

## DECLARATION

I declare that the thesis entitled '**Some Aspects of Scale Invariant Analysis on Cantor Sets**' has been prepared by me under the guidance of **Dr. Dhurjati Prasad Datta**, Professor (Retd.), Department of Mathematics, University of North Bengal. The thesis has not been submitted, in whole or in part, in any previous application for a degree or fellowship previously.

Date : 25.05.2023

*Soma Sarkar*

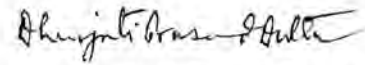
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## CERTIFICATE OF SUPERVISOR

I certify that **Soma Sarkar** has prepared the thesis entitled, '**Some Aspects of Scale Invariant Analysis on Cantor Sets.**' for the award of Ph.D degree of the University of North Bengal, under my guidance. She has carried out the work at the Department of Mathematics, University of North Bengal.

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# Certificate of Originality

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Some Aspects of Scale Invariant Analysis on Cantor Sets A THESIS SUBMITTED TO THE UNIVERSITY OF NORTH BENGAL FOR THE AWARD OF DOCTOR OF PHILOSOPHY IN MATHEMATICS BY SOMA SARKAR Under the Supervision of DHURJATI PRASAD DATTA Professor (Retd.) DEPARTMENT OF MATHEMATICS UNIVERSITY OF NORTH BENGAL RAJA RAMMOHANPUR DIST. DARJEELING PIN 734013, INDIA 2023

Chapter 1 Introduction The study of fractals and other complex systems is of considerable interest in contemporary time [1]. The relevance and usefulness of fractal geometric concepts and constructions in modeling various natural structures and processes are now well established. Fractals are generally considered to be sets or geometric figures that are highly irregular in nature, so that the theory of Classical Calculus and Analysis fail to yield any satisfactory results. Historically, such non-smooth sets and figures were studied in past, for instance, Cantor sets, Von Koch continuous but no-where differentiable curves, Sierpinski triangle or gasket etc. However, these objects were considered mostly as general mathematical curiosities and counter-examples only having almost no relevance in general mathematics and its real world applications and were even regarded as pathological and/or mathematical monsters. The scenario, however, is changed dramatically with the ground breaking work of Benoit Mandelbrot [2] who systematically initiated and established the theory of fractal geometry as a new field of mathematical and scientific research. A Cantor set is a compact, perfect, totally disconnected subset of the real number system  $\mathbb{R}$ . Recently there have been a lot of interest in developing a framework of analysis on a Cantor like fractal sets [6, 11–13]. The present thesis is a part of an ongoing project that aims at developing a general framework of scale invariant analytical framework that would be suitable to construct not only a rigorous analysis on fractal subsets of  $\mathbb{R}^n$ , but would also be suitable for having new insights into the dynamics of general nonlinear dynamical systems [20]. 1

Soma Sarkar  
25.05.2023

Dhurjati Prasad Datta  
25.05.23

## DEDICATION

*To*

*My respected grand father **Late Rabindra Nath Sarkar** and My father-in-law **Late Ram Gopal Bhowmik** whose lives of perseverance incessantly inspire me and my family.*

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