# CHAPTER - I

# REVIEW OF THE PREVIOUS WORKS

# AND

# SCOPE AND OBJECT OF THE PRESENT WORK

## 1.1. Radio Frequency Probes and Plasma Diagnostics:

The possibility of the use of r.f. signal as a dignostic probe in the study of the electrical gas discharge phenomena was first suggested by Vander Pol (1919). For a partially ionised non-equilibrium cold plasma the radio frequency current density , $I_{r.f.}$, that flows through it, is given by

$$I_{r.f.} = \frac{e^2 n x_o}{m} \left[ \frac{\nu_c}{\nu_c^2 + \omega^2} - j \frac{\omega}{\nu_c^2 + \omega^2} \right]$$

where m is the electron mass, e the electronic charge, n the number density and $\nu_c$ the collision frequency of the electrons in the plasma, and $\omega$ the angular frequency of the applied electric field $x_o \, e^{j\omega t}$ . In the derivation above, the Maxwell distribution is usually assumed for electrons while the motion of ions is neglected because of their larger inertia. Further the constancy of the electron collision frequency $\nu_c$ is also assumed, regardless of the electron velocity and electric field strength. Hence the complex conductivity is given by

$$\sigma_c = \frac{I_{r.f}}{X_o} = \frac{e^2 n}{m \nu_c} \left[ \frac{\nu_c^2}{\nu_c^2 + \omega^2} - j \frac{\omega \nu_c}{\nu_c^2 + \omega^2} \right]$$

or $\sigma_c = \sigma_r - j\sigma_i$

where $\sigma_r = \dfrac{e^2 n \nu_c}{m(\nu_c^2 + \omega^2)}$ and $\sigma_i = \dfrac{e^2 n \omega}{m[\nu_c^2 + \omega^2]}$

It may thus be seen that both $\sigma_r$ and $\sigma_i$ are functions of (i) probe frequency $\omega$, (ii) electron density $n$ and (iii) the collision frequency $\nu_c$ which itself is a function of pressure. The value of the conductivity is maximum when $\nu_c = \omega$ in which case $\sigma_r/\sigma_i = 1$. Thus, by measuring the variation of conductivity with frequency of the r.f. field, both the electron concentration and collision frequency can be obtained. The conductivity of ionised air was measured by Childs (1932) by substitution of a resistance of known value for the leakage resistance of the ionized gas, the oscillation frequency being 1 MHz. Appleton and Chapman (1932) studied the variation of the radio frequency conductivity of ionised air with pressure at a frequency of the order of 1000 MHz, using a Lecher wire system coupled to the condenser within which the discharge tube was placed, the radio frequency current being rectified by means of a galena crystal and detected by the galvanometer. As the conductivity increases, the galvanometer deflection falls and Appleton and Chapman observed that the conductivity attains a maximum value at a certain pressure and then decreases in accordance with the theory; but they have not reported any absolute value of the conductivity for the gas investigated, namely, air. Similar study was made in case of sulphur dioxide and Xenon by Imam and Khastgir (1937) in the pressure range 10 – 120 cm. of mercury using radio waves of $\lambda$ = 481 cm.

The simple theory outlined above has been modified by Margenau (1946) by taking into consideration the proper velocity distribution and employing Boltzman transport equation.

The modified expression for $\sigma$ is given by,

$$\sigma = \frac{4}{3} \frac{ne^2 \lambda_e}{\sqrt{2\pi m k T_e}} \cos \omega t + \frac{n e^2 \lambda_e^2 \omega}{3 k T_e} \sin \omega t ,$$

for values of $\nu_c >> \omega$.

Later on several authers developed the theory considerably. Dawson and Oberman (1962, 1963) have developed an elementary model to calculate the high frequency electrical conductivity of plasma. H.L. Berk (1964) showed how the plasma model suggested by Dawson and Oberman can be adopted to yield a kinetic description of electrical transport processes, which is uniformly valid for high and low frequency theories, as well as for finite wavelengths.

The theory of the electrical conductivity of a gas which is either fully ionised or weakly ionised has been well established for a number of years. But very little quantitative information exists, principally because of the mathematical complications which arise when the electron-electron interaction is included in the Boltzman equation. Recently Johnson (1967) calculated the electrical conductivity for a variety of assumed electron - moleculе collision frequencies. The results differ by only a few percent from those obtained using an approximation suggested by Frost (1962,1964). A simple procedure , requiring no numerical integrations, has been given there relating electron temperature to electrical conductivity for a partially ionised gas.

Sen and Ghosh (1966) studied the properties of ionized gases experimentally by using radio frequency probe. The radio frequency conductivity ( $\sigma_r$ ) of the ionized air and nitrogen has been determined at various pressures and also at various values of discharge current. They observed that $\sigma_r$ increases with pressure and attaining a maximum value gradually decreases. The maximum value of $\sigma_r$ occurs at the same value of pressure for different discharge currents for the same gas. In the following year in our laboratory Gupta and Mandal (1967) studied the radio frequency conductivity for a field embedded plasma.

Nagata (1966) presented a simple technique for measuring the plasma conductivity. The method was based on the observation that Hall current and Hall voltage are related simply to an electrical resistance. This method may also be applied to the measurement of electron density in high pressure plasma. An improved probe method of measuring the electrical conductivity of low temperature plasma is set out by Khozhabiev and Yarin (1966). They presented experimental data regarding the effect of layers near the electrodes on the probe readings.

From a study of complex conductivity of mercury vapour at microwave frequencies Adler (1949) has shown plots of $\sigma_r$ and $\sigma_i$ with current or pressure when the other is fixed. Using the theoretical expression of Margenau, Adler calculated the values of the electron density in the discharge space and compared the values thus obtained with those obtained experimentally using Langmuir probe measurements. Adler found that the theoretical and experimental values agree closely and that $\sigma$ varies linearly with the discharge current. Aleksandrov and Yalsenko (1965) studied the complex conductivity of neon plasma by the Q-meter method. The results are given regarding measurements of the active and reactive components of the conductance of the parallel plate capacitor containing between its electrodes the plasma of a positive gas discharge column. The frequency range was 0.5 - 25 MHz., the discharge currents were 5 - 100 mA, and various gas pressures were used. The experimental results were in good agreement with the theoretcially calculated values.

There exists a large number of experimental and theoretical studies regarding radio frequency probing for magnetised plasma. Since the presence of a magnetic field changes the various characteristics of a discharge, it is natural to suppose that the conductivity of an ionised gas will also change in presence of a magnetic field. Conductivity of ionized gases such as air, nitrogen and hydrogen in a magnetic field was measured by Ionescu and Mihul (1935) for pressure greater than $10^{-3}$ mm. of Hg. who found that maxima other than those due to free electrons could be obtained. With very intense fields, only the vibration due to free electrons remained, the others disappearing and the values of the magnetic field giving maximum conductivity varied with pressure. A theory regarding the variation of radio frequency conductivity with magnetic field was proposed by Appleton and Boohariwala (1935) who showed that the real part of radio frequency conductivity in a magnetic field is given by

$$\sigma_{r.H} = \frac{ne^2}{m} \; \frac{\nu_c(\omega^2 + \omega_b^2 + \omega_c^2)}{(\omega^2 + \omega_b^2 + \omega_c^2)^2 - 4\omega^2 \omega_b^2}$$

where n is the number of electrons per unit volume and $\nu_c$ the collision frequency, $\omega$ is the angular frequency of the applied field and $\omega_b = eH/m$. A general theory regarding the variation of radio frequency conductivity of ionized gases and its variation with pressure and magnetic field has been worked out by Gilardini (1959) who derived the expression for the conductivity of an ionised gas under the following assumptions:

(a) when the distribution function is predominantly spherically symmetrical in velocity space but not necessarily Maxwellian.

(b) when the electron collision frequency is an arbitrary function of electron velocity, the value of the complex conductivity is given by

$$\sigma = \frac{e^2 n}{m} \frac{1}{\nu_c + j\omega}$$

In presence of magnetic field he defined two conductivities; a conductivity $\sigma_c$ for the right-handed polarization and a conductivity $\sigma_o$ for the left handed polarization where

$$\sigma_c = \frac{e^2 n}{m} \left[ \frac{1}{\nu_c + j(\omega - \omega_b)} \right]$$

and

$$\sigma_o = \frac{e^2 n}{m} \left[ \frac{1}{\nu_c + j(\omega + \omega_b)} \right]$$

and the conductivity in the direction of the field is given by ,

$$\sigma_{H} = \frac{1}{2} ( \sigma_c + \sigma_o )$$

i.e., $$\sigma_{rH} = \frac{e^2 n}{m} \left[ \frac{\nu_c}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{\nu_c}{\nu_c^2 + (\omega + \omega_b)^2} \right]$$

and after simplification it reduces to the result obtained earlier by Appleton and Boohariwalla.

Later on several authors (Wu, 1965; Oberman and Shure, 1963; Schweilzer and Milchner, 1967; Green et al, 1965 ) studied the ionised gas in presence of magnetic field and developed the theory considerably. Complex conductivity of a plasma in a steady magnetic field has been studied by Pradhan and Das-Gupta (1967). They derived an expression for the complex conductivity tensor of a homogeneous classical plasma in an external uniform magnetic field using the Kubo theory of transport phenomena and obtained exact relations between the conductivity tensor in ~~the~~ presence of the magentic field and in its absence. In the same year, as mentioned earlier, Gupta and Mandal (1967) of our department studied elaborately the radio frequency conductivity of magnetized air and carbondioxide plasma for a wide range of pressure. It was observed that conductivity decreases in presence of magnetic field for all values of pressure and the pressure at which conductivity becomes a maximum increases with the increase of magnetic field. The results were explained fairly well by an extension of the theory put forward by Gilardini (1959) and the quantitative agreement was also satisfactory.

Most of the radio frequency probing experiments described above give accounts of the plasma diagnostic technique using essentially capacitor probes. , i.e. plasma under study was placed inside the parallel plates of a capacitor which was either a part of the oscillator or forms a part of a tank circuit. Consequently the theories of measurement were based on the interaction of parallel radio frequency electric field with plasma.

Measurements based on the interaction of solinoidal r.f. electric field with plasma has some advantages over the methods described earlier in a way that one can use weak high frequency fields, which do not induce any appreciable excitation in the plasma. This method of investigation requires the use of coil probes and the solenoidal electric field is induced within the plasma due to the r.f. magnetic field generated by the coil probe. The method is often termed as induction method (Donskoi et al, 1963) and attracted attention of many plasma physicists in the present and past few decades because of its obvious advantages over conventional probe techniques particularly when the attention is focussed on the electrical conductivity of plasma.

The use of magnetic (coil) probes, however , in contrast to that of the radio frequency coil probe is more well known.

The magnetic probes are used to sample the magnetic fields in or around the plasma. A good review of the theory and application of magaetic probes is available in an article by Lovberg and Huddlestone (1965). However, a very brief review on the use of magnetic probes is given in the next section in order to get a proper understanding of the technique and purpose of the method in contrast to the radio frequency coil probe method mentioned above, the review of which will appear in detail following the next section. Though the purposes of the magentic (coil) probes are different, it offers shielding problems similar to the r.f. coil probe technique.

1.2. Magnetic (coil) Probes:

Magnetic probes have been treated by Lovberg (1959) who pointed out their widespread use in mapping current distributions in plasma accelerators and Pinches. The probe usually consists of a few turns of wire arranged in loop, which may be a millimeter in diameter or larger, as required for the measurement. These magentic probes operate on the principle that a time varying magnetic field induces a voltage in the loops; the magnetic field can be determined from a measurement of induced voltage (Glasstone and Loveberg, 1960; Colgate et al, 1958). Field sensitive elements, such as Hall current probes, may be made as small as 1 mm. in diameter and grouped in X-Y-Z arrays to measure three dimensional field configurations (Pollock et al 1960). Current density contours and the pressure of hydrodynamic instabilities in dense plasmas are measured by a linear array across current channels. The data can be displayed by rapid sampling. One difficulty in connection with the use of magnetic probes is that they generate a voltage proportional to dB/dt rather than B. This problem is usually solved by integrating the probe signal with a passive circuit integrator (Krall and Trivelpiece, 1973). The output signal of such a probe signal is

$$V = \frac{nA}{RC} \cdot B \times 10^{-8} \text{ volts.}$$

when n = number of turns in the loop

A = area of the loop, $cm^2$

B = magnetic field strength, Gauss.

R = resistance ($\Omega$) of the series R - C combination

C = capacitance (F) ofthe series R - C combination.

Care must be taken to shield magnetic probes electrostatically so that electric fields associated with the discharge could be avoided. Such shielding limits the maximum frequency of magnetic field fluctuations that can be detected by the probe assembly.

Another kind of coil system that measures the rates of change of enclosed current channels is the Rogowasky Loop or circle (Golovin et al, 1958; Cooper, 1963). The assembly consists of two sets of coils, one around the entire experimental region and other around only the current channel or a part of it. The difference in induced voltage represents the currents not enclosed, such as wall currents. The coils may be segmented, with leads brought out separately, to indicate current profiles. It is shown by Bergland et al (1963) that in the design of magnetic probes for plasma field measurements, attention must be given to the space resolving power of the coils. A technique is described by them, which by mechanical means gives an exact alignment of the individual coils in a multiple probe without any need for further manual adjustment. Integration of the signals by means of transistorised Miller integrators placed at the probe and of the transmitting cables, gives a good signal-to-noise ratio. The described system has an overall sensitivity of 1 mv $G^{-1}$ at an RC time of 8 mS, and the band-width extends to over 2 MHz.

Various magnetic probe measurements have been carried out by Cilliers et al (1963) to investigate the behaviour of the plasma in a theta type discharge. With ''balanced'' probes the amount of trapped field at the second implosion could be estimated. This technique has been used to estimate electron temperature.

1.3. <u>Electrical Conductivity and Radio Frequency Coil Probe.</u>

The knowledge of electrical conductivity of a plasma sometimes serves to enlighten the state of the plasma and hence to corelate other plasma parameters. In certain applications of plasma Physics (e.g., MHD generator) the knowledge of electrical conductivity is of direct importance in its own right.

The electrical conductivity of highly ionized gases plays an important role in high temparature gas dynamics (specially for large diamension as in celestial gas dynamics) through the interaction of magnetic fields with the gas motion (magnetohydrodynamics). Measurement of electrical conductivity of a plasma can be made using variety of methods depending on the nature of the plasma (e.g., discharge plasma, shock induced plasma, plasma jets and other flow facility plasmas etc.). The plasma conductivity is mostly determined by conventional probes (electrodes). The inadequacy of the usual probe method becomes obvious in several circumstances. It was observed by Lin et al (1955) that in hot ionized gas the probe method is accompanied by difficulties arising from the existance of a cold boundary layer around the probe. In the case of cold plasma, the probe current gives little information on the conductivity. In that case an attempt for indirect measurement of conductivity may be made by measuring the electron density; but evaluation of conductivity becomes still difficult due to the fact that no exact method of measurement of collision frequency has yet been found.

The probe method is not applicable to a field-free plasma such as after-glow plasma, diffusion plasma and so on. Further, in the case of flowing plasma, the probe method should not be used because the inserted probe may appreciably disturb the dynamics of the flow. In some cases the plasma jet may even destroy the diagnostic probe. Hence coil probe technique has become very much popular to deal with conductivity problems in variety of circumstances. It has been observed that with ion densities in the range $10^{13} - 10^{15}$ $cm^{-3}$ it is one of the few techniques that can be used. The basic principle involved in most of the coil probe diagnostic techniques lies in the fact that magnetic field associated with a solinoidal radio-frequency electric field induces solenoidal current into the plasma under study, and the effect is reflected back into the probe coil wound around (and some times inserted in) the plasma. Hence this method is often termed as induction method or magnetic flux method by different authors.

13.1. In the coil probe method devised by Lin et al (1955) for the determination of electrical conductivity profiles of highly ionised argon produced by shock waves, however, no radio-frequency source was employed. In their method the information was obtained from the search coil (probe) pick up of electromagnetic distrubances produced by the passage of shock waves through it. Possibly, the paper of Lin et al (1955) represents the first record of coil probe experiment for the determination of electrical conductivity of a plasma. While experimenting on the electrical conductivity of shock-produced argon plasma by conventional probe method they were encountered with the following difficulties.

It was shown by them that for small degrees of ionization the electrical conductivity of a plasma may be approximately given by

$$\sigma = \text{Const.} \; \frac{T^{1/2}}{\rho^{1/2}} \cdot e^{-q_1/2kT}$$

where $\rho$ is the density of the gas, T is the absolute temperature, $q_1$ is the first ionization potential of the gas, and k is the Boltzman constant. According to the results of their probe experiments it was found that the relation between $\sigma$ and gas temperature showed somewhat exponential character; however, the indicated conductivity was as much as thousand times smaller than the theoretical value. They also observed that for some range of gas pressure the conductivity was found to be approximately proportional to the gas density instead of being inversely proportional to the square root as predicted by the theory. The discrepancy was attributed to the non-equilibrium effects. It was conjectured by them that the gas next to the probe and the shock tube walls was cooler than the gas away from these surfaces. It was thought that these cool boundary layers might greatly increase the apparent gas resistance. Instead of trying to explore these effects quantitatively, they rather concentrated on developing a new method.

To eliminate the inherent surface effects associated with probe measurements an experiment utilising the interaction between a magnetic field and the conducting gas behind the shock wave was designed. In their method an axially symmetric magnetic field was introduced into the glass section of the shock tube. A d.c. magnetic field was provided by a short solenoid, the axis of which was common to that of the shock tube. The probe coil ( a small search coil) was placed slightly ahead of the field coil to pick up the electromagnetic disturbances produced by the passage of the shock wave through the magnetic field. The direct computation of electrical conductivity calls for the knowledge of the geometry of the arrangement and gas velocity and involves severe mathematical complications; however, it was much easier to experimentally obtain the response of the apparatus to moving metallic rod. This calibration was expected to serve to evaluate a geometric function which was difficult to calculate otherwise. The lack of exact simulation of the metal and the moving gas was easily taken into account in their analysis. According to them it was observed that conductivity distribution $\sigma(\xi)$ (They ignored the effect of radial non-uniformity and the term ''distribution'' meant axial distribution) could be obtained by solving an integral equation of the first kind with the metallic rod response function $V_e$ ( $S - \xi$ ) as the <u>kernel</u>, where $S$ is the position of the shock front with respect to the probe at a given time t, and $\xi$ represents the axial co-ordinate of any point with respect to the shock front. It is now worthwhile to describe the constructional features of their probe coil which has got some ingenuity in avoiding the electrostatic effect associated with the shock phenomenon. During the early stage of the coil probe experiment, where a single 50-turn coil was used, electrostatic effects were noticed. ~~Drum~~ During the experiment it was observed that even when the steady magnetic field was put off, large signals were found to pass through the search coil during each shock. This was, as termed by them, was due to electrostatic effects. Actually those pick-ups were due to the formation of finite capacitance between the search coil and the gas inside the shock tube. This is observed by many observers including the present author (Ghosal et al, 1976) and this is termed as stray capacitance effect. To get rid of this effect Lin et al devised a centre-tapped search coil arrangement. The search coil was a 50 - turn 8 mm. long single-layered coil made of enamelled wire, the coil was actually made up of two 25- turn coils wound side by side and connected in series (Fig.1.1). The junction point of the two coils was grounded, and the two ends of the combined coil were connected to the push-pull input of a Tektronix (512) oscilloscope. The value of the shunt input resistance (1000 Ω) was so chosen to give approximately critical damping of the search coil circuit. It may easily be understood from the figure that the capacitive pick- up from the two ends of the coil cancelled out by the push-pull arrangement, while the magnetic pick-up was unaffected. The experimental determination of the electrical conductivity was claimed to have agreed with the limit of accuracy of the theory and the discrepancy at lower temperatures was attributed to the non-equilibrium effects. Later, Lamb and Lin (1957) took measurements in shock wave air plasma utilizing the same method and the results corroborated with the theoretical predictions.

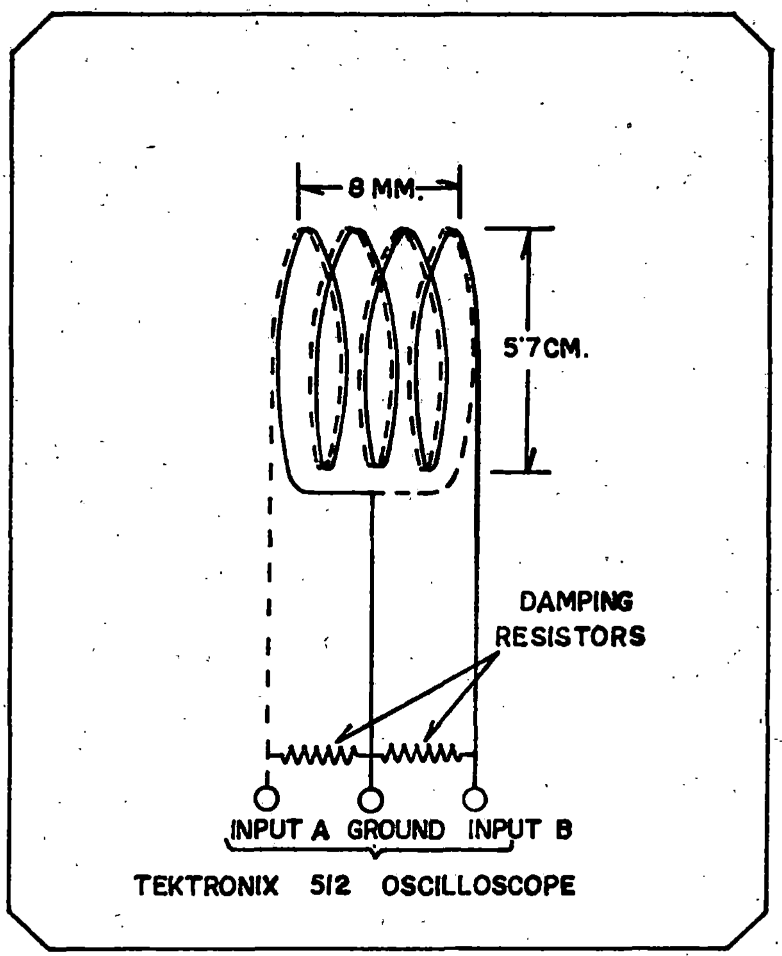



FIG. 1·1 . SCHEMATIC DIAGRAM OF THE SEARCH COIL USED IN THE APPARATUS.
THE CENTERTAPPED DESIGN IS INTENDED TO MINIMIZE ELECTROSTATIC PICKUP DUE TO THE SUDDEN APPEARANCE OF (SLIGHTLY) CHARGED GAS IN THE SHOCK TUBE.

1.3.2. A few years later K.B. Persson (Persson 1960,1961) developed a new coil probe method for measuring the conductivity in a high electron density plasma. The theoretical aspect of their technique was endowed with simplicity and elegance which compensates for the technical rigour of their measurement method. Here the coil probe was made to send solenoidal electric field into the plasma in order to collect information from inside. This may be termed as active probing in order to distinguish it from the passive probing arrangement of the experiment of Lin et al, where the probe was used as a pick-up device only, making the method unsuitable for stationary plasma. This active coil probe method was based on the interaction between the solenoidal electric field and a circular cylindrical plasma column and this formed a basis to design a pulse-operated bridge suitable for transient measurements of electrical conductivity in high electron density after-glow plasma. A cylindrical plasma with rotational symmetry, long in comparison with its radius r , was confined within a tube made of non-conducting material. Another tube carried a solenoid which was long relative to its own radius but short in comparison to the length of the plasma column. The uniform pitchers of the solenoid were such that the resonance frequency of the coil was much larger in comparison to the probing frequency. The stray capacitance effect was eliminated by applying a thin tin-oxide coating inside the second tubing. The thickness of the shield was so adjusted that the alternating field in one hand could penetrate into the plasma (thickness very small in comparison to the skin-depth at the relevent frequency) and on the other hand the axial electric field associated with the plain solenoid could automatically be short-circuited. The tin-oxide coating was applied in order to ensure that only the time varying magnetic field generated by the solenoid was allowed to penetrate into the plasma. The theory of their measurements was quite straight-forward and simple. The magnetic flux generated by the solenoidal plasma currents was easily evaluated since the length of the interaction zone was large in comparison with the diameter of the plasma. The plasma magnetic field induced a voltage $\Delta V$ in solenoid and could be treated as a perturbation quantity. Quite justly the solenoid and the plasma were considered as a transformer with the solenoid as the primary and the plasma as a one-turn secondary winding with the resistances $R_s$ and $R_p$ and inductances $L_s$ and $L_p$ respectively, and with the mutual inductance $M_{sp}$, the perturbation voltage $\Delta V$ satisfies the differential equation,

$$\left( R_s + L_s \frac{\partial}{\partial t} \right) \left( R_p + L_p \frac{\partial}{\partial t} \right) \Delta V = - M_{sp}^2 \frac{\partial^2 V}{\partial t^2}$$

where V was the voltage impressed on the primary winding. The effect of $L_p$ can completely be neglected if skin-depth is assumed to be large in comparison to the diameter of the plasma column and in that case the differential equation which involves the measurable quantity $\Delta V$ can be written as

$$\frac{\partial \Delta V}{\partial t} + \frac{R_s}{L_s} \Delta V = - \frac{M_{sp}^2}{R_p L_s} \cdot \frac{\partial^2 V}{\partial t^2}$$

where $\frac{M_{sp}}{R_p L_s}$ may be expressed in terms of the geometry of the arrangement and some plasma parameters, viz., $\omega_p$ (plasma frequency ) and $\nu_m$ (electron-atom collision frequency) as:

$$M_{sp}^2 / R_p L_s = r_p^4 \langle \omega_p^2 \rangle / 8 r_s^2 c^2 \nu_m$$

where the average $\langle \omega_p^2 \rangle$ was defined as

$$\langle \omega_p^2 \rangle = \frac{8}{r_p^4} \int_0^{r_p} r \, dr \int_0^{r_p} \omega_p^2 \, r \, dr.$$

In his paper the last differential equation was solved both for sinusoidal and Gaussian-like pulse voltages. The steady state solution obtained for the first case with applied frequency $\omega$ was

$$\frac{\Delta V}{V} = \frac{r_p^2 \omega^2}{8 r_s^2 c^2 \nu_m \left[ j\omega + \frac{R_s}{L_s} \right]} \langle \omega_p^2 \rangle = \frac{\mu_0 r_p^4 \omega^2}{8 r_s^2 \, j\omega + \frac{R_s}{L_s}} \langle \sigma \rangle$$

where $\langle \sigma \rangle$ defined the suitable average conductivity. A measurement of the ratio $\Delta V/V$ yielded information about the average conductivity. The proportionality constant could not easily be evaluated exactly due to the presence of end effects (though could be done in principle). It was however easily determined experimentally by replacing the plasma with some electrolyte with known conductivity. It is to be noted at this point that this calibration technique utilising electrolytic solutions was used by various authors afterwards.

It is observed that for transient or decaying plasma the continuous sinusoidal probing was accompanied with difficulties. This demanded for low Q of the measurement circuit or higher probe frequency. Neither of the two alternatives could be used without sacrificing the available range for measurements. The difficulty was overcome by replacing the sinusoidal signal V in the last differential equation with a suitable pulse function. This was conveniently done by expanding V in terms of Gaussian function of the form $V_o \exp[-(t/\tau)^2/2]$. It was shown that the unbalanced signal could be written as

$$V = -V_o \frac{R_s}{R_p}\left(\frac{M_{sp}}{L_s}\right)^2 \sum_{i=2}^{\infty}\left(\frac{L_s}{\tau R_s}\right)\exp\left[-\frac{1}{2}\left(t/\tau\right)^2\right]H_i(t/\tau)$$

where $H_i(t/\tau)$ is the Hermite polynomial of the i th degree. It was further shown that the maximum of the unbalanced pulse (when $(t/\tau)^2 = 3$) was most convenient for the measurements on the plasma and hence,

$$\left(\frac{\Delta V}{V_o}\right)_{max} \cong \frac{2}{\tau}\frac{M_{sp}^2}{R_p R_s} \simeq -\frac{2\pi}{8}\frac{N^2 r_p^4}{R_s l_s \tau^2}\langle \sigma \rangle$$

It is now worthwhile to describe in brief the operation of the pulse-operated bridge designed by the author for measurements in transient plasmas (Fig.1.2). Two single-layered solenoids were made as identical as possible. They were located symmetrically in separate compartments of a shielding box of brass. One of those solenoids was wound around the tube containing plasma. The solenoids were coupled together forming the bridge which was symmetric with respect to ground or the shielding box. The pulse applied to the bridge was supplied through toroidal ferrite core transformer which was designed to deliver very symmetric pulse with respect to the shield. The current pulse through the primary of the transformer was generated by high current thyratron sweep arrangement. The pulse delivered by the secondary, however, resembled Gaussian pulse (Pulse Width $\simeq 1\mu$sec.).

The discharge tube was inserted in one of the solenoids. The bridge constituting the symmetric transformer and the two solenoids and other resistive and reactive elements was balanced by several trimming arrangements. The basic principle on which the operation of the bridge was based was that, provided the bridge is well balanced beforehand, it is driven off-balance if the plasma is introduced in one of the solenoids. The resulting resistively unbalanced signal, when observed and recorded, could yield informations of plasma conductivity. The automatic operation and recording arrangements and time adjustments by various triggering mechanisms were explained in his paper. The measurements were made for highly ionized after-glow plasma and the author

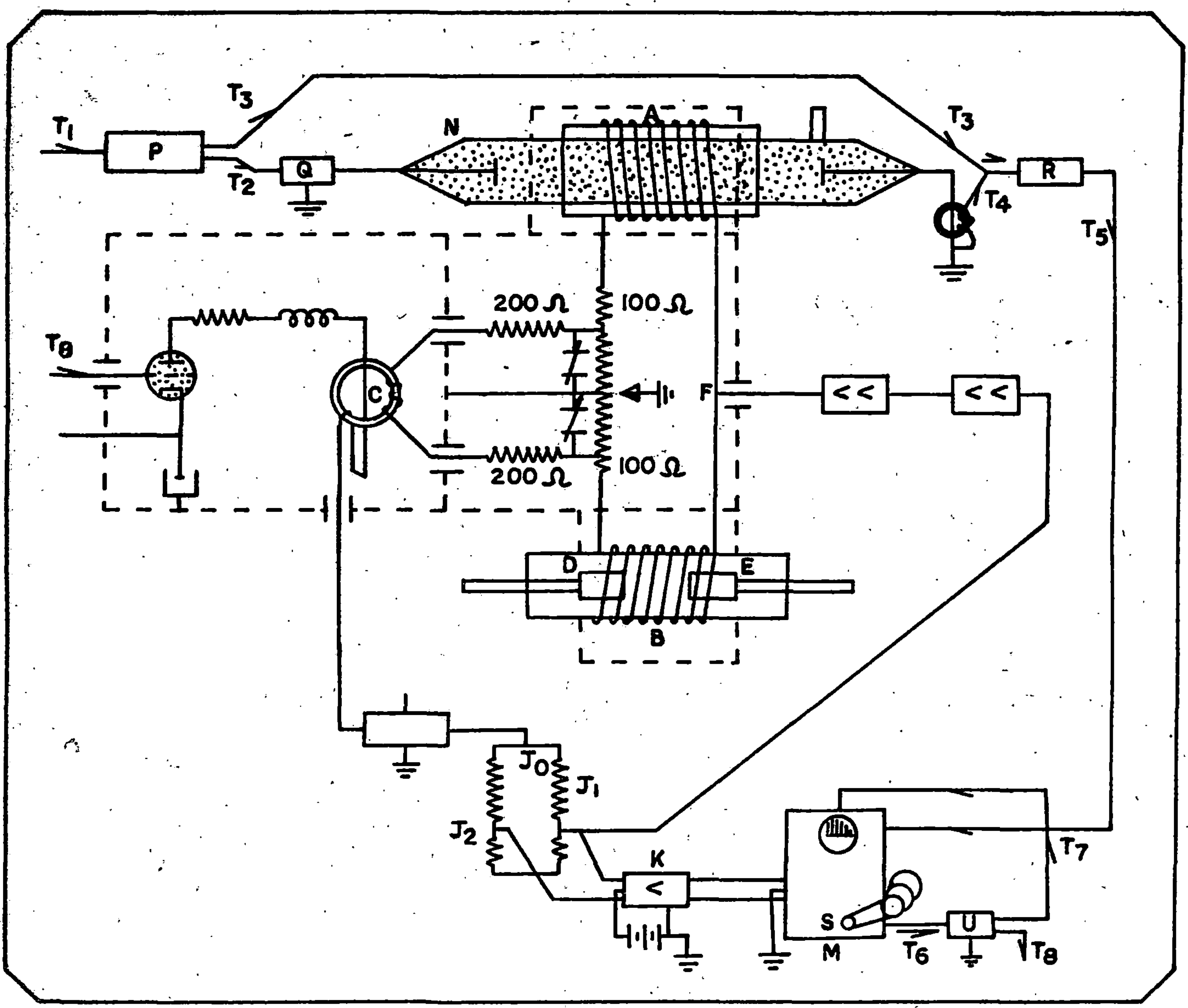



FIG. 1·2 . SCHEMATIC DIAGRAM OF THE PULSE OPERATED BRIDGE WITH ITS AUXILIARY CIRCUITS FOR MEASUREMENTS ON TRANSIENT PLASMAS.

obtained the temporal conductivity distribution. It was also indicated that linearity of the interaction (together with the electronic circuitry) was checked by the use of electrolytes. It is to be noted here that the use of electrolytic solution for calibration of the coil probe apparatus was also reported by Blackman (1959), a year before the work of Persson.

1.3.3. Blackman, in his paper entitled ''Magnetohydrodynamic Flow Experiments of a steady state Nature'' described a method in which the inductance of a coil surrounding a plasma is reduced by the shielding effect in the electrically conducting plasma. The reduced inductance increased the frequency of a circuit resonant at about 50 Mc/s and the shift was detected by a radio receiver. Savic and Boult (1962) intelligently utilised the above idea, that the inductance of a coil surrounding a conducting fluid changes, to devise a frequency modulation circuit for the measurement of gas conductivity and boundary layer thickness in a shock tube. They first theoretically calculated the change of inductance of a short coil due to the presence around its axis of a cylindrical, moving electrically conducting fluid and it was shown by them that the hydromagnetic influence could be kept small. The calibration and design of the frequency modulation circuit was based on those calculations. It was observed that conductivities of gases behind strong shocks in shock tubes could be measured with fair accuracy and adequate time resolution for the detection of local variations in flow structure. A different technique was adopted by Rosa (1961) in his well known MHD experiments in determining the conductivity of a flowing plasma. In his method the coil was embedded in the insulator wall of the MHD generator. The coil was resonated with a condenser at about 300 KHz and the damping of the circuit due to the exhaust of the gas through the insulating tube was measured to determine the gas conductivity. In this experiment also calibration was effected by placing various salt solutions in the channel.

1.3.4. Olson and Lary (1961,1962,1963) suggested a different approach where the coil probe was immersed within the plasma instead of being wound around it. The first two papers (Olson and Lary, 1961; Lary and Olson, 1962) of their series discussed the theoretical aspects of the technique. Their third paper (Olson and Lary, 1962) was concerned mainly with the instrumental methods employed in the measurement of conductivity of the positive column of a hydrogen glow discharge and 4500°F rocket nozzle flow. Later on, immersive coil probe technique was reported by some authors (Moulin and Masse, 1964;Stubbe, 1968 and very recently Jayakumar et al, 1977). In spite of the obvious disadvantages of immersive probe methods Olson and Lary pleaded for some of its advantages. According to them the non-immersive method appeared to be rather insensitive to variations in plasma conductivity and were affected by the properties of the wall surrounding the plasma and by stray capacitive effects. The method described by them was little influenced by electrostatic effects and claimed to have recorded variations of plasma conductivity of as small as 1 mho/m or less. The basic principle involved for this technique was however the same as other methods i.e., this method depended upon the interaction of the conducting fluid with imposed r.f. magnetic field. In particular, r.f. impedance of a solenoid was affected by the presence of a conducting medium in the neighbourhood of the solenoid. In the case of a coil wound arround a conductor, the r.f.magnetic field of the coil induces an azimuthal electric field which causes azimuthal current to flow through the conductor. This causes an increase in the apparent resistance of the coil. Similar results are expected to be obtained if the conductor surrounds the finite coil rather than being surrounded by it, as may be verified by applying the same argument to the external return flux. The technique involved the use of small movable probe containing the inductor (coil). Since the magnetic field diminishes rapidly with the distance from the coil, the conductivity in the immediate vicinity of the probe was sampled. The probe could be moved about in the plasma in order to resolve spatial variation of plasma conductivity.

The experimental apparatus designed to record the change in coil resistance as a function of conductivity and impressed frequency had some novelty of its own. The coil probe formed a part of the tank circuit of a Colpitts type oscillator. A vacuum tube supplied the power to sustain the oscillation of the tank circuit. At the on-set of the plasma, power dissipation due to the azimuthal currents induced by the coil in the conducting medium, resulted in a resistive loading of the tuned circuit which changed the grid current of the oscillator tube, the measurement of which was achieved through the use of a grid-dip meter. The grid current varied almost linearly with the power dissipated in the coil conductor circuit. As indicated earlier electrolytic solutions, whose conductivities were in the same range as plasmas were used to calibrate the conductivity probe. The calibration was further verified by the use of semiconductors for double check. First the measurements were made in the positive column of a glow discharge in hydrogen. An 8 mm. (outer dia.) probe was immersed in a 25 mm. dia. plasma column. Special precaution was taken to eliminate the heating effect of the coil involving change of coil dimension and resistance, which would otherwise lead to erroneous results. To ensure this the coil temperture was held constant by a cooling flow of dry nitrogen. A thermo-couple was used to monitor the probe temperature. The performance of the coil probe (after suitable modifications) was also investigated for a MHD flow environment. A cool boron nitrite probe was inserted into a subsonic MHD flow with temperatures in the neighbourhood of 4500°F. The same calibration method was used in this experiment also.

1.3.5. Donskoi, Duaev and Prokof'ev (1963) of U.S.S.R. used the same induction technique of non-immersive type to measure electrical conductivity of heated gas streams. The streams were obtained as a result of burning of ethyl alcohol in oxygen containing 0.01 % potassium. The pressure in the combustion chamber was maintained at 4 atm. The gas was supplied for 6-12 Sec., thus necessiating semi-transient measurements. The method was based on measurements of the parameters of tank circuit whose coil surrounded the stream of the heated gas. According to them when the respective positions of the coil and the plasma stream are kept constant, the parameters of the circuit (effective inductance, circuit resistance, Q factor etc.) depend upon the magnitude and distribution of electrical conductivity over the cross-section of the stream and also on the impressed frequency. Therefore, by measuring the parameters of the circuit for given dimensions of the coil and the stream at different frequencies , one can determine the magnitude and the distribution of electrical conductivity over the cross-section either by calculation or by calibration curves. In practice they have chosen the second method i.e., the non-absolute method as was done by previous authors. It was also assumed by them that distribution of gas streams was uniform . Accordingly they suggested that the observation of the change of one circuit parameter (viz., Q of the circuit) would only suffice to determine the conductivity provided the apparatus was well calibrated beforehand. In actual experiment the tank circuit was fed by a radiofrequency oscillator and the output taken from the coil was amplified and fed to the oscilloscope faciliating semi-transient measurements. They thought that the change of output was due to the change of Q-factor only, but in practice the change of inductance and servere severe capacitive effects might have played an important role. However, since the method was based on previous calibration with electrolytes the results obtained may be expected to be not far from the true values. It is to be noted here that if the capacitative effects were prominent and the stream had a radial distribution, the results would have been far from the true azimuthal value of the electrical conductivity.

1.3.6. Another technique was described by Korits and Keck(1964) for measuring the electrical conductivity of hypersonic wakes and any other conducting medium by measurement of Joule losses produced by the oscillatory magnetic field of a circular coil surrounding it. The technique was similar in principle to that developed by Lin, Resler and Kantrowitz (1955) to measure the conductivity behind shock waves in argon but the drawback of the <u>passive</u> probing arrangement was removed by substituting it by <u>active</u> probing method (i.e. by impressing radio frequency field to the coil). Consequently the method had the advantage that it may be employed in cases where the medium is stationary. Originally the apparatus was designed specifically to investigate the conductivity in the wake of hypersonic pallets; but later it was also used to measure the conductivity of electrolytic solutions, electrical discharges, flames, and plasmas produced in shock tubes. The results of their measurements for shock wave air plasma was corroborated by that of Lamb and Lin (1957).

The actual circuit consisted of a symmetrical r.f. bridge (The bridge developed by Persson (1961) may be recalled), two arms of which contained identical coils. When a conducting medium was inserted into one of the coils, the effective impedance of the coil changed and the bridge became unbalanced. They chose the probe frequency in such a way so that associated displacement current was negligible and the change of impedance of the coil was entirely resistive. Further for the complete penetration of the field the skin-depth was required to be much greater than the radius $r_\omega$ of the plasma column. Mathematically the following two relations were satisfied :

$$\sigma \gg \omega \epsilon \qquad \text{(for neglegible displacement current)}$$

and $$\sigma \ll 2(\mu \omega r_\omega^2)^{-1} \qquad \text{(for complete penetration of the field)}$$

where $\epsilon$ and $\mu$ are the dielectric constant and magnetic permeability of the medium and $\omega$ is the impressed frequency. Under the above conditions the change of the impedance of the coil was

$$\Delta R = P/I^2$$

where I is the rms current in the coil and P is the power dissipated in the medium. They studied the coil-plasma (conductor) interaction first in a somewhat generalised way and later made relevant approximations. Since they intended to measure the conductivity of wakes behind the hypersonic pallets they considered both the radial (r) and axial (z) non-uniformity of the conductivity. Thus assuming cylindrical symmetry, P was given by

$$P = \int_0^{\infty} \int_{-\infty}^{+\infty} \sigma \, E_{\phi}^2 (r,z) . \, 2 \pi r \, dr \, dz$$

where $E_{\phi}$ (r,z) is the induced solenoidal electric field at any point (r,z) within the coil. This was calculated in a standard way to yield

$$E_{\phi}(r,z) = \frac{\mu N}{2\pi} \left( \frac{r_c}{r} \right)^{1/2} k^3 \, C \, (k^2) \, \frac{\partial I}{\partial t}$$

N being the number of turns of the coil, $C(k^2)$ is the complete elliptic integral (Jahnke and Emde, 1945) which for small values of k may be approximated as

$$C(k^2) = \frac{1}{16} \pi \left( 1 + \frac{3}{4} k^2 + .......... \right)$$

where $k^2 = 4r \, r_c / \left[ (r_c + r)^2 + z^2 \right]$, $r_c$ being the radius of the coil.

Consequently, R may be written as

$$R = \frac{3}{4r_c} \left[ \frac{\pi \mu \omega N}{4} \right]^2 \int_0^{+\infty} \sigma \, (r) \, r^3 \, dr$$

where $\bar{\sigma}(r)$ is some suitable axial average value of conductivity. To interpret the above equation in terms of their measurements they introduced a concept of effective radius $\bar{r}_\omega$ of the conducting column, in terms of which the above equation was written as

$$\Delta R = \frac{2}{r_c} (\pi \mu \omega N/_{16})^2 \bar{\sigma}(o) \; \bar{r}_\omega{}^4$$

where $$\bar{r}_\omega = \left[ \frac{1}{\bar{\sigma}(o)} \int_0^{+\infty} r^4 \; \frac{d\bar{\sigma}(r)}{dr} \; dr \right]^{1/2}$$

The equation for $\Delta R$ is equivalent to a model where

$$\sigma < \bar{r}_\omega = \text{const.} \text{ and } \sigma > \bar{r}_\omega = 0$$

It is interesting to note here that in contrast to others the formulae for $\Delta R$ given by them could be used for absolute determination (without calibration) of $\bar{\sigma}(o)$ provided $\bar{r}_\omega$ could be obtained by some other means. Unfortunately they desired the equation only to show that

$$\Delta R \propto \bar{\sigma}(0)$$

and obtained the conductivity results using calibration with electrolytes, the hazard of which was indicated previously (and later it will be discussed in some detail). The measurement of conductivity of stationary plasma was straightforward and simple but the wake measurements were difficult due to the unknown factor $r_\omega$ ; but previously extensive studies on wake radii measurements were made by Taylor et al (1963) using optical technique, so it was possible to estimate the conductivity from the experimental results, calibration curves and the data of Taylor et al (1963). It was, however, assumed that the wake radius was equal to the plasma radius.

1.3.7. Tanaka and Hagi (1964, 1964) in a couple of papers (appeared in the same issue of Japanese Journal of Applied Physics) re-examined the tentatively adopted conductor approximation for plasma by various authors (viz., Tanaka and Usami, 1962; Gourdin, 1963; Khvashchtevski, 1962, and others mentioned above). Conductor approximation for plasma means that when an electrical a.c. is impressed upon it, the plasma is assumed to offer no reactance and a.c. conductivity essentially becomes d.c. conductivity. In their first paper, they critically analysed the interaction of alternating currents with plasma in general and showed that if the change of magnetic flux through a coil due to a presence of plasma inside it could be measured, the d.c. conductivity is possible to be obtained even in presence of displacement current, for a wide range of frequencies. Their analysis started from the well known expression for a.c. conductivity ( $\sigma_{a.c.}$ ) for partially ionized non-equilibrium cold plasma, (Sengupta, 1961; Heald and Wharton, 1965),

$$\sigma_{a.c} = \frac{ne^2}{m\nu}\left(\frac{\nu^2}{\nu^2+\omega^2} - j\,\frac{\omega\nu}{\nu^2+\omega^2}\right)$$

where m is the electron mass, e the electronic charge, n the electron number density, $\nu$ the electron-atom collision frequency and $\omega$ is the angular frequency of the applied radio frequency ~~and is the angular frequency of the applied radio frequency~~ field. In the above equation the imaginary part appears due to inherent plasma reactance. Actually the phase lag which is apparent from the equation is due to the mass inertia. It may thus be seen that the conductor approximation is valid if $\omega \ll \nu$ ; in that case,

$$\sigma_{a.c} = \sigma_0 = \frac{ne^2}{m\nu}$$

whereas for the another extreme case ( $\omega \gg \nu$ ) the plasma impedance is purely reactive. They also studied the a.c. conductivity for intermediate frequencies where both resistive and reactive parts were dominating. Finally they solved the Maxwell equations in cylindrical form for uniform plasma. It was found that magnetic field* H(r) at any radial postion r could be given by the equation

$$H(r) = \frac{H(R)}{J_0(\beta R)}\, J_0(\beta r)$$

where

$$\beta^2 = -\frac{\omega^2}{c^2}\,\frac{ne^2}{m\nu}\,\frac{4\pi\nu}{\nu^2+\omega^2}\left(1+j\frac{\nu}{\omega}\right)$$

---

* According to the authors H(r) is the magnetic field in the radial direction but actually field direction should be longitudinal.

and R is the plasma radius. The reduction of magnetic flux $\phi$ due to the presence of plasma is evident from the above equation for H(r). If $\phi_o$ denotes the magnetic flux in the absence of the plasma, the reduction ratio $\alpha$ could be written as

$$\alpha = \frac{\phi}{\phi_o} = \frac{2}{\beta R} \quad \frac{J_1(\beta R)}{J_o(\beta R)}$$

The relation between $\sigma_o$ and reduction ratio $\phi/\phi_o$ was obtained numerically and plotted taking $\gamma = \nu/\omega$ as a parameter. This revealed an '' interesting aspect '' of the problem. It could be seen from the graphs that the d.c. conductivity obtained from the magnetic flux change is insensitive to the value of the parameter $\gamma = \nu/\omega$. Thus it was argued that provided the successful evaluation of the quantity $\phi/\phi_o$ is known , $\sigma_o$ could be determined reliably since no detailed knowledge of $\nu$ is required. In the second paper of Tanaka and Hagi (1964), an example of magnetic flux measurement in the plasma and evaluation therefrom of the plasma conductivity was given. Basically the method was very similar to that of Blackman (1959). The plasma tube was inserted in a single turn coil of a LC oscillator. From the observed shift $\Delta f$ of the resonance frequency at the onset of the plasma, magnetic flux penetrating the coil was evaluated and finally plasma coductivity was determined. It may be apparent now that all the active coil probe experiments mentioned in this section utilizes the change in either inductance or resistance of coil probe due to insertion of plasma, in one way or other. But theoretically each of them viewed the problem from different angles leading to some uniqueness of every experiment. Tanaka and Hagi viewed the inductance change effect in a different manner. According to them if plasma is conductive, the applied solenoidal r.f. field will induce eddy currents which will flow around the plasma dissipating energy in the region where they flow, by which magnetic flux is excluded from that region. The argument is similar to that given for the physical significance of skin effect. Thus the effective inductance of the network is reduced, resulting in a change in resonance frequency.

Measurements were carried out for two hot cathode type discharge tubes, one filled with 1 torr of argon and the other with 8 torr of neon. An ordinary Hartley type LC oscillator (frequency range 100 - 150 MHz), with a single turn coil, was employed. The radius of the coil (2.25 cm) was kept much larger than the radius of the discharge tube in order to minimize the stray capacity between the coil and the tube. The coil output and the output from a standard signal generator was fed to a mixer circuit (Fig.1.3). The resonance frequency shift $\Delta f$ due to the insertion of the plasma tube in L was then determined by observation of the <u>beat</u> between the oscillator signal and the standard signal. The observed frequency shifts $\Delta f$ for different discharge currents were given in Figs. 1.4 (a) and 1.4 (b) from which it could be observed that in accordance with their theoretical prediction $\Delta f$ was found to be positive and increasing with the discharge

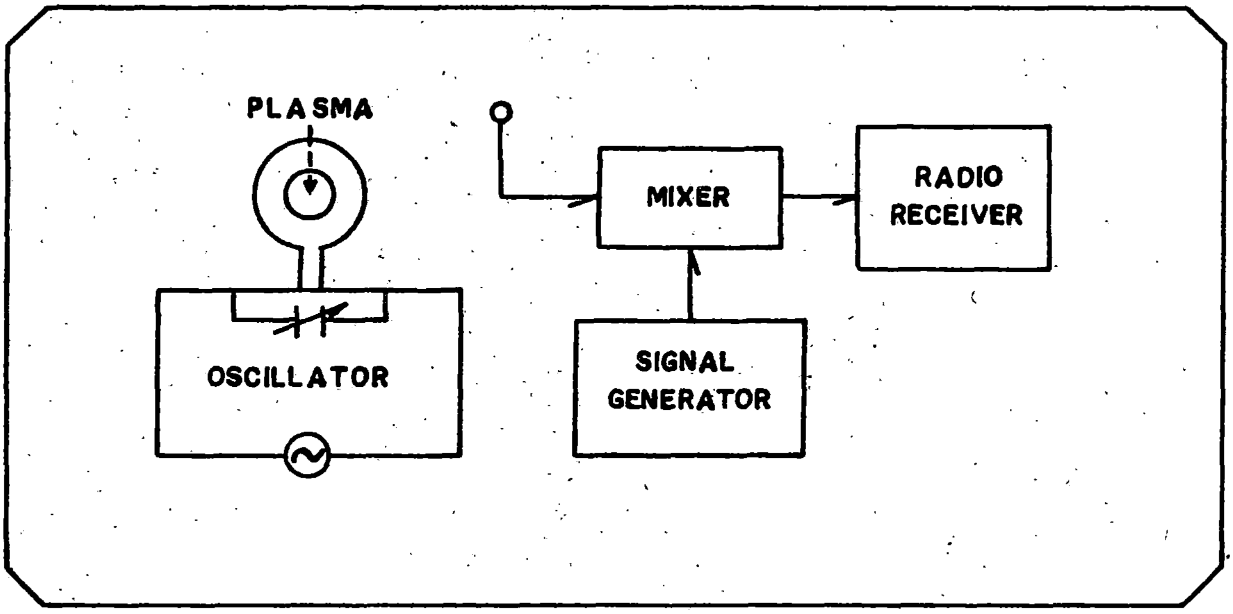
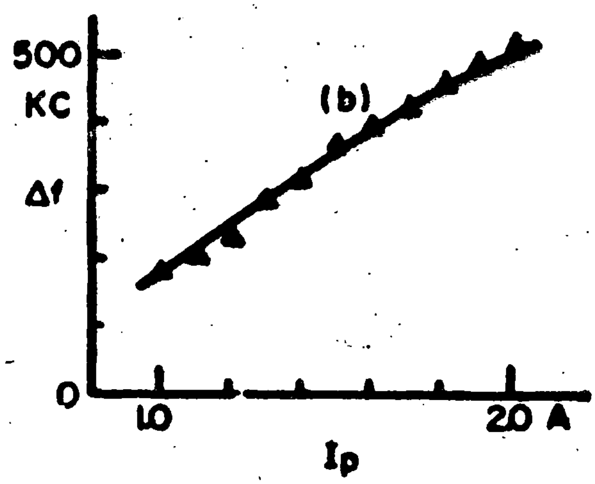
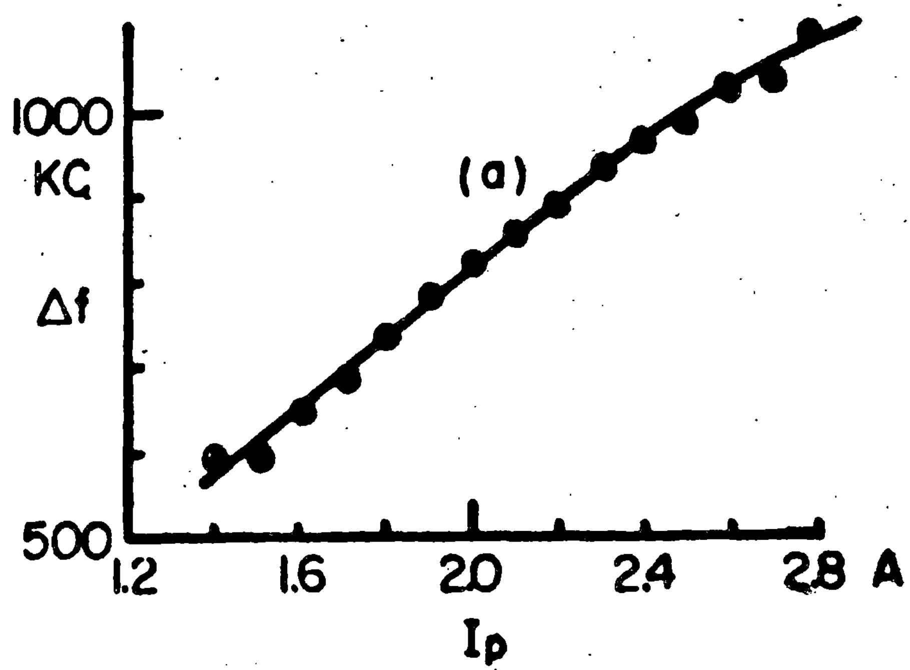



FIG. 1·3. SCHEMATIC DIAGRAM SHOWING THE METHOD.




(a)




(b)

**Fig.1.4. Frequency shift v.s. discharge current:**
**(a) Ar at 154 Mc**
**(b) Ne at 119 Mc**

Reproduced from the paper of H. Tanaca and M. Hagi (1964), J. Appl. Phys. Japan, 3 , p. 339

current $I_p$ , as long as it was in the Ampere range. But it was also observed (not shown in the figures) that for extremely small values of $I_p$ , $\Delta f$ was found to be negative. Previously the same author (Tanaka and Usami, 1962) observed the negative change in 'f' more markedly in the case of a fluorescent lamp. According to them the observed negative change in 'f' is an open question. However the present author's comment on this observation will appear in the introduction of the next chapter. As may be seen below this effect was also discussed by Akimov and Konenko (1966) and Hausler (1957) quite at length.

To determine conductivity from the knowledge of $\Delta f$ they obtained on the basis of a simple model of uniform plasma , the relation between the flux reduction factor and $\Delta f$ as

$$\frac{\Delta f}{f} = k' \frac{S_1}{S_1+S_2} \left( 1 - \frac{\phi_1}{\phi_0} \right)$$

where $(S_1 + S_2)$ represents the cross-sectional area of the coil, $S_1$ being the central area occupied by plasma and $S_2$ the remainder , and $\phi_0$ and $\phi_1$ represent the magnetic flux penetrating $S_1$ before and after insertion of the plasma respectively and $k'$ is a constant characteristic tor the apparatus used. They did not at this stage utilize the usual calibrating technique using electrolytes and semiconductors to avoid the unkown factor $k'$ , instead determined it using a tricky method. In order to determine $k'$, aluminium foils in the shape of cylinder of various diameters were inserted in the coil. Since the skin depth was of the order $10^{-3}$ cm. for aluminium at the relevant frequency, the magnetic field was completely excluded out of the area $S_1$.

$$\text{Hence } \phi_1 = 0$$

$$\text{and } \quad \frac{\Delta f}{f} = k' \frac{S_1}{S_1 + S_2}$$

Thus $k'$ was found to be 0.2.

According to the present author, however, this method was also a calibration method in another form, and since no guard was taken against the formation of capacitance between the coil and the aluminium foil, in all probability $\Delta f$ values were obtained with severe uncertainties. Further, the criticism made by the authors against the calibration methods adopted earlier may be said to be applicable to their own method also. The major discrepancy observed between the conductivity results obtained by standard probe method and the coil probe method may be due to this. There should, however, remain intrinsic discrepancies between the results obtained by two different techniques. This may be understood (Chapter IV) if the effect of radial distribution of conductivity is taken into account.

1.3.8. It is now relevant to mention a very important work of Akimov and Konenko (1966) who investigated the validity of the two similar well-known coil probe methods for measuring plasma conductivities and also explored various other possibilities. Though they particularly discussed the work of Blackman (1959) and Donskoi et al (1962), their comments are also applicable to those who studied electrical conductivity by observing either change in resonance frequency 'f' or change in quality factor Q of the oscillator coil in which the plasma was inserted. In almost all these experimetns in order to calibrate the apparatus, the plasma was replaced by electrolytes with known conductivity and made use of the calibration curves { $\Delta f = \Delta f(\sigma)$ or $\Delta Q = \Delta Q(\sigma)$ } thus obtained. Akimov and Konenko, following their own observation questioned the reliability of this calibration method. The experimental apparatus, similar to that of Tanaka and Hagi (1964), consisted of a Colpitts oscillator used to excite the tank circuit, standard frequency (s.f) generator and a mixer. The frequency shift was measured by the observation of beats. The measurements were made in electrolytes and in the positive column of an argon-mercury gas arc lamp. They obtained the quantity $\Delta f/f_o$ ('$f_o$' being the frequency of the oscillator before insertion of the test object) for electrolytes of different conductivities and also for plasma at different discharge currents enabling them to plot conductivity versus $\Delta f/f$ graphs (Fig.1.5) both for electrolytes and plasmas.

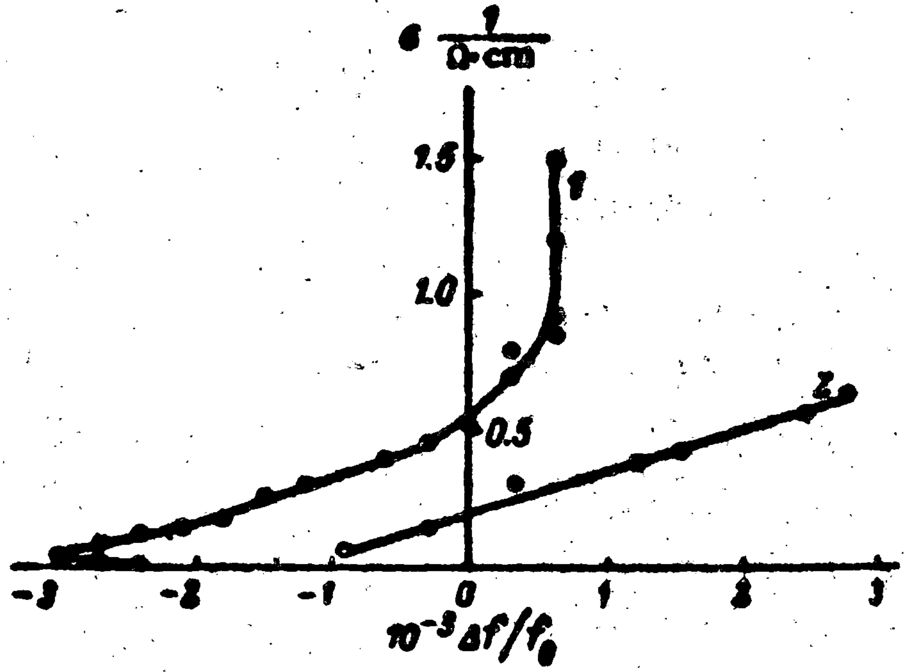



Fig. 1.5. The relative change in the frequency of the measuring oscillator and the corresponding conductivity of the plasma (1) and the electrolyte (2) at a frequency of 80 Mc. The capacitance of the oscillator circuit $C_c$ = 10 μμf.

Reproduced from the paper of A.V.Akimov and O.R. Konenko (1966), Sov. Phys. Tech. Phys., 10, 8, p. 1126

The plasma conductivity was obtained by ordinary probe measurements. It may be observed from the figure that in contrast to the prediction of Tanaka and Hagi (1964) (but in conformation with experimental observations) the test object in the coil can decrease the oscillator frequency for some ranges of conductivities. According to the authors and also according to Hausler (1957), the reduction of frequency was due to the capacitive effect of the test object on the work coil. The presence of conducting body in the vicinity of a coil increases its stray capacity (see chapter III also) and consequently the oscillator frequency decreases. Another aspect may be observed from the graphs that the plasma and its simulating electrolytes do not have the same effect on the measurement circuit. The discrepancy between the plasm conductivity averaged over the cross-sections and the calibrating curves was attributed by them to the radial non-uniformity of the plasma in the arc. According to them, due to the skin effect, the method gives informations of the peripheral region of the plasma only where the conductivity is much smaller than the average value; but according to the present author even if the skin depth is much larger than the plasma radius the discrepancy is expected to remain, since probe method and coil method give information on moments of conductivity distribution of different orders (see chapter IV).

1.3.9. All the authors (mentioned above and also others viz., Hollister (1964); Murino and Bonomo (1964), etc) using immersive or non-immersive coil probes determined in some way the average plasma conductivity because test plasma was radially inhomogeneous. Some of them, though aware of the fact that the test plasma was radially non-uniform, did not explore the type of average they were getting. The importance of the work of Ciampi and Talini (1967, 1969) lies in the fact that they studied the interaction of solenoidal electric field with a radially non-homogeneous plasma in a very generalised way and obtained the expression for meaningful averages of conductivity in relevance to the measurement techniques adopted by themselves or by earlier authors. The average was defined to be the conductivities equal to that of a fictitious homogeneous plasma imagined the insertion of which into the coil would produce the same change of coil impedance parameters, that is observed for the true plasma. Since normally in the experiments both the change of inductance and resistance of the coil are measured, they obtained the expressions for two relevant average conductivities. For this they first expressed the impedance of a solenoid of length l in terms of the electric field $E_R$, magnetic field $H_R$ and coil parameters (length l and radius R, number of turns N etc.) and also in terms of the applied frequency $\omega$, coil inductance etc. as,

$$Z = (8\pi^2 N^2 R/_{cl}) \cdot (E_R / H_R) = i\omega\lambda S$$

where the parameter

$S = (2 / i\mu K_o R) \cdot (E_R / H_R)$ , $K_o$ being the wave number, is generally a complex number depending on the medium characteristics and the probing frequency. The physical significance of the term $S( = \beta - i\alpha)$ is that it signifies the algebric reduction of the coil impedance due to the presence of the conducting fluid. This quantity may be experimentally determined, $\beta$ represents the contribution of the medium to the coil inductance and $\alpha$ the resistive contribution due to the energy loss in the medium. To express the dependance of 'S' on the characteristics of a radially inhomogeneous medium, the Maxwell's equations were solved using cylindrical co-ordinates to obtain the equation

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + \left( \frac{k^2}{R^2} - \frac{1}{r^2} \right) E = 0$$

where the dimensionless quantity

$$k^2 (r) = (k_o^2 R^2 / \mu) (\epsilon + 4\pi\sigma'(r) / i\omega)$$

is same as the quantity $(\beta r)^2$ defined by Tanaka and Hagi (1964). It may easily be observed that for homogeneous plasma i.e., for $\sigma'(r)$ = const., ($\sigma'(r)$ being the r.f. conductivity) the equation for 'S' reduces to

$$S = 2 J_1 (k) \Big/ k J_o (k)$$

which may now be compared to the expression obtained by Tanaka and Hagi (1964) (equation 4 of the first paper) and consequently 's' represents in that case of the so called flux reduction ratio.

They solved the problem (i.e., the differential equation given above) numerically by taking the conductivity profile of the form

$$\sigma(r) = \sigma_o \left[1 - m\left(\frac{r}{R}\right)^n\right]$$

They analysed the dependance of S with conductivity $\sigma_0$ for different values of m and n i.e., for various conductivity profiles. In their analysis they however, made a low frequency approximation i.e., $\omega/\nu \ll 1$, so that the effect of displacement current could be neglected and therefore the r.f. conductivity $\sigma'(r/R)$ was replaced by the d.c. conductivity $\sigma(r/R)$. It was argued that if the conductivity profiles is known beforehand, a measurement of $\alpha$ or $\beta$ at any frequency gives the value of the on-axis conductivity $\sigma_0$. It was also observed that for unknown profiles the $\alpha$, $\beta$ measurements could give informations about the plasma conductivity through data which are proportional to $\sigma_0$. This observation was found to be valid for some range of $\sigma_0$. It seems that the physical significance of this range of $\sigma_0$ in their derivation was a bit obscured. However, to the present author, the range of $\sigma_0$ is actually determined by the requirement of complete skin penetration of the r.f. field at the measuring frequency. Hence in the mentioned range, the measurement of $\alpha$ ( or $1-\beta$ ) for the unknown plasma and for a homogeneous medium of conductivity $\bar{\sigma} = h\sigma_0$ gives the same result at any frequency. Therefore, according to them, with reference to a resistive (or inductive) measurement the plasma 'simulates' a homogeneous medium and $\bar{\sigma}$ can be interpreted as a spatial average conductivity. Thus two averages $\sigma_*$ and $\sigma_{**}$ were obtained according to the resistive and inductive measurements respectively:

$$\sigma_* = \frac{4}{R^4} \int_0^R \sigma(r)\, r^3\, dr$$

$$\sigma^2_{**} = \frac{3}{4}\left[\frac{4}{R^4}\int_0^R \sigma(r) r^3\, dr\right]^2 + \frac{3}{4}\cdot\frac{6}{R^6}\int_0^R r^5\, dr\left[\frac{4}{r^4}\int_0^r \sigma(r) r^3\, dr\right]^2$$

Experiment was performed for a flow facility plasma utilising Q-factor measurements and employing calibration with electrolytes ($H_2SO_4$ solutions) to obtain the first average conductivity $\sigma_*$. Later they (Ciampi and Talini, 1969) extended the theory and measurements taking the effect of collision frequency $\nu$ into account.

With due regard to the depth of the theoretical aspects of the problem treated by Ciampi and Talini, it is felt by the present author that the expression of the two meaningful averages and the relevant frequency and conductivity ranges could be obtained in a more simpler way, by considering the probe coil and the conducting medium to form a transformer (Ghosal, Nandi and Sen, 1976, 1978) the primary and the single-turn secondary being the coil and the medium itself respectively. In this way the average $\sigma_*$ was obtained yielding the same result (using equation(2) of reference Ghosal et al, 1978) given by Ciampi and Talini. It is also observed, though not reported, that in the same way the expression for $\sigma^2_{**}$ could be obtained. Further, withholding the consideration of the sensitivity, accuracy and difficulty of the measurement technique, the frequency and conductivity range doublets could also be obtained by simple arguments by requiring that $\omega$ should be very small compared to the collision frequency so that the displacement current may be neglected ( i.e. r.f. conductivity = d.c. conductivity) and $\sigma$ should be sufficiently small so that the skin depth is very large compared to the plasma radius.

1.3.10. It is also felt in conjunction with the observations of Stokes(1965,1969) that on the basis of average conductivity model no information concerning the nature of the conductivity profile could be obtained from the Q-factor measurements alone, and in fact a completely false picture of the character of the conducting region such as peak conductivity and the effective extent of the conducting region, can be obtained if the measurements are interpreted in terms of an average conductivity model (see also chapter V of the present thesis).

As for example, for profiles of the types treated by Ciampi and Talini, the difference between the azimuthal average and the on-axis conductivity can be as much as a factor of 5 and if the profile constants 'm' and 'n' are allowed to be varied indefinitely the aforementioned factor may be extremely high. Further the choice of the profile demands that the plasma fills the available volume; this may be a valid assumption for an ordinary discharge plasma but the assumption will be fatal for other situations such as solid metal arcs, flow facility plasma, plasma jets, etc. In these cases the errors can be very much greater since the plasma conductivity may fall to zero some distance away from the confining wall. Temperature measurements on an argon plasma jet made by Moskvin and Chesnokova (1965) indicate a peak conductivity of roughly 3000 mho/m, falling approximately to zero at a radius of about 3.3 mm. Stokes (1969) theoretically calculated the azimuthal average that should be obtained for the Moskvin-Chesnokova plasma stream assumed to be exhausting along a 2 cm. diam. tube. This is given to be approximately 100 mho/m. Thus it is seen that the on-axis conductivity is 30 times larger than the apparent average. Given below also the results obtained by the present author (Ghosal, Nandi and Sen, 1978) for the azimuthal average $\bar{\sigma}_\phi$, volume average $\bar{\sigma}_{vol}$ and the on-axis conductivity $\sigma_o$ of a mercury arc plasma for direct comparison.

Table 1.1

| Discharge current I (amps) | $\bar{\sigma}_\phi = \frac{4}{R^4}\int_0^R \sigma(r) r^3 dr$ (mhos/cm) | $\bar{\sigma}_{vol} = \frac{2}{R^2}\int_0^R \sigma(r) r dr$ (mhos/cm) | $\sigma_o$ (mhos/cm) | $\frac{\sigma_o}{\bar{\sigma}_\phi}$ | $\frac{\sigma_o}{\bar{\sigma}_{vol}}$ |
|---|---|---|---|---|---|
| 2.1 | 0.89 | 1.90 | 6.26 | 7.03 | 3.29 |
| 3.1 | 1.26 | 3.70 | 18.05 | 14.32 | 4.88 |
| 4.0 | 1.78 | 5.33 | 26.55 | 14.91 | 4.98 |
| 5.0 | 1.94 | 6.01 | 31.00 | 15.72 | 5.07 |

It may be observed from the above table that at 5 amps. discharge current the on-axis conductivity can be about 16 times the azimuthal average value.

However, the use of induction probing for measuring plasma conductivities, is by no means as fruitless as would appear from the above comments, instead the method can be fruitfully utilized to determine major characteristics of the plasma if the aforementioned measurement is supplemented with additional information using an approach of the present author (Chapter IV and Chapter V of the present thesis) or that of Goldenburg et al (1940).

1.3.11. In the preceding sections methods for determination of average azimuthal electrical conductivity have been discussed where the effect of plasma medium on either the coil resistance or the probe coil inductance was utilized for the purpose. Mikoshiba and Smy (1969), on the other hand described a newer approach of measuring plasma conductivity by utilizing the dependence of the mutual inductance of two coils upon the conductivity of the medium lying between them. Out of the several advantages of the method highlighted by the authors in their paper, it seems to the present author that, '' that it can be used over a continuous and wide range of conductivity can be measured (upto $10^6$ mhos/m) '' is the most significant one. This improvement is the outcome of the inherent sensitivity of their apparatus which was not endowed with other electronic accessories. In the former type of single coil measurements the reflected impedance of the oscillator coil was small and sensitivity was usually achieved by using mixing techniques or '' Q spoiling methods''. With the two coil method described by these authors, one coil acts as a transmitter, the other as a receiver. The signal induced by the receiving coil is much less than that applied to the transmitting coil with the result that the relative reaction of the induced currents upon the receiver coil is much more pronounced and a very simple measurement of signal attenuation is sufficient for conductivity measurement.

The two coil technique was developed in order to measure the conductivity of a shock precursor plasma. The necessary theory was developed by considering a shock tube geomentry with two radial conductivity distributions that might be expected in a precursor plasma, viz., uniform and annular conductivity profiles. Depicted in the diagram (Fig.1.6) the coil configurations as used by them. The field coil consisted of one turn. Multiturn was avoided due to the apperance of undesirable resonance arising from winding capacitances. Field coil loading due to the presence of plasma in the immediate vicinity was minimized by connecting a suitably large resistor in series with the coil. The output was taken from a similar single turn search coil and was fed directly to oscilloscope probe (high input impedance). Stray capacitive effects were also observed by them, particularly at high frequencies. To solve this problem, a second search coil of similar construction to the first search coil was placed on the other side of the field coil with respect to the first one and both search coil signals were fed into a difference amplifier. Through it

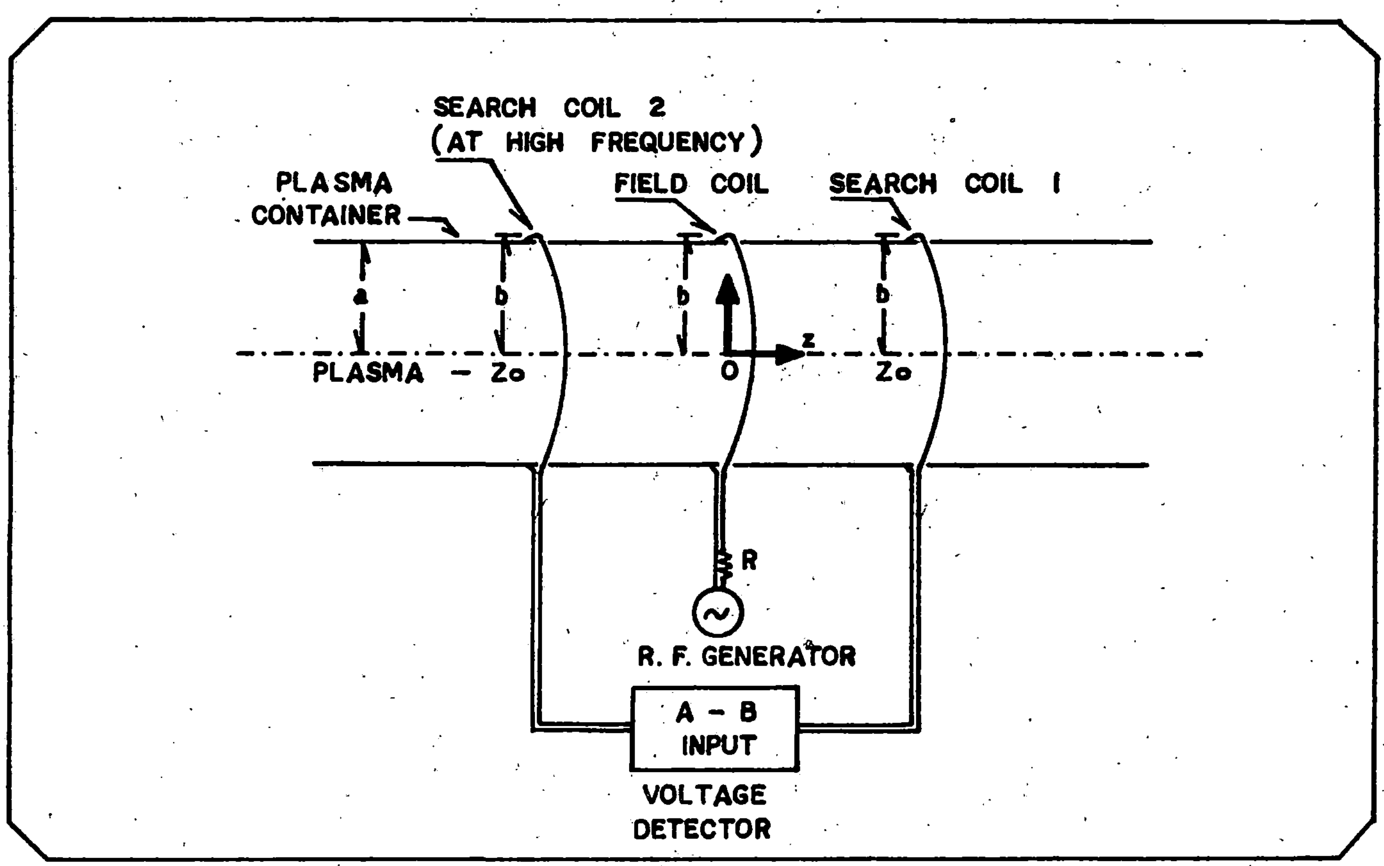



FIG. I·6 . TWO COIL rf PROBE ASSEMBLY FOR CONDUCTIVITY MEASUREMENTS. SEARCH COIL I RECEIVES THE rf SIGNAL TRANSMITTED FROM THE FIELD COIL THROUGH PLASMA, FROM WHICH CONDUCTIVITY IS CALCULATED. SEARCH COIL 2 IS USED (AT HIGH FREQUENCIES) TO BALANCE OUT THE CAPACITIVE CURRENENTS IN THE SEARCH COILS.

suffers from the weakness that the conductivity results are some sort of averages depending on the choice of the conductivity distribution model, the work may be said to be a significant advancement to other methods (discussed in earlier sections) in that it needs no calibration and also in that it provides a very wide range of measurable conductivity $\sigma_r + i\sigma_i$ .

1.3.12. Basu and Maiti (1973) elaborately studied the non-immersive coil probe method for a hot-cathode low-pressure dc discharge plasma where the electron-atom collision frequency is comparable to the probe frequency. The situation corresponds to the case where the conductor approximation is no longer valid and the plasma is characterised by a complex conductivity. Tanaca and Hagi (1969), as discussed in article 1.3.7 was the first who focussed particular attention to the problem of coil probe conductivity measurements on plasma which shows impedance characteristics at working frequencies; but the problem of Tanaca and Hagi was how to obtain the ''d.c conductivity '' when the immaginary part of the conductivity is non-zero. However, the intention of Basu and Maity was different. They obtained both the real and imaginary part of the plasma conductivity by measuring the two parameters (instead of one, as done by Tanaka and Hagi) viz. resistive and inductive parts of the reflected impedance of the probe coil.

The complex conductivity is, in general, related to plasma parameters by the following expression (Heald and Wharton 1965):

$$\sigma_r + i\sigma_i = -\frac{4\pi}{3}\,\epsilon_0\,\omega_p^2 \int_0^\infty \frac{1}{\nu(v)+i\omega}\,\frac{df_o(v)}{dv}\,v^3\,dv$$

where v is the electron velocity, $f_o(v)$ is the equilibrium ~~where~~ distribution function, $\omega$ is, the angular frequency of the applied r - f field, $\nu(v)$ is the electron atom collision frequency of momentum transfer etc. The authors considered two situations where $\nu$ is independent of electron velocity and where $\nu$ is dependent on v. In the first case the above equation can readily be simplified to yield:

$$\sigma_r + i\sigma_i = \epsilon_0\,\omega_p^2/(\nu + i\omega)$$

For the second situation also ( $\sigma_r + i\sigma_i$ ) satisfies the last equation if $\nu$ is replaced by some suitable effective momentum transfer collision frequency $\nu_{eff.}$. It was shown by them that both $\nu_{eff}$ and N (electron density) can be expressed in terms of two parameters $\alpha$ and $\beta$ which are proportional to the real part and imaginary part of the coil impedance in presence and in absence of plasma respectively. They obtained the following relations:

$$\nu_{eff.} = \omega\alpha / (1 - \beta)$$

and $$N = \frac{8m}{e^2\mu_e^2 a^2}\cdot\left(1 - \beta + \frac{\alpha^2}{1-\beta}\right),$$ a being the radius of the plasma column.

Experimentally they measured the resistance and reactance of the coil with a Q- meter used in conjunction with a low power r - f oscillator. Measuring the Q of the coil and noting the capacitance required for tuning the circuit both in presence and in absence of plasma it was possible to obtain two important plasma parameters $\nu_{eff}$ & N .

Though theoretically the two cases where $\nu$ is independent of electron speed v and where $\nu = \nu(v)$ was considered quite at length the present author notes that it was overstressed since the experimental observations cannot distinguish the above two situations and the observed collision fre-quencies are always the effective momentum transfer collision frequencies in some way or other. Besides, the expressions of $\nu_{eff}$ and N were derived on the basis of uniform conductivity model which is far from being true in most cases. They were of course self-conceited by stating that these obtained parameters are the values averaged over the cross-section of the plasma, but proper understanding is only possible provided the nature of these averages are known theoretically. However, if the conductivity profile is known beforehand by some other means the above approach after little modification might yield more meaningful results.

SCOPE AND OBJECT OF THE PRESENT WORK.

Measurement of radio-frequency conductivity in a plasma using an inductor having the discharge tube as a core material has been proposed by various authors. The measurement of radio-frequency conductivity of a low density plasma such as produced in a glow discharge has been carried out in this laboratory with the plasma acting as a lossy dielectric within the tuning condenser in the secondary circuit (Sen and Ghosh 1966, Sen and Gupta 1969;). In the present investigation it has been shown that the non-immersive radio-frequency coil probe method which was suggested earlier by various authors but not used in the case of an arc plasma can be utilised to find the average azimuthal conductivity of an arc plasma by studying the change in the band-width of a coil wound around an arc tube due to the presence of the plasma column within it. It has been shown that the capacitance effect which is important in the case of glow discharge, is not at all important in the case of arc plasma and the present method is suitable for azimuthal conductivity measurement in an arc plasma. The method could be advantageously used for some anisotropic plasmas as well.

It is well known that a plasma within a tube cannot be regarded as uniform with regard to radial electron density distribution or conductivity and in the case of glow discharge the radial distribution of the charge density is cylindricallly symmetric and can be represented by the Bessel function which is known as Schottky model. The Schottky model, as applied to glow discharge, can also be assumed to be valid in the case of a low pressure arc. The validity or otherwise of these assumed models has been put to some experimental tests in the case of glow discharge by the probe method but no elaborate investigation in this regard has been carried out in the case of arc plasma. In the present investigation it has been observed that in case of arc plasma there is a distribution of electron density or electrical conductivity which is not governed by Bessel function. The investigation was started by assuming a generalised radial conductivity distribution and measuring experimentally a quantity which is a function of this assumed conductivity distribution. In the next step a radial distribution function has been obtained which gives the closest approach to the experimental results.

The problem of plasma conductivity measurement using radio-frequency coil probes has been investigated by various authors. In all those experiments the probe, a single-layer coil of suitable geometry was either inserted in or wound around the test plasma. The basic principle involved in the coil probe diagnostic techniques lies in the fact that magnetic field associated with a solenoidal radio-frequency current through a coil wound around or inserted in the plasma generates an impedance reflected back into the probe coil in terms of its changes in impedance parameters. In those experiments instruments were calibrated by replacing the test plasma with elecrolytes of known conductivities. Since the plasma is never uniform within the confining walls, the obtained conductivities were some sort of averages defined to be the conductivities equal to that of a fictitious homogeneous plasma, the insertion of which into the coil would produce the same change of coil impedance parameters. Since normally in the experiments either the change in inductance or resistance of the coil were measured, the obtained conductivities were averages of two distinct types. Most of the authors experimentally obtained the average conductivities of plasma of various types but did not look into the structure of these averages. It is felt by the present author in conjuction with the observation of Stokes (1965, 1969) that on the basis of average conductivity model no information concerning the nature of the conductivity profile could be obtained from the resistance measurements alone, and in fact an incomplete picture of the character of the conducting region, such as peak conductivity and effective extension of the conducting region can be obtained if the measurements are interpreted in terms of an average conductivity model. It can be said that these conductivity measurements can give practically no information regarding the basic structure of the conducting medium. One cannot even predict the conductivity at the axis or effective width of the conducting region with any accuracy on the basis of these measurements. In a detailed analysis it has been shown by the present author that from the results of simultaneous measurements of asimuthal electrical conductivity (coil probe method) and longitudinal electrical conductivity (Langmuir probe method) of a plasma it is possible to obtain valuable informations on the major characteristics of the conducting medium with a very good accuracy such as, maximum extension of the plasma, lower and upper boundaries of the peak conductivity and the obtainable informations are independent of the choice of the profile form.

When an arc is formed within a tube, the current density is not uniform throughout the cross-section but is maximum at the axis and minimum at the periphery. This phenomenon gives rise to selective self-heating at the axis of the arc plasma. The arc continuously absorbs power from the source and gives it away to the surroundings. One might therefore be tempted to consider that the mechanism of selective self-heating might be employed to determine the thermal conductivity of the plasma. There are justifications in neglecting the effect of radiation and convection in the case of low temperature arcs but nevertheless it is worthwhile to mention that in this case the process of heat flow requires close observations. In a weakly ionized plasma both the electronic and molecular contributions to thermal conductivity are to be considered. One might predominate substantially over the other depending on the electron temperature, temperature gradients (electron temperature and gas temperature), etc. There might be present another mechanism of heat flow other than thermal conduction, radiation and convection which arises owing to the fact that electron density distribution may cause diffusion and energy might be carried away by the electrons. In contrast to the case of high pressure arc this mechanism of heat flow might play a significant role in case of low pressure arc. The theory and the experimental results presented here have made it possible to calculate separately the contribution by the different processes and it has been observed that the major part of the heat loss is due to ambipolar diffusion and the loss due to conduction by electrons, ions and neutral particles is comparatively small. Further from the experimental results obtained it has been possible to calculate the collision cross-section of electrons with the mercury atoms for electron energies less than 1 eV.

In the last experiment of the present series a small steady longitudinal magnetic field has been used in conjunction with the coil probe and it is seen that the conductivity is tensorial in this situation. By simultenous measurements of azimuthal and axial conductivities in presence and in absence of the magnetic field it has been possible to obtain experimentally a very important plasma parameter viz., the electron-atom collision frequency ~~of electrons~~ of a mercury arc plasma. The relevant theory has been developed taking the effect of radial distribution into account. It has been shown that the experimental results are in good agreement with the theory.

The proposed line of investigation presented above and the experimental results obtained therein are expected to reveal the nature of the physical processes occuring in an arc plasma specially with regard to the conduction of heat and electricity in a dense ionised region. It has been proposed to utilise high current arc plasma as a source in the generation of high frequency electro-magnetic radiation and the study of the physical processes will be useful in the design and fabrication of such sources. If the loss processes are well understood then means may be found to reduce the loss, so that greater percentage of input energy can be converted to visible radiation and effiency of arcs as light sources can be enhanced. A detailed study of the various processes will be useful in developing a generalised theory of an arc plasma.

REFERENCES:


1. Adler, F.P. (1949), J.Appl.Phys., 20, 1125
2. Akimov, A.V. and Konenko, O.R. (1966), Sov. Phys.Tech.Phys., 10, 1126
3. Aleksandrov, A.F. and Yalsenko, I.M. (1965), High Temp.(USA), 3, 321
4. Appleton, E.V. and Chapman, E.W. (1932), Proc.Phys.Soc.Lond., 44, 246
5. Appleton, E.V. and Boochariwala, D.B. (1935), Proc.Phys.Soc. 47, 1374
6. Basu, J. and Maiti, J.N. (1973), J.Appl.Phys., 44, 3975
7. Basu, J. (1976), J.Appl.Phys., 47, 4234
8. Basu, J. and Hore, K. (1977), J.Appl.Phys., 48, 4812
9. Bergland, S. et al (1963), J.Sci.Intrum.(G.B) 40, 250
10. Berk, H.L. (1964), Phys.Fluids, 7, 257
11. Blackman, V.H. (1959), AIOSR TN - 59 -681
12. Childs, E.C. (1932), Phil. Mag., 13, 873
13. Ciampi, M. and Talini, N. (1967), J. Appl.Phys., 38, 3771
14. Ciampi, M. and Talini, N. (1969), Ibid., 40, 5394
15. Cilliers, W.A. et al (1963), Nucl.Fusion, 3, 78
16. Colgate, S., Farguson, J. and Furth, H. (1958), Proc. 2nd. U.N.Conf. on peaceful uses of Atomic Energy (Geneva) 32, 129, 140

17. Cooper, J. (1963), Plasma Phys. (J. Nucl. Energy C), 5 285

18. Dawson, J. and Oberman, C. (1962), Phys. Fluids, 5, 517

19. Dawson, J. and Oberman, C. (1963), Ibid., 6, 394

20. Donskoi et al (1963), Sov. Phys. Tech. Phys. 7, 805

21. Frost, L.S. and Phelps, A.V. (1962), Phys. Rev., 127, 1621

22. Frost, L.S. and Phelps, A.V. (1964), ibid., 136A, 1538

23. Ghosal, S.K.; Nandi, G.P. and Sen, S.N. (1976), Int. J. Electronics, 41, 509

24. Ghosal, S.K., Nandi, G.P. and Sen, S.N. (1978), Int. J. Electronics, 44, 409

25. Gilardini, A. (1959), Nuovo Cimento Suppl. Sec. 10, 13, No. 1-2

26. Glasstone, S. and Lovberg, R.H. (1960), Controlled Thermonuclear Reactions, Van Nostrand, Princeton, N.J. (USA)

27. Golovin. I.N. et al (1958), Proc. 2nd U.N. Conf. On Peaceful Uses of Atomic Energy (Geneva), 32, 72

28. Goldenberg, S.A. et al (1964), High Temp., 2, 616

29. Gourdin, M.C. (1963), Symposium on Magnetoplasmadynamic Electrical Power Generation, Session III, p. 35

30. Green, H.S. and Leipnik, R.B. (1965), I.J.Engg. Sci. (G.B), 3, 491

31. Gupta, R.N. and Mandal, S.K. (1967), Indian J.Phys. 41, 251

32. Hausler, R.S. (1957), Zs.angew phys., 9, 66 - 68

33. Heald, M.A. and Wharton, C.B. (1965), Plasma diagnostics with microwaves, John Wiely and sons. Inc. New York.

34. Hollister, D.D. (1964), AIAA J., 2 1568

35. Imam, A. and Khastagir, S.R. (1937), Phil.Mag., 33, 858

36. Ionescu, V. and Mihul, C. (1935), J.Phys.Radium, 6, 35

37. Jahnke, E. and Emde, F. (1945), Tables of Functions (Dover Publications, Inc., N.Y.) pp.73

38. Jayakumar, R.; Chakravarty, D.P. and Rohatgi, V.K. (1977), Rev.Sci.Instrum., 48, 1706

39. Johnson, L.C. (1967), Phys. Fluids (USA), 10, 1080

40. Khoshabiev, M.B. and Yarin, L.P. (1966), High Temp. (USA), 4, 576

41. Khvashchtevski, S. (1962), Nukleonika, 7, 369

42. Koritz, H.E. and Keck, J.C. (1964), Rev.Sci.Instrum., 35, 201

43. Krall, N.A. and Trivelpiece, A.W., (1973), Principles of Plasma Physics, Mc Graw-Hill Book Company.

44. Lamb, L. and Lin, S.C. (1957), J.Appl.Phys.28,754

45. Lary, E.C. and Olson, R.A. (1962), Paper presented at the 3rd Symposium on Engineering Aspects of Magnetohydrodynamics, Rochester, N.Y.

46. Lin, S.C. et al (1955), J.Appl.Phys., 26, 95

47. Lovberg, R.H. (1959), Ann.of Phys., 8, 3311

48. Lovberg, R.H. (1965), Magnetic probes. In ''Plasma Diagnostic Techniques''(R.H. Huddlestone and S.L.Leonard, eds.) Academic Press, New York.

49. Margenau, H. (1946), Phys.Rev., 69, 508

50. Mikoshiba, S. and Smy, P.R. (1969), Rev.Sci.Instrum., 40, 1187

51. Moskvin, Yu. V. and Chesnokova, N.N. (1965), High Temp., 3, 335

52. Moulin, T. and Masse, J. (1964), Symposium International Sur la production MHD d'Energie Electrique, Paris

53. Murino, P. and Bonomo, R. (1964), XIX Congresso Nazionale ATI, Seina, p. 44.

54. Nagata, M. (1966), Elec.Engg.Japan (USA), 86, 40

55. Oberman, C. and Shure, F. (1963), Phys.Fluids, 6, 834

56. Olson, R.A. and Lary, E.C. (1961), USA Res.Lab.Rept. M – 1282 – 1

57. Olson, R.A, and Lary, E.C. (1962), Rev.Sci.Instrum, 33 , 1350

58. Olson, R.A, and Lary, E.C. (1963), AIAA J., 1, 2513

59. Persson, K.B. (1960), Gen.Elec.Res.Lab. Rept.No. 60-RL-2545c

60. Persson, K.B. (1961), J.Appl.Phys. 32, 2631

61. Pollock, H.;Goldman, L. and Westendrop, W. (1960), Bull.American Phys. Soc., 5 , 337

62. Pradhan, T. and Dasgupta, B. (1967), Phys.Rev., 160, 184

63. Rosa, J.R. (1961), Phys.Fluids, 4, 182

64. Savic, P. and Boult, G.T. (1962), J.Sci.Inst. 39, 258

65. Schweitzer, S. and Milchner, M. (1967), Phys.Fluids, 10, 799

66. Sen, S.N. and Ghosh, A.K. (1966), Ind.J.Pure and Appl.Phys. 4, 70

67. Sen, S.N. and Gupta, R.N. (1969), Ind.J.Pure and Appl.Phys. 7, 462

68. Sen, S.N. and Jana, D.C. (1978), Ind.J.Phys.,52B,288

69. Sengupta, D.L. (1961), Proc.IEEE,49,1872

70. Stokes, A.D. (1965), Proc.Inst.Elec. Engrs.,112,1585

71. Stokes, A.D. (1969), J.Appl.Phys.,40,1973

72. Stubbe, E.J. (1968), Proc.IEEE, 56,1483

73. Tanaca, H. and Usami, S. (1962), Bull.Fac.Eng. Yokohama Nat.Univ., 11 , 65

74. Tanaca, H. and Hagi, M. (1964), J.Appl. Phys. Japan, 3, 335

75. Tanaca, H. and Hagi, M. (1964), Ibid., 3, 338

76. Taylor, R. L. et at (1963), AIAA J., 1 ,2186

77. Uman, M.A. (1964), Introduction to Plasma Physics, Mc Graw Hill, Inc.

78. Vander Pol, B. (1919), Phil.Mag., 38,352

79. Wu, C.S. (1965), Phys.Rev.,138,A51

80. Wu, C.S. (1965), Phys,Rev.,140,A118