Chapter 1

Introduction

'Compact objects' are systems where very high density phases of strongly interacting matter are expected to exist. Neutron stars are compact objects whose masses are comparable to that of the Sun but radii compressed to about 10 km only! Naturally, these gravitationally bound, highly dense objects provide us cosmic laboratories for studying the properties of matter at high densities. As gravity also comes into effect at such a high density, these compact objects are also the meeting points of all the four fundamental forces.

At the final stage of a star's evolution, when all its thermonuclear fuel is exhausted, it cannot withstand the gravitational pull and starts collapsing. As it contracts appreciably from its original size, the density increases and at a sufficiently high density (of the order of nuclear matter density), it produces nonthermal pressure via degenerate fermion and particle interactions to support it against further collapse and becomes a 'compact star'. In 1932, shortly after the discovery of the particle "neutron" by Chadwick, Landau first predicted the possibility of a neutron star; a cold, dense star, whose principal composition are neutrons [1]. In a neutron star, the gravitational attraction is counter balanced by the pressure due to the degenerate neutron gas. The typical mass of a neutron star is about $1 - 2\ M_{\odot}$ ($M_{\odot} = Solar\ mass = 1.989 \times 10^{33}\ gm$), radius $\sim 10\ km$ and mean density $\sim 10^{14}\ gm\ cm^{-3}$. In 1934, Baade and Zwicky [2] pointed out that neutron stars could be formed during supernova explosion. In 1939, Oppenheimer and Volkoff [3] built a relativistic theory of neutron stars. Our knowledge on these compact objects has improved considerably over the last few decades, thanks to the new generation satellite data as well as our current understanding of basic physics. A good introduction to the physics of neutron stars is given in a book written by Shapiro and Teukolosky [1]. For recent developments in this field, we may refer to some of the recent papers, [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] & [14].

The energy-density inside a cold neutron star may range from a very low density at the suface to about $10^{15}\ gm\ cm^{-3}$ at the core of the star. This wide density range may broadly be grouped into three main regimes: (i) below neutron drip density ($\rho_{drip} \sim 4 \times 10^{11}\ gm\ cm^{-3}$) regime, (ii) neutron drip density to nuclear density ($\rho_{nuc} \sim 2.8 \times 10^{14}\ gm\ cm^{-3}$) regime, and (iii) supernuclear density regime. To construct a stellar model we need an equation of state (EOS), i.e., $p = p(\rho)$, where, p and ρ denote pressure and energy-density of the stellar material, respectively. Although we have reasonably well-established theoretical knowledge of equations of state of matter upto nuclear matter density, the EOS at supernuclear density remains a field of active research till date [1]. As we cannot reach such a high density in laboratories, we need to build acceptable theories consistent with the observed compact star properties. The macroscopic properties, such as mass and radius of compact objects, are influenced by the composition of its stellar material. As the density inside the core of these compact objects may exceed the normal nuclear matter density (ρ_{nuc}) many exotic phases may exist inside such stars which include condensation of pions and kaons; occurrence of hyperons and more interestingly, a transition from hadronic to quark matter. It can also have mixed phases. Recent progress in high energy particle physics coupled with new observational data has given tremendous impetus in understanding the physics of highly dense compact objects.

Neutron stars are observed as radio pulsars, X-ray pulsars and X-ray bursters. In 1968, Hewish *et al* [15] for the first time observed a pulsar, which was soon identified as a rotating neutron star. Since then, more than thousand pulsars have been discovered [16]. Since more and more observational data are now available, we hope to improve our theoretical understanding of these compact objects.

The gravitational fields for such compact objects are so strong that a general relativistic treatment for such compact objects is required. Consequently, the model for such compact objects are constructed from Einstein's field equations of general relativity:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}, \tag{1.1}$$

where, $G_{\mu\nu}$ is Einstein tensor, $T_{\mu\nu}$, energy-momentum tensor of the stellar matter, $g_{\mu\nu}$, the metric tensor, $R_{\mu\nu}$, the Ricci tensor, and R, the Ricci scalar.

Initial temperature of a neutron star is about 10^{12} K; but very rapidly it cools down below $10^6 K$ [8, 14], mainly due to neutrino emission. As the temperature decreases, matter in a neutron star becomes strongly degenerate due to very high density and hence, neutron stars, in general, are treated as cold ($T = 0$) stars. The themal effects may be treated as small perturbations which will have very little effect on the gross structure of these stars. Also, to develop a theory for these objects, one starts with a non-rotating, static configuration primarily because of the complexity of the theory. For radio pulsars this is a good approximation as most of them are known to be slow rotators [17].

The gross structure of neutron stars has so far been studied by using a variety of equations of state based upon phenomenological as well as realistic models of nuclear forces e.g, [6], [10], [14], [18], [19] & [20]. Our objective here is to develop a model of stars which seem to be more compact than ordinary neutron stars. Specifically, we are interested in those pulsars whose estimated mass and radius can not be explained with normal neutron star equations of state. Obviously, the crucial thing here is the accuracy of the estimated mass and radius of a star. Although, mass of a compact star can be estimated from binary pulsar observations with very high accuracy [21], there is no direct method to determine the radius of a compact star. However, recent observations of some pulsars in the low-mass X-ray binaries (LMXBs) have been able to put some constraints on their compactness (mass to radius ratio). In some cases, the calculations for radius are also claimed to be very accurate [22]. There are also some proposals that the radius of a neutron star will be directly measured by the observation of gravitational waves from coalescing neutron star/black hole binaries [23], [24]. These observations will hopefully be made in the near future and will be very important to develop detailed models for compact stars. The conclusions that can be drawn at this stage is that some recently observed values for masses and radii are not consistent with the normal neutron star equations of state.

The conjecture, made independently by Witten [25] and Farhi and Jaffe [26], that quark matter (and not ^{56}Fe) might be the true ground state of hadrons, has led to the discussion of an entirely new class of compact stars, popularly called 'strange stars'. These are composed of deconfined u, d & s quarks. Strange stars are expected to form during the collapse of the core of a massive star after the supernova explosion [27]. Another possibility is that some rapidly spinning neutron stars in low mass x-ray binaries (LMXBs) can accrete sufficient mass to undergo a phase transition to become a strange star [28]. The concept of strange stars has later been supported by many authors, e.g., [29], [30], [31], [32] & [33]. Glendenning *et al* [34] and Benvenuto & Althaus [35] analyzed strange matter hypothesis and showed that if Witten's [25] hypothesis is true, we can have an entirely new class of stable white dwarfs called strange dwarfs. Strange matter hypothesis has been used by Madsen [37] to explain the pulsar glitches mechanism. Datta *et al* [38] showed that the very high frequencies in kilohertz quasi-periodic brightness oscillations (kHz QPOs) observed in certain X-ray burst sources (e.g., 4U 1636-53 with frequency 1.22 kHz) can be understood if one considers (non-magnetized) strange star X-ray binary (SSXB) system rather than neutron star X-ray binary (NSXB) sytem.

Quantum chromodymamics (QCD), the theory of strong interactions, predicts a denconfined quark-gluon phase under extreme conditions of temperature and/or density. In high-energy heavy-ion collisions, a few microseconds ($10^{-5}s$) after the Big Bang or in the core of a very compact star we surely have such extreme conditions. The analysis of pulse timing, brightness, surface temperature and rotational mode instabilities etc. of very well known pulsars also strongly favour such a phase transition. In a recent paper, Kapoor and Shukre [17] took up a good number of modern neutron star equations of state and examining each EOS systematically (making use of the Rankin relation [36]) concluded that they are not good enough to explain the observed mass-radius constraints of known pulsars. They even suggested that pulsars are not neutron stars; rather they are strange stars. Madsen [39] also claimed that millisecond pulsars are most likely to be strange stars rather than neutron stars. These claims have already been supported by various investigators who showed that a number of pulsars, earlier thought to be neutron stars, are actually good strange star candidates. From the semi-empirical mass-radius estimations of X-ray pulsar Her X-1 [40] and X-ray burster 4U 1820-30 [41], Dey *et al* [42] proposed that they are good strange star candidates. Millisecond pulsar SAX J 1808.4-3658 [43], X-ray source 4U 1728-34 [44] and PSR 0943+10 [45] have also been proposed to be strange stars. Very recently, an isolated star RX J185635-3754 has been studied giving it a mass $M = 0.9 \pm 0.2\ M_{\odot}$ and radius $b = 6^{-1}_{+2}\ km$ [47] which may also belong to the class of strange stars as can be seen from the mass-radius curve obtained by Li *et al* [43]. In Table 1.1, we give a list of compact stars whose interior may have strange or other exotic matter.

As the maximum mass and radius of both neutron stars and strange stars are very similar, it is very difficult to distinguish between them. However, the mass-radius curve of a strange star is different from that of a neutron star, vide e.g., Alcock *et*

Pulsar	M ($M_\odot$)	b (km)
Her X-1 ([40])	0.98 ± 0.12	6.7 ± 1.2
4U 1820-30 ([41])	0.8 - 1.8	< 10
4U 1728-34 ([44])	~ 1.1	< 10
SAX J 1808.4-3658 ([43])	1.435(SS1) & 1.323(SS2)	7.07 (SS1) & 6.55(SS2)
PSR 0943+10 ([45])	—	—
PSR 1937+21 ([46])	< 2.4	< 11.5
RX J1856-37 ([47])	0.9 ± 0.2	6^{-1}_{+2}

Table 1.1: Candidates for compact stars composed of strange or other exotic matter. (Li *et al* [43] predicted two masses and radii for SAX J 1808.4-3658 considering two sets of EOS, SS1 & SS2).

al [30, 31, 32]. Neutron star mass generally decreases to a minimum value as radius increases. But strange star mass increases with radius and there is no minimum mass.

As we do not observe quarks as free particles, the quark confinement seems to be a valid proposition, even if the mechanism is not well understood. In 1974, Chodos *et al* [48] proposed a phenomenological model for quark confinement, known as MIT bag model, in which quarks are assumed to be confined in a bag and the confinement is caused by a Universal pressure 'B', called the bag pressure, on the surface of the bag. In this model quark matter energy density has a term denoted by the bag constant B which is the difference in energy densities of the vacua for hadronic matter and the quark matter. The value of bag constant B has not yet been fixed and a representative value of B is 57 $MeV\ fm^{-3}$. Most of the thermodynamical treatments of the strange matter has been carried out in the frame work of MIT bag model. An alternative description of confinement in which the masses of the quarks depend upon the baryon number density was introduced by Fowler *et al* [49]. Using the quark mass density dependent model, Chakrabarty *et al* [50] studied strange quark stars and obtained results different from that of MIT bag model. But, later Benvenuto and Lugones [51] pointed out that the results were different because of a wrong thermodynamical treatment made in [50]. They studied the stellar properties and also the stability of strange matter at $T = 0$, using the quark mass-density-dependent model for non interacting quarks and obtained mass-radius relations for strange stars very similar to that of MIT bag model. Lugones and Benvenuto [52] did the same calculation for $T > 0$, and showed that at finite temperature too (below a certain critical temperature above which strange matter is unstable), the mass density dependent model gives very similar mass-radius relationship as in MIT bag model. Recently, Peng *et al* [53] modified the thermodynamical treatment discussed in [51] and updated the quark mass scaling and showed that the EOS thus obtained gives dimensionally smaller and less massive strange stars compared to the results of the model in [51] or in MIT bag model. Developments on strange stars has been reviewed by Cheng *et al* [54]. Very recently, Dey *et al* [42] developed a new model for strange stars where the quark interaction is described by an interquark vector potential originating from gluon exchange and a density dependent scalar potential which restores chiral symmetry at a high density. It has been claimed that the model has the ability to describe very compact stars.

The possibility that scalar fields present in the early Universe could condensate to form stars, known as boson stars, has also been explored in recent years. In cosmology, boson stars have been studied in the light of missing non-baryonic dark matter problem [55]. In 1968, Kaup [56] and later Ruffini and Bonazzola [57] first introduced the idea of objects made up of non-interacting massive scalar particles. These stars are supported against collapse by Heisenberg's uncertainty principle, unlike neutron stars which are stabilized by Pauli's exclusion principle. Typically, boson stars are gravitationally bound spherically symmetric configurations of complex scalar fields minimally coupled to gravity as given in general relativity. The physical nature of the spin-0 particle out of which a boson star is formed is still not very clear. As of now, we do not have enough experimental support to predict the main constituent of a boson star. Most of the recent calculations on boson stars are based on the work formulated by Colpi *et al* [58], who described a boson star by considering a self-interacting complex scalar field Φ and a potential of the form $\frac{\Lambda}{4}|\Phi|^4$, where Λ is the coupling constant. The concept of a boson star was later extended to the case of a combined boson-fermion star by Henriques *et al* [59] and Jetzer[60] to model cold stellar objects composed of both bosons and fermions, parametrized by the central densities of the bosons and fermions.

In view of these varied developments, we wish to study compact objects in a different perspective. As mentioned earlier, the construction of all these highly dense stellar objects demands an EOS, for which one has to depend crucially on the microscopic behaviour of matter at high density and/or temperature. As physics of high density matter is still poorly understood, we shall follow a different technique to study these objects. In our method, we shall reconstruct the supernuclear equations of state directly from the observed properties of compact stars. In 1992, Lindblom [61] developed a method to reconstruct the EOS for nuclear matter from neutron star masses and radii. In a recent paper, Harada [62] also developed a direct method to construct super nuclear EOS from any two of the known properties of compact stars. Although, our idea is similar, the methodology is totally different. We shall prescribe a geometry and then look for suitable matter content that can support such a geometry. Based on a purely geometric approach, macroscopic properties such as mass and radius are the only input parameters required here to determine a class of possible equations of state.

The model was first considered by Vaidya and Tikekar [63]. To find the interior solution of a compact object filled with a perfect fluid-like matter in hydrostatic equilibrium, they made an ansatz for one of the metric coefficients of the relevant metric. The ansatz makes the Einstein's equations easy to solve and also gives physically acceptable solutions capable of describing a class of supercompact objects. Exact solutions of Einstein's equations for this class of stars have been obtained for some specific values of a parameter λ, e.g., $\lambda = 2$ [63], 7 [64], & 14 [65]. Maharaj & Leach [66] obtained solutions of the same for a set of discrete values of λ. The general solution, for any value of λ, was obtained by Mukherjee *et al* [67]. The general solution greatly enhanced the applicability of the model to realistic cases. In chapter 2 we shall present some of the solutions reported where the authors used the ansatz given by Vaidya and Tikekar [63] and in particular, in section 2.3.4 we shall discuss the solution obtained by Mukherjee *et al* [67]. This general solution will be used extensively by us in the chapters to follow.

We have extended the work of Mukherjee *et al* [67] to the case of a static charged sphere. Earlier work in this field were done by Tikekar [103], Patel *et al* [104], Tikekar and Singh [105] & Patel and Kopper [115]. Essentially we have obtained an exact static spherically symmetric solution of the Einstein's field equations for a source consisting of a perfect fluid and a non-null electromagnetic field whose exterior is described by the usual Reissner-Nordstrom metric [68]. In our method we assumed (in curvature coordinates) a special functional form for the metric function g_{rr} given by Vaidya and Tikekar [63] and then with a particular choice for the electric field intensity E, we determined the remaining metric coefficient g_{tt} in a trigonometric form. Physical significance of the solution has been analysed by calculating the energy density ρ, pressure p and charge density σ of the system. It was found that the solution satifies both strong and weak energy conditions. Where relevant we have regained the uncharged limit of quantities of interest. In particular, we have shown that the causal signals in the charged case are permitted over a wider interval than in the case for uncharged stars.

Although it seems unlikely that there exits a charged astrophysical object, the possibility of a compact star acquiring a net charge by accretion from the surrounding medium has been pointed out recently by Treves and Turolla [69]. Similar observations have also been reported recently by Mak *et al* [70]. These are some of the areas where our general solution may have applications. Recently, Ivanov [71] has reviewed the known solutions for static charged fluid spheres in general relativity which includes our solution. The solution has been discussed in chapter 3.

In chapter 2, we have shown that the solution given in [67] satisfies the desired physical and regularity conditions. However, the relevant question here is whether the model can describe a realistic star. To answer this question and to show the physical applicability of the model, we have considered some interesting pulsars and from observational data we have shown how the relevant high density equations of state can be reconstructed from the model in a very simple manner. This has also enabled us to study some other properties of these stars. We have considered, in particular, the X-ray pulsar Her X-1 and millisecond pulsar SAX J 1808.4-3658 as their masses and radii are well estimated. These pulsars are interesting because their estimated radii are rather small and these small values cannot be explained by neutron star EOS. This has led various investigators to identify them as strange stars or quark-diquark stars. Employing the strange matter EOS given by Dey *et al* [42], Li *et al* [43] claimed that the low mass X-ray binary (LMXB) pulsar SAX J1808.4-3658 is a strange star. In chapter 4 we have shown that if SAX is a strange star, the relevant EOS can be generated from the solution of Mukherjee *et al* [67].

The equation of state (EOS) in our model, for a suitable choice of the parameter λ, also covers the EOS for a quark-diquark star, a star whose interior has deconfined u, d quarks and diquarks. In chapter 5, we have studied Her X-1 and showed that if Her X-1 is a quark-diquark star, the relevant EOS obtained by Horvath and Pacheco [72] can also be generated by this model. Interestingly, although, the agreement of two equations of state are excellent, the boundary conditions assumed in the two cases are different which led us to make conclusions different from those of Horvath and Pacheco [72].

To study a star one should also check if the stellar configuration is gravitationally stable. Following the earlier work of Knutsen [156] and Tikekar and Thomas [157], which uses the method developed by Chandrasekhar [73], we have shown that the configurations presented in chapter 4 & 5 to describe SAX J1808.4-3658 and Her X-1, respectively, are stable with respect to small radial oscillations. This has been discussed in chapter 6.

We have observed that the Vaidya-Tikekar [63] model has a scaling property which allows the solution to describe a family of stars of equal compactness ($\frac{M}{b}$) where, M and b are the mass and radius of the star, respectively. In the MIT bag model, scaling property with respect to the bag constant B in the EOS for strange matter has been reported earlier by Witten [25]. The strange matter EOS formulated by Dey *et al* [42] can also be approximated to a linear form as shown by Gondek-Rosińska *et al* [74]. The linearised EOS has the form $p = a(\rho - \rho_0)$, where, ρ and p are energy density and pressure of the star, respectively and a and ρ_0 are two parameters. In the bag model too, exactly the same type of EOS was obtained by Zdunik [75]. The linear form of the EOS allows to scale all the physical parameters of the star with some powers of ρ_0 for a fixed value of a; analogous to the scaling law with respect to the bag constant B. Thus, scaling property in compact stars like strange stars may be thought as due to the EOS being a linear one. But, we have shown that the scaling property in compact stars may rather be thought as a general feature of a spherical distribution in hydrostatic equilibrium. We have demonstrated this by analysing the Tolman-Oppenheimer-Volkoff (TOV) equations and the boundary conditions that are used while solving these equations. A special class of these scaling property is explicitly exhibited by the solution obtained by Mukherjee *et al* [67]. In chapters 4 & 5, we have shown that the solution [67] can be applied to describe strange stars as well as stars whose interior might have other exotic components. Moreover, although for a large value of the parameter λ the EOS becomes almost linear in this model, the same is not true for a smaller value of λ. Scaling property, nevertheless, applies in all the cases. The scaling behaviour in the Vaidya-Tikekar model [63] as well as it's physical implications have been discussed in chapter 7.

In case of compact stars, different possible models have been discussed so far: ordinary neutron star with a deconfined quark core (quark stars having a thin layer of hadronic matter) and bare quark stars, i.e., quark stars with no crust. Also in an ordinary neutron star it may not always be possible to use a single EOS to describe the entire star. The possibility of a compact star having a quark-diquark core surrounded by a low-density envelope of nucleon has already been discussed by Kastor and Traschen [76]. In chapter 8 we have presented a new core-envelope treatment, relevant for compact stars having a deconfined quark core surrounded by less compact hadronic matter, using the solutions obtained by Mukherjee *et al* [67]. Earlier, using TOV equations, Lindblom [77] analysed the features of the mass-radius curve of a relativistic stellar model constructed from an EOS with a first order phase-transition. The basic difference in our approach compared to the earlier workers [76, 77] is that, in our model, we do not make any ad hoc assumption about the EOS. We have constructed the model for a compact star exhibiting a phase transition somewhere inside the star by choosing two different values of the parameter λ (which occur in the Vaidya-Tikekar [63] model) for the two layers separated by a surface where a phase transition may take place and implementing necessary boundary conditions across the surface. It has been found that the core region gives a much softer EOS compared to the envelope region, as expected. Also, we have shown that the central density reached in case of a core-envelope model is larger than that of a star composed of less dense baryonic matter.

The solution obtained by Mukherjee *et al* [67] can also be applied in gravitational collapse problem. The formation of cold compact stellar objects are usually preceded by a period of radiative collapse. The problem of gravitational collapse was first investigated by Oppenheimer and Snyder [78], who considered the contraction of a spherically symmetric dust cloud. In that case the exterior space-time was described by the Schwarzschild metric and the interior space-time was represented by a Friedman-like solution. Later on, Vaidya [79] derived the line element which describes correctly the exterior gravitational field of a spherically symmetric radiating mass. This has enabled many investigators to model the interior of a radiating star by matching the interior solution to the exterior space-time given by Vaidya (de Oliveira *et al* [80, 81, 82], Kramer [83] and Govender *et al* [84]). The junction conditions for a spherically symmetric shear-free radiating star were derived by Santos [85]. The crucial result that follows from the work of Santos [85] is that the pressure on the boundary of a radiating sphere is non-vanishing in general. Subsequently, different models of radiative gravitational collapse with heat flow were found by utilizing these junction conditions, e.g., [86]. In particular, special attention was given to models in which an initial static stellar configuration started collapsing by dissipating energy in the form of a radial heat flux (Bonnor *et al* [87]). The initial static configuration was taken to be an exact solution of the Einstein field equations before it starts collapsing. In contrast to earlier methods, we started with a final static configuration and looked for solutions for earlier evolutionary stages. In chapter 9, we have presented a simple model of radiative gravitational collapse with a radial heat flux which can describe qualitatively the stages close to the formation of a superdense cold star. Starting with the final static general solution for a cold star, as given by Mukherjee *et al* [67], we have shown that the model can generate solutions for the earlier stages of the star [89].

In this thesis we have ignored two aspects of a neutron star: (a) deformation of the star due to its rotation and (b) the effects of a strong magnetic field. In case of Her X-1, Li *et al* [40] have pointed out that rotation of Her X-1 has negligible effects on its $(M - b)$ relation. Also, considering the rotational effect upto Ω^2 (where Ω is the angular velocity observed at infinity) in case of a slowly rotating boson-fermion star, de Sousa and Silveira [90] observed that field equations remain the same as in the case of a boson-fermion star with no rotation. Hence, they concluded that star properties such as mass and radius of a static model do not differ considerably from that of a slowly rotating boson-fermion star. Thus, although we assumed a static model for Her X-1, error in our results due to rotation, if any, should be small. Considering strange star EOS under the MIT bag model, Phukon [91] showed that the presence of a magnetic field less than 10^{18} G, does not change the EOS considerably. He found no noticable change in the maximum mass and radius of a strange star for a magnetic field less than 10^{18} G. We may, thus, presume that our results would not differ considerably in the presence of a magnetic field atleast for SAX J1808.4-3658 (magnetic field $\sim 10^8$ G) and Her X-1 (magnetic field $\sim 10^{12}$ G).

In chapter 10 we have concluded by summarizing our results and discussing avenues for further work.