P R E F A C E

This thesis which contains five chapters is concerned with some linear and non-linear problems of thin elastic plates placed on elastic foundation.

The first chapter contains one linear problem on dynamical response of an orthotropic infinite plate placed on elastic foundation and subjected to time dependent concentrated force. The first work on the problem of foundation characterization was done by Winkler (1867) who assumed the foundation to act like a fluid, the reaction at any point being directly proportional to the deflection at that point. But in the case of an elastic continuum such a hypothesis approximates but crudely the actual behaviour of the continuum. Subsequently, many investigators such as Vlasov (1949), Pasternak (1954), Soklnikoff (1956), Kerr (1964), Hermann (1967), Fletcher (1971) and others have proposed more complicated foundation models in attempts to provide a more realistic representation of real materials. These investigators considered static loading and observed that Winkler type of foundation could be adjusted to give a reasonable estimate of the maximum deflection under the load point, but the shape of the overall deflection curve could not be well represented. In the paper of this chapter dynamic loading is considered with the foundation having the properties of a semi-infinite elastic body and a comparative study has been made with Winkler type of foundation. It is observed that Winkler type of foundation gives a reasonable estimate of maximum deflection under the load point in the case of dynamic loading too. Hence for all the problems in the subsequent chapters, analysis has been made for foundations of Winkler type.

The second chapter contains three problems on elastic stability of thin plates of various shapes and under different types of loading. Stability of thin elastic plates having complicated shapes can be investigated with the help of various approximate numerical techniques such as finite difference method, finite element method etc. For such numerical techniques reference may be made to the works of Salvadori (1957), Weingarten (1957), Bradley (1963 But these numerical methods are time consuming. On the other hand, elastic stability problems for complicated shapes can easily be investigated with less labour following complex variable method given by Laura and Shahady (1969) and the results obtained by different numerical techniques applied to boundary value problems involving irregular shapes can also be verified by this method.

The first paper of this chapter deals with the critical buckling conditions of a thin elastic equilateral triangular plate placed on elastic foundation with clamped edges, and under the action of uniform compression parallel to one of the edges. The same problem without the elastic foundation was investigated by Bradley (1963) by finite difference method. In the present paper, the solution is obtained by the use of conformal mapping and Galerkin's procedure. In the second paper of this chapter critical buckling conditions of a non-homogeneous simply supported rectangular plate under the action of combined bending and compression and placed on an elastic foundation have been investigated with the help of error function.

For the same problem of homogeneous rectangular plates without elastic foundation reference may be made to the works of Johnson and Noel (1953) and Timoshenko and Gere (1961). The third paper of this chapter is devoted to thermal buckling of thin plates placed on elastic foundation. Two cases are considered : a heated equilateral triangular plate and an elliptic plate. Assuming the temperature distribution to be a function of the plate thickness, the solution is obtained by complex variable theory. For thermal buckling of circular and rectangular plates reference may be made to the works of Klosner and Forray (1958), Nowacki (1962) and Mansfield (1964).

The third chapter contains six non-linear problems of thin plates of various shapes and under different types of loading. In solving these large deflection problems, the approximate method proposed by Berger (1955) has been followed. Berger's method is based on the neglect of the second strain invariant of the middle surface strains in the expression corresponding to the total potential energy of the system. Although no complete explanation of this method is given, the results obtained so far for different problems are in good agreement with those obtained from practical analysis. Following this approximate method, many non-linear plate problems have been solved with remarkable ease by many investigators such as Iwinski and Nowinski (1957), Nowinski (1958), Nash and Modeer (1959), Basuli (1961), Sinha (1963), Banerjee (1967) and others. It is to be noted that Berger's method is still an intriguing subject. (e.g. a recent note in Jour. of Appl. Mech., June, 1974, P. 521).

The first paper of this chapter is devoted to the large deflection of a uniformly loaded circular plate placed on elastic foundation and supported at several points along the boundary. The problems of circular and rectangular plates without any edge discontinuity were investigated by Sinha (1963). In the present paper the solution is obtained in terms of an infinite series involving Bessel's functions. A particular case, where the number of supports is two, has been treated fully. The second paper deals with the large deflection of a clamped circular plate on elastic foundation under a concentrated load at the centre of the plate. The corresponding problem without any elastic foundation was treated by Basuli (1961) and the problem of an orthotropic plate was investigated by Banerjee (1967). In the second paper, the deflections are obtained involving Bessel's functions and the theoretical results have been verified experimentally. The third paper is devoted to the large deflections of a clamped circular plate on elastic foundation and under symmetrical loads. The corresponding problem without any elastic foundation was investigated by Banerjee (1967). The fourth paper deals with the large deflections of a simply supported semi-circular plate placed on elastic foundation and subjected to a uniform load. The deflections have been obtained in terms of an infinite series involving Bessel's functions and Lommel's functions. The fifth paper of this chapter is concerned with the large deflections of a simply supported equilateral triangular orthotropic plate on elastic foundation and under uniform load. Trilinear co-ordinates as shown by Sen, B. (1968) has been used in solution of this problem.

The sixth paper of this chapter deals with the large deflection of a heated clamped elliptic plate placed on elastic foundation. Berger's technique of neglecting the second invariant of the middle surface strains has been extended by Basuli (1968) to the large deflection problems of heated rectangular, circular and right-angled triangular plates without any elastic foundation and under uniform pressure and stationary temperature distribution. Assuming stationary temperature distribution the deflection in the present paper is obtained in terms of Mathieu function of the first kind and of zero order.

The fourth chapter contains only one non-linear problem on large deflections of clamped elliptic plates exhibiting rectilinear orthotropy. The plate is placed on an elastic foundation of the Winkler type and is subjected to uniform loading. The investigation is based on von Karman's (1910) differential equations extended to rectilinear orthotropy. Following von Karman's equations Weil and Newmark (1956) and Nash and Cooley (1959) investigated by numerical methods the large deflections of isotropic elliptic plates under uniform load and without elastic foundation. In the present paper the stress function is found from the compatibility equation and the final solution is obtained by Galerkin's method. Results for isotropic elliptic plates and circular plates have been deduced from those obtained for orthotropic elliptic plates. These results are in excellent agreement with the known results for isotropic plates. For circular plates without elastic foundation reference may be made to the works of Way (1934) and Schmidt (1968). For circular plates with foundation reference may be made to the works of Bolton (1972).

The fifth and the last chapter of this thesis contains only one problem on large amplitude free vibrations of irregular plates placed on elastic foundation. Following Berger's equations the approximate values of the lower natural frequency for circular, square, and cornered plates with simply supported and clamped edges are obtained by the conformal mapping technique. Theoretical results thus obtained have been verified experimentally. The same problem without elastic foundation was treated by Nash and Modeer (1960) and Chu and Herrmann (1956) who followed Berger's technique and von Karman's equations respectively. For non-linear vibration problems of different elastic plates reference may be made to the works of Nowinski (1963) who followed von Karman's field equations to obtain his results.