C H A P T E R - I.

DYNAMICAL RESPONSE OF AN ORTHOTROPIC INFINITE PLATE PLACED ON ELASTIC FOUNDATION AND SUBJECTED TO TIME DEPENDENT CONCENTRATED FORCE.

Introduction :

The analysis of a thin elastic plate supported by a flexible elastic foundation has numerous engineering applications. At the present time, the accepted method of approximate analysis represents the supporting material as a Winkler (1867) foundation. Winkler approximated the foundation by a series of closely packed linear springs attached to a rigid base. The springs are assumed to act independently so that the foundation acts like a fluid, the reaction at any point being directly proportional to the deflection at that point. But in the case of a coherent subgrade such a hypothesis approximates but crudely the actual behaviour of the subgrade.

The elastic modulus and Poisson's ratio of the foundation material are often known and a better approximation of the foundation reaction can be obtained on the assumption that the foundation has the properties of a semi-infinite elastic body. Numerous investigators such as Kerr (1964), Herrmann (1967), Fletcher (1971) have attempted to provide a more realistic representation of the behaviour of the elastic continuum in terms of the elastic constants of the foundation material. All these investigators considered the loading to be static.

The object of this paper is to study the dynamic response of an orthotropic infinite rectangular plate, such as reinforced concrete, under the action of a time-dependent concentrated force and placed on an elastic foundation. These types of problems are of practical importance for the design of concrete paving slabs for roads, aircraft runways, bed plates for machine tools, engines etc. Livesley (1953) has discussed the case in which the loading is static and the foundation reaction is directly proportional to the local plate deflection in details for both infinite and semi-infinite rectangular plate of various boundary conditions. In this paper the foundation is assumed to have the properties of a semi-infinite elastic body. Winkler type of foundation is also considered for comparison. Owing to the complexities of mathematics involved, the problem has been confined to axial symmetrical cases. The solutions are obtained by a semi-inverse method with the help of Laplace and Hankel transforms.

Analysis :

Consider an infinitely large orthotropic plate in a state of axial symmetry. The plate is placed on an elastic foundation and subjected to the action of a dynamic surface loading. Following Timoshenko and Woinowsky-Krieger (1959, P.365) the differential equation for equilibrium of an element of the plate can be written in rectangular co-ordinates as

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = q(x,y,t) - p(x,y,t) \quad \ldots (1.1)$$

where $D_x = \dfrac{E_x' \; h^3}{12}$, w is the deflection in z - direction,

$H = \frac{E'' h^3}{12} + \frac{G\ h^3}{6}$; $D_y = \frac{E'_y\ h^3}{12}$, ρ = density of the plate material, h = plate thickness, q (x, y, t) denotes the given surface loading, p (x, y, t) the reaction of the foundation and E'_x , E'_y , E'' and G are elastic constants of the material.

For an orthotropic material such as reinforced concrete

$$H = \sqrt{D_x D_y}$$

and introducing $x = x_1$, $y_1 = \left(\frac{D_x}{D_y}\right)^{\frac{1}{4}} y$ as new variables in eq.(1.1) one gets

$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right)^2 W + \frac{\rho h}{D_x}\frac{\partial^2 w}{\partial t^2} = \frac{q(x_1, y_1, t)}{D_x} - \frac{p(x_1, y_1, t)}{D_x} \quad \ldots (1.2)$$

Putting $x_1 = r_1 \cos\theta_1$, $y_1 = r_1 \sin\theta_1$ in eq.(1.2) one gets for axial symmetry

$$\left(\frac{\partial^2}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial}{\partial r_1}\right)^2 W + \frac{\rho h}{D_x}\frac{\partial^2 w}{\partial t^2} = \frac{q - p}{D_x} \quad \ldots (1.3)$$

Multiplying both sides of eq.(1.3) by $D_x\, r_1\, J_o(\alpha\, r_1)$ and integrating between the limits 0 to ∞ one gets

$$\overline{w}\left[D_x \alpha^4 K(\alpha) + 1\right] + \rho h K(\alpha)\frac{\partial^2 \overline{w}}{\partial t^2} = K(\alpha)\, Q(\alpha)\, F(t) \quad \ldots (1.4)$$

where $\overline{w} = \int_0^\infty w r_1 J_o(\alpha\ r_1)\, dr_1$, J_o denoting the Bessel function of zero order ; the term $K(\alpha) = \frac{w}{p}$ depends on the nature of subgrade ; the term $Q(\alpha) = \overline{q}(r_1) = \int_0^\infty q r_1 J_o(\alpha r_1) dr_1$ depends on the intensity of loading ; and $q = q(r_1)\, F(t)$.

Eq. (1.4) can be written as

$$\frac{\partial^2 \overline{w}}{\partial t^2} + a^2 \overline{w} = C\ F(t) \quad \ldots (1.5)$$

where $a^2 = \dfrac{D_x \alpha^4 K(\alpha) + 1}{\rho h K(\alpha)}$; $C = \dfrac{Q(\alpha)}{\rho h}$

Taking the intensity of loading to be periodic

$$F(t) = \sin \omega_0 t \qquad \ldots \qquad \ldots (1.6)$$

The initial conditions are

$$W = \frac{\partial W}{\partial t} = 0 \quad \text{at} \quad t = 0 \qquad \ldots \qquad \ldots (1.7)$$

One gets from eq.(1.5) after applying Laplace transform method

$$\bar{W} = \frac{C\,\omega_0}{a^2 - \omega_0^2} \left[\frac{\sin \omega_0 t}{\omega_0} - \frac{\sin at}{a} \right] \qquad \ldots (1.8)$$

If the load is concentrated at the origin

$$Q(\alpha) = \frac{P}{2\pi} \qquad \ldots \qquad \ldots (1.9)$$

and in the case of an isotropic semi-infinite medium [Ref.5, P.280]

$$K(\alpha) = \frac{1}{K_0 \alpha} \qquad \ldots \qquad \ldots (1.10)$$

K_0 being the elastic constant of the medium defined by $K_0 = \dfrac{E_0}{2(1-\nu_0^2)}$,

E_0 being the modulus of elasticity and ν_0 the Poisson's ratio of the medium.

Considering eqs.(1.9) and (1.10) and applying Hankel inversion theorem one gets from eq.(1.8) for semi-infinite medium

$$W(r_1, t) = \frac{P}{2\pi} \int_0^\infty \frac{\sin \omega_0 t \; \alpha J_0(\alpha r_1)\, d\alpha}{\beta} - \frac{P\omega_0}{2\pi} \int_0^\infty \frac{\sin (\psi/\rho h)^{\frac{1}{2}} t \; (\rho h)^{\frac{1}{2}} \alpha J_0(\alpha r_1)\, d\alpha}{\psi^{\frac{1}{2}} \beta} \qquad \ldots (1.11)$$

where $\beta = D_x \alpha^4 + K_0 \alpha - \rho h \omega_0^2$

$\psi = D_x \alpha^4 + K_0 \alpha$

and for Winkler type of foundation having the reaction K per unit area per unit deflection

$$W(r_1,t) = \frac{P}{2\pi}\int_0^\infty \frac{\sin\omega_o t\ \alpha J_o(\alpha r_1)d\alpha}{\beta_1} -$$

$$- \frac{P\omega_o}{2\pi}\int_0^\infty \frac{\sin(\psi_1/\rho h)^{\frac{1}{2}}t\ (\rho h)^{\frac{1}{2}}\alpha J_o(\alpha r_1)d\alpha}{\psi_1^{\frac{1}{2}}\ \beta_1} \quad \ldots (1.12)$$

where $\beta_1 = D_x\ \alpha^4 + K - \rho h\ \omega_o^2$

$\psi_1 = D_x\ \alpha^4 + K$

The distribution of pressure $p(r_1,t)$ at any point is readily obtained from eq.(11)

$$p(r_1,t) = \frac{PK_o}{2\pi}\int_0^\infty \frac{\sin\omega_o t\ \alpha^2 J_o(\alpha r_1)d\alpha}{\beta} -$$

$$- \frac{P\omega_o K_o}{2\pi}\int_0^\infty \frac{\sin(\psi/\rho h)^{\frac{1}{2}}t\ (\rho h)^{\frac{1}{2}}\alpha^2 J_o(\alpha r_1)d\alpha}{\psi^{\frac{1}{2}}\beta} \quad \ldots (1.13)$$

After integrating eqs.(1.11),(1.12) and (1.13) numerically one gets finally the following results :

For semi-infinite medium

$$\frac{W}{h} = \frac{P\sin\omega_o t}{2\pi\ K_o l_o h}\Big[\frac{J_o(0.58\ r_1/l_o)}{0.71-\mu^2}\left\{0.39-0.74\,\mu\ \frac{\sin(0.84\ w_n t)}{\sin w_o t}\right\} +$$

$$+ \frac{J_o(3.4\ r_1/l_o)}{137-\mu^2}\left\{15.23-1.3\,\mu\ \frac{\sin(11.7\ w_n t)}{\sin w_o t}\right\}\Big]$$

$$\ldots (1.14)$$

where $l_o^3 = \frac{D_x}{K_o}$, $\frac{K_o}{\rho h l_o} = w_n^2$, $\mu = \frac{w_o}{w_n}$

and for Winkler foundation

$$\frac{W}{h} = \frac{P \sin w_0 t}{2\pi K l^2 h} \left[\frac{J_0(0.58\, r_1/l)}{1.13 - \mu^2} \left\{ 0.89 - 0.85\, \mu \, \frac{\sin(1.06\, w_n t)}{\sin w_0 t} \right\} + \right.$$

$$\left. + \frac{J_0(3.4\, r_1/l)}{134.6 - \mu^2} \left\{ 15.28 - 1.3\, \mu \, \frac{\sin(11.7\, w_n t)}{\sin w_0 t} \right\} \right] \quad \ldots (1.15)$$

where $l^4 = \frac{D_x}{K}$, $\frac{K}{\rho h} = w_n^2$

The distribution of pressure is obtained as

$$p(r_1, t) = \frac{P \sin w_0 t}{2\pi l_0^2} \left[\frac{J_0(0.58\, r_1/l_0)}{0.71 - \mu^2} \left\{ 0.52 - 0.43\, \mu \, \frac{\sin(0.84\, w_n t)}{\sin w_0 t} \right\} + \right.$$

$$\left. + \frac{J_0(3.4\, r_1/l_0)}{137 - \mu^2} \left\{ 0.52 - 4.3\, \mu \, \frac{\sin(11.7\, w_n t)}{\sin w_0 t} \right\} \right] \quad \ldots (1.16)$$

For an isotropic infinite plate under the concentrated load P at the origin, the maximum static deflection is obtained from eq.(1.14)

$$W = 0.214 \frac{P l_0^2}{D} \quad \ldots \quad \ldots (1.17)$$

compared to the corresponding result obtained by Timoshenko, [Ref. 5, P. 280]

$$W = 0.192 \frac{P l_0^2}{D} \quad \ldots \quad \ldots (1.18)$$

NUMERICAL RESULTS

Steady state amplitudes at the origin for various values of frequency ratio are calculated both for semi-infinite medium and Winkler model by eqs.(1.14) and (1.15) respectively. These results are presented in Fig. 1-1.

From Fig.1.1 it is observed that Winkler foundation as well as semi-infinite medium give practically the same dynamic response under the load point when the disturbing frequency is higher than the first natural frequency. When the disturbing frequency is lower than the first natural frequency, Winkler model gives a somewhat lower value of amplitude compared to the semi-infinite medium.

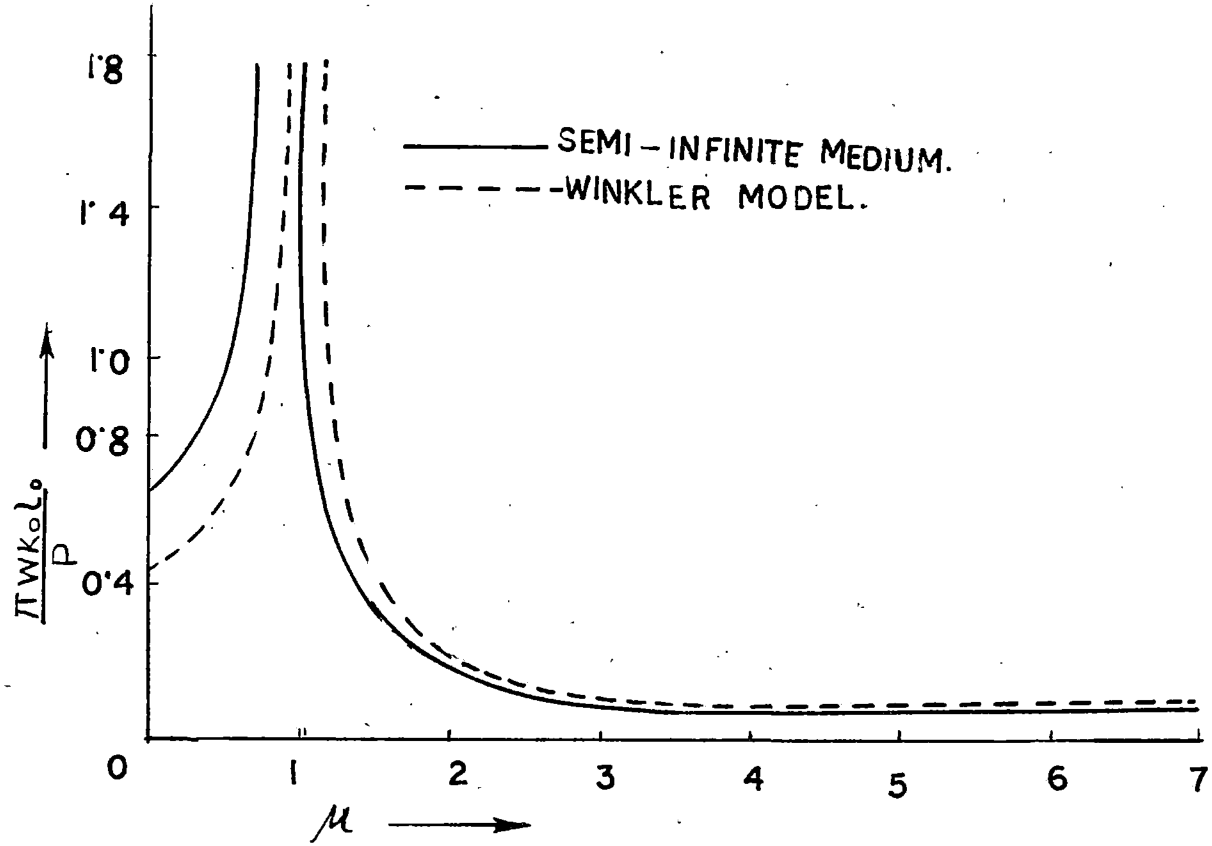

FIG. 1.1 AMPLITUDE-FREQUENCY CURVE.