ABSTRACT

This thesis studies the nonlinear dynamic (free vibration) behavior of thin plate and shell structures of different geometrical shapes with or without thermal loading. In many situations, geometric characteristics of structures combined with oppressive operational conditions induce large deflections i. e. deflections that are of the same order as the plate or shell thickness and small compared to the in-plane dimensions of the structures; even within elastic limit of the structural material and thus nonlinear effects come into play. It is worth mentioning that any effort to restrict the deflections appropriate to the linear theory results in uneconomic structures. Further, stress-strain characteristics of structural material with or without thermal loading also induce non-linearity in the behavior of structures. Structural components undergoing large deflections (nonlinear deformations) may exhibit strain-hardening or strain-softening behavior. The advantage of extra strength of strain-hardening as well as desired level of natural frequency of vibrations may be achieved by properly proportioning and designing geometric elements of the structures and its end conditions. These facts warrant to investigate nonlinear behavior of structures or structural components there of in order to achieve efficient and economic utilization of material. For example, the knowledge of nonlinear dynamic behavior of structures will help to attain desired level of frequency of vibrations by properly proportioning and designing geometric elements.

The structures are assumed to be made of homogeneous, isotropic and elastic materials. In some problems material properties of materials are assumed constant and

temperature-independent. In some other problems, materials having temperature-dependent material properties are considered.

Further, continuum model has been used to derive the basic governing differential equations as and whatever required, in the sense of von Karman classical large deflection theory or on the basis of Berger approximations i.e. the energy contribution due to second strain invariant of the middle surface is neglected in the total potential energy expression. The basic governing differential equations for the resulting system has been solved by Galerkin error minimizing technique incorporating prescribed boundary conditions to obtain relative nonlinear frequency of vibrations.

Numerical results to study the effects of different parameters, as occurred in the analysis, on nonlinear dynamic behavior of such structures have been presented. The present study reveals some interesting nonlinear dynamic behavior which may prove useful to the designers. A few of the important observations are mentioned in the following paragraphs.

It is observed that thin plates viz. circular, triangular and parabolic plates with clamped immovable edges exhibit strain-hardening type of non-linearity valid within the proportional limit of the plate material.

It is interesting to note that a triangular plate with the geometric configuration having aspect ratio around 1.0 and skew angle around 30^0 is more prone to develop dynamic instability with or without thermal loading. Similarly, parabolic plate having aspect ratio around 1.4 is more prone to develop dynamic instability with or without thermal loading.

Thermal loading influences the dynamic behavior of thin plates in various ways. Thin plates of various shapes with clamped immovable edges viz. circular, triangular and parabolic plates exhibit similar behavior under thermal loading characterized by constant surface temperatures T_u and T_b, measured from stress free temperature, for upper surface and lower surface respectively. For example, the stiffness and hence the frequency of vibration, both linear as well as nonlinear, decreases with increase of the average surface temperature $\left(\frac{T_u + T_b}{2}\right)$ and after certain stage linear frequency becomes zero i.e. thermal instability of the structure occurs based on linear theory; but nonlinear frequency is still non-zero which makes $\frac{\omega_{NL}}{\omega_L}$ infinity. So, thermal instability is delayed due to additional stiffness associated with large deflection as per nonlinear theory.

Thermal loading does influence the temperature-dependent material properties in various ways; but the temperature dependency of material properties does not alter the dynamic characteristics of thin isotropic circular plates with clamped immovable edges significantly within the range of temperature causing thermal instability. However, the temperature dependency of material properties reduces the nonlinear frequency of vibrations and such reduction is not appreciable compared to the effects of large deflection and direct heating.

Axisymmetric thin shallow spherical shells with clamped immovable edge exhibit both strain-hardening and strain-softening types of behavior. A very shallow thin spherical shell tends to behave like a circular plate and exhibit strain-hardening type of nonlinearity. With increase of nondimensional geometric parameter $\left(\frac{R^2}{2R_0 H_0}\right)$ i.e. ratio of the rise of the shell to the shell thickness at the center of the shell, the behavior of shallow spherical shell changes from strain-hardening type to strain-softening type and relative nonlinear frequency decreases to yield the lowest peak and this peak corresponds to the geometrical configuration of thin shallow spherical shell which is most liable to undergo snap buckling. If the shell geometry matches the transition state configuration nonlinear frequency becomes equal to linear frequency. With further increase of $\frac{R^2}{2R_0H_0}$ the resistance of shallow spherical shell to snap buckling and the relative nonlinear frequency increases and finally it approaches unity. Nonlinear frequency approaches linear frequency as the thickness parameter(τ) decreases algebraically i.e. the thickness of the shell increases towards the edges.

It is worth mentioning that the overall dynamic behavior obtained for thin shallow spherical shell with clamped immovable edges based on Berger's approximation matches approximately with those obtained in the present study based on classical large deflection theory in the von Karman sense.

Finally, it can be mentioned that this study reveals some interesting dynamic behavior of thin isotropic plate and shell structures which may be useful to the designers, For example, the knowledge of dynamic behavior of such structures will help to achieve desired level of frequency of vibrations by properly proportioning the geometric elements. This will also help to utilize the advantage of extra strength due to strain-hardening and to reduce the failure probability by taking proper precaution against structural instability.