CHAPTER - I

A COMPREHENSIVE STUDY OF THE EARLY RESEARCHES IN THE AREA UNDER INVESTIGATION

With the advent of modern civilisation the applications of elastic and plastic properties of solids are gaining momentum day- by-day because of their manifold practical uses in engineering design and in modern technology. Attempts have been made by numerous research-ers to study the elastic behaviour of matter. Robert Hook (1635-1710) gave a simple but highly restricted relation between stress and strain of the elastic substance. But the extensive study in this field was made in the tweentyeth century. Linear analysis of the problems relating to static and dynamic behaviour of elastic plates and shells has long attracted attention, resulting in a wealth of papers published by numerous authors. An extensive bibliography on the subject has been presented by Gontkevich [1]. Moreover, Leissa [2] in his monogram has reviewed a major part of the works done on vibration of plates. Unfortunately, the linear classical theory is no longer applicable in most of the cases of practical interest and this lead to the non-linear analysis of problems. Considerable interest has been shown in the past in the analysis of the vibrations of plates and shells associated with their symmetrical and unsymmetrical bending characters. The interest had been engendered by the widespread use of such plates and shells in engineering design. Symmetrical bending of circular and rectangular plates of variable thickness have been investigated by different authors. Outstanding research workers in this field are H. Holzer [3], R. G. Olson [4] and H. D. Conway [5]. Investgations of deflections of plates resting on elastic foundations with various boundary conditions have been used by different authors including V. Lewe [6], H. M. Westgaard [7], H. J. Fletcher and C. J. Therne [8]. S. Dutta [9] analyzed large deflection of a clamped circular plate on elastic foundation under non-uniform but symmetrical loads. The solutions are given in the form of an infinite series involving Bessel's functions.

There are situations, e.g., use of soft filaments in aerospace structures, building activities in cold regions, foundations of heavy duty machines, underwater and embedded structures, etc., in which the effect of the supporting medium has to be considered for adequate analysis. There is a recent study by Nath and Jain /172/ in which an axisymmetric shallow spherical shell supported on an elastic foundation and undergoing moderately large deformations has been analyzed by using Chebyshev series and the Houbolt technique /173/. Later on Y. Nath, O. Mahrenholtz and K. K. Varma /136/ investigated the moderately large dynamic response of a doubly curved shallow spherical shell of rectangular plan form, supported on a two parameter elastic subgrade and subjected to uniformly distributed step and sinusoidal loadings. Von Karman-Donnel type non-linear partial differential equations of motion are employed and solved by using finite difference and Houbolt time marching techniques. Two boundary conditions are considered. The influence of stiffnesses and mass of the elastic subgrade on the moderately large amplitude of response of the shell has been investigated.

Some investigations considering thermal effect on plates and shells resting on elastic foundation have been made by W.Nowachi /10/ and M. C. Pal /11/.

Outstanding research workers in this field are J.L. Nowinski [12], W.A.Nash and I.D.Cooley [13], S.Way [14]. The symmetrical and unsymmetrical bending properties of plates and shells and their dynamic characteristics pose great difficulties in solving the particular problem of practical interest due to their highly non-linear behaviours. Attempts have been made by a number of workers to solve the necessary differential equations linearizing those through proper approximations. Such an approach provides exact solutions of the problems still the results so obtained do not claim non-trivially exact for truely complicated plate geometry which are actually used in practice. Approximate solutions of such problems may be determined from the two von karman [15] field equations. As these equations involve the deflection and membrane stress function as two dependent variables coupled together, the solutions of complex problems require considerable computations . Many works have been done using karman equations among which the works of Chu and Herrmann [16] and Yamaki [17] need special mention. Several techniques have been used to solve them; For example, S.Levy [18] substituted a double Fourier series solutions in the equations for rectangular plates and evaluated the coefficients . Chi-Teh-Wang [19] wrote the equations for rectangular plates in a finite difference form and solved them by the method of successive approximations. S.Way solved the circular plate equations by substituting a power series solution into the energy expression determining the coefficients by setting the first variation of the strain- energy equal to the first variation of the potential energy due to the external loading for any variation of each coefficient. In fact, it is very difficult, though not impossible, to obtain an appropriate approximate solution for a truely complicated problem.

In 1955 H.M.Berger [20] offered an alternative method or an approach, which may be applied to solve more complicated problems. Of all the methods or approaches for solving the non-linear problems Berger's approach is relatively simple and accurate to some extent. In this approximate method Berger assumed that the second strain invariant in the expression of the total potential energy of the system has been neglected in deriving a simple fourth order order differential equation from two coupled fourth order differential equations for deflections. Although no complete explanation of such assumption has been drawn still the method yields solutions which are in excellent agreement with the experimental results. Berger's equations are practically quasi non-linear. Berger's technique of neglecting the second strain in-variant has been successfully applied by many workers and obtained satisfactory results. Applying Berger's method Iwinski and Nowinski [21] solved problems of orthotropic plates. B. Banerjee, S.N. Sinharay, G. C. Sinharay and many other researchers published a series of papers [22 - 31] using Berger's technique.

P. Biswas [32 - 33] applied the same technique to solve problems on elastic plates and shells under a temperature distribution. Later on, Nash and Modeer [34] extended Berger's method to a

dynamic case and it has subsequently been used by the others [35 -37].
M. M. Banerjee with his co-workers published a large number of papers [38 - 45] based on berger's hypothesis. Most of their works are mainly related to problems concerning the variation of thickness of plates and shells.

Neglecting in-plane inertia, Nash and Modeer [46] and Chen [47] showed that the use of such equations for simply-supported plates yields results which are in excellent agreement with those obtained from karman equations [16].

The non-linear vibration of plates neglecting transverse shear deformations and rotatory inertia have been studied and their effects were included in the studies of Wu and Vinson [48], Singh, Das and Sundararanjan [49], Kanaka Raju and Venkateswara Rao [50], Kozlemyakina and Morgalvski [51], Sathyamoorthy [52].

Yu and Lai [53] determined the influence of transverse shear on the non-linear vibration of sandwich plates. Thickness shear flexibility was included in the analysis of Ambartsumyan [54] , Wu and Vinson

[55], Singh, Das and Sundararanjan [56]. Non-uniform plates were analyzed by Huang and Meng [57], Huang [58], Ramachandran [59], Ramachandran and Reddy [60], Huang, Woo, Walker and Kanakaraju [61-62].

Ramaiah and Kumar made a thorough study of annular plates [63]. The Ritz method was used with algebraic polynomial deflection functions to obtain frequency parameter for all the nine combinations of simple boundary conditions for various ratios of flexural rigidities and of boundary radii (b/a). Simple approximate formulas expressed the orthotropic frequencies in terms of flexural rigidity ratios and the frequencies of corresponding modes in the axisymmetric case. A simplified method based on the assumption that the radial bending moment is small at a nodal circle was shown to be especially useful for estimating frequencies of modes having a large number of nodal circles.

Orthotropic circular plates having concentric isotropic cores have been analyzed by Woo, Krimser and Huang [64] and Rao and Ganapathi

/65/. Axisymmetric frequencies were given for cases having clamped and simply-supported boundaries. Annular corrugated disks have been represented by orthotropic plates for the theoretical analysis of the case in which the outside boundary is free and the inside one clamped. Rubin /66/ used the frobenius method to study annular sector plates with radial edges simply-supported. Vibrations of circular polar orthotropic plates have also been studied by Padovan, Lestngi /67/, Imer & Zimmermann /68/ and Pardoen /69/. The case of a material with principal axes of orthotropy that are straight but not orthogonal has been investigated by Nair and Durvasula /70/. They used the Ritz method and presented extensive numerical results for square and skew plates having various combinations of boundary conditions. Orthotropic rhombic plates have also beem studied by Srinivasan and Ramachandran/71/.

Dickinson/72/ analyzed the orthotropic, clamped square plate subjected to hydrostatic loading by means of Bolotin's method. Plates having two parallel edges clamped and the other two free analyzed.

The effects of residual stresses upon frequencies of a free plate with a weld longitudinally along its centre have been examined by R.F.D.Porter Goff /73/. Extensive numerical results for simply-supported plates having different rotational springs along various edges and subjected to bi-axial in-plane loads have been obtained by Laura & Romanelli /74/. Jones & Mazumder /75/ addressed the problem of hydrostatically loaded, elliptic plate having clamped or simply-supported edges. Numerical results were presented for a wide range of loading parameters and for various aspect ratios. Natural frequencies of plates having elliptical holes have been studied by P.K.Dutta & R.L.Carlson /76/. Rhombic plate problems have been studied by Srinivasan and Ramachandran /77/; trapezoidal plates have been studied by Greentham & Bailey /78/. Triangular and quadrilateral plates have been analyzed by C.W.Bert, Banerjee and others/79-81/. Vibrations of thick, solid, circular plates have been studied by Chandrasekharan and Kunukkasseril /82-83/, Y.K.Cheung /84/ and Soni & Amba Rao /85/.

In the case of von Karman equations investigators followed two alternatives; a small group used Chu and Herrman approach while the other group worked with Timoshenko /86/ approach.

Crawford and Atluri /87/ examined the effect of initial stresses on the non-linear vibration of simply-supported rectangular plates considering in-plane inertia negligible. Errigen /88/ analyzed the non-linear axisymmetric vibrations of circular membrane using the initial membrane strain as a perturbation parameter and in the year 1955 he examined the non-linear oscillations of viscoelastic plates with heriditary damping included in the stress- strain relations. A number of investigators analyzed the non-linear oscillations of anisotropic plates. Yu /89/, Yu and Lai /90/, Shahin /91/ studied the non-linear vibrations of sandwich plates, while Alwar and Adimurthy /92/ studied the response of sandwich panels to pulse excitations. Yu /93/ investigated the non-linear vibration of layered plates and shells.

Hassert and Nowinski /94/, Wu and Vinson /95/, Sathyamoorthy and

Pandalai[96] and Ramachandran [97] treated a rectangular plate with special reference to rectangular orthotropy. J.N.Reddy [98-100], Reddy & Liu [101], Reddy & Bert [102], Reddy & N.D.PHAN [103],Reddy &D. Frederick & others [104] published a series of papers dealing with various orthotropic laminated, cross-ply and composite plates. Nowinski and Paul [105] analyzed orthotropic circular plates. Nowinski and Ismail [106] and Vendhan and Dhoopar [107] analyzed orthotropic triangular plates; while Sathyamoorthy and Pandalai analyzed rectilinearly orthotropic skew plates. Huang and Woo [108] used a Ritz-Kantorovich method to analyze cylindrically anisotropic circular and annular plates, while Venkateswara Rao, Kanaka Raju and Raju used a finite element methods to analyze orthotropic circular plates. Mayberry and Bert [109] experimentally investigated the vibrations of various laminated anisotropic rectangular plates and compared their results with an orthotropic plate analysis.

Wu and Vinson extended[55] this analysis by including the shear flexibility. Bennett [110], Chandra and Basava-Raju [111] investigated respectively, the forced and free vibrations of angle-ply laminated rectangular plates, while Chandra and Basava-Raju /112/ investigated free vibrations of cross-ply laminated rectangular plates. Bennett, Bert and Schimidt investigated the non-linear oscillations of an arbitrary laminated rectangular plates. Yen and Lee /113/used the Lindstedt-Poincare technique to determine the non-linear vibrations of a circular membrane including the effects of the longitudinal inertia. They found that their solutions invalid when the linear fundamental in-plane frequency is twice the fundamental transverse frequency; this occurs when the initial strain is about 0.4882. Then they obtained an expansion valid for this internal resonance case; however, they did not indicate what material these results can be applied to. We note that for most metals the initial strain can not be made to exceed about 0.05. Chobotov and Binder /114/ used a combination of a Ritz- Galerkin procedure and a perturbation technique to determine the effect of sinusoidal excitations on the non-linear response of circular membranes.

A number of investigators studied the dynamics of spinning disks because of their use as saw- blades and turbine wheels. The impotance of flexural waves in causing spinning disk failures was recognized a long time ago by Richards(1872). Besides them a large number of workers namely Cambell, Von Freudeudrick, Tobias,Arnold, Willam, Krauter, Bulkely, Efstathiades, Mc Elman Advani, Bhattacherjee studied the various aspects of spinning disks and obtained satisfactory results. Nowinski and Woodall /115/ studied the non-linear vibrations of spinning disks.

In 1961 (oct.) H.N.Chu published a paper on the influence of large amplitude on flexural vibrations of a thin circular cylindrical shell in the journal of Aerospace sciences. In 1962, J.Nowinski submitted an article on transverse non-linear vibrations of cylindrical orthotropic shells. In these papers on Nowinski and Chu the derivation of field equations were same but those are obtained independently. The only exception to note here that the works of Chu is a special case of that of Nowinski. Chu deals with isotropic material whereas Nowinski deals with orthotropic materials. The results obtained by these authors for isotropic case are in close agreement with each other. Lateron, Nowinski (1963) revised his paper starting with derivation of the general field equations using in contrast to the process followed by Chu where those equations are derived as, Euler-Lagrange equations from Hamilton's principle, the balance of momenta and the compatibility condition. Actually, Chu (1961) derived the counterpart of the works of Crawford and Atluri for circular cylindrical shells.

Cylindrical shells were studied experimentally by Olson [116], Matsuzaki and Kobayashi [117] and theoretically by Chu, Evensen, Nowinski, Goodier, Mclvor, Bieniek, Fan Lackman, Evensen and Fulton, Mayers and Wrenn, Mclvor and Lovell and others [118-123]. Nowinski studied the non-linear transverse vibrations of orthotropic cylindrical shells. Nowinski [124] also studied the response of a cylindrical shell to tranverse non-linear oscillations. Nash and Modeer [125] studied elaborately the theory of thin elastic shells.

M.M.Banerjee [126] analyzed the non-linear free vibrations of shallow shells cylindrical in shape at an elevated temperature. Mazumder.J. published a series of papers [127-132] with his co-workers on the non-linear analysis ofplates and shallow shells on various planforms utilizing different techniques among which the application of the method of constant deflection contour lines is notable. These worke drew the attention of the present author to study the constant deflection contour lines method in the analysis of plate and shell structures with more complicated problems.

Leissa and Kadi [133] enlighted the curvature effects on shallow shell vibrations. Jones and Mazumder [134] studied the transverse vibrations of shallow shells by the method of constant deflection contour lines. Sinharay and Banerjee [135] studied the large amplitude free vibrations of a shallow shell and analyzed the spherical and cylindrical shells after modifying Berger's equations. Timoshenko and Woinowinsky Krieger [136] studied various aspect of the theory of plates and shells. Banerjee M.M., Biswas.P., Sikder

S [137-139] studied the temperature effect on the dynamic response of spherical shells. One should also note the analysis of different shell structures investigated by different authors. Of such studies mention should be made of studies by Mclvor and Sonstegard [140], Grossman, Koplik and Yu [141] on spherical shell. Hemispherical shells were studied by Jordan [142], while conical shells were studied by Sun and Lee [143], square shells were studied by Chanban and Ashwell [144], axisymmetric shells and solids were studied by Nagaranjan & Popov [145]. Shells of revolution were studied by Stricklin, Martinej, Tillerson, Heng and Haisler [146]. Lehar, Batterman, Belytschko, Hsieh, Kankaraju and Venkateswara Rao [147-150] studied various shells structures. Shallow shells were studied by Alekseva, El-Zaouk and Dym [151] and Singh,Das and Sundararanjan [152]. Curved panels were studied by Hayes and Miles [153], Greenspon [154], Cummings [155], Rehfield, Girl and Sparrow [156]. Homogeneous and layered plates and shells were studied by Wu and Witmer [157],cylindrical panels were studied by Wu and Witmer [158], Volmir and Kulterbaov /159/, Ramachandran and Murthy /160/. Sahinpoor /161/analyzed the combined radial axial large amplitude oscillations of hyperelastic cylindrical tubes while Thurman and Moti /162/ analyzed the non-linear oscillations of a cylinder containing flowing fluid. The post buckling of thin elastic shells was examined by Lange and Newell /163/. Majumder, J. and Bucco, D. /164/ studied the transverse vibrations of viscoelastic shallow shells. Majumder, J. published a series of papers on elastic-plastic plates and shells.

From the above studies it appears to the present author that the following studies are not complete and need elaborate investgations.

(i) Use of von karman equations in non-homogeneous elastic plate problems appears to be rare and no literature is available to the present author where such problem has been dealt.

(ii) No literature is available where a comparative study on Berger's approach with its modification mentioning their merits and demerits has been attempted.

(iii) Use of non-homogeneous elastic-plastic plates in modern space shuttle structures, in civil and aero-space engineering are very wide and no literature is found where elastic-plastic plates of variable thickness or of variable flexural rigidity have been analyzed.

(iv) Modern structures are expected to experience severe vibrations in which damping or resisting forces are always called into play and as a result of which damped vibrations are obvious. No paper has been found where non-linear damped oscillations of elasto-plastic shallow shells are analyzed.

(v) Modern structures are usually set upon elastic subgrade. The effect of foundation on large deflections of plates and shells have been studied by numerous authors using different approaches, but the application of constant deflection contour lines to solve such problems and to obtain exact solutions appears to be rare.

(vi) The effect of temperature field in the non-linear vibrations are very common and only a few literatures are available on the topic.

(vii) Critical reviews on the existing methods which are frequently used in the linear and non-linear analysis of plate and shell problems are very rare and need such studies.

The present auther has much attraction to make a thorough investigations on the vibrational aspects of elastic and elasto-plastic plates and shells including their static behaviours and to find out shortcomings in the early investigations inthe field under consideration. For this purpose the present author studied some linear and non-linear static and dynamic behavioure of elastic and elastic-plastic plates and shells using classical field equations with suitable techniques. A critical survey on the non-linear analysis reveals that no unified method exists which can be used to find out approximate solutions for all the problems; rather it is the escence of the present study that the method of constant deflection contour lines may perhaps, convenently be used for this purpose.