Chapter V

Boughn Effect and Some Twin Paradoxes

5.1 Introduction

The relativistic time dilation (RTD) of a clock is governed by the so called Lorentz factor γ of the Lorentz transformation (LT), where $\gamma = (1 - v^2/c^2)^{-1/2}$ with v and c being the speeds of the clock and light in free space respectively. A counter-intuitive aspect of the RTD lies in its reciprocity. Simply stated, two observers in relative motion, will observe each other's clock to run equally slow compared to their own. This enigmatic feature of RTD is seen to be best exemplified in the posing of the so called twin paradox. In the standard twin parable, one twin Albert (A) stays on earth while the other Barbara (B) leaves the earth and travels in a fast rocket in uniform speed to a distant star. Subsequently she turns around and returns with the same speed to meet her stay-at-home sibling to find that Albert has aged more. This will happen since B's (biological) clock runs at a slower rate due to RTD caused by her speed required for the trip. The paradox comes about when B considers herself to be at rest and pretends that A is doing all the moving. Hence B can claim that A's clock should run slow and therefore expects that it is A who should be younger at their reunion after the trip. In order to resolve the paradox, it is often argued that the conditions experienced by the twins are asymmetric. While the earth bound twin may be considered to lie in an inertial frame[1], the rocket-bound one is a non-inertial observer by virtue of her direction reversing deceleration during the turn around. Thus since the rules of special relativity (SR) holds good only for an inertial observer, A's conclusion must be correct and therefore the traveller twin B will indeed be younger. Besides since there is a basic asymmetry between the motion of the siblings, one should not be surprised by the asymmetric outcome

[1] One ignores here the non-inertiality of the earth's frame due to the motion of the earth about the sun and also the former's spin about its axis.

concerning their ages after the trip. However, although correct, this qualitative argument explaining away the paradoxical element (asymmetric aging) of the twin problem may lead to a misconception that the turn around acceleration (or the force causing the acceleration) is the direct cause of the differential aging. In an interesting article Gruber and Price[1] have clarified the relationship of acceleration of the rocket twin and time dilation by giving an example of "no asymmetric aging" in spite of one of the twins arbitrarily large acceleration. There is also a converse situation as discussed by Boughn[2] in connection with an interesting variation on the twin paradox. It is shown therein that spatially separated twins can age differently although their history of acceleration remains the same. (In what follows, this counter-intuitive effect of SR first elaborately discussed by Boughn, will play one of the key roles in various issues to be addressed in this context. We here after will refer it to as the Boughn effect (BE)).

It follows therefore that the acceleration *per se* cannot be the direct cause of the asymmetric aging in the usual version of the paradox even as the turn-around acceleration of B is necessary for an unambiguous comparison of the ages of the twins at one location. (In some variation of the paradox even the turn-around acceleration is avoided by posing the problem in a closed universe setting[3, 4]).

It is now generally understood that at the heart of the paradox lies the question of relativity of distant simultaneity of SR. Indeed in a paradox, incorrect arguments are given at the time of posing the problem. The incorrect arguments come about when the time dilation formula of SR is freely used from the perspectives of both the twins and the question of applicability of the formulas with respect to the traveller twin who changes inertial frames (because of her turn around) is ignored. Indeed while standard relativistic time dilation formula (TDF) is correct within one inertial frame of Albert, the same formula does not hold with respect to the non-inertial frame attached to Barbara. In the abrupt turn-around scenario however, relativistic TDF is valid separately in the inertial frames of B in its outward and return journeys. But change of inertial frames by B produces another effect linked with the "relativity of simultaneity" of SR, in which the key to the resolution of the paradox lies. What happens when a change of reference frame takes place is best exemplified in the BE mentioned earlier.

It is clear that any resolution of the twin paradox must involve a demonstration of equal differential aging from the perspectives of A and B . From A's point of view there is only RTD, while from B's perspective, in addition to the time dilation of A's clock[2] this BE comes into play. The latter then more than compensates (to be discussed) for the time dilation of A's clock to produce the differential aging predicted from A's perspective, hence dissolving the paradox.

In this chapter we show that apparently situations can be created (or kinematical worlds can be constructed) where the usual time dilation effect of relativity is absent but BE exists. Paradoxes of different nature then comes into existence in the canonical twin problem scenario. All this along with its resolution and some more ramifications of the issue are discussed in sections(5.3), (5.4) and (5.5). As a bonus the reader will find in it a *quantitative* explanation of the ordinary twin paradox in terms of BE which to our knowledge has not yet been available in the literature. The last section (Sec.(5.6)) will provide the summary followed by some concluding remarks. In order to set the stage we will briefly reproduce in Sec.(5.2), the arguments due to Boughn, that will explain BE.

[2]In the text we have implicitly assumed that each observer (A, B, etc.) carries a clock so that the age of an observer also means the clock reading and hence terms like differential aging or time-offset between observers' clocks can be used interchangeably.

5.2 Boughn Effect and Twin Paradox

Boughn, as mentioned in the previous section, presents a form of twin paradox in SR where although the twins experience equal amount of acceleration for the same time, they age differently. According to Boughn's parable, twins P and Q on board two identical rockets, initially at rest at a distance x apart in an inertial frame S, start travelling with equal amount of acceleration for some time and eventually come to rest (when all their fuel has been expended) with respect to another inertial frame S' moving with a constant velocity v with respect to S along the positive x-direction.

Using the Lorentz transformation (LT), Boughn has obtained a counter-intuitive result that after the acceleration phase is over, the age of the forwardly placed (with respect to direction of acceleration) twin Q becomes more than that of P. The result is surprising since in the problem it has been assumed that P and Q throughout have identical experiences, but their presynchronized (biological) clocks go out of synchrony.

The amount of this desynchronization or the age difference can be quantified as follows:

Let us first write down LT

$$\begin{aligned} x_k' &= \gamma(x_k - vt_k), \\ t_k' &= \gamma(t_k - vx_k/c^2), \end{aligned} \tag{5.1}$$

where x_k and t_k refer to the space-time coordinates of the observer k (k stands for P and Q) with respect to a frame S and the prime refers to the corresponding coordinates in S'.

From the time transformation of Eq.(5.1), it follows that

$$t_Q' - t_P' = \gamma[(t_Q - t_P) - v(x_Q - x_P)/c^2]. \tag{5.2}$$

Consider an event simultaneous to P and Q, say a common birthday in S of P and Q so that $t_Q = t_P$. For this event putting $t_Q - t_P = 0$ in equation (5.2) one obtains,

$$t'_Q - t'_P = -\gamma v x/c^2, \tag{5.3}$$

implying in S', birthday of Q occurs before that of P and hence Q becomes older than P in their new abode (S') by the above precise amount. Hence the two clocks (twins) separated by a proper distance x that are synchronized in their rest frame S becomes unsynchronized by an amount

$$\delta t_{desync} = -\gamma v x/c^2, \tag{5.4}$$

after their arrival in S'.

The paradox associated with the fact that spatially separated twins, in spite of their identical history of accelerations with respect to S, age differently, can be solved as soon as one recognizes that age difference of two observers at different locations or time offset between two spatially separated clocks does not have any unequivocal meaning. Two clocks can only be compared unambiguously only at one spatial point; once that is done in Boughn's scenario after P and Q arrive in S' (for example P may walk towards Q for comparing their watches), the histories of their acceleration with respect to S then fail to be identical[5, 6, 7].

However the desynchronization δt_{desync} is real in the sense that it refers to desynchronization in relation to the Einstein convention of synchronization[3]. We have already called this departure from Einstein synchrony as BE.

Incidentally in a recent paper[6] we have shown how BE can account for the differential aging from the traveller's point of view resolving the paradox. That

[3]Incidentally Selleri[8] has noted in different words in this context that after identical acceleration, the two clocks readings define a natural (absolute) synchronization, which is different from Einstein's synchrony; the latter can only be established by resynchronizing them artificially (see also[9]).

has however been done in an indirect way by making an appeal to the principle of equivalence of general relativity. However, this is not necessary. Boughn paradox can directly be used to understand how BE can offset the reciprocal time dilation effect of A-clock with respect to the observer B, providing the correct asymmetric aging (that A is older than B) that one obtains from A's point of view. In this chapter we have just stopped short of showing this for the usual twin paradox, however we have provided a couple of templates for solutions in some other worlds, which the readers can make use of to complete the exercise.

We now proceed by noting that whenever the standard twin paradox is posed, the symmetrical time dilation effect of SR is only highlighted and a possible role of BE in contributing to the differential aging of the twins is suppressed. To understand the importance of BE more clearly we look for some possible contrasting scenarios where BE is readily evident while the other effect viz RTD of SR is either hidden or is truly absent. Surely as we shall see, this converse situation will again invite interesting paradoxes. All this will be the subject matter of the next section. These novel paradoxes and their resolutions will hopefully provide more clarity and insight into the century old twin paradox.

5.3 Separating RTD from BE and a Paradox

As we have indicated in the last section, the best way to highlight the importance of the role of BE in resolving the twin paradox is to remove the other effect viz RTD altogether from the twin problem. This can be conceived in the following way. We consider LT under small velocity approximation such that the world is essentially classical. When the relative velocity v between S and S' is very small compared to the speed of light c, such that v^2/c^2 terms can be neglected in comparison to unity, the so called Lorentz factor γ can be assumed to be 1. In this approximation the well-known relativistic effects like length contraction and time dilation are absent. This is in conformity with classical kinematics and hence in this small velocity regime the world is expected to be classical or non-relativistic. However contrary to the common belief (see for example[10, 11, 12]), for such small velocities, LT does not go over to Galilean transformation (GT), it becomes instead the so called approximate Lorentz transformation (ALT)[13, 14, 15, 16]:

$$\begin{aligned} x' &= (x - vt), \\ t' &= (t - vx/c^2), \end{aligned} \tag{5.5}$$

which expresses coordinates (x') and time (t') of S' in terms of those (x and t) of S. The inverse transformation under the same approximation is given by

$$\begin{aligned} x &= (x' + vt'), \\ t &= (t' + vx'/c^2). \end{aligned} \tag{5.6}$$

The above transformation equations, that have been obtained by putting $\gamma = 1$ in LT or in its inverse, represent the Galilean (classical) world since one may verify from the transformations (5.5) and (5.6) that moving rods do not contract and also there is no RTD effect for moving clocks. Relative velocity or velocity transformation formulas for small velocities are also Galilean in character. The only apparent non-Galilean feature of ALT lies in the space-dependent terms in the time transformations of (5.5) and (5.6)[4]. This is expected since in the relativity theory, synchronization of distant clocks in a given inertial frame is performed using light signal following the convention of standard synchrony (or the Einstein synchrony) according to which the one-way-speed (OWS) of light is assumed to be the same as its two-way-speed (TWS) along a given direction. It is well known that the so-called

[4] Note that these space-dependent terms cannot be dropped since for any preassigned small velocity, x or x' can be taken to be arbitrarily large so that vx/c^2 or vx'/c^2 may not be neglected.

"relativity of simultaneity" of special relativity is the direct result of this "stipulation" of equality of TWS and OWS of light. It is understandable that a mere small velocity approximation cannot alter this conventional ingredient embedded in LT. Indeed one can verify by simple kinematical calculation that ALT still represents Einstein synchrony[14].

One recalls that when the time dilation effect of relativity is obtained from LT, the moving clock is assumed to be at a fixed position (say at $x' = 0$) in some inertial frame S', which moves with velocity v with respect to the observer's frame S and hence the term vx'/c^2 of LT does not have any contribution to the time dilation effect. However when it comes to the question of resolution of the paradox the role of this term, which is linked to clock synchronization issue, has to be brought into fore.

Now the world described by ALT, is clearly a non-relativistic one and if one ignores its history[5], there is no trace of time dilation of clocks-in-motion and hence apparently the twin paradox should not exist. However one who is aware of BE which is linked to the phase terms in ALT, discovers a differential aging from the perspective of Barbara although Albert does not expect Barbara's (biological) clock to go slow. Let us see how this contradiction comes about a bit more clearly.

Consider the abrupt turn around scenario of the standard twin parable. Assume that the turn around of B takes place when the distance between the twins (with respect to S') measures L' say. Now, just before the deceleration phase starts, one may consider another observer Alfred ($\bar{A}$) of the same age as that of Barbara (i.e it is assumed that $\bar{A}$'s clock is synchronized with B's in S') and at same location

[5]Here the equations have been obtained by putting $\gamma = 1$ in LT or its inverse. This could have been obtained under the same approximation from Zahar transformation[17] (see later) which describes a classical world with Einstein synchrony[9, 18, 19, 20]

of A comoving with respect to B such that, like in Boughn's scenario, $\tilde{A}$ and B both undergo the same but arbitrarily large negative acceleration with respect to S', which moves with constant velocity v with respect to S. From S' frame, B and $\tilde{A}$ may be considered as Boughn's twins accelerating from rest along the negative x-direction (i.e now $\tilde{A}$ is forwardly placed with respect to B) and settles in some inertial frame S'' moving with velocity $-w$ (say) with respect to S' (and $-v$ with respect to S).

BE therefore tells us that with respect to Einstein synchronized clocks in S'', there is a desynchronization effect between the clocks (or ages) of Alfred and Barbara,

$$t''_B - t''_{\tilde{A}} = \delta t_{desync} = \gamma_w w L'/c^2, \tag{5.7}$$

which has been obtained from Eq.(5.4) replacing γ, v and x by γ_w, $-w$ and L' respectively. Note that here $\gamma_w = (1 - w^2/c^2)^{-1/2} \approx 1$ and $w = 2v/(1 + v^2/c^2) \approx 2v$ in the classical regime. Hence

$$\delta t_{desync} \approx 2vL/c^2, \tag{5.8}$$

since there is no length contraction effect.

The above desynchronization also corresponds to a synchronization gap between the Einstein synchronized reference frames S' and S''. The presence of this synchronization gap between instantaneously comoving inertial frames for an accelerated observer is the reason why such frames cannot be meshed together. Because of the (instantaneous) turn-around Barbara switches her inertial frame and because of desynchronization, the clocks of Alfred and Barbara no longer represent the Einstein synchronized coordinate clocks of S''. Instead of not turning around if Barbara would continue to move forward covering the same length of journey with uniform speed as she would do after the turn-around, coordinate clocks (Einstein synchronized) of S' frame of Barbara could be used to measure the coordinate time and connect the same with the proper time of Albert through TDF for the entire trip. However if during the second phase of the trip someone playfully tamper with the synchronization, any coordinate time measurement following it will then be erroneous and hence a calculation to obtain the proper time τ_A of A from this measurement (by applying TDF on it) will give wrong result. In order to get the correct answer the remedy is to first undo the mischief by getting back to the Einstein synchronization that was adopted before and then one is free to use TDF in order to obtain proper time from the coordinate time. Let us now see what is the corresponding situation if we consider Barbara's turn-around. In this case the second leg of Barbara's journey corresponds to the inertial frame S''. The adoption of Einstein synchronization in this frame can be equated with the deliberate alteration of synchronization just discussed in connection with the uniform motion scenario of Barbara, since the standard of simultaneity in S'' is thus made different from that in S' which corresponds to the earlier leg of Barbara's trip.

It is clear that the proper time and coordinate time of a clock are connected by TDF provided the latter refers to a *uniform* synchronization. We then ask if there is any way so that one can continue with the standard of simultaneity (synchrony) of S' in S''. The answer is in the affirmative and is provided by Boughn's thought experiment. From the symmetry of the problem it is evident that clocks of Alfred and Barbara initially synchronized in S' continue to remain synchronized with respect to S' even when they arrive stationary in S'' after the turn-around acceleration. From B's perspective one can easily obtain the round-trip time τ_B in B-clock for A's journey (see later), but this does not correspond to the coordinate time for the same in S'. Clearly a correction term δt_{desync}, is to be added to τ_B to obtain the said coordinate time. This correction is equivalent to the process of restoration of the synchronization mentioned in Barbara's non turn-around example. Assuming the relevant time dilation factor to be unity, the proper time of A-clock obtainable from the coordinate time, automatically gets modified by the same amount. Thus Barbara should predict the proper time τ_A of A after the round trip to be $\tau_B + \delta t_{desync}$ instead of just τ_B. The correction term to τ_B is the result of BE, which exists even if RTD is ignored. Note that this is a "distance" effect[6] and takes place just after the turn around and this is happening since we have assumed that the clocks of both S' and S'' are synchronized following Einstein synchrony. On the other hand if we had adopted absolute synchrony in the Galilean world directly (and not by taking the small velocity approximation of LT preserving the Einstein synchrony) one does not have to deal with the space-dependent term in the time transformation (since in this case $t' = t$). Summarizing one observes that since under the small velocity approximation, one essentially deals with a classical world, B's clock does not run slow compared to A and hence A does not predict differential aging. But since relativity of simultaneity is preserved in the approximation, although A's clock does not run slow with respect to B's clock (as we have put $\gamma = 1$) B predicts a differential aging due to BE of amount $2vL/c^2$ which can be made arbitrarily large by increasing B's length of journey for any preassigned small uniform velocity v of B. Time dilation of clocks in SR refers to the *rate* of ticking of clocks in relative motion, on the other hand BE is the result of an offset of the initial setting of spatially separated clocks when an observer (in this case Barbara) changes from one inertial frame to another. Resolution of the standard twin paradox depends on a beautiful interplay of these two relativistic effects — one overcompensating the other, so that both the twins finally agree on their age

[6]For this reason the initial acceleration of Barbara at the time of departing from Albert and the final deceleration required to reunite with Albert do not have any BE since in those phases of Barbara's trip the distance of separation between them tends to zero.

difference.

In the present setting apparently there is no time dilation effect that the differential aging due to BE predicted by Barbara is required to be balanced! This results in contradicting claims by Albert and Barbara regarding their ages (Albert does not predict any age difference, which Barbara disagrees) signifying a paradox.

The fallacy hinges on the fact that although the time dilation factor γ can be assumed to be arbitrarily close to unity, BE can be made arbitrarily large by increasing the length of the trip. The resolution of the paradox however is not a difficult job. We should first recognize that the problem arises as we are comparing two relativistic effects of different nature. While, as we have observed the RTD effect refers to clock rate, BE refers to the time-offset, which is an integrated effect. Indeed if one increases L arbitrarily the integrated effect of time dilation of B's (biological) clock leading to differential aging with respect to A also becomes arbitrarily large. Hence although $\gamma \approx 1$ in the approximation, the accumulated effect of time dilation of B's clock cannot be neglected. The remedy of the problem therefore lies in not neglecting the time dilation effect in the first place. All this therefore suggests that in order to ascertain unequivocal differential aging, ALT will not work and hence one should get back to the full LT; then only A also predicts a differential aging (for large L) in spite of arbitrarily slow trip of B.

5.4 Yet Another!

Surprisingly the problem does not end here with the suggested remedy. There are deeper questions and the fallacy seems to persist. To understand this we begin by asking what happens if ALT is obtained via a different route? For example consider the Zahar transformation (ZT),

$$\begin{aligned} x' &= (x - vt), \\ t' &= \gamma^2(t - vx/c^2). \end{aligned} \tag{5.9}$$

As indicated in the footnote number(5), the above transformation represent a classical (Galilean) world with Einstein's synchrony[7]. This Galilean or classical world is supposed to be endowed with a preferred (ether) frame S, where light propagation is assumed to be isotropic. In any other frame S', however it will not be so. The TWS of light will be different in different directions in S' as one would expect in a classical world. Note that with respect to S, as expected, the moving rods do not contract and clocks in motion do not run slow. The effect of Einstein synchrony however is manifested through the phase term of the time transformation of Eq.(5.9) with consequent apparent length contraction and time dilation effects with respect to S'.

In the small velocity regime even this apparent length contraction and time dilation effects which are artifacts of the Einstein synchrony go away and one obtains the approximate Zahar transformation (AZT)[14, 20] which is the same as ALT.

Now, as before, in this classical world there is BE with respect to Barbara but there is no time dilation effect with respect to Albert. But the problem here is that there is no time dilation of moving clocks with respect to Albert is not an approximate result, hence there is no possibility of non-null differential aging from A's perspective, that can compete (as has been the case in the relativistic world) with BE. It therefore appears that the resolution of the problem for the world described by ALT, as discussed in the preceding paragraph, falls through in the case of AZT although algebraically the latter is the same as ALT, only their histories

[7]See chapter II for detailed discussions of the CS-thesis in both classical (due to Zahar) and relativistic worlds.

are different.

The answer to this paradox lies in the details of the workings of some effects similar to the relativistic ones (such as time dilation and length contraction effects and BE) from the perspectives of both the twins. If done properly (using the full transformation equation) both A and B will agree on their predictions, no matter whether the world is classical or relativistic. This we will not do here. However one will be able to verify it by following the steps outlined in the next section.

5.5 What is Wrong?

We begin this section by asking what is wrong with the transformation Eq.(5.5) representing ALT (i.e the same as AZT). Cannot it by itself (not as an approximation of LT) represent even a hypothetical kinematic world with its characteristic (whatever) behavior of moving rods and clocks and synchronization scheme? A mathematical transformation can lead to results, which may not be supported by the empirical world, but here the mathematical consistency of the Eqs.(5.5) and (5.6) seems to be at stake.

To understand this, it is enough to note that Eq.(5.5) itself may represent a hypothetical world (kinematical) but Eq.(5.6) is not its inverse, although the latter has been obtained as an approximation of LT representing transformation of coordinates of S' in term of those of S.

The inverse of Eq.(5.5) instead is given by

$$\begin{aligned} x &= \gamma^2(x' + vt'), \\ t &= \gamma^2(t' + vx'/c^2). \end{aligned} \tag{5.10}$$

We shall show below that not only algebraically, but also from the twin paradox point of view Eq.(5.5) and Eq.(5.10) represent a consistent kinematical world (World

1).

Similarly if one starts with Eq.(5.6), the corresponding inverse transformation would have been

$$x' = \gamma^2(x - vt),$$
$$t' = \gamma^2(t - vx/c^2). \tag{5.11}$$

Clearly the pair of transformations (5.11) and (5.6) represent another kinematical world (World 2) different from world 1. In this case also observers A (stationary in S) and B can be shown to agree in their predictions of the differential aging.

We show below step by step how do the twins living in world 1 and 2 make unequivocal predictions regarding their age differences. As mentioned in the last section, the reader may follow these steps as a template to resolve the twin paradox put in different worlds including the usual relativistic one. We start by considering world 2 first.

World 2

Step 1:

The transformation equations (5.6) and (5.11) suggest that A (an observer in S) does not predict any time dilation effect, however B predicts a time dilation for a clock stationary in S. These two observations may be summarized as follows:

Time Dilation Formulas:

$$TDF1: \quad \Delta t_B(A) = \Delta t_A(A), \tag{5.12}$$

$$TDF2: \quad \Delta t_A(B) = \gamma^{-2}\Delta t_B(B). \tag{5.13}$$

In the above we have used a notation scheme where $\Delta t_B(A)$ $[\Delta t_A(B)]$ denotes the B $[A]$-clock reading for a time interval between two events occurred at its position as inferred by the observer A $[B]$ drawn from its own coordinate clocks' records for the interval, $\Delta t_A(A)$ $[\Delta t_B(B)]$ and its knowledge of the relevant time dilation effect. Indeed the time intervals $\Delta t_B(A)$ or $\Delta t_A(B)$ are based on one clock measurements and hence they refer to proper times of B and A respectively.

As regards the notations $\Delta t_B(B)$ or $\Delta t_A(A)$, a clarification is needed. While, for example $\Delta t_A(B)$ refers to the difference between one clock (A) reading for two events, $\Delta t_B(B)$ refers to in general, the observed difference in readings (for the same events) recorded in two spatially separated (synchronized) clocks stationary with respect to the frame of reference attached to B. However when $\Delta t_B(B)$ concerns measurement of the round trip time of an object or a clock (A say), it also refers to a single clock (B) measurement.

Similarly one has two length contraction formulas (LCF) from the perspective of A and B. These formulas which follow from the transformation equations (for space) (5.6) and (5.11) may be written as

$$LCF1: \qquad L_B(A) = \gamma^{-2} L_B(B), \tag{5.14}$$

$$LCF2: \qquad L_A(B) = L_A(A). \tag{5.15}$$

Where $L_A(A)$ and $L_B(B)$ are the rest lengths of rods in S and S' respectively and $L_B(A)$ and $L_A(B)$ are the corresponding observed lengths from the other frames (A and B respectively). However we shall have no occasion to use Eq.(5.14) since the only distance of interest is that of the distant star from A which is clearly a rest length in A i.e. $L_A(A)$. Hence

$$L_A(A) = L, \qquad L_A(B) = L', \tag{5.16}$$

according to our definitions of L and L'.

Step 2:

A-clock time for B's up and down travel of distance $2L$ is

$$\Delta t_A(A) = 2L/v, \tag{5.17}$$

and using the above value for $\Delta t_A(A)$ the B-clock time for the same as calculated by Albert using relevant time dilation formula (Eq.(5.12)) is obtained as

$$\Delta t_B(A) = 2L/v. \tag{5.18}$$

Step 3:

Differential aging with respect to A is therefore given by

$$\delta t(A) = \Delta t_A(A) - \Delta t_B(A) = 0. \tag{5.19}$$

The following steps will lead to the same (null) differential aging from B's perspective.

Step 4:

From Barbara's point of view, A makes the round trip and Barbara measures the time for this trip as $\Delta t_B(B)$. This is nothing but the B-clock time as calculated by Albert, $\Delta t_B(A)$ which is given by Eq.(5.18) Hence

$$\Delta t_B(B) = 2L/v. \tag{5.20}$$

This can also be seen in the following way. According to B, A also travels a distance $2L$ for the latter's round trip as there is no length contraction effect with respect to B (see Eq.(5.15)). The speed of A with respect to B is also v as the transformation equations (5.6) and (5.11) honour the reciprocity of relative velocity. Hence the travel time $\Delta t_B(B)$ is again calculated as $2L/v$.

Step 5:

The same time interval in A-clock as calculated by B by the *näive* application of TDF2 (Eq.(5.13) alone on $\Delta t_B(B)$) is obtained as,

$$\Delta\bar{t}_A(B) = \gamma^{-2}2L/v. \tag{5.21}$$

This is however incorrect since desynchronization of distant clocks due to BE has not been taken into account and hence we have put a bar sign on t, to be removed later after correction.

Step 6:

The above expression must be corrected by taking into account the BE. To calculate this effect we first split the frame of reference (K) attached to B into two inertial frames S' and S'' which move with velocities v and $-v$ respectively with respect to S. Clearly B is at rest with these frames in its onward and return journeys.

Writing the transformation equations connecting the space-time coordinates of S and S'' as

$$\begin{aligned} x'' &= \gamma^2(x + vt), \\ t'' &= \gamma^2(t + vx/c^2), \end{aligned} \tag{5.22}$$

one readily obtains transformation equations between S' and S'' as

$$\begin{aligned} x'' &= \gamma_w(x' + wt'), \\ t'' &= \gamma_w(t' + wx'/c^2), \end{aligned} \tag{5.23}$$

where

$$w = 2v/(1 + v^2/c^2), \tag{5.24}$$

represents the relative speed of S'' with respect to S' and

$$\gamma_w = (1 - w^2/c^2)^{-1/2} = (1 + v^2/c^2)/(1 - v^2/c^2). \qquad (5.25)$$

As discussed earlier, Alfred ($\bar{A}$) and Barbara separated by a length L' in S' after deceleration arrives in the final frame of reference S'' producing a temporal offset (desynchronization) between their clocks which is given by (obtained from transformation for time in Eq.(5.23))

$$\delta t_{desync} = \gamma_w w L'/c^2 = \gamma_w w L/c^2, \qquad (5.26)$$

The last equality follows from Eqs.(5.15) and (5.16) which states that $L' = L$.

Step 7:

Going back to Eq.(5.20), leading to Eq.(5.21) one now discovers that the application of Eq.(5.13) on $\Delta t_B(B)$ to obtain $\Delta t_A(B)$ is a mistake (which has been pointed out earlier, see Sec.(5.3)) and one needs to add this desynchronization effect (Eq.(5.26)) to $\Delta t_B(B)$ before the application of TDF2 (Eq.(5.13)). Having done so and reapplying TDF2, one should obtain, removing the bar sign on Δt in Eq.(5.21),

$$\Delta t_A(B) = \gamma^{-2}(2L/v + \gamma_w w L/c^2). \qquad (5.27)$$

Which by using Eqs.(5.24) and (5.25) gives

$$\Delta t_A(B) = 2L/v. \qquad (5.28)$$

This again leads to the null differential aging from B's perspective as well,

$$\delta t(B) = \Delta t_A(B) - \Delta t_B(B) = 0. \qquad (5.29)$$

thus resolving the paradox.

World 1

Step 1:

Transformation equations (5.5) and (5.10) representing World 1 gives the following results for time dilation and length contraction:

$$TDF1: \quad \Delta t_B(A) = \gamma^{-2}\Delta t_A(A), \tag{5.30}$$

$$TDF2: \quad \Delta t_A(B) = \Delta t_B(B), \tag{5.31}$$

$$LCF1: \quad L_B(A) = L_B(B), \tag{5.32}$$

$$LCF2: \quad L_A(B) = \gamma^{-2}L_A(A). \tag{5.33}$$

According to our definitions of L and L' one can write the last equation as

$$L' = \gamma^{-2}L. \tag{5.34}$$

Step 2:

A-clock time for B's up and down travel of distance $2L$ is as before

$$\Delta t_A(A) = 2L/v, \tag{5.35}$$

and the same recorded in B's clock as interpreted by A can be obtained by applying TDF1 (Eq.(5.30)) on $\Delta t_A(A)$. Hence

$$\Delta t_B(A) = \gamma^{-2}2L/v. \tag{5.36}$$

Step 3:

From the last two relations, the differential aging with respect to A now comes out to be

$$\delta t(A) = \Delta t_A(A) - \Delta t_B(A) = 2Lv/c^2. \tag{5.37}$$

Step 4:

With respect to B, A travels a distance $2L'$ for its round-trip with speed v hence, the time recorded in B's clock for A's round-trip is given by

$$\Delta t_B(B) = 2L'/v = \gamma^{-2}2L/v. \tag{5.38}$$

where we have made use of Eq.(5.34).

Step 5:

B may try to calculate the corresponding time as recorded by A by näively applying only TDF2 given by Eq.(5.31) on $\Delta t_B(B)$ and obtains,

$$\Delta\bar{t}_A(B) = \gamma^{-2}2L/v. \tag{5.39}$$

Step 6:

As mentioned before the above expression is incorrect (hence we have put the bar sign on Δt) since BE has not been taken care of. To calculate this effect we again split the reference frame K attached to B into two inertial frames S' and S'' representing the inertial frames of B in its forward and return journeys. The transformation equations between coordinates of S' and S'' remain the same as that in world 2.

$$\begin{aligned} x'' &= \gamma_w(x' + wt'), \\ t'' &= \gamma_w(t' + wx'/c^2), \end{aligned} \tag{5.23}$$

where w and γ_w are as defined by Eqs.(5.24) and (5.25).

Hence following the previous arguments leading to Eq.(5.26), we have

$$\delta t_{desync} = \gamma_w wL'/c^2, \tag{5.40}$$

which after using Eq.(34) can be written as

$$\delta t_{desync} = \gamma_w w \gamma^{-2} L/c^2. \tag{5.41}$$

Step 7:

Following arguments given in step 7 (world 2) but now using Eq.(5.31) after making the correction due to Boughn effect, we find that

$$\Delta t_A(B) = \Delta \bar{t}_A(B) + \gamma_w w \gamma^{-2} L/c^2 = \gamma^{-2}(2L/v + \gamma_w w L/c^2) = 2L/v. \tag{5.42}$$

Hence

$$\delta t(B) = \Delta t_A(B) - \Delta t_B(B) = 2L/v - \gamma^{-2} 2L/v = 2Lv/c^2, \tag{5.43}$$

which is the same as $\delta t(A)$. Hence in this case also the paradox does not exist.

5.6 Summary and Concluding Remarks

According to BE two presynchronized clocks that accelerate identically from one inertial frame to another along the direction of their spatial separation should get desynchronized. It has been remarked in the literature that the standard twin paradox can be explained in terms of this effect which, if taken care of properly, may be seen to overcompensate for the apparent slowing down of clocks of the stay-at-home twin with respect to that of the traveller one[2]. However the actual demonstration of unequivocal prediction for differential aging by both the twins using BE seems to be a non-trivial exercise. It is however clear that the answer to the twin paradox depends on an interesting interplay of two special relativistic effects viz RTD and BE.

In a bid to isolate the role of BE in the standard twin paradox one may try to study it in the classical (small velocity) regime where γ, hence the time dilation factor is assumed to be unity. In this regime, since relativity of distant simultaneity persists, BE continues to take part in the twin problem. But since one does not expect any time dilation, one ends up with a new fallacy. Since now as if Albert predicts no differential aging (in absence of RTD) which Barbara contradicts because of her knowledge of BE. We thus have a converse situation here. Recall that in posing the usual twin paradox, one emphasizes on the RTD effect only but BE is overlooked. But now in the new problem, one highlights BE and RTD is ignored. The paradox however is a mild one which gets resolved as soon as one takes into account the integrated effect of time dilation which was previously ignored under the small velocity approximation ($v^2/c^2 << 1$). However the fallacy reappears if one considers ZT which describes a classical world with Einstein synchrony. In this world there is no time dilation and length contraction effects with respect to the preferred frame S from the beginning. In the small velocity regime though, the transformation equations are the same as ALT. The question then arises as to what, in absence of RTD (from Albert's perspective) can compensate BE which still exists in the classical world because of adopted synchrony.

The answer lies in the details of the workings of the transformation equations in producing time dilation effects and BE from *both the twins'* perspectives. Here we have worked out these details for two mathematically consistent hypothetical kinematical worlds to show that the twins in any case make unequivocal predictions.

We end chapter by briefly addressing a much talked about issue regarding the often made claim in the literature that the full solution of the twin paradox lies in the realm of general relativity (GR)[21, 22, 23, 24, 25, 26, 27]. As correctly pointed out by Builder[28], it is indeed strange to first deny by some authors the applicability of SR in the resolution of the twin paradox and then use conclusions derived from SR itself by means of the principle of equivalence of GR. The essence of any general relativistic solution of the problem lies in introducing an equivalent pseudo gravitational potential to be experienced by the traveller twin at the time of her direction reversing acceleration. A consequent gravitational time offset effect then provides the extra aging of the stay-at-home twin required to make the correct prediction by Barbara. Now, since as we have mentioned that BE can directly be used to resolve the paradox, the use of pseudo-gravitational field, to explain the problem of equivocal prediction of differential aging by the twins must be a trivial exercise. After all no true gravitational field exists in the problem; hence in order to resolve the issue, introduction of GR in the essentially flat space-time (with vanishing Riemann tensor) is utterly misleading[5, 28, 29, 30]. This conclusion now is strengthened by the fact that twin paradoxes can be devised, as has been shown earlier, in some hypothetical kinematical worlds characterized by the existence of BE, which in turn is an outcome of relativity of distant simultaneity. The unequivocal predictions of differential aging (or its absence) by the twins of these worlds can be explained by appropriate use of this non-special relativistic BE in addition to the time dilation effects. It has been shown elsewhere by the present authors[6] that the gravitational time offset-effect of GR (in the case of uniform gravity) follows from the equivalence principle provided one uses the full machinery of SR and is therefore essentially special relativistic in origin. On the other hand the two worlds discussed in the last section are only theoretical constructs hence it is not possible to replace BE of these hypothetical worlds by equivalent gravitational fields which may act as a "physical agent" responsible for producing the extra aging of the stationary sibling. Yet we have seen how the resolution of the paradox comes about from purely kinematical considerations.

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Part II