ABSTRACT

In this work we have studied nonlinear partial differential equations and their exact solutions. In, general two sets of equations, namely the Generalized Yangs equations and the Generalized Charaps equation have been included in this study.

In general, two sets of equations, namely the Generalized Yang equations(2.1) and the Generalized Charap's equations(2.5) have been studied from the mathe matical point of view. There are two aspects in this venture. First, no sophisticated formalism has been used.Simple process of speculating the nature of the solution (in the form of choosing an ansatz) and checking the consistency thereafter has been adopted. The mathematical tools used for Painleve' Analysis also did not go beyond that being taught at graduation level of study. Graphical representations have been offered for explicit solutions. The target was to demonstrate, in continuation to the previous study mentioned in the Review Chapter, that one can approach nonlinear equations using simple techniques as well. And thus, the general practice of reducing non-linear equations to linearones based upon several assumptions can be avoided. In some situations we could get explicit solutions. In some other we got complex solutions. And for other situations we could reach very nearby of the exact solutions. However ultimately the expression could not be inverted for getting explicit solutions. The work offers two aspects. One, is to achieve exact solutions and the second, is to learn how can the simple procedure with which one can approach exact solutions.

For the Generalized Yangs equation we can comment the following:-

One of the patterns of mathematical challenges involved in nonlinear coupled partial differential equations and a strategy for overcoming the challenges have been indentified. The equations are Poisson – like. The strategy was originally used by D. Ray and it is basically a search for a transformations to a set of two variables so that the dependent variables can be expressed in terms of those two variables which are themselves mutually conjugate Laplace solutions. It may be a pleasure to see how a

strategy works in some situation and does not work will in some other (apparently) similar situation. However importance of the transfer of a strategy to a similar situation cannot be neglected.

We also see that the Reduced Generalized Yang's equation satisfy Painleve' criteria for $k^{'}=1$ and does not satisfy the same for $k^{'}=\frac{1}{2},\frac{1}{4}$. Interestingly, the solutions in closed form are obtainable for $k^{'}=1$ and not obtainable for $k^{'}=\frac{1}{2},\frac{1}{4}$. This again establishes the strength of the relationship between integrability and existence of Painleve property. One question, however, remains. How can one get some information for the equations like (5.32), (5.34) etc ? One answer could be the application of the Krylov-Bogoliubov-Mitroploskii (KBM) [[43],44]] method with the subsequent developments [45].

For the generalized Charap equations we can can comment the following:-

Here we have described a new class of exact solutions for different values of the coupling constants in a generalized form (Equations 2.5a to 2.5c) of the celebrated Chiral equations of field theory due to Charap[13]. The generalized form was first proposed by Saha and Chanda[35]. In most of the situations the solutions could not be arrived at as the equations led to non-integrable expressions. In one situation the solutions were exact and complex. In the rest of the situations the solutions are explicit. The importance of such type of study is the opportunity that may be available to a theoretician in dealing with coupled partial differential equations which relate to her/his area of interest. (i) Complex solutions for both the equations i.e the Generalized Charap's equation and the Yang's equation are in terms of Laplace solutions. The solutions reported for Yang's equations expressed in terms of ζ which satisfied the Laplace equation in four dimension . The solutions reported for the Generalized Charap's equations are expressed in terms of X and Y which are mutually conjugate Laplace solutions.

(ii) In both the cases $\phi\phi^*$, $\psi\psi^*$, $\chi\chi^*$ can be expressed in the real form in a very straight forward way.

From the Graphical representation we observed the existence of solitary solutions which are self-focussing in nature, multiple solitary solutions (time-independent) , chaotic solutions with periodic regularity and square well potential which reverse periodically.