* *CHAPTER III*

FINE-HYPERFINE INTERACTION

* A part of the contents of this chapter has already been published in Ref.(75).

III.1. Introduction :

The spin-averaged properties of heavy quarkonia seem to be described fairly accurately by a Schrödinger equation with a quark-antiquark potential, which has been determined empirically, with some guidance from quantum chromodynamics. The spin-independent potential is determined accurately at least in the range 0.1 fm < r <1 fm. The potential is in general agreement with the calculation of lattice gauge theory. The situation is, however, less satisfactory when one considers the spin-dependent effects, *e.g.* fine-hyperfine splittings. The fine-structures of χ-states, which have been measured fairly accurately presents, in particular, a case where the simple-minded theoretical results seem to fail. This may appear a bit surprising, since the fine-hyperfine interactions are essentially short-distance effects and one may expect a QCD-motivated potential to yield good results. However, fine-hyperfine interactions are relativistic in origin and the failure of the standard non-relativistic potentials may be due to the fact that the spin-dependent potentials are not related to the non-relativistic potentials in the simple way normally assumed. The method of studying the spin-dependent interactions have been developed by Eichten and Feinberg,[41] Buchmüller,[44,76] Gromes[35,42] and others.[8,39,46,48,49,77] Their prescription will be summarized in the next section. The Breit-Fermi interaction potential which is obtained within the

framework of Bethe-Salpeter equation are also discussed. In section III.3, we consider a modification of the Breit-Fermi interaction potential which leads to a better agreement with the experimental results. The calculated results are given in section III.4. Theoretical results obtained by some other authors have been compared with our results in this section. The possible modification of the results due to the inclusion of a pseudoscalar exchange potential is also studied. Our conclusions are summarized in the last section.

III.2. Breit-Fermi potential :

The general form of the nearly non-relativistic $Q\bar{Q}$ potential, as defined by the Wilson loop, can be written to lowest order in $(v/c)^2$ as

$$V(r) = V_{SI}(r) + \frac{S.L}{2M_Q^2}\left(-\frac{dV_{SI}}{rdr} + \frac{4}{r}\frac{dV_2(r)}{dr}\right) + \frac{1}{3M_Q^2}S_{12}V_3(r) + \frac{2}{3M_Q^2}S_1.S_2\,V_4(r)\,, \qquad (3.1)$$

where

$$S = S_1 + S_2 \quad \text{and} \quad S_{12} = \left(3(S_1.\hat{r})(S_2.\hat{r}) - S_1.S_2\right) \qquad (3.2)$$

and V_{SI} is the spin-independent potential, and V_2, V_3, V_4 are the spin-dependent ones, related to the correlation functions of colour electric and colour magnetic fields which are different from those that determine the spin-independent potential. The potentials V_2, V_3, V_4 may not, in principle, be simply related to

V_{SI}. However, it has been noted that a conceptually simple picture may be obtained if one assumes relations similar to those that emerge when one considers the Breit-Fermi interaction potentials obtained in the non-relativistic limit of a Bethe-Salpeter equation with vector and scalar exchange kernels. Thus one makes the following identification :

$$V_{SI}(r) \simeq V_{nR}(r) = V_v(r) + V_s(r) \tag{3.3}$$

$$V_2(r) = V_v(r) \tag{3.4}$$

$$V_3(r) = -\frac{d^2V_v(r)}{dr^2} + \frac{dV_v(r)/dr}{r} \tag{3.5}$$

$$V_4(r) = \nabla^2 V_v(r) \quad , \tag{3.6}$$

where $V_v(r)$ and $V_s(r)$ represent the vector and scalar exchange potentials respectively. With the usual non-relativistic reduction of the Bethe-Salpeter equation, the fine-hyperfine energies are given by

$$\langle V_{SD}(r) \rangle = a \langle L.S \rangle + b \langle S_{12} \rangle + c \langle S_1.S_2 \rangle \ ,$$

where,

$$a = \frac{1}{2M_Q^2} \left\langle \frac{3dV_v/dr - dV_s/dr}{r} \right\rangle \tag{3.7}$$

$$b = \frac{1}{3M_Q^2} \left\langle \frac{dV_v/dr}{r} - \frac{d^2V_v}{dr^2} \right\rangle \tag{3.8}$$

$$c = \frac{2}{3M_Q^2} \langle \nabla^2 V_v \rangle \qquad (3.9)$$

and $\langle S_1.S_2 \rangle = \frac{1}{2} [\, S(S+1) - \frac{3}{2} \,]$

$$\langle L.S \rangle = \frac{1}{2} [\, J(J+1) - L(L+1) - S(S+1) \,]$$

$$\langle S_{12} \rangle = \frac{-\langle L.S \rangle^2 - (1/2)\langle L.S \rangle + (1/3)\langle L^2 \rangle \langle S^2 \rangle}{(2L+3)(2L-1)} \,.$$

It can be seen that both scalar and vector potentials enter into the spin-orbit interaction term whereas the spin-spin and tensor interactions depend only on the vector part. The spin-dependent mass-splitting formula can be written conveniently in the matrix form as

(i)
$$\begin{vmatrix} M(^3S_1) \\ M(^1S_0) \end{vmatrix} = \begin{vmatrix} 1 & 1/4 \\ 1 & -3/4 \end{vmatrix} \begin{vmatrix} M_0(nS) \\ c(nS) \end{vmatrix} \quad \text{for S-states}$$

(ii)
$$\begin{vmatrix} M(^3P_2) \\ M(^3P_1) \\ M(^3P_0) \\ M(^1P_1) \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1/10 & 1/4 \\ 1 & -1 & 1/2 & 1/4 \\ 1 & -2 & -1 & 1/4 \\ 1 & 0 & 0 & -3/4 \end{vmatrix} \begin{vmatrix} M_0(nP) \\ a(nP) \\ b(nP) \\ c(nP) \end{vmatrix} \quad \text{for P-states}$$

(iii)
$$\begin{vmatrix} M(^3D_3) \\ M(^3D_2) \\ M(^3D_1) \\ M(^1D_2) \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1/7 & 1/4 \\ 1 & -1 & 1/2 & 1/4 \\ 1 & -3 & -1/2 & 1/4 \\ 1 & 0 & 0 & -3/4 \end{vmatrix} \begin{vmatrix} M_0(nD) \\ a(nD) \\ b(nD) \\ c(nD) \end{vmatrix} \quad \text{for D-states}$$

Here $M_0(nL)$ represents the spin-averaged mass of the system. Thus for P-states, the centre of gravity of the triplet state is given by

$$M.\,(\,^3P_{COG}\,) = [\,M(^3P_0) + 3\,M(^3P_1) + 5\,M(^3P_2)\,]\,/\,9\,.$$

From the known masses of ^3P_J states of $b\bar{b}$ and $c\bar{c}$ systems, it is customary to define the fine-structure ratio r as

$$r = \frac{(M_2 - M_1)}{(M_1 - M_0)} \qquad (3.10)$$

where M_J are the masses of the χ-states with total spin J. The experimental values give $r < 0.8$ for all χ-states. In the limit of a coulomb potential, $r = 0.8$. Experimentally $r = 0.67 \pm 0.05$ for $b\bar{b}(1P)$ and $r = 0.48 \pm 0.01$ for $c\bar{c}(1P)$. Therefore, as expected, the $b\bar{b}$ system is more coulombic in nature than $c\bar{c}$ system.

The simple prescription, Eqs. (3.3)-(3.6), however, imposes severe constraints on the potentials $V_{nR}(r)$ and $V_v(r)$ and it is not surprising that the potentials fail to account for the observed fine-hyperfine splittings. It is now recognised that the hyperfine structure of P-states, in particular, cannot be explained[78] within the framework mentioned above. Igi and Ono[50,79] have considered the radiative corrections to the potential, as given by Gupta, Radford and Repko[48,49] for the spin-dependent interactions and found that the recent data on h_b and h_c cannot be accounted for. Pantaleone and Tye,[47,80] using the potentials

derived by the method of Eichten and Feinberg,[41] have also reached similar conclusions. They even suggested a reexamination of the experimental results. Gupta *et al.*[81] have also noted that both $b\bar{b}$ and $c\bar{c}$ data cannot be fitted with the same set of potential parameters. It appears that the spin-dependent interactions are not yet understood and further investigation is essential.

III.3. Modified Breit-Fermi potential :

While looking for a modification, it will be useful to work within the framework of Breit-Fermi form, although the validity of the Eqs. (3.3) - (3.6), in particular, will have to be examined critically. We intend to undertake such a study. As a first step, we consider a simple empirical variation of the relations (3.3)-(3.6), aimed at fitting the fine-hyperfine data, without losing contact with the fairly successful non-relativistic treatment.

We note that while the static potential has to be equal to the sum of V_v and V_s (Gromes[35,42] sum rule), V_2 does not have to be equal to V_v. This provides a possible direction for introducing a modification. We write the non-relativistic $Q\bar{Q}$ potential as in the previous chapter, *e.g.*,

$$V_{nR} = f(r)V_{QCD}^{(2)}(r) + \left(1-f(r)\right)V_L(r)$$

where

$$f(r) = \frac{(1 + e^{-r_0/s})}{(1 + e^{(r-r_0)/s})} \quad ,$$

and $V^{(2)}_{QCD}(r)$ is the 2-loop QCD potential. The long-range part V_L is not well-determined and we choose

$$V_L(r) = Ar + B\,(1 + \ln\Lambda_{\overline{MS}}\, r) \quad . \tag{3.11}$$

The motivation for including the $\ln\Lambda_{\overline{MS}}\, r$ term may be mentioned here. As has been pointed out by Olsson and Suchyta,[82] the spin-orbit contribution of the confinement potential can be separated in the observed fine-structure of 1P states and the results are

$$\begin{aligned} \alpha_{conf}\,(b\bar{b}) &= -7.7 \pm 2.1 \text{ MeV} \\ \alpha_{conf}\,(c\bar{c}) &= -25.9 \pm 1.3 \text{ MeV} \quad , \end{aligned} \tag{3.12}$$

while a linear confinement term kr, with $k \sim 0.2$ GeV^2 gives the values $\alpha_{conf}(b\bar{b}) = -2.8$ MeV and $\alpha_{conf}(c\bar{c}) = -18$ MeV. Thus if the Breit-Fermi form is accepted, a modification in the long-range potential is necessary, and this can be done conveniently in the intermediate range of r values. Olsson and Suchyta[83] considered a modification of the form $V_L(r) = Ar - \alpha/r$, which is supported by lattice gauge calculations.[84] Lüsher[85] also suggested a similar form from flux-tube considerations. We, however, prefer the form (3.11) because it leads to a milder behavior for the corresponding spin-orbit term at short-distances. We now look for a simple modification of the standard Breit-Fermi relations by choosing V_{nR}

$= V_2(r) + V_s(r)$, where

$$V_2(r) = f(r)V_{QCD}^{(2)}(r) - \Lambda_{\overline{MS}}^3 r^2(1-\beta/r), \qquad r \leq \beta$$

$$= 0 \qquad r > \beta \qquad (3.13)$$

$$V_s(r) = \left(1 - f(r)\right)V_L(r) + \Lambda_{\overline{MS}}^3 r^2(1-\beta/r), \qquad r \leq \beta$$

$$= \left(1 - f(r)\right)V_L(r) \qquad r > \beta \qquad (3.14)$$

$$V_3(r) = -d^2V_2(r)/dr^2 + \frac{dV_2(r)/dr}{r} \qquad (3.15)$$

$$V_4(r) = \nabla^2 V_2(r) \ . \qquad (3.16)$$

The potentials chosen have the following features :

(1) The non-relativistic potential V_{nR} remains unchanged. There is no discontinuity in V_{nR}, and the discontinuity in dV_{nR}/dr occurs at a large value of $r = \beta = 1.835$ fm, so that the short-range spin-dependent results are not appreciably disturbed.

(2) $V_2(r)$ differs from $V_v(r)$, the difference indicating a possible contribution from non-perturbative interactions. As $r \to 0$, one regains the one-gluon-exchange term.

(3) The Woods-Saxon function has been introduced[66] to interpolate between the perturbative and the confining regions. One may hope that the fine-hyperfine splittings could be fitted by adjusting these parameters. The expection, however, is not fulfilled. The

obvious conclusion is that within the framework of Breit-Fermi scheme, the potentials $V_v(r)$ and $V_L(r)$ need some modifications. We have given above a simple modification with an extra parameter β.

(4) At $r = \beta < \Lambda_{\overline{MS}}^{-1}$, $f(r)V_v(r)$ is already negligibly small, where a cut-off is introduced. The cut-off is needed anyway to eliminate the unphysical perturbative QCD singularity at $r = \Lambda_{\overline{MS}}^{-1}$. The cut-off should not affect the calculated results with our choice of $\Lambda_{\overline{MS}} = 196$ MeV.

(5) The potentials $V_2(r)$ and $V_s(r)$ and $V_t = V_{nR}$ have been shown in Fig.3.1. Whether the potential $V_2(r)$ should still be called the vector exchange potential is not clear, but the extensive modification of the QCD potentials needed even for a modest fit with the experimental fine-hyperfine results is an indication of the constraints on the potentials.

III.4. Fine-hyperfine splittings :

With the choice of V_{nR} as in above, we have solved the Schrödinger equation numerically and have also evaluted the quantities relevant for fine-hyperfine splitting replacing V_v by V_2 in Eqs. (3.7) - (3.9). The values of the parameters M_b, M_c, $\Lambda_{\overline{MS}}$, A, B, r_o and s are the same as in the previous chapter. To determine fine-hyperfine splittings, we have used only one extra parameter β =1.835 fm.

The results obtained may be summarized as follows :

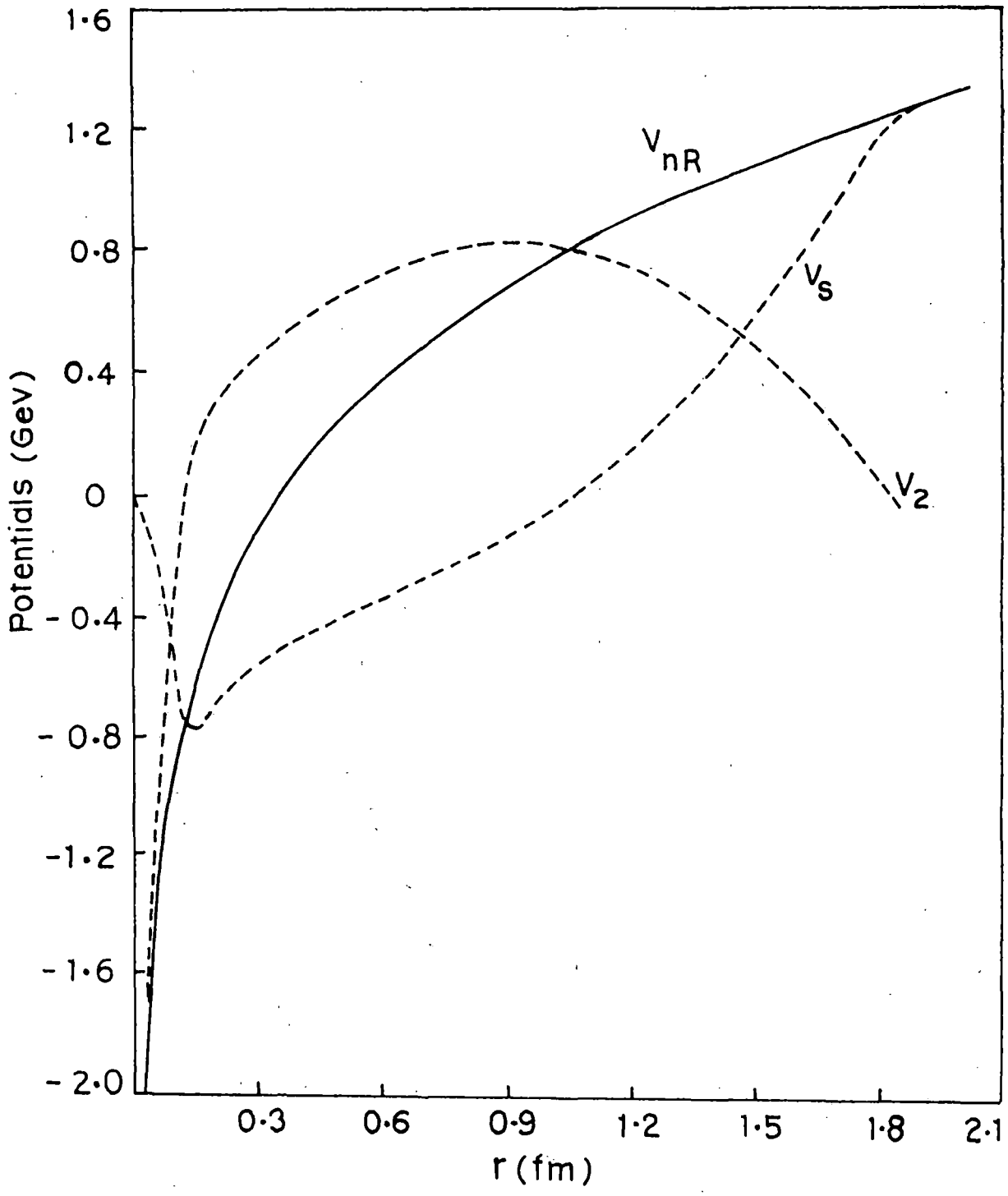

Fig.3.1. The $Q\bar{Q}$ potentials $V_2(r)$, $V_s(r)$ and the total potential $V_{nR} = V_2(r) + V_s(r)$.

(1) The non-relativistic potential V_{nR} gives the spin-averaged spectra fairly accurately, excepting for the centre of gravity mass for the 1P states, which is about 15 MeV less than the experimental value. In the case of Martin potential,[86] the lowest mass Y P-state is obtained 40 MeV below the COG value. The absence of a coulomb-like singularity raises the S-states with respect to P-states. This problem exists with Cornell potential also, although not recognized by the earlier workers. The reason, as given by Jacobs *et al.*,[87] is that earlier workers used the ψ, 1P, ψ' masses while one considers, in the present context, the COG values of the split levels. For ψ and ψ', this change means a difference of about 25-30 MeV, whereas the shift for 1P state is much smaller, leading to the observed disagreement. We have not tried to remedy this, because the fine-hyperfine splittings are not likely to be much altered by this small shift of the COG value. We have not considered the spin-independent relativistic corrections. For the $b\bar{b}$ states, these are anyway very small.

(2) To study the spin-dependent interactions, we first consider the Breit-Fermi potentials without any modification, *viz.* $V_2 = V_v$ and $V_s = \left(1-f(r)\right) V_L$. The calculated values for a,b,c for the five levels (1S,2S,1P of $c\bar{c}$ and 1P,2P of $b\bar{b}$), for which the experimental results are available, have been shown in Table 3.1. It is obvious that the naive choice $V_2 = V_v$ and $V_s = \left(1-f(r)\right) V_L$ do not agree at all with the experimental results. Note, in particular, that with $V_2 = V_v$, one gets negative values for a and c

Table 3.1. Results for unmodified Breit-Fermi interaction (i) without and (ii) with pseudoscalar exchange contribution.

State	a(MeV)		b(MeV)		c(MeV)	
	Expt.	Theory	Expt.	Theory	Expt.	Theory
$b\bar{b}$(1P)	14.1 ± 0.4	5.64	12.0 ± 0.9	(i) 9.44 (ii)12.16	5.4 ± 2.2	(i)-4.87 (ii)-5.12
$b\bar{b}$(2P)	10.3 ± 1.2	6.97	9.8 ± 1.4	(i) 7.67 (ii)13.54		(i)-3.16 (ii)-0.35
$c\bar{c}$(1S)					116 ± 5	(i)32.37[a] (ii)167.82[a]
$c\bar{c}$(2S)					92 ± 5	(i)26.04[a] (ii)145.65[a]
$c\bar{c}$(1P)	34.9 ± 0.3	-18.41	40.1 ± 0.8	(i) 8.64 (ii)30.18	0 ± 0.9	(i) -5.99 (ii)-1.25

[a]No $\delta^3(r)$ term was considered for evaluating c.

for $c\bar{c}$ states, while the experimental values are positive. A modification of the naive choice is definitely called for.

(3) With the choice of Eqs.(3.13)-(3.16), we get the results shown in Table 3.2. The overall agreement for the fine-hyperfine splittings of the P-states for both $b\bar{b}$ and $c\bar{c}$ systems is satisfactory, but leaves scopes for improvement. It has been possible to get a positive value (though smaller than the experimental value) for c for Y(1P) state. This may be taken as indicative of the trend of the modification needed to explain these data. The fine interaction results are in general agreement with the experimental results. It may be pointed out that our attempt to get a positive c for 1P $b\bar{b}$ state has led to a little worsening of the agreement for the values of a and b. This can be seen by increasing the value of β a little, say to β = 1.86 fm where we get a = 19.33 MeV, b = 12.9 MeV and c = 1.94 MeV for $b\bar{b}$ 1P state. While smaller values of a and b can be obtained by choosing a smaller β, the value for c becomes negative. Igi and Ono[79] and Pantaleone and Tye[80] obtained negative values for c. None of the calculations could accurately fit the value of b for the 1P($c\bar{c}$) state. Also, the value of c(2S) is generally lower than the experimental results. We have not considered any $\delta^3(r)$ term in the spin-spin potential. Our values for a are slightly higher than the experimental results. It is possible that the $\ln\Lambda_{\overline{MS}}\, r$ term in the confining potential cannot describe accurately the necessary modification in the potential in the distance scales scanned by 1P or 2P $b\bar{b}$ states.

Table 3.2. Expectation values of the spin-orbit (a), tensor (b) and spin-spin (c) potentials obtained in different models.

State	Values of	Experimental values in MeV	Pantaleone *et al.*[47,80] (MeV)	Igi and Ono[50,79] with $\Lambda_{\overline{MS}}$ = 200 MeV (MeV)	Ours Eqs. (3.13) -(3.16) (MeV)
$b\bar{b}(1S)$	c		35.0	39.5	58.83[b]
$b\bar{b}(2S)$	c		19.0	21.5	30.8[b]
	a	14.1 ± 0.4	13.0	14.3	19.07
$b\bar{b}(1P)$	b	12.0 ± 0.9	9.0	11.76	12.86[b]
	c	5.4 ± 2.2	-0.4,-0.5[a]	-0.772	1.74
	a	10.3 ± 1.2	9.3	9.38	13.88
$b\bar{b}(2P)$	b	9.8 ± 1.4	6.7	7.92	9.98
	c		-0.3,-0.4[a]	-0.542	-0.88
$c\bar{c}(1S)$	c	116 ± 5	101.0	140	122.69[b]
$c\bar{c}(2S)$	c	92 ± 5	69.0	84	34.78[b]
	a	34.9 ± 0.3	48.0	40.5	33.22
$c\bar{c}(1P)$	b	40.1 ± 0.8	46.0	43.6	30.97
	c	0 ± 0.9	-3.6,-1.4[a]	-3.49	0.98

[a] New values obtained.[80]

[b] No $\delta^3(r)$ term considered for evaluating c.

(4) Table 3.3 and Table 3.4 show the fine-hyperfine structure of $b\bar{b}$ and $c\bar{c}$ systems. We have compared our results with recent calculations by Gupta, Radford and Suchyta III[88] (GRS), Schmitz, Beavis and Kaus[89] (SBK), Fulcher[51] (FUL) and also the experimental values[28]. However, GRS use a non-singular potential by considering different values of the scalar-vector mixing parameter for different flavours which need a justification. The potential chosen by SBK includes a running coupling constant $\alpha(r)$ with some modification to avoid a singularity. Fulcher's work is based on a one-loop perturbative potential supplemented by a linear confining potential and a long-range spin-orbit potential. The pseudoscalar partner η_c of ψ meson in our case is very close to the experimental value whereas the $^1S_0$ partner of $\psi'(\eta_c')$ is too high. In the Y system, the predicted centre of gravity value of 1P state is too low if we fix the value of 1^3S_1 energy level at 9460 MeV. SBK also found similar differences but the results of FUL agree with the experiments. However, our result for the COG of 2P states is very close to the experimental value and it almost agrees with FUL's result whereas in SBK's calculation, the COG is shifted upward by approximately 20 MeV. In case of ψ mesons, the spin-averaged χ-level predicted by us is about 20 MeV below its observed value. The energy levels of D-state of $b\bar{b}$ and $c\bar{c}$ systems are listed in Table 3.5. Here, our results are compared with the calculated results of FUL[51] and also of Sebastian *et al.*[90] (SGZ) who used the potential proposed by Gupta, Radford and Repko. In the Y system, almost all the potential models predict nearly the

Table 3.3. Energy level splittings for the Υ system.

State	Experimental value (MeV)	GRS[88] (MeV)	FUL[51] (MeV)	SBK[89] (MeV)	OURS (MeV)
$1^3S_1(\Upsilon)$	9460.32 ± 0.22	9460.0	9460	9461	9460.21
$1^1S_0(\eta_b)$		9412.2	9420		9401.38
$2^3S_1(\Upsilon')$	10023.29 ± 0.31	10015.8	10006	10023	10037.70
$2^1S_0(\eta_b')$		9992.5	9983		10006.90
$3^3S_1(\Upsilon'')$	10355.3 ± 0.5	10357.8	10355		10362.95
$3^1S_0(\eta_b'')$		10339.7	10336		10343.14
$4^3S_1(\Upsilon''')$	10580 ± 3.5				10596.51
$4^1S_0(\eta_b''')$					10582.46
$1^3P_2(\chi_b^2)$	9913.2 ± 0.6	9913.9	9908	9908	9904.02
$1^3P_1(\chi_b^1)$	9891.9 ± 0.7	9893.3	9895	9880	9873.60
$1^3P_0(\chi_b^0)$	9859.8 ± 1.3	9861.8	9874	9849	9835.24
$1^1P_1(h_b)$	9894.8 ± 1.5	9900.1	9901		9884.50
$2^3P_2(\chi_b'^2)$	10269.0 ± 0.7	10269.8	10268	10292	10269.66
$2^3P_1(\chi_b'^1)$	10255.2 ± 0.4	10253.7	10256	10270	10247.89
$2^3P_0(\chi_b'^0)$	10235.3 ± 1.1	10228.8	10239	10245	10219.04
$2^1P_1(h_b')$		10259.0	10262		10257.66

Table 3.4 Energy level splittings for the ψ system.

State	Experimental value (MeV)	GRS[88] (MeV)	SBK[89] (MeV)	OURS (MeV)
$1^3S_1(\psi)$	3096.93 ± 0.09	3096.9	3107	3099.37
$1^1S_0(\eta_c)$	2979.6 $^{+1.7}_{-1.6}$	2981.1	2982	2976.68
$2^3S_1(\psi')$	3686.0 ± 0.1	3689.7	3692	3671.50
$2^1S_0(\eta_c')$	3594.0 ± 5.0	3619.1		3636.72
$1^3P_2(\chi_c^2)$	3556.3 ± 0.4	3553.5	3561	3535.57
$1^3P_1(\chi_c^1)$	3510.6 ± 0.5	3507.0	3490	3487.71
$1^3P_0(\chi_c^0)$	3415.1 ± 1.0	3412.2	3412	3408.04
$1^1P_1(h_c)$	3525.4 ± 0.8[a]	3518.5		3504.47

[a]Baglin et al. (Ref.27).

Table 3.5. Predicted D-state energy level splittings for Y and ψ systems in MeV.

State	$b\bar{b}$ system			$c\bar{c}$ system	
	FUL[51]	SGZ[90]	OURS	SGZ[90]	OURS
1^3D_3	10162	10153	10163.3	3830	3783.9
1^3D_2	10160	10149	10152.4	3822	3782.6
1^3D_1	10155	10143	10141.5	3801	3782.1
1^1D_2	10160	10150	10153.8	3822	3800.0
2^3D_3		10451	10436.6		
2^3D_2		10446	10432.0		
2^3D_1		10440	10425.2		
2^1D_2		10447	10433.6		

same spin-averaged D-state B.E. to within ± 10 MeV. For the 1D level of $c\bar{c}$, the experimental value of 1^3D_1 state is already known, i.e. $M(^3D_1)$ = 3769.9 ± 2.5 MeV. From the Table 3.5, we see that our results are marginally better than SGZ's results, but leaves scope for improvement. Moreover, using the theoretical value of the radiated photon energy, we have studied the E1 transition rates for both $b\bar{b}$ and $c\bar{c}$ systems. These are listed in Table 3.6 and Table 3.7.

(5) While we are convinced of the need for a modification of the naive BF interaction potentials, the question whether this could come as a contribution from a possible pseudoscalar exchange potential[35,42] should also be examined. The pseudoscalar exchange makes no contribution in the non-relativistic limit but contributes the following $(v/c)^2$ correction to the fine-hyperfine interaction :

$$H_{PS} = \frac{1}{3M_Q^2} S_1 . S_2 \nabla^2 V_{PS}(r) + \frac{1}{3M_Q^2} \left(d^2V_{PS}/dr^2 - \frac{dV_{PS}/dr}{r} \right) S_{12} . \qquad (3.17)$$

We choose a fairly general $V_{PS}(r)$, i.e. ,

$$V_{PS}(r) = E/r + F \ln\Lambda_{\overline{MS}}\, r + Gr + Hr^2 \quad , \qquad (3.18)$$

and try for a fit with the experimental results for b and c. Note that r^2 term does not contribute to b and the $1/r$ term contributes only a $\delta^3(r)$ term to c, which we neglect for the time being. The results, as shown in Table 3.1 disagree with the experimental

Table 3.6. E1 transition rates for Y states.

Transition	Experimental ω (MeV)	Calculated ω (MeV)	Γ_{E1} (Expt.) (KeV)	Γ_{E1} (Theo.) (KeV)
$2^3S_1 \rightarrow 1^3P_0$	162.3 ± 1.3	200.42	1.29 ± 0.31	2.40
1^3P_1	130.7 ± 0.7	162.76	2.01 ± 0.49	3.86
1^3P_2	109.5 ± 0.6	132.79	1.98 ± 0.48	3.50
$3^3S_1 \rightarrow 1^3P_0$	483.8 ± 1.4	512.27		0.01
1^3P_1	453.2 ± 0.9	477.80	0.04 ± 0.03	0.03
1^3P_2	432.8 ± 0.8	448.77	0.06 ± 0.05	0.04
$3^3S_1 \rightarrow 2^3P_0$	119.3 ± 1.1	142.91	1.22 ± 0.3	2.54
2^3P_1	99.6 ± 0.4	114.42	3.08 ± 0.6	3.90
2^3P_2	85.9 ± 0.7	92.87	3.26 ± 0.7	3.48
$1^3P_0 \rightarrow 1^3S_1$	391.7 ± 1.3	367.88		23.13
1^3P_1	422.5 ± 0.7	404.73		30.80
1^3P_2	442.9 ± 0.6	433.87		37.94
$2^3P_0 \rightarrow 1^3S_1$	741.5 ± 2.3	730.66		6.45
2^3P_1	764.8 ± 0.8	757.41		7.19
2^3P_2	776.8 ± 0.7	777.55		7.78
$2^3P_0 \rightarrow 2^3S_1$	205.0 ± 2.3	179.73		8.35
2^3P_1	229.7 ± 0.9	208.03		12.95
2^3P_2	242.3 ± 0.8	229.34		17.35

Table 3.7. E1 transition rates for ψ states.

Transition	Experimental ω (MeV)	Calculated ω (MeV)	Γ_{E1} (Theo.) (KeV)
$2^3S_1 \rightarrow 1^3P_0$	261.0	254.0	64.36
1^3P_1	171.8	179.19	67.79
1^3P_2	127.7	133.41	46.63
$1^3P_0 \rightarrow 1^3S_1$	303.2	294.69	170.08
1^3P_1	388.7	366.72	329.69
1^3P_2	429.4	409.29	458.35
$3^3S_1 \rightarrow 1^3P_0$		558.98	0.36
1^3P_1		490.52	0.72
1^3P_2		448.64	0.92
$3^3S_1 \rightarrow 2^3P_0$		149.55	37.48
2^3P_1		111.94	47.16
2^3P_2		118.48	93.20
$2^3P_0 \rightarrow 1^3S_1$		685.78	64.11
2^3P_1		717.64	73.47
2^3P_2		712.15	71.79
$2^3P_0 \rightarrow 2^3S_1$		184.01	124.50
2^3P_1		220.88	215.34
2^3P_2		214.52	197.27

results. The parameters have been chosen as E = 1.297, F = 3.2 GeV, G = -2.052 GeV^2 and H = 0.1636 GeV^3. We note, in particular, that (1) pseudoscalar exchange term does not modify the spin-orbit interaction and hence the value of a remains unacceptably low, (2) c for $b\bar{b}(1P)$ is still negative and (3) c for 1S and 2S states can be altered by including a term $k\delta^3(r)S_1.S_2$ with k = 0.073 GeV^{-2}, giving for the $c\bar{c}$ system, the values $c(1S)$ = 116 MeV and $c(2S)$ = 117.6 MeV, the latter being about 25 MeV larger than the experimental value. Our conclusion is that the addition of a pseudoscalar exchange potential alone is not sufficient.

III.5. Conclusions :

We have shown that the naive Breit-Fermi interactions (Eqs.3.3-3.6) are not consistent with the fine-hyperfine splittings of heavy quarkonia. The formal relationships may perhaps be maintained if the vector and scalar potentials are modified at least in the intermediate range of r values. The modification cannot be described by a pseudoscalar exchange potential. It appears that the measurement of the splittings of a few more levels should provide sufficient additional constraints and help in deciding if the Breit-Fermi form of interaction is at all valid for heavy quarkonia.