Chapter 10

Concluding Remarks

In this chapter we would like to summarize our results and point out areas for future work. We have shown many physical applications of a class of solutions obtained by Mukherjee *et al* [67] based on an ansatz made by Vaidya and Tikekar [63] for the geometry of a static spherically symmetric star. Within the framework of Vaidya-Tikekar model [63], we have studied various possibilities of cold compact stars, namely, (i) charged stars, (ii) quark stars (strange stars), (iii) combined boson-fermion stars (quark-diquark stars) and (iv) stars with a phase transition. Our results may be outlined as follows:

- We have shown that the solution is very useful in describing ultra-compact objects. We have developed a direct method to reconstruct super nuclear equations of state; e.g., strange matter EOS and quark-diquark EOS, by considering only two values of the observables of a star, namely mass and radius, and a parameter λ. The parameter λ appears from the geometry of the star and for a given mass and radius, characterizes the EOS of the star. To demonstrate the physical applicability of the solution, we have considered a couple of interesting cases. We have shown that if SAX J1808.4-3658 is a strange star, the relevant EOS obtained by Dey *et al* [42] can be reconstructed from the solution given in [67] for the masses and radii given in [43]. Similarly, if Her X-1 is a quark-diquark star as suggested by Horvath and Pacheco [72], the relevant EOS can also be obtained in this model. Although a realistic description of a star, a very complex system, is not expected by this simple description, we have been able to capture the gross features of the composition of the star. This will provide a useful first step in our search for a complete understanding of the composition of very compact stars and of the physical processes that lead to the observed compactness of these objects.

- We have verified the stability of the relevant stellar configurations obtained by using this model. Earlier, the stability of Vaidya-Tikekar [63] model were analysed for some restricted values of the parameter λ, namely, $\lambda = 2$ [156] & 7 [157], because of the non-availability of solutions for other values of λ. But, using the general solution obtained by Mukherjee *et al* [67], the stability of the model can be verified for different choices of the parameter λ. We have verified the stability of the model for large values of the parameter λ, namely, $\lambda = 53.34,\ 100\ \&\ 230.58$, which were found to be relevant for the description of compact objects in our study.

- We have observed that the Vaidya-Tikekar [63] model has a scaling property which allows the solution to describe a family of stars of equal compactness. Following the observations made by Witten [25] and Zdunik [75] the scaling property in compact stars like strange stars might be thought of being associated with the equation of state and its linearity. But we have shown that a cold compact star should have a scaling property, whenever one can identify a parameter with the dimension of a length in the EOS. The result does not depend on the specific form of the equation of state. It is rather a general feature of spherical distribution that is in hydrostatic equilibrium. We have shown that Vaidya-Tikekar [63] model can be applied to describe strange stars as well as stars whose interior might have other exotic components. Moreover, although for a large value of the parameter λ the EOS becomes almost linear in this model, same is not true for a smaller value of λ. Scaling property, however, applies to all the cases.

- We have extended the solution to the case of a static charged spherical distribution of matter and obtained a new class of solutions for such bodies. Physical properties of such charged bodies have been analysed. The solution may be useful, in core-envelope model, in describing the outer envelope of a star which may become charged due to accretion [69], [70].

- The model has been used to describe stars having two layers of different material composition, say, a deconfined quark core surrounded by a comparatively less dense baryonic envelope. This has been done by choosing solutions for two different values of the parameter λ for the two layers separated by a surface where a phase transition may take place and satisfying necessary boundary conditions across the surface. This model can be used in the case of compact stars where phase transition is a possibility.

- The model has also been utilised to study the late evolutionary stages of a collapsing star, becoming ultimately a cold compact star.

Although, we have demonstrated some applications of the solution of Mukherjee *et al* [67], the applicability of these class of solutions may be much wider. The solution, being simple and analytic, can provide a description of the gross features of cold compact stars. One will, no doubt, need to consider a more complex description for a realistic star. We would like to point out here some areas where the model may have further applications as well as areas where the model can be improved upon.

- In some cases, pressure anisotropy may be a feature to be considered [165]. However, the anisotropy can be studied by generalizing the model and at present, we are looking into this possibility.

- In our study, we have neglected the rotational effects on the gross observational properties of compact objects. It has been pointed out by Li *et al* [40] (in case of Her X-1), and by de Sousa and Silveira [90] in case of combined boson fermion stars, that the effects are small. However, for a more realistic description, we should extend the model to the case of a rotating star and investigate the effects of rotation on the physical properties of the star.

- Although considering a strange matter EOS under the MIT bag model, Phukon [91] showed that gross features of the star remain almost unaltered for a magnetic field of strength upto 10^{18} G, it will be interesting to investigate the deformation in the metric coefficients due to the presence of a strong magnetic field in this model.

- Finally, since the model can describe some of the general features of interesting X-ray pulsars like Her X-1, in terms of analytic functions, it may be useful in explaining some special features like quasi periodic oscillations, bursts etc., by a simple perturbative approach.

To summarize, based on available data on pulsars, we have been able to study various properties of compact objects, making use of a simple model. Knowing fully well that the model is too simple, we have emphasized on what can be achieved from the model, rather than what we cannot know from the model. We hope that this simple analytic model will be able to provide some guidance to more realistic and detailed numerical study to understand present and future observational data on compact objects.