CHAPTER - VII

PERIODICITY IN YEARLY RAINFALL

CHAPTER - VII

PERIODICITY IN YEARLY RAINFALL

7.1. INTRODUCTION

The rhythmic behaviour of precipitation in India and elsewhere has more importance now a days. While this information is of interest, it gives an insight into the temporal variation of the rainfall process. A study of the time scale variations would start with time-domain and frequency domain analyses of data in time series. These are also the initial steps in bulding stochastic models of time series. The nature of relationship between these rhythmic behaviours in a time series of rainfall data becomes more clear through this type of analysis. The first step of the prediction of rainfall with time series analysis technique involves a search for periodicities in the historical records. Periodicities ,if present,represent potential predictive power for the nature of rainfall during a future time period. Major researches for periodicities of rainfall time series and their association with periodicities,luni-solar and solar-cycle phenomena are now very common in climatological investigation. Other than these periodicities several short term cycles which are known as quasi-biennial,quasi-triennial and quasi-five year etc. periodicity show significant results in mostof the raingauge stations or meteorological sub-divisions in India. Suitable statistical analysis of time series may prove to be useful in the sense that the fluctuations are unexplainable on the physical basis,past history may give some indication about the future.

In this study an attempt has been made to examine the actual nature of the historical precipitation time series and to detect the periodicity in yearly rainfall data at the two stations in Cooch Behar district.

7.2. Autocorrelation :

The most conventional method of computing the power spectrum through the autocovariance function is known as the lagged product method ,referred to equally spaced time series of finite length. In this section, the time domain analysis has been applied to examine the randomness in the annual rainfall of the two stations in Cooch Behar district **viz.** Dinhata and Cooch Behar. In this situation,available records of this natural phenomenon are limited in length by 88 years. These are spaced at equal time interval as in year.

We have already discussed in chapter IV of this study that the annual rainfall at the two stations i.e.at Cooch Behar and at Dinhata can be considered to follow the normal distribution. Maximum likelihood estimators have been applied to estimate the autocorrelations for lags 0 to K (K<N) and are given by

$$r_m = \frac{C_m}{C_o}$$

Where $C_m = \frac{1}{N} \sum_{t=1}^{N-m} (X_t - \overline{X})(X_{t+m} - \overline{X})$

and $\overline{X} = \frac{1}{N} \sum_{t=1}^{N} X_t$

r_m = autocorrelation coefficient for lag m.

N = total number of yearly rainfalls

X_t = variate representing tth year's rainfall

$\overline{X}$ = mean of variate X

m = period of lag takes the values from 0,1,2......K.

Tables - **7.1** and **7.2** show the autocorrelation coefficients for the two time series of rainfall data separately. We have computed here the autocorrelation coefficients up to lag 25 which are used to test the randomness of the annual rainfall of the two stations in Cooch Behar district.

The standard errors of sample autocorrelation function are estimated for each of the coefficients and are defined by

$$\text{Estimate of S.E}(r_m) = \frac{\left[1 + 2\sum_{i=0}^{m-1} r_i^2\right]^{1/2}}{\sqrt{N}}$$

The computed standard errors of each of the autocorrelation coefficients are also given in Tables- **7.1** & **7.2** for the annual rainfall at Cooch Behar station and Dinhata station respectively.

7.2.1. TEST OF SIGNIFICANCE OF AUTOCORRELATION.

Table **7.1** shows the standard errors of autocorrelation coefficients which are based on the order of N. On the basis of the asymptotic normal distribution, the approximate two-tailed critical values for the autocorrelation are $\pm\ 2\sqrt{1/N}$ which are known as Bartlett band at the 5 percent level of significance. Here we have N = 88,so the approximate Bartlett band is 2.13 at 5 percent level. The calculated values of the autocorrelation coefficients do not fall out side this band. The autocorrelation function has the same shape throughout. This is the shape as characterized by a random process. But in this process,the critical values are the same throughout. And if we consider the 2 SD band the critical values may be changed according to the order of lag of autocorrelation. However in this case also,we have the standard error of each order of lag of autocorrelation coefficient as stated in the table. So,we can easily get the 2 SD of each order of lag of autocorrelation coefficient and it is seen that the same results are obtained. It is to be noted that the two series appear to satisfy the stationarity condition.

7.3. PERIODOGRAM :

To investigate the existence of any periodic or any quasi-periodic fluctuations, the series is subject to periodogram analysis. The usual procedure adopted in periodogram analysis is to locate the frequencies $\{w_k\}$ by using search technique based on a function called the periodogram.

For N observational data X_1, X_2,.......,X_N the periodogram I(w) is defined for all w in thw range $-\pi \leq w \leq \pi$ by

$$I(w) = \frac{2}{N}\left| \sum_{t=1}^{N} X_t e^{-i2\pi wt} \right|^2$$

or alternatively $I(w) = \frac{2}{N}\left| \{A(w)\}^2 + \{B(w)\}^2 \right|$

where $A(w) = \sum_{t=1}^{N} X_t \cos 2\pi wt$, $B(w) = \sum_{t=1}^{N} X_t \sin 2\pi wt$

Although $I(w)$ is defined for all w in $(-\pi, \pi)$ we cannot evaluate it numerically as a continuous function of w. We compute it only at a discrete set of frequencies .e.,e evaluate

$$I_k = I(w_k), \quad w_k = \frac{k}{N}, \quad k = 0,1,2,\ldots\ldots,\frac{N}{2}$$

and then plot I_k *against* $\frac{k}{N}$

[We should note here that some authors call it spectogram. Theycall the graph of I_k *againet the periods* $\frac{2\pi N}{k}$ *the periodogram]*

Now we consider the standardized form of the periodogram

$$I^*(W_k) = I(W_k)/S^2$$

Where S^2 *is the variance of the time series and the standardized periodogram is also plotted against the angular frequencies,*$\frac{k}{N}$*. The true frequencies may be located by noting the positions of the peaks in the graph of the standardized periodogram ordinates.*

Figure **7.1***(a) shows the standardized periodogram of the annual rainfall time series of Cooch Behar,plotting the standardized periodogram versus the angular frequencies. And Figure* **7.1***(b) shows the same for Dinhata station. From this graphical representation ,there are observed five peaks on the spectrum of the annual raifall at Cooch Behar as well as at Dinhata station. Besides these peaks there are also some small peaks.*

7.3.1. TEST OF SIGNIFICANCE OF SPECTRAL PEAKS.

The significance test of spectral power estimates may be determined by several methods.

For testing the significance of spectral peak, we consider the test criterion, suggested by Fuller (1976). Therefore for testing the hypothesis for detecting the periodicity at α percent level of significance and holding simultaneously for standardized periodogram values is given by

$$b_m = -\pi^{-1} \log_e \beta$$

Where $$\beta = 1 - (1-\alpha)^{1/m} \approx \frac{\alpha}{m}$$

If we put $\alpha = 0.05$ $m = 44$, we get,

$$b_{44} = 2.158$$

If any standardized periodogram estimate is greater than the test value, i.e. 2.158, it indicates the presence of significant periodicity in the time series. And the period, P, corresponding to any spectral estimate is given by

$$P = \frac{1}{W_k} = \frac{N}{K}$$

Where W_k is the angular frequency corresponding to the significant spectral peak in question.

A list of significant spectral peaks with their corresponding periods, identified by spectral analysis for both the stations are given in Table **7.3** and Figures 7.1 (a & b) illustrate the standardized periodogram of annual rainfall at Cooch Behar and Dinhata respectively. The standardized periodogram values of the remarkable spectral peaks are 3.201, 2.626, 2.293, and

2.299 corresponding to the periods 2.667,3.52,4.889 and 22.0 years respectively. All the values of standardized periodogram are greater than the hypothetical value, 2.158. Hence these show the presence of peaks, significant at 5 percent level,at the station of Cooch Behar corresponding to the wave periods in the range 2.667 to 22 years. These are indications of the existence of prominent periodicities in the time series of rainfall at Cooch Behar station.

Now we come to consider the wave period in the time series of rainfall at Dinhata where five remarkable peaks are under considerationin this analysis.The spectral peaks,in consideration,with their corresponding periods for Dinhata station are given in the same table (Table **7.3**) . Here the values of standardized periodogram of remarkable spectral peaks are 2.301, 2.275, 5.631, 4.117 and 3.438 corresponding to the periods, 2.75, 3.25, 3.52, 14.67 and 29.33 years respectively. All these values of the standardized periodogram are greater than the hypothetical value (2.158). These are highly significant at the 5 percent level. The significant spectral peaks indicate the prominent periodicities in the yearly rainfall at Dinhata.

The extent and significance of 2.0 – 2.9 years frequency band have concentrated in this region. The significant peaks at the approximately quasi-biennial oscillation (2.2 years) are particularly characteristics of Cooch Behar district.

The significant quasi-triennial (3.0 – 3.9 years) periodicity is concentrated also. Stations with significant quasi-quadrennial (4.0 to 4.9 years) oscillation is also identified in the time series. The quasi-quadrennial oscillation is approximately quasi-five year oscillation as the 4.889 years has been considered as 5 years oscillation.

The spectral analysis has revealed some interesting evidence for the 14.67 and 29 years periodicities in the yearly rainfall at Dinhata while 22.0 years periodicity in the annual rainfall at Cooch Behar. But the existence of the 14.67 and 29 years periodicities at Cooch Behar and that of 22.0 years periodicity at Dinhata is not statistically significant.

However the periodicity of 14.67 years in the annual rainfall series at Dinhata and that of 22.0 years at Cooch Behar have been found statistically significant in this homogeneous meteorological region.

So, slightly different pattern for the 10-11 year solar cycle occurs at the location at Dinhata but the same wave band is not significant at Cooch Behar station. The yearly rainfall records at Cooch Behar have yielded an evidence for slightly different pattern of luni-solar cycle.

The presence of 29 years wave is significant at the 5 percent level in the rainfall of Dinhata station while the presence of such a period at the other station is not significant. Moreover, some neighbouring spectral estimates may also be considered significant but to overcome this problem, we try to employ the procedure of smoothing the periodogram in the next sub-section.

7.4. SMOOTH SPECTRUM

Smoothing the periodogram is to smooth the neighbouring autocovariances with a time domain moving average. Every frequency domain window (moving average) has a time domain representation and reversely,every time domain window has a frequency domain representation.

However,the new spectral estimates are obtained by smoothing the raw estimates of periodogram by a set of weights.

Humming estimates the periodogram by Tuckey-Hamming window and the estimated smooth periodogram is given by

$$P(W) = \frac{1}{\pi} (C_o + 2 \sum_{K=1}^{N} \lambda_k C_k \text{Cos } 2\pi w)$$

Where λ_k is the lag window with truncation point M defined at discrete points k. In this computation the Tukey-Hanning window has been used as

$$\lambda_k = .54 + .46 \text{ Cos } (\frac{\pi k}{M}), \ K \leq M .$$

Various band widths have been used in this computation. As various wave lengths are detected in the previous section, progressively larger band widths are employed to estimate the smoothed spectral density.

The chosen default is Q^2 where Q is the cube root of the number of observations. Here Q^2 is approximately 20.

Following the computation procedure mentioned above,we have obtained the smooth spectral estimates at the angular frequency W_k.

The smooth spectrum has been standardized. So, we have,

$$P^*(W_k) = \frac{P(W_k)}{S^2}$$

Where S^2 is the variance of the series of X_t . The standardized spectrums are used to detect the peak of the power spectram for truncation points (M) . There are three distinct wave periods in the smoothed spectral density of the rainfall at Cooch Behar station. But there are four remarkable spectral peaks in the rainfall at Dinhata station. The frequency scale is linear since the bandwidth is independent of frequency.

The standardized smooth spectral density functions corresponding to their angular frequencies are illustrated in Figures **7.2(a & b)** for the stations Cooch Behar and Dinhata respectively.

Table **7.4** also presents the remarkable high spectral peaks corresponding to their respective periods for both the stations.

In the next sub-section, we adopt a test statistic to test the hypothesis for the presence of periodicity in the individual rainfall series.

7.4.1. TEST OF SIGNIFICANCE OF SPECTRAL PEAKS.

For testing the presence of periodicity in the rainfall series, an α significance level holding simultaneously for M values of $P^*(w_k)$ is given by

$$b_M = \frac{\chi^2_{d,1-\beta}}{\pi d}$$

where $\beta = \frac{\alpha}{M}$

and 'd' denotes the degrees of freedom of the spectral window. The degrees of freedom of the lag window is 2.5164(N/M). The test criterion is used to compute the critical limit under the null hypothesis. The test has been used by Helmut (1986).

Now in our assignment, we have to put $\alpha = 0.05$, $M = 20$ and $N = 88$, then we obtain

$$b_{20} = 0.$$

The standardized smooth spectral densities are employed here, to detect the presence of periodicities in the individual rainfall series.

If any of the spectral estimates lies outside this critical point, it means the presence of significant periodicity in the rainfall series.

The standard smoothed spectrum density values of the remarkable spectral peaks are 1.771, 1.265 and 1.257 corresponding to the periods 2.667, 3.52 and 5.176 respectively in the annual rainfall series at Cooch Behar station. All these values are greater than the hypothetical value of Chi-square at the 5 percent level of significance. The results give evidence for low band-limited signals near quasi-biennial, quasi-triennial and quasi-five year in the rainfall of this station. It is obvious from the Figure **7.2(a)** that periodicities greater than quasi-five year bands are not statistically significant in the climatological time series at Cooch Behar station.

But it is interesting to note that the evidence to be presented later is more convincing for the presence of high band-limited signal near 18.6 year luni-solar cycle. The standardized smoothed spectral density values of the remarkable spectral peaks are 0.958, 1.672, 0.805 and 1.999 corresponding to the periods 2.378, 3.52, 8.80 and 17.60 respectively in the annual rainfall series at Dinhata. All these values of the spectral peaks are also greater than the hypothetical value of Chi-square at the 5 percent level of significance. So,this smoothed spectrum consists of one signal induced by the highly resonant 17.6 year quasi-standing wave and additionally there are three smaller cycle terms of near 9 years, quasi-triennial and quasi-biennial in the annual rainfall at Dinhata station.

In this analysis,there is a band-limited power near 17.6 years periodicity at Dinhata station which yields the evidence of slightly different pattern of 18.6 luni-solar cycle but it is not identified in the rainfall at Cooch Behar. And the analysis of the smooth power spectrum has reported evidence for a signal with period 8.8 years in the rainfall at Dinhata ,but this period is not found in the annual rainfall at Cooch Behar station while the existence of the quasi-five year oscillation in the rainfall at Cooch Behar has been reported by this analysis.Most of the procedures of analysis are discussed by Jenkins & Watts (1968), Priestley (1981) and Chatfield (1982).

So,there are some significant differences observed between the simple periodogram and the smooth spectral estimates. But it is also indicated that the dominance of short duration fluctuations has been confirmed by the two methods of analysis in the individual rainfall series of the two stations. While a long duration periodicity cannot be ruled out in the annual rainfall in

Cooch Behar district.

7.5. CONCLUSION.

The spectral analysis technique has a very long history during which various approaches have been developed for the computation of the spectral estimates of the observed data. The simple periodogram analysis and the autocorrelation transformation technique have been used in this study.

The quasi-biennial oscillation in the annual rainfall in this region has been indicated prominently. This is in conformity with the well known fact that quasi-biennial oscillations exist in the several meteorological phenomena over India. The quasi-triennial and quasi-five year periodicities are also identified in the annual rainfall series at these homogeneous meteorological stations.

The spectral analysis of yearly total precipitation records at Cooch Behar yields evidence for 17.6 year cycle signal. The periodicity of 17.6 years may approximately be considered as presence of 18.6 years luni-solar cycle. The 10 -11 years solar cycle in the rainfall series have not been detected as such in the analysis though approximately similar result,the periodicity of 14.67 years has been found as evidence for a significant signal.

TABLE 7.1. Autocorrelation function & its plot of yearly rainfall at Coochbehar

AUTOCORRELATIONS

1- 12	.19	0.0	.17	.10	.11	-.02	.01	-.06	-.11	.11	.20	-.16
ST.E	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12	.12	.12
13-24	.04	.24	.09	.08	.05	.02	.02	-.01	-.02	.04	-.08	-.02
ST.E	.12	.12	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13
25-25	-.C2											
ST.E.	.13											

PLOT OF AUTOCORRELATIONS

```
                      -1.0 -0.8 -0.8 -0.4 -0.2  0.0 0.2 0.4 0.6 0.8 1.0
LAG          CCRR.      +....+....+....+....+....+....+....+....+....+....+

1            0.190                          +    Ixxxx
2           -0.004                          +    I   +
3            0.171                          +    Ixxxx
4            0.105                          +    Ixxx +
5            0.115                          +    Ixxx +
6           -0.022                          +   xI    +
7            0.007                          +    I    +
8           -0.059                          +   xI    +
9           -0.114                          + xxxI    +
10           0.107                          +    Ixxx +
11           0.204                          +    Ixxxxx+
12          -0.161                          +xxxxI    +
13           0.045                               Ix   +
14           0.201                          +    Ixxxxx+
15           0.093                          +    Ixx   +
16           0.081                          +    Ixx   +
17           0.052                          +    Ix    +
18           0.015                          +    I     +
19           0.024                          +    Ix    +
20          -0.009                          +    I     +
21          -0.019                          +    I     +
22           0.042                          +    Ix    +
23         - 0.055                          +   xI     +
24         - 0.017                          +    I     +
25         - 0.017                          +    I     +
```

TABLE 7.2. Autocorrelation function and its plot of yearly rainfall at Dinhata.

AUTOCORRELATIONS

1- 12	.12	.09	.20	.09	-.05	-.21	.04	-.08	-.16	.09	.12	-.13
ST.E.	.11	.11	.11	.11	.11	.11	.12	.12	.12	.12	.12	.12
13-24	-.C7	.02	.01	-.09	-.07	-.02	.01	-.06	.07	-.02	.02	-.08
ST.E.	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13	.13
25- 25	.C6											
ST.E	.13											

PLOT OF AUTOCORRELATIONS

```
                 -1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2  0.4  0.6  0.8  1.0
LAG       CCRR.   +....+....+....+....+...+...+....+....+....+....+
                                           I
1         0.121                        +   Ixxx+
2         0.093                        +   Ixx +
3         0.203                        +   Ixxxxx
4         0.092                        +   Ixx   +
5        -0.052                        +  xI     +
6        -0.205                        +xxxxI    +
7         0.040                        +    Ix   +
8        -0.082                        +  xxI    +
9        -0.163                        +xxxxI    +
10        0.085                        +    Ixx +
11        0.123                        +    Ixxx +
12       -0.135                        + xxxI     +
13       -0.066                        +  xxI     +
14        0.023                        +    Ix    +
15        0.015                        +    I     +
16       -0.094                        +  xxI     +
17       -0.068                        +  xxI     +
18       -0.016                        +    I     +
19        0.014                        +    I     +
20       -0.059                        +   xI     +
21        0.067                        +    Ixx   +
22       -0.019                        +    I     +
23        0.015                        +    I     +
24       -0.076                        +  xxI     +
25        0.064                        +    Ixx   +
```

TABLE: 7.3.

Spectral peaks of standardized periodogram

	Cooch Behar		Dinhata	
	Periodogram values	Period	Periodogram values	Period
1st peak	2.299*	22.00	3.438*	29.33
2nd peak	2.293*	4.889	4.117*	14.67
3rd peak	2.626*	3.52	5.631*	3.52
4th peak	3.201*	2.667	2.275*	3.25
5th peak			2.301*	2.75

* Significant at 5% level.

TABLE: 7.4

Spectral peaks of standardized smooth spectrum.

	Cooch Behar		Dinhata	
	Spectrum values	Period	Spectrum values	Period
1st peak	1.257*	5.176	1.999*	17.60
2nd peak	1.265*	3.52	0.805*	8.80
3rd peak	1.171*	2.667	1.672*	3.52
4th peak			0.958*	2.378

*Significant at 5% level.

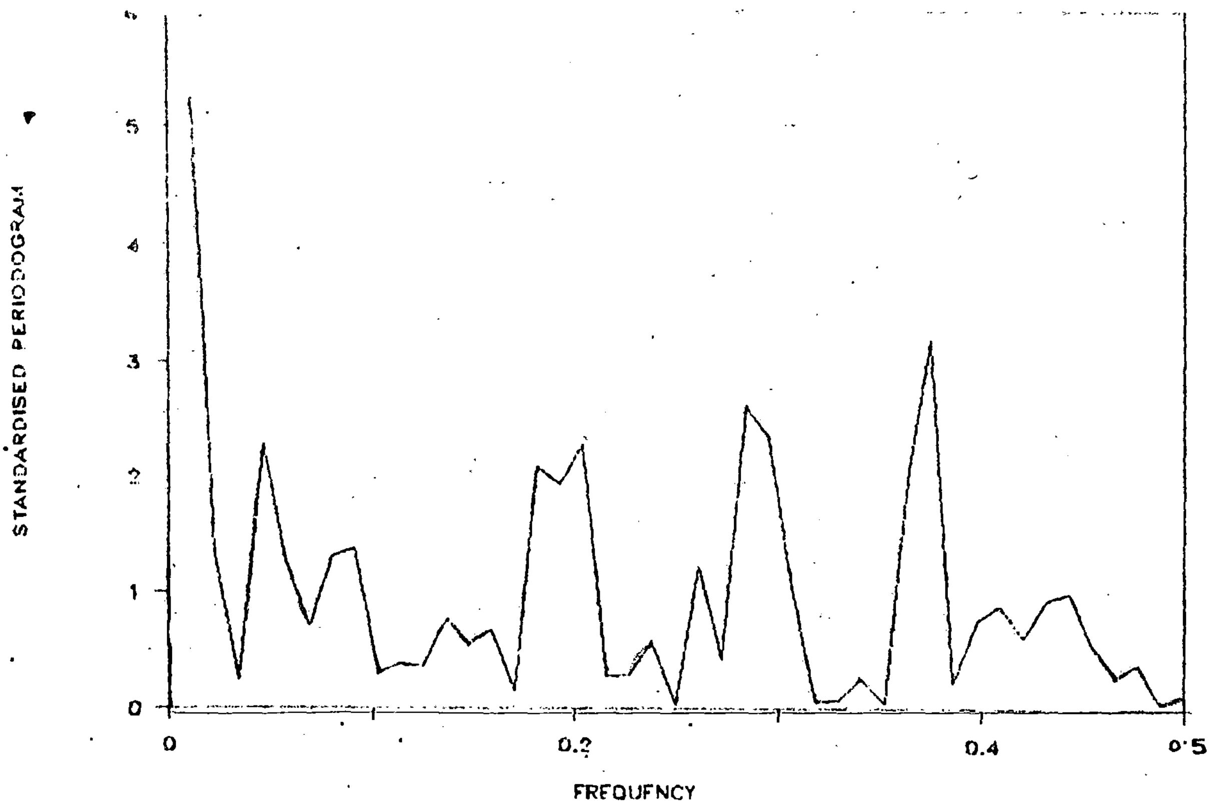

Figure : 7:1(a). Standardised Periodogram of annual rainfall at Cooch Behar.

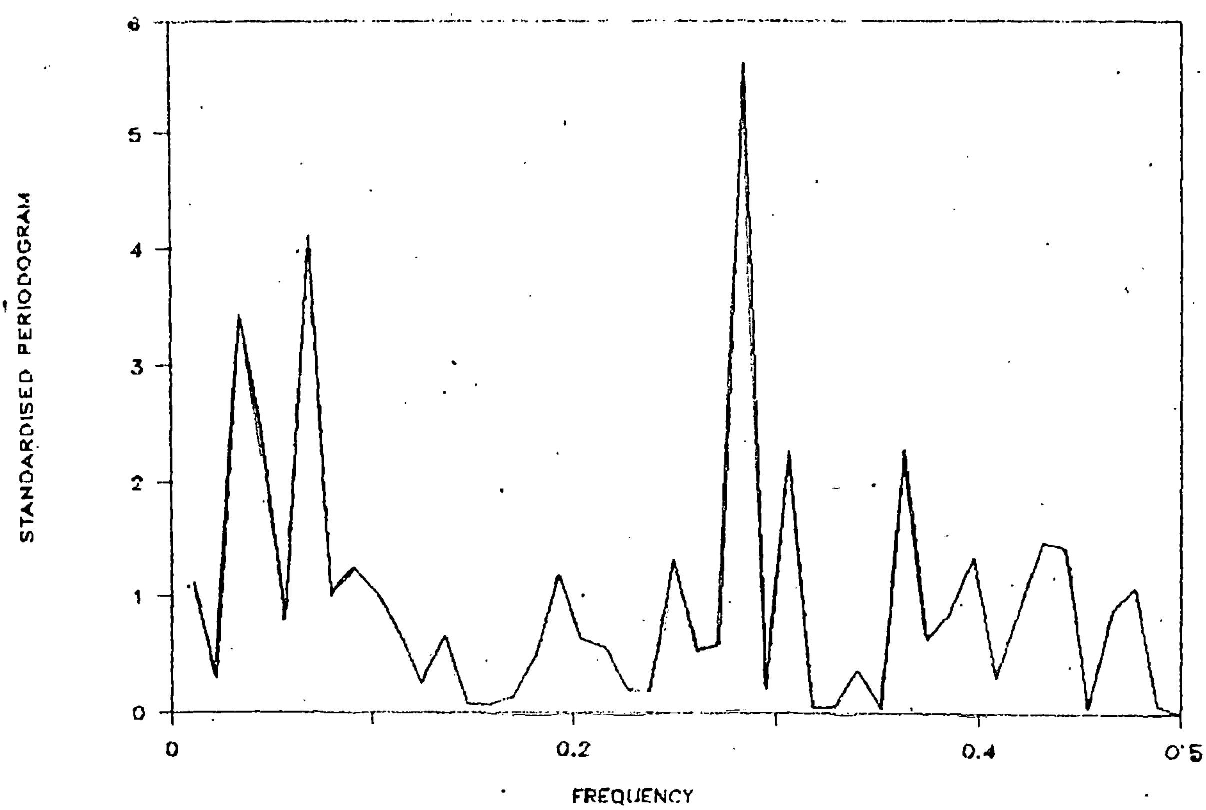

Figure. 7.1(b). Standardised Periodogram of annual rainfall at Dinhata.

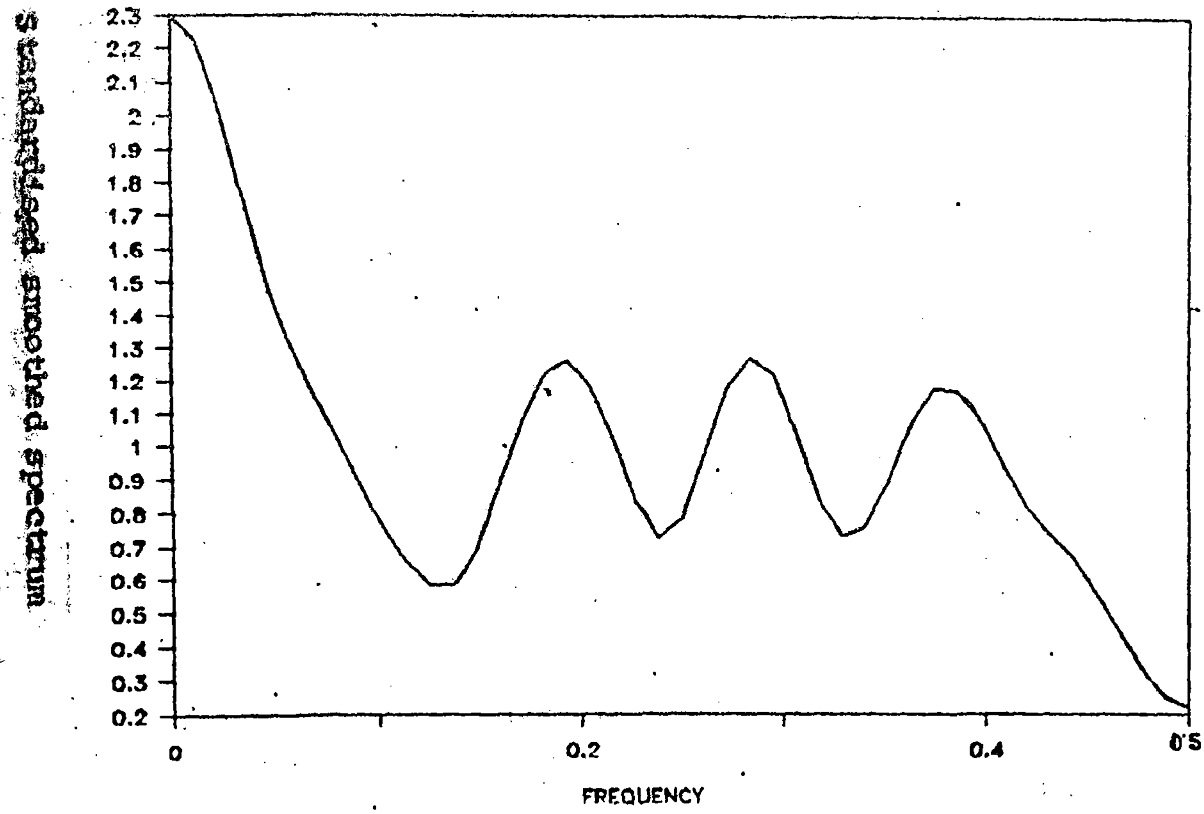

Figure. 7.2(a). Standardised smoothed spectrum of annual rainfall at Cooch Behar.

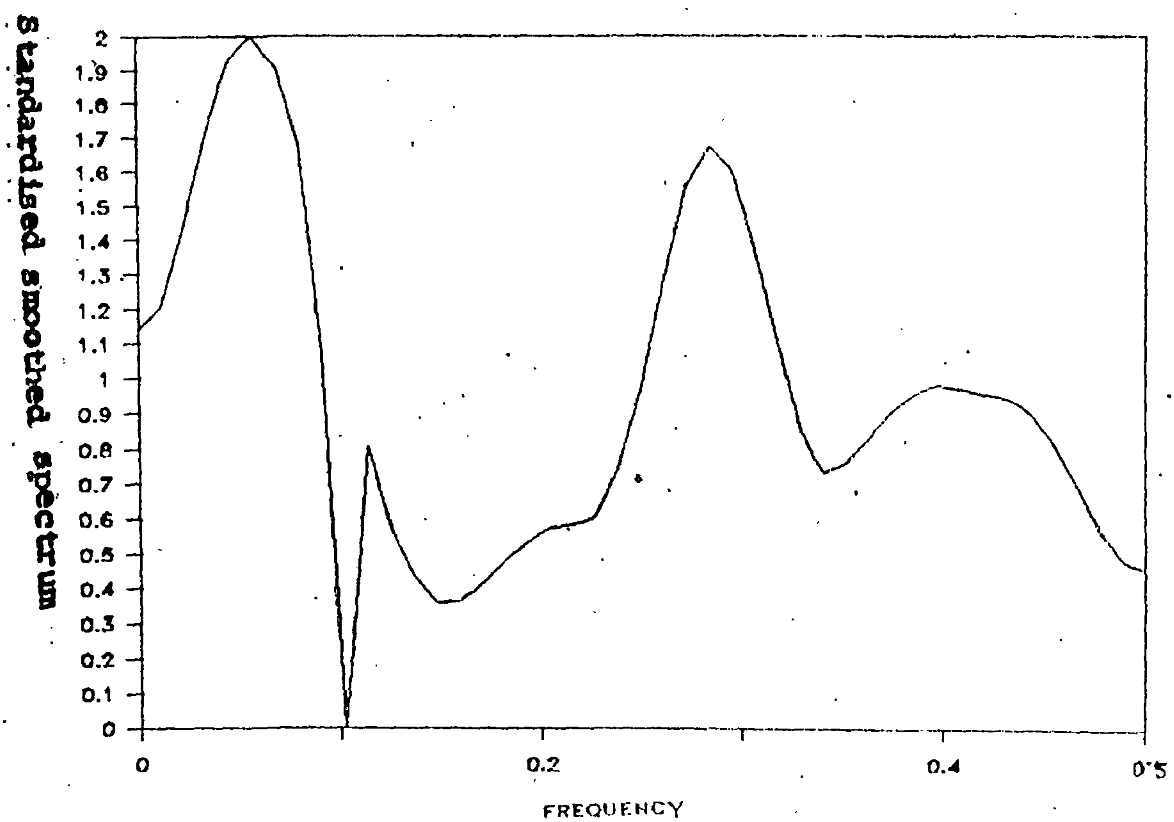

Figure. 7.2(b). Standardised smoothed spectrum of annual rainfall at Dinhata.