FLUID FLOWS WITH OR WITHOUT DUSTY SUSPENSION
AND
MASS TRANSFER AND FREE CONVECTION

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1 9 9 1

Gopal Chandra Mandal, M. Sc.

Dedicated to my
Eldest Brother

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(Gopal Chandra Mandal)

Assistant Scientific officer,
Hydraulic Study Department,
Calcutta Port Trust,
20, Garden Reach Road,
Calcutta, India - 700 043

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CHAPTER – I

INTRODUCTION

1.1 Introductory remarks

The contents of this thesis are arranged in four chapters. Chapter I is of a review nature and deals with a general introduction to the thesis. Chapter II is devoted to the study of the unsteady flows of elastico-viscous liquid under various configurations. These problems are of general and continuing interest and are discussed here to study the flow-characteristics of elastico-viscous liquid under a variety of situations. It consists of two parts. In the first part of this chapter, an analysis is made to investigate the unsteady flow of a non-Newtonian fluid between two oscillating plates, while, in the second part, the oscillating flow of visco-elastic Oldroyd fluid in a long circular tube in presence of magnetic field is explained.

Chapter III is concerned with the unsteady flows of dusty fluid under several physical and geometrical circumstances. This chapter is divided into two sections. Section A deals with the dusty Newtonian fluid flows, while, in section B, the dusty non-Newtonian fluid flows are considered. These problems are becoming increasingly important due to their varied applications in the fields of science and technology. These studies reveal the detailed structure of the flow and estimate the surface characteristics, such as, skin-friction coefficients, particle velocity of the liquid, etc.. Section A is composed of two parts. In the first part, the rotational motion of a dusty viscous fluid contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface is analysed. The second part of this section is concerned with the study of the semi-infinite dusty viscous fluid flow in a rotating system when its horizontal boundary is suddenly stopped from its state of steady motion. Section B consists of three parts. In the first part, the unsteady flow of the dusty visco-elastic liquid between two oscillating plates is investigated. The second part of this section is concerned with the study of the unsteady axi-symmetric rotational flow of elastico-viscous fluid with a uniform distribution of dust particles. In the last part of this section, the unsteady flow of a dusty elastico-viscous liquid in the Ekman layer is analysed.

The last chapter i.e. chapter IV of the thesis is devoted to the study of the diffusion of the matter through the Newtonian fluid, the free convection and the mass transfer in the Newtonian fluid. It is divided into five parts. In the first part of this chapter, an exact analysis of dispersion of the solute in an electrically conducting fluid flowing between two parallel plates porous walled channel in the presence of a uniform transverse magnetic field is made by using a generalized dispersion model. Dispersion coefficients are evaluated as functions of time. Unsteady free convective flows in presence of mass transfer through a porous medium bounded by a vertical porous plate are subjects of discussion in the subsequent four parts of the last chapter. Part two and three are devoted to study the effects of magnetic parameter on the flow field while the effects of rotation are analysed in the last two parts.

Before we discuss various problems, we present below a brief survey of the literature on the non-Newtonian fluids, the dusty fluid motion, the diffusion/dispersion of the solutes in fluids flowing under the laminar-flow conditions, the free convection and the mass transfer in a porous medium with a view to presenting the work of this thesis in its proper perspective.

1.2 Non-Newtonian fluids

A 'Newtonian' fluid is one for which a linear relation exists between stress and the spatial variation of velocity. If changes in the fluid density are not important, the constant of proportionality is called viscosity, a characteristic constant of the material at a given temperature and pressure. So a 'Newtonian' fluid is characterised by a linear relation between stress and rate of strain of the form

$$p_{ij} = (\lambda\Delta - p) \delta_{ij} + 2\mu e_{ij} , \qquad (1.1)$$

$$e_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i}) , \qquad (1.2)$$

where p_{ij} , e_{ij} , P and Δ denote the stress tensor, strain-rate tensor, pressure and dilatation respectively; λ, μ are coefficients of the medium. Its theory has been extensively investigated during the last century. The relationship (1.1) explains reasonably well most of the phenomena like drag, lift, skin-friction, separation etc. occurring in the flows of fluids. However, it fails to explain the occurrence of Weissenberg effect [1], Merrington effect [2] and Poynting effect [3]. In order to explain these effects, the stress-rate of strain relation should be generalised so that the constitutive equation becomes non-linear. The fluids under this category are termed as non-Newtonian fluids. Non-Newtonian fluids form an extremely wide class of different materials, whose only common features are fluidity and a failure to obey Newton's viscous law [4]. The exceptions to Newton's viscous law are not of rare occurrence, in fact the so-called non-Newtonian fluids are to be found close at hand everywhere. The fluids like blood, honey, condensed milk, liquid lubricants, printing inks, starch, rasin pastes, plastics, high polymers, salad dressings, butter, whipped cream and doughs, egg white, paints, certain varieties of oil and many other materials of industrial importance fall under the category of non-Newtonian fluids [5]. But the behaviour of such fluids is not so readily amenable to theoretical analysis due to the non-linearity of the stress-strain rate relations governing the fluid model and the dependence of the coefficients occurring in them on physical properties of the fluids. Moreover, experiments are not many to throw sufficient light on the flow phenomena of this type of fluids.

The study of non-Newtonian fluids has become essntial due to their importance in modern industries. This has led to the formulation of various theories of non-Newtonian fluids. We present below a brief discussion of the constitutive equations for three models of non-Newtonian fluids, viz. i) Model B due to Oldroyd ii) Walters'liquid B' model iii) Kuvshiniski's model which are directly related to our present thesis.

(a) Elastico-viscous liquids (Model B due to Oldroyd)

The class of liquids which possesses a certain degree of elasticity in addition to viscosity is known as the elastico-viscous liquid. Thus, when an elastico-viscous liquid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to viscous dissipation. In a normal inelastic viscous liquid we are concerned only with the rate of strain, but in elastic liquids we can not neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the reverse flow that follows the removal of stress. Only in elastico-viscous liquid there is a degree of recovery from the strain when the stress is removed whereas in other fluids the whole strain remains.

Evidences of liquid elastcity in an 1.5 percent starch solution can be observed by Hess's experiment [6] and by the recoil of air bubbles in a mixture of Polymethyl Methacrylate and Cyclohexanone (made by dissolving 3 gms. perspex in 100 ml of solvent) contained in a bottle which has been suddenly turned and then brought to rest.

Oldroyd [7] formulated a non-linear theory of a class of isotropic incompressible elastico-viscous liquids with the following rheological equation of state between the stress tensor p_{ik} with the rate-of-strain tensor e_{ik} :

$$p_{ik} = -p\,\delta_{ik} + p'_{ik}\,, \tag{1.3}$$

where

$$p'_{ik} + \lambda_1\left(\frac{D}{Dt}\right)p'_{ik} + \mu_0 p'_{ij}\,e_{jk} - \mu_1\left(p'_{ij}e_{jk} + p'_{jk}e_{ij}\right)$$
$$+ \nu_1\, p'_{j\ell}e_{j\ell}\delta_{ik} = 2\,\eta_0\Big[e_{ik} + \lambda_2\left(\frac{D}{Dt}\right) e_{ik} - 2\,\mu_2 e_{ij}e_{jk}$$
$$+ \nu_2\, e_{j\ell}\, e_{j\ell}\delta_{ik}\Big], \tag{1.4}$$

and e_{ij} given by the equation (1.2).

In equation (1.4), η_0, λ_1 and λ_2 denote the coefficient of viscosity, stress relaxation time and strain retardation time respectively. The other five material constants μ_0, μ_1, μ_2, ν_1 & ν_2 are all of dimensions of time. The quantity $(\frac{D}{Dt})$ denotes the total derivative following a fluid element taking into account its translational and rotational motion. It may be noted that the memory of the liquid is accounted for through the stress relaxation time λ_1 and rate-of-strain retardation time λ_2 and the linearity of the Newtonian costitutive equation is broken through introduction of quadratic terms in the strain rate components and the product of the stress rate and strain rate components.

Oldroyd, in his paper [8], considered two particular types of liquid viz., liquid A and liquid B. General model of liquid B is goverened by equation (1.4) with

$$\left.\begin{array}{l} \eta_0 > 0,\ \lambda_1 = \mu_1 > \lambda_2 = \mu_2 \geq 0 \\ \qquad \mu_0 = \nu_1 = \nu_2 = 0 \end{array}\right\} \tag{1.5}$$

It was observed by Oldroyd that the model represented by the constitutive equation (1.4) along with (1.5) exhibits Weissenberg climbing effect when sheared at a finite uniform rate between two co-axial cylinders and has a distribution of normal stress equivalent to an extra tension along streamlines with isotropic state of stress in the plane normal to the streamlines. Thus the constitutive equation (1.4) subject to (1.5) retains essentially the rehological properties of a liquid. This model can predict normal stress and time-dependent visco-elastic behaviour which are in accord with physical observation. However, this model fails to account for the variation of apparent viscosity with rate of shear. Further, the consitutive equation (1.4) along with (1.5) holds for low rates of shear.

It was experimentally found by Oldroyd, Strawbridge and Toms [9] that a solution of a mixture of Polymethyl Methacrylate in pyridine obeys the constitutive equation subject to (1.5) and for this solution $\lambda_1 = 0.065$ (sec.) and $\lambda_2 = 0.015$ (sec.) with $\nu_0 = 7.9$ poises and density 0.98 gm/ml. Thus for a real elastico-viscous liquid, the restriction $\lambda_1 > \lambda_2$ is a valid one.

(b) Walters' liquid B'

The constitutive equation for Walters' liquid B' [10] (at small rates of shear) are given by

$$p_{ik} = - p\, g_{ik} + p'_{ik} \quad , \tag{1.6}$$

$$p'^{ik}(x,t) = 2 \int_{-\infty}^{t} \psi\,(t - t')\, \frac{dx^i}{dx'^m} \frac{dx^k}{dx'^r} e^{(1)mr}(x',t')\,dt' \tag{1.7}$$

where p'^{ik} is the deviatoric stress tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed co-ordinate system x^i, x'^i the position at time t' of the element which is instanteneously at the point x^i at time t, $e^{(1)}_{ik}$ the rate of strain tensor and

$$\psi(t - t') = \int_0^\infty \frac{N(\tau)}{\tau} e^{-(t-t')/\tau} \, d\tau , \qquad (1.8)$$

$N(\tau)$ being the distributive function of the relaxation time τ. It may be noted that the elastico-viscous liquid of model B due to Oldroyd is the special case of Walters' liquid B', obtained by substituting

$$N(\tau) = \eta_0 \frac{\lambda_2}{\lambda_1} \delta(\tau) + \eta_0 \frac{\lambda_1 - \lambda_2}{\lambda_1} \delta(\tau - \lambda_1) \qquad (1.9)$$

in equations (1.7) and (1.8). $\delta(\tau)$ denotes a Dirac delta function.

It has been shown by Walters [11] that in the case of liquids with short memories (i.e. short relaxation times), the equation of state can be simplified to

$$p'^{ik} = 2\eta e^{(1)ik} - 2K_0 \frac{\delta}{\delta t} e^{(1)ik} , \qquad (1.10)$$

where $\eta \left(= \int_0^\infty N(\tau) \, d\tau \right)$ is the limiting viscosity at small rates of shear, $K_0 \left(= \int_0^\infty \tau N(\tau) \, d\tau \right)$ is the elastic coefficient and $\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor.

It is interesting to note that the mixture of Polymethyl Methacrylate in pyridine at 25° C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits well in the above model. For this mixture, the relaxation spectrum as by Walters is

$$\begin{aligned} N(\tau) &= \sigma \eta_0 \delta(\tau) + \frac{1 - \sigma}{\beta} \eta_0 , \quad (0 \le \tau \le \beta) \\ &= 0 \qquad \text{for } \tau > \beta \end{aligned} \qquad (1.11)$$

where $\sigma = 0.13$, $\eta_0 = 7.9$ poises (gm/cm.sec) and $\beta = 0.18$ sec. .

(c) Elastico-viscous fluid (model due to Kuvshiniski)

The constitutive equations for Kuvshiniski's liquid [12] are given by

$$\left(1 + \lambda \frac{D}{Dt}\right) p'_{ik} = 2\mu e_{ik} \tag{1.12}$$

where

$$\frac{D}{Dt}(p'_{ij}) = \frac{\partial p'_{ij}}{\partial t} + v_m \frac{\partial p'_{ij}}{\partial x_m} \tag{1.13}$$

and $$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) . \tag{1.14}$$

The stress tensor is given by

$$p_{ij} = - p\,\delta_{ij} + p'_{ij} \tag{1.15}$$

where p is static pressure, δ_{ij} the Kronecker delta and p'_{ij} is deviatoric stress tensor usually related to the rate of strain, e_{ij}, by the equation of state (1.12). Also, physically λ means, if the motion stops suddenly, the shear stress will decay as $e^{-t/\lambda}$; μ is coefficient of viscosity, v_i is the velocity of liquid particle.

1.3 Unsteady flow of elastico-viscous liquid

(a) Flow between parallel plates

An extensive liturature exits on flow of non-Newtonian fluids between two parallel plates as these problems are of vital importance in certain industrial processes. Sharma [13] considered plane Poiseuille flow through tube and case of a parallel plate visco-meter. Agarwal and Jain [14] extended the paper of Sharma [13] by introducing magnetic field in the system. He considered the fluid to satisfy the Oldroyd model. Frater [15] discussed the flow of an elastico-viscous liquid between torsionally oscillating discs. The flow of an elastico-viscous Walters' liquid B' near an oscillating infinite plate and between two infinite oscillating plates with a phase difference, with the same frequency and different amplitudes under a transverse magnetic field relative to the fluid and fixed relative to one of the plates was considered by Gulati [16]. The plane Poiseuille flow, plane Couette flow and the Couette flow of micropolar fluids was investigated by Rajagopalan [17]. Siddappa and Hegde [18] discussed the exact solution for oscillating motion of a visco-elastic fluid bounded by two infinite plates, one fixed and the other executing simple harmonic oscillations in its own plane. The case, in which both the plates oscillates was also analysed. Johri [19] discussed the problem of an elastico-viscous flow (Rivlin-Ericksen model) induced by circular oscillations of two infinite parallel discs. Later, Johri [20] studied the problem of unsteady slow flow of Oldroyd B-liquid between two infinite parallel discs. The flow was induced by the elliptic harmonic oscillations of the discs.

In part one of chapter II of the present thesis, an analysis is made on the flow characteristics of a certain class of non-Newtonian fluids, viz. elastico-viscous liquid proposed by Kuvshiniski [12], flowing between two oscillating parallel plates. Analytical expressions for velocity field and skin-friction on the upper plate wall are obtained by the technique of Laplace transform. The effects of elastic elements in the liquid, the amplitude of steady state oscillations and the phase difference of the velocity fields, the skin-friction at the upper plate are presented graphically/numerically.

(b) Flow in a pipe due to applied pressure gradient

Oscillating flow of viscous fluid in a long circular tube under the influence of a pressure gradient was investigated theoretically and experimentally by Richardson and Tyler [21] and theoretically by Sexl [22].

Many common liquids such as oils, certain paints, blood, polymer solutions, some organic liquids and many new materials of industrial importance exhibits both viscous and elastic properties. However, this sub-class shows diverse behaviour in response to applied stress. A number of rheological models were proposed to explain such a diverse behaviour. Many scientists carried out their research on flow problems in a tube under applied pressure gradient in different elastico-viscous liquid models. Flow of a Maxwell fluid through a tube under the application of an axial pressure gradient

$$\text{(i)}\ -\frac{1}{\rho}\frac{\partial p}{\partial z} = Ke^{\alpha^2 t} \qquad \text{(ii)}\ -\frac{1}{\rho}\frac{\partial p}{\partial z} = Ke^{-\alpha^2 t},$$

was considered by Soundalgekar [23]. In the first case formation of boundary layer was observed while the layer was not noticed in the second case. Both the cases could have been dealt by the author [23] as a single case. The unsteady flow of an Oldroyd fluid through a tube under a pressure gradient $-\frac{1}{\rho}\frac{\partial p}{\partial z} = a_0 + f(t)$ was studied by Ghosh [24]. The pulsating flow of a second-order fluid through a circular pipe was analysed by Ramacharyulu [25] while the flow problem for the same fluid under time-dependent exponentially increasing or decreasing pressure gradient was studied by Soundalgekar [26]. Later Soundalgekar [27] extended his work [26] in Walters' liquid B' under the same geometry and same physical situations. But none of the authors examined the effects of magnetic field in this geometry. In the part two of chapter II we study the flow behaviour of visco-elastic Oldroyd fluid in the presence of magnetic field in a long circular tube. Considering the flow field under the influence of an oscillating pressure gradient, the expression for velocity of liquid is obtained analytically. A comparative study for the effects of magnetic field in Newtonian and Oldroyd fluid is made.

1.4 Rotating fluid flows

The stimulus for scientific research on fluid system in rotating environments is originated from geophysical and fluid engineering applications. Many aspects of the motion of terrestrial and planetary atmospheres are influenced by the effects of rotation. The broad subjects of oceanography, meteorology, atmospheric science all contain some important and essential aspects of rotating flows. Rotating flow theory is utilised in detemining the viscosity of fluids and in the construction of turbines and other centrifugal machines. Also the study of flows through porous media in a rotating system is of considerable interest in many scientific and engineering applications. viz., to the petroleum engineers concerned with the movement of oil and gas through reservoir; to the hydrologist in his study of migration of underground water etc.. The complete literature pertaining to rotating fluids is enormous and an excellent review can be found in the monograph by Greenspan [28]. Rotation in a fluid system produces two effects, viz., the coriolis and the centrifugal forces, on the fluid particles. The balance between the coriolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. In considering flows in rotating environment we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the later case it is preferable to use an inertial co-ordinate system fixed in space. On the other hand the flow behaviour in the former case can be described in a co-ordinate system which rotates with the fluid, and in this frame of reference the fluid is at rest.

In a steadily rotating system, a balance is struck between coriolis and frictional forces in a thin layer over horizontal boundaries. This layer, called the Ekman layer, was first noticed by Ekman [29] and plays a very fundamental role in the rotating fluid flows. In this thesis we consider certain problems in rotating system due to their varied applications in the field of technology.

1.5 Dusty fluid flows

Fluid flows with particulate suspensions, when the suspended matter may consist of solid particles, liquid droplets, gas bubbles or combinations of these, are commonly termed as dusty gas flows or dusty fluid flows. They are also referred to as two-phase flows, since they involve a composite of two-phase or two materials with different distinguishable properties – one phase being the fluid medium which is a continuous phase and the other phase being the particulate suspensions which are scattered throughout the fluid medium and hence known as the dispersive phase or discrete phase or simply particulate phase.

The flows of fluids with suspended material particles abound in nature, classical examples being pollution of air and contamination of water. The earth's atmosphere is a predominantly gaseous envelope of air surrounding the earth and it contains solid particles and liquid droplets. Besides it is also being constantly polluted by a number of dust particles like carbon, sulpher and many other toxic elements which arise as inevitable consequence and natural by-products of industrialization.

Dusty gas flows assume importance in such engineering problems as fluidization (flow throuh packed beds), sedimentation, powder technology, flows in rocket tubes where small carbon or metalic fuel particles are present, aerosol filtration, gas purification and slurries. Further, problems concerned with atmospheric fallout, rain errosion of guided missiles and aircraft icing are some of the areas where the dynamics of dusty gases play a prominent role.

A knowledge of dusty fluid flows is useful to some extent in understanding the rheology of blood flows through capillaries, where red blood cells can be regarded as rigid particles embedded in the plasma. Another biological situation where the study of two-phase flows assumes importance is the phenomena of particle deposition in the respiratory tract.

The formulation of the fundamental equations of the dusty fluid flows is guided in a reasonably simple manner under certain basic assumptions : (i) the fluid is an incompressible viscous fluid; (ii) dust particles are spherical and undeformable, all having equal radius and mass ; (iii) the density of the dust particles is high compared with the fluid density and the mass fraction of the dust particles is not extremely high so that the volume occupied by the dust particles is negligible ; (iv) the volume-fraction of the dust particles is so small that the interaction between individual dust particles may be neglected and so the fluid phase contributes the entire pressure ; (v) dust particles form a cloud of pseudo-fluids with negligible viscosity; (vi) the thermal and gravitational forces are neglected ; (vii) the radius of the dust particles being small, the only force acting due to interaction between dust and fluid particles is the drag force given by Stokes' law, where it is also supposed that the Reynolds number of the relative motion of the dust and fluid is small compared to unity ; (viii) the distortion of the flow field around the dust particles is neglected.

Under these assumptions, following Saffman's formulation [30], the equations of continuity, equations of motion of viscous fluid and the dust particles are

$$u_{i,i} = 0, \tag{1.16}$$

$$\frac{\partial N}{\partial t} + (N v_i)_{,i} = 0 , \tag{1.17}$$

$$\rho \left(\frac{\partial u_i}{\partial t} + u_k u_{i,k} \right) = p_{ik,k} + KN (v_i - u_i), \tag{1.18}$$

$$m \left(\frac{\partial v_i}{\partial t} + v_k v_{i,k} \right) = K (u_i - v_i) , \qquad (1.19)$$

where u_i, v_i are the local velocity vectors of liquid and dust particles respectively, p_{ik} the stress tensor defined by equation (1.1) in which the rate of strain tensor e_{ik} defined in terms of the velocity field u_i by

$$e_{ik} = \frac{1}{2} (u_{i,k} + u_{k,i}) ,$$

ρ the density, K the Stokes resistance coefficient (for spherical particles of radius d, it is $6 \pi \mu d$), N the number density of a dust particle and m the mass of a dust particle.

Based upon the theoretical model proposed by Saffman [30], numerous authors investigated a number of dusty gas flow problems under several physical and geometrical circumstances. Kazakevich and Krapivin [31] and Sproull [32] performed experimental works on dusty fluid and observed that the aerodynamical resistance of the dusty gas is less than that of a clean gas. Soo [33,34,35,36] made pioneering works in the two-phase problems including the problems of dusty fluid flows. Marble [37], Pai [38], Jain [39] and Goddard [40] reviewed the subject of dynamics of dusty fluids. Rudinger and Chang [41] and Rudinger [42] considered the effects of volume-fraction of dust particles on the flow of dusty gases and also analysed unsteady two-phase flow. Wallis [43] studied one-dimensional two phase flow. Davidson [44] gave some features on the flow of fluid-solid mixtures. Torobin and Gauvin [45,46], Crooke [47], Hinch [48] and Purcell [49] put forward some noble ideas on the fundamental aspects of the flow of dusty fluids.

In section A of chapter III of the present thesis, we consider two problems on unsteady flow of dusty viscous liquid.

Among many fluid mechanical problems which are of direct technological interest, one of the most important classes is that involving the motion of particles in polymer solutions or melts and other non-Newtonian fluids. A considerable amount of both theoretical and experimental work in this line of study exists in literature. Leal [50] examined the slow motion of slender rod like axi-symmetric particles on a second order fluid while Goddard [51,52] discussed the stress field of slender and rod like particles respectively flowing in a non-Newtonian fluid. Recently, Leal [53] presented a report on the studies of motion of small particles in non-Newtonian fluids.

There is another class of flow problems which concerns with the study of motion of non-Newtonian fluids having suspension of undeformable small solid particles in it. Very little is known [54,55,56,57,58], so far as we are aware, about the dynamics of dusty non-Newtonian fluid although it has applications on the problems of transport of solid particles suspended in non-Newtonian fluids through pipes and channels, polluted oil extraction, polymer extrusion, paint spraying, boundary layer growth of non-Newtonian fluids having suspended dust particles, etc. and it is the subject of investigation presented in section B of chapter III of the thesis.

(a) Dusty fluid flows through a circular cylinder or between two co-axial cylinders

The classical problem on the laminar flow of a dusty gas between two rotating cylinders was investigated by Michael and Norey [59]. They assumed that the relaxation time of the dust, τ, to be small and obtained solutions by considering the ratio of the time scales, on which the gas velocity and the mass concentration of dust change, to be both large and small. Dusty fluid flows between two rotating co-axial cylinders were investigated by Nath [60] and Tiwari [61] under different physical situations. Gupta and Gupta [62] investigated the Couette flow of a dusty gas between two infinite co-axial cylinders. Crooke and Walsh [63] developed a method for the construction of solutions for the flow of a viscous, incompressible gas with suspended dust particles when the flow domain possesses special geometry. In first, second and third sections, they derived a set of linear partial differential equations representing two-dimensional flow and obtained solutions to those equations for rectangular and circular geometries. In the fourth section, they presented the general solution for two-dimensional flow through arbitrary cross-sections in terms of eigen function expansions for the geometry of these cross-sections. Later, oscillatory flow of a dusty viscous fluid in a cylinder was investigated by Arunachalam et al. [64].

In part one (section A) of chapter III, the rotational motion of a dusty viscous fluid contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface is considered. This paper extends the analysis of

Bhattacharyya [65] to observe the effects of dust particles on the fluid flow.

Dusty non-Newtonian fluid flows between two oscillating cylinders were investigated by Mukherjee et al. [57] and Mukherjee and Maiti [58]. In part two (section B) of chapter III we investigate the flow of elastico-viscous liquid embedded with particles in an oscillating cylinder. Explicit expressions are obtained for the velocities of liquid and dust particles by the technique of Laplace transform. Numerical computation of the velocity fields are carried out for different values of mass concentration, relaxation time of dust particles and elastic elements in the liquid.

(b) Dusty fluid flows in a rotating system

The study of dusty fluid flows in a rotating system is of interest to many research workers in fluid mechanics due to its varied practical importance. These studies have some bearing on the problems of atmospheric pollution. Oscillatory flow of a dusty gas in a rotating system was considered by Datta and Jana [66]. Unsteady flow of a dusty gas due to non-torsional oscillation of a plate in a rotating frame was investigated by Jana and Datta [67]. An exact solution for the flow of an incompressible viscous dusty gas induced by two infinitely extended parallel plates when the lower plate is oscillating harmonically and the upper one is at rest in a rotating frame of reference was studied by Mitra [68]. In the second part (section A) of chapter III, the unsteady flow of semi-infinite dusty fluid on a horizontal plate in a rotating frame is considered when the horizontal plate is impulsively brought to rest from the state of uniform motion. Due to complexity of the problem, we restrict ourselves to the case of large time only.

Boundary layer flow of viscous fluid over a flat plate in a rotating frame of reference has gained considerable attention due to its varied and wide applications in the areas of geophysics and astrophysics. Gupta and Pop [69] and Gupta [70] analysed the boundary layer growth in a liquid with suspended particles.

But these investigations are confined to the cases of dusty viscous fluid flows. However, the study of dusty non-Newtonian fluid flows over a flat plate in a rotating system is of considerable importance in various technical problems. In the third part of chapter III (section B), we extend the analysis of Gupta and Pop [69] to cover a wider class of elastico-viscous liquid, viz. Walters' liquid B ' (short memories). The initial value problem is solved by Laplace transform technique and the qualitative features of the unsteady boundary layer with particular reference to the effect of rotation through inertial oscillations and the establishment of the Ekman boundary layer are discussed. It is found that the frequency of inertial oscillations decreases with the increase of either mass concentration or elastic element.

(c) Dusty non-Newtonian fluid flows between parallel plates

The problem of unsteady flow of liquid between parallel plates is of common interest as it has varied applications in many engineering and technological problems. Moreover, the equations of motion are greatly simplified so that one can find an exact solution of the problem. Vimala [71] studied the flow of dusty gas induced by two infinite flat plates oscillating in their own planes. Subsequently, Mukherjee et al. [72], Mathur et al. [73] and Kishore and Pandey [74] analysed the flow of a dusty gas between two parallel flat plates under different conditions. Recently, Mitra [68] discussed the oscillatory flow of a dusty gas between two parallel flat plates in a rotating frame while Mitra and Bhattacharyya [75] investigated the unsteady flow of a dusty gas between two parallel flat plates, one being at rest and other begins to oscillate harmonically in its own plane. However, so far as we are aware, the work on dusty non-Newtonian fluid flows between parallel plates is not reported in literature.

In part one of chapter III (section B) of the present thesis, we discuss the unsteady flow of dusty elastico-viscous liquid between two oscillating parallel flat plates. The analytical expressions for the velocities of the dusty liquid and dust particles are obtained by the technique of Laplace transform. The effects of the elastic element in the liquid, the mass concentration and the relaxation time of the dust particles on the velocity fields, the skin-friction at the lower plate and the volume flow rate in between the plates presented numerically/graphically. It is found that both skin-friction at the lower plate wall and volume flow rate in between the plates increase in the presence of the elastic element in the liquid.

1.6 Unsteady convective diffusion

We frequently encounter with the problems of dispersion of soluble matter in laminar flow in chemical industries, tracer analysis, chromatography and physiological systems. When a solute is released in a solvent which flows steadily under laminar conditions through a circular tube, it spreads out longitudinally about a plane moving at the mean speed of the flow under the combined effect of lateral molecular diffusion and longitudinal convection, and longitudinal molecular diffusion. Extensive investigations were made, both theoretically and experimentally, by many authors to analyse the dispersion of soluble matter in laminar flow.

Taylor [76] investigated the dispersion of a soluble matter in an incompressible viscous fluid flowing in a circular tube under laminar conditions considering the unsteady convective diffusion in a steady flow by the dispersion model

$$\frac{\partial C_m}{\partial t} + u_m \frac{\partial C_m}{\partial x} = K \frac{\partial^2 C_m}{\partial x^2} , \qquad (1.20)$$

where t is time, x is the axial co-ordinate, C_m is the area average concentration, u_m is the average velocity and K is the dispersion coefficient which depends on the physical parameters but not on t or x. The analysis of Taylor [76] on his conceptual dispersion model is applicable only for large values of time t. On the other hand, Lighthill [77], neglecting axial molecular diffusion, obtained an exact solution of the unsteady convective diffusion equation which is asymptotically valid for small t. To be more precise, Taylor's dispersion model is valid if the time after the injection of the solute exceeds $0.5a^2/D$ ('a' is the tube radius, D the molecular diffusivity) while Lighthill's model takes account of the initial action of diffusion in front of the concentration distribution and is valid for time less than about $0.1\ a^2/D$. However, Gill [78] and Gill and Ananthakrishnan [79] obtained an exact solution for the local concentration using the series expansion of the dispersion model. Later, Gill and Sankarasubramanian [80] constructed a generalised dispersion model for the above steady flow, which is valid for all t, by involving an infinite set of time-dependent coefficients and obtained exact solution for the local concentration C. Since K is a function of time t in this model, this might account for the considerable amount of scatter which is generally found in experimental data on dispersion. Subsequently, using the generalised dispersion model, they [81,82,83] studied, respectively, the dispersion of a non-uniform slug, non-uniformly distributed time-variable continuous source and the interphase mass transfer in fully developed time-dependent flow. Recently, Krishnamurthy and Subramanian [84] formulated convective diffusion theory for the predictive modelling of field-flow fractionation columns used for the separation of colloidal mixture. Jayaraj and Subramanian [85] used the truncated versions of generalised dispersion theory to study the relaxation phenomena in field-flow-fractionation. Annapurna and Gupta [86] and Gupta [87] analysed the unsteady magnetohydrodynamic convective diffusion in electrically conducting fluid flowing in a parallel plate channel.

The first paper of the last chapter of this thesis deals with an exact analysis of the dispersion of solute in an elctrically conducting fluid flowing between two parallel plate porous channel in the presence of uniform transverse magnetic field. Using a generalised dispersion model, which is valid for all time after the injection of solute in the flow, the dispersion coefficients as functions of time are evaluated. The behaviour of the coefficients in the dispersion equation is explained on physical grounds and also numerical calculations are carried out to get a physical insight into the problem. The results of the present investigation are likely to have applications to situations where tracers are used for measuring the flow rate in a porous-walled channel.

1.7 Free convection and mass transfer in a porous medium

Fluid flow due to density differences in the force field is generally called free convection. Such external forces are gravity forces, and the density difference, a very simple case, is the result of the temperature drop between the solid surface and the fluid. Free convection flow is not of rare occurrence in nature. In fact trade winds are due to convection currents set up in the atmosphere due to unequal heating. Also land and sea breezes arise in a similar manner. Studies on free convection have long considered the problem of unsteady free convection flow past an infinite vertical plate as one of the fundamental problem in heat transfer owing to its practical applications. Free convection effects on the Stokes problem for an infinite vertical plate was investigated by Soundalgekar [88]. This problem is better known as

Stokes problem for the vertical plate. Later, Pop and Soundalgekar [89] investigated the free convection flow past an accelerated vertical infinite plate. However, in nature, along with the free convection currents caused by the temperature differences, the flow is also affected by chemical composition differences and gradients or by material or phase constitutions. This can be seen in our everyday life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by the temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as the mass transfer flow. When a mixture of gases or liquids is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from a region of high concentration to regions of low concentration. There is also a mass transfer associated with convection in which mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level. Due to applications in various technological problems and in agricultural science, effects of mass transfer on the unsteady free convective flow past an infinite porous plate with constant or variable suction were studied by Soundalgekar [90], Soundalgekar and Wavre [91,92], Soundalgekar [93] and Raptis et al. [94].

Flows through porous media are very much prevalent in nature and threfore, the study of flow through porous media has become of principal interest in many scientific and engineering applications. Also, porous media are very widely used for a heated body to keep its temperature. To make the heat insulation of the surface more effective it is necessary to study the free convection and mass transfer effects on the flow through porous medium. A number of workers studied both steady/unsteady free convection and mass transfer flows in a rotating/non-rotating systems through porous medium bounded by an infinite vertical porous plane. Raptis et al. [95,96] investigated the steady two-dimensional free convection and mass transfer flow of an incompressible viscous fluid through a porous medium bounded by a vertical infinite porous plate. Raptis et al. [95] investigated the problem when the plate was subjected to constant temperature while in a follow up paper Raptis et al. [96] investigated the case when the plate was subjected to a constant heat flux. Effects of Grashof number, modified Grashof number and permeability of the porous medium on the velocity and rate of heat transfer were discussed when the surface was subjected to a constant suction velocity. But in both the papers, free-stream velocity was considered to be zero. Unsteady free convective flow and mass transfer through a porous medium bounded by an infinite vertical limiting surface with constant suction and time-dependent temperature was studied by Raptis [97]. In further follow up programmme, Raptis [98] considered the effects of free convection and mass transfer flow through a very porous medium bounded by an infinite vertical porous plate when there was free-stream velocity. The porous plate was subjected to a constant suction, the temperature and the species concentration at the plate were considered to be constant and the flow was steady.

It is known that MHD flows have received considerable attention because of their practical applications. On the other hand the significance of suction/injection for the boundary layer control in the field of aerodynamics and space science is well recognised. A theoretical analysis of steady free convective and mass transfer flow was presented by Raptis and Kafousias [99] when a viscous, incompressible and electrically conducting fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite vertical porous plate. A magnetic field of uniform strength was applied perpendicular to the plate and constant heat flux at the plate was assumed. In part two and three of chapter IV, we have considered two problems on the free convection and mass transfer through porous medium bounded by an infinite vertical porous plate in presense of transverse magnetic field. In part two, the temperature at the plate is considered to vary with time about a non-zero constant mean while it is considered to be constant at the far field. In part three, we consider a situation where the concentration fluctuates with time about a non-zero constant mean. Analytical expressions for the velocity, temperature and concentration fields are obtained. The influence of the various parameters entering into the problem on the flow are discussed in both part two and part three.

The study of flows through porous media in a rotating system is of considerable interest in many scientific and engineering applications; viz., to the petroleum engineer concerned with the movement of oil and gas through reservoir; to the hydrologist in his study of migration of underground water etc. A number of workers [100,101,102,103] considered both steady and unsteady free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate for a fluid rotating with constant angular velocity in different physical situations. Raptis [100] considered the steady case when there was a constant heat flux at the plate while the effects of Hall current was observed by Raptis and Ram [101]. In a recent paper, Kumar and Varshney [102] considered the unsteady free convection flow of an incompressible, viscous fluid through a porous medium past an oscillating porous plate in a rotating system. Temperatures at the plate and at the far field were assumed to be constant (but different). In a more recent paper, Mahato and Maiti [103] considered a similar problem where temperature at the plate was assumed to be fluctuating with time. In part four of chapter IV, we consider free convection and mass transfer flow in a porous medium bounded by infinite vertical plate in a rotating system when there is constant heat flux at the vertical wall and species concentration oscillates with time. Expressions for the velocity, temperature and concentration fields are obtained with the help of perturbation technique and the effects of the various parameters on the velocity field are discussed. In a physically realistic situation we may assume fluctuations in velocity in far field flow region. Unsteady free convection flow and mass transfer of viscous fluid through a porous medium bounded by an infinite vertical porous plate in a rotating system when there is an oscillating free-stream velocity, is the subject of discussion in the last part of the last chapter of the thesis.

References

[1] Weissenberg, K. (1947) Nature, Vol.159,310.

[2] Merrington, A. C. (1943) Nature, Vol.152,663.

[3] Poynting, J. H. (1909) Proc. Roy. Soc., Lond., Vol. A82, 546.

[4] Astarita, G. and Marrucci, G. (1974) Principles of non-Newtonian Fluid Mechanics, Mc Graw-Hill Book Company Ltd., London.

[5] Harris, J. (1977) Rheology and non-Newtonian Flow, Longman Inc., New York.

[6] Reiner, M. (1949) Deformation and Flow,Lewis,London.

[7] Oldroyd, J. G. (1958) Proc. Roy. Soc., London, Vol. A245, 278.

[8] Oldroyd, J. G. (1950) Proc. Roy. Soc.,London, Vol. A200, 523.

[9] Oldroyd, J. G., Strawbridge, D. J. and Toms, B. A. (1951) Proc. Phys., Soc., Vol. B64, 44.

[10] Walters, K. (1960) Quart. J. Mech. Appl. Math., Vol.13, 444.

[11] Walters, K. (1962) J. Mecanique, Vol. 1, 474.

[12] Kuvshiniski, E. V. (1951) J. Exptl. Theor. Phys. (USSR), Vol. 21, 88.

[13] Sharma, S. K. (1959) ZAMM, Vol. 39, 313.

[14] Agarwal, J. P. and Jain, M. K.	(1962)	ZAMP, Vol. 13, 152.
[15] Frater, K. R.	(1964)	J. Fluid Mech., Vol. 19, 175.
[16] Gulati, S. P.	(1966)	J. Phys. Soc., Japan, Vol.21,1411.
[17] Rajagopalan, Renuka.	(1968)	J. Indian Inst. Sci., Vol. 50, 57.
[18] Siddappa, B. and Hegde, S.	(1972)	Prog. Math., Vol. 6, 15.
[19] Johri, A. K.	(1974)	Agra Univ. Res. Sci. J., Vol.23, 15.
[20] Johri, A. K.	(1978)	Indian J. Pure Appl. Math., Vol.9, 481.
[21] Richardson, E. C. and Tyler, E.	(1929)	Proc. Phys.Soc., London, Vol.42,1.
[22] Sexl, T.	(1930)	Z. Phys., Vol. 61, 346.
[23] Soundalgekar,V. M.	(1965)	A.I.A.A. J., Vol. 3, 868.
[24] Ghosh, A. K.	(1969)	Acta Tech. Academiae Sci. Hungaricae, Vol. 66, 455.
[25] Ramacharyulu, N. Ch. P.	(1971)	Bull. Tech. Univ. Istanbul, Vol.24, 19.
[26] Soundalgekar,V. M.	(1972)	Indian J. Phys., Vol. 46, 250.
[27] Soundalgekar,V. M.	(1973)	Chem. Engng. Sci., Vol. 28, 654.
[28] Greenspan, H. P.	(1968)	The Theory of Rotating Fluids, Camb. Univ. Press.

[29] Ekman, V. W. (1905) Arch. Math. Astr. Phy., Vol. 2, 1.

[30] Saffman, P. G. (1962) J. Fluid Mech., Vol. 13, 120.

[31] Kazakevich, F. P. and Krapivin, A. M. (1958) Izv. Vyssh. Uchaben. Zavedenii, Energetika, Vol. 1, 101.

[32] Sproull, W. T. (1961) Nature, Vol. 190, 976.

[33] Soo, S. L. (1961) A.I.Ch.E.J., Vol. 7, 384.

[34] Soo, S. L. (1965) I and EC Fundamentals, Vol.4, 426.

[35] Soo, S. L. (1967) Fluid dynamics of multiphase systems, Blaisdell Pub. Co., Waltham, Mass.

[36] Soo, S. L. (1969) Appl. Sci. Res., Vol.21, 68.

[37] Marble, F. E. (1963) Proc. Fifth AGARD Combust. Propul. Colloquium, 175.

[38] Pai, S. I. (1970) Techn. Note – BN – 668, Institute for Fluid Dynamics and Applied Mathematics,University of Maryland.

[39] Jain, A. C. (1975) Dynamics of dusty gases, Engineering and Environmental Fluid Mechanics, Ed. A. C. Gupta and A. K. Gupta , I.I.T., Kanpur, 3.1.

[40] Goddard, J. D. (1977) Continuum models of discrete systems, June 26 – July 2, 605.

[41] Rudinger, G. and Chang, A. (1964) Phys. Fluids, Vol. 7, 1747.

[42] Rudinger, G. (1965) A.I.A.A. J., Vol.3, 1217.

[43] Wallis, G. B. (1969) One-dimensional two-phase flow, McGraw-Hill Book Comp., New York.

[44] Davidson, J. F. (1969) J. Fluid Mech., Vol. 39, 375.

[45] Torobin, L. B. and Gauvin, W. H. (1959) Can. J. Chem. Eng., Vol.37, 129, 167, 224.

[46] Torobin, L. B. and Gauvin, W. H. (1960) Can. J. Chem. Eng., Vol.38, 142, 189.

[47] Crooke, P. S. (1975) ZAMM, Vol.55, 272.

[48] Hinch, E. J. (1977) J. Fluid Mech., Vol.83, 695.

[49] Purcell, E. M. (1978) J. Fluid Mech., Vol.84, 551.

[50] Leal, L. G. (1975) J. Fluid Mech., Vol.69, 305.

[51] Goddard, J. D. (1976) J. Fluid Mech., Vol.78, 177.

[52] Goddard, J. D. (1976) J. Non-Newtonian Fluid Mech., Vol.1, 1.

[53] Leal, L. G. (1979) J. Non-Newtonian Fluid Mech., Vol.5, 33.

[54] Srivastava, L. P. (1971) Istanbul Teknik Universitesi Bulteni, Vol.24, 19.

[55] Sharma, C. L. and Dube, S. N. (1976) Bull. de L' Acad. Polo. des Sci. Series des Sci. Tech., Vol. XXIV, 145.

[56] Bagchi, S. and Maiti, M. K. (1980) Acta Ciencia Indica, Vol. VIm, 130.

[57] Mukherjee, S., (1984) Indian J. Technol., Vol.22, 41.
Maiti, M. K.
and
Mukherjee, S.

[58] Mukherjee, S. (1986) Indian J. Technol., Vol.24, 698.
and
Maiti, M. K.

[59] Michael, D. H. (1968) Quart. J. Mech. and Appl. Math., Vol.21, 375.
and
Norey, P. W.

[60] Nath, G. (1970) Proc. Nat. Acad. Sci., India, Vol.40(A), 257.

[61] Tiwari, V. B. (1973) J. Sci. Res., Vol.23, 145.

[62] Gupta, R. K. (1977) Proc. Ind. Nat. Sci. Acad., Vol.43A, 56.
and
Gupta, S. C.

[63] Crooke, P. S. (1974) Powder Technology, Vol.9, 111.
and
Walsh, R. A.

[64] Arunachalam, P.V., (1976) Proc. 21st. ISTAM, 185.
Naidu, K. B.
and
Nirmala, C.

[65] Bhattacharyya, P. (1981) Indian J. Math., Vol.23, 65.

[66]	Datta, N. and Jana, R. N.	(1976)	Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie,Vol. 20, 71.
[67]	Jana, R. N. and Datta, N.	(1977)	Bull. Math. de la Soc.Sci.Math.de la R.S. de Roumanie, Vol 21, 309.
[68]	Mitra, P.	(1981)	Mechanique Appliquee, Vol. 26, 47.
[69]	Gupta, A. S. and Pop, I.	(1975)	Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie, Vol. 19, 291.
[70]	Gupta ,A. S.	(1977)	Analde Universitatii, Bucurasti, Mathematica, Vol. 26, 45.
[71]	Vimala, C. S.	(1972)	Def. Sci. J., Vol.22, 231.
[72]	Mukherjee, S. R., Tewari, V. D. and Bhattacharjee, S.	(1973)	Prog. of Maths., Vol. 7, 37.
[73]	Mathur, A. K., Sacheti, N. C. and Bhatta, B. S.	(1976)	The Mathematics Education, Vol. 10.
[74]	Kishore, N. and Pandey, R. D.	(1977)	J. Scientific Res., Vol. XXVIII(2), 151.
[75]	Mitra, P. and Bhattacharyya, P.	(1982)	Rev. Roum. Sci. Techn. – Mec. Appl., Vol.27, 57.

[76] Taylor, G. I. (1953) Proc. Roy. Soc., Lond., Vol. A219, 186.

[77] Lighthill, M. J. (1966) J. Inst. Math. Appls., Vol.2, 97.

[78] Gill, W. N. (1967) Proc. Roy. Soc., Lond., Vol.A298, 335.

[79] Gill, W. N. and Ananthakrishnan, V. (1967) A. I. Ch. E., Vol. 13, 801.

[80] Gill, W. N. and Sankarasubramanian, R. (1970) Proc. Roy. Soc., Lond., Vol.A316, 341.

[81] Gill, W. N. and Sankarasubramanian, R. (1971) Proc. Roy. Soc., Lond., Vol.A322, 101.

[82] Gill, W. N. and Sankarasubramanian, R. (1970) Proc. Roy. Soc., Lond., Vol.A327, 191.

[83] Sankarasubramanian, R. and Gill, W. N. (1973) Proc. Roy. Soc., Lond., Vol.A333, 115.

[84] Krishnamurthy, S. and Subramanian, R. S. (1977) Seperation Science, Vol.12, 347.

[85] Jayaraj, K. and Subramanian, R. S. (1978) Seperation Science and Technology, Vol.13, 791.

[86] Annapurna, N. and Gupta, A. S. (1979) Proc. Roy. Soc., Lond., Vol.A367, 281.

[87] Gupta, P. S. (1980) Chem. Engg. Commun., Vol.7, 301.

[88] Soundalgekar,V.M. (1977) Trans. ASME J. Heat Transfer, Vol.99, 499.

[89] Pop, I. and Soundalgekar, V. M. (1980) ZAMM, Vol.60, 167.

[90] Soundalgekar,V.M. (1976) Proc. Indian Acad. Sci., Vol.84A, 194.

[91] Soundalgekar,V.M. and Wavre, P. D. (1977) Int. J. Heat Mass Transfer, Vol.20,1363.

[92] Soundalgekar,V.M. and Wavre, P. D. (1977) Int. J. Heat Mass Transfer, Vol.20,1375.

[93] Soundalgekar,V.M. (1979) Trans. ASME J. Appl. Mech., Vol.46, 757.

[94] Raptis, A. A., Perdikis, C. and Tzivanidis, G. J. (1981) J. Eng. & Appl. Sci., Vol.1, 139.

[95] Raptis, A., Tzivanidis, G. and Kafousias, N. (1981) Lett. Heat Mass Transfer,Vol.8,417.

[96] Raptis, A. A., Kafousias, N. G. and Massalas, C. V. (1982) ZAMM, Vol.62, 489.

[97] Raptis, A. A. (1983) Energy Research, Vol.7, 385.

[98] Raptis, A. (1984) Arch. Mech., Vol.36, 307.

[99] Raptis, A. and Kafousias, N. (1982) Can. J. Phys., Vol.60, 1725.

[100] Raptis, A. (1983) Int. Comm. Heat Mass Transfer, Vol.10, 141.

[101] Raptis, A. and Ram, P. C. (1984) Int. Comm. Heat Mass Transfer, Vol.11, 385.

[102] Kumar, Komal. and Varshney, C. L. (1987) Reg. J. Energy Heat Mass Transfer, Vol.9, 1.

[103] Mahato, J. P. and Maiti, M. K. (1988) Indian J. Technol., Vol.26, 255.

CHAPTER - II

UNSTEADY FLOW OF ELASTICO-VISCOUS LIQUID

PART ONE

UNSTEADY CHANNEL FLOW OF A NON-NEWTONIAN FLUID *

2.1 Introduction

A number of workers studied non-Newtonian fluid flow between two parallel plates due to its applications in various fields of science and technology. Sharma [1] considered plane Poiseuille flow through tube and case of a parallel plate visco-meter. Agarwal and Jain [2] extended the paper of Sharma [1] by introducing magnetic field in the system. He considered the fluid to satisfy the Oldroyd model. Frater [3] discussed the flow of an elastico-viscous liquid between torsionally oscillating discs. The plane Poiseuille flow, plane Couette flow and the Couette flow of micropolar fulids was investigated by Rajagopalan [4]. Subsequently, Johri [5] discussed the problem of an elastico-viscous flow (Rivlin-Ericksen model) induced by circular oscillations of two infinite parallel discs. Later, Johri [6] studied the problem of unsteady slow flow of Oldroyd B-liquid between two infinite parallel discs. The flow was induced by the elliptic harmonic oscillations of the discs.

Here we consider the flow of visco-elastic fluid (Kuvshiniski model [7]) between two oscillating parallel flat plates as we are aware of very little work [8,9] in this model.

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2.2 Basic equations and their solutions

We consider the flow of an incompressible elastico-viscous liquid bounded by two infinite parallel flat plates executing simple harmonic oscillations with a frequency ω in their own planes $y = \pm h$. We assume that the fulid was at rest for $t \leq 0$. Since the plates are infinite, the flow is a parallel flow in which the stream lines are along the x-axis and the velocities are functions of distance y and time t only.

Under these assumptions, the equations (1.12) – (1.15) reduce to

$$\left(1 + \lambda_o \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \quad , \tag{2.1}$$

where u is the velocity in the direction of x, λ_o is the relaxation time and ν is the kinematic coefficient of viscosity.

The flow is generated by the non-torsional oscillations of the plates given by

$$u = a \sin \omega t \text{ at } y = \pm h \quad , \tag{2.2}$$

where a is the amplitude of oscillation and ω is the imposed oscillations. For a liquid at rest for all $t \leq 0$, it may be assumed that the initial state of stress is zero. The initial conditions are

$$u\,(y,t) = 0, \quad \frac{\partial u(y,t)}{\partial t} = 0 \text{ at } t = 0 \text{ for all } y. \tag{2.3}$$

Introducing non-dimensional parameters $u' = \frac{u}{h\,\omega}$, $t' = t\omega$, $y' = \frac{y}{h}$, $\alpha = \lambda_0 \omega$, $R = \frac{\nu}{h^2 \omega}$, $a' = \frac{a}{h\omega}$ in equations (2.1) – (2.3)

and dropping dashes, we have

$$\left(1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = R \frac{\partial^2 u}{\partial y^2} \quad , \tag{2.4}$$

$$u = \frac{\partial u}{\partial t} = 0 \text{ at } t = 0 \text{ for all } y \quad , \tag{2.5}$$

$$u = a \sin t \text{ at } y = \pm 1 \;, \; t > 0. \tag{2.6}$$

Using the Laplace transform in equation (2.4) subject to initial and boundary conditions in (2.5) and (2.6), it turns out taht the solution of the problem can be readily represented by the Laplace inversion integral in the form

$$u\,(y,t) = \frac{1}{2\pi i}\, a \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{1}{1 + p^2} \, \frac{\cosh My}{\cosh M} \, e^{pt} \, dp \;, \tag{2.7}$$

$p > 0$, γ is greater than the real part of the singularities of the integrand and

$$M = \sqrt{[p\,(1 + \alpha\, p)\, /\, R\,]} \quad .$$

Using the theory of residues, we evaluate the integral (2.7) and the expression for the velocity field becomes

$$\begin{aligned} u\,(y,t) = {} & \frac{a}{E_3^2 + E_4^2} \, [\, (E_1E_3 + E_2E_4\,)\sin t \\ & + (\, E_2E_3 - E_1E_4\,) \cos t\,] \\ & + a\,\pi\, R \sum_n \sum_{i=1}^{2} \frac{(-1)^n(2n+1)}{{p_n^i}^2 + 1}\, Q(p_n^i)\,.\, \cos\left(\frac{2n+1}{2}\right)\pi y \; e^{p_n^i\, t} \;, \end{aligned} \tag{2.8}$$

where

$E_1 = \cosh Cy \cos Dy,$

$E_2 = \sinh Cy \sin Dy,$

$$E_3 = \cosh C \ \cos D ,$$

$$E_4 = \sinh C \ \sin D ,$$

$$C, D = \sqrt{[\sqrt{(1 + \alpha^2)} \mp \alpha] / \sqrt{2R}} ,$$

p_n^i 's are the roots of the quadratic

$$\alpha \, {p_n^i}^2 + p_n^i + R \left(\frac{2n + 1}{2}\right)^2 . \, \pi^2 = 0 \tag{2.9}$$

and

$$Q \, (p_n^i) = \frac{1}{(1 + 2 \alpha \, p_n^i)} .$$

The first term of Eqn. (2.8) (under bracket) represents the steady state velocity field and the last infinite sum corresponds to the transient component of the solution which decays exponentially as t→ ∞ and the final steady state oscillations is achieved. u (y,t) may also be represented by

$$u \, (y,t) = A \sin (t + \theta) + Tr. , \tag{2.10}$$

where A is amplitude of oscillations, θ is phase difference with the oscillations of the plate and Tr. is the transient part of the velocity profile.

The skin-friction on the upper plate wall is

$$\tau_{xy} = \left[\left(1 - \alpha \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial y} \right]_{y = 1}$$

$$= \frac{a}{E_3^2 + E_4^2} [(E_9 + \alpha E_{10}) \sin t + (E_{10} - \alpha E_9) \cos t]$$

$$+ a \pi R \sum_n \sum_{i=1}^{2} \frac{2n + 1}{{p_n^i}^2 + 1} . Q(p_n^i) \, e^{p_n^i t} . \left(\frac{2n + 1}{2}\right) \pi . (\alpha p_n^i - 1), \tag{2.11}$$

where

$$E_9 = E_5 \, E_7 + E_6 \, E_8 ,$$

$$E_{10} = E_5 \, E_8 - E_6 \, E_7 ,$$

$$E_5 = C \, E_3 + D \, E_4 ,$$

$$E_6 = C\,E_4 - D\,E_3 \quad ,$$
$$E_7 = \sinh C \cos D \quad ,$$
$$E_8 = \cosh C \sin D \quad .$$

2.3 Discussion

To have a physical insight into the problem, velocity distribution is calculated for different values of α. Figure 2.1 shows that maximum velocity is shifted towards the axis of the channel in presence of elastic elements in the liquid but for Newtonian liquid the case is reversed. It can be remarked that flow of elastico-viscous liquid has a boundary layer character near the plate walls. Figure 2.1 also depicts that effect of elasticity is more prominent near the axis of the channel.

From table 2.1, it may be concluded that amplitude of steady state oscillations of the velocity profile increases with the increase in the elastic parameter. There will be a phase lag of oscillations on the axis of the channel. Phase lag increases with the increase in the value of elastic parameter.

Table 2.2 shows that the effect of elastic element is to increase the magnitude of shear stress on the plate wall.

Table 2.1: Amplitude (A) and phase difference (θ) of the velocity profile at $y = 0$.

α	A	θ
0	0.0999999	-4.9999×10^{-4}
1	0.1000499	-5.0016×10^{-4}
2	0.1000521	-5.0036×10^{-4}
3	0.1001500	-5.0049×10^{-4}
4	0.1002002	-5.0066×10^{-4}
5	0.1002504	-5.0082×10^{-4}

Table 2.2: Shear stress (τ_{xy}) at the upper plate.

t	$\alpha = 1$	$\alpha = 2$	$\alpha = 2.25$
1	-0.0897863	0.2881691	0.3091253
1.5	7.00048×10^{-3}	-0.1274758	-0.1462511
5	0.0275514	0.1739753	0.1973641

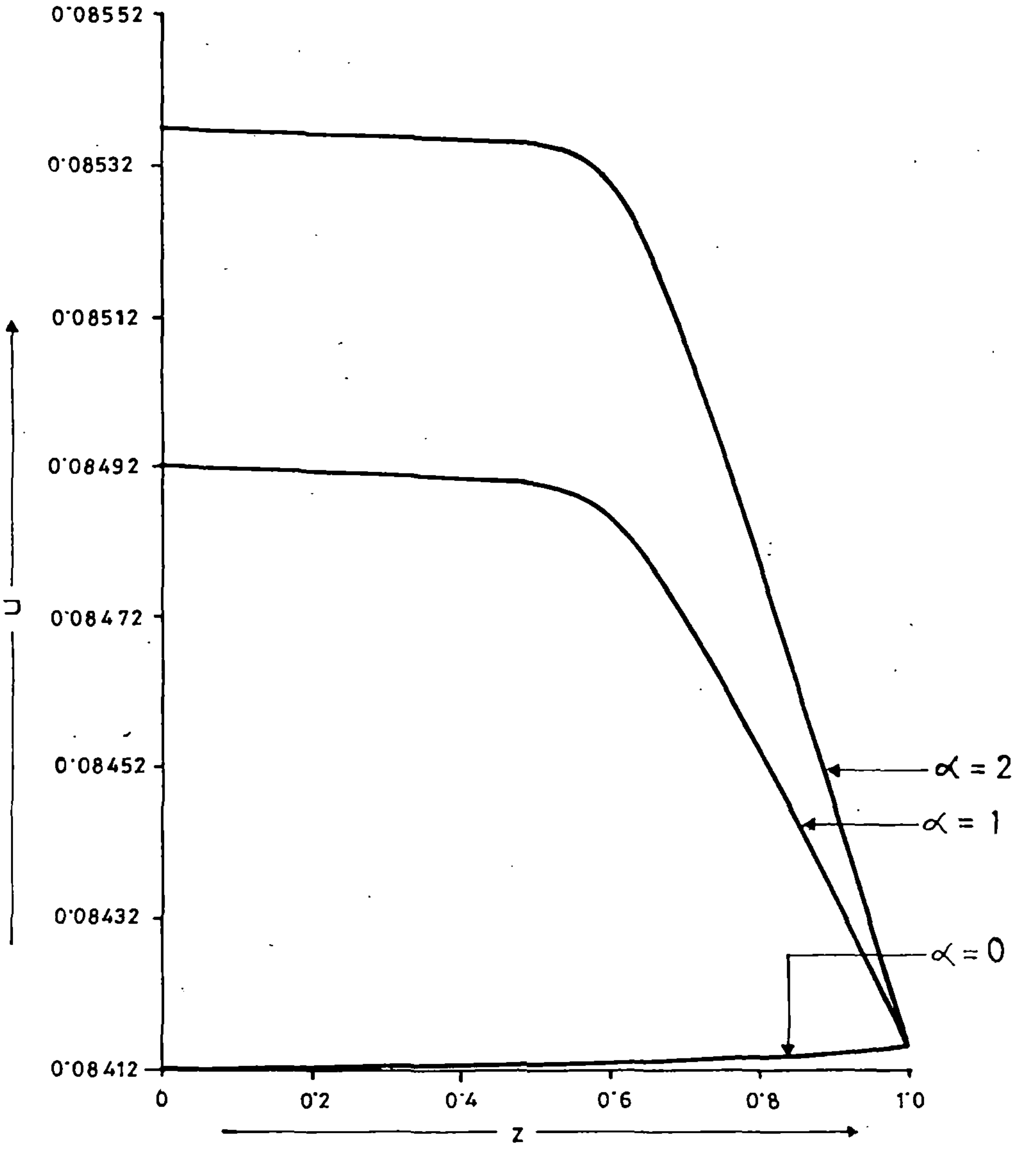

Fig. 2·1. Velocity profile of liquid particles at t=1 when a=0·1 , R = 1000 .

PART TWO

OSCILLATING FLOW OF VISCO-ELASTIC OLDROYD FLUID IN A LONG CIRCULAR TUBE IN PRESENCE OF MAGNETIC FIELD *

2.4 Introduction

Oscillating flow of viscous fluid in a long circular tube under the influence of a pressure gradient was investigated theoretically and experimentally by Richardson and Tyler [10] and theoretically by Sexl [11].

Many common liquids such as oils, certain paints, blood, polymer solutions, some organic liquids and many new materials of industrial importance exhibit both viscous and elastic properties. Based on the macroscopic rheological behaviour of real materials, Oldroyd [12,13] formulated rheological equations of state for a class of incompressible visco-elastic liquids and initiated the study of visco-elastic flows. Based upon Oldroyd's equation of state, the flow phenomena have received a considerable attention for various configurations. Johri and Singhal [14] investigated the problem of oscillating flow of visco-elastic liquid of Maxwell type in a long circular tube under the influence of periodic pressure gradient.

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The object of the present investigation is to discuss the oscillating flow of an incompressible electrically conducting visco-elastic (Oldroyd) liquid in a long circular tube in the presence of magnetic field of uniform strength. The effects of magnetic field and elastic elements in the liquid are shown in tabular form. In the presence of magnetic field, the velocity of Newtonian liquid is more than that of visco-elastic Oldroyd type liquid and decreases with the increase in elastic elements in the liquid. The effect of elasticity on the shear stress at the wall of tube is to increase its magnitude. Both the volume flow rate and shear stress increase with the increasing magnetic field. It is noted that if the oscillation be of high frequency, a boundary layer is formed close to the wall of tube. Such type of flow occurs e.g. under the influence of a reciprocating piston.

2.5 Mathematical formulation and solution of the problem

We consider axisymmetric flow of an electrically conducting (of conductivity σ) elastico-viscous liquid obeying (1.3) – (1.5) in presence of magnetic field of uniform strength (B_o) within a circular cylinder of radius 'a'. The time-dependent excitation is caused by the prescribed time-dependent pressure gradient.

Referring the problem to cylindrical polar coordinates (r,θ,x), the non-dimensional form (dropping dashes) of the governing equation of motion in the direction of x is

$$(1 + \alpha_1 \frac{\partial}{\partial t}) \frac{\partial u}{\partial t} =- (1 + \alpha_1 \frac{\partial}{\partial t}) \frac{\partial p}{\partial x}$$

$$+ (1 + \alpha_2 \frac{\partial}{\partial t}) (\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r})$$

$$- M^2 (1 + \alpha_1 \frac{\partial}{\partial t}) u , \qquad (2.12)$$

where

$$x' = \frac{x}{a} , \quad t' = \frac{t\nu}{a^2} , \quad u' = \frac{ua}{\nu} , \quad p' = \frac{pa^2}{\rho\nu^2} , \quad r' = \frac{r}{a},$$

and $\alpha_1 = \lambda_1 \frac{\nu}{a^2}$, $\alpha_2 = \lambda_2 \frac{\nu}{a^2}$ are elastic parameters and $M = \frac{B_0^2 \sigma a^2}{\rho \nu}$ is the magnetic parameter (ν is the kinematic coefficient of viscosity).

We assume $-\frac{\partial p}{\partial x} = \mathrm{Re}(K e^{i\omega t})$. (2.13)

The boundary condition for the problem is

$u = 0$ on $r = 1$. (2.14)

We assume the velocity component in the form

$u = \mathrm{Re} f(r) e^{i\omega t}$, (2.15)

where $f(r)$ is a function of r to be determined.

On substituting (2.15) in (2.12), we get

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \frac{(i\omega + M^2)(1 + i \omega \alpha_1)}{(1 + i \omega \alpha_2)} f + \frac{K(1 + i \omega \alpha_1)}{(1 + i \omega \alpha_2)} = 0, \qquad (2.16)$$

whence the solution satisfying the condition (2.14) will be given by

$$u (r,t) = - \frac{K}{(i\omega + M^2)} \left\{ \frac{J_0 (r K_1)}{J_0 (K_1)} - 1 \right\} e^{i\omega t} , \qquad (2.17)$$

where

$$K_1^{\,2} = - \frac{(\, i\omega + M^2 \,)(\, 1 + i\,\omega\,\alpha_1)}{(\, 1 + i\,\omega\,\alpha_2)} ,$$

$$= K_2^{\,2}\ (\cos 2\alpha \ - i \sin 2\alpha \,),$$

$$K_2^{\,2} = \frac{1}{(1+\omega^2\alpha_2^2)} \left[\left\{ \omega^2\,\alpha_1 - M^2 - \omega^2\,\alpha_2(1 + M^2\,\alpha_1)^2 \right\}^2 + \omega^2 \left\{ 1 + M^2\,\alpha_1 - \alpha_2(\,M^2 - \omega^2\,\alpha_1) \right\}^2 \right]^{1/2},$$

$$\tan 2\,\alpha = \frac{\omega\,\{\,1 + M^2\,\alpha_1 - \alpha_2(\,M^2 - \omega^2\,\alpha_1)\,\}}{\{\,\omega^2\,\alpha_1 - M^2 - \omega^2\,\alpha_2\,(\,1 + M^2\alpha_1)\,\}} .$$

The volume flow rate is given by

$$Q = - \frac{\pi\,K}{(\,i\omega + M^2\,)} \left[\frac{2\,J_1(K_1)}{K_1\,J_0(K_1)} - 1 \right] e^{i\omega t}$$

$$= - \frac{\pi\,K}{(i\,\omega + M^2)} \left[\frac{K_1^2}{8} - \frac{K_1^4}{32.3} + \,.........\right] e^{i\omega t} . \qquad (2.18)$$

For small values of K_2, i.e., for slow vibrations,

$$u = \frac{K_2^2\,(\,1 - r^2\,)\,\cos\left\{\omega\,t - (\,2\,\alpha + \zeta\,)\right\}}{4\,(M^4 + \omega^2\,)^{3/2}} \qquad (2.19)$$

where $\tan\zeta = \dfrac{\omega}{M^2}$.

So, in this case, there is a phase lag ($2\alpha + \zeta$) from the phase of prescribed pressure gradient.

Now for large values of K_2, we use asymptotic expression

$$J_0(z) \sim \sqrt{\frac{2}{\pi z}} \quad e^{\,i\,(z - \frac{\pi}{4})}$$

whence

$$u = -\frac{K}{M^4 + \omega^2}\Big[R \Big\{ M^2 \cos(\omega t + R_1) + \omega \sin(\omega t + R_1)\Big\}$$

$$- \Big\{ M^2 \cos \omega t + \omega \sin \omega t \Big\} \Big] \quad , \qquad (2.20)$$

where

$$R_1 = K_2(r - 1)\cos \alpha,$$

$$R = \frac{1}{(r)^{1/2}} e^{-K_2(1 - r)\sin \alpha} \; .$$

It is evident from (2.20) that for large values of K_2, the first term within third brackets in (2.20) decreases rapidly and for greater distances only the second term, which does not depend on the distance (1−r) from the wall of the cylinder, remains, since at large distance from the wall fluid moves as if it were frictionless. The flow has a distinct boundary layer character.

The non-dimensional shear stress on the wall of the tube is given by

$$\tau\big|_{r=1} = \Big[1 - (\alpha_1 - \alpha_2)\frac{\partial}{\partial t}\Big]\Big(\frac{\partial u}{\partial r} - \frac{u}{r}\Big)$$

$$= \frac{KK_1}{i\omega + M^2} \; \frac{J_1\,(K_1)}{J_0\,(K_1)} \Big[1 - (\alpha_1 - \alpha_2)\,i\omega\Big] . e^{i\omega t} . \qquad (2.21)$$

2.6 Numerical results and discussion

The effects of magnetic field and elastic elements over flow profile are shown in tabular form (tables 2.3 − 2.5). It is seen from table 2.3 that in presence of magnetic field, the velocity of Newtonian liquid is more than that of visco-elastic Oldroyd type liquid and it decreases with the increase in elastic elements. The effect of elasticity is to increase the shear stress on wall of the cylinder (table 2.4). It is clear from the table 2.5 that both the volume flow rate and shear stress increase with the increasing magnetic field.

Table 2.3: Effect of elastic parameters on the velocity profile when $M^2 = 1$, $\omega = 1$.

u	$r=0$	$r=0.2$	$r=0.4$	$r=0.6$	$r=0.8$	$r=1.0$
$\alpha_1=2, \alpha_2=0.2$	0.433673	0.412273	0.357052	0.265013	0.143392	0
$\alpha_1=1, \alpha_2=0.05$	0.434042	0.413166	0.358905	0.271357	0.151360	0
$\alpha_1=0, \alpha_2=0$	− 0.687967	−0.655684	−0.558585	−0.474683	−0.282026	0

Table 2.4: Effect of elasticity on the shear stress when $M^2 = 1$, $\omega = 1$.

	$\alpha_1 = 0,\ \alpha_2 = 0$	$\alpha_1 = 1,\ \alpha_2 = 0.05$	$\alpha_1 = 2,\ \alpha_2 = 0.2$
$\tau\big\|_{r=1}$	0.5705354	0.7308212	− 0.7327728

Table 2.5: Effect of magnetic parameter on shear stress and volume flow rate when $\alpha_1 = 1$, $\alpha_2 = 0.05$, $\omega = 1$.

	$M^2 = 1$	$M^2 = 4$	$M^2 = 9$
$\tau\big\|_{r=1}$	0.7308212	2.2408276	3.2824201
Q_λ	-8.3337×10^{-4}	−0.5246432	− 3.3630692

REFERENCES

[1] Sharma, S. K. (1959) ZAMM, Vol.39, 313.

[2] Agarwal, J. P. and Jain, M. K. (1962) ZAMP, Vol.13, 152.

[3] Frater, K. R. (1964) J. Fluid Mech., Vol. 19, 175.

[4] Rajagopalan, Renuka. (1968) J. Indian Inst. Sci., Vol.50, 57.

[5] Johri, A. K. (1974) Agra Univ. Res.Sci.J.,Vol.23, 15.

[6] Johri, A. K. (1978) Indian J.Pure Appl.Math.,Vol.9,481.

[7] Kuvshiniski, E. V. (1951) J. Exptl. Theor. Phys. (USSR), Vol.21, 88.

[8] Soundalgekar, V. M. (1981) Indian J.Theoret.Phys.,Vol.29, 23.

[9] Gupta, Prem Chand., Nair, B. K. and Sharma, R. G. (1982) Indian J.Theoret.Phys.,Vol.30, 79.

[10] Richardson, E. C. and Tyler, E. (1929) Proc. Phys. Soc., Lond., Vol.42, 1.

[11] Sexl, T. (1930) Z. Phys., Vol.61, 346.

[12] Oldroyd, J. G. (1950) Proc. Roy.Soc., Lond.,Vol.A200,523.

[13] Oldroyd, J. G. (1951) Quart. J. Mech. Appl. Math., Vol.4, 271.

[14] Johri, A. K. and Singhal, R. K. (1980) Acta Ciencia Indica, Vol.VIm, 63.

CHAPTER - III

DUSTY FLUID FLOWS

SECTION - A

DUSTY NEWTONIAN FLUID FLOWS

PART ONE

ROTATIONAL MOTION OF A DUSTY VISCOUS FLUID CONTAINED IN THE SEMI-INFINITE CIRCULAR CYLINDER DUE TO AN INITIALLY APPLIED IMPULSE ON THE SURFACE *

3.1.1 Introduction

The unsteady motion of fluid resulting due to the pure rotation of a solid boundary or due to the application of a uniformly distributed shear stress along a solid boundary is of both theoretical and practical significance in fluid mechanics. A number of workers studied both steady and unsteady two-dimensional axi-symmetric rotational flow of a viscous fluid in view of its growing importance in various technical problems. Khamrui [1] analysed the slow steady motion of an infinite viscous fluid due to the rotation of a circular cylinder. Iben [2] considered the non-stationary, plane circular-symmetric flow of a viscous fluid which forms itself within as well as outside a rotating infinitely long circular cylinder. Bhattacharyya [3] studied the rotational motion produced in an enclosed fluid, contained in a circular cylinder of infinite depth. The disturbance was generated on the surface of the fluid by an impulsive couple. Later Mukherjee and Bhattacharyya [4] studied the rotational flow of viscous fluid due

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to the rotation of a circular cylinder or by the action of shearing stress on the boundary.

For quite a number of reasons, be they scientific or practical engineering ones, particles are added to fluids. Mixtures of fluid and solid lumps or particles are common in various fields of engineering-hydraulic, mechanical and chemical- and of geophysics and considerations of their motion raise many puzzling dynamical questions. Saffman [5] proposed dusty fluid model in terms of a large number density N (x,t) of undeformable spherical particles suspended in an incompressible fluid. Using the formulation of Saffman, many authors studied a number of dusty gas problems and the results were well documented in a review by Marble [6].

In the present investigation we extend the analysis of Bhattacharyya [3] to observe the effects of dust particles on the fluid flow. It is observed that the effect of mass concentration of dust particles on the flow field is to increase the velocity field of dusty fluid. This problem may have some bearing on the problems of transport of solid particles by air or water and motion of solid particles in a rocket motor exhaust.

3.1.2 Formulation of the problem

Initially the liquid and dust particles are at rest. Consider the flow of a dusty liquid in a long circular cylinder of radius 'a'. Disturbance is set up by an impulse of the shearing force on the surface. Referring the problem to cylindrical polar co-ordinates (r,θ,z), we take the z-axis along the axis of the cylinder and the origin on the surface of the fluid. The symmetry consideration gives

$$\left.\begin{aligned} & u_1 = u_3 = 0,\ v_1 = v_3 = 0,\ u_2 = u_2\,(r,z,t), \\ & v_2 = v_2(r,z,t) \text{ and } \frac{\partial}{\partial\theta} = 0\ , \end{aligned}\right\} \qquad (3.1.1)$$

where u_2 and v_2 are circumferential velocities of liquid and dust particles respectively. Since the distribution of dust particles is uniform, the number density N of the particles equals N_0, a constant throughout the motion. Using equations (1.16) – (1.19), the linearised equations of motion of dusty fluid and that of dust particles become

$$\rho\,\frac{\partial u_2}{\partial t} = \mu\left(\frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r}\frac{\partial u_2}{\partial r} - \frac{u_2}{r^2} + \frac{\partial^2 u_2}{\partial z^2}\right) + KN_0(v_2 - u_2), \qquad (3.1.2)$$

$$m\,\frac{\partial v_2}{\partial t} = K\,(u_2 - v_2) \qquad . \qquad (3.1.3)$$

Initial and boundary conditions are

$$\left.\begin{aligned} & u_2 = 0 \text{ at } t \le 0 \text{ for all } z\ , \\ & u_2 \longrightarrow 0 \text{ as } z \to \infty\ , \\ & u_2\big|_{r=a} = 0,\ z \ge 0, \end{aligned}\right\} \qquad (3.1.4)$$

$P_{\theta z}\big|_{z=0} = \mu\,\left(\frac{\partial u_2}{\partial z}\right)_{z=0}$ is prescribed as function of r and t, $r \le a$, $t \ge 0$.

Introducing the following non-dimensional quantities

$$u = \frac{u_2\,a}{\nu}\ ,\ v = \frac{v_2\,a}{\nu}\ ,\ t_1 = \frac{t}{\tau}\ ,\quad r_1 = \frac{r}{(\nu\tau)^{1/2}},\quad z_1 = \frac{z}{(\nu\tau)^{1/2}}\ .$$

$a_1 = \dfrac{a}{(\nu\tau)^{1/2}}$, $f = \dfrac{mN_0}{\rho}$ (mass concentration of dust particles),

$\tau = \dfrac{m}{K}$ (relaxation time of dust particles)

in (3.1.2) – (3.1.4), we get (dropping suffices)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} + f(v-u) \quad , \tag{3.1.5}$$

$$\frac{\partial v}{\partial t} = (u - v) \quad , \tag{3.1.6}$$

$$u = 0 \text{ at } t \le 0 \text{ for all } z, \tag{3.1.7}$$

$$u \longrightarrow 0 \text{ as } z \longrightarrow \infty, \tag{3.1.8}$$

$$u\Big|_{r=a} = 0, \qquad z \ge 0 \quad , \tag{3.1.9}$$

$$p_{\theta z}\Big|_{z=0} = \frac{\partial u}{\partial z}\Big|_{z=0} = F(r)\delta(t) \qquad \text{(prescribed).} \tag{3.1.10}$$

3.1.3 Method of solution

We solve the present problem by using technique of Laplace tranform. Taking Laplace transform of equations (3.1.5), (3.1.6) and using (3.1.7), we get

$$p\bar{u} = \frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r} - \frac{\bar{u}}{r^2} + \frac{\partial^2 \bar{u}}{\partial z^2} + f(\bar{v} - \bar{u}) \quad , \tag{3.1.11}$$

$$p\bar{v} = \bar{u} - \bar{v} \, , \tag{3.1.12}$$

where

$$\bar{u} = \int_0^\infty u\, e^{-pt} dt, \qquad \bar{v} = \int_0^\infty v\, e^{-pt} dt, \quad Re(p)>0.$$

Eliminating $\bar{v}$ from (3.1.11) and (3.1.12) we get

$$\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r} - \frac{\bar{u}}{r^2} + \frac{\partial^2 \bar{u}}{\partial z^2} - \left[\frac{f}{p+1} + 1\right] p\, \bar{u} = 0. \tag{3.1.13}$$

The solution of the above equation (3.1.13) can be expressed in the form

$$\bar{u}\,(r,z,p) = \sum_{n=1}^{\infty} \bar{u}_n \, , \tag{3.1.14}$$

where

$$\bar{u}_n = J_1(\alpha_n r)\,\bar{\Phi}_n(z,p) \tag{3.1.15}$$

and α_n can be determined from the equation (3.1.9) as the positive root of

$$J_1(\alpha_n a) = 0. \tag{3.1.16}$$

On substituting the value of $\bar{u}_n$ from (3.1.15) in (3.1.13) we get

$$\frac{d^2\bar{\phi}_n}{dz^2} - \left\{ \alpha_n^2 + p\left(1 + \frac{f}{1+p}\right) \right\} \bar{\phi}_n = 0. \tag{3.1.17}$$

Solution of (3.1.17) subject to Laplace transform of the condition (3.1.8)

$$\bar{\phi}_n = A_n \exp\left[-z \left\{ \alpha_n^2 + p\left(1 + \frac{f}{1+p}\right) \right\}^{1/2} \right], \tag{3.1.18}$$

where A_n is independent of z for all n.

Transformed shearing stress on the surface is given by

$$\bar{p}_{\theta z}\Big|_{z=0} = F(r), \quad r \leq a$$

$$= \sum_{n=1}^{\infty} C_n J_1(\alpha_n r), \tag{3.1.19}$$

where

$$C_n = 2a^{-2}\,[J_2(\alpha_n a)]^{-2} \int_0^a r\,F(r)\,J_1(\alpha_n r)\,dr. \tag{3.1.20}$$

Again from (3.1.14), (3.1.15) and (3.1.18) we have

$$\bar{p}_{\theta z}\Big|_{z=0} = -\sum_{n=1}^{\infty} J_1(\alpha_n r)\,A_n \sqrt{\left(\alpha_n^2 + p\left(1 + \frac{1}{1+p}\right)\right)}. \tag{3.1.21}$$

We now compare the expressions (3.1.19) and (3.1.21) to obtain

$$A_n = -C_n \left[\alpha_n^2 + p\left(1 + \frac{f}{1+p}\right) \right]^{-1/2}. \tag{3.1.22}$$

The expression for velocity becomes

$$u = -\sum_{n=1}^{\infty} C_n J_1(\alpha_n r)\, \mathcal{L}^{-1}\Big[\Big\{\alpha_n^2 + p\,\Big(1 + \frac{f}{1+p}\Big)\Big\}^{-1/2} \times \exp\Big\{ -z < \alpha_n^2 + p(1+f/(1+p)) >^{1/2} \Big\}\Big], \qquad (3.1.23)$$

where $\mathcal{L}^{-1}$ is the operator for inverse Laplace transform.

Inverse Laplace transform of (3.1.23) presents some difficulties and we restrict ourselves to calculate the velocity expression for large values of time 't' only. For large time, $p \ll 1$ and hence

$$\sqrt{\Big[\alpha_n^2 + p\Big(1+\frac{f}{1+p}\Big)\Big]} \approx \sqrt{\big[\alpha_n^2 + p(1+f)\big]}\ .$$

Then

$$\mathcal{L}^{-1}\Big[\Big\{\alpha_n^2 + p\Big(1+\frac{f}{1+p}\Big)\Big\}^{-1/2} e^{<-z\sqrt{\{\alpha_n^2 + p(1+\frac{f}{1+p})\}}>}\Big]$$

$$= \frac{1}{(1+f)^{1/2}}\, e^{-\frac{\alpha_n^2 t}{1+f}}\, \mathcal{L}^{-1}\Big[\frac{1}{p^{1/2}}\, e^{<-z(1+f)^{1/2}(p)^{1/2}>}\Big]$$

$$= \frac{1}{(1+f)^{1/2}}\, e^{-\frac{\alpha_n^2 t}{1+f}} \Big[(\pi t)^{-1/2}\, e^{-z^2(1+f)/4t}\Big]\ .$$

Equation (3.1.23) is then given by

$$u = \frac{-1}{\{\pi t(1+f)\}^{1/2}} \exp\{-z^2(1+f)/4t\} \times \sum_{n=1}^{\infty} C_n J_1(\alpha_n r) \exp\Big(-\frac{\alpha_n^2 t}{1+f}\Big)\ . \qquad (3.1.24)$$

This solution satisfies initial and boundary conditions given by (3.1.7) – (3.1.10).

3.1.4 Particular cases

CASE I. Motion due to impulsive shearing force applied within a circular area on the surface :

We take

$$F(r) = \epsilon r \quad \text{for} \quad 0 \leq r \leq b,$$
$$\qquad = 0 \quad \text{for} \quad b < r \leq a, \qquad (3.1.25)$$

where ϵ is a constant.

Relation (3.1.25) corresponds to the situation where the applied force is acting within a circular area $r = b$, the rest of the surface being kept free from the impulse.

From (3.1.20) we get

$$C_n = 2 \epsilon b^2 J_2^{-2}(\alpha_n a) J_2(\alpha_n b) / \alpha_n a^2 . \qquad (3.1.26)$$

On substituting the values of C_n in (3.1.24), we get the expression for velocity as

$$u = - \frac{2 \epsilon b^2}{a^2 \{\pi t (1 + f)\}^{1/2}} \exp \left\{ -z^2 (1 + f)/4t \right\}$$
$$X \sum_{n=1}^{\infty} J_2(\alpha_n b) J_2^{-2}(\alpha_n a) J_1(\alpha_n r) \alpha_n^{-1} \exp \left(- \frac{\alpha_n^2 t}{1+f} \right). \qquad (3.1.27)$$

In absence of dust particles ($f = 0$), the expression for velocity profile (equation (3.1.27)) becomes same as was deduced by Bhattacharyya [3] in equation (26) (when made non-dimensional).

CASE II. Flow due to applied impulsive force distributed over the circumference of the circle $r = b$; $b < a$:

We take

$$F(r) = S \delta(r - b) \qquad (3.1.28)$$

where S is constant and δ is Dirac delta function.

From (3.1.20) we get

$$C_n = 2 S b J_1 (\alpha_n b) / a^2 J_2^2 (\alpha_n a). \qquad (3.1.29)$$

On substituting the value of C_n in (3.1.24), we get the expression for velocity as

$$u = \frac{-2 S b}{a^2 (\pi t)^{1/2} (1 + f)^{1/2}} \exp\{ - z^2 (1 + f) / 4t \}$$

$$\times \sum J_1(\alpha_n r) J_1(\alpha_n b) J_2^{-2}(\alpha_n a) \exp\{ - \alpha_n^2 t/(1 + f)\}. \qquad (3.1.30)$$

In case of clean fluid (f=0), the expression for velocity profile is same as that was deduced by Bhattacharyya [3] in equation (32) (when made non-dimensional).

3.1.5 Discussion

In order to illustrate the effects of mass concentration of dust particles on the flow field for two particular cases, numerical calculations are carried out and depicted in Figures 3.1 and 3.2. Figures 3.1 and 3.2 reveal that magnitude of velocity of dusty fluid increases with the increase of mass concentration of dust particles. It is clear from both the figures that maximum velocity occurs near the axis of the cylinder for a fixed f and as f increases, maximum velocity shifts towards the wall of the cylinder.

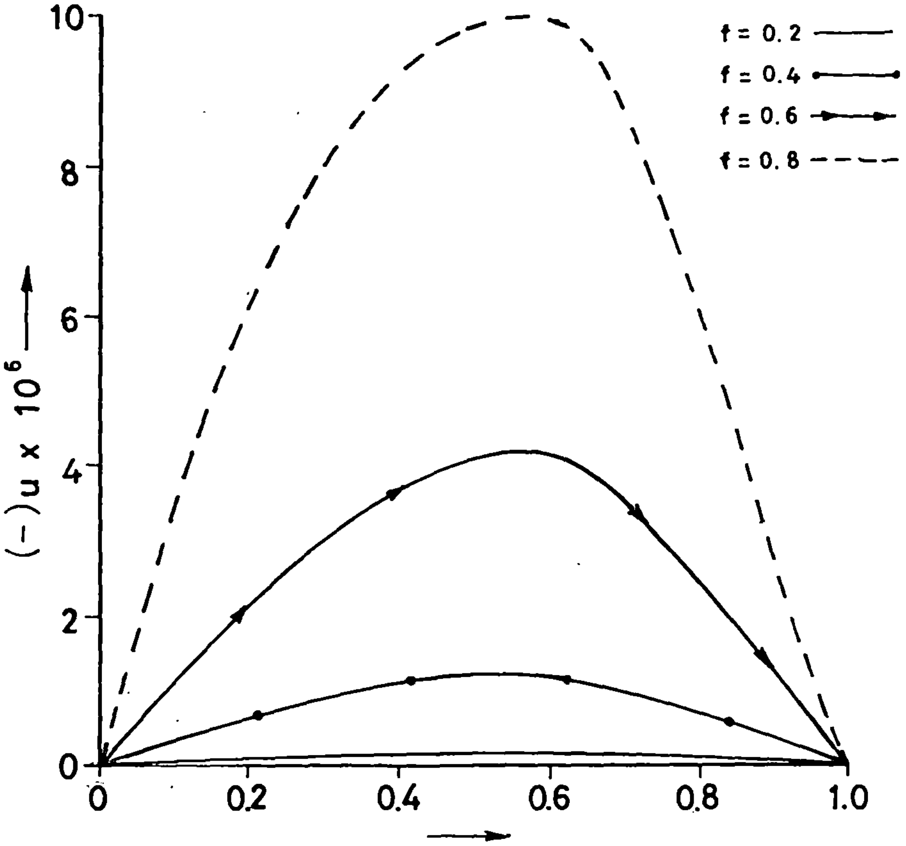

Fig.-3.1. Velocity distribution of dusty fluid for different values of f when t=1, ϵ=1, z=1, a=1, b=0.5

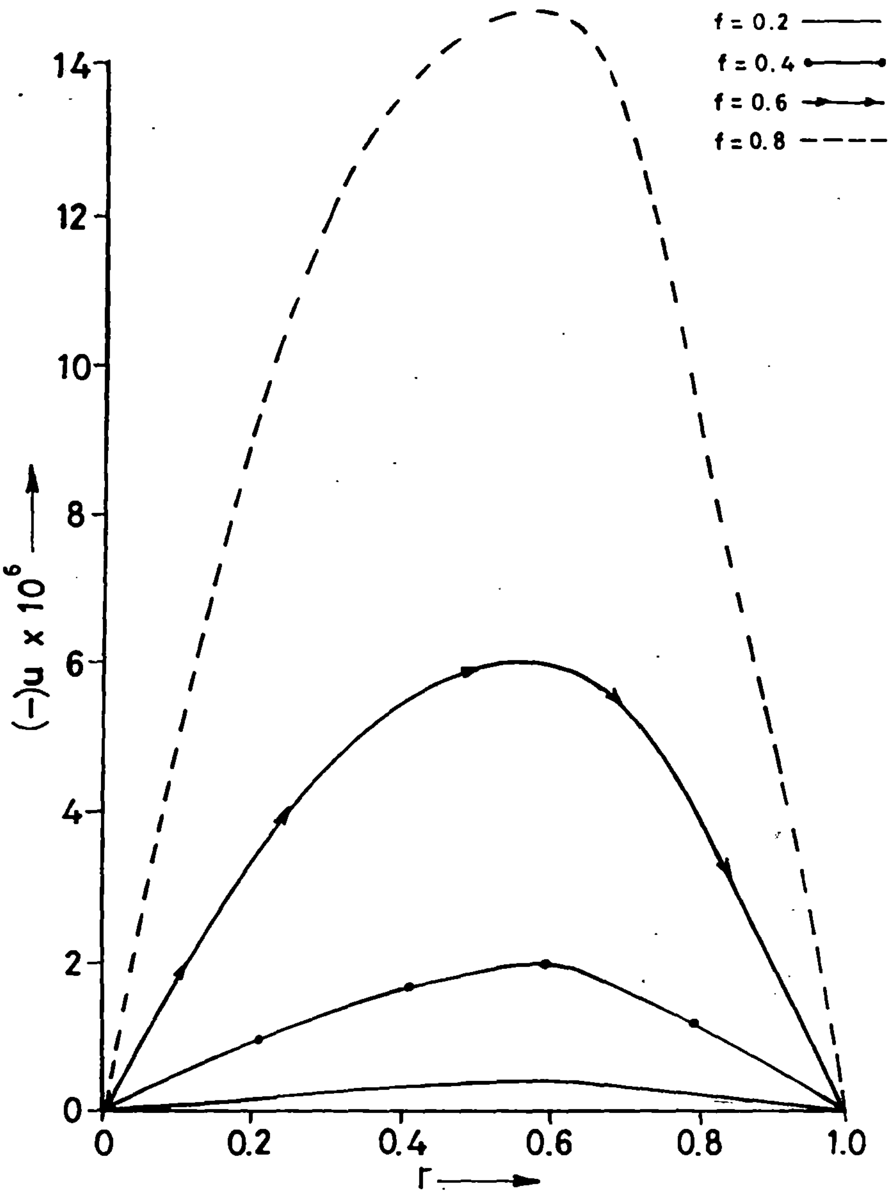

Fig.- 3.2. Velocity distribution of dusty fluid for different values of f when t=1, s=1, z=1, a=1, b=0.5

PART TWO

EFFECT OF DUST PARTICLES ON THE FLOW OF INCOMPRESSIBLE FLUID IN A ROTATING SYSTEM

3.1.6 Introduction

The mechanical behaviour of dusty fluids has received greater attention during the recent past in the field of fluid dynamics. Problems dealing with the influence of dust particles on viscous flows find place in several branches. Some such flows are those of dissolved micromolecules, fibre suspensions, red corpuscles and other bodies in blood.

The momentum equations given by Saffman [5] characterizing the dusty fluid flow was discussed by Michael and Miller [7] for the flow in the semi-infinite space over flat plate. A comprehensive review of the dynamics of dusty gases was given by Marble [6]. Later, Ramana Prasad and Ramacharyulu [8] studied the unsteady flow of an incompressible viscous fluid with uniform distribution of dust particles between two parallel plates when one of which is impulsively stopped from the state of uniform motion. Little work seems to have been done on the flow of a dusty gas in a rotating system although this has bearing on the pollution problems as well as on the motion of aerosol over the rotating earth. Gupta and Pop [9] studied the unsteady boundary layer flow generated in a viscous dusty liquid bounded by an infinite flat plate. Here we consider the flow of viscous incompressible dusty fluid over a flat plate which is impulsively brought to rest from a state of uniform motion parallel to itself while both the liquid and the plate are in a state of rigid body rotation about an axis normal to the plate.

3.1.7 Formulation of the problem

We consider an infinite plate lying along the plane z=0 and situated in a viscous liquid in which there is a distribution of dust particles with a small bulk concentration. Initially the plate was rotating in unison with the liquid with a uniform angular velocity Ω about z-axis and was moving with a constant velocity. When the steady state was reached, the moving plate was impulsively brought to rest. The aim of the present paper is to investigate the subsequent motion.

Following Saffman [5], the momentum equations for the liquid in a rotating frame of reference are

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + 2 \Omega v + \frac{K N_0}{\rho} (u' - u) , \qquad (3.1.31)$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial z^2} - 2 \Omega u + \frac{K N_0}{\rho} (v' - v) \qquad (3.1.32)$$

along x and y directions respectively. Similarly, the momentum equations for dust particles along x and y directions are

$$\frac{\partial u'}{\partial t} = \frac{K}{m} (u - u') + 2 \Omega v' , \qquad (3.1.33)$$

$$\frac{\partial v'}{\partial t} = \frac{K}{m} (v - v') - 2 \Omega u' , \qquad (3.1.34)$$

where (u,v,0) and (u',v',0) are components of velocities of liquid and dust particles respectively, m is the mass of a dust particle, K is the Stokes resistance coefficient. The other symbols have their usual meanings. Equations (3.1.31) and (3.1.32) are combined as

$$\frac{\partial q}{\partial T} = \frac{\partial^2 q}{\partial Z^2} - (2i\omega + f\ )q + fq' \ , \tag{3.1.35}$$

where

$$q = \frac{u + iv}{U} \ , \quad q' = \frac{u' + iv'}{U} \ , \ T = \frac{t}{\tau} \ , \ Z = \frac{z}{(\nu t)^{1/2}} ,$$

$\Omega\tau = \omega$, $f = \frac{mN_0}{\rho}$ is the mass concentration, $\tau = \frac{m}{K}$ is known as the relaxation time of dust.

Equations (3.1.33) and (3.1.34) are combined as

$$\frac{\partial q'}{\partial T} = q - (\ 2i\omega + 1)\ q' \ . \tag{3.1.36}$$

3.1.8 Steady state solution

The equations of steady state motion for fluid and dust particles are given by

$$\frac{d^2 q}{dZ^2} - (2i\omega + f)q + fq' = 0 \tag{3.1.37}$$

and

$$q' = \frac{1}{1 + 2i\omega}\ q \ . \tag{3.1.38}$$

It is clear from equation (3.1.38) that, in a rotating system, the fluid and dust particles do not move with same velocity whereas, in the absence of rotation, fluid and dust particles move with same velocity.

The boundary conditions are

$$\left.\begin{array}{l} q = 1 \text{ for } Z = 0 \quad \text{and} \\ \\ q \longrightarrow 0 \quad \text{as} \quad Z \longrightarrow \infty \ . \end{array}\right\} \tag{3.1.39}$$

The steady state velocities are given by

$$q = e^{-MZ} \tag{3.1.40}$$

and

$$q' = \frac{1}{1+2i\omega}\, e^{-MZ} \tag{3.1.41}$$

------------------with the proviso that the real part of M is taken positive where

$$M = \left(2i\omega \left(1 + \frac{f}{1+2i\omega}\right)\right)^{1/2}.$$

3.1.9 Flow when the moving plate is impulsively brought to rest

The fluid velocity $q(Z,T)$ and dust velocity $q'(Z,T)$ satisfy the equations (3.1.35) and (3.1.36) respectively. The initial conditions are

$$q(Z,0) = e^{-MZ}, \tag{3.1.42}$$

$$q'(Z,0) = \frac{1}{1+2i\omega}\, e^{-MZ}. \tag{3.1.43}$$

The boundary conditions is

$$q(0,T) = 0 \text{ for } T > 0. \tag{3.1.44}$$

The boundary conditions on dust velocity can not be prescribed as dust particles may slip on the plate z = 0.

Let $\bar{q}$ and $\bar{q}'$ be the Laplace transforms of q and q′ respectively, defined by

$$\bar{q} = \int_0^\infty e^{-pt} q(Z,T)\,dT ,$$

$$\bar{q}' = \int_0^\infty e^{-pt}\, q'(Z,T)\,dT, \quad Re(p) > 0 .$$

With the help of equations (3.1.42) and (3.1.43), the Laplace transforms of equations (3.1.35) and (3.1.36) (after rearrangement) give

$$\frac{d^2\bar{q}}{dZ^2} - N^2\,\bar{q} = -K'\,e^{-MZ} \quad , \tag{3.1.45}$$

$$\bar{q}\,' = \frac{1}{(1 + p + 2i\omega)} \left[\bar{q} + \frac{e^{-MZ}}{1 + 2i\omega} \right] , \tag{3.1.46}$$

where

$$N^2 = (p + 2i\omega)\,\frac{1 + p + f + 2i\omega}{1 + p + 2i\omega} \quad .$$

and

$$K' = 1 + \frac{f}{(1 + 2i\omega)(1 + p + 2i\omega)} \quad .$$

The transformed boundary conditions are

$$\left.\begin{array}{l} \bar{q} = 0 \text{ on } Z = 0 \quad \text{and} \\ \bar{q} \longrightarrow 0 \text{ as } Z \longrightarrow \infty \quad . \end{array}\right\} \tag{3.1.47}$$

The solution of equation (3..1.45) subject to the boundary conditions in (3.1.47) is given by

$$\bar{q} = \frac{K'}{M^2 - N^2}\left(e^{-NZ} - e^{-MZ} \right) \tag{3.1.48}$$

with the proviso that the real part of N is taken positive.

Using equation (3.1.48) in equation (3.1.46) we find the velocity of dust particles at the plate z = 0 as

$$q'(0,T) = \frac{1}{1 + 2i\omega}\,\mathcal{L}^{-1}\left[\frac{1}{(1 + p + 2i\omega)}\right] , \tag{3.1.49}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform operator. Evaluation of the inverse transform of (3.1.49) and separating the resulting expression into real and imaginary parts, we have

$$\frac{u'(0,T)}{U} = \frac{e^{-T}}{1 + 4\omega^2}\left[\cos 2\omega T - 2\omega \sin 2\omega T \right] , \tag{3.1.50}$$

$$\frac{v'(0,T)}{U} = \frac{-\,e^{-T}}{1 + 4\omega^2}\left[2\omega \cos 2\omega T + \sin 2\omega T \right] \quad . \tag{3.1.51}$$

Inversion of (3.1.48) presents some difficulties and we restrict ourselves to large values of time T which correspond to small values of p.

Inverting (3.1.48), we have

$$q(Z,T) = \frac{(1+2i\omega)^2+f}{(1+2i\omega)(1+f+2i\omega)}\left[\frac{\eta\exp\{-(2i\omega T+\eta^2/4T)\}}{2(\pi T)^{1/2}(2i\omega T-\eta^2/4T)}\right], \quad (3.1.52)$$

where

$$\eta = \sqrt{1+\frac{f}{1+2i\omega}}\,.Z = \frac{1}{(1+4\omega^2)^{1/2}}(A_3-iB_3)Z\,.$$

Separating (3.1.52) into real and imaginary parts, we have

$$\frac{u}{U} = A(A_7A_8+B_7B_8)\,, \quad (3.1.53)$$

$$\frac{v}{U} = A(A_7B_8-A_8B_7)\,, \quad (3.1.54)$$

where

$$A = \frac{2Z\exp\{-A_4/4T(1+\omega^2)\}}{\{(1+f-4\omega^2)^2+4\omega^2(2+f)^2\}(A_5^2+B_5^2)}\sqrt{\frac{(1+4\omega^2)T}{\pi}}\,,$$

$$A_1 = Z^2(1+f)-16\omega^2T^2\,,$$

$$B_1 = 8\omega T^2+2\omega Z^2\,,$$

$$A_2 = -Z^2(1+f)-16\omega^2T^2\,,$$

$$B_2 = 8\omega T^2-2\omega Z^2\,,$$

$$A_3 = \frac{1}{(2)^{1/2}}\left[((1+f+4\omega^2)^2+4f^2\omega^2)^{1/2}+(1+f+4\omega^2)\right],$$

$$B_3 = \frac{1}{(2)^{1/2}}\left[((1+f+4\omega^2)^2+4f^2\omega^2)^{1/2}-(1+f+4\omega^2)\right],$$

$$A_4 = A_1 + 2\omega B_1 ,$$

$$B_4 = B_1 - 2\omega A_1 ,$$

$$A_5 = A_2 + 2\omega B_2 ,$$

$$B_5 = B_2 - 2\omega A_2 ,$$

$$A_6 = A_3A_5 - B_3B_5 ,$$

$$B_6 = B_3A_5 + A_3B_5 ,$$

$$A_7 = A_6 \cos B_4 - B_6 \sin B_4 ,$$

$$B_7 = A_6 \sin B_4 + B_6 \cos B_4 ,$$

$$A_8 = (1 + f - 4\omega^2)^2 + 8\omega^2 (2 + f) ,$$

$$B_8 = - 2\omega f (1 + f - 4\omega^2) .$$

3.1.10 Discussion

From the expression (3.1.50) and (3.1.51) for the velocity of dust particles at the plate wall, it may be remarked that inertial oscillations take place with very small amplitude.

We have calculated numerically the values of u and v from the expressions in (3.1.53) and (3.1.54) for different values of f and graphs are drawn to show the effects of dust particles on velocity profile of liquid particle in the directions of x and y respectively. Figures 3.3 and 3.4 show that motion is highly of oscillating nature. Also, flow reversal takes place in different layers both for u and v. Amplitude of oscillation steadily increases with the distance from the plate upto a certain distance and then rapidly decreases and tends to zero as it is expected. As f increases, the change of phase of oscillations is quicker near the plate wall.

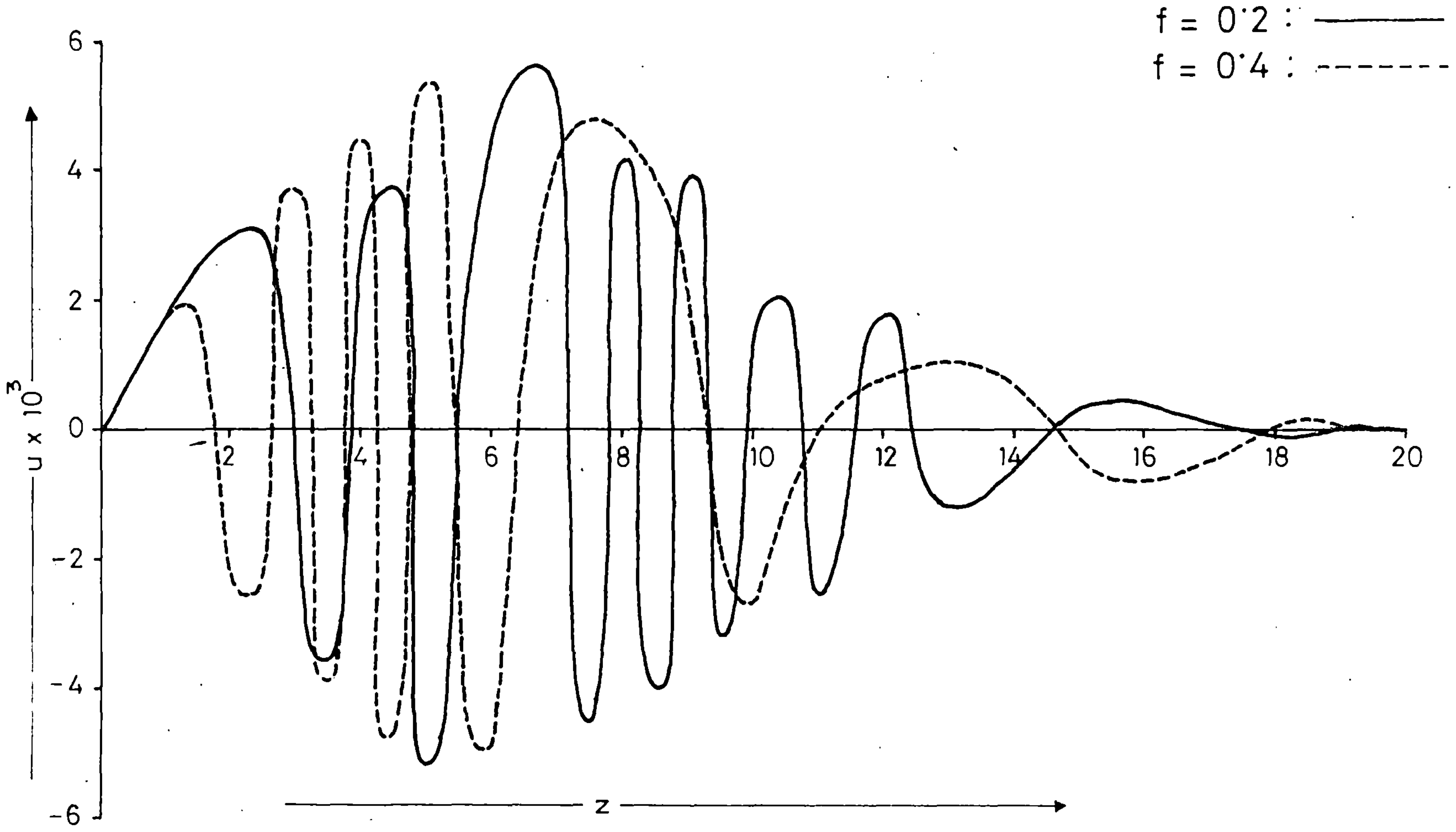

Fig. 3'3. Plot of $u \times 10^3$ against z for different values of f when $t = 40$, $\omega = 1$.

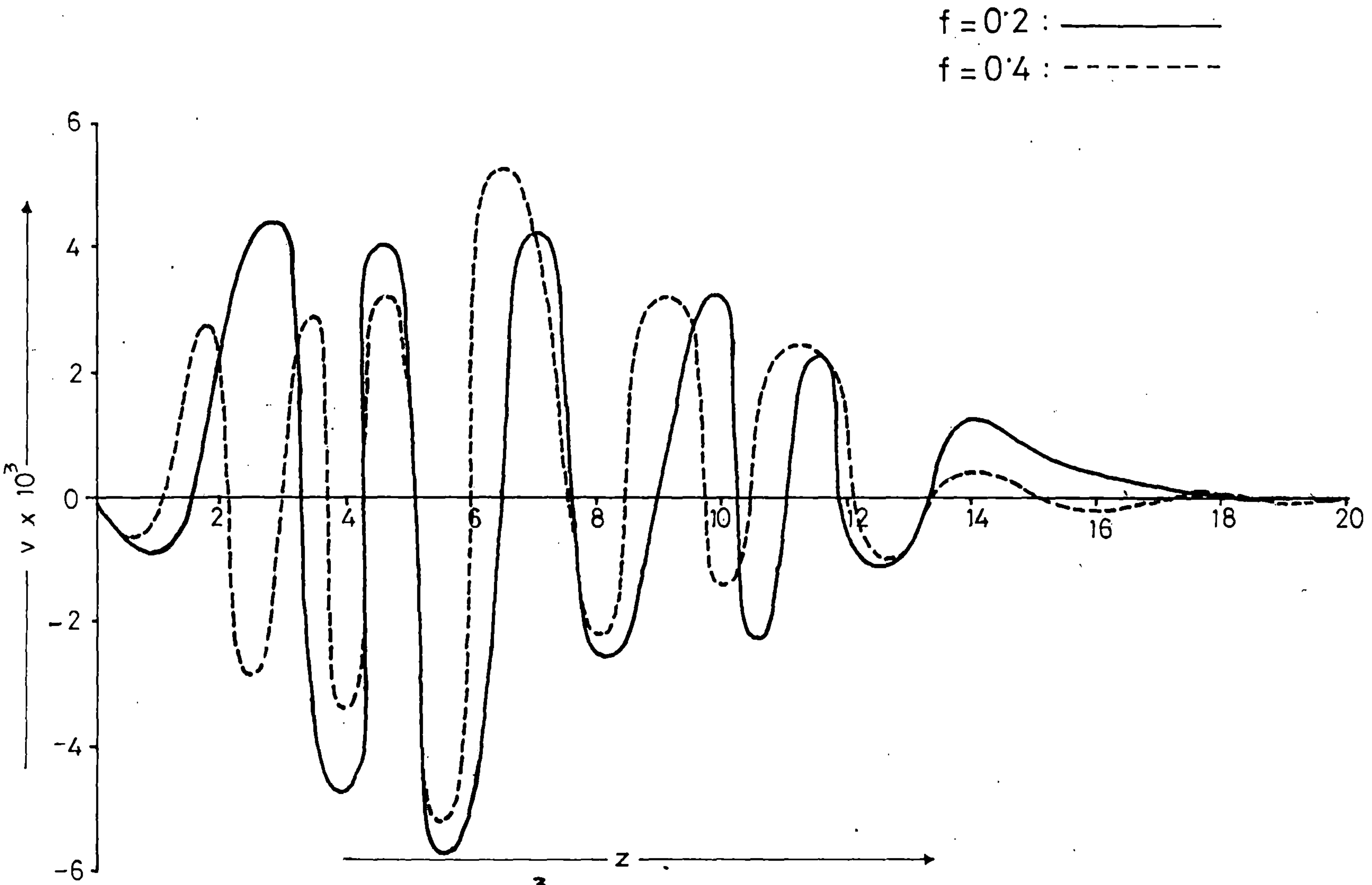

Fig. 3·4. Plot of $v \times 10^3$ against z for different values of f when t = 40, $\omega = 1$.

SECTION - B

DUSTY NON-NEWTONIAN FLUID FLOWS

UNSTEADY FLOW OF DUSTY VISCO-ELASTIC LIQUID BETWEEN TWO OSCILLATING PLATES *.

3.2.1 Introduction

Interests in problems of flow of a dusty gas i.e. a mixed system of fluid and particles have increased enormously in recent years. A model equation describing the motion of such mixed system has been given by Saffman [5]. Based upon Saffman's model numerous authors [6,10,11] investigated a number of dusty gas flow problems in different situations.

There is another class of flow problems which concern with the study of the flow of dusty non-Newtonian fluids such as latex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava. However, the study on this class of problems and rheological aspects of such flows have not received much attention. Little work is reported in literature [12,13,14,15] on the flow of dusty non-Newtonian fluid, although this has some bearing on the problems of petroleum industry and chemical engineering interest. This consideration provides motivation for the present study.

In the present investigation, we consider the unsteady laminar flow of a visco-elastic liquid [16] containing uniformly

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small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane. The velocity fields for the liquid and dust particles are obtained explicitely by using the technique of Laplace transform. The effect of elastic element in the liquid, the mass concentration and the relaxation time of dust particle on the velocity profiles are studied graphically. The skin-friction at the lower plate wall and the total volume-flow in between the plates are also obtained.

This problem is likely to have some industrial and chemical engineering applications on the problems of transport of solid particles suspended in visco-elastic fluids through channels.

3.2.2 Basic equations and their solutions

We suppose that the dusty visco-elastic liquid fills the space between two infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency w in its own plane. In our analysis we take a co-ordinate system such that the x-axis coincides with the lower fixed plate and the z-axis is perpendicular to it. The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and let it be N_0. Since the plates are infinite, the velocities will depend on z and time t only.

Using equations (1.12) – (1.19) we get the equations of motion of dusty visco-elastic liquid as (dropping dashes)

$$\left(1 + \alpha \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = R \frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\left(1 + \alpha \frac{\partial}{\partial t}\right)(v - u), \qquad (3.2.1)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}(u - v), \qquad (3.2.2)$$

where u and v are velocities of liquid and dust particles respectively in the direction of x and $u' = \frac{u}{h\ w}$,

$v' = \frac{v}{hw}$, $t' = tw$, $z' = \frac{z}{h}$, $\alpha = \lambda_0 w$, $R = \frac{\nu}{h^2 w}$,

f(mass concentration) $= \frac{m\ N_0}{\rho}$, τ(relaxation time)$= \frac{m\ w}{K}$.

The initial and boundary conditions in non-dimensional form are

$t \leq 0$:

$$u = \frac{\partial u}{\partial t} = 0 \text{ for all } z, \qquad (3.2.3)$$

$t > 0$:

$$\left.\begin{array}{lll} u = a \sin t & \text{at} & z = 1, \\ u = 0 & \text{at} & z = 0. \end{array}\right\} \qquad (3.2.4)$$

Taking Laplace transforms of (3.2.1) and (3.2.2), using (3.2.3), we get

$$p(1 + \alpha\ p)\ \bar{u} = R \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{f}{\tau}(1 + \alpha\ p)\ (\bar{v} - \bar{u}), \qquad (3.2.5)$$

$$v = \frac{1}{1 + p\ \tau}\ \bar{u}, \qquad (3.2.6)$$

where

$$\bar{u} = \int_0^\infty u e^{-pt} dt, \qquad \bar{v} = \int_0^\infty v e^{-pt} dt,$$

$\text{Re}(p) > 0$.

Transformed boundary conditions are

$$\left.\begin{aligned} \bar{u} &= \frac{a}{1+p^2} \quad \text{at } z = 1 , \\ \bar{u} &= 0 \qquad\quad \text{at } z = 0 . \end{aligned}\right\} \tag{3.2.7}$$

Substituting (3.2.6) into (3.2.7) we get

$$R\,\frac{\partial^2 \bar{u}}{\partial z^2} - \bar{u}\left[\frac{F + p\tau}{p\tau + 1}\right] p(1 + \alpha p) = 0, \tag{3.2.8}$$

where f+1 = F.

Thus the solution of (3.2.8) is

$$\bar{u} = A \cosh Mz + B \sinh Mz, \tag{3.2.9}$$

where

$$M^2 = \frac{(F + p\tau)\, p(1 + \alpha p)}{R(p\tau + 1)} .$$

On using boundary conditions in (3.2.7) and taking inverse transform, we get

$$u = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{a}{1+p^2}\, \frac{\sinh Mz}{\sinh M}\, e^{pt} dp , \tag{3.2.10}$$

where γ is greater than the real part of the singularities of the integrand in (3.2.10). The integrand is an integral function of p and has singularities at p = ± i and at the zeros of sinh M. Calculating the residues and simplifying further, we obtain the expression for u as

$$\begin{aligned} u = {} & \frac{a}{E_3^2 + E_4^2}\Big[(E_1E_4 - E_2E_3)\cos t + (E_1E_3 + E_2E_4)\sin t\Big] \\ & + 2R\pi \sum_{n=0}^{\infty} \sum_{i=1}^{\infty} (-1)^{n+1} \frac{n}{1 + p_n^{i\,2}}\, Q(p_n^i) \sin(n\pi z)\, e^{p_n^i t} , \end{aligned} \tag{3.2.11}$$

where

$$E_1 = \sin N_2 z \cosh N_1 z \quad ,$$

$$E_2 = \cos N_2 z \sinh N_1 z \quad ,$$

$$E_3 = \sin N_2 \cosh N_1 \quad ,$$

$$E_4 = \cos N_2 \sinh N_1 \quad ,$$

$$N_1, N_2 = \frac{1}{2}\Big[\Big[\Big\{(F - \tau\alpha) + \tau(\tau + F\alpha)\Big\}^2 + \Big\{\tau(F - \tau\alpha) - (\tau + F\alpha)^2\Big\}\Big]^{1/2}$$

$$\pm \Big\{\tau(F - \tau\alpha) - (F\alpha + \tau)\Big\}\Big]^{1/2} \times \frac{1}{\{R(1 + \tau^2)\}^{1/2}} ,$$

and p_n^i 's are those roots of the cubic equation

$$p_n^{i^3} \cdot \tau\alpha + p_n^{i^2}(\tau + \alpha F) + p_n^i(F + n^2\pi^2 R\tau) + R n^2 \pi^2 = 0 , \tag{3.2.12}$$

which are having negative real parts,

$$Q(p_n^i) = (1 + p_n^i\tau)^2 / \Big[\Big\{p_n^i \tau (1 + \alpha p_n^i) + (1 + 2\alpha p_n^i) \times (F + \tau p_n^i)\Big\} - \tau p_n^i (F + \tau p_n^i)(1 + \alpha p_n^i)\Big] . \tag{3.2.13}$$

By the convolution theorem, we obtain from (3.2.6)

$$v = \frac{a}{E_3^2 + E_4^2}\,\frac{\tau}{\tau^2 + 1}\Big[(E_1E_3 + E_2E_4)\Big\{(\frac{1}{\tau}\sin t - \cos t) + e^{-t/\tau}\Big\}$$

$$+ (E_1E_4 - E_2E_3)\Big\{(\sin t + \frac{1}{\tau}\cos t) - \frac{1}{\tau}e^{-t/\tau}\Big\}\Big]$$

$$+ 2R\pi \sum_{n=0}^{\infty}\sum_{i=1}^{3}\Big[\frac{(-1)^{n+1}}{p_n^i\tau + 1}\,\frac{n}{p_n^{i^2} + 1}\,Q(p_n^i)\sin(n\pi z)$$

$$\times (e^{p_n^i t} - e^{-t/\tau})\Big] . \tag{3.2.14}$$

The dimensionless shearing stress τ_p at the lower plate due to the dusty visco-elastic liquid is

$$\tau_p = \left[\left(1 - \alpha \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial z} \right]_{z=0}$$

$$= \frac{a}{E_3^2 + E_4^2} \Big[\cos t \left\{ (E_1E_4 - E_2E_3) - \alpha \, (N_2E_3 + N_1E_4) \right\}$$

$$+ \sin t \left\{ (E_1E_3 + E_2E_4) + \alpha \, (N_2E_4 - N_1E_3) \right\} \Big]$$

$$+ 2 R \pi \sum_{n=0}^{\infty} \sum_{i=1}^{3} \Big[(-1)^{n+1} . \frac{n^2 \pi}{(p_n^{i^2} + 1)} Q(p_n^i) e^{p_n^i t} (1 - \alpha p_n^i) . \qquad (3.2.15)$$

The volume flow of dusty visco-elastic liquid discharged per unit breadth of the plate is given by

$$Q_v = 2 \int_0^1 u \, dz = \frac{2a}{(N_1^2 + N_2^2)\,(E_3^2 + E_4^2)} \Big[(E_4 \cos t + E_3 \sin t)$$

$$X(N_1 \sin N_2 \sinh N_1 - N_2 \cos N_2 \cosh N_1 + 2 N_2)$$

$$+(E_4 \sin t - E_3 \cos t)\,(N_1 \cos N_2 \sinh N_1 + N_2 \sin N_2 \cosh N_1) \Big]$$

$$- 2 \pi R . \sum_n \sum_{i=1}^{3} \frac{1}{(1 + p_n^{i^2}) \pi} . Q(p_n^i) . e^{p_n^i t} , \qquad (3.2.16)$$

where n = 1, 3, 5,..... .

3.2.3 Discussion

The present analysis reveals that the solution contains three pertinent non-dimensional parameters viz. α (elastic parameter of the liquid particle), τ (relaxation time of dust particle), and F (= (f+1) , f is mass concentration of dust particle). The behaviour of these parameters, therefore, yields a physical insight into the problem. Numerical computation is made to observe the effects of these parameters on velocity profiles, skin-friction at the lower plate and volume flow in between the plates.

Figures 3.5 and 3.6 depict the velocity profiles of liquid and dust particles against z for different values of mass concentration and elastic parameter when τ is fixed . It is interesting to note that both u and v increase with the increase in f in the case of Newtonian fluid ($\alpha = 0$) but behave in a reverse fashion when the fluid is visco-elastic. It is also to be noted that velocity profiles of dusty fluid and dust particles decrease with the increase in elastic parameter. Figure 3.7 reveals that u decreases with the increase in relaxation time τ (with F fixed) when $\alpha = 0$ but in the presence of elastic element u increases with increase in τ. Figure 3.8 shows that the effect of relaxation time is to increase the velocity of dust particles irrespective of whether the fluid is Newtonian or non-Newtonian. It is also to be remarked that the influence of relaxation time (τ) is more on the velocity of dust particles than that of dusty liquid. But the mass concentration has very little effect on both u and v irrespective of whether the fluid is Newtonian or non-Newtonian.

Finally, for some representative values of F and τ, skin-friction at the lower plate and the volume flow in between the plate walls are calculated numerically for different values of α. Table 3.1 reveals that the magnitude of shear stress and total flux increase with the increase in the value of elastic parameter.

Table 3.1 : Shear stress (τ_p) at the lower plate and volume flow (Q_v) in between the plates.

	τ_p			Q_v		
α	t = 5	t = 10	t = 15	t = 5	t = 10	t = 15
1.0	−1.2426252	0.2952193	1.4101103	−0.4795342	−0.2721468	0.3251387
2.0	−1.5263597	1.1345457	2.1700154	−0.4795486	−0.2722571	0.3252040
2.25	−1.5834031	1.3425213	2.3920024	−0.4795531	−0.2722841	0.3252201

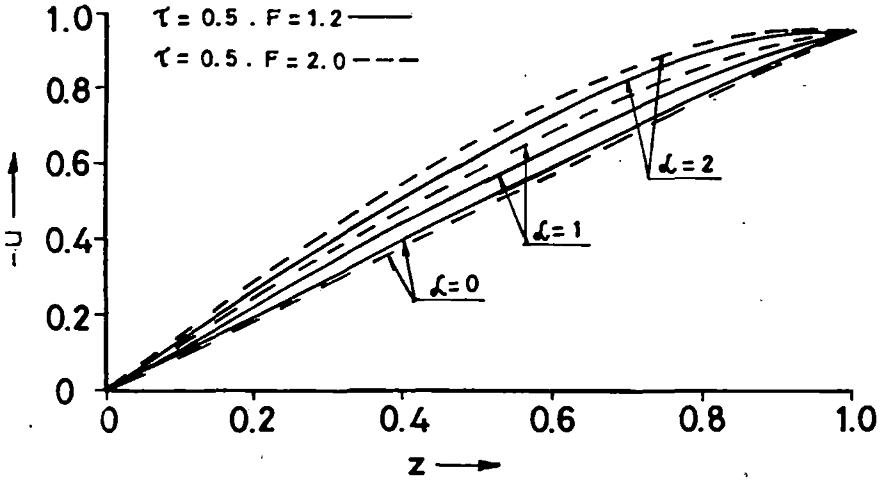

Fig.- 3.5. Velocity profile of dusty fluid (u) against z at t = 5 & $\tau = 0.5$

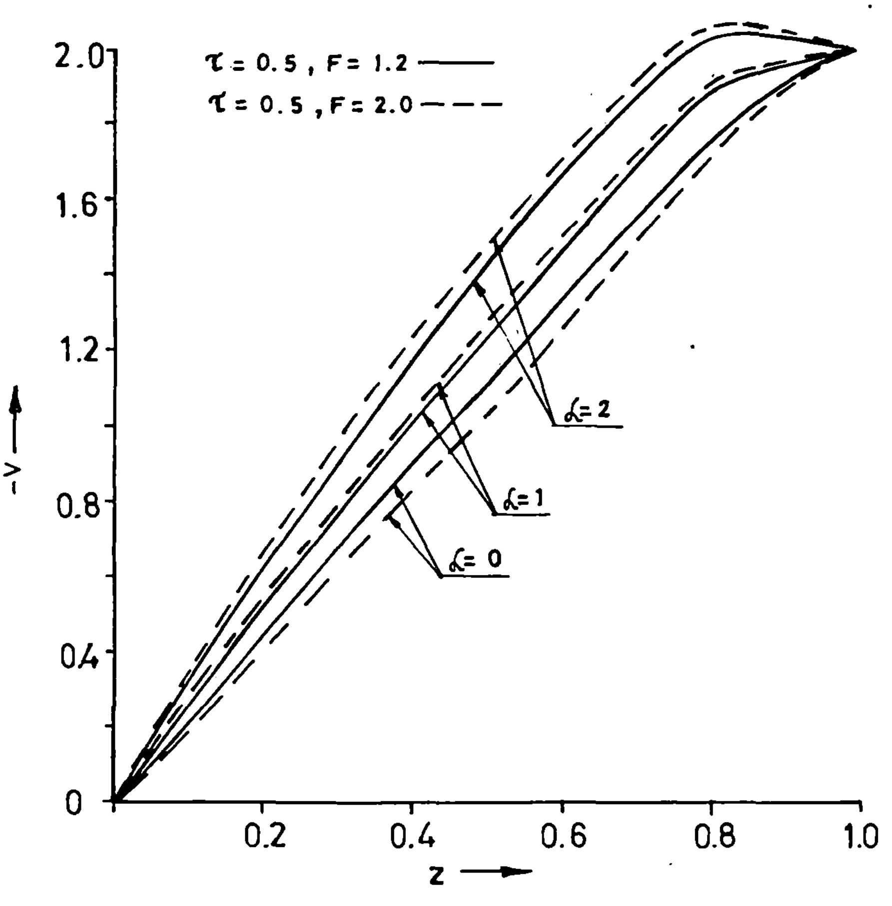

Fig.-3.6. Velocity profile of dust particles (v) against z at t = 5 & $\tau = 0.5$

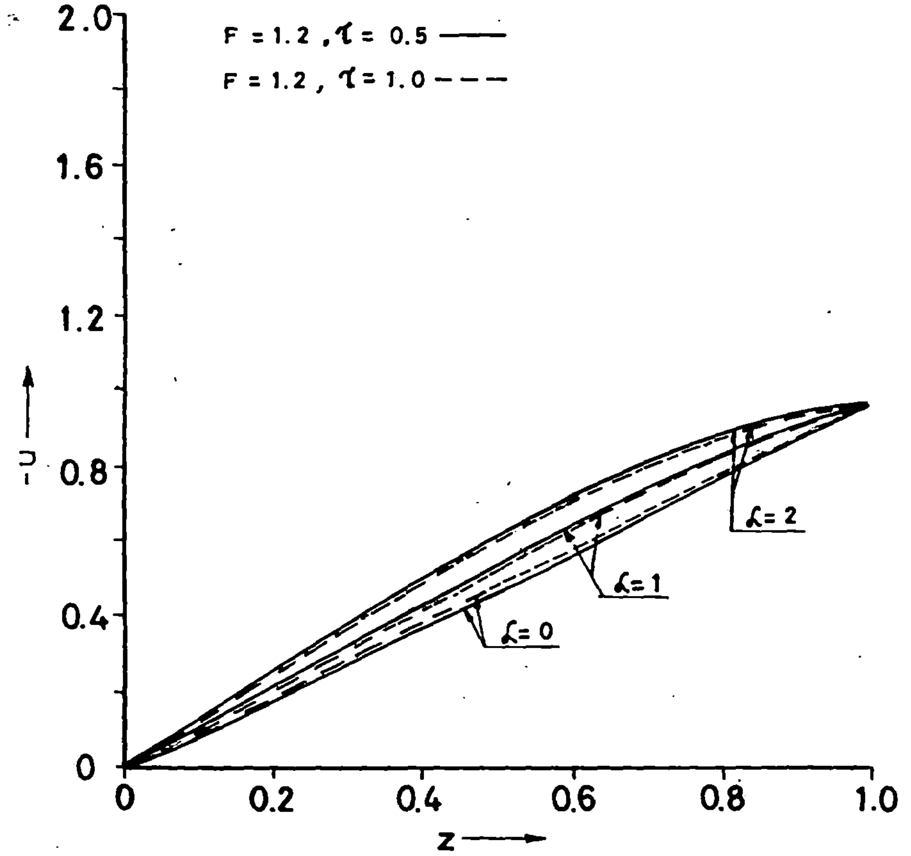

Fig.-3.7. Velocity profile of dusty fluid (u) against z at t = 5 & F = 1.2

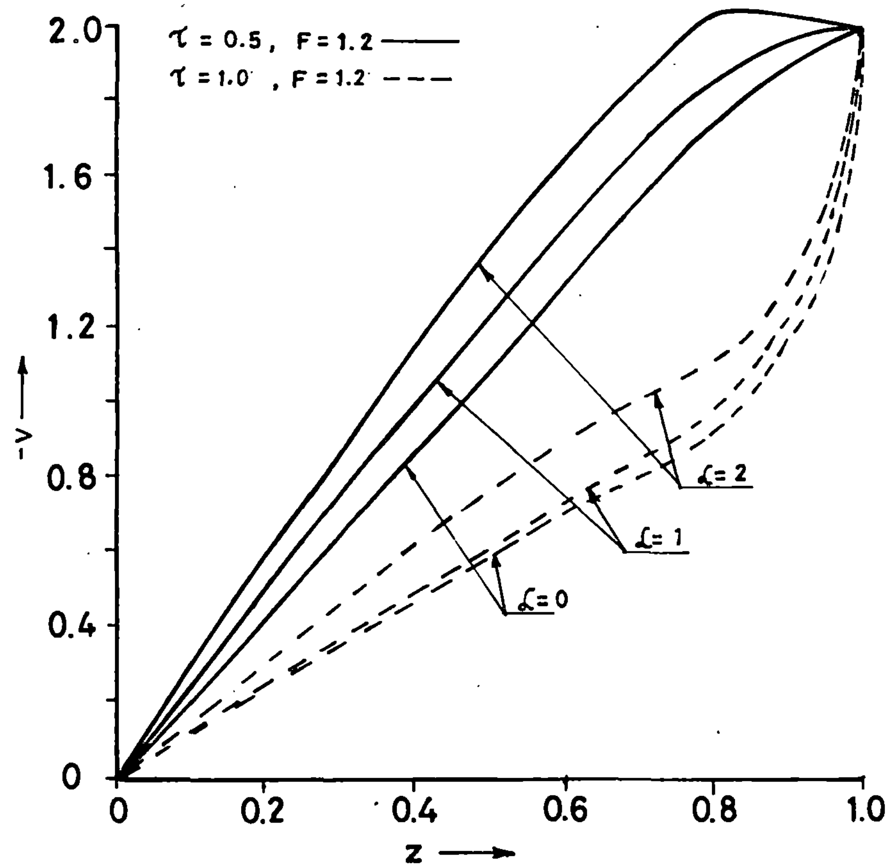

Fig.-3.8. Velocity profile of dust particles (v) against z at t = 5 & F = 1.2.

PART TWO

UNSTEADY AXISYMMETRIC ROTATIONAL FLOW OF DUSTY ELASTICO-VISCOUS LIQUID*

3.2.4 Introduction

The study of rotational transient flow of non-Newtonian fluids in both semi-infinite field and bounded field is of practical need for certain industrial processes to have the description of fluid mechanical phenomena exhibited by non-Newtonian materials. Srivastava [17] and Tandon [18] analysed the propagation of small disturbances in an Oldroyd fluid contained in a semi-infinite circular cylinder due to the slow rotation of a disc at the base. Srivastava [17] considered the radius of the disc to be same as the radius of the cylinder while Tandon [18] considered it to be smaller. Rao and Rao [19] investigated the rectilinear oscillations of a circular cylinder about a mean position along a diameter in an infinitely extended micropolar fluid. Tandon and Chandra [20] discussed the unsteady motion inside and outside of an infinite cylinder which suddenly starts rotating impulsively about an axis in an incompressible Oldroyd's two-parametric fluid, not three-parametric one as claimed by the authors. Recently Mukherjee and Mukherjee [21] considered the unsteady axisymmetric rotational flow of

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elastico-viscous liquid due to the time-dependent rotation of a circular cylinder.

However, studies on dusty non-Newtonian fluid flows and rheological aspects of such flows have not received much attention though the studies of dusty non-Newtonian fluid flows are likely to have some industrial and chemical engineering applications on the problems of polluted oil extraction, polymer extrution and paint spraying. Based upon the theoretical model proposed by Saffman [5], Srivastava [12] analysed the unsteady flow of dusty Rivlin-Ericksen fluid through a channel. Bagchi and Maiti [14] studied the unsteady flow of dusty elastico-viscous liquid through a channel with arbitrary time-varying pressure gradient.

Here we study the rotational flow of dusty elastico-viscous liquid. The expressions for the velocity fields of the liquid and the dust particles are obtained explicitely. The effect of elastic element in the liquid, the mass concentration and the relaxation time of dust particles on the velocity profiles of liquid and dust particles are studied graphically. This problem is likely to have some bearing on the problems of transport of solid particles suspended in non-Newtonian fluids through pipes.

3.2.5 Mathematical formulation of the problem

In the present problem, it is assumed that the particles are spherical in shape and uniform in size and the bulk-concentration (concentration by volume) of dust is very small. Following Saffman, it is assumed that steady Stokes law of resistance between the particles and fluid is applicable. However the mass concentration of dust can be of the order of unity by allowing the ratio of the density of the dust and fluid to be large. For sufficiently small particles, the velocity of sedimentation will be small compared with a characteristic velocity of the flow and can be neglected.

Initially the liquid and dust particles are at rest. We consider the flow of a dusty elastico-viscous liquid in an infinitely long circular cylinder of radius 'a' which oscillates with constant frequency about the axis of the cylinder. In the cylindrical polar co-ordinate system (r,θ,z), the z-axis is chosen along the axis of the cylinder. The physics of the problem suggests

$u_i \equiv (0,u(r,t),0)$

and

$v_i \equiv (0,v(r,t),0)$,

where u_i and v_i are the local velocity vectors of liquid and dust particles respectively.

Using equations (1.12) – (1.19), we get the equations of motion of dusty elastico-viscous liquid [16] as

$$\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right) + \frac{KN_0}{\rho}\left(1+\lambda_0\frac{\partial}{\partial t}\right)(v-u), \tag{3.2.17}$$

$$\frac{\partial v}{\partial t} = \frac{K}{m}(u-v) \quad , \tag{3.2.18}$$

where ν is the kinematic viscosity of the liquid and the number density of dust particles is $N = N_0$, a constant throughout the motion, λ_0 is the relaxation time, K is the Stokes resistance coefficient.

Initial and boundary conditions for the problem are

$$\left.\begin{aligned} u(r,t) &= \frac{\partial u(r,t)}{\partial t} = 0 \,, \\ v(r,t) &= \frac{\partial v(r,t)}{\partial t} = 0 \,, \end{aligned}\right\} \text{at } t = 0 \text{ and for all } r \qquad (3.2.19)$$

$$\left.\begin{aligned} &u = u_0\, e^{-\,i\Omega t} \text{ on } r=a \,, \\ &u \text{ is finite on } r=0 \,, \end{aligned}\right\} t > 0 \qquad (3.2.20)$$

where u_o is the characteristic velocity and Ω is the imposed oscillation.

Using the non-dimensional variables

$$\bar{u} = \frac{u}{u_o} \,,\ \bar{v} = \frac{v}{u_o} \,,\ \bar{r} = \frac{r}{a} \,,\ \bar{t} = \frac{t\,\nu}{a^2} \,,\ \bar{\Omega} = \frac{\Omega a^2}{\nu}$$

equations (3.2.17) – (3.2.20) in non-dimensional form are written as (dropping bars)

$$(1 + \alpha \frac{\partial}{\partial t})\frac{\partial u}{\partial t} = (\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}) + \beta(1 + \alpha \frac{\partial}{\partial t})(v - u), \qquad (3.2.21)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}\,(u - v), \qquad (3.2.22)$$

$$\left.\begin{aligned} u &= \frac{\partial u}{\partial t} = 0, \\ v &= \frac{\partial v}{\partial t} = 0, \end{aligned}\right\} \text{at } t = 0 \text{ and for all } r. \qquad (3.2.23)$$

$$\left.\begin{aligned} &u = e^{-i\Omega t} \text{ on } r=1, \\ &u \text{ is finite on } r = o, \end{aligned}\right\} t > 0 \qquad (3.2.24)$$

where

$\alpha \left[= \dfrac{\lambda_0 \nu}{a^2} \right]$ the non-dimensional parameter,

$f \left[= \dfrac{m\ N_0}{\rho} \right]$ the mass concentration of dust particles,

$\tau \left[= \dfrac{m\,\nu}{Ka^2}\right]$ the dimensionless relaxation time of dust particles and $\beta = \dfrac{f}{\tau}$.

3.2.6 Solution of the problem

Using the Laplace transform technique in equations (3.2.21) and (3.2.22) subject to initial and boundary conditions in equations (3.2.23) and (3.2.24), it turns out that the expressions for the velocity profile of liquid and dust particles can be represented by the Laplace inversion integral in the form

$$u = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{1}{(p+i\Omega)}\;\frac{I_1\left[r\left\{\dfrac{p(1+\alpha p)(1+f+p\tau)}{(p\tau+1)}\right\}^{1/2}\right]}{I_1\left[\left\{\dfrac{p(1+\alpha p)(1+f+p\tau)}{(p\tau+1)}\right\}^{1/2}\right]}\,.\,e^{pt}\,dp\;, \tag{3.2.25}$$

$$v = \frac{1}{\tau}\,e^{-t/\tau}\int_0^t u(r,\lambda)\,e^{\lambda/\tau}\,d\lambda \qquad , \tag{3.2.26}$$

where γ is greater than the real parts of the singularities of the integrand and $\mathrm{Re}(p) > 0$.

On evaluating equations (3.2.25) and (3.2.26), we have the expressions for the velocity profile of liquid as

$$u = \frac{I_1[rA]}{I_1[A]}\,e^{-i\Omega t} - 2\sum_n\sum_j \frac{\beta_n}{Q(p_{nj})}\,\frac{1}{(p_{nj}+i\Omega)}$$

$$\times\;\frac{J_1(\gamma\beta_n)}{J_0(\beta_n)}\,e^{p_{nj}t} \tag{3.2.27}$$

and of dust particles as

$$v = \frac{I_1[rA]}{I_1[A\ ]} \frac{e^{-i\Omega t} - e^{-t/\tau}}{(1 - i\Omega\tau\ )} - 2 \sum_n \sum_j \frac{\beta_n}{Q(p_{nj})} \frac{1}{(p_{nj} + i\Omega\ )}$$

$$\times \frac{J_1(r\beta_n)}{J_0(\beta_n)} \frac{1}{(1 + p_{nj}\tau)} \left(e^{p_{nj}t} - e^{-t/\tau} \right), \qquad (3.2.28)$$

where

$$A = \left\{ \frac{-i\,\Omega\ (1 - i\,\alpha\,\Omega\ )(1 + f - i\,\Omega\,\tau\ )}{(1 - i\,\Omega\,\tau\ )} \right\}^{1/2},$$

β_n's are the roots of

$$J_1(\beta) = 0 \qquad (3.2.29)$$

and p_{nj}'s are the roots of the cubic equation

$$\frac{p_n(1 + \alpha\ p_n)(1 + f + p_n\tau\ )}{(\ p_n\ \tau + 1)} = -\beta_n^2\ ,\ n = 0,1,2,\ldots, \qquad (3.2.30)$$

$$Q(p_{nj}) = \Big[\Big\{ (\ 1 + 2\,\alpha\ p_{nj})(1 + f + p_{nj}\tau) + p_{nj}\tau(\ 1 + \alpha\ p_{nj}) \Big\}$$

$$\times\ (p_{nj}\tau + 1) - \tau\ p_{nj}(\ 1 + \alpha\ p_{nj})(1 + f + p_{nj}\tau\ ) \Big] / (1 + \tau p_{nj})^2.$$

$$(3.2.31)$$

It is clear from equation (3.2.30) that all roots of p_n (for any $n = 0,1,2,\ldots.$) are either negative or one negative and other two complex. From the physics of the problem we consider those values of p_{nj} in equations (3.2.27) and (3.2.28) for which $e^{p_{nj}t} \longrightarrow 0$ as $t \longrightarrow \infty$.

The non-dimensional skin-friction on the wall of the cylinder is given by

$$\tau_{r\theta}\Big|_{r\ =\ 1} = (1 + i\alpha\Omega)\Big[\frac{A I_1'[A]}{I_1[A]} - 1\Big] e^{-i\Omega t}$$

$$- 2 \sum_n \sum_j \frac{\beta_n^2 (1 - \alpha\ p_{nj}\ )}{Q(p_{nj})\ (p_{nj} + i\ \Omega\ )}\ e^{p_{nj}t}\ . \qquad (3.2.32)$$

It is evident from equations (3.2.27) and (3.2.28) that velocity of liquid and dust particles become same as the relaxation time tends to zero, i.e., when the dust particles become very fine. In absence of elastic parameter and dust particles, the expression for the velocity profile of liquid particles is same as that obtained by Mukherjee and Bhattacharyya [4] (if it is made dimensionless).

3.2.7 Discussion

The analysis of the present study reveals that the solution contains three pertinent flow parameters, viz., α (the dimensionless elastic parameter), f (the mass concentration of dust particles) and τ (the relaxation time of dust particles). The behaviour of these parameters, therefore, yeilds a physical insight into the problem. Keeping this in view, the numerical computations of real part of equations (3.2.27) – (3.2.29) are carried out to represent graphically the velocity fields, skin-friction at the plate walls for different values of α, f, τ.

The velocity of liquid and dust particles are depicted in figures 3.9 – 3.12 against r for different values of α, f and τ. Figures 3.9 and 3.10 show the effect of f on u and v (with τ fixed) while figures 3.11 and 3.12 depict the variation of u and v due to the change of relaxation time of dust particles (with f fixed) for different values of elastic parameter. From figures 3.9 and 3.11, it is seen that u increases with increasing α for fixed τ and f, i.e., the effect of elastic element in the liquid is to increase the velocity of liquid particles. Also it is observed that both mass concentration (f) and relaxation time (τ) increase the velocity of liquid for any α. Figure 3.10 shows that flow occurs in reverse direction, (i.e., in the direction of decreasing θ) for α = 0,1,2 and τ = 0.5. As f increases both forward flow and back flow exist and the region of forward flow increases with the increase in f for any value of α. It is evident from figure 3.12 that as τ increases, the magnitude of the velocity of dust particles increases with fixed f.

Table 3.2 shows that the magnitude of skin-friction increases with the increase in elastic parameter for f=0.2, τ=0.5 at t = 5. The negative values of skin-friction indicate that the shearing stress acts in the decreasing θ direction at t = 5.

Table 3.2 : Skin-friction on the wall at t=5, f=0.2 and τ =0.5.

α	$\tau_{r\theta}\|_{r=1}$
1.0	− 0.0273578
1.5	− 0.053213
2.0	− 0.912364
2.5	− 1.563151
3.0	− 2.251595

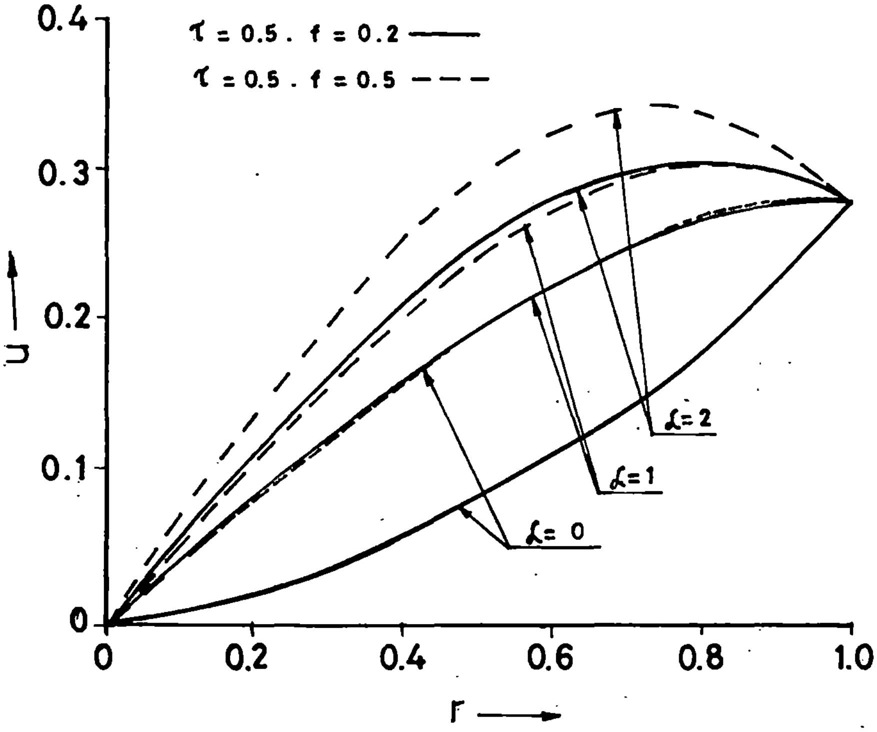

Fig.-3.9. Velocity profile of liquid particles at t = 5 when τ=0.5

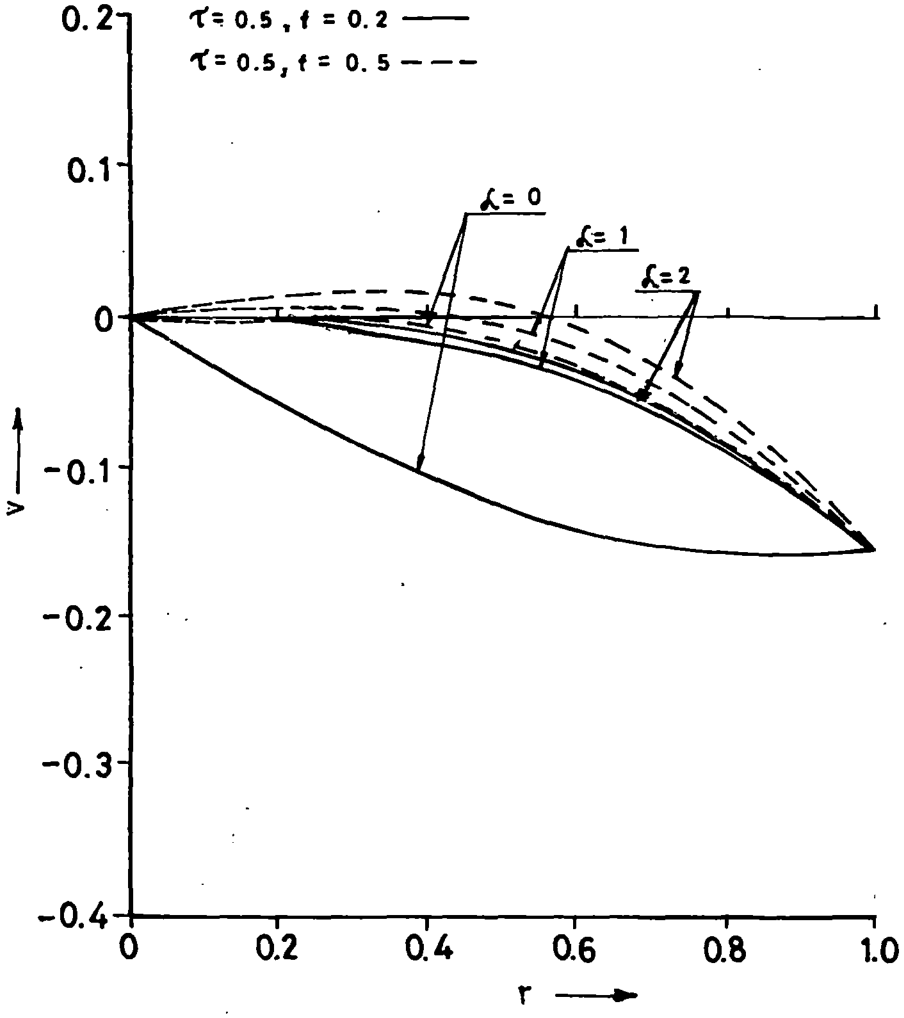

Fig.-3.10. Velocity profile of dust particles at t = 5 when $\tau = 0.5$

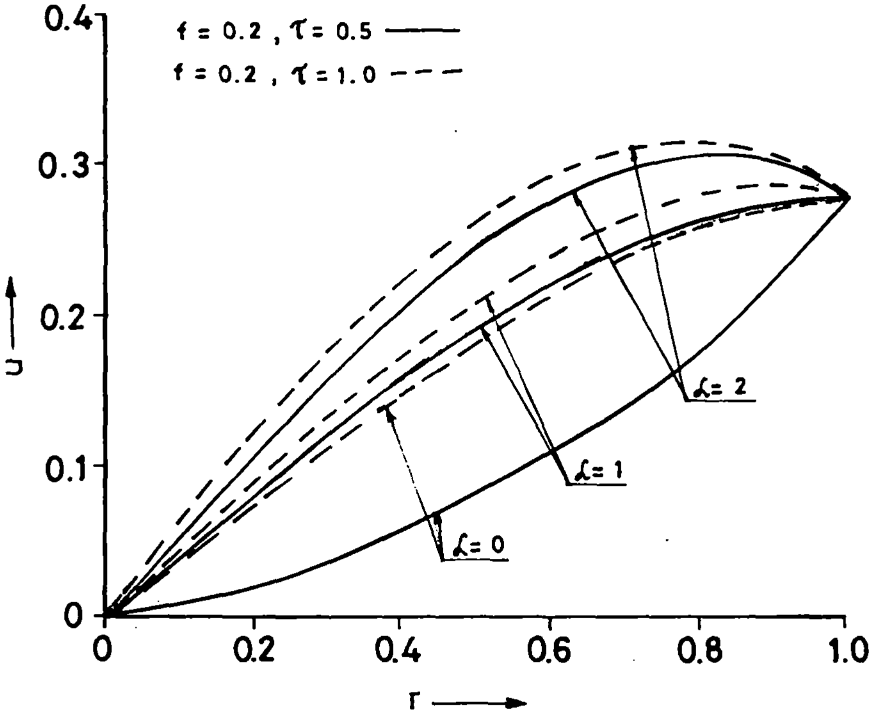

Fig.-3.11. Velocity profile of liquid particles at t = 5 when f = 0.2

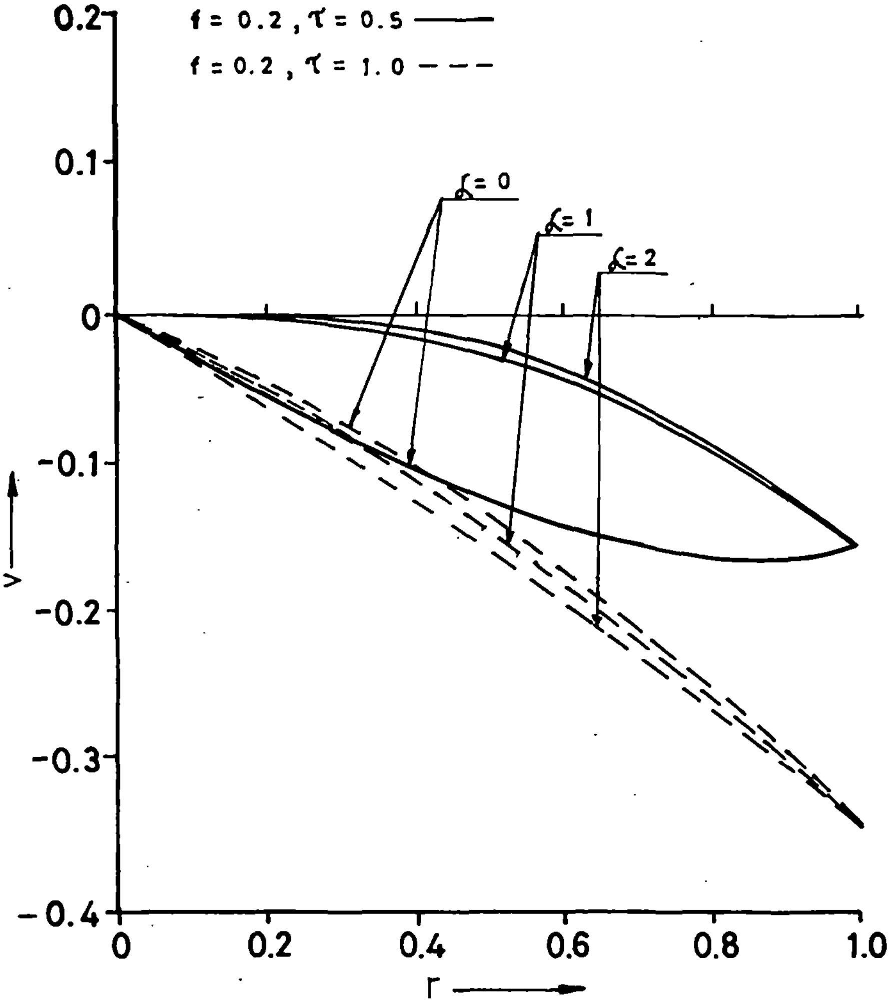

Fig.-3.12. Velocity profile of dust particles at t = 5 when f = 0.2

PART THREE

UNSTEADY FLOW OF A DUSTY ELASTICO-VISCOUS LIQUID IN THE EKMAN LAYER *

3.2.8 Introduction

Multiphase flow problems are of current interest in fluid dynamics. In particular, fluid mechanical problems involving gas particle mixtures arise in various fields of engineering. Based on the theoretical model proposed by Saffman [5], many authors investigated a number of dusty gas flow problems in various geometries. Regarding the plate problems, Michael and Miller [7], Liu [22,23], Healy and Yang [24] investigated a number of dusty gas flow problems. But little attention is paid to the flow of a dusty gas in rotating system although this has some bearing on the pollution problem as well as on the motion of aerosol over the rotating earth. Recently, Gupta and Pop [9] investigated the unsteady boundary layer flow in a rotating viscous liquid bounded by an infinite flat plate when there is a suspension of dust particles in the liquid. Very recently Jana et al. [25] investigated the unsteady flow in the Ekman layer of an elastico-viscous liquid.

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In the present investigation, we extend the analysis of Gupta and Pop [9] to cover a wider class of elastico-viscous liquid, viz. Walters' liquid B' (with short memory) [26] and, in particular, to observe (qualitatively) the effects of elastic element and mass concentration on the flow field.

3.2.9 Mathematical formulation and asymptotic analysis

We consider an infinite plate coinciding with the plane $z=0$ and rotating in unison with a dusty elastico-viscous liquid occupying the region $z > 0$ with a uniform angular velocity Ω about the z-axis for time $t \leq 0$. At time $t > 0$, the plate starts moving with a uniform velocity U in its own plane relative to the rotating frame of reference. The horizontal homogeneity of the problem demands that the physical quantities depend on z and t only. The equation of continuity of the liquid then gives $w \equiv 0$ everywhere in the flow, where (u,v,w) are components of the liquid velocity at a point.

The equation of continuity of dust particles is given as

$$\frac{\partial N}{\partial t} + (N v_i')_{,i} = 0 \quad ,$$

where N is the number density of dust particle and v_i' $(u', v', 0)$ is the velocity of dust particle. Since the distribution of dust particles is uniform, the number density of the particles $N = N_0$, a constant throughout the motion.

Using (1.6) and (1.10) in equations (1.18) and (1.19), we get the equations of motion for the liquid and dust particle in a rotating frame of reference as

$$\frac{\partial u}{\partial t} - 2\,\Omega\, v = \nu \left(1 - \frac{K_0}{\rho\nu}\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2} + \frac{K N_0}{\rho}(u' - u) \quad , \qquad (3.2.33)$$

$$\frac{\partial v}{\partial t} + 2\,\Omega\, u = \nu\,\left(1 - \frac{K_0}{\rho\nu}\,\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} + \frac{K\,N_0}{\rho}(v' - v)\,, \tag{3.2.34}$$

$$\frac{\partial u'}{\partial t} - 2\,\Omega\, v' = \frac{K}{m}(u - u')\,, \tag{3.2.35}$$

$$\frac{\partial v'}{\partial t} + 2\,\Omega\, u' = \frac{K}{m}(v - v')\,, \tag{3.2.36}$$

where m is the mass of a dust particle, K is Stokes resistance coefficient, K_0 the elastic coefficient, $\nu = \frac{\eta}{\rho}$, η the limiting viscosity at small rates of shear and ρ the density. Equations (3.2.33) – (3.2.36) are combined as

$$\frac{\partial q}{\partial T} - \left(1 - K_1\frac{\partial}{\partial T}\right)\frac{\partial^2 q}{\partial Z^2} + (2i\omega + f)q - fq' = 0, \tag{3.2.37}$$

$$\frac{\partial q'}{\partial T} + (2i\omega + 1)q' - q = 0, \tag{3.2.38}$$

where

$$q = \frac{u + iv}{U}\ (= u_1 + iv_1),\ q' = \frac{u' + iv'}{U}\ (= u'_1 + iv'_1)\ ,$$

$$T = \frac{t}{\tau}\ ,\ Z = \frac{z}{(\nu\tau)^{1/2}}\ ,\ \tau = \frac{m}{K}\ ,\ \Omega\,\tau = \omega,\ K_1 = \frac{K_0}{\rho\nu\tau}\ \text{and}$$

$f = \frac{mN_0}{\rho}$ is the mass concentration of dust particles and τ is the relaxation time of dust particles.

The initial and boundary conditions are

$$q = q' = 0 \quad \text{for } T \leq 0, \tag{3.2.39}$$

$$q = 1 \text{ at } Z = 0 \text{ for } T > 0, \tag{3.2.40}$$

$$q \longrightarrow 0 \text{ as } Z \longrightarrow \infty\ . \tag{3.2.41}$$

Taking Laplace transforms of (3.2.37) and (3.2.38) and using (3.2.39), we get

$$(1 - K_1 p)\,\frac{d^2\bar{q}}{dZ^2} - (2i\omega + f + p)\bar{q} + f\bar{q}' = 0, \tag{3.2.42}$$

$$(p + 2\, i\, \omega + 1)\, \bar{q}' = \bar{q} \ , \qquad (3.2.43)$$

where

$$\bar{q}\,(Z,p) = \int_0^\infty q(Z,T)\, e^{-pT}\, dT \ ,$$

$$\bar{q}'\,(Z,p) = \int_0^\infty q'\,(Z,T)\, e^{-pT}\, dT \ , \ \mathrm{Re}(p) > 0.$$

Eliminating $\bar{q}'$ from equations (3.2.42) and (3.2.43) and then solving for $\bar{q}$ with the help of transformed boundary condition, we have

$$\bar{q}\,(Z,p) = \frac{1}{p}\, e^{-MZ} \ , \qquad (3.2.44)$$

where

$$M = \frac{1}{(1 - K_1 p)^{1/2}} \left[p + 2\, i\, \omega + f - \frac{f}{1 + p + 2\, i\, \omega} \right]^{1/2}$$

with the proviso that real part of M is taken positive.

The dust velocity at the plate Z = 0 is given by equations (3.2.43) and (3.2.44) as

$$q'\,(0,T) = \mathcal{L}^{-1} \left[\frac{1}{p(p + 1 + 2\, i\, \omega)} \right] . \qquad (3.2.45)$$

Taking inverse transformation of equation (3.2.45) and separating real and imaginary parts, we have

$$u'_1(0,T) = \frac{1}{1+ 4\omega^2} \left[1-e^{-T}\cos 2\omega T+2\omega e^{-T}\sin 2\omega T \right] , \qquad (3.2.46)$$

$$v'_1(0,T) = \frac{1}{1+ 4\omega^2} \left[e^{-T}\sin 2\omega T-2\omega(1-e^{-T}\cos 2\omega T) \right] . \qquad (3.2.47)$$

It is interesting to note from equations (3.2.46) and (3.2.47) that the dust particles do not stick to the plate but move relative to it. However, such a velocity slip is compatible with the assumption that the bulk concentration of the dust particles is small since we can then imagine that the particles nearest to the plate will in general be several particle diameters away from it. It is also remarked that the velocity of dust particles on the plate is unaffected by the presence of elastic parameter of the fluid.

To investigate the asymptotic nature of the solution for large time, we assume $p << 1$. Equation (3.2.44) can then be approximated by

$$\bar{q}\,(Z,p) = \frac{1}{p} e^{-[A(\,p + B/A)]^{1/2}\,Z}\,, \tag{3.2.48}$$

where

$$A = \alpha + i\beta\,, \tag{3.2.49}$$

$$B = 2\,i\,\omega\,\left(1 + \frac{f}{1 + 2\,i\,\omega}\right), \tag{3.2.50}$$

so that $\alpha = (1 + 4\omega^2 + 4\,\omega^2 f\, K_1)\,/\,(1 + 4\,\omega^2)$,

$\beta = 2\,\omega\, K_1(1 + 4\omega^2 + f\,)\,/\,(1 + 4\,\omega^2)$,

Equation (3.2.48) gives, on using the table of the inverse Laplace transforms due to Campbell and Foster [27],

$$q\,(Z,T) = \frac{1}{2}\Big[\,e^{\sqrt{B}\,Z}\,.\mathrm{erfc}\left(\frac{\sqrt{A}}{2\sqrt{T}}\,Z + \frac{\sqrt{BT}}{\sqrt{A}}\right) + e^{-\sqrt{B}\,Z}$$

$$\mathrm{X}\ \mathrm{erfc}\left(\frac{\sqrt{A}}{2\sqrt{T}}\,Z - \frac{\sqrt{BT}}{\sqrt{A}}\right)\Big]\,. \tag{3.2.51}$$

It is known that

$$\mathrm{erfc}\,(Z) \simeq Z^{-1}\,\pi^{-1/2}\,\exp\,(-\,Z^{2}) \text{ as } |\,Z\,| \longrightarrow \infty\,, \tag{3.2.52}$$

$$\mathrm{erfc}\,(-Z) = 2 - \mathrm{erfc}\,(Z).$$

Using equations (3.2.51) and (3.2.52) and separating $q(Z,T)$ into real and imaginary parts, we get the asymptotic expressions for $u_1(Z,T)$ and $v_1(Z,T)$ for large T as

$$u_1(Z,T) = e^{-Z\alpha_1} \cos \beta_1 Z - \frac{Z}{2\sqrt{\pi T}} \cdot \frac{1}{(\alpha_5^2 + \beta_5^2)}$$

$$\times \left\{(\alpha_2\alpha_5 + \beta_2\beta_5) \cos \alpha_4 + (\beta_2\alpha_5 - \alpha_2\beta_5) \sin \alpha_4 \right\}$$

$$\times \exp \left\{ - (\frac{\alpha Z^2}{4T} + (\alpha_3^2 - \beta_3^2) T) \right\} , \qquad (3.2.53)$$

$$v_1(Z,T) = e^{-Z\alpha_1} \sin \beta_1 Z - \frac{Z}{2\sqrt{\pi T}} \cdot \frac{1}{(\alpha_5^2 + \beta_5^2)}$$

$$\times \left\{(\beta_2\alpha_5 - \alpha_2\beta_5) \cos \alpha_4 - (\alpha_2\alpha_5 + \beta_2\beta_5) \sin \alpha_4 \right\}$$

$$\times \exp \left\{ - (\frac{\alpha Z^2}{4T} + (\alpha_3^2 - \beta_3^2) T) \right\} , \qquad (3.2.54)$$

where

$$\alpha_1, \beta_1 = \left(\frac{\omega}{1 + 4\omega^2}\right)^{1/2} \left[\left\{ (1 + 4\omega^2 + f)^2 + 4 \omega^2 f^2 \right\}^{1/2} \pm 2\omega f \right]^{1/2},$$

$$\alpha_2, \beta_2 = \frac{1}{(2(1 + 4\omega^2))^{1/2}} \left[\left\{ (1 + 4\omega^2 + 4\omega^2 f K_1)^2 + 4\omega^2 K_1^2 \right.\right.$$

$$\left.\left. \times (1+f+4\omega^2)^2 \right\}^{1/2} \pm (1 + 4\omega^2 + 4\omega^2 f K_1) \right]^{1/2},$$

$$\alpha_3 = (\alpha_1\alpha_2 + \beta_1\beta_2) / (\alpha_2^2 + \beta_2^2),$$

$$\beta_3 = (\beta_1\alpha_2 - \alpha_1\beta_2) / (\alpha_2^2 + \beta_2^2),$$

$$\alpha_4 = (\beta + 2 \alpha_3\beta_3 T) ,$$

$$\beta_4 = 0,$$

$$\alpha_5 = (\alpha_3^2 - \beta_3^2)T - \alpha Z^2/4T,$$

$$\beta_5 = 2\alpha_3\beta_3 T - \beta Z^2/4T.$$

The dimensionless skin-friction can be calculated from equation (3.2.51) as

$$- \frac{\partial q}{\partial Z}\bigg|_{Z=0} = (\alpha_1 + i\beta_1).\mathrm{erf}((\alpha_3 + i\beta_3)\sqrt{T}) + \frac{\alpha_2 + i\beta_2}{\sqrt{\pi T}} e^{-BT/A} . \qquad (3.2.55)$$

3.2.10 Discussion

The first terms in equations (3.2.53) and (3.2.54) represent the velocity components for the Ekman boundary layer (modified by the presence of dust particles) on the plate, which is established in the final steady state. The boundary layer thickness is clearly of order $(\alpha_1)^{-1}$ and gradually it becomes thinner with the increase in f, the mass concentration. This distribution is independent of K_1. Another distinctive feature of the above asymptotic solution is that the second terms in equations (3.2.53) and (3.2.54) confirm the existence of inertial oscillations which decay exponentially with time. The effect of rotation menifests itself through these oscillations with frequency, $(2\alpha_3\beta_3)$. The effects of elastic element, mass concentration on the frequency of inertial oscillations are shown in table 3.3 (a,b). It reveals that the frequency decreases with increase in either mass concentration or elastic element in the liquid.

As $T \longrightarrow \infty$, the equation (3.2.55) gives the steady state skin-friction as $(\alpha_1 + i\beta_1)$. In absence of dust parameter, it reduces to $(1 + i)\ \omega^{1/2}$, which agrees with the classical result for Ekman spiral near a plate in a rotating frame. It is of interest to have an estimate of the time which elapses from the start of the plate in the rotating frame till the steady state is reached. It is clear from the equation (3.2.55) that the steady state is reached after a time T_0 where

$$\mathrm{erf}((\alpha_3 + i\beta_3)T_0) \simeq 1 .$$

Since $\mathrm{erf}(x) \simeq 1$ when $|x| \gtrsim 2$, it follows that

$$T_0 \simeq \frac{2}{(\alpha_3^2 + \beta_3^2)^{1/2}} \quad .$$

The effects of mass concentration and elastic parameter on T_0 can be seen from table 3.4 (a,b). It reveals that the effect of mass concentration is to decrease the time to reach the steady state while it increases as the elastic element increases.

Table 3.3:(a) Effect of elastic element on frequency of inertial oscillations when $f = 0.1$, $\omega = 1.0$.

K_1	0.5	1.0	1.5	2.0	2.5
$2\alpha_3\beta_3$	1.4004176	0.8837880	0.6260497	0.4809395	0.4193873

(b) Effect of mass concentration on frequency of inertial oscillations when $K_1 = 0.5$, $\omega = 1.0$.

f	0.10	0.20	0.30	0.35	0.40
$2\alpha_3\beta_3$	1.4004176	1.3872846	1.3747694	1.3699121	1.3609122

Table 3.4:(a) Effect of elastic element on the time (T_0) to reach the steady state when $f = 0.1$, $\omega = 1.0$.

K_1	0.5	1.0	1.5	2.0	2.5
T_0	1.5906671	1.9306756	2.2767133	2.6301002	3.0001219

(b) Effect of mass concentration on the time (T_0) to reach the steady state when $K_1 = 0.5$, $\omega = 1.0$.

f	0.10	0.15	0.20	0.25	0.30
T_0	1.5906671	1.5851219	1.5768748	1.5751921	1.5728127

REFERENCES

[1] Khamrui, S. R. (1957) Bull. Cal. Math. Soc., Vol.47, 61.

[2] Iben, H. Ebene (1974) ZAMM, Vol.54, 213.

[3] Bhattacharyya, P. (1981) Indian J. Math., Vol.23, 65.

[4] Mukherjee, S. and Bhattacharyya, P. (1982) Indian J. Math., Vol.24, 113.

[5] Saffman, P. G. (1962) J. Fluid Mech., Vol.13, 120.

[6] Marble, F. E. (1963) Fifth AGARD Combust. Propul. Colloq.,Pergamon Press,Oxford,175.

[7] Michael, D. H. and Miller, D. A. (1966) Mathematika, Vol.13,97.

[8] Ramana Prasad,V.V. and Ramacharyulu, N. Ch. P. (1981) Def. Sci. J., Vol.31, 231.

[9] Gupta, A. S. and Pop, I. (1975) Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie, Vol.19, 291.

[10] Vimala, C. S. (1972) Def. Sci. J., Vol.22, 231.

[11] Kishore, N. and Pandey, R. D. (1977) J. Scientific Res., Vol.XXVIII(2), 151.

[12] Srivastava, L. P. (1971) Istanbul Teknik Universitesi Bulteni, Vol.24, 19.

[13] Sharma, C. L. and Dube, S. N. (1976) Bull. de L' Acad. Polo. des Sci. Series des Sci. Tech., Vol. XXIV, 145.

[14] Bagchi, S. and Maiti, M. K. (1980) Acta Ciencia Indica, Vol.VIm, 130.

[15] Mukherjee, S., Maiti, M. K. and Mukherjee, S. (1984) Indian J. Technol., Vol.22, 41.

[16] Kuvshiniski,E. V. (1951) J. Exptl. Theor. Phys. (USSR), Vol.21, 88.

[17] Srivastava, P. N. (1964) Indian J. Math., Vol.6, 29.

[18] Tandon, P. N. (1969) J. Sci. Engng. Res., Vol.13, 280.

[19] Rao, S. K. L. and Rao, P. Bhujanga. (1972) Int. J. Engng. Sci., Vol.10, 185.

[20] Tandon, P. N. and Chandra, S. (1972) Labdev J. Sci. & Tech., Vol.10-A, 162.

[21] Mukherjee, S. and Mukherjee, S. (1983) Indian J. Pure & Appl. Maths., Vol.14, 1534.

[22] Liu, J. T. C. (1966) Phys. Fluids, Vol.9, 1716.

[23] Liu, J. T. C. (1967) Astronautica Acta, Vol.13, 369.

[24] Healy, J. V. and Yang, H. T.	(1972)	Astronautica Acta, Vol.17, 851.
[25] Jana, R. N., Gupta, A. S. and Datta, N.	(1982)	Rheologica Acta, Vol.21, 733.
[26] Walters, K.	(1962)	J. Mecanique, Vol.1, 474.
[27] Campbell, G. A. and Foster, R. M.	(1948)	Fourier Integrals for practical applications, Van Nostrand, New York.

CHAPTER - IV

CONVECTIVE DIFFUSION

AND

FREE CONVECTION AND MASS TRANSFER

PART ONE

EXACT ANALYSIS OF UNSTEADY MHD CONVECTIVE DIFFUSION IN A POROUS CHANNEL*

4.1 Introduction

Taylor [1] investigated the dispersion of soluble matter in a solvent flowing under laminar conditions in a circular tube. The analysis of Taylor on his conceptual model is applicable only for large values of time t. On the other hand, Lighthill [2], neglecting axial molecular diffusion, obtained an exact solution of the unsteady convective diffusion equation which is asymptotically valid for small t. However, recently Gill and Sankarasubramanian [3,4,5,6] developed a generalized dispersion model to study the unsteady convective diffusion systems, which are valid for all t, by involving an infinite set of time-dependent coefficients that can be determined from first principles and obtained exact solution for the local concentration C. Krishnamurthy and Subramanian [7], using generalized dispersion model, formulated convective diffusion theory for the predictive modelling of field-flow fractionation columns used for the seperation of colloidal mixture.

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The purpose of the present investigation is to study the dispersion of soluble matter in an incompressible electrically conducting viscous fluid in a porous-walled parallel plate channel permeated by transverse magnetic field. We desire to obtain the dispersion coefficients in MHD flow in a porous-walled channel valid for all time t and also to find the dependencies of these coefficients on the cross-flow Pe'clet number P and the magnetic parameter M, defined for the problem considered. The results of the present investigation are likely to have applications to situations where tracers are used for measuring the flow rate in a porous-walled channel, in drilling technology and in environmental pollution problems.

4.2 Mathematical formulation

We consider the laminar flow under a uniform pressure gradient Pr of an electrically conducting incompressible viscous fluid (of conductivity σ and permeability μ_e) between two infinite electrically non-conducting porous-walled parallel plate channel permeated by transverse magnetic field H_o along y-axis. This flow, studied by Hartmann [8], has a velocity u(y) along x-axis (parallel to the plates) given by,

$$u(y) = Z\left[M \coth M - \frac{M \cosh (My/b)}{\sinh M}\right], \qquad (4.1)$$

$$Z = \frac{Pr}{\mu_e^2 \sigma H_o^2}, \quad M = \mu_e H_o b(\sigma/\rho\nu)^{1/2}. \qquad (4.2)$$

The average velocity $\bar{u}$ of the flow is

$$\bar{u} = \frac{1}{2b}\int_{-b}^{b} u(y)dy = Z\ (M \coth M - 1). \qquad (4.3)$$

In addition to this, some fluid is injected with a velocity v in the lower porous plate and removed the same at the opposite plate at the same rate.

If a solute diffuses in the above fully developed flow, the concentration C(t,x,y) of solute satisfies

$$\frac{\partial C}{\partial t} + u(y)\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] \qquad (4.4)$$

along with the following initial and boundary conditions

$$\left.\begin{aligned} C(0,x,y) &= C_o \qquad \text{for} \quad |x| \le \tfrac{1}{2}x_s \\ &= 0 \qquad \text{for} \quad |x| > \tfrac{1}{2}x_s \end{aligned}\right\}, \qquad (4.5)$$

$$-D\frac{\partial C(t,x,\pm b)}{\partial y} + vC(t,x,\pm b) = 0, \qquad (4.6)$$

where D is the molecular diffusivity assumed to be independent of C, x_s is the length of the slag input of solute and since the amount of solute is finite

$$C(t,\infty,y) = 0. \qquad (4.7)$$

Since there is no net solute flux at the walls, the total mass of the solute in the channel is conserved and the equation (4.6) is consistent with this statement.

Introducing the dimensionless variables

$$\theta = \frac{C}{C_o},\ U(y) = \left[\frac{u(y)}{\bar{u}} - 1\right],\ X = \frac{Dx}{b^2\bar{u}},$$

$$Y = \frac{y}{b},\ P = \frac{bv}{D},\ P_e = \frac{b\bar{u}}{D},\ T = \frac{Dt}{b^2}$$

in equations (4.4) – (4.7) we get

$$\frac{\partial\theta}{\partial T} + U(y)\,\frac{\partial\theta}{\partial X} + P\,\frac{\partial\theta}{\partial Y} = \left[\frac{1}{P_e^2}\,\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right] \qquad (4.8)$$

with the conditions

$$\left.\begin{aligned}\theta(0,X,Y) &= 1 \quad \text{for} \quad |X| \le \tfrac{1}{2}X_s\\ &= 0 \quad \text{for} \quad |X| > \tfrac{1}{2}X_s\end{aligned}\right\}, \qquad (4.9)$$

$$\frac{\partial\theta(T,X,\pm1)}{\partial Y} = P\theta(T,X,\pm1), \qquad (4.10)$$

$$\theta(T,\infty,Y) = 0. \qquad (4.11)$$

Define a new axial co-ordinate moving with the average velocity of flow in the dimensionless form as

$$X_1 = X - T. \qquad (4.12)$$

4.3 Solution

The solution of equations (4.8) – (4.11) are formulated as a series expansion in $\frac{\partial^k\theta_m}{\partial X_1^k}$ such that

$$\theta = \sum_{k=0}^{\infty} f_k(T,Y)\,\frac{\partial^k\theta_m}{\partial X_1^k}, \qquad (4.13)$$

where $\theta_m = \frac{1}{2}\int_{-1}^{1}\theta\,dY.$ $\qquad (4.14)$

Upon substituting equation (4.13) into (4.8), after transforming to (T,X_1,Y) coordinate system and integrating equation (4.8) with respect to Y from −1 to 1 followed by the use of equation (4.10), we obtain generalised dispersion equation for $\theta_m\,(T,X_1)$ as

$$\frac{\partial \theta_m}{\partial T} = \sum_{i=1}^{\infty} K_i(T) \frac{\partial^i \theta_m}{\partial X_1^i} , \qquad (4.15)$$

where

$$K_i(T) = \frac{\delta_{i2}}{P_e^2} - \frac{1}{2} \int_{-1}^{1} U(Y) f_{i-1}(T,Y)\,dY, \quad (i = 1,2,3,\ldots) \qquad (4.16)$$

and the differential equations for the function $f_k(T,Y)$ defined in equation (4.14) are

$$\frac{\partial f_k}{\partial T} + P \frac{\partial f_k}{\partial Y} = \frac{\partial^2 f_k}{\partial Y^2} - U f_{k-1} + \frac{1}{P_e^2} f_{k-2} - \sum_{i=1}^{k} K_i f_{k-i} ,$$

$$(k=0,1,2,\ldots\ldots) \qquad (4.17)$$

with $f_{-1} = f_{-2} = 0$.

The initial and boundary conditions on the functions $f_k(T,Y)$ are

$$\frac{\partial f_k}{\partial Y} - P f_k = 0 \text{ at } Y = \pm 1, \qquad (4.18)$$

$$f_k(0,Y) = 0 , \qquad (4.19)$$

$$\int_{-1}^{1} f_k \, dY = 2\delta_{k0}, \quad (k = 0,1,2,\ldots). \qquad (4.20)$$

The function $f_o(T,Y)$ is independent of the velocity field and can be easily solved. The equation for $f_o(T,Y)$ is derived from equation (4.17) by setting $k = 0$:

$$\frac{\partial f_o}{\partial T} + P \frac{\partial f_o}{\partial Y} = \frac{\partial^2 f_o}{\partial Y^2} , \qquad (4.21)$$

$$\frac{\partial f_o}{\partial Y}(T,\pm 1) = P f_o(T, \pm 1), \qquad (4.22)$$

$$f_o(0,Y) = 0, \qquad (4.23)$$

$$\int_{-1}^{1} f_o(T,Y)dY = 2. \tag{4.24}$$

Using the method of separation of variables the solution of equation (4.21) and (4.22), (4.23), (4.24) are obtained as

$$f_o(T,Y) = \sum_{n=o}^{\infty} A_n e^{-\lambda_n^2 T} . \phi_n(Y), \tag{4.25}$$

$$\lambda_n^2 = b_n^2 + \frac{P^2}{4}, \tag{4.26}$$

$$b_o^2 = \frac{-P^2}{4}, \quad b_n^2 = \frac{n^2\pi^2}{4}, \quad n = 1,2,..... \tag{4.27}$$

and

$$\phi_n(Y) = e^{(P/2)Y} (\cos b_n Y + H_n \sin b_n Y) \tag{4.28}$$

and

$$\left.\begin{aligned} H_n &= -\frac{2b_n}{P} \quad (n \text{ odd}), \\ &= \frac{P}{2b_n} \quad (n \text{ even}). \end{aligned}\right\} \tag{4.29}$$

When n = 0, equation (4.28) may be written as

$$\phi_o(Y) = e^{PY}. \tag{4.30}$$

The expansion coefficients A_n are obtained by using the orthogonality property of the set of eigen function ϕ_n and

$$\begin{aligned} A_n &= \frac{1}{\lambda_n^4} (P^2 b_n \sin b_n \cosh \frac{P}{2}), \quad (n \text{ odd}) \\ &= \frac{1}{\lambda_n^4} (2P b_n^2 \cos b_n \sinh \frac{P}{2}), \quad (n \text{ even}). \end{aligned} \tag{4.31}$$

It is noted that A_0 is independent of the initial distribution and is given as

$$A_o = \frac{P}{\sinh P} . \tag{4.32}$$

Asymptotically as $T \longrightarrow \infty$, $f_o(\infty,Y)$ approaches a steady state distribution given by

$$f_o(\infty,Y) = \frac{P}{\sinh P} e^{PY} . \tag{4.33}$$

From equation (4.16), we have

$$K_1(T) = - \frac{1}{2} \int_{-1}^{1} U(Y) f_o(T,Y)\, dY$$

$$= K_1(\infty) + \frac{1}{2} \frac{M/\sinh M}{M \coth M-1} \sum_{n=1}^{\infty} A_n\, e^{-\lambda_n^2 T} . C_n , \tag{4.34}$$

where

$$K_1(\infty) = - \frac{1}{(M \coth M - 1)} + \frac{1}{2} \frac{MP/\sinh P}{(M \coth M - 1) \sinh M}$$

$$X \left\{ \frac{\sinh(P+M)}{(P+M)} + \frac{\sinh(P-M)}{(P-M)} \right\} \tag{4.35}$$

and

$$C_n = - \frac{2Mb_n}{P} \sin b_n \left[\frac{\cosh(\frac{P}{2}+M)}{(\frac{P}{2}+M)^2 + b_n^2} - \frac{\cosh(\frac{P}{2}-M)}{(\frac{P}{2}-M)^2 + b_n^2} \right] , (n \text{ odd})$$

$$= M \cos b_n \left[\frac{\sinh(\frac{P}{2}+M)}{(\frac{P}{2}+M)^2 + b_n^2} - \frac{\sinh(\frac{P}{2}-M)}{(\frac{P}{2}-M)^2 + b_n^2} \right] , \qquad (n \text{ even}).$$

From equation (4.34), it is interesting to note that $K_1(T)$ depends on T even though velocity field is independent of T.

Putting k=1 in equation (4.17) we get an expression for f_1 as

$$\frac{\partial f_1}{\partial T} + P\frac{\partial f_1}{\partial Y} = \frac{\partial^2 f_1}{\partial Y^2} - \left[U(Y) + K_1(T)\right] f_0(T,Y) \tag{4.36}$$

along with the conditions

$$\frac{\partial f_1}{\partial Y}(T,\pm\ 1) = Pf_1(T,\pm\ 1) \tag{4.37a}$$

$$\text{and} \int_{-1}^{1} f_1 dY = 0,\ f_1(0,Y) = 0. \tag{4.37b}$$

Using Duhamel's theorem, solution for equation (4.36) is written as

$$f_1(T,Y) = f_1(\infty,Y) + \sum_{n=1}^{\infty} S_n(T)\ \phi_n(Y), \tag{4.38}$$

where

$$f_1(\infty,Y) = L_1 + L_2 e^{PY} + \frac{L_3}{P}\left(Y - \frac{1}{P}\right)e^{PY} - \frac{L_4 e^{PY}}{M^2 - P^2}\left(\cosh MY - \frac{P}{M}\sinh MY\right), \tag{4.39}$$

$$S_n(T) = -\frac{1}{E_n}\left[\sum_{s=1}^{\infty}\left(F\,A_s'\,d_{sn} + 2GB_{sn}\left(e^{-b_{ns}T} - e^{-\lambda_n^2 T}\right)\right) + \sum_{s=1}^{\infty}\sum_{m=1}^{\infty} D_{smn}\, e^{-\lambda_n^2 T}\left\{e^{-(\lambda_n^2 + \lambda_m^2)} - 1\right\}\right] + \sum_{m=1}^{\infty} B_m e^{-\lambda_m^2 T} - \frac{B_n'}{E_n} e^{-\lambda_n^2}\ T, \tag{4.40}$$

$$L_3 = \frac{P}{\sinh P}\left\{ K_1(\infty) + \frac{1}{M\coth M - 1}\right\}, \tag{4.41}$$

$$L_4 = \frac{PM/(M\coth M - 1)}{\sinh M \sinh P}, \tag{4.42}$$

$$L_1 = \frac{e^{-P}}{P}\left[\frac{L_3}{P} + \frac{L_4}{M^2 - P^2}(M \sinh M - P \cosh M)\right], \qquad (4.43)$$

$$B_m = \frac{1}{1+G_m^2}\left[\frac{2L_1 P}{\lambda_m^2} + \frac{2L_3}{P\lambda_m^2}\cos b_m \sinh\frac{P}{2}\right.$$

$$- \frac{L_4}{M^2 - P^2}\left\{(1 - \frac{P}{m})\, 4M \cos b_m \sinh(\frac{P}{2} + M)\right.$$

$$\left. - (1 + \frac{P}{m})\frac{4M \cos b_m \sinh(\frac{P}{2} - M)}{(P - 2M)^2 + 4b_m^2}\right\}, \quad (m \text{ even})$$

$$= \frac{1}{1 + G_m^2}\left[\frac{4b_m L_1}{\lambda_m^2}\sin b_m \cosh\frac{P}{2} - \frac{4L_3 b_m}{\lambda_m^2 P^2}\sin b_m \cosh\frac{P}{2}\right.$$

$$+ \frac{L_4}{M^2 - P^2}\left[\frac{(1 - \frac{P}{M})8Mb_m \sin b_m \cosh(\frac{P}{2} + M)}{P\left\{(P + 2M)^2 + 4b_m^2\right\}}\right.$$

$$\left. + (1 + \frac{P}{M})\frac{8M b_m \sin b_m \cosh(\frac{P}{2} - M)}{P\left\{(P - 2M)^2 + 4b_m^2\right\}}\right], \quad (m \text{ odd}) \qquad (4.44)$$

$$L_2 = -\frac{L_1 P}{\sinh P} + \frac{L_3}{P}(\frac{2}{P} - \coth P) + \frac{L_4 P}{(M^2 - P^2)^2}$$

$$\times \left\{(M + \frac{P^2}{M}) \sinh M \coth P + 2P \cosh M\right\}, \qquad (4.45)$$

$$E_n = (1 + G_n^2), \qquad (4.46)$$

$$F = \frac{1}{(M \coth M - 1)} + K_1(\infty), \qquad (4.47)$$

$$G = \frac{1}{2}\frac{M/\sinh M}{M \coth M - 1}, \qquad (4.48)$$

$$A'_s = -\frac{U_s}{\lambda_s^2}\ , \tag{4.49}$$

$$\left.\begin{aligned} d_{ns} &= (1 + G_n^2) \quad \text{if } n = s \\ &= 0 \qquad\qquad \text{if } n \neq s \end{aligned}\right\}, \tag{4.50}$$

$$B_{ns} = \frac{A_n\ f_{ns}}{\lambda_n^2}\ , \tag{4.51}$$

$$f_{ns} = 4M \sinh M \left[\frac{1 + G_n\ G_s}{M^2 + (b_n - b_s)^2} - \frac{1 - G_n\ G_s}{M^2 + (b_n + b_s)^2} \right],$$

(n odd, s odd)

$$= 4M \sinh M \left[\frac{1 + G_n\ G_s}{M^2 + (b_n - b_s)^2} + \frac{1 - G_n\ G_s}{M^2 + (b_n + b_s)^2} \right],$$

(n even, s even)

$$= -\ 4 \cosh M \left[\frac{(b_n + b_s)(1 - G_n G_s)}{M^2 + (b_n + b_s)^2} + \frac{(b_n - b_s)(1 + G_n G_s)}{M^2 + (b_n - b_s)^2} \right],$$

(n odd, s even)

$$= -\ 4 \cosh M \left[\frac{(b_n + b_s)(1 - G_n G_s)}{M^2 + (b_n + b_s)^2} - \frac{(b_n - b_s)(1 + G_n G_s)}{M^2 + (b_n - b_s)^2} \right]$$

(n even, s odd) (4.52)

$$b_{ns} = (\lambda_n^2 + \lambda_s^2), \tag{4.53}$$

$$U_s = -\frac{1}{(1 + G_s^2)} \left[\left\{ \frac{1}{M \coth M - 1} + K_1(\lambda) \right\} \sum_{n=1}^{\infty} A_n\ e^{-\lambda_n^2 \lambda} . d_{ns} \right.$$

$$- \frac{MP/\sinh P \sinh M}{(M \coth M - 1)} e_s - \frac{M/(M \coth M - 1)}{\sinh M} \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \lambda} . f_{ns} \Bigg],$$

(4.54)

$$e_s = \frac{2Mb_s}{P} \sin b_s \left\{ \frac{\cosh (\frac{P}{2} - M)}{(\frac{P}{2} - M)^2 + b_s^2} - \frac{\cosh (\frac{P}{2} + M)}{(\frac{P}{2} + M)^2 + b_s^2} \right\}, \quad (s \quad \text{odd})$$

$$= M \cos b_s \left\{ \frac{\sinh (\frac{P}{2} + M)}{(\frac{P}{2} + M)^2 + b_s^2} - \frac{\sinh (\frac{P}{2} - M)}{(\frac{P}{2} - M)^2 + b_s^2} \right\}, (s \text{ even})$$

(4.55)

$$D_{nms} = - G \frac{A_n \, B_m \, C_n \, d_{ms}}{(\lambda_m^2 + \lambda_n^2)}, \qquad (4.56)$$

$$B'_n = \frac{2 \, P \, G \, e_n}{\sinh P} . \qquad (4.57)$$

Now setting k = 2 in equation (4.16) and using (4.38) we get the expression for $K_2(T)$ as

$$K_2(T) - \frac{1}{P_e^2} = K_2(\infty) + \frac{1}{2} \frac{M/\sinh M}{M \operatorname{Coth} M - 1} \sum_{n=1}^{\infty} S_n(T) C_n , \qquad (4.58)$$

where

$$K_2(\infty) = - \frac{1}{2(M \coth M - 1)} \Bigg[2L_2 \frac{\sin P}{P} + \frac{2L_3}{P^2} (\cosh P - \frac{2}{P} \sinh P)$$

$$+ \frac{L_4}{M^2 - P^2} (\frac{P}{M} - 1) . \frac{\sinh (P + M)}{(P + M)} - \frac{L_4}{M^2 - P^2} (\frac{P}{M} + 1) . \frac{\sinh (P - M)}{(P - M)}$$

$$- \frac{L_2 M}{\sinh M} \left\{ \frac{\sinh (P + M)}{(P + M)} + \frac{\sinh (P - M)}{(P - M)} \right\}$$

$$- \frac{L_3 M}{P \sinh M} \left\{ \frac{\cosh (P + M)}{(P + M)} - \frac{\sinh (P + M)}{(P + M)^2} + \frac{\cosh (P - M)}{(P - M)} \right.$$

$$- \frac{\sinh(P - M)}{(P - M)^2}\Bigg\} + \frac{L_3 M}{P^2 \sinh M}\left\{\frac{\sinh(P + M)}{(P + M)} + \frac{\sinh(P - M)}{(P - M)}\right\}$$

$$+ \frac{L_4 M/\sinh M}{2(M^2 - P^2)}\left\{\frac{\sinh(P+2M)}{(P + 2M)} + \frac{\sinh(P - 2M)}{(P - 2M)} + \frac{2\sinh P}{P}\right\}$$

$$- \frac{L_4 P/\sinh M}{2(M^2 - P^2)}\left\{\frac{\sinh(P+2M)}{(P + 2M)} - \frac{\sinh(P - 2M)}{(P - 2M)}\right\}\Bigg]. \qquad (4.59)$$

Gill and Sankarasubramanian [3] have shown that the equation (4.15) can be truncated after the term involving K_2 without causing serious error. So the resulting model for the mean concentration θ_m can be written as

$$\frac{\partial \theta_m}{\partial T} = K_1(T)\,\frac{\partial \theta_m}{\partial X_1} + K_2(T)\,\frac{\partial^2 \theta_m}{\partial X_1^2} . \qquad (4.60)$$

Since a slug is being considered, θ_m will have to satisfy

$$\left.\begin{aligned} \theta_m(0, X_1) &= 1, \quad |X_1| \le \tfrac{1}{2}\, X_s \\ &= 0, \quad |X_1| > \tfrac{1}{2}\, X_s \end{aligned}\right\} \qquad (4.61)$$

$$\theta_m(T, \infty) = 0. \qquad (4.62)$$

The solution of equation (4.60) satisfying initial and boundary conditions is given as

$$\theta_m(T, X_1) = \frac{1}{\sqrt{\pi X_1}}\, e^{-\xi^2/4\zeta} , \qquad (4.63)$$

where

$$\xi(T, X_1) = X_1 + \int_0^T K_1(\tau)\, d\tau ,$$

$$\zeta = \int_0^T K_2(\tau)\, d\tau .$$

4.4 Discussion

Following Gill and Sankarasubramanian [3], we confine ourselves to calculate the dispersion coefficients upto the order K_2. Higher order dispersion coefficients may be determined by adopting the foregoing procedure but the computation will be more and more complicated. Moreover, K_3 and higher order dispersion coefficients decrease in magnitude at a very rapid rate.

For small p, equation (4.35) may be approximated as

$$K_1(\infty, P) \approx \frac{1}{(M \coth M - 1)}\left(MP \coth M + \frac{P^2}{3} + \frac{P^2}{2M^2}\right). \quad (4.64)$$

Equation (4.64) shows that as P ⟶ 0 i.e. the case of no cross-flow at channel walls, $K_1(\infty)$ becomes zero and which is the expected result.

For large P, equation (4.35) is approximated as

$$K_1(\infty) \approx \frac{1}{(M \coth M-1)}\left[M\left(1+\frac{M^2}{P^2}\right) \coth M-1 - \frac{M^2}{P}\right]. \quad (4.65)$$

The present analysis reveals that the dimensionless solute concentration in the porous-walled channel depends on three pertinent flow parameters, viz. P (cross-flow Pe'clet number), P_e(axial Pe'clet number) and M (magnetic parameter). Equation (4.60) clearly shows that the solute concentration is convected downstream in the porous-walled channel with a time-dependent velocity $-K_1(T)$ and spreads axially with respect to its centre of gravity with a time-dependent dispersion coefficient $K_2(T)$. To get a physical insight into the problem, numerical calculations are carried out of the equations (4.34), (4.35) and (4.59).

Figure 4.1 shows that the transient approach of $K_1(T)$ to its steady state value for various representative values of P (fixed M) and M (fixed P). Figure 4.1 depicts that $K_1(T)$ increases as T increases and approaches to the steady state nearly at time T = 5 for different values of M and P. Figure 4.1 also clearly shows that the relaxation time required for $K_1(T)$ to reach its asymptotic steady state decreases with increasing P and M. As expected $K_1(T)$ is equal to zero for P = M = 0 at all time.

Figure 4.2 shows the graph of asymptotic $K_1(\infty)$ against P for different values of M. It reveals that $K_1(\infty)$ increases with increasing values of both P and M.

Figure 4.3 shows the behaviour of $K_2(\infty)$ as a function of P for various values of M and it can be remarked from the figure that $K_2(\infty)$ decreases as P increases with fixed M and $K_2(\infty)$ increases as M increases with fixed P.

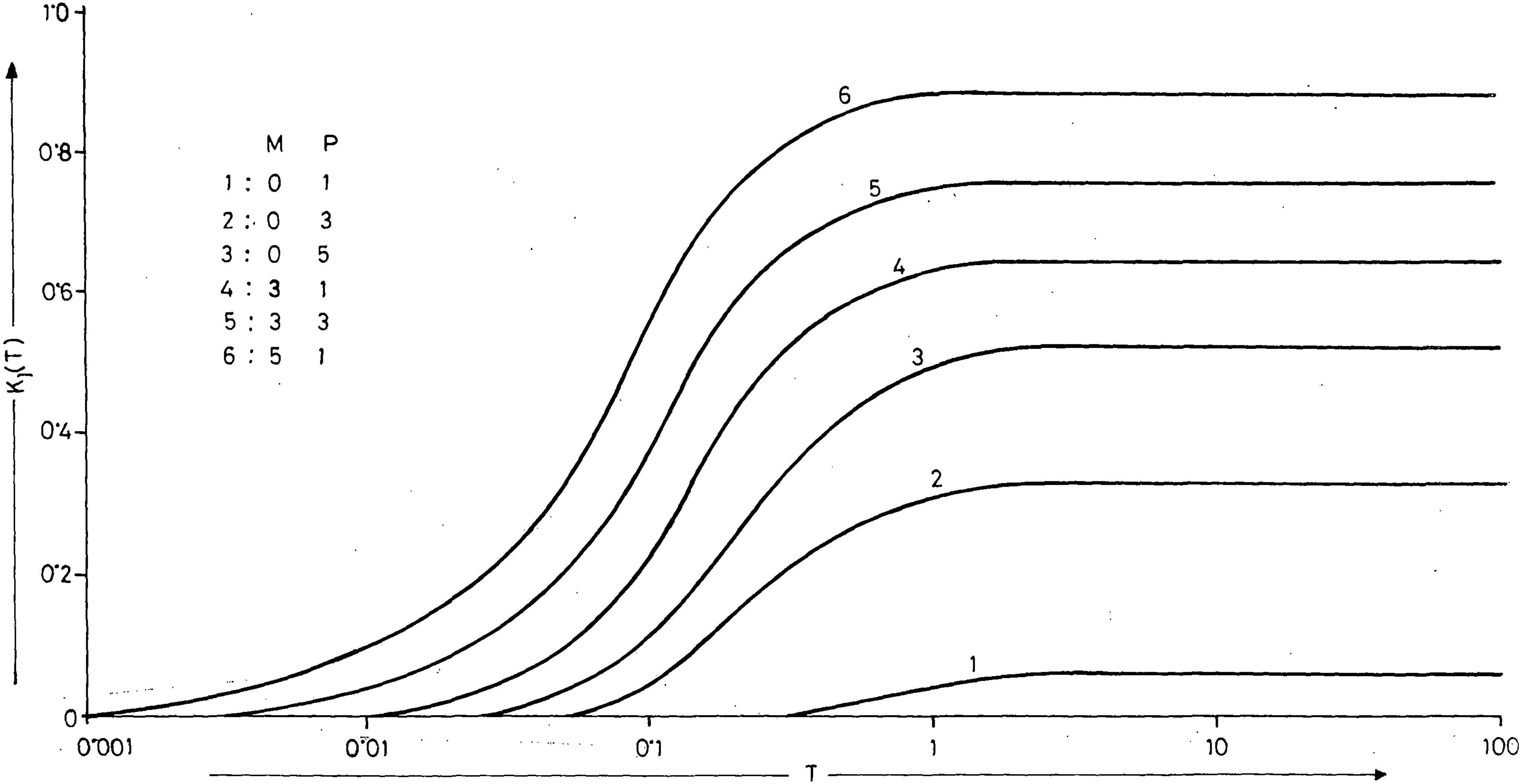

Fig. 4·1. Plot of the dimensionless convective coefficient $K_1(T)$ against T for different values of P and M.

(Semi-log).

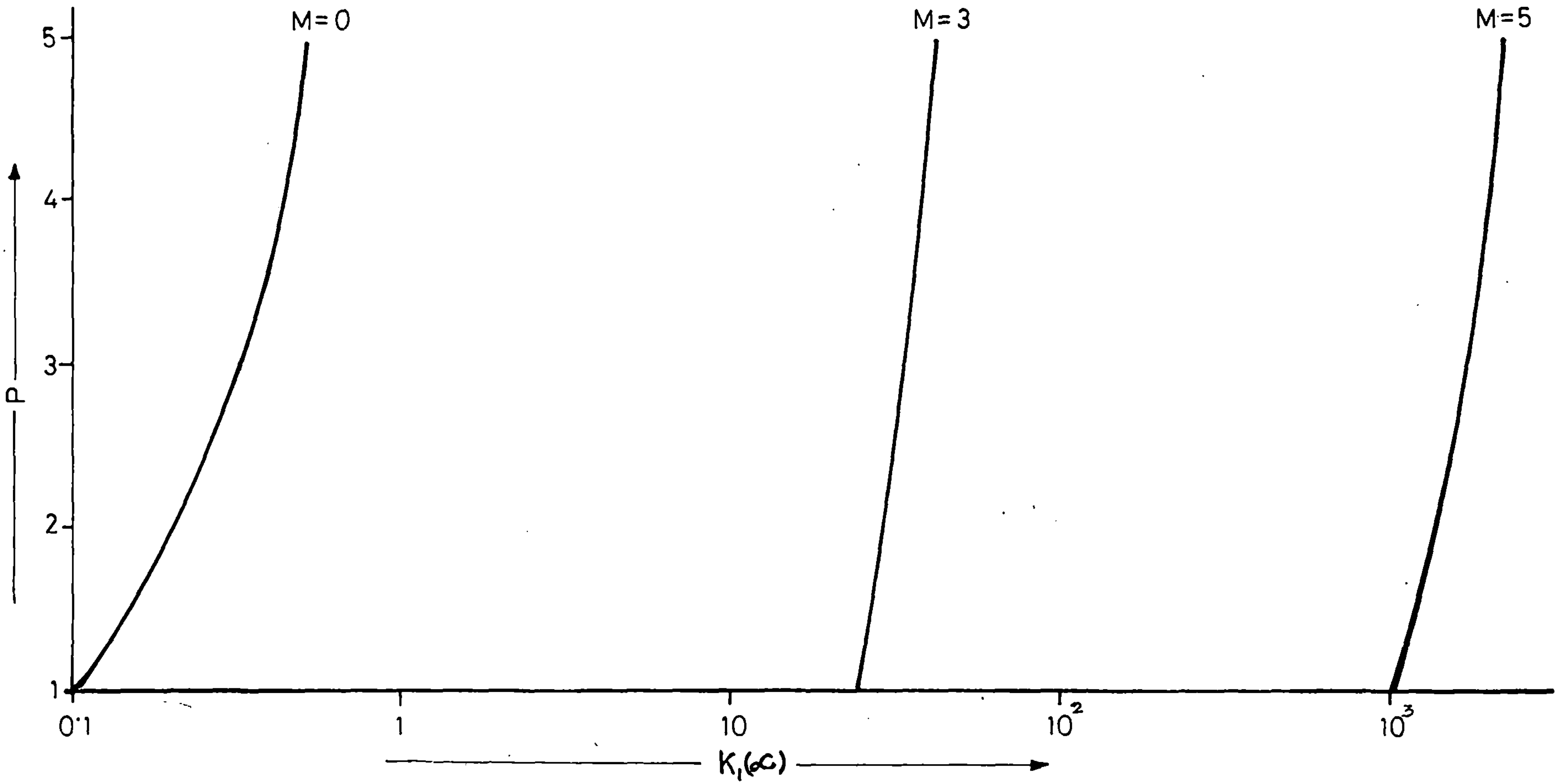

Fig. 4·2. Plot of the dimensionless steady-state convective coefficient $K_1(\infty)$ against **P** for different values of M.

(Semi - log)

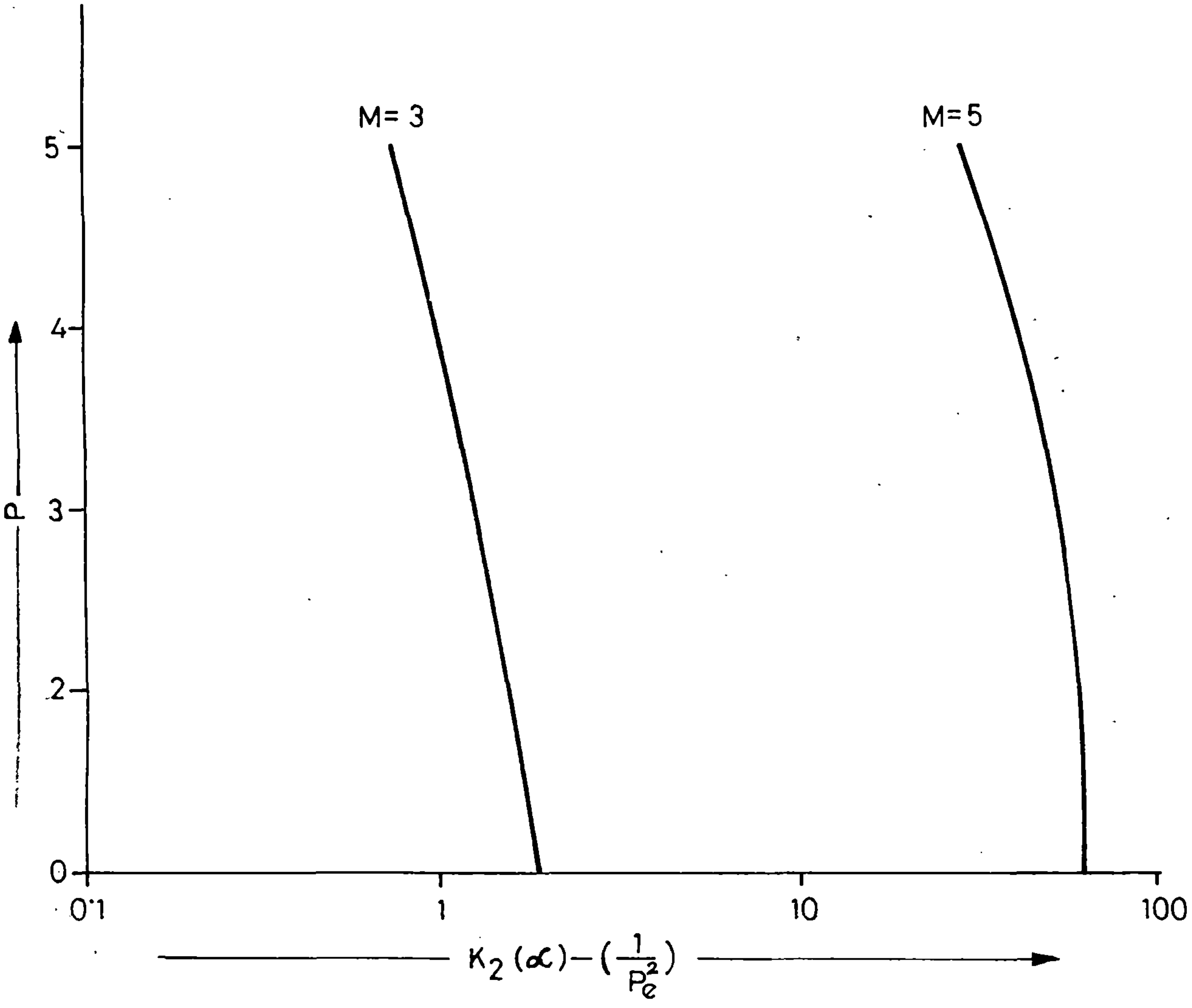

Fig. 4·3. Plot of the steady-state dispersion coefficient $K_2(\alpha) - (\frac{1}{P_e^2})$ against P for M = 3, 5.

(Semi - log).

UNSTEADY FREE CONVECTION MHD FLOW PAST AN INFINITE VERTICAL PLATE WITH CONSTANT SUCTION AND MASS TRANSFER *

4.5 Introduction

There are many transport processes occurring in nature due to temperature differences. This difference causes the density difference. This can be seen in our every day life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as mass transfer flow.

The Stokes problem for a vertical plate was solved by Soundalgekar [9] by taking into account the presence of free convection currents. The effect of mass transfer on the free convective flow in Stokes problem for an infinite vertical plate was studied by Georgantopoulos et al. [10]. In these papers the

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plate was assumed to be impermeable and fluid was non-conducting. It is known that mhd flows have received considerable attention [11, 12, 13, 14, 15] because of their practical applications. Therefore it is of interest to make an investigation in order to analyse the effects of magnetic field on the free convection flow with mass transfer of an electrically conducting viscous fluid past a vertical plate subjected to a constant suction.

4.6 Mathematical formulation

We consider the unsteady flow in a porous medium of an electrically conducting viscous fluid past an infinite vertical porous plate. The temperature at the plate varies with time about a non-zero constant mean while the temperature at the free stream is constant. Also the species concentration on the plane surface and at the free stream are constant. The x-axis is taken along the plate in vertically upward direction and z-axis is taken normal to the plate and pointing towards the medium. A magnetic field of uniform strength is applied transversely in the direction of flow. The governing equations for flow are given by

$$\frac{\partial w}{\partial z} = 0, \tag{4.66}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = g\beta\,(T - T_\infty) + g\beta^*\,(C - C_\infty) + \nu\frac{\partial^2 u}{\partial z^2}\,.$$

$$-\frac{\nu}{K}u - \frac{\sigma B_0^2}{\rho}u\,, \tag{4.67}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p}\frac{\partial^2 T}{\partial z^2}, \qquad (4.68)$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2}, \qquad (4.69)$$

where u, w are the corresponding velocity components along and perpendicular to the plate, υ the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion, T the fluid temperature, T_∞ the fluid temperature at infinity, K^* the thermal conductivity, ρ the density of fluid, C_p the specific heat at constant pressure, K the permeability of porous medium, σ the electrical conductivity, B_o the magnetic induction, D the diffusivity, C the species concentration. The equation of continuity (4.66) gives

$$w = \text{Constant} = -w_o, \qquad (4.70)$$

where w_o is the steady constant velocity of suction at the surface. The relevant boundary conditions are

$$\left.\begin{array}{l} u = 0,\ T = T_w + \epsilon\,(T_w - T_\infty)e^{i\omega t},\ C = C_w \text{ at } z = 0 \\ u \longrightarrow 0,\ T \longrightarrow T_\infty,\ C \longrightarrow C_\infty \text{ as } z \longrightarrow \infty \end{array}\right\}. \qquad (4.71)$$

In a physically realistic situation we cannot ensure perfect insulation in any experimental set-up. There will always be some fluctuation in the temperature. The plate temperature is assumed to vary harmonically with time. Since ϵ is very small, the plate temperature varies only slightly from the mean value T_w.

Introducing the following non-dimensional parameters

$$\bar{z} = \frac{w_0 z}{\nu}, \quad \bar{u} = \frac{u}{w_0}, \quad \bar{t} = \frac{t w_0^2}{\nu}, \quad \bar{T} = \frac{T - T_\infty}{T_w - T_\infty} .$$

$\bar{C} = \dfrac{C - C_\infty}{C_w - C_\infty}$, $S_c = \dfrac{\nu}{D}$ (Schmidt number),

$P = \dfrac{\mu C_p}{K^*}$ (Prandtl number), $\bar{K} = \dfrac{w_0^2 K}{\nu^2}$ (permeability parameter), $M = \dfrac{\mu\sigma B_0^2}{w_0^2}$ (magnetic parameter),

$G_r = \dfrac{\nu g\beta (T_w - T_\infty)}{w_0^3}$ (Grashof number), $G_m = \dfrac{\nu g\beta^* (C_w - C_\infty)}{w_0^3}$ (modified Grashof number) in equations (4.66) and (4.71) we get (dropping bars)

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial z} = G_r T + G_m C + \frac{\partial^2 u}{\partial z^2} - \frac{u}{K} - Mu, \tag{4.72}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{P} \frac{\partial^2 T}{\partial z^2} , \tag{4.73}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} . \tag{4.74}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} u = 0,\ T = 1 + \epsilon e^{i\omega t},\ C = 1 \text{ at } z = 0 \\ u \to 0,\ T \to 0,\ C \to 0 \text{ as } z \to \infty \end{array}\right\} . \tag{4.75}$$

4.7 Solution of the problem

In order to obtain a solution of the above coupled non-linear system of equations, we expand u, T and C in the following manner,

$$u = u_o + \epsilon\, e^{i\omega t}\, u_1 + \ldots\ldots \tag{4.76}$$

$$T = T_o + \epsilon\, e^{i\omega t}\, T_1 + \ldots\ldots \tag{4.77}$$

$$C = C_o + \epsilon\, e^{i\omega t}\, C_1 + \ldots\ldots \tag{4.78}$$

Substituting equations (4.76) – (4.78) into equations (4.72) – (4.74), equating harmonic and non-harmonic terms, we get

$$u_o'' + u_o' - (M + \frac{1}{K})\, u_o = -\, G_r\, T_o - G_m\, C_o\ , \tag{4.79}$$

$$u_1'' + u_1' - (M + \frac{1}{K} + i\omega)\, u_1 = -\, G_r T_1 - G_m C_1\ , \tag{4.80}$$

$$T_o'' + PT_o' = 0, \tag{4.81}$$

$$T_1'' + PT_1' - i\omega P T_1 = 0, \tag{4.82}$$

$$C_o'' + S_c\, C_o' = 0, \tag{4.83}$$

$$C_1'' + S_c\, C_1' - i\omega S_c\, C_1 = 0, \tag{4.84}$$

with the corresponding boundary conditions

$$\left.\begin{array}{l} u_o\ (0) = 0,\ T_o\ (0) = 1,\ C_o\ (0) = 0 \\ u_1\ (0) = 0,\ T_1\ (0) = 1,\ C_1\ (0) = 0 \\ u_o(\infty) \longrightarrow 0,\ T_o\ (\infty) \longrightarrow 0,\ C_o\ (\infty) \longrightarrow 0 \\ u_1(\infty) \longrightarrow 0,\ T_1\ (\infty) \longrightarrow 0,\ C_1\ (\infty) \longrightarrow 0 \end{array}\right\} . \tag{4.85}$$

Thus the solution of the problem is

$$u = L_1 e^{-M_2 z} + L_2 e^{-Pz} + L_3 e^{-S_c z}$$

$$+ \epsilon(e^{-M_4 z} - e^{-P_1 z})e^{i\omega t} \tag{4.86}$$

$$T = e^{-Pz} + \epsilon\, e^{-P_1 z} \cdot e^{i\omega t} , \qquad (4.87)$$

$$C = e^{-S_c z} , \qquad (4.88)$$

where

$$L_1 = \frac{G_r}{P^2 - P - M_1} + \frac{G_m}{S_c^2 - S_c - M_1} ,$$

$$L_2 = -\frac{G_r}{P^2 - P - M_1} ,$$

$$L_3 = -\frac{G_m}{S_c^2 - S_c - M_1} ,$$

$$L_4 = \frac{G_r}{P_1^2 - P_1 - M_2} ,$$

$$P_1 = \frac{1}{2}\left(- P + \sqrt{P^2 + 4i\omega P}\right) ,$$

$$M_1 = \left(M + \frac{1}{K}\right),$$

$$M_2 = \frac{1}{2}\left(1 + \sqrt{1 + 4M_1}\right) ,$$

$$M_3 = \left(M + \frac{1}{K} + i\omega\right),$$

$$M_4 = \frac{1}{2}\left(1 + \sqrt{1 + 4M_3}\right).$$

4.8 Discussion

In order to point out the effects of the permeability parameter K, Grashof number G_r, modified Grashof number G_m, magnetic parameter M and Schmidt number S_c on the velocity field, numerical calculations are carried out for $t = \frac{\pi}{4}$, $\omega = 2$ and $\epsilon = 0.1$ and are depicted in figure 4.4. Two values of Schmidt number S_c

(= 0.30 and 0.60) are considered here to represent species H_e and H_2O in air. The Grashof number G_r represents the effect of free convection currents and the case $G_r > 0$ corresponds physically to an externally cooled surface. The velocity variations in the case of externally cooled surface are shown in drawings 4 and 5 of figure 4.4. We conclude from these curves that greater cooling of the surface (as G_r increases from 3 to 5) results in an increase in velocity. It is seen from drawings 3 and 4 in figure 4.4 that an increase in permeability parameter K increases the velocity. It is observed from drawings 1, 2 and 3 that velocity is much influenced by mass transfer parameter G_m and magnetic parameter M. The velocity increases with the increase in modified Grashof number G_m while decreases with the increase in magnetic parameter. Also an increase in Schmidt number S_c leads to decrease the fluid velocity.

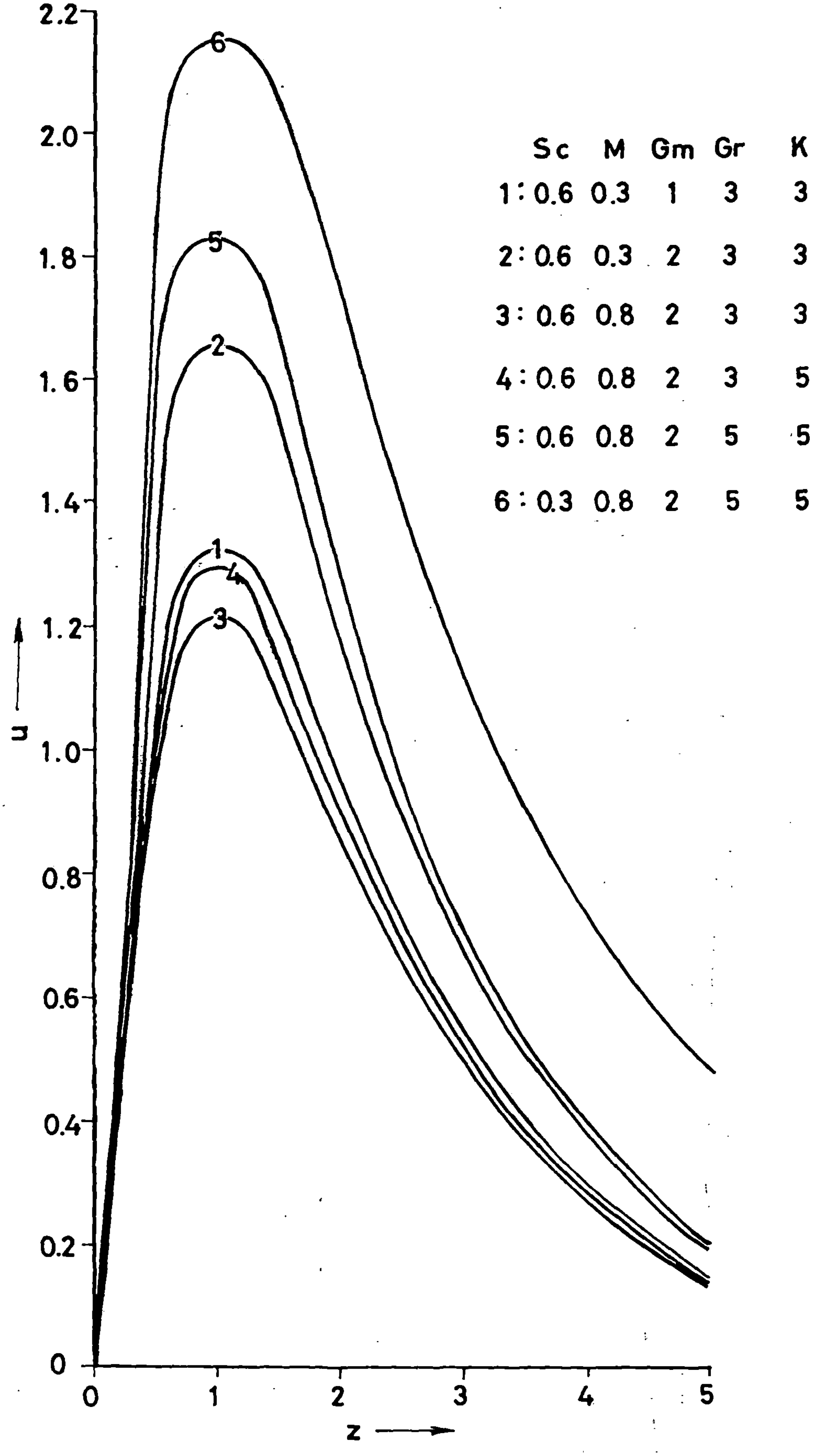

Fig.-4.4. u against z when $\omega = 2$, $\epsilon = 0.1$, $t = \pi/4$

PART THREE

EFFECTS OF MAGNETIC FIELD ON THE FREE CONVECTION FLOW AND MASS TRANSFER THROUGH A POROUS MEDIUM*

4.9 Introduction

Flows which arise in fluids due to interaction of the force of gravity and density difference caused by the simultaneous diffusion of thermal energy and chemical species, is known as free convection and mass-transfer flow. On the other hand results of the effects of magnetic field on the flow in presence of free convection and mass-transfer are also useful in steller atmosphere. For this reason, Georgantopoulos and Nanousis [16] studied the effects of mass transfer on the free convection flow of an electrically conducting viscous fluid past an impulsively started infinite vertical surface, when the magnetic Reynolds number of the flow was small. Raptis [17] considered exactly the same problem, when the magnetic Reynolds number of the flow was not small. Raptis and Kafousias [18] analysed the steady free convective and mass transfer flow in an electrically conducting fluid through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Hence it is of interest to

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make an investigation in order to analyse the effects of magnetic field on the free convective flow with mass transfer when the concentration of species oscillate in time about a constant mean. The plate is subjected to a constant normal suction velocity and the heat flux at the plate is also constant. Analytical expressions for the velocity field, concentration field and temperature field are given and numerical calculations are carried out to enable a discussion of the results, which are shown on graphs.

4.10 Mathematical analysis

We consider the unsteady flow of viscous fluid through a porous medium bounded by a vertical porous plate in the presence of free convection and mass transfer. Magnetic field of uniform strength is applied transversely to the direction of flow. The concentration of species is considered to be oscillating with time about a constant mean when the heat flux at the plate wall is constant. The x-axis is taken along the plate in the direction opposite to the direction of gravity and y-axis is taken to be normal to the plate pointing towards the medium.

Under the above consideration, the governing equations of motion are

$$\left(\frac{\partial u}{\partial t} - v_o \frac{\partial u}{\partial y}\right) = g\beta^* (C - C_\infty) + g\beta (T - T_\infty)$$

$$+ \frac{\nu}{K^*}(U - u) + \frac{\sigma B^2}{\rho}(U - u) + \nu \frac{\partial^2 u}{\partial y^2}, \qquad (4.89)$$

$$\frac{\partial C}{\partial t} - v_o \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} , \qquad (4.90)$$

$$- v_o \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} , \qquad (4.91)$$

with the boundary conditions

$$\left.\begin{array}{l} u = 0,\ \frac{\partial T}{\partial y} = -\frac{q}{K},\ C = C_v(1 + \epsilon\ e^{i\omega t}) \text{ at } y = 0, \\ u = U,\ T = T_\infty,\ C = C_\infty \text{ as } y \longrightarrow \infty , \end{array}\right\} (4.92)$$

where u is the velocity component in x-direction, T the temperature of the fluid, T_∞ the temperature of fluid far away from the plate, β the volumetric coefficient of thermal expansion, C the species concentration, C_∞ the species concentration far away from the plate, β^* the volumetric coefficient of expansion with concentration, ν the kinematic viscosity of the fluid, K^* the permeability of the porous medium, K the thermal conductivity, C_p the specific heat at constant pressure and D the molecular diffusivity, q the constant heat flux per unit area and v_o is the suction velocity.

In river bed, we may come across some situation where fluctuations in concentration is possible in presence of earth's magnetic field. The concentration at the plate $z = 0$ is assumed to vary harmonically with time. Since ϵ is very small, the concentration at the plate varies only slightly from the mean value.

The non-dimensional quantities introduced in the above equations are as follows

$$\bar{u} = \frac{u}{U}\ ,\ \bar{y} = \frac{y\, v_o}{\upsilon}\ ,\ \bar{t} = \frac{t\, v_o^2}{\upsilon}\ ,\ \bar{w} = \frac{w\upsilon}{v_o^2}\ ,$$

$S_c = \frac{\upsilon}{D}$ (Schmidt number), $\bar{K}^* = \frac{v_o^2\, K^*}{\upsilon^2}$,

$G_r = \frac{g\beta q v^2}{v_0 U\, K}$ (Grashof number), $\theta = \frac{T - T_\infty}{qv} K\, v_0$

$\bar{C} = \frac{C - C_\infty}{C_w - C_\infty}$, $G_m = \frac{\nu g\beta^*(C_w - C_\infty)}{\nu_0^2 \rho}$ (modified Grashof number), $M^2 = \frac{\sigma B^2 \upsilon}{v_o^2\, \rho}$ (magnetic parameter),

$P = \frac{\mu\, C_p}{K}$ (Prandtl number).

Substituting the above non-dimensional quantities into equations (4.89) – (4.91) we get (on dropping bars)

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y} + G_m\, C + G_r\, \theta + \left\{ \frac{1}{K^*} + M^2 \right\} (1-u) + \frac{\partial^2 u}{\partial y^2}\ , \tag{4.93}$$

$$\frac{\partial C}{\partial t} = \frac{\partial C}{\partial y} + \frac{1}{S_c}\ \frac{\partial^2 C}{\partial y^2}\ , \tag{4.94}$$

$$\frac{d^2\theta}{dy^2} + P\, \frac{d\theta}{dy} = 0 \tag{4.95}$$

and the boundary conditions (4.92) become

$$\left.\begin{aligned} &u = 0,\ C = 1 + \frac{C_w}{C_w - C_\infty}\, \epsilon\, e^{i\omega t},\ \frac{\partial\theta}{\partial y} = -1 \text{ on } y = 0 \\ &u = 1,\ C = 0,\ \theta = 0 \text{ as } y \longrightarrow \infty\ . \end{aligned}\right\} \tag{4.96}$$

Solution of equation (4.95) is obtained with the help of boundary condition in (4.96) as

$$\theta = \frac{1}{P} \; e^{-Py}. \tag{4.97}$$

In order to solve equations (4.93) and (4.94) we use the method of perturbation, considering ϵ to be very small.

$$\text{Let } u = u_0 + \epsilon \, u_1 \, e^{i\omega t} , \tag{4.98}$$

$$C = C_0 + \epsilon \, C_1 \, e^{i\omega t} . \tag{4.99}$$

Substituting equations (4.98) and (4.99) in equations (4.93) and (4.94) we get

$$\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} - L \, u_0 + \frac{G_r}{P} \, e^{-Py} + (L + G_m \, C_0) = 0, \tag{4.100}$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - (L + i\omega) u_1 + G_m \, C_1 = 0, \tag{4.101}$$

$$\frac{d^2 C_0}{dy^2} + S_c \, \frac{dC_0}{dy} = 0, \tag{4.102}$$

$$\frac{d^2 C_1}{dy^2} + S_c \, \frac{dC_1}{dy} - i\omega S_c C_1 = 0 \tag{4.103}$$

with the boundary conditions

$$u_0 = 0, \; u_1 = 0, \; C_0 = 1, \; C_1 = \frac{C_v}{C_v - C_\infty} \text{ on } y = 0, \tag{4.104}$$

$$u_0 = 1, \; u_1 = 0, \; C_0 = 0, \; C_1 = 0 \text{ as } y \longrightarrow \infty \tag{4.105}$$

where $L = (M^2 + \dfrac{1}{K^*})$.

Thus the solution of the problem is

$$u = 1 + A_1 \, e^{-B_1 y} + A_2 \, e^{-Py} + A_3 \, e^{-S_c y} + \epsilon \, A_4 \, e^{-(B_2 + B_3) y} \, e^{i\omega t} \tag{4.106}$$

and

$$C = e^{-S_c y} + \epsilon\, e^{i\omega t} \cdot \frac{C_v}{C_v - C_\infty}\, e^{-B_3 y} , \qquad (4.107)$$

where

$$A_1 = \frac{G_r}{P(P^2 - P - L)} + \frac{G_m}{S_c^{\,2} - S_c - L} - 1,$$

$$A_2 = -\frac{G_r}{P(P^2 - P - L)} ,$$

$$A_3 = -\frac{G_m}{S_c^{\,2} - S_c - L} ,$$

$$A_4 = \frac{G_m\, C_v}{\left\{ B_3^{\,2} - B_3 - (L + i\omega) \right\} (C_v - C_\infty)} ,$$

$$B_1 = \frac{1 + \sqrt{1 + 4\,L}}{2},$$

$$B_2 = \frac{1 + \sqrt{1 + 4\,(L + i\omega)}}{2},$$

$$B_3 = \frac{S_c + \sqrt{S_c^{\,2} + 4\, i\omega\, S_c}}{2} .$$

4.11 Discussion

Numerical calculations are carried out and are depicted in figure 4.5 to observe the effects of various parameters G_m, M and K^* on the velocity profile. It is seen from the figure that velocity increases with the increasing value of both G_m and K^* while it decreases as magnetic parameter increases.

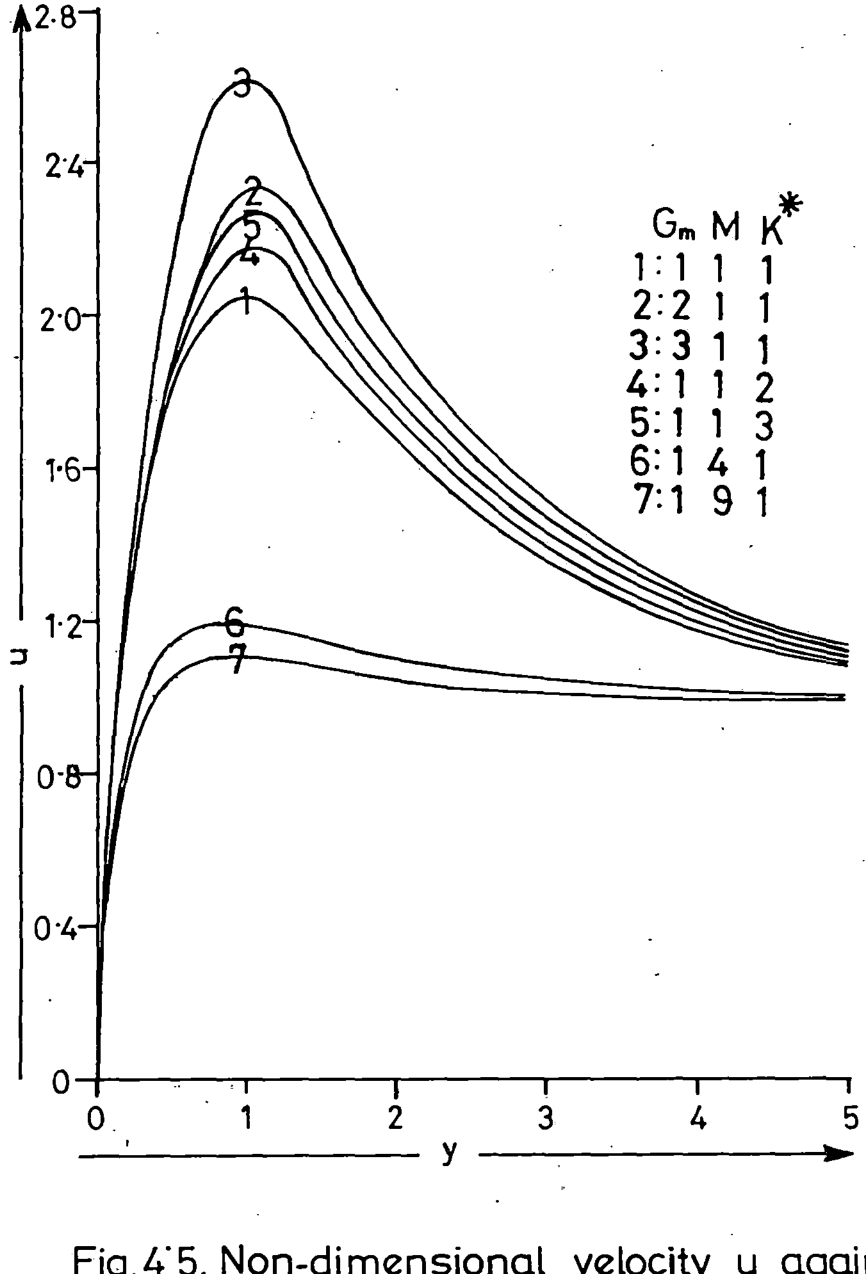

Fig. 4·5. Non-dimensional velocity u against y when $Sc = 0{\cdot}6$, $\omega = 2$, $t = \pi/4$, $P = 0{\cdot}71$, $\epsilon = 0{\cdot}1$ & $Gr = 4$.

MASS TRANSFER AND FREE CONVECTION FLOW PAST A VERTICAL POROUS PLATE IN A ROTATING LIQUID*

4.12 Introduction

The investigation of the flow through a porous medium is important to understand the character of the porous medium and to make it more effective. Therefore the study of flows through porous media has become principal interest in many scientific and engineering applications. In nature, the flow of fluids is caused not only by the temperature differences but also by concentration. So in recent years analytical solutions to such problems of flow have been presented by many authors. Gebhart and Pera [19] studied the laminar fluid flows due to the interaction of the forces of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species. Raptis [20] analysed the unsteady free convection and mass transfer flow of a viscous fluid through a porous medium due to the fluctuation of the surface temperature about a constant non-zero mean value. Recently Mahato and Maiti [21] analysed the unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium, bounded by an infinite vertical porous surface, in a rotating system.

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In the present work we study the unsteady free convection flow and mass transfer through a porous medium for a rotating fluid bounded by an infinite vertical porous plate where there is constant suction and constant heat flux. The concentration on the surface fluctuates with time about a non-zero constant mean and the concentration at the free stream is constant. The analytical expressions for the velocity, temperature and concentration are obtained. The influence of the various parameters entering into the problem on the velocity field are discussed. This study is likely to have some bearing on the geophysical and fluid engineering problems.

4.13 Mathematical analysis

We consider the unsteady flow of an incompressible viscous fluid by the presence of free convection and mass transfer through a porous medium bounded by an infinite vertical porous plate. We also consider a cartesian co-ordinate system rotating uniformly with the fluid in a rigid state of rotation with a constant angular velocity Ω about z-axis and the vertical plate is assumed to coincide with the plane $z = 0$ and z-axis is normal to the plate and pointing towards the medium.

The equations which govern the problem are :

Momentum equations

$$\rho\left(\frac{\partial u}{\partial t} - 2\Omega v + w\frac{\partial u}{\partial z}\right) = (T - T_\infty)\,\beta g\rho + \mu\frac{\partial^2 u}{\partial z^2} - \frac{\mu}{K}\,u$$

$$+ \rho g\beta^*\,(C - C_\infty)\ , \qquad (4.108)$$

$$\rho \left(\frac{\partial v}{\partial t} + 2\Omega u + w \frac{\partial u}{\partial z} \right) = \mu \frac{\partial^2 v}{\partial z^2} - \frac{\mu}{K} v \ . \qquad (4.109)$$

Diffusion equation

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} \qquad (4.110)$$

Energy equation

$$w \frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p} \frac{\partial^2 T}{\partial z^2} \qquad (4.111)$$

where u, v, w are the velocity components in x, y and z directions respectively, T the temperature of the fluid, T_∞ the temperature of the fluid far away from the plate, β the volumetric coefficient of thermal expansion, C the species concentration, C_∞ the species concentration far away from the plate, β^* the volumetric coefficient of expansion with concentration, μ the viscosity of the fluid, K the permeability of the porous medium, K^* the thermal conductivity, ρ the density, C_p the specific heat at constant pressure and D the chemical molecular diffusivity.

The boundary conditions of the problem are

$$u = 0 = v, \ \frac{\partial T}{\partial z} = -\frac{q}{K^*}, \ C = C_v(1 + \epsilon \, e^{i\alpha t}), \ \text{at } z = 0$$

$$(4.112)$$

$$u, v \longrightarrow 0, \ T \longrightarrow T_\infty, \ C \longrightarrow C_\infty \ \text{ as } z \longrightarrow \infty \qquad (4.113)$$

where q is the constant heat flux per unit area at the plate.

In river bed, we may come across some situation where fluctuations in concentration is possible. The concentration at the plate $z = 0$ is assumed to vary harmonically with time. Since ϵ is very small, the concentration at the plate varies only slightly from the mean value C_v. For constant suction, we have $w = - w_o$.

Using $W = u + iv$ and introducing the following non-dimensional quantities

$$\bar{u} = \frac{u}{w_o}, \quad \bar{v} = \frac{v}{w_o}, \quad \bar{z} = \frac{w_o z}{\nu}, \quad \theta = \frac{(T-T_\infty) K^* w_o}{q\nu},$$

$$S_c = \frac{\nu}{D}, \quad G_r = \frac{g\nu^2 q\beta}{K^* w_o^4}, \quad G_m = \frac{\nu g\beta^*(C_w - C_\infty)}{W_o^3},$$

$$\bar{K} = \frac{w_o^2}{\nu^2} K, \quad \bar{\alpha} = \frac{\alpha\nu}{w_o^2}, \quad \bar{C} = \frac{C - C_\infty}{C_v - C_\infty}, \quad E = \frac{\Omega\nu}{w_o^2},$$

$$\bar{t} = \frac{t w_o^2}{\nu}, \qquad p = \frac{\mu C_p}{K^*}$$

in equations (4.108) – (4.111) (dropping bars) we get

$$\frac{\partial W}{\partial t} + 2iEW - \frac{\partial W}{\partial z} - \frac{\partial^2 W}{\partial z^2} + \frac{W}{K} - G_m C - G_r \theta = 0, \qquad (4.114)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \qquad (4.115)$$

$$\frac{d^2\theta}{dz^2} + P\frac{d\theta}{dz} = 0 \qquad (4.116)$$

and the boundary conditions are

$$W = 0, \ \frac{d\theta}{dz} = -1, \ C = 1 + \epsilon \frac{C_w}{C_w - C_\infty} e^{i\alpha t} \text{ on } z = 0, \qquad (4.117)$$

$$W \longrightarrow 0,\ \theta \longrightarrow 0,\ C \longrightarrow 0 \text{ as } z \longrightarrow \infty\ . \qquad (4.118)$$

To solve the system of equations (4.114) and (4.115) we assume

$$W = W_o(z) + \epsilon\, W_1\ e^{i\alpha t} + \ldots\ , \qquad (4.119)$$

$$C = C_o + \epsilon\, C_1\ e^{i\alpha t} + \ldots\ . \qquad (4.120)$$

Substituting (4.119) and (4.120) in equations (4.114) and (4.115) we get

$$W_o'' + W_o' - MW_o + G_m C_o + \frac{G_r}{P} e^{-Pz} = 0\ , \qquad (4.121)$$

$$W_1'' + W_1' - (M + i\alpha)\, W_1 + G_m C_1 = 0\ , \qquad (4.122)$$

$$C_o'' + S_c C_0' = 0\ , \qquad (4.123)$$

$$C_1'' + S_c C_1' - i\alpha\, S_c\, C_1 = 0\ , \qquad (4.124)$$

where

$$\theta = \frac{1}{P} e^{-Pz} \text{ and } M = 2iE + \frac{1}{K}\ .$$

The boundary conditions (4.117) and (4.118) now become

$$W_o = W_1 = 0,\ C_o = 1,\ C_1 = \frac{C_W}{C_w - C_\infty} \text{ on } z = 0\ , \qquad (4.125)$$

$$W_o = W_1 = 0,\ C_o = 0,\ C_1 = 0 \text{ as } z \longrightarrow \infty\ . \qquad (4.126)$$

Thus the solution of the problem is

$$W(z,t)=R_1\ e^{-L_1 z}+R_2\ e^{-Pz}+R_3\ e^{-S_c z}+\epsilon\ e^{i\alpha t}\ R_4\ (e^{L_2 z}+e^{-L_3 z})\ , \qquad (4.127)$$

$$C(z,t) = e^{-S_c z} + \epsilon\, e^{i\alpha t}\, \frac{C_w}{C_w - C_\infty}\, e^{-L_3 z}\,, \qquad (4.128)$$

where

$$R_1 = \frac{G_r}{P(P^2 - P - M)} + \frac{G_m}{S_c^2 - S_c - M}\,,$$

$$L_1 = \frac{1 + \sqrt{1 + 4M}}{2}\,,$$

$$R_2 = -\,\frac{G_r}{P(P^2 - P - M)}\,,$$

$$R_3 = -\,\frac{G_m}{S_c^2 - S_c - M}\,,$$

$$R_4 = \frac{G_m C_w}{(C_w - C_\infty)(L_3^2 - L_3 - (M + i\alpha))}\,,$$

$$L_2 = -\,\frac{1 + \sqrt{1 + 4(M + i\alpha)}}{2}\,,$$

$$L_3 = \frac{S_c + \sqrt{S_c^2 + 4\, i\alpha\, S_c}}{2}\,.$$

4.14 Discussion

It is seen from equation (4.127) that the steady state flow has multiple-layer character. The first layer is due to the temperature distribution, the second layer is due to the concentration of the species and the third layer is due to the porosity of the medium (modified by rotation). It can be also remarked that as the porosity (K) of the medium increases the depth of penetration increases.

In order to have a physical insight into the problem, graphs are drawn for the primary and secondary velocity components for different values of E, K and G_m for fixed values of P, ϵ, α and S_c. We see from the figures (Figs. 4.6 and 4.7) that u increases as G_m increases while it decreases as rotation parameter E increases and v follows the same as u in both the cases of increasing G_m ad E. The primary velocity u decreases as porosity K increases while reverse effect is seen for secondary velocity v.

Figure 4.8 shows the variation of concentration of species with z for different values of Schmidt number S_c. It clearly shows that the concentration of species is more near the vertical porous plate and decreases slowly as it moves far away from the plate.

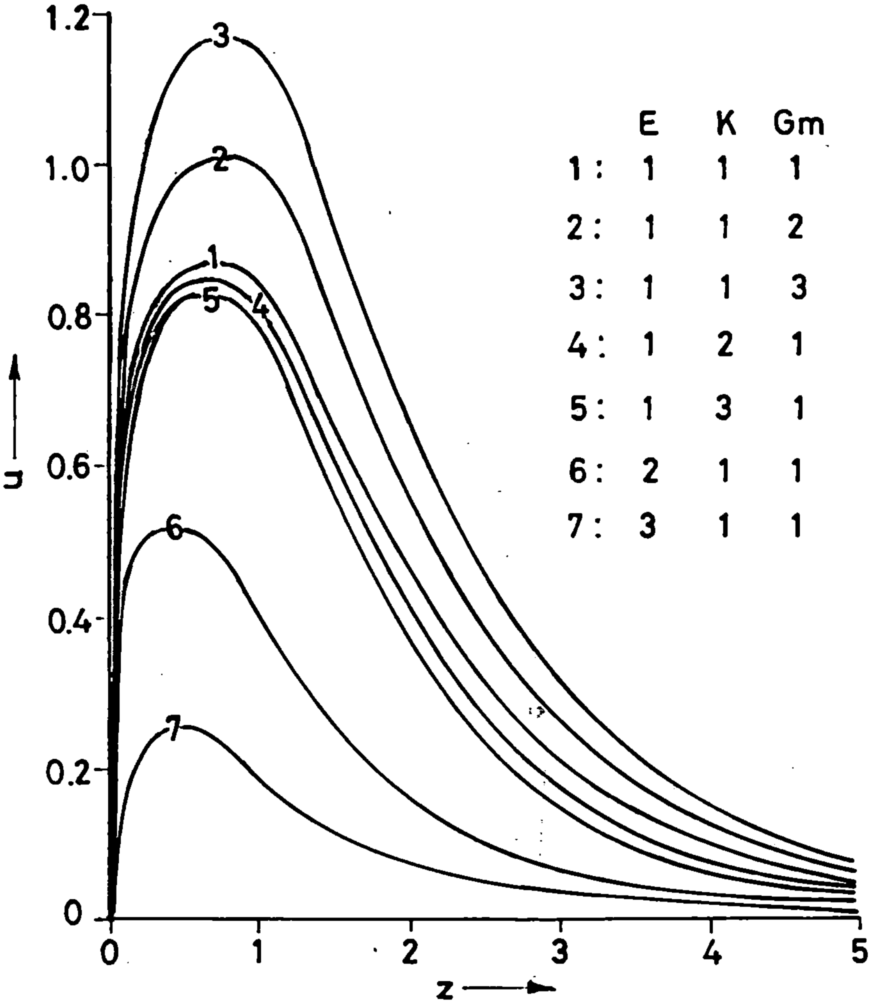

Fig.-4.6. Primary velocity profiles (u) when Sc = 0.60, P = 0.71, ϵ = 0.01 & α = 2.00.

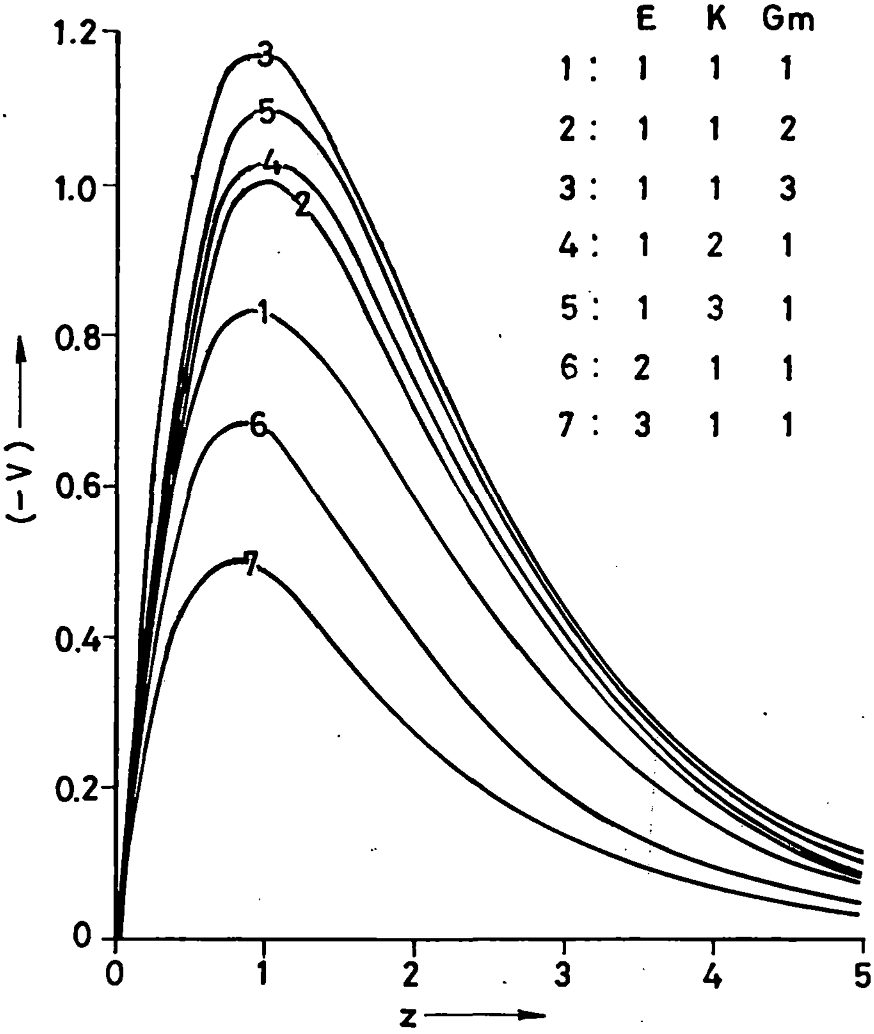

Fig.- 4.7. Secondary velocity profiles (v) when $Sc = 0.60$, $P = 0.71$, $\epsilon = 0.01$, $\alpha = 2.00$.

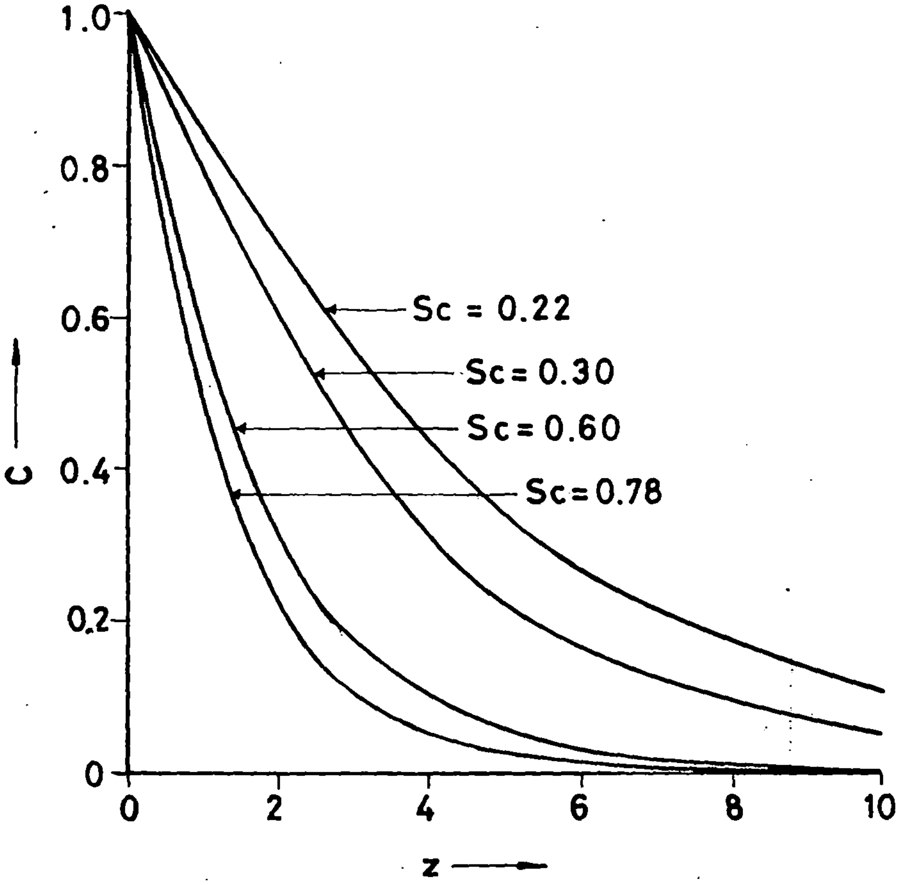

Fig.-4.8. Concentration (C) for different values of Sc when $\alpha = 2$

PART FIVE

FREE CONVECTION FLOW AND MASS TRANSFER THROUGH A POROUS MEDIUM IN A ROTATING SYSTEM

4.15 Introduction

In recently published papers by Raptis et al. [22, 23], Raptis [24] and Raptis and Perdikis [25], an analytical study of the free convection flow and mass transfer through a very porous medium bounded by an infinite porous plate was given. It is also known that the flow through a porous medium in a rotating system, is one of the most considerable and contemporary subjects, because it finds great applications in geothermy, geophysics, petroleum industry and ground water technology. Raptis [26] studied steady free convection and mass transfer through a porous medium bounded by an infinite vertical porous plate for a fluid rotating with a constant angular velocity when the heat flux at the plate was constant. Very recently Mahato and Maiti [21] investigated the unsteady free convective flow and mass transfer in a rotating porous medium bounded by an infinite porous plate when the temperature at the plate was fluctuating with time.

In the present work an attempt is made to study the unsteady free convection flow and mass transfer during the motion of viscous fluid through a porous medium bounded by an infinite porous plate in a rotating system when there is an oscillating free stream velocity. In a physically realistic situation we may assume fluctuations in velocity far field flow region. The effects of various parameters on the primary and secondary velocity profiles and temperature distribution are analysed.

4.16 Mathematical analysis

We consider that the vertical infinite porous plate rotates in unison with viscous semi-infinite fluid occupying the porous region with constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. Certesian coordinate system is chosen such that x, y axes respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z = 0$ while z-axis is normal to it. The fluid is subjected to constant suction along the plate $z = 0$ with velocity $w = -w_o$. Since the plate is infinite in length, all the physical quantities, except pressure p, are functions of z and t only. Consequently the equations expressing the conservation of momentum and energy and mass transfer are

$$\frac{\partial q}{\partial t} - w_o \frac{\partial q}{\partial z} + 2i\Omega q = \frac{\partial q^*}{\partial t} + 2i\Omega q^* + g\beta\,(T - T_\infty) + g\beta^*(C - C_\infty)$$

$$+ \frac{1}{\rho}\,\frac{\mu}{K}\,(q^* - q) + \frac{\mu}{\rho}\,\frac{\partial^2 q}{\partial z^2}\ , \qquad (4.129)$$

$$\frac{\partial T}{\partial t} - w_o \frac{\partial T}{\partial z} = \frac{K^*}{\rho C_p}\,\frac{\partial^2 T}{\partial z^2}\ , \qquad (4.130)$$

$$\frac{\partial C}{\partial t} - w_o \frac{\partial C}{\partial z} = D\,\frac{\partial^2 C}{\partial z^2}\ , \qquad (4.131)$$

with the boundary conditions

$$\left.\begin{array}{l} q = 0,\ T = T_w,\ C = C_w \quad \text{at } z = 0 \\ q = q^* = q'\,(1 + \epsilon\, e^{-i\alpha t}),\ T = T_\infty,\ C = C_\infty \ \text{as } z \longrightarrow \infty \end{array}\right\} \tag{4.132}$$

where $q = u + iv$, u and v are velocity components in the directions of x, y directions respectively; ρ the density; g the acceleration due to gravity; β the coefficient of volume expansion; T the temperature of the fluid; T_w is the surface temperature; T_∞ the fluid temperature far away from the plate; K the permeability coefficient of the medium; K^* the thermal conductivity and C_p the specific heat at constant pressure.

We introduce the following non-dimensional quantities

$$\bar{z} = \frac{w_0\, z}{\nu},\ \bar{t} = \frac{w_0^2\, t}{\nu},\ \bar{q} = \frac{q}{q'},\ \bar{\alpha} = \frac{\nu\alpha}{w_0^2},$$

$$\bar{K} = \frac{w_0^2}{\nu^2} K,\ \bar{T} = \frac{T - T_\infty}{T_w - T_\infty},\ G_r = \frac{\nu g\beta (T_w - T_\infty)}{q'\, w_0^2}$$ (Grashof

number), $G_m = \dfrac{\nu g\beta^* (C_w - C_\infty)}{w_0^2\, q'}$ (modified Grashof number),

$\bar{q}^* = \dfrac{q^*}{q'}$, $E\ \dfrac{\Omega\nu}{w_0^2}$ (rotation parameter),

$\bar{C} = \dfrac{C - C_\infty}{C_w - C_\infty}$, $P_r = \dfrac{\mu\, C_p}{K^*}$ (Prandtl number),

$S_c = \dfrac{\nu}{D}$ (Schmidt number).

In view of the above non-dimensional quantities, equations (4.129)' – (4.132) reduce to (dropping the dashes)

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial z} + 2iEq = \frac{\partial q^*}{\partial t} + 2iEq^* + G_r\ T + G_m\ C + \frac{\partial^2 q}{\partial z^2} + \frac{1}{R}\ (q^* - q), \tag{4.133}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial z} = \frac{1}{P_r}\ \frac{\partial^2 T}{\partial z^2}\ , \tag{4.134}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial z} = \frac{1}{S_c}\ \frac{\partial^2 C}{\partial z^2}\ , \tag{4.135}$$

with boundary conditions

$$\left.\begin{array}{l} q = 0,\ T = 1,\ C = 1 \text{ at } z = 0 \\ q = q^* = 1 + \epsilon\ e^{-i\alpha t},\ T = 0,\ C = 0 \text{ as } z \longrightarrow \infty \end{array}\right\}. \tag{4.136}$$

To solve equations (4.133) – (4.135) under the boundary conditions (4.136), we assume

$$q = q_0(z) + \epsilon\ q_1(z)\ e^{-i\alpha t}\ , \tag{4.137}$$

$$T = T_0(z) + \epsilon\ T_1(z)\ e^{-i\alpha t}\ , \tag{4.138}$$

$$C = C_0(z) + \epsilon\ C_1(z)\ e^{-i\alpha t}\ . \tag{4.139}$$

Substituting (4.137) – (4.139) in (4.133) – (4.135) and separating the harmonic and non-harmonic terms, we have

$$q_1'' + q_1' + q_1\ (i\alpha - 2iE - \frac{1}{R}) = (i\alpha - 2iE - \frac{1}{R}), \tag{4.140}$$

$$q_0'' + q_0' - q_0\ (2iE + \frac{1}{R}) = -2iE - G_r\ e^{-P_r z} - G_m\ e^{-S_c z} - \frac{1}{R}\ , \tag{4.141}$$

$$T_1'' + P_r\ T_1' + i\alpha P_r\ T_1 = 0, \tag{4.142}$$

$$T_0'' + P_r\ T_0' = 0\ , \tag{4.143}$$

$$C_1'' + S_c C_1' + i\alpha S_c C_1 = 0 , \tag{4.144}$$

$$C_o'' + S_c C_o' = 0 . \tag{4.145}$$

Changed boundary conditions are

$$q_o = 0,\ q_1 = 0,\ T_o = 1,\ T_1 = 0,\ C_o = 1,\ C_1 = 0 \text{ at } z = 0, \tag{4.146}$$

$$q_o = 1,\ q_1 = 1,\ T_o = 0,\ T_1 = 0,\ C_o = 0,\ C_1 = 0 \text{ as } z \longrightarrow \infty . \tag{4.147}$$

On solving equations (4.140) – (4.145) with the boundary conditions in (4.146) and (4.147), we get the solutions for the velocity profile, temperature distribution and mass concentration as

$$q = 1 - L_1 e^{-P_r z} - L_2 e^{-S_c z} + L_3 e^{-R_1 z} + \epsilon [1 - e^{-M_2 z}] e^{-i\alpha t}, \tag{4.148}$$

$$C = e^{-S_c z}, \tag{4.149}$$

$$T = e^{-P_r z} , \tag{4.150}$$

where

$$L_1 = \frac{G_r}{(P_r - R_1)(P_r - R_2)} ,$$

$$L_2 = \frac{G_m}{(S_c - R_1)(S_c - R_2)} ,$$

$$L_3 = L_1 + L_2 - 1 ,$$

$$R_1, R_2 = \frac{1}{2} [1 \pm \sqrt{1 + 4 M_1}] ,$$

$$M_2 = \frac{1}{2} [1 + \sqrt{1 - 4(i\alpha - M_1)}] ,$$

$$M_1 = \frac{1}{K} + 2iE.$$

The real and imaginary part of q give the velocity expressions in the directions of x and y respectively. We are not presenting the mathematical expressions of u and v as discussions are made through numerical results.

4.17 Discussion

Equation (4.148) gives the expression of the composite velocity distribution and this exhibits the existence of multiple boundary layer. The steady state field is controlled by three layers viz. thermal layer [O $(1/P_r)$], concentration layer [O $(1/S_c)$] and suction layer (modified by rotation). Thermal layer is due to interaction of the thermal field and velocity field while the concentration layer arises due to the interaction of the concentration field and the velocity field.

Primary velocity u and secondary velocity v are depicted in figures. 4.9 and 4.10 for different values of G_r, G_m, S_c, K, E when $\alpha = 2$, $\epsilon = 0.1$, $P_r = 0.71$, $t = \pi/4$.

The Prandtl number P_r is taken equal to 0.71, which corresponds to air. Schmidt number S_c = 0.6, 0.3 are chosen in such a way as to represent H_2O vapour and H_e respectively at low concentration in air at approximately 25°C and 1 atm.

Figure 4.9 depicts that primary velocity decreases with increase in permeability parameter K, Grashof number G_r and Schmidt number S_c while it increases with increasing modified Grashof number G_m and rotation parameter E. This shows that permeability of porous medium exert retarding influence on the primary flow but rotation parameter enhance the velocity.

Figure 4.10 reveals that secondary velocity decreases as permeability parameter K, rotation parameter E and Schmidt number S_c increase and it increases as Grashof number G_r and modified Grashof number G_m increase.

Figure 4.11 shows the variation of temperature for different fluids. For mercury, the temperature remains almost stationary near the plate. The slope of the curve increases in case of air while in case of water the curve becomes very steep. This behaviour may be explained as induction of heat is quicker in case of small Prandtl number than those having large Prandtl number, as such water is more effective to maintain the temperature of its surroundings for a long time than air or mercury.

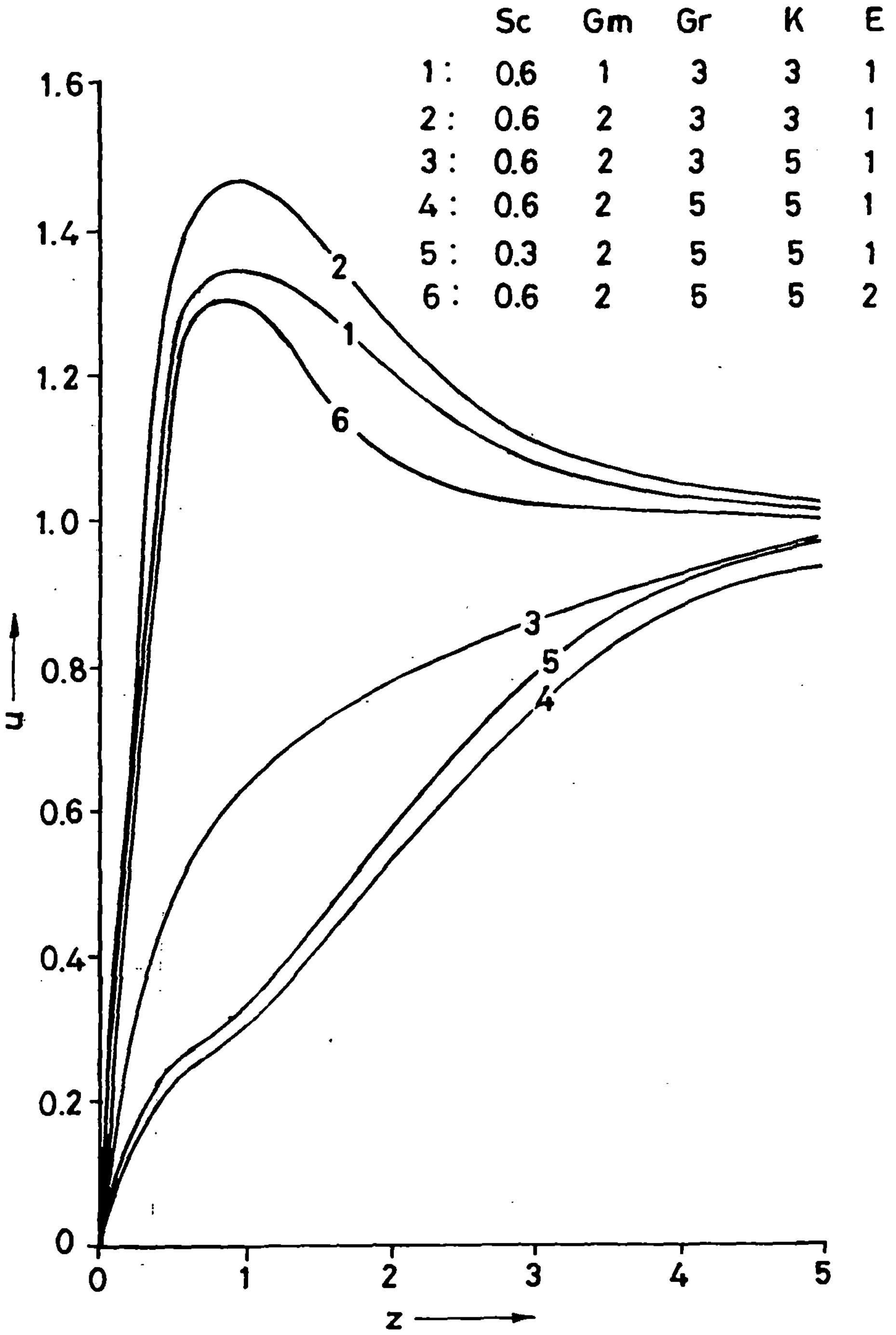

Fig.- 4.9. Plot of primary velocity profile u against z when $P_r = 0.71$, $\alpha = 2$, $\epsilon = 0.1$, $t = \pi/4$.

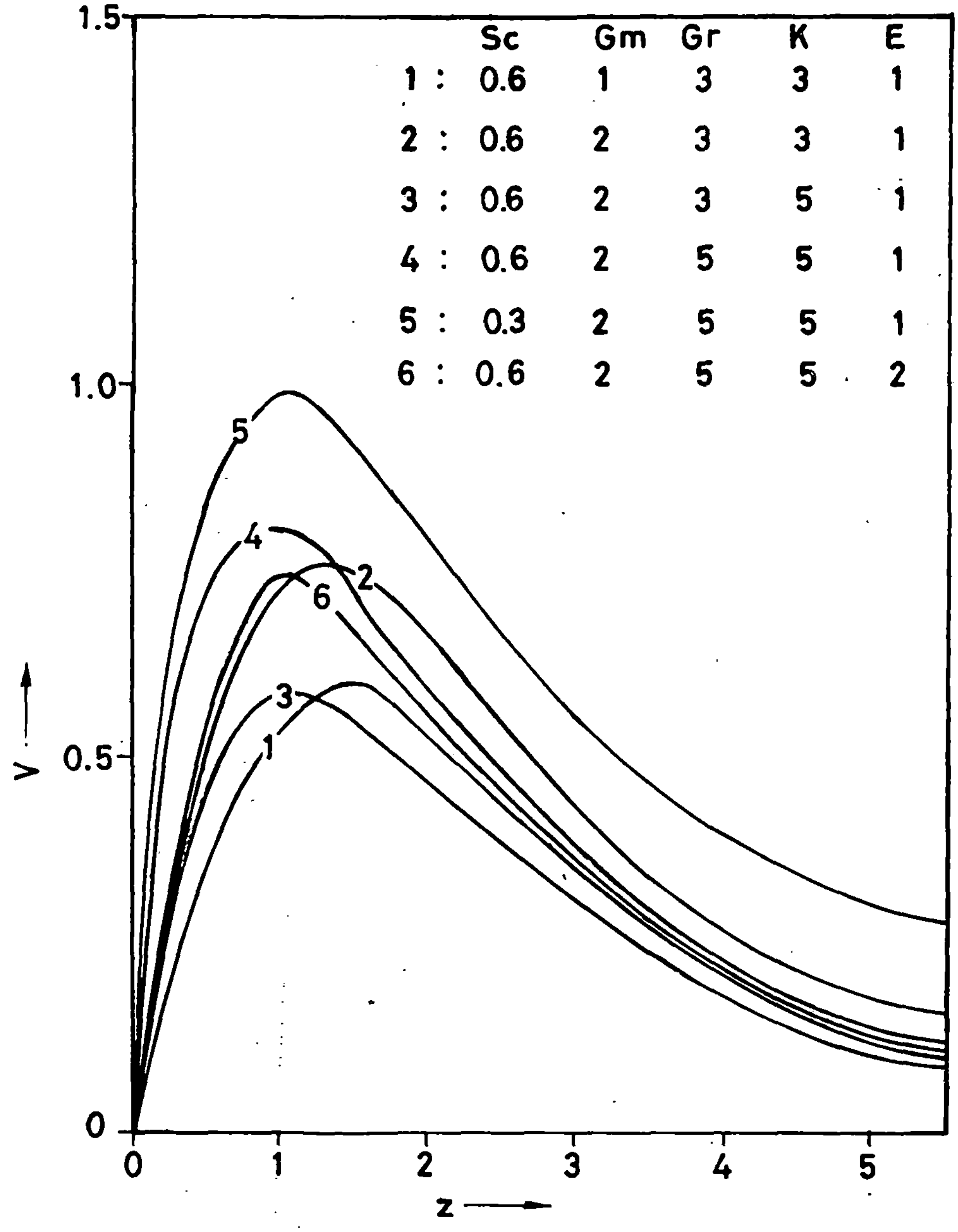

Fig.- 4.10. Plot of secondary velocity profile v against z when $P_r = 0.71$, $\alpha = 2$, $\epsilon = 0.1$, $t = \pi/4$.

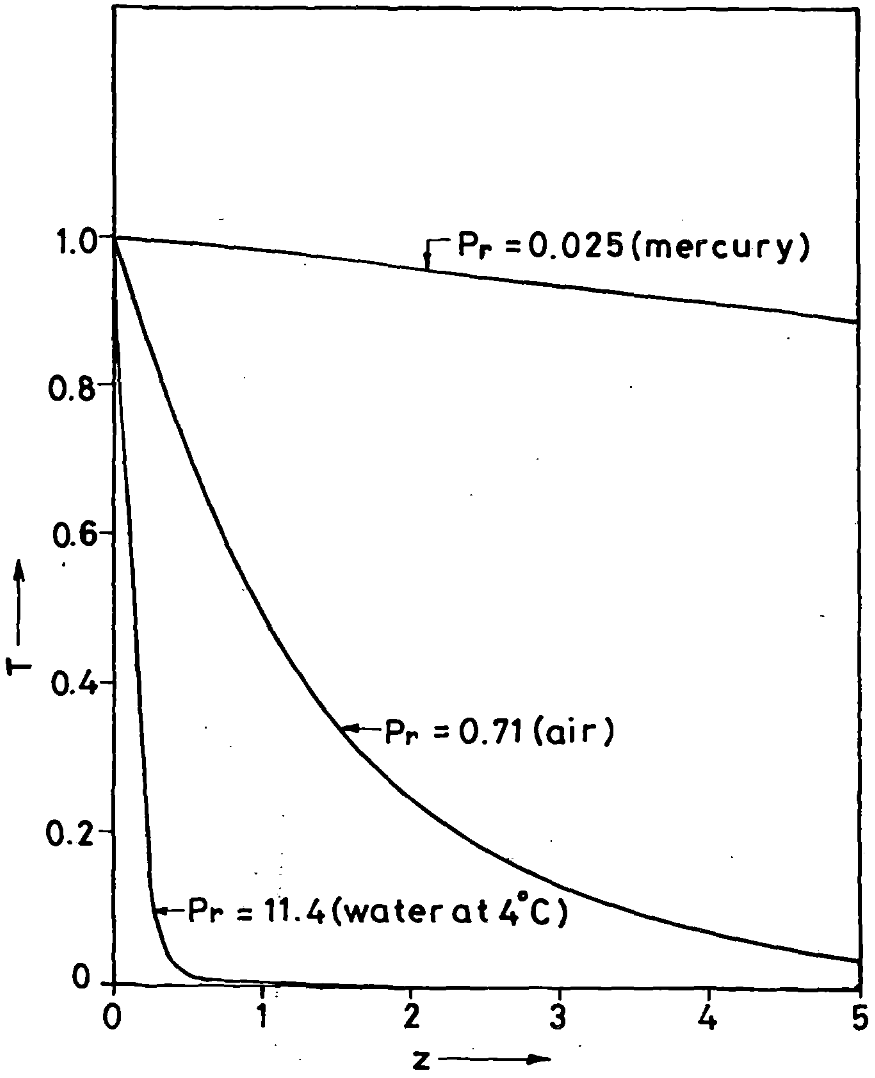

Fig.- 4.11. Plot of temperature (T) against z for different values of Pr.

REFERENCES

[1] Taylor, G.I. (1953) Proc.Roy.Soc.Lond.,Vol.A219, 186.

[2] Lighthill, M.J. (1966) J.Inst.Math.Appls.,Vol.2, 97.

[3] Gill, W.N. and Sankarasubramanian, R. (1970) Proc.Roy.Soc.Lond.,Vol.A316, 341.

[4] ——————— (1971) Proc.Roy.Soc.Lond.,Vol.A322, 101.

[5] ——————— (1972) Proc.Roy.Soc.Lond.,Vol.A327, 191.

[6] Sankarasubramanian, R. and Gill, W.N. (1973) Proc.Roy.Soc.Lond.,Vol.A333, 115.

[7] Krishnamurthy, S. and Subramanian, R.S. (1977) Separation Science,Vol.12, 347.

[8] Hartmann, J. (1937) K. danska vidensk.Selsk.Skr., Vol.15, 6.

[9] Soundalgekar, V.M. (1977) Trans. ASME.J.Heat Transfer, Vol.99, 499.

[10] Georgantopoulos, G.A., Nanousis, N.D. and Goudas, C.L. (1979) Astroph.Space Sci.,Vol.66, 13.

[11] Raptis, A. and Kafousias, N. (1982) Energy Research, Vol.6, 241.

[12] Raptis, A.A. and Tzivanidis, G.J. (1982) Wärme und Stoffübertragung. Vol. 16, 145.

[13] Raptis, A.A. (1983) Journal of the Franklin Institute, Vol.316, 445.

[14] Singh, Nand Lal (1985) I.J.Theo.Phys.,Vol.33, 17.

[15] Takhar, H.S., Raptis, A.A. and Perdikis, C.P. (1986) Acta Mech.,Vol.65, 287.

[16] Georgantopoulos, G.A. and Nanousis, N.D. (1980) Astrophys. Space Sci.,Vol.67, 229.

[17] Raptis, A.A. (1983) Astrophys. Space Sci.,Vol.92, 135.

[18] Raptis, A. and Kafousias, N. (1982) Canadian J.Phys.,Vol.60, 1725.

[19] Gebhart, B. and Pera, L. (1971) Int.J.Heat Mass Transfer, Vol.14, 2025.

[20] Raptis, A. (1983) Int.J.Eng.Sci.,Vol.21, 345.

[21] Mahato, J.P. and Maiti, M.K. (1988) Indian J.Technol.,Vol.26, 255.

[22] Raptis, A., Tzivanidis, G. and Kafousias, N. (1981) Lett.Heat Mass Transfer,Vol.8, 417.

[23] Raptis, A., Kafousias, N. and Massalas, C. (1982) ZAMM, Vol.6, 489.

[24] Raptis, A. (1984) Arch. Mech.,Vol.36, 307.

[25] Raptis, A. and Perdikis, C. (1987) Energy Research, Vol.11, 423.

[26] Raptis, A. (1983) Int.Comm.Heat Mass Transfer, Vol.10, 141.