Abstract

Fuzzy mathematics became known in the second half of the 20^{th} century after L.A. Zadeh published the 1st article on fuzzy sets [42] in 1965. The article generated lots of research and implicitly leads to the founding of a new branch in mathematics. Fuzzy mathematics may be regarded as a parallel to the classical mathematics or as a natural continuation of the latter, but in most cases, it is considered a new mathematics, very useful in solving problems expressed through a vague language. The study of fuzzy mathematics is organized on several sub domains. Among them, we can mention: the fuzzy logic, the fuzzy arithmetic, the fuzzy topology etc. Compared to other research domains, fuzzy topology gained interest after its introduction by C. L. Chang [7] in 1968, who studied a number of the basic concepts, including fuzzy continuous maps and compactness. Goguen [14] in 1973, presented the fundamental ideas of basis, sub basis and product in an investigation of compactness. After the introduction of compactness in fuzzy topological spaces by C.L. Chang, a series of different notions of fuzzy compactness have been presented by T.E. Gantner, R.C. Steinlage, R.H. Warren [12], R. Lowen [21, 22], Wang Guojun [40], Gunther Jager [18] etc. and they were compared in detail by R. Lowen [23]. In 1980, Pu and Liu introduced a new concept of so called Q-neighbourhood which could reflect the features of neighbourhood structures in

fuzzy topological spaces and by this new neighbourhood structure the Moore - Smith convergence was established [26]. In the same year, B. Hutton and I. Reilly presented separation axioms in fuzzy topological spaces [17]. Fuzzy closure operators and fuzzy closure systems have been studied by Mashour and Ghanim [24], G. Gerla [13], Bandler and Kohout [2], R. Belohlavek [3], whereas fuzzy interior operators and fuzzy interior systems have appeared in the studies of R. Belohlavek and T. Funiokova [4], Bandler and Kohout [2].

Kazimierz Kuratowski $(1896-1980)$, a Polish mathematician, introduced the concept of closure operators known in mathematical circles as the Kuratowski closure operators, which was fundamental for the development of topological space theory and irreducible continuum theory between two points. Kuratowski closure operator generates a topological space in which it coincides with the closure of a set. After the initiation of Kuratowski closure operators, many mathematicians studied various combinations of Kuratowski closure axioms. Dr. M. Singha and Prof. S. De Sarkar (2012), in their article "*On K*Ω *and Relative closure operators in* $P(X)^{\mathbb{N}}$"[30], defined KΩ-closure operators in $P(X)^{\mathbb{N}}$ which generates a topology like structure and is same as so called sequential topology introduced by Prof. M.K. Bose and Prof. I. Lahiri [5]. In [5], Prof. M.K. Bose and Prof. I. Lahiri introduced the concept of sequential topological spaces and they developed separation axioms in sequential topological spaces upto regularity. Encountering some problems in the definition of regularity in a sequential topological space while going through [5], N. Tamang, M. Singha and S. De Sarkar, presented an article entitled "*Separation Axioms in Sequential Topological Spaces in the Light of Reduced and Augmented Bases*"[32] in which they studied the separation axioms in a sequential topological space with some modified definitions so as to solve the problems encountered. Apart from this, the studies of sequential topology comprises of Semi open and Weakly Semi open sequential sets by S. Das, M. Singha and S. De Sarkar [9], monotonic sequential operators by M. Singha and S. De Sarkar [31] etc.

After going through the above mentioned topics on fuzzy topology and that on sequential topology, it is natural to ask whether the study of the concept of sequential topology is possible in case of fuzzy sets. In this thesis, we investigate various notions of a sequential topological space like separation axioms, operators, mappings, compactness etc. in the fuzzy setting.

The thesis comprises of seven chapters. **Chapter** 1 is introductory in nature and to provide a suitable background for the rest of the chapters, it consists of basic definitions and results from the theory of fuzzy topological spaces.

Chapter 2 is devoted to the introduction of fuzzy sequential sets and fuzzy sequential topology. Different notions of a general topological space, like basis, subbasis, closure and interior of a set, derived set, etc. are studied in the setting of fuzzy sequential topological spaces. The concept of Q-neighbourhoods is also introduced. Given a fuzzy topological space, one can always associate a fuzzy sequential topology with the given space as one of its components and given a fuzzy sequential topological space, it is possible to construct countably many fuzzy topological spaces, called components of the given space. Many pleasant properties of a fuzzy sequential topology are compared with that of its components and many properties of a fuzzy topological space are compared with that of the associated fuzzy sequential topology. A variant of Yang's result [19] in a general topological space, is also achieved in a fuzzy sequential topological space. The contents of this chapter is published in [Singha M., Tamang N. and De Sarkar S., *Fuzzy Sequential Topological Spaces*, International Journal of Computer and Mathematical Sciences, **3** (4) (June 2014), 2347-8527.]

In Chapter 3, we present separation axioms in a fuzzy sequential topological space, where fs-T_0, fs-T_1, fs-Hausdorff, fs-regular and fs-normal spaces are studied. A necessary and sufficient condition for a space to be each of these spaces is established. Further, the results relating the separation axioms in component fuzzy topological spaces with that of the corresponding fuzzy sequential topological space are also obtained. The results of this chapter is published in [Tamang N., Singha M. and De Sarkar S., "*Separation Axioms in Fuzzy Sequential Topological Spaces*", Journal of Advanced Studies In Topology, **4** (1) (2013), 83-97.]

Closure and interior operators on an ordinary set belong to the very fundamental mathematical structures with direct applications on many fields like topology, logic etc. The approach in

Chapter 4 is inspired by the importance of closure and interior operators. In this Chapter, the concepts of fs-closure and fs-interior operators on a set are presented. Other studied notions are the concepts of fs-closure and fs-interior systems. With an fs-closure system (fs-interior system), one can always associate an fs-closure operator (fs-interior operator) and vice-versa. An fs-closure and an fs-interior operator on a set may not induce the same fuzzy sequential topologies in general. So the question arises whether there is any condition under which, these operators induce the same fuzzy sequential topologies. The answer is in affirmative and a necessary and sufficient condition under which an fs-closure and an fs-interior operator on a set induce the same fuzzy sequential topologies is obtained.

In general set theory, composition of closure (interior) operators is again a closure (interior) operator and hence it also induces a topology. It is natural to ask whether the result is true in case of fs-closure and fs-interior operators and if the answer is in affirmative, is there any relation among the topologies induced by the composition and that induced by the participants to the composition? The answers to these questions are also given in **Chapter** 4 and a relation between such fuzzy sequential topologies is obtained. The relation between the collections of fs-closure and fs-interior operators is also investigated. Further, the Relative fs-closure operators and fs-connectors connecting two fuzzy topologies on a set are introduced and studied. The contents of this Chapter are published in [Tamang N., Singha M. and De

Sarkar S., *FS-closure operators and FS-interior operators*, Ann. Fuzzy Math. Inform., **6** (3) (November 2013), 589-603.] and [Tamang N., Singha M. and De Sarkar S., *Composition of Fuzzy Sequential Operators with Special Emphasis on FS-Connectors*, Palestine Journal of Mathematics, **4** (1) (2015), 37-43.]

Chapter 5 deals with the study of the concepts of continuity and compactness in fuzzy sequential topological spaces. Some characterizations of continuity are given. Other notions like open maps, closed maps and homeomorphisms are also studied. Two kinds of compactness, fs-compactness and Ωfs-compactness in a fuzzy sequential topological space are introduced, studied and the Chapter has been concluded by investigating the behavior of the product of these compact spaces.

One major area of research in general topology during the last few decades that mathematicians have been pursuing is to investigate different types of generalized open sets, generalized continuous functions and study their structural properties. **Chapter** 6 and **Chapter** 7 is devoted to the study of different types of generalized open sets, generalized continuous functions in the setting of a fuzzy sequential topological space. Their interrelations have been shown using a diagram in **Chapter** 7. Further, various properties of these sets and functions have been studied in both **Chapter** 6 and **Chapter** 7. Finally, **Chapter** 7 is concluded showing a decomposition of fs-continuity.