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Thermodynamic Behaviour of Viscous Variable Chaplygin Gas in a Flat Universe

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In this manuscript we have studied the thermodynamic stability of the viscous variable Chaplygin gas (VVCG) model. The equation of state $p = -\frac{B}{\rho}$ is employed, where $B = B_0 V^{-\frac{n}{3}}$ and B_0 is a positive universal constant. The bulk viscosity coefficient $\xi = \xi_0 \rho^{\frac{1}{2}}$ has also been included in our research. When a fluid satisfies the established stability requirements, we obtain the intriguing conclusion that $(\frac{\partial p_{eff}}{\partial V})_S < 0$ and $(\frac{\partial p_{eff}}{\partial V})_T < 0$ throughout the evolution for $\xi_0 < 0$, when n without a certainty, are limited to the negative values. Additionally, the third law of thermodynamics is assured to be valid in this instance, as is the positivity of thermal heat capacity (c_V). Also we have discussed that how thermal variables depend on either temperature or volume. A new general equation of state for the VVCG model as a function of volume and temperature is obtained during the thermal process, which also shows that the cosmos is thermodynamically stable. Furthermore, the thermal and caloric equation-of-state parameters coincide at large volume in the limit $T \to 0$.

I. INTRODUCTION

Fundamental physics and cosmology theories now face a fresh challenge because of the finding of the universe's cosmic acceleration [1–3]. According to NASA's findings [4, 5], the mass of all stars and galaxies combined together is less than 5% of the total mass of the universe. Moreover, cold dark matter particles, non-baryonic in nature and clustering gravitationally, constitute about 22% of the universe's total energy density, while approximately 73% is attributed to a gravitationally repulsive component, known as dark energy. The existence of one of the strangest and most enigmatic substances, known as dark energy, in the cosmic fluid is sometime blamed for the universe's late acceleration. One of the most challenging theoretical problems in present cosmology, however, is the nature of this dark energy, and many start searching fervently for various tenable solutions. We ignore the specifics and point out that many forms of Chaplygin gas (CG) are also strong candidates for the same, saving readers from having to endure the repetition of those arguments and their opposites (for outstanding evaluations in this area) [6, 7]. The stability of the system is a crucial concern in this context. Among various considerations, our primary motivation

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is to determine whether the well-established stability criteria impose stringent constraints on the parameter values of the system. Upon comparison with these criteria, we find that several assertions made in previous studies are categorically invalidated. Recently, the stability issue has been addressed in a number of publications [8, 9]. The present work extends that discussion by focusing specifically on the VMCG model.

Recent developments in Chaplygin-type gas cosmology [10, 11] suggest that dark energy can be effectively modeled using a novel matter field. However, such an equation of state (EoS) is not well suited for describing the primordial universe [11, 12]. Notably, this EoS gives rise to a component that initially behaves like dust and subsequently evolves to mimic a cosmological constant (Λ). The CG [13–18] is of special interest as a candidate for dark energy. The original model has been expanded to the generalized Chaplygin gas (GCG) [19–23] in order to preserve agreement with observational data. Other modifications result in models such as the modified Chaplygin gas (MCG) [24] and the modified cosmic Chaplygin gas (MCCG) [25]. Despite being compatible with facts, a viable dark energy model should be able to approximate the cosmological constant model under certain circumstances [26]. Most of the models based on CG successfully address the problem of late-time cosmic acceleration. But they often fail to solve the original singularity issue. The following represents the equation of state (EoS) for the CG:

$$p = -B/\rho, \tag{1}$$

where B is a constant. According to a recent presentation and constraint of a variable Chaplygin gas (VCG) model utilizing SNeIa gold data [27, 28], the value of B is determined by the scale factor of our chosen metric. The relationship above is now understood to be $B = B_0 V^{-\frac{n}{3}}$, where the universal constant B_0 is positive. The VCG equation of state reduces to the standard CG equation of state when $n = 0$. It is possible for n to have a positive or negative value. A gold sample of 157 SNeIa data was used by Guo *et al.* [27] to show that $n = -3.4$ is the best fit value. In another article [28], the authors employed the X-ray gas mass fraction in 26 galaxy clusters [29] and the gold sample of 157 SNeIa data to restrict on VCG, and fond that the best fits value of $n = 0.5^{+1.0}_{-1.1}$. This finding supports a CG model that resembles a phantom and allows the density of dark energy to increase with time. Significantly, recent observations indicate that the data might be well-fitted by the extremely negative state equation, $\omega = -1$ [30–32]. The value of n, however, is shown to be between $(-1.3, 2.6)$ and $(-0.2, 2.8)$ in another investigation [33].

The thermodynamical stability of the GCG model was recently examined by Santos *et al.* [34]. In this study, the thermodynamical behavior of VCG will be examined, and we describe the temperature of the equation as well as the integrability condition equation. Temperature and volume are used to obtain all thermal values. Here, we demonstrate that the CG satisfies the third rule of thermodynamics. Additionally, as a function of temperature or volume, we found that a CG is properly described by a novel universal equation of state. The behavior of the VCG is expected to be comparable to that of the CG. Accordingly, we affirm that CG may provide a consistent framework for understanding dark matter and cooling energy as the universe expands. Returning to the requirement for CG stability, we observe that n ought to be negative. Remarkably, according to Guo *et al*. [27], the best fit value of the probability contour was $n = -3.4$. It is possible for n to have a positive or a negative best fit value [33].

It has been shown that bulk viscosity plays a critical role in cosmology [35–37]. The importance of viscosity may have been hinted at early on the CG, which was first hypothesized in [38] and then validated in [39–44]. Both the VMCG and VMCCG models account for time-dependent energy density, and are discussed in detail in [37, 42]. The VCG model is also examined within the context of a non-flat FRW universe [40]. This concept provides a logical foundation for comprehending dark energy and dark matter. We investigate the combined impacts of CG and bulk viscosity on a flat FRW universe. The standard Friedmann equations, modified to include bulk viscosity, are further influenced by the inclusion of the Chaplygin gas (CG) component. The model for the bulk viscous coefficient is $\xi = \xi_0 \rho^{\frac{1}{2}}$ [45]. We use the thermodynamic criteria described in [46] to determine if the conditions $(\frac{\partial p}{\partial V})_S < 0$, $(\frac{\partial p}{\partial V})_T < 0$ and $c_V > 0$ hold. We follow the methodology of Santos et al. [34] in the study of generalized and modified Chaplygin gas models [47]. To guarantee instantaneous thermodynamic stability, these prerequisites must be met. We examine several cosmological parameters in the VGCG model, such as the effective pressure p_{eff}, effective equation of state ω_{eff}, effective deceleration parameter q_{eff} and the adiabatic square speed of sound as a function of volume and temperature [34, 47, 48] and [9, 12]. The interplay between the VGCG and $f(R,T)$ gravity is further examined in the FRW framework [49], where the modified Friedmann equations incorporate time-dependent contributions from the CG and dark energy in terms of pressure and energy density.

II. BASIC FIELD EQUATIONS OF FRW COSMOLGY

The following is the known expression for the Friedmann-Robertson-Walker metric equation:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{2}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ and $a(t)$ is the expansion rate of the universe. Here $k = -1$, 0 and $+1$ represent, respectively open, flat and closed universes. The dimensionless coordinates r, θ and ϕ are referred to as co-moving coordinates. Since we are studying in a flat spacetime at this instance $(k = 0)$, the Einstein equation can be expressed as,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{3}$$

where $c = 1$, $8\pi G = 1$ and $\Lambda = 0$ are the parameters. From Eq. (2) and Eq. (3) the following assertion is obtained with a bulk viscous fluid [41, 43]:

$$T_{\mu\nu} = (\rho + p_{eff})u_\mu u_\nu - p_{eff}g_{\mu\nu}, \tag{4}$$

where u^μ and ρ stand for the four-velocity vector and energy-density vector, respectively. $T_{\mu\nu}$ and $g_{\mu\nu}$ represent the energy-momentum tensor and the metric tensor of the universe, respectively. Therefore, the total effective pressure of the viscous fluid can be written as,

$$p_{eff} = p + \Pi = -\frac{B}{\rho} - \sqrt{3}\xi_0\rho = -\frac{B_0 V^{-\frac{n}{3}}}{\rho} - \sqrt{3}\,\xi_0\rho, \tag{5}$$

where the bulk viscosity coefficient $\xi = \xi_0\rho^{\frac{1}{2}}$ and the viscous pressure $\Pi = -3\xi H$. The energy density

$$\rho_{vvcg} = \frac{U}{V}, \tag{6}$$

is the ratio of the internal energy U and the volume V. We have the thermodynamics relation [50]:

$$\left(\frac{\partial U}{\partial V}\right)_S = -p_{eff}. \tag{7}$$

From Eqs. (1), (5), (6) and (7), we get

$$\left(\frac{\partial U}{\partial V}\right)_S = B_0\left(\frac{V^{(1-\frac{n}{3})}}{U}\right) + (\sqrt{3}\xi_0)\frac{U}{V}. \tag{8}$$

The internal energy U can be expressed as,

$$U = \left[\frac{B_0}{(1-\sqrt{3}\xi_0-\frac{n}{6})}V^{2-\frac{n}{3}} + bV^{2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}, \tag{9}$$

where the integration constant b is arbitrary. The aforementioned expression can also be written as

$$U = \left(\frac{2B_0V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}} V\left[1+\left(\frac{\epsilon}{V}\right)^N\right]^{\frac{1}{2}}, \quad \text{for} \quad \epsilon = \left[\frac{bN}{2B_0}\right]^{\frac{1}{N}}, \tag{10}$$

where $N = 2(1-\sqrt{3}\xi_0-\frac{n}{6})$. Consequently, another way to express the energy-density in the VVCG model is as follows:

$$\rho_{vvcg} = \left(\frac{2B_0V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1+\left(\frac{\epsilon}{V}\right)^N\right]^{\frac{1}{2}}. \tag{11}$$

We discuss the thermodynamic behavior of this model.

A. Pressure

We calculate the effective pressure using the viscous parameter ξ_0 and volume V. From Eq. (5) we get

$$p_{eff} = \rho_{vvcg}\left[(-\sqrt{3}\xi_0) - \frac{N}{2\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right]. \tag{12}$$

It can also be stated as,

$$p_{eff} = -\left(\frac{2B_0V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]^{\frac{1}{2}}}\left[1+\frac{\sqrt{3}\xi_0}{N/2}\left[1+\left(\frac{\epsilon}{V}\right)^N\right]\right]. \tag{13}$$

Effective pressure is expressed by the equation above. A negative total effective pressure results from the bulk viscous pressure phase becoming more dominant and negative than the thermodynamic pressure. Dark energy is characterized by negative pressure (or tension), which is responsible for the observed accelerated expansion of the universe. Given the circumstances, we now

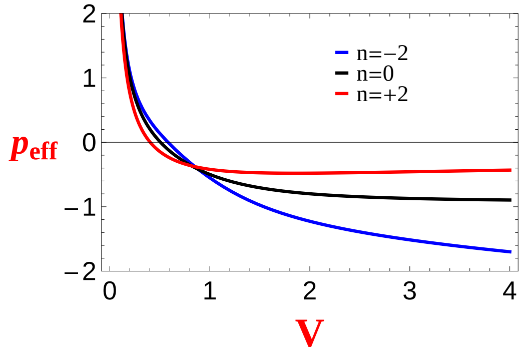

FIG. 1. Plots of p_{eff} versus V for the values of $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$.

obtain the following relations:
a) For $n \neq 0$ and $\xi_0 = 0$, the above Eq. (13) gives the variable CG model [34],

$$p = -\left(\frac{NB_0V^{-\frac{n}{3}}}{2}\right)^{\frac{1}{2}}\left[1+\left(\frac{\epsilon}{V}\right)^N\right]^{-\frac{1}{2}}. \tag{14}$$

The thermodynamic pressure equals the effective pressure. Both density and pressure typically fall as volume increases in an expanding universe.

b) For $\xi_0 = 0$ and $n = 0$, the results are reduced to the CG model [8]. The thermodynamic behavior of such a system was already examined by Santosh *et al.* [34]. Accordingly,

$$p = -\frac{B_0^{\frac{1}{2}}}{\left[1+\left(\frac{\epsilon}{V}\right)^2\right]^{\frac{1}{2}}}. \tag{15}$$

As seen in Fig. 1, we plot effective pressure against volume for a given ξ_0. We observe that the effective pressure decreases with volume as $n \leqslant 0$. Figure 1 shows that effective pressure is always negative for $n < 0$.

The critical volume (V_c) for the zero pressure condition, $p_{eff} = 0$, is obtained as,

$$V_c = \epsilon\left[\frac{(-\sqrt{3}\xi_0)}{1-\frac{n}{6}}\right]^{\frac{1}{N}}. \tag{16}$$

Alternatively,

$$V_c = \left[\frac{bN(-\sqrt{3}\xi_0)}{2B_0(1-\frac{n}{6})}\right]^{\frac{1}{N}}. \tag{17}$$

A viable decelerated universe is indicated by a positive effective pressure (p_{eff}) when the magnitude of the critical volume is greater than the volume V, i.e. $V_c > V$ and $p_{eff} = 0$ for $V = V_c$. The effective pressure turn is negative when $V > V_c$, which suggests that the cosmos is accelerating. In the analysis, we obtain a new scale of V_c when a dust-dominated universe enters into the

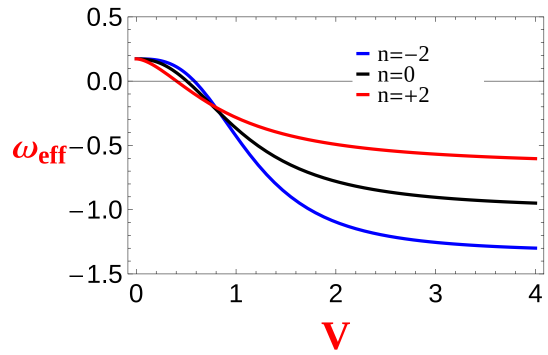

FIG. 2. Plots of ω_{eff} versus V for the values of $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$.

acceleration phase. We discover that ϵ and V_c are of the same order of magnitude. The decelerated universe is represented by $V \ll \epsilon$, whereas the very vast volume with a conceivable accelerated universe is represented by $V \gg \epsilon$.

B. Equation of state parameters

In this section we determine the equation of state (EoS) for the bulk viscous fluid, which is defined as,

$$\omega_{eff} = \frac{p_{eff}}{\rho_{vvcg}}. \tag{18}$$

Equations (11) and (13) are used to rewrite the aforementioned expression as,

$$\omega_{eff} = \left[(-\sqrt{3}\xi_0) - \frac{N/2}{\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]} \right]. \tag{19}$$

In a viscous cosmology, the characteristics of the cosmic fluid and its bulk viscosity shape the dynamic relationship between the effective EoS parameter and volume. Instead of being constant, the effective EoS parameter changes when the cosmic scale factor (a) changes. The impact of bulk viscosity on the EoS is minimal during early eras when the cosmos is dense. Bulk viscosity may become dynamically significant as the universe expands, pushing the effective EoS toward negative values $(\omega_{eff} < -1/3)$, thereby leading to accelerated expansion, as illustrated in Fig. 2. We examine a few special cases:

(i) When volume is small, $V \ll \epsilon$, i.e. $\epsilon/V > 1$, the effective pressure is p_{eff} = $(-\sqrt{3}\xi_0)\rho_{vgcg}$ and ω_{eff} = $-\sqrt{3}\xi_0$. It is observed that the effective pressure depends on viscous coefficient parameter ξ_0. If the value of ξ_0 is set to zero, $p_{eff} \approx 0$, the universe is dominated by dust and EoS does not depend on n. For $\xi_0 > 0$, $\omega_{eff} < -1$, indicating a 'Big Rip' scenario which corresponds to a phase of uncontrolled accelerated expansion.

(ii) For large volume $V \gg \epsilon$, i.e $\frac{\epsilon}{V} \ll 1$ and if $\xi_0 = 0$, then we get $\omega_{eff} \approx -1 + \frac{n}{6}$. If $n = 0$, then $\omega_{eff} \approx -1$ which is leading to an accelerated expansion, i.e. Λ Cold Dark Matter (ΛCDM). If n is positive valued, ω_{eff} lies in the region $0 > \omega_{eff} > -1$, in this case a Big Rip does not occur, and the universe evolves toward a quiescent phase. The phantom-like model: $\omega_{eff} < -1$, is obtained for $n < 0$. At $n = -2$ and $\xi_0 = -0.1$, ω_{eff} is more negative, which can be seen in Fig. 2.

C. Deceleration parameter

The deceleration parameter in a cosmology where a viscous fluid predominates and dynamically changes with the volume of the universe, which is represented by the scale factor. As a result of cosmic acceleration driven by viscosity, the deceleration parameter changes from a positive (decelerating) value in the early universe to a negative (accelerating) value in the later universe. In order to explain the observable expansion history, this dynamic behavior offers an alternative to the cosmological constant (Λ). Radiation and matter densities were quite high in the early cosmos. Based on the specification, the effective deceleration parameter that corresponds to the bulk viscosity of the VVCG model can be expressed as,

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\frac{p_{eff}}{\rho_{vvcg}}. \tag{20}$$

The equation can be written using the viscous parameter ξ_0 using Eq. (19) as,

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\left[(-\sqrt{3}\xi_0) - \frac{N/2}{\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]}\right]. \tag{21}$$

Figure 3 graphically shows the variation of q_{eff} with V. We examine some critical aspects of the

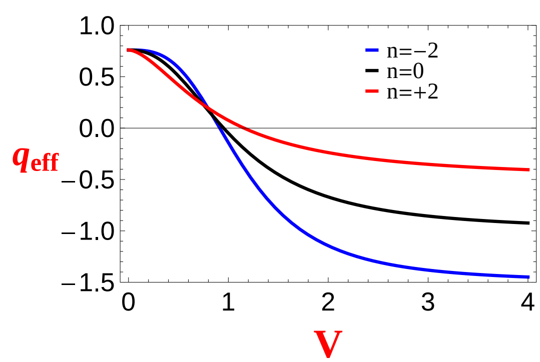

FIG. 3. Plot of q_{eff} versus V for the values of $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$.

relation defined in Eq. (21):

(i) The early universe corresponds to $V \ll \epsilon$. Equation (21) reduces to $q_{eff} \approx \frac{1-3\sqrt{3}\xi_0}{2}$, i.e. q_{eff} is positive for $\xi_0 \leq 0$, which indicates that the universe is decelerated. If viscosity coefficient (ξ_0) vanishes, the deceleration parameter $q_{eff} = \frac{1}{2}$ that results in a matter-dominated universe. Because of the large energy density and initial insignificant negative pressure associated with bulk viscosity, indicating q_{eff} is positive and a decelerating phase.

(ii) In the late universe, the volume is very large, $V \gg \epsilon$, and Eq. (21) reduces to $q_{eff} \approx -1 + \frac{n}{4}$, which also depends on n. When n is zero, $q_{eff} \approx -1$, universe is accelerated. Energy density falls as the universe gets bigger and its volume rises. However, the negative viscous pressure may start to dominate. As the cosmic volume expands, the negative bulk viscous pressure becomes more significant. In order to move from a decelerating to an accelerating phase, the deceleration parameter falls and exceeds the threshold, $q_{eff} = 0$ for $n = 4$. This corresponds to a constant expansion rate.

In this instance, the $flip$ volume (V_f) will occur when the effective deceleration parameter is zero. In many viscous models, the deceleration parameter gets closer to $q_{eff} = -1$ as the scale factor gets closer to infinity. This is equivalent to a de Sitter universe, which behaves like a cosmological constant and experiences exponential growth driven by a constant vacuum energy density. Consequently, the flip volume expression can be stated as,

$$V_f = \epsilon \left[\frac{2(1-3\sqrt{3}\xi_0)}{4-n}\right]^{\frac{1}{N}} = \left[\frac{bN(1-3\sqrt{3}\xi_0)}{B_0(4-n)}\right]^{\frac{1}{N}}. \tag{22}$$

According to Fig. 3, the cosmos accelerates as the volumes rises. The equation above demonstrates that the value of V_f must be genuine when $\xi_0 < 0$; otherwise, there won't be any $flip$. Accordingly, the universe accelerates when $V > V_f$ and decelerates when $V < V_f$. Therefore, we determine that two scales of volume are V_c and V_f, respectively from the zero-pressure condition. A transition from deceleration to acceleration may occur at a critical value of the effective deceleration parameter. We may obtain the following relation:

$$\frac{V_f}{V_c} = \left[\frac{(3\sqrt{3}\xi_0 - 1)(6-n)}{3\sqrt{3}\xi_0(4-n)}\right]^{\frac{1}{N}}. \tag{23}$$

D. Speed of sound

This analysis focuses on the stability criterion of the VVCG model. The definition of sound speed in a viscous fluid is

$$v_s^2 = \left(\frac{\partial p_{eff}}{\partial \rho_{vvcg}}\right)_S = \left[(-\sqrt{3}\xi_0) + \frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right]. \tag{24}$$

The velocity of sound in the cosmic fluid is a dynamically changing quantity that is governed by the bulk viscosity and thermodynamic parameters of the fluid rather than being a constant in viscous cosmology. The relation between sound speed and volume is driven by the expansion of the universe, as opposed to static fluids, where density and elastic characteristics control sound speed. Sound speed is comparatively stable in the early cosmos because adiabatic processes dominate the fluid's characteristics. In the late universe, bulk viscosity increases with increasing volume, causing the sound speed to dynamically vary and possibly turn negative. This is a prerequisite for the fluid to function as a source of rapid expansion as shown in Fig. 4. We also know that the range of speed of sound must follow the criterion: $0 < v_s^2 < 1$. This range was the focus of our investigation. It lowers to $v_s^2 = -\sqrt{3}\xi_0$ when volume is very small ($V \ll \epsilon$), i.e. at the early

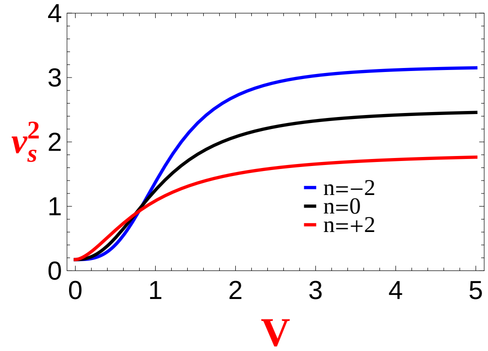

FIG. 4. Plots of v_s^2 versus V for $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$.

universe. When volume is very large ($V \gg \epsilon$), Eq. (24) yields

$$v_s^2 = 1 - 2\sqrt{3}\xi_0 - \frac{n}{6}. \tag{25}$$

In the above equation, both ξ_0 and n are negative, $v_s^2 > 0$, i.e. the thermodynamical stability condition is satisfied, leading to a phantom type of universe [51, 52]. If $\xi_0 > 0$ and $n > 0$, as seen in Fig. 4, Eq. (25) yields an imaginary speed of sound, which causes a perturbative cosmology [53]. For holographic DE, Myung [54] observed that $v_s^2 < 0$, whereas for basic CG and tachyon matter, it is non-negative. According to Panigrahi and Chatterjee [55], with varying MCG, v_s^2 might be either positive or negative.

III. THERMODYNAMIC STABILITY

We examine the thermodynamic stability conditions of the fluid throughout the evolution of the universe. The criteria for thermodynamic stability are described in [46]. In both adiabatic and isothermal expansion, the pressure decreases as (a) $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ and (b) $c_V > 0$. Differentiating Eq. (13) wrt volume, the above equation can be expressed in terms of pressure as,

$$\begin{aligned}\left(\frac{\partial p_{eff}}{\partial V}\right)_S &= -\frac{p_{eff}}{V}\frac{1}{\left[N + 2\sqrt{3}\xi_0\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]\right]} \\ &\quad\times\left[\frac{n\xi_0}{\sqrt{3}}\left[1 + \left(\frac{\epsilon}{V}\right)^N\right] + N\left[\frac{n}{6} + \left(\frac{\epsilon}{V}\right)^N\left(\sqrt{3}\xi_0 - \frac{N/2}{\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]}\right)\right]\right].\end{aligned} \tag{26}$$

(a) Firstly, when the volume is very small ($V \ll \epsilon$), the above expression can be written as $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -(1 - \sqrt{3}\xi_0)\frac{p_{eff}}{V}$. According to earlier research, in the early universe, $p_{eff} = (-\sqrt{3}\xi_0)\rho_{vgcg}$. Therefore,

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -(1 - \sqrt{3}\xi_0)(-\sqrt{3}\xi_0)\frac{\rho_{vgcg}}{V} \tag{27}$$

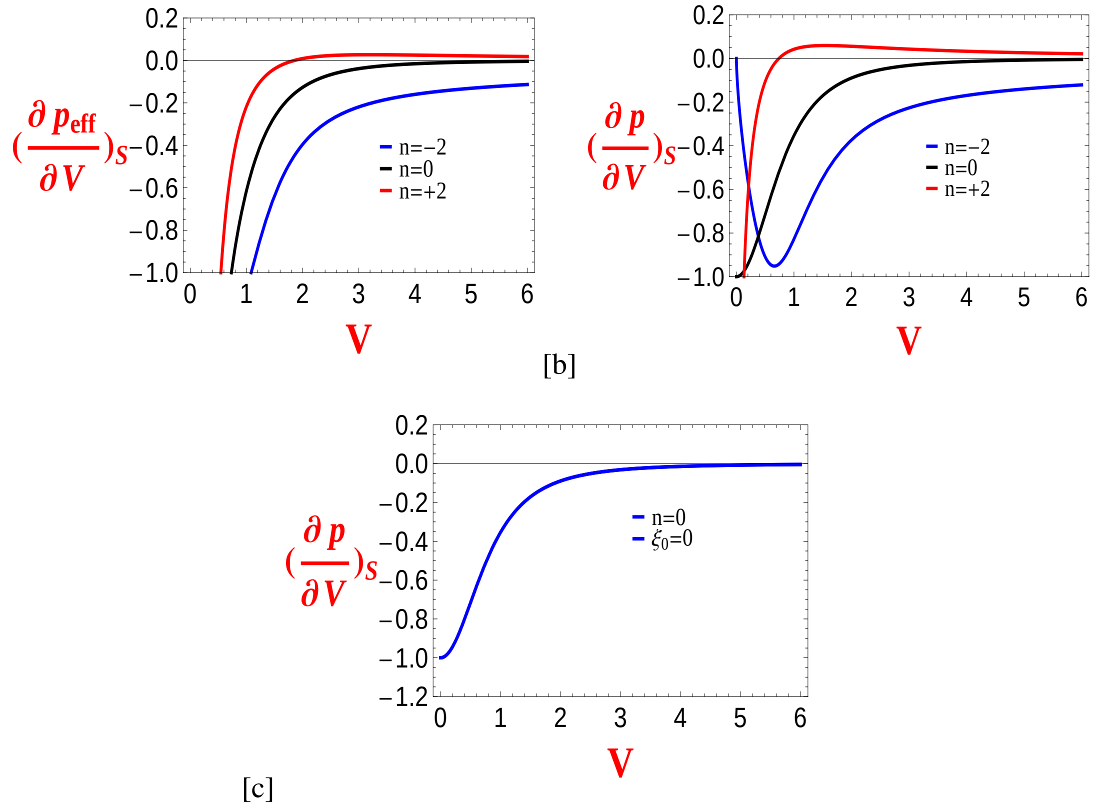

FIG. 5. Plots of $\left(\frac{\partial p}{\partial V}\right)_S$ versus V for (a) the VVCG model $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$; (b) The variable CG model $\xi_0 = 0$, $B_0 = 1$ & $b = 1$ and (c) the CG model $\xi_0 = 0$, $n = 0$, $B_0 = 1$ and $b = 1$.

solely depends on ξ_0 and is independent of n. The pressure in this evolution is negative. Hence, we would get $(\frac{\partial p}{\partial V})_S < 0$ for $\xi_0 < 0$.

(b) Secondly, Eq. (26) reduces to $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -\frac{np_{eff}}{6V}$ for large volumes ($V \gg \epsilon$). In order to make $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$, n must be negative because p_{eff} is negative at the late stage of evolution. The reliance of n is evident in the latter instance, as we have seen in Fig. 5 of different CG models. The following particular scenarios will be discussed in order to limit the parameters utilized here:

(i) If we put ξ_0 = 0 and $n \neq 0$, we find that Eq. (26) becomes similar to the one proposed by Panigrahi [8]. We get

$$\left(\frac{\partial p}{\partial V}\right)_S = \frac{p}{6V}\left[(6-n)\left(1 - \frac{1}{\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]}\right) - n\right]. \tag{28}$$

From Eq. (28), we can see throughout the evolution $\left(\frac{\partial p}{\partial V}\right)_S < 0$ for $n \leq 0$. Figure 5(b) shows that the positive value of n is not compatible in the VCG model. It is feasible to draw the conclusion that in order to achieve thermodynamically stable development, the positive value of n needs to be removed. The characteristics of the graphs for $n = 0$, $n = -2$ and

$n = +2$ are initially somewhat different, but both $n = 0$ and $n = -2$ yield $\left(\frac{\partial p}{\partial V}\right)_S < 0$ throughout the evolution. The impact of n is the reason for such observations.

(ii) If we put $\xi_0 = 0$ and $n = 0$, the Eq. (26) reduced to as given by

$$\left(\frac{\partial p}{\partial V}\right)_S = \frac{p}{V}\left[\left(1 - \frac{1}{\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]}\right)\right], \tag{29}$$

which corresponds to CG. The graphical representation for $\xi_0 = 0$ and $n = 0$ (CG model), is shown in Fig. 5(c), which shows that $\left(\frac{\partial p}{\partial V}\right)_S < 0$ throughtout the evolution and leading to the stability of fluid.

IV. THERMAL EOS

Using the thermodynamics relation, we also verified that the specific heat was positive at constant volume. Entropy and temperature can be used to express the specific heat as,

$$c_V = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \rho_{vvcg}}{\partial T}\right)_V, \tag{30}$$

where T and S represent temperature and entropy, respectively. The equation is utilized to determine the fluid temperature $T = \left(\frac{\partial U}{\partial S}\right)_V$ [54], which can be expressed as,

$$T = \left(\frac{\partial U}{\partial b}\right)_V \left(\frac{\partial b}{\partial S}\right)_V. \tag{31}$$

Using Eq. (9), the expression for temperature can be written as,

$$T = \frac{1}{2}V^{\sqrt{3}\xi_0}\left[\frac{2B_0 V^N}{N} + b\right]^{\frac{1}{2}}\left(\frac{\partial b}{\partial S}\right)_V. \tag{32}$$

Since b is a universal constant, we find that $\left(\frac{db}{dS}\right)_V = 0$. This implies that, regardless of the pressure and volume, the temperature is zero. However, in the case of CG, the temperature does change with expansion. Therefore, we must consider $\frac{db}{dS} \neq 0$. Lacking specific information on how b depends on S, we assume that $\frac{db}{dS} > 0$ [8, 9], which allows us to derive positive temperatures that decrease through adiabatic expansion. Equation (9) can be obtained by dimensional analysis.

$$[b] = [U]^2[V^{(-\sqrt{3}\xi_0)}]^2. \tag{33}$$

Since we are aware that $[U] = [T][S]$, the equation above can be expressed as,

$$[b] = [T]^2[S]^2[V]^{(-2\sqrt{3}\xi_0)}. \tag{34}$$

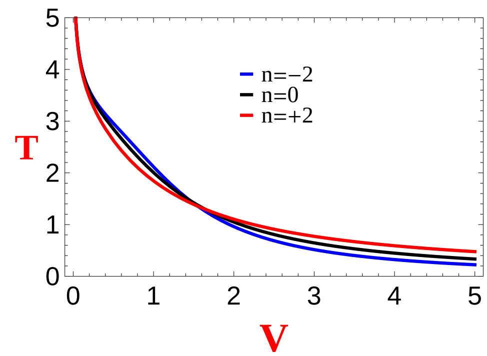

FIG. 6. Plots of T versus V for $\xi_0 = -0.1$, $B_0 = 1$ and $b = 1$.

It is challenging to solve for b from Eq. (34). Consequently, the expression for b in a trial case can be expressed as τ and v,

$$b = \left(\tau v^{-\sqrt{3}\xi_0}\right)^2 S^2. \tag{35}$$

In this case, v represents the dimension of volume and τ (a constant) represents merely the temperature. Presently, we get

$$\left(\frac{db}{dS}\right)_V = 2\left(\tau v^{-\sqrt{3}\xi_0}\right)^2 S. \tag{36}$$

Using Eqs.(35) and (36), Eq. (32) becomes

$$T = V^{-\left((N+\frac{n}{3})-1\right)}\rho^{-1}\left(\tau v^{-\sqrt{3}\xi_0}\right)^2 S. \tag{37}$$

The expression above can be streamlined to

$$T = V^{\sqrt{3}\xi_0}\left(\tau v^{-\sqrt{3}\xi_0}\right)\left[1+\left(\frac{V}{\epsilon}\right)^N\right]^{-\frac{1}{2}}. \tag{38}$$

Putting the value of b in Eq. (37), the entropy is obtained as,

$$S(T) = \left(\frac{2B_0 V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}} \frac{\left(\frac{V^{1-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)}{\left[\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{-\sqrt{3}\xi_0}}\right)^2 - 1\right]^{\frac{1}{2}}}. \tag{39}$$

This equation may also be written as,

$$S(T) = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}}\left(\frac{V^{1-\frac{n}{6}}}{T}\right)\frac{\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]^{\frac{1}{2}}}. \tag{40}$$

In contrast to the standard model, where temperature and volume are inversely related, the

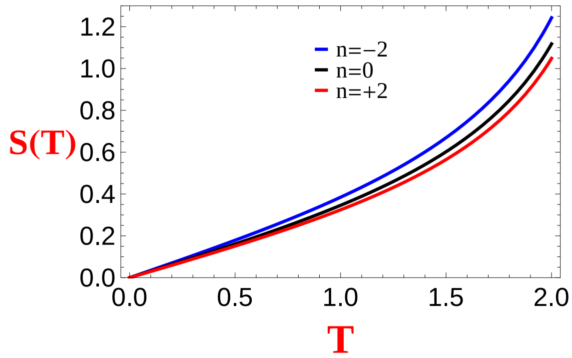

FIG. 7. Plots of $S(T)$ versus T for $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$, $v = 1$.

VVCG model has a different relationship between temperature and volume. This thermodynamic relation is altered by the introduction of a dissipative, heat-producing component in the form of a bulk viscous fluid. Figure 6 indicates that the temperature decreases to zero as the volume approaches the infinitely large value. The third law of thermodynamics is obeyed by the VVCG model in this regard. A perfect and non-viscous fluid expands adiabatically under a conventional cosmological model, which involves no heat exchange and constant entropy. When viscosity is included, the system generates entropy and becomes irreversible. However, the relationship between temperature and entropy is more complicated and relies on the cosmic epoch, whereas in a cosmological model with a viscous fluid, entropy continuously rises throughout time since viscosity is irreversible. Throughout cosmic history, cooling brought on by the expansion of the universe has had varying effects on the evolution of entropy. In the early cosmos, viscosity was important, for example, in the quark-gluon plasma phase. According to the causal thermodynamics-based models, such as the Israel-Stewart theory, the temperature may have risen exponentially during some stages, while viscous dissipation caused the comoving entropy to increase significantly. Figure 7 shows that the entropy increases with temperature in the VVCG model.

From Eq. (40) we see that the condition $0 < TV^{-\sqrt{3}\xi_0} < \tau v^{-\sqrt{3}\xi_0}$ must hold for positive and finite entropy. This condition is confirmed as it meets the constraints $\tau > T > 0$ and $v < V < \infty$, where v represents the lowest volume and τ represents the highest temperature. Therefore, the universe with VVCG represents a model of the universe that first decelerates and then accelerates. It is a thermodynamically stable system with $\xi_0 < 0$, positive squared velocity of sound, and positive heat capacity throughout its evolution. If $T \to 0$, then Eq. (40) yields $S = 0$, indicating that the third law of thermodynamics is satisfied. Substituting Eqs. (38) and (40) into Eq. (30), it follows that

$$c_V(T) = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}} V^{\frac{N}{2}} \frac{\frac{TV^{-\sqrt{3}\xi_0}}{\left(\tau v^{-\sqrt{3}\xi_0}\right)^2}}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]^{\frac{3}{2}}}. \tag{41}$$

In terms of entropy S, the aforementioned equation can also be written as,

$$c_V(T) = \frac{S}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]}. \tag{42}$$

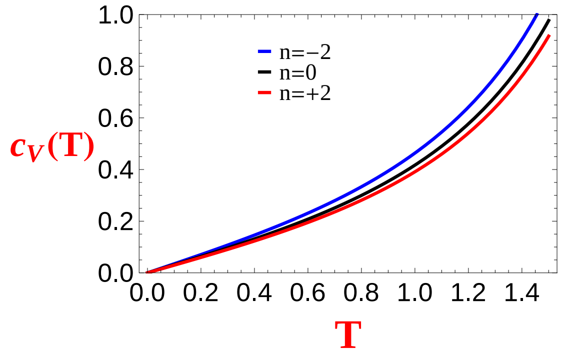

FIG. 8. Plots of $c_V(T)$ versus T for $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$, $\upsilon = 1$.

Thermal heat capacity in cosmic viscous fluids can change with temperature because of particle formation, non-equilibrium thermodynamics, and the relativistic character of the early universe. Because of dissipative effects, viscous cosmic fluids differ from ideal fluids in that their heat capacity deviates from constant values, especially in the early universe when expansion rates and temperatures were extremely high. Since heat is not conserved in a comoving volume, the expansion of the universe is non-adiabatic due to the bulk viscosity in a CG. The cosmic expansion rate and the non-ideal fluid behavior are thus closely related to the thermal characteristics of the fluid. When $0 < TV^{-\sqrt{3}\xi_0} < \tau\upsilon^{-\sqrt{3}\xi_0}$ (alternatively, $\tau > T > 0$ and $\upsilon < V < \infty$ and $\xi_0 < 0$) then $c_V > 0$ for all n. We found that in this model, the specific heat capacity vanishes at zero temperature–in agreement with the third law of thermodynamics. We also saw that the thermal heat capacity c_V and the entropy S have positive values. Thus, for $\xi_0 < 0$ and $n < 0$, the VVCG model is thermodynamically stable. The plot of thermal heat capacity versus volume for a given $\xi_0 = -0.1$ and different values of n is displayed in Fig. 8. If we enter $\xi_0 = 0$ in Eq. (42), we obtain an expression that is comparable to c_V, which was discovered by Panigrahi [8], and once again we get the same expression for c_V at $n = 0$ that was obtained by Santosh *et al.* [34]. Using Eqs. (9), (35), and (40) we calculate the internal energy in this model as,

$$U(T) = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}} V^{1-\frac{n}{6}} \left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}}. \tag{43}$$

The internal energy of a VVCG in a viscous fluid varies with temperature as a consequence of dissipative bulk viscosity and its unconventional equation of state. Unlike an ideal gas, whose internal energy depends only on temperature, the VVCG shows a more intricate dependence involving cosmological parameters and the functional form of viscosity. As the universe expands, the fluid cools, and in stable models the internal energy remains a well-defined function of temperature. Consequently, non-physical behaviors such as divergent temperatures or unphysical zero-temperature states are avoided. In contrast to a perfect VVCG, where the internal energy can be temperature-independent due to adiabatic evolution, bulk viscosity requires an explicit temperature dependence, as shown in Fig. 9. This is a prerequisite for thermodynamic stability. For the stability of the VVCG model, we looked at the isothermal condition $(\frac{\partial p}{\partial V})_T < 0$. Applying

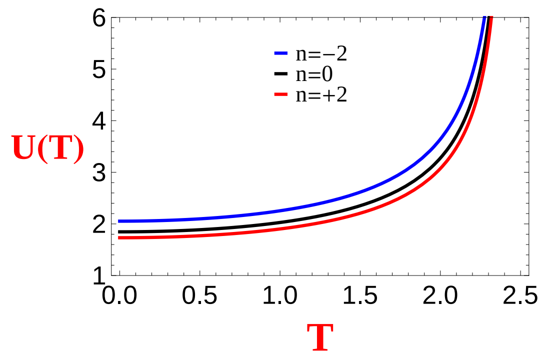

FIG. 9. Plots of $U(T)$ versus T for $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$, $v = 1$.

$p = p(V, T)$ from thermodynamic relations and resolving Eqs. (13) and (35), we obtain

$$p_{eff}(T) = -\left(\frac{2B_0 V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}}\left[\sqrt{3}\xi_0 + \frac{N}{2}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]\right]. \tag{44}$$

The aforementioned statement can also be expressed in terms of entropy,

$$p_{eff}(T) = -\left(\frac{TS}{V}\right)\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{-\sqrt{3}\xi_0}}\right)^2\left[\sqrt{3}\xi_0 + \frac{N}{2}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]\right], \tag{45}$$

and the density in terms of temperature is expressed as,

$$\rho_{vvcg}(T) = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}} V^{-\frac{n}{6}}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}}. \tag{46}$$

A combination of its unique equation of state (EoS) and dissipative bulk viscosity causes pressure in a viscous fluid described by the VVCG model to vary with temperature in a non-trivial manner. As the universe expands, its energy is irreversibly dissipated into heat due to the bulk viscosity. This mechanism stops the expansion from being entirely adiabatic and influences the temperature evolution of the fluid. The Hubble expansion rate and the bulk viscosity coefficient are included in the equation governing the temperature evolution, indicating that the fluid temperature and pressure are inseparable due to cosmic dynamics. However, viscous theories have been put up to explain the present fast expansion of the universe. The required negative pressure in this model to drive acceleration is provided by a bulk viscous pressure inside the component of dark energy as depicted in Fig. 10. The variability of ρ_{vvcg} and p_{eff} when both T and V vary is plotted in Fig. 11.

In the VVCG model the variation of energy density with temperature seems to be complex and it dependents on the thermodynamic assumptions made. Figure 12 shows that the relationship between energy density and temperature is proportional, meaning that it rises as the temperature

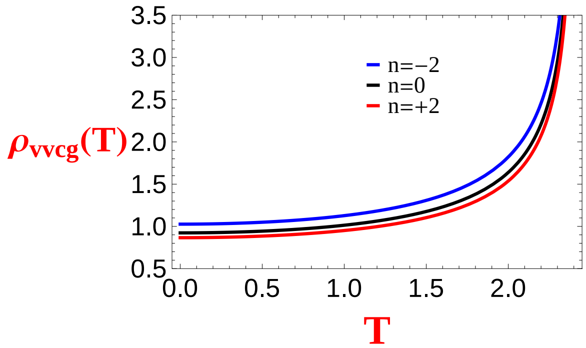

FIG. 10. Plots of $\rho_{vgcg}(T)$ versus T for $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$, $\upsilon = 1$.

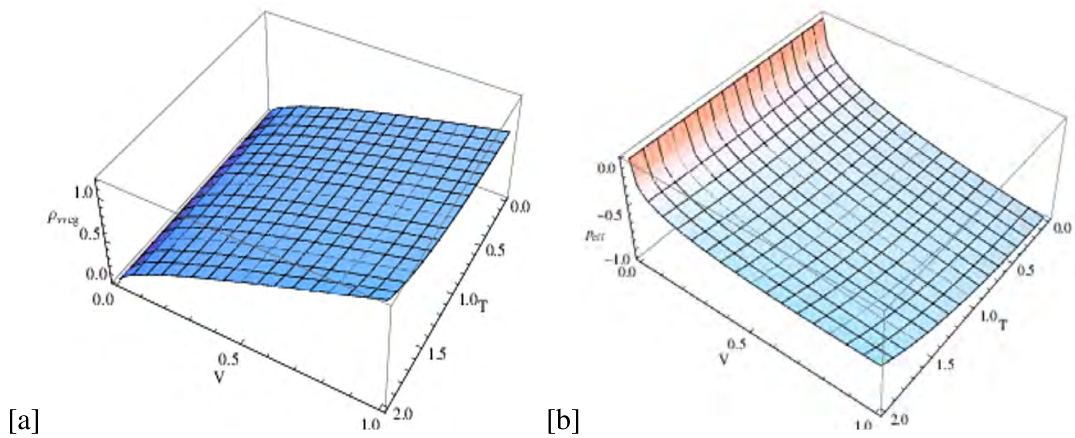

FIG. 11. Plots of [a] ρ_{vvcg} and [b] p_{eff} when T and V vary. We set $\xi_0 = -0.1$, $n = -2$, $\tau = 2.73$, $B_0 = 1$ and $\upsilon = 1$ in the VVCG model.

does. The form of the relevant thermal EoS parameter of VVCG is

$$\omega_{eff}(T) = -\left[\sqrt{3}\xi_0 + \frac{N}{2}\left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]\right]. \tag{47}$$

A viscous fluid composed of a VCG has an EoS that changes with temperature due to dissipative bulk viscosity, in addition to simply relating pressure and energy density. This relation is constructed to ensure thermodynamic stability, which is particularly essential for cosmological applications, rather than being assumed a priori. Both temperature and volume must be taken into account when determining the viscous VCG pressure in order to attain thermodynamic stability. This ensures that the fluid is cooling as the universe expands, which is a prerequisite for a workable cosmological model. It has been found that the parameter in the equation of state is a function of temperature in the VVCG model, as shown in Fig. 12. In the early universe, when temperatures were extremely high, $T \to \tau$, Eq. (47) indicates that the cosmos is dominated by dust, as $\omega_{eff} = 0$ and $p_{eff} = 0$. The ΛCDM model is shown by the Eq. (47), which yields $\omega_{eff} = -1$ when the universe is in its late stage and the temperature is extremely low, $T \to 0$.

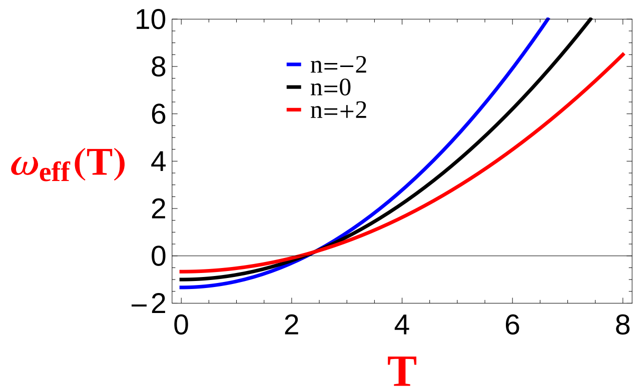

FIG. 12. Plots of $\omega_{eff}(T)$ versus T for $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$ and $v = 1$.

We will now analyze Eq. (44) for $\left(\frac{dp_{eff}}{dV}\right)_T \leq 0$. Consequently, we obtain

$$\left(\frac{dp_{eff}}{dV}\right)_T = -\frac{\left[V^{-1-\frac{n}{6}}\left(-\frac{B_0}{n+6\sqrt{3}\xi_0-6}\right)^{\frac{1}{2}}\right]}{18\sqrt{2}\,[1-\Upsilon^2]^{\frac{3}{2}}}[1-\Upsilon^2]\left[\sqrt{3}n^2\Big[1-\Upsilon^2\Big] - 6\sqrt{3}n\Big[1-(1-2\sqrt{3}\xi_0)\Upsilon^2\Big] + 108\xi_0\Upsilon^2\left((1-\sqrt{3}\xi_0) - \frac{\sqrt{3}\xi_0}{[1-\Upsilon^2]}\right)\right], \tag{48}$$

where $\Upsilon = \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)$. Figure 13 indicates that $\left(\frac{dp_{eff}}{dV}\right)_T < 0$ for the negative values of n and ξ_0, throughout the evolution, which is the thermodynamics stability condition. In this case, for $n < 0$ and $\xi_0 < 0$, the value of $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ should be negative, implying that the isobaric curves in our VVCG model do not overlap with the isotherms within the thermodynamic state diagram. This distinction represents a meaningful advancement in our analysis. Since $n < 0$ and $\xi_0 < 0$, we infer that both $\left(\frac{\partial p_{eff}}{\partial V}\right)_S$ and $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ are negative, which is consistent with thermodynamic stability. Assuming that the initial conditions at $V = V_0$ are $\rho = \rho_0$, $p_{eff} = p_0$ and $T = T_0$, we obtain from Eq. (9):

$$b = V_0^{2(1-\sqrt{3}\xi_0)}\left(\rho_0^2 - \frac{2B_0V_0^{-\frac{n}{3}}}{N}\right). \tag{49}$$

The energy density ρ and the effective pressure p_{eff} as a function of volume V are obtained using Eqs. (11), (12) and (49), we get

$$\rho = \left(\frac{2B_0V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1 + \frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1 - \frac{2B_0V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{\frac{1}{2}}, \tag{50}$$

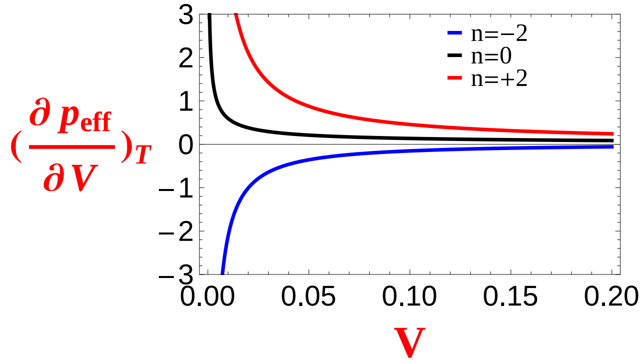

FIG. 13. Plots of $\left(\frac{\partial P_{eff}}{\partial V}\right)_T$ versus T for $\xi_0 = -0.1$, $T = 1$, $\tau = 2.73$, $B_0 = 1$ and $v = 1$.

and

$$p_{eff} = -\left(\frac{2B_0V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1+\frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1-\frac{2B_0V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{\frac{1}{2}} \times\left[\sqrt{3}\xi_0+\frac{N}{2}\left[1+\frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1-\frac{2B_0V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{-1}\right]. \quad (51)$$

Equations (44), (50), and (51) may now be expressed as functions of the reduced parameters η, v, P, κ, t and τ^* so that

$$\eta = \frac{\rho}{\rho_0}, \quad v = \frac{V}{V_0}, \quad P = \frac{p_{eff}}{B_0^{\frac{1}{2}}}, \quad \kappa = \frac{2B_0}{N\rho_0^2}, \quad t = \frac{T}{T_0}, \quad \tau^* = \frac{\tau}{T_0}. \quad (52)$$

Also it is feasible to represent Eqs. (44), (50) and (51) in the reduced units. At $P = P_0$, $V = V_0$ and $T = T_0$, we have $t = 1$ and $v = 1$. Then

$$P_0 = \left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{6}}\left[\frac{2\sqrt{3}\xi_0}{N}\frac{V_0^{\frac{n}{6}}}{\kappa^{\frac{1}{2}}}+\frac{\kappa^{\frac{1}{2}}}{V_0^{\frac{n}{6}}}\right] = \left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{6}}\left[\frac{\frac{2\sqrt{3}\xi_0}{N}}{\left[1-\frac{1}{(\tau^*)^2}V_0^{-2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}}+\left[1-\frac{1}{(\tau^*)^2}V_0^{-2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}\right]. \quad (53)$$

Thus, κ and τ^* can be calculated as follows:

$$\kappa = V_0^{\frac{n}{3}}\left(1-\frac{1}{(\tau^*)^2}V_0^{-2\sqrt{3}\xi_0}\right), \quad \text{and} \quad \tau^* = \frac{V_0^{-\sqrt{3}\xi_0}}{\left(1-\kappa V_0^{-\frac{n}{3}}\right)^{\frac{1}{2}}}. \quad (54)$$

We have observed that τ^* depends on ξ_0, κ, V_0 and n. All of the aforementioned equations reduce to the equations obtained by Panigrahi [8] for $\xi_0 = 0$, and to that of Santos *et al.* [34] for $n = 0$ and $\xi_0 = 0$. Since $\kappa = \frac{2B_0}{N\rho_0^2}$ at the present epoch, $\rho_0 = \left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}}$. Considering the Planck era temperature $\tau = 10^{32}K$ and the present epoch temperature $T_0 = 2.7K$, the ratio $\tau^* = \frac{\tau}{T_0} = 3.7 \times 10^{31}$. Thus, the ratio κ will be

$$\kappa = V_0^{\frac{n}{3}} \left(1 - \frac{1}{(3.7 \times 10^{31})^2} V_0^{-2\sqrt{3}\xi_0}\right) \approx V_0^{\frac{n}{3}}. \tag{55}$$

Once again, using Eq. (46) for the present epoch when T is extremely low, i.e. $T \to 0$,

$$\rho_0 = \left(\frac{2B_0}{NV_0^{\frac{n}{3}}}\right)^{\frac{1}{2}} \approx \left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}}. \tag{56}$$

For large volume, Eq. (11) yields the same conclusion. Therefore, according to Eq. (35), at the present epoch the energy density of the universe filled with the VVCG must be nearly equal to $\left(\frac{2B_0}{N\kappa}\right)^{\frac{1}{2}}$.

V. CONCLUSIONS

We have investigated the thermodynamic behavior of the VVCG model. We consider the value $n = -2, 0, +2$, as reported by Panigrahi and Chatterjee [55]. Previous studies showed that n may be positive or negative valued [33]. Despite the fact that recent cosmological observations support the existence of dark energy in the universe, it is extremely challenging to weigh the advantages of its various forms, at least based on the observational data. Actually, most of the models fit within the observed energy budget. Consequently, we investigate whether the gas described by the equation behaves as a thermodynamically closed system for appropriate parameter values that are consistent with observational constraints, particularly for various types of CG. The consistency of the gas in this case has been confirmed using its stability requirements, and the standard recommendation has been adhered to: $\left(\frac{dp_{eff}}{dV}\right)_S < 0$, $\left(\frac{dp_{eff}}{dV}\right)_T < 0$ and $c_V > 0$. The VVCG is illustrated here for the third law of thermodynamics to be satisfied. Additionally, we have determined that all thermal quantities are solely dependent on temperature and characterizes the viscosity variable with $\xi_0 = -0.1$. In addition, we find a novel effective equation of state, deceleration parameter, and speed of sound that fully characterize the CG as a function of temperature and volume of the viscous variables. Fascinatingly, this requires that the additional parameter n added to the VVCG model must be negative valued. This conclusion is in line with the findings of Sethi *et al.* [33], who found that the best fitted value of n lies in the range $(-1.3, 2.6)$. However, Lu *et al.* [56] found that $n > 0$. Our analysis suggests that this result is implausible, as such a condition renders the model thermodynamically unstable. As illustrated in Fig. 1, for $\xi_0 < 0$ and negative values of n, the model predicts that the pressure becomes increasingly negative with increasing volume. Furthermore, the model shows that, in a dust-dominated universe, the effective equation of state approaches $p_{eff} = 0$. At late times, the effective equation of state approaches $\omega_{eff} \approx -1 + \frac{n}{6}$. For $n = 0$ we recover the ΛCDM model. As mentioned, thermodynamic stability considerations favor negative values of n, which correspond to a phantom-like regime and may lead to a Big Rip scenario. Shown in Fig. 2 is the phenomenon mentioned above. However, the phantom-like evolution is eventually found to be consistent with the CMB anisotropy measurements and the SNe Ia observations [51, 52]. We have also studied the deceleration parameter in context of thermodynamics. The flip is feasible for $n < 4$, as demonstrated by our calculation of the flip volume. We obtain $q_{eff} = \frac{1}{2}$ for a dust-dominated universe, while for $n < 0$ the universe accelerates at a late stage when $q_{eff} < 0$. This is in line with the observation of [27]. From our calculation of the speed of sound a phantom-like universe is resulted when both ξ_0 and n take negative values. This indicate that the thermodynamical stability condition is met by the speed of sound [51, 52]. We are primarily concerned with the thermodynamic stability of the chosen gas model. Our analysis, as illustrated in Fig. 5, shows that $\left(\frac{dp_{eff}}{dV}\right)_S < 0$ throughout the evolution, which is a requirement of stability, but for the negative values of n. Since c_V is always positive for $n < 0$, the thermal heat capacity (c_V) will be calculated in this context. Thus, the thermodynamic stability requirements of the viscous fluid are examined, demonstrating that the fluid remains thermodynamically stable throughout its evolution.

It is evident that both $S = 0$ and c_V of the VVCG model disappear at $T = 0$, according to the third law of thermodynamics. Additionally, the thermal and caloric EoS show that $0 < T < \tau$. Using the bulk viscosity $(\xi = \xi_0\rho^{\frac{1}{2}})$, we have examined the behavior of a viscous cosmological model in terms of the volume V of the universe. When volume is small (at an early time), which corresponds to high energy density, the universe is dominated by the initial fluid pressure (such as, radiation or matter). Although viscosity may not dominate the dynamics, it can significantly moderate the rapid expansion of the universe following a Big Bang like phase. Energy density still dominates cosmic expansion, but the model is most viscous at early periods, which influences entropy formation. The universe expands nearly freely when volume is large, i.e. in late time, which corresponds to low energy density and is dominated by residual matter/radiation or the cosmological constant. If dark energy or pressure predominates, the model asymptotically approaches a de Sitter-like phase.

Furthermore, we have calculated all thermal quantities related to temperature. The thermal EoS depends explicitly only on temperature. When $\xi_0 < 0$, the gas undergoes adiabatic expansion, and its volume rises with temperature. In this instance, Eq. (47) reduces to $\omega_{eff} = 0$ as $T \to \tau$, suggesting that the cosmos is dominated by dust. Equation (48) shows that for $n < 0$ and $\xi_0 < 0$, $\left(\frac{dp_{eff}}{dV}\right)_T < 0$ throughout the evolution, which satisfy the thermodynamic stability condition, as illustrated in Fig. 13. The isobaric curves of the VVCG model, on the other hand, naturally attains negative values for $n < 0$, indicating that the criterion of thermodynamic stability is satisfied. Motivated by these theoretical insights, the model is used to explore the thermodynamic nature of the universe.

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[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998).
[2] R. A. Knop *et al.*, Astrophys. J. **598**, 102 (2003).
[3] R. Amanullah *et al.*, Astrophys. J. **598**, 102 (2003).
[4] D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).
[5] D. N. Spergel *et al.*, Astrophys. J. Suppl. **170**, 377 (2007).
[6] I. P. Neupane and H. Trowland, Int. J. Mod. Phys. D **19**, 364 (2010).
[7] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **15**, 2105 (2006).

[8] D. Panigrahi Int. J. Mod. Phys. D **24**, 1550030 (2015).
[9] D. Panigrahi and S. Chatterjee, J. Cosmol. Astropart. Phys **1605**, 052 (2016).
[10] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B, **511** 265 (2001).
[11] M. C. Bento, O. Bertolami and A. A. Sen, Rev. Phys. D, **66** 043507 (2002).
[12] D. Panigrahi and S. Chatterjee, JCAP **10**, 002 (2011).
[13] M. Makler *et al.*, Phys. Lett. B **555**, 1, (2003).
[14] H. Sandvik *et al.*, Rev. Phys. D **69**, 123524 (2004).
[15] Z. H. Zhu Astron. Astrophys. J. **423**, 421 (2004).
[16] Y. Wang *et al.* Phys. Rev. D, **87** 083503 (2013).
[17] M. R. Setare, Phys. Lett. B **648**, 329 (2007).
[18] M. R. Setare, Int. J. Mod. Phys. D **18**, 419 (2009).
[19] N. Bilic, G. B. Tupper and R. D. Viollier, Phys. Lett. B, **535** 17 (2002).
[20] D. Bazeia, Phys. Rev. D **59**, 085007 (1999).
[21] L. Xu, J. Lu and Y Wang, Eur. Phys. J. C **72**, 1883 (2012).
[22] M. R. Setare, Phys. Lett. B **654**, 1 (2007).
[23] M. R. Setare, Eur. Phys. J. C **52**, 689 (2007).
[24] U. Debnath, A. Banerjee and S. Chakraborty, Class. Quan. Grav. **21**, 5609 (2004).
[25] H. Saadat and B Pourhassan, Astrophys. Space Sci. **344**, 237 (2013).
[26] H. K. Jassal, J. S. Bagla and T. Padmanabhan, Mon. Not. Roy. Astron. Soc. **405**, 2639 (2010).
[27] Z. K. Guo and Y. Z. Zhang, Phys. Lett. B **645**, 326 (2007).
[28] Z. K. Guo and Y. Z. Zhang, arXiv:astro-ph/0509790.
[29] S. W. Allen *et al.*, Mon. Not. R. Astron. Soc. **353**, 457 (2004).
[30] G. R. Bengochea, Phys. Lett. B **695**, 405 (2011).
[31] B. Novosyadlyj *et al.*, Phys. Rev. D **86**, 083008 (2012).
[32] A. V. Astashenok *et al.*, Phys. Lett. B **709**, 396 (2012).
[33] G. Sethi, S. K. Singh and P. Kumar, Int. J. Mod. Phys. D **15** 1089 (2006).
[34] F. C. Santos, M. L. Bedran and V. Soares, Phys. Lett. B **636**, 86 (2006).
[35] H. Saadat, Int. J. Theor. Phys. **51**, 1317 (2012).
[36] I. Brevik and S.D. Odintsov, Phys. Rev. D **65**, 067302 (2002).
[37] I. Brevik *et al.*, Phys. Rev. D **84**, 103508 (2011).
[38] X. H. Zhai *et al.*, arXiv:astro-ph/0511814.
[39] H. Saadat and B. Pourhassan, Astrophys. Space Sci. **343**, 783 (2013).
[40] Y. D. Xu *et al.*, Astrophys. Space Sci. **337**, 493 (2012).
[41] H. Saadat and H. Farahani, Int. J. Theor. Phys. **52**, 1160 (2013).
[42] A. R. Amani and B. Pourhassan, Int. J. Theor. Phys. **52**, 1309 (2013).
[43] B. Pourhassan, Int. J. Mod. Phys. D **22**, 1350061 (2013).
[44] J. Sadeghi and H. Farahani, Astrophys. Space Sci. **347**, 209 (2013).
[45] P. Thakur, Int. J. Phys. **92**, 537 (2018).
[46] L. D. Landau and E. M. Lifschitz, *Statistical Physics* (Butterworth-Heinemann, London, 1984).
[47] F. C. Santos, M. L. Bedran and V Soares, Phys. Lett. B **646**, 215 (2007).
[48] M. L. Berdran and V. Soares, Prog. Theor. Physics **123**, 51 (2010).
[49] E. H. Baffou, M. J. S. Houndjo and I. G. Salako, Int. J. Geo. Met. Mod. Phys. **14**, 1750051 (2017).
[50] F Reif, *Fundamentals of Statistical and Thermal Physics* (Levant Books, Kolkata, 2010).
[51] L. P. Cimento and R. Lazkov, Phys. Rev. Lett. **91**, 211301 (2003).
[52] C. Kaconikhon, B. Gumjudpai and E. N. Saridakis, Phys. Lett. B **695**, 10 (2011).

[53] J. C. Fabris and J. Martin, Phys. Rev. D **55**, 5205 (1997).
[54] Y. S. Myung, Phys. Lett. B **652**, 223 (2007).
[55] D. Panigrahi and S. Chatterjee, J. Cosmol. Astropart. Phys. **2016**, 052 (2016).
[56] J. Lu, Phys. Lett. B **680**, 404 (2009).