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ABSTRACT

The beginning of the twentieth century saw the rise of a new branch of mathematics, called the theory of hyperstructures due to F. Marty, a French mathematician. Hyperoperations which map ordered pair of elements of the underlying set to a nonempty subset of it instead of binary operations play main role and gave birth to that theory. For the last three decades a number of communications in this regard have been made only in algebraic flavor, but from the beginning of the twenty first century few contributions are found in the field of topological hyperstructures.

This thesis comprises six chapters, each of which deals with topological studies on various algebraic hyperstructures and ended with a summary that speaks about the entire chapter. The first one is introductory that provides basic results as ready references and inculcates concepts regarding algebraic hyperstructures and topology that have been used directly or indirectly in later chapters.

To generalize conventional topological groups, topological hypergroups were considered [26] (D. Heidari, B. Davvaz, and S. M. S. Modarres. Topological hypergroups in the sense of Marty. *Comm. Algebra*, 42(11):4712–4721, 2014). Some results of topological groups fail to hold in the new setting, e.g., unlike in case of topological group, translation of open sets in topological hypergroup, in general, may fail to be open. To get rid of such obstacles towards the extension of the study, in the second chapter, the domain of thoughts has been restricted to a particular subclass of hypergroups, known as complete hypergroups and topological complete hypergroup along with its subhypergroups and quotient hypergroups are studied. The article [60] (M. Singha, K. Das, and B. Davvaz. On topological complete hypergroups. *Filomat*, 31(16):5045–5056, 2017) includes the main results of this chapter.

In the third chapter, different uniform structures along with Roelcke Uniformity have been studied on a subclass of topological hypergroups, known as topological polygroups. Further, it has also been showed that different uniformities coincide on balanced polygroups and the topology induced by each of the uniformities is compatible with the underlying one. The published paper [61] (M. Singha, K. Das, and B. Davvaz. Uniformities on OCP-polygroups. *J. Hyperstructures*, 7(2):104–123, 2018) is based on this chapter.

In the fourth chapter, prenorm is defined on a polygroup and showed that such prenorm can be rediscovered from a bounded real-valued function defined on the polygroup. It is also showed that first countability and T_1-property of topological polygroups ensure the existence of the aforementioned prenorm which induces a metametric on the polygroup. This metametric is so efficient that can generate all the open sets in the underlying topological polygroup. The significant results of this chapter have been published in [59] (M. Singha and K. Das. A variant of Birkhoff-Kakutani theorem on topological polygroups. *Palestine J. Math.*, 11(3):598–603, 2022).

Chapter 1 and Chapter 2 deal with topological hypergroups $(\mathscr{H}, *, \tau)$, where the members of τ are complete parts. But, imposition of such condition may lead to triviality, i.e., if $(\mathscr{H}, *, \tau)$ is a T_0 topological hypergroup such that the members of τ are complete parts, then, $\mathscr{H}$ is a group. To get rid of such a situation, topological hypergroups are studied in Chapter 4 considering the lower topology on the co-domain of the hyperoperation. Here, semi-topological and $S-$topological polygroups are also studied using the notions of semi-open sets and semi-continuity, respectively. Some results of this chapter have been published in [57] (M. Singha and K. Das. On topologized polygroups. *J. Tri. Math. Soc.*, 22(Dec-2020):33–42, 2022).

The hyperring is a more general structure that meets the ring-like axioms. If $+$ and $\cdot$ are two hyperoperations on a nonempty set R such that $(R, +)$ is a hypergroup and $\cdot$ is an associative hyperoperation, which is distributive over $+$, then the triplet $(R, +, \cdot)$ is a hyperring. If $\cdot$ is a binary operation, then, R is an additive hyperring. A particular instance of this sort of hyperring is the Krasner hyperring. In the last chapter, a topological study has been made over Krasner hyperrings to obtain some significant results, especially the isomorphism theorems. This chapter is based on the publication [58] (M. Singha and K. Das. Topological Krasner hyperrings with special emphasis on isomorphism theorems. *Appl. Gen. Topol.*, 23(1):201–212, 2022).

Hope this theory will penetrate into more deep and pave in different ways with huge applications like the theory of hyperstructure.