tau, \eta)$ by solving the system of Equations (3.87a)–(3.87d) and then obtain the diffusion coefficients $K_i(\tau)$ from (3.86a)–(3.86c).

We next proceed to determine $K_1(\tau)$. It can be readily shown that the solution of (3.87a) subject to (3.89)–(3.91) is given by

$$f_0(\tau, \eta) = 1. \dots (3.92)$$

Substitution from (3.74) and (3.92) in (3.86a) then gives

$$K_1(\tau) = -\frac{1}{P} \left[2 + 2\varepsilon \cos \lambda \tau + \frac{1 - \cosh 2\delta L_1}{\delta L_1 \sinh 2\delta L_1} + \frac{\varepsilon}{\delta} (A \cos \lambda \tau - B \sin \lambda \tau) \right], \dots (3.93)$$

where

$$\left. \begin{aligned} P &= \frac{\sinh 2\delta L_1 - \sinh \delta L_1}{\sinh 2\delta L_1}, \\ A &= \frac{1}{2} \left\{ \frac{1 - \cosh 2\delta L_2}{L_2 \sinh 2\delta L_2} + \frac{1 - \cosh 2\delta L_3}{L_3 \sinh 2\delta L_3} \right\}, \\ B &= -\frac{i}{2} \left\{ \frac{1 - \cosh 2\delta L_2}{L_2 \sinh 2\delta L_2} - \frac{1 - \cosh 2\delta L_3}{L_3 \sinh 2\delta L_3} \right\}. \end{aligned} \right\} \dots (3.94)$$

With the help of Duhamel’s theorem and using (3.74) and (3.93) respectively, we solve equation (3.87b). The result is as follows

$$f_1(\tau, \eta) = \frac{1}{\delta P L_1 \sinh 2\delta L_1} \left[\frac{\sinh(1-\eta)\delta L_1}{-\delta L_1} - \frac{(1 - \cosh 2\delta L_1)}{4} \eta^2 \right]$$

$$\begin{aligned}
& + \frac{\varepsilon}{2P\delta} \left[\frac{\sinh(1-\eta)\delta L_2}{\delta L_2^2 \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{\sinh(1-\eta)\delta L_3}{\delta L_3^2 \sinh 2\delta L_3} e^{-i\lambda\tau} - (A \cos \lambda\tau - B \sin \lambda\tau) \right. \\
& \left. \times \frac{\eta^2}{2} \right] - \frac{\varepsilon}{2P\delta} \left\{ (2C + A) \cos \lambda\tau + (2\bar{D} - B) \sin \lambda\tau \right\} \eta + \frac{1}{2\delta P L_1 \sinh 2\delta L_1} \\
& \times \left[\frac{(\cosh 2\delta L_1 - 1)}{\delta^2 L_1^2} + \frac{1}{6} (1 - \cosh 2\delta L_1) \right] - \frac{(1 + \cosh 2\delta L_1)}{2\delta P L_1 \sinh 2\delta L_1} \eta + \frac{\varepsilon}{2P\delta} \times \\
& \left[\frac{(\cosh 2\delta L_2 - 1)}{\delta^3 L_2^3 \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{(\cosh 2\delta L_3 - 1)}{\delta^2 L_3^3 \sinh 2\delta L_3} e^{-i\lambda\tau} + \frac{1}{3} (A \cos \lambda\tau - B \sin \lambda\tau) \right] \\
& + \sum_{n=1}^{\infty} \left[\frac{(1 - \cosh 2\delta L_1) \cos \lambda_n}{\delta P L_1 \sinh 2\delta L_1} \left(\frac{\delta L_1}{\lambda_n^2 + \delta^2 L_1^2} + \frac{1}{\lambda_n^2} \right) e^{-\lambda_n^2 \tau} - \frac{\varepsilon \cos \lambda_n}{2 P \delta} \times \right. \\
& \left. \left\{ \frac{(\cosh 2\delta L_2 - 1)}{L_2^2 \sinh 2\delta L_2} \frac{(i\lambda e^{i\lambda\tau} + \lambda_n^2 e^{-\lambda_n^2 \tau})}{(\lambda_n^2 + \delta^2 L_2^2)(\lambda_n^2 + i\lambda)} + \frac{(\cosh 2\delta L_3 - 1)}{L_3^2 \sinh 2\delta L_3 (\lambda_n^2 + \delta^2 L_3^2)} \right. \right. \\
& \left. \left. \frac{(\lambda_n^2 e^{-\lambda_n^2 \tau} - i\lambda e^{-i\lambda\tau})}{(\lambda_n^2 - i\lambda)} - \frac{2}{\lambda_n^2} \left(\frac{A(\lambda^2 \cos \lambda\tau - \lambda_n^2 \lambda \sin \lambda\tau)}{\lambda_n^4 + \lambda^2} + \frac{A\lambda_n^4 e^{-\lambda_n^2 \tau}}{\lambda_n^4 + \lambda^2} \right. \right. \right. \\
& \left. \left. \left. \frac{B(\lambda_n^2 \lambda \cos \lambda\tau + \lambda^2 \sin \lambda\tau - \lambda_n^2 \lambda e^{-\lambda_n^2 \tau})}{\lambda_n^4 + \lambda^2} \right) \right\} \right], \quad \dots (3.95)
\end{aligned}$$

where

$$\lambda_n = n\pi,$$

$$C = \frac{1}{2} \left\{ \frac{\cosh 2\delta L_2}{L_2 \sinh 2\delta L_2} + \frac{\cosh 2\delta L_3}{L_3 \sinh 2\delta L_3} \right\}, \quad \bar{D} = -\frac{i}{2} \left\{ \frac{\cosh 2\delta L_3}{L_3 \sinh 2\delta L_3} - \frac{\cosh 2\delta L_2}{L_2 \sinh 2\delta L_2} \right\}.$$

Substituting (3.74) and (3.95) in (3.86b), we obtain the diffusion coefficient $K_2(\tau)$ as follows

$$K_2(\tau) = \frac{1}{Pe^2} + S + D_2(\tau), \quad \dots (3.96)$$

where S and $D_2(\tau)$ represent the steady and time dependent part of the diffusion coefficient respectively.

The expression for S and $D_2(\tau)$ are given as

$$\begin{aligned} S = & \frac{1}{P\delta \{ \sinh 2\delta L_1 - \sinh \delta L_1 \}} \left[\frac{2 \sinh^2 \delta L_1}{\delta^2 L_1^3} - \frac{(1 - \cosh 2\delta L_1)}{2L_1} \left\{ \frac{1}{3} - \frac{1}{2 \sinh 2\delta L_1} \right. \right. \\ & \times \left. \left. \left(\frac{2 \sinh^2 \delta L_1}{\delta L_1} + \frac{4 \sinh^2 \delta L_1}{L_1^3 \delta^3} - \frac{4 \cosh \delta L_1 \sinh \delta L_1}{L_1^2 \delta^2} \right) \right\} - \frac{\varepsilon^2 \sinh 2\delta L_1}{8\delta^2 \sinh 2\delta L_2 \sinh 2\delta L_3} \right. \\ & \times \left(\frac{\sinh 2(L_2 + L_3)\delta}{L_2 + L_3} - \frac{\sinh 2(L_2 - L_3)\delta}{L_2 - L_3} \right) \left(\frac{1}{L_2^2} + \frac{1}{L_3^2} \right) + \frac{(1 + \cosh 2\delta L_1) \cosh \delta M}{\delta L_1^2 \sinh 2\delta L_1} \\ & \times \left(\frac{\sinh \delta L_1}{L_1 \delta} - \cosh \delta L_1 \right) + \frac{(\cosh 2\delta L_1 - 1)}{L_1} \left(\frac{1}{\delta^2 L_1^2} - \frac{1}{6} \right) \left(1 - \frac{\sinh^2 \delta L_1}{\delta L_1 \sinh 2\delta L_1} \right) - \\ & \frac{\varepsilon^2 \sinh 2\delta L_1}{2\delta^3 \sinh 2\delta L_2 \sinh 2\delta L_3} \left\{ \frac{(\cosh 2\delta L_2 - 1) \sinh^2 \delta L_3}{L_3 L_2^3} + \right. \\ & \left. \frac{(\cosh 2\delta L_3 - 1) \sinh^2 \delta L_2}{L_2 L_3^3} \right\} \left. \right]. \quad \dots (3.97) \end{aligned}$$

$$\begin{aligned} D_2(\tau) = & \frac{1}{\delta P^2 L_1 \sinh 2\delta L_1} \left[\frac{\varepsilon e^{i\lambda\tau}}{2\delta L_1 \sinh \delta L_2} \left(\frac{\sinh 2(L_2 + L_1)\delta}{2(L_2 + L_1)\delta} - \frac{\sinh 2(L_2 - L_1)\delta}{2(L_2 - L_1)\delta} \right) \right. \\ & \left. + \frac{\varepsilon e^{-i\lambda\tau}}{2\delta L_1 \sinh \delta L_3} \left(\frac{\sinh 2(L_1 + L_3)\delta}{2(L_1 + L_3)\delta} - \frac{\sinh 2(L_1 - L_3)\delta}{2(L_1 - L_3)\delta} \right) + \varepsilon \cos \lambda\tau \left\{ \frac{(1 - \cosh 2\delta L_1)}{6} \right. \right. \end{aligned}$$

$$\left. -\frac{2\sinh^2 \delta L_1}{\delta^2 L_1^2} \right\} + \frac{\varepsilon e^{i\lambda\tau} (1 - \cosh 2\delta L_1)}{8\sinh \delta L_2} \left\{ \frac{\sinh^2 \delta L_2}{\delta L_2} + \frac{4\sinh^2 \delta L_2}{\delta^3 L_2^3} - \frac{2\sinh 2\delta L_2}{L_2^2 \delta^2} \right\}$$

$$+ \frac{\varepsilon e^{-i\lambda\tau} (1 - \cosh 2\delta L_1)}{8\sinh \delta L_3} \left\{ \frac{4\sinh^2 \delta L_3}{\delta^3 L_3^3} - \frac{2\sinh 2\delta L_3}{\delta^2 L_3^2} + \frac{\sinh^2 \delta L_3}{\delta L_3} \right\} \left] - \frac{\varepsilon}{2\delta P^2} \times$$

$$\left[\frac{2e^{i\lambda\tau} \sinh^2 \delta L_2}{\delta^2 L_2^3 \sinh 2\delta L_2} + \frac{2e^{-i\lambda\tau} \sinh^2 \delta L_3}{\delta^2 L_3^3 \sinh 2\delta L_3} - \frac{e^{i\lambda\tau}}{\delta L_2^2 \sinh 2\delta L_1 \sinh 2\delta L_2} \right] \times$$

$$\left\{ \frac{\sinh 2(L_2 + L_1)\delta}{2(L_2 + L_1)\delta} - \frac{\sinh 2(L_2 - L_1)\delta}{2(L_2 - L_1)\delta} \right\} - \frac{e^{-i\lambda\tau}}{\delta L_3^2 \sinh 2\delta L_1 \sinh 2\delta L_3}$$

$$\left\{ \frac{\sinh 2(L_3 + L_1)\delta}{2(L_3 + L_1)\delta} - \frac{\sinh 2(L_1 - L_3)\delta}{2(L_1 - L_3)\delta} \right\} - \frac{\varepsilon e^{2i\lambda\tau}}{2\delta L_2^2 \sinh^2 2\delta L_2} \left\{ \frac{\sinh 4\delta L_2}{4\delta L_2} - 1 \right\}$$

$$+ \frac{2\varepsilon \cos \lambda\tau \sinh^2 \delta L_3}{\delta^2 L_3^3 \sinh 2\delta L_3} e^{-i\lambda\tau} - (A \cos \lambda\tau - B \sin \lambda\tau) \left\{ \frac{2}{3} - \frac{1}{\sinh 2\delta L_1} \left(\frac{\sinh^2 \delta L_1}{\delta L_1} \right. \right.$$

$$\left. \left. + \frac{2\sinh^2 \delta L_1}{L_1^3 \delta^3} - \frac{\sinh 2\delta L_1}{L_1^2 \delta^2} \right) \right\} - \frac{\varepsilon e^{i\lambda\tau}}{2\sinh 2\delta L_2} \left\{ \frac{2\sinh^2 \delta L_2}{\delta L_2} + \frac{4\sinh^2 \delta L_2}{L_2^3 \delta^3} \right.$$

$$\begin{aligned}
& -\frac{2\sinh 2\delta L_2}{L_2^2 \delta^2} \left\{ \frac{\varepsilon e^{-i\lambda\tau}}{2\sinh 2\delta L_3} \left[\frac{2\sinh^2 \delta L_3}{\delta L_3} + \frac{4\sinh^2 \delta L_3}{L_3^3 \delta^3} - \frac{2\sinh 2\delta L_3}{L_3^2 \delta^2} \right] \right. \\
& + \frac{2}{3} \varepsilon \cos \lambda\tau \left. \right\} - \frac{1}{P} \left[\frac{(1 + \cosh 2\delta L_1)}{2P\delta L_1 \sinh 2\delta L_1} \left\{ \frac{\varepsilon e^{i\lambda\tau}}{2\sinh 2\delta L_2} \left(\frac{\sinh 2\delta L_2}{L_2^2 \delta^2} - \frac{2\cosh^2 \delta L_2}{\delta L_2} \right) \right. \right. \\
& + \frac{\varepsilon e^{-i\lambda\tau}}{2\sinh 2\delta L_3} \left(\frac{\sinh 2\delta L_3}{L_3^2 \delta^2} - \frac{2\cosh^2 \delta L_3}{\delta L_3} \right) \left. \right\} + \frac{\varepsilon}{2P\delta \sinh 2\delta L_1} \left(\frac{\sinh 2\delta L_1}{L_1^2 \delta^2} - \right. \\
& \left. \frac{2\cosh^2 \delta L_1}{L_1 \delta} \right) \left\{ (2C + A) \cos \lambda\tau + (2\bar{D} - B) \sin \lambda\tau \right\} + \frac{\varepsilon^2}{2P\delta} \left\{ (2C + A) \cos \lambda\tau \right. \\
& + (2\bar{D} - B) \sin \lambda\tau \left. \right\} \left\{ \frac{e^{i\lambda\tau}}{2\sinh 2\delta L_2} \left(\frac{\sinh 2\delta L_2}{L_2^2 \delta^2} - \frac{2\cosh^2 \delta L_2}{\delta L_2} \right) + \frac{e^{-i\lambda\tau}}{2\sinh 2\delta L_3} \right. \\
& \left. \left. \times \left(\frac{\sinh 2\delta L_3}{L_3^2 \delta^2} - \frac{2\cosh^2 \delta L_3}{\delta L_3} \right) \right\} \right] + \frac{1}{P} \left[\frac{\varepsilon (\cosh 2\delta L_1 - 1)}{2\delta P L_1 \sinh 2\delta L_1} \left(\frac{1}{\delta^2 L_1^2} - \frac{1}{6} \right) \right. \\
& \left. \times \left\{ \frac{e^{i\lambda\tau} \sinh^2 \delta L_2}{\delta L_2 \sinh 2\delta L_2} + \frac{e^{-i\lambda\tau} \sinh^2 \delta L_3}{\delta L_3 \sinh 2\delta L_3} - 2\cos \lambda\tau \right\} - \frac{\varepsilon}{P\delta} \left(1 - \frac{\sinh^2 \delta L_1}{\delta L_1 \sinh 2\delta L_1} \right) \right] \\
& \left\{ \frac{(\cosh 2\delta L_2 - 1)}{\delta^2 L_2^3 \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{(\cosh 2\delta L_3 - 1)}{\delta^2 L_3^3 \sinh 2\delta L_3} e^{-i\lambda\tau} + \frac{1}{3} (A \cos \lambda\tau - B \sin \lambda\tau) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\varepsilon^2}{2P\delta} \left\{ \frac{(\cosh 2\delta L_2 - 1)}{\delta^2 L_2^3 \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{(\cosh 2\delta L_3 - 1)}{\delta^2 L_3^3 \sinh 2\delta L_3} e^{-i\lambda\tau} \right. \\
& + \frac{1}{3} (A \cos \lambda\tau - B \sin \lambda\tau) \left. \left\{ \frac{\sinh^2 \delta L_2}{\delta L_2 \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{\sinh^2 \delta L_3}{\delta L_3 \sinh 2\delta L_3} e^{-i\lambda\tau} \right. \right. \\
& \left. \left. - 2 \cos \lambda\tau \right\} \right] + \sum_{n=1}^{\infty} \frac{1}{P} \left[\frac{\cos \lambda_n (1 - \cosh 2\delta L_1)}{\delta P L_1 \sinh 2\delta L_1} \left(\frac{\delta L_1}{\lambda_n^2 + \delta^2 L_1^2} + \frac{1}{\lambda_n^2} \right) e^{-\lambda_n^2 \tau} \right. \\
& \left. - \frac{\varepsilon \cos \lambda_n}{2P\delta} \left\{ \frac{(\cosh 2\delta L_2 - 1)(i\lambda e^{i\lambda\tau} + \lambda_n^2 e^{-\lambda_n^2 \tau})}{L_2^2 (\lambda_n^2 + \delta^2 L_2^2) (\lambda_n^2 + i\lambda) \sinh 2\delta L_2} + \frac{(\cosh 2\delta L_3 - 1)}{L_3^2 (\lambda_n^2 + \delta^2 L_3^2)} \right. \right. \\
& \left. \left. \times \frac{(\lambda_n^2 e^{-\lambda_n^2 \tau} - i\lambda e^{-i\lambda\tau})}{(\lambda_n^2 - i\lambda) \sinh 2\delta L_3} - \frac{2}{\lambda_n^2} \left(A \frac{\lambda^2 \cos \lambda\tau - \lambda_n^2 \lambda \sin \lambda\tau}{\lambda_n^4 + \lambda^2} + A \frac{\lambda_n^4}{\lambda_n^4 + \lambda^2} \times \right. \right. \right. \\
& \left. \left. \left. e^{-\lambda_n^2 \tau} - B \frac{(\lambda_n^2 \lambda \cos \lambda\tau + \lambda^2 \sin \lambda\tau)}{\lambda_n^4 + \lambda^2} + B \frac{\lambda_n^2 \lambda e^{-\lambda_n^2 \tau}}{\lambda_n^4 + \lambda^2} \right) \right\} \right] \times \left[\frac{L_1 \delta \cos \lambda_n}{(\lambda_n^2 + L_1^2 \delta^2)} \right. \\
& \left. \times \frac{\cosh 2\delta L_1 - 1}{\sinh 2\delta L_1} + \frac{\varepsilon \delta L_2 \cos \lambda_n (\cosh 2\delta L_2 - 1)}{2(\lambda_n^2 + L_2^2 \delta^2) \sinh 2\delta L_2} e^{i\lambda\tau} + \frac{\varepsilon \delta L_3 \cos \lambda_n}{2(\lambda_n^2 + L_3^2 \delta^2)} \right. \\
& \left. \times \frac{(\cosh 2\delta L_3 - 1)}{\sinh 2\delta L_3} e^{-i\lambda\tau} \right]. \quad \dots (3.98)
\end{aligned}$$

3.11 RESULTS AND DISCUSSION

It is seen that an exact solution to the convective diffusion equation can be constructed in a rather simple way by using the series expansion proposed by Gill

and Sankarasubramanian [2]. This exact solution involves a dispersion model that includes third order and higher order derivative of θ_m w.r.to X . Fortunately the coefficient $K_{i+2}(\tau)$ ($i=1,2,3,4,\dots$) can be calculated from equation (3.86c) and are very small compared to $K_2(\tau)$. Therefore, to a good approximation, the higher order terms in equation (3.85) can be neglected in determining θ_m . Neglecting $K_3(\tau)$ and higher order dispersion coefficients equation (3.85) can be written as

$$\frac{\partial \theta_m}{\partial \tau} = K_1(\tau) \frac{\partial \theta_m}{\partial X} + K_2(\tau) \frac{\partial^2 \theta_m}{\partial X^2} \quad \dots (3.99)$$

The solution of equation (3.99) subject to (3.88a), (3.88 b) is obtained as

$$\theta_m = \frac{1}{2} \left[\operatorname{erf} \frac{\frac{1}{2} X_s + X_1}{2T_0^{\frac{1}{2}}} + \operatorname{erf} \frac{\frac{1}{2} X_s - X_1}{2T_0^{\frac{1}{2}}} \right],$$

where

$$X_1 = X + \int_0^\tau K_1(\eta) d\eta, \quad T_0 = \int_0^\tau K_2(\eta) d\eta.$$

It reveals that the dimensionless solute concentration in oscillating hydro magnetic Couette flow in a rotating system mainly depends on Hartman number M and rotation parameter K . So we have computed dispersion coefficient $K_2(\tau)$ for several values of M and K . It is interesting to note that the dispersion of solute in unsteady flow arising out of rotation of entire system gives rise to dispersion coefficient, which consists of both steady and fluctuating part. We have computed S the steady part of dispersion coefficient for various values of M (Hartman no.) and K (rotation parameter) with $Sc=1000$ (Schmidt number) and plotted in Fig.-3.12. From Fig.-3.12. it is evident that S increases as M increases and attains maximum nearly at $M=1$ and then slowly decreases with increasing M . Fig.-3.12. also reveals that value of S decreases as K (rotation parameter) increases. Naturally, the effect of

moderate or large values of K and M has less or no effect on the steady part S of dispersion coefficient. The fluctuating part due to τ of dispersion coefficient $K_2(\tau)$ is evaluated for different values of τ , M , K for $Sc=1000$. Fig.-3.13. shows the plot of $D_2(\tau)$ with τ for several values of M . The plot of $D_2(\tau)$ is oscillating in nature and amplitude of oscillation is more in case of higher values of M . It is also seen that moderate or large values of M has more effect on fluctuating part than steady part. Fig.-3.14. shows the variation of $D_2(\tau)$ with τ for different values of K and amplitude of oscillation decreases as K increases. Clearly the effect of rotation parameter K on the fluctuating part of dispersion coefficient is much more than steady part. Present analysis reveals the striking results that the dispersion of solute in oscillating hydro magnetic Couette flow is affected by the unsteadiness of the flow due to rotation.

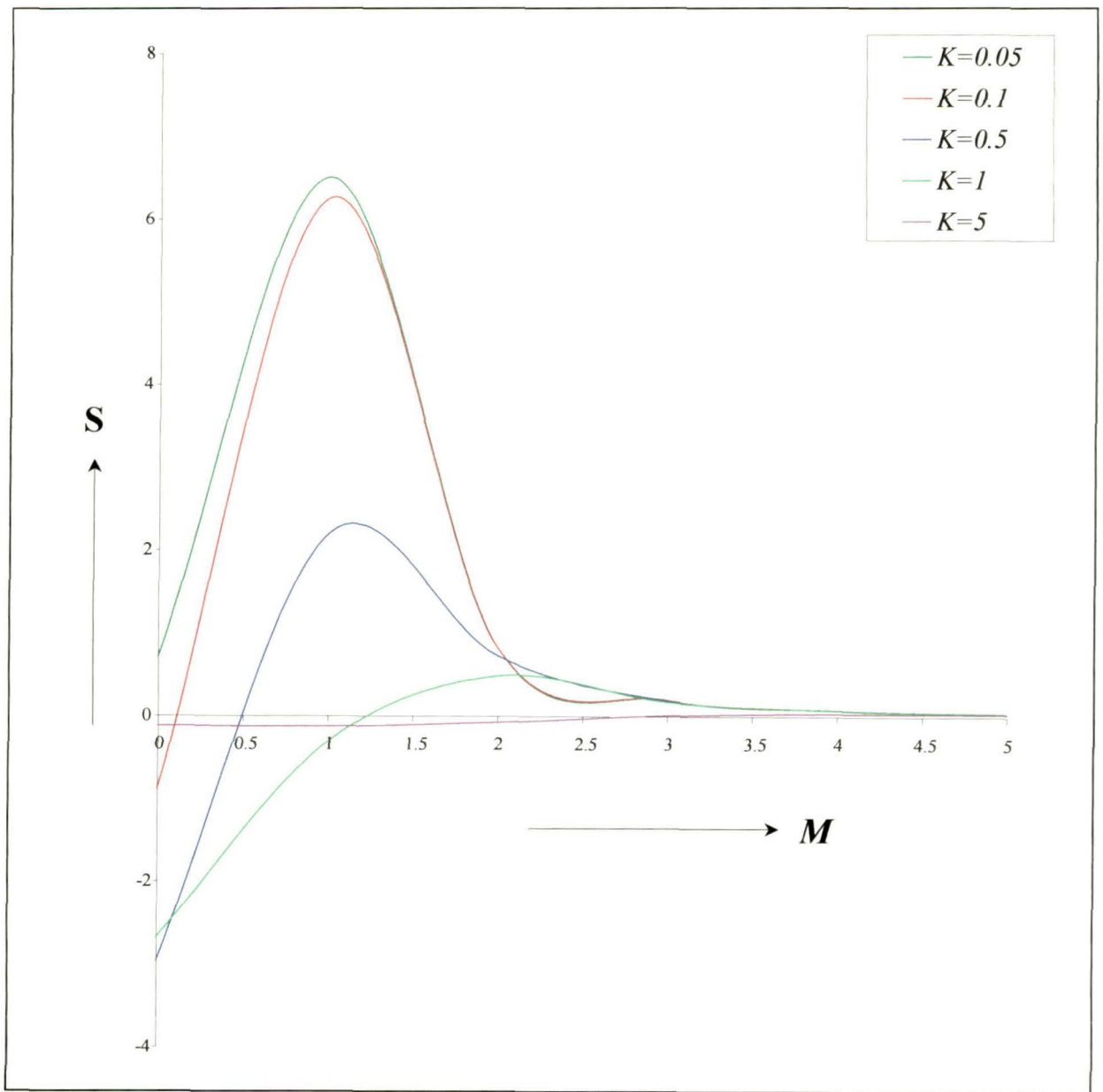


Fig - 3.12 Plot of S against M for different values of K .

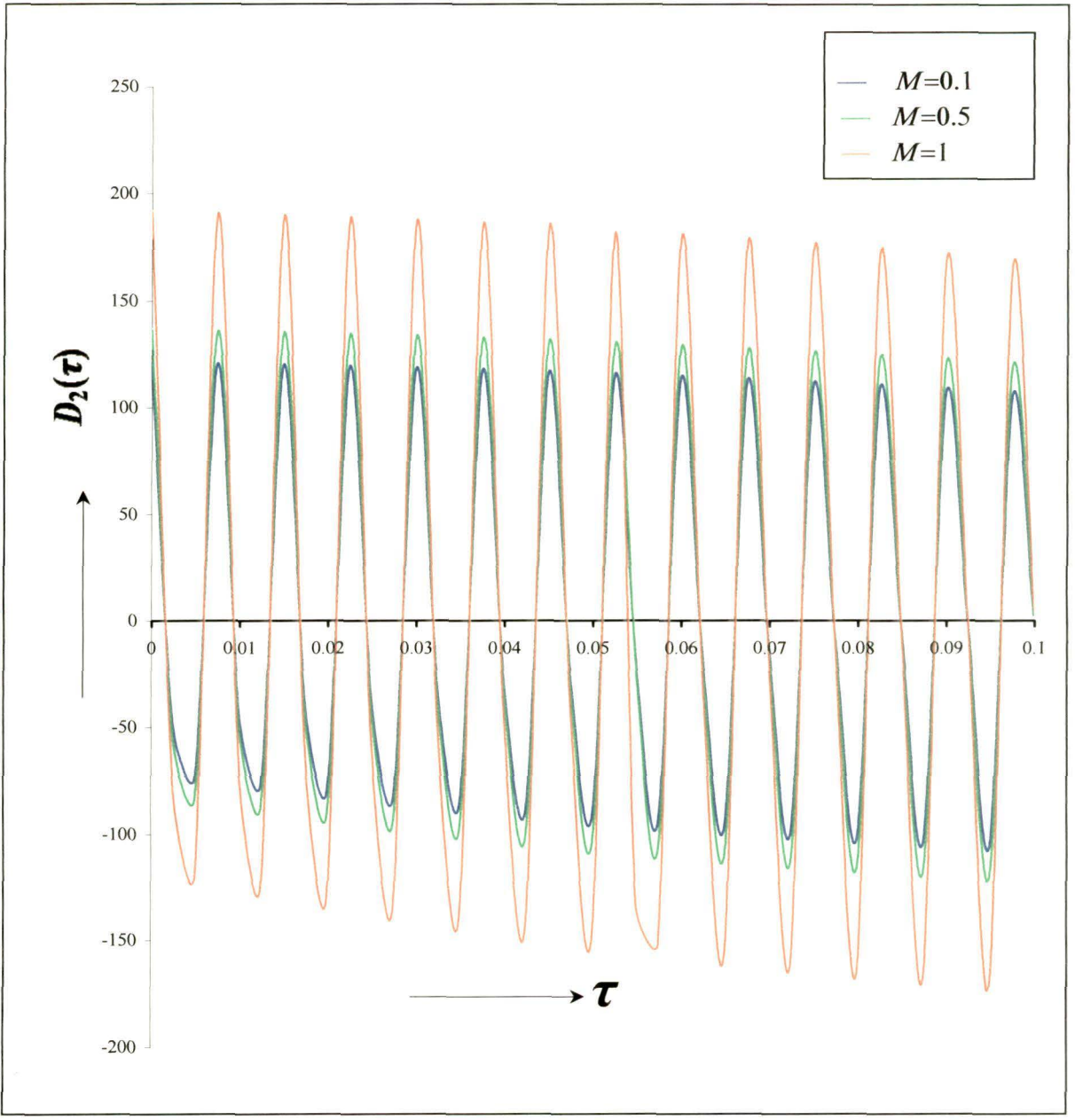


Fig. 3.13 Plot of $D_2(\tau)$ vs. τ for different values of M .

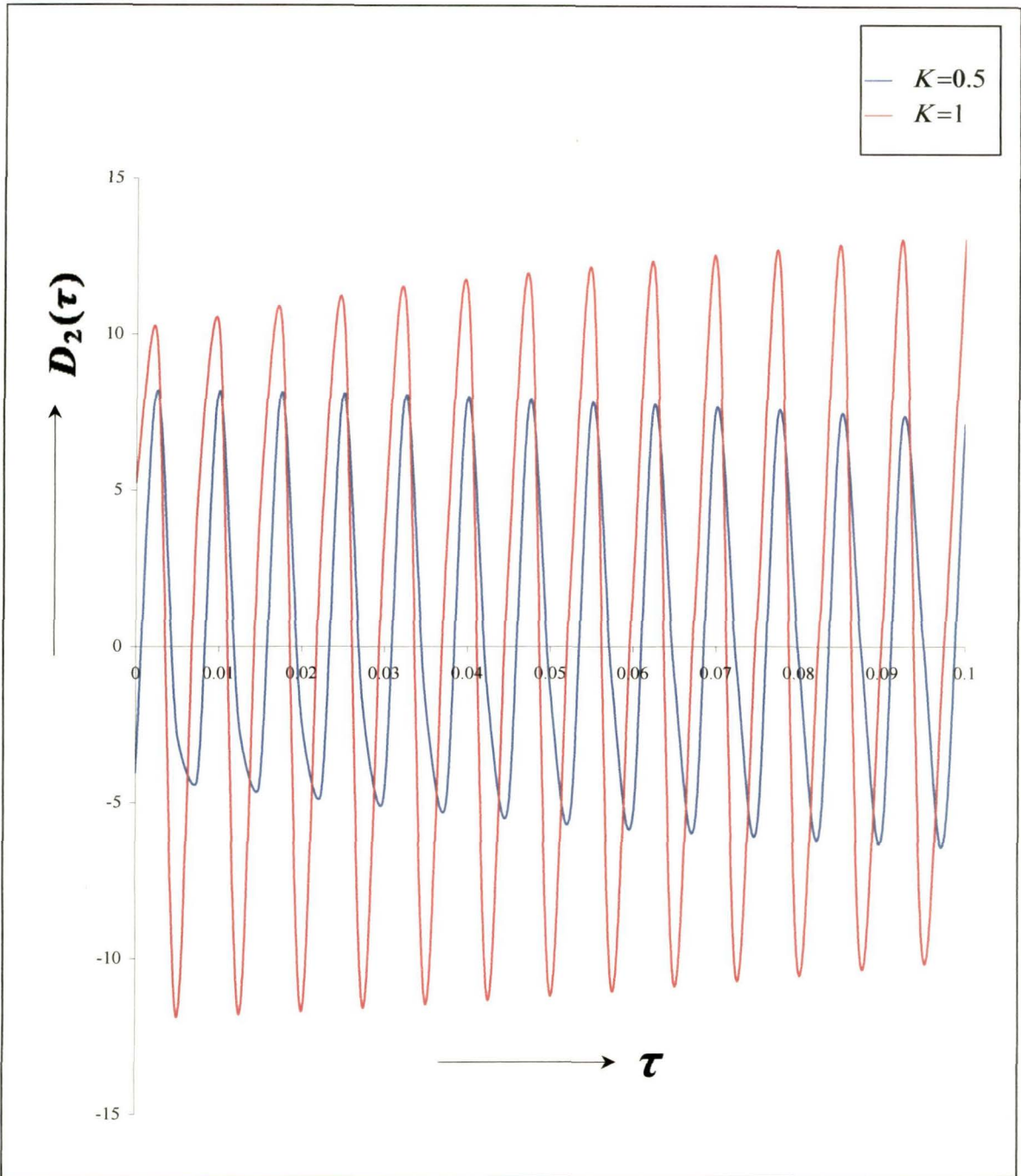


Fig. 3.14 Plot of $D_2(\tau)$ vs. τ for different value of K .

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