SOME ASPECTS OF MODIFIED THEORIES OF GRAVITY AND DARK ENERGY MODELS OF THE UNIVERSE

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By

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Guide

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DECLARATION

I declare that the thesis entitled "**Some Aspects of Modified Theories of Gravity and Dark Energy Models of The Universe**" has been prepared by me under the guidance of **Dr. Bikash Chandra Paul**, Associate Professor, Department of Physics, University of North Bengal. No part of the thesis has formed the basis for the award of any degree or fellowship previously.

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Abstract

In modified theories of gravity cosmological models are obtained which accommodate both early and late universe. We consider a Lagrangian density in the Einstein-Hilbert action which is a polynomial function of scalar curvature (R) . It is known that the modification of the action with higher power of scalar curvature terms that are relevant during the early epoch permit inflation and the curvature inverse term (R^{-1}) that are relevant at the present epoch permit late time acceleration. In the thesis cosmological models are obtained considering terms of the scalar curvature of both the types. As the field equations corresponding to the above action are highly non-linear numerical technique is adopted to understand the evolution of the universe from the beginning to the present epoch. It is found from the numerical analysis that the gravitational theories considered here permit both the early inflation and late time accelerating phase. The models obtained here are new and interesting. The models have predictive power for future evolution which is numerically analyzed. The duration of the late accelerating phase is found to depend on the coupling parameters of the theory.

In cosmology, primordial black holes (in short, PBH) are considered important objects. Primordial black holes are considered as candidates for dark matter also. The probability of creation of a universe with or without primordial black hole pairs are estimated in a general scenario considering a universe with usual four or higher dimensions in the framework of a non-linear theory of gravity. Using a semi-classical approach proposed by Bousso and Hawking and the Hartle-Hawking 'No boundary' proposal, the probability measure is obtained. Gravitational instantons are Euclidean solution of field equations. Using the gravitational instantons we estimate probability for two different topologies: (a) S^{D-1} topology which does not accommodate primordial black hole pair and (b) $S^1 S^{D-2}$ topology which accommodates a pair of primordial black holes. The probability for quantum creation of an inflationary universe with a pair of primordial black holes is then evaluated assuming a gravitational action which is described by a polynomial function of scalar curvature with or without a cosmological constant (Λ). We note a new class of instanton solution in R^4 theory which are relevant for cosmological model building.

Gauss-Bonnet term in the Einstein-Hilbert action is a high energy modification of gravity which generally does not have any dynamical effect in 4-dimensions. However, when coupled with a dilaton field Gauss-Bonnet term may play an important role in the dynamics of the evolution of the universe. In the framework of Einstein Gauss-Bonnet gravity coupled with a dilaton field in four dimensions we obtain cosmological solutions which admit an Emergent Universe (EU). For different dilatonic field coupling parameter of the Gauss-Bonnet terms and the corresponding potentials for the dilatonic field are determined. It is noted that the Gauss- Bonnet terms, coupled with a dilatonic field, might have played a crucial role in the dynamics of the early and late universe. A very interesting case is noted where the Gauss-Bonnet term dominates initially in the asymptotic past regime, decreases subsequently but

dominates the cosmic dynamics again in late time. It is also noted that the Einstein static universe permitted here is unstable and an asymptotic EU might follow.
Mukherjee *et al.* first obtained such EU model in a flat universe with a non-linear equation of state (in short, EOS) $p = A\rho - B\rho^{\frac{1}{2}}$. It consists of two parameters, namely A and B which we call EOS parameters. The allowed range of values of these parameters and additionally of K, which appears as an integration constant, are determined using the Observed Hubble Data ($H(z) - z$) data, BAO (Baryon Acoustic Oscillation) peak parameter and CMB (Cosmic Microwave Background) shift parameter (WMAP7 data).
Recently modified Generalized Chaplygin Gas (in short, MCG) is considered as a prospective candidate for dark energy which will be taken up here. The free parameters of the model namely, A, B and α corresponding to the EOS are determined using the observed data. The allowed range of values of B parameter is estimated considering the Cold Dark Matter model where MCG plays the role of dark energy and Unified Dark Matter Energy model where MCG plays the role of both dark energy and dark matter.

Preface

This thesis is the outcome of my research work as a Ph.D student in the Department of Physics, University of North Bengal, West Bengal, India. The thesis contains 8 chapters. The first chapter is an introduction which reviews some of the earlier works on modified gravity and dark energy models of the universe and serves to describe the motivation of the work. A brief note on the aim of the work as well as a summary is to be found here. Chapter $2-7$ are based on works already reported to journals and published as research articles as indicated in the following.

Chapter 2 is based on the paper:

> "Accelerating Universe in Modified Theories of Gravity ", B. C. Paul, P. S. Debnath, and **S. Ghose**, Phys. Rev. D. **79**, 083534 (2009);

Chapter 3 is based on the paper:

> "Probability for primordial black holes in a multidimensional universe with nonlinear scalar curvature terms", B. C. Paul, A. Saha, and **S. Ghose**, Phys. Rev. D. **78**, 084007 (2008);

Chapter 4 is based on the paper:

> "Emergent Universe Scenario in the Einstein-Gauss-Bonnet Gravity with Dilaton ", B. C. Paul and **S. Ghose**, Gen. Rel. Grav. **42**, 084007 (2010);

Chapter 5 is based on the paper:

> "Constraints on Exotic Matter for An Emergent Universe ", B. C. Paul, P. Thakur and **S. Ghose**, Mon. Not. Roy. Astron. Soc. **407**, 415 (2010);

Chapter 6 is based on the papers:

> "Emergent Universe from A Composition of Matter, Exotic Matter and Dark Energy", B. C. Paul, **S. Ghose** and P. Thakur Mon. Not. Roy. Astron. Soc. **413**, 686 (2011); "Observational Constraints on the Model Parameters of a Class of Emergent Universe ", **S. Ghose**, P. Thakur and B. C. Paul Mon. Not. Roy. Astron. Soc. **421**, 20 (2012);

Chapter 7 is based on the paper:

> "Modified Chaplygin Gas and Constraints on its B parameter from CDM and UDME Cosmological models ", P. Thakur, B. C. Paul and **S. Ghose**, Mon. Not. Roy. Astron. Soc. **397**, 1935 (2009);

Finally, some concluding remarks and future plan find their places in chapter 8.

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Souvik Ghose

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Dedicated to my family.

Chapter 1

Introduction

1.1 Introduction

Modern Cosmology is passing through a remarkable transition from speculative science to experimental one in recent times. This is primarily due to the high precision observations putting tight constraints on the theoretical models at the same time opening new windows to understand the evolution of the universe in a better way. Most of the cosmological models today are based on a theory commonly known as Big-Bang model conjectured by Lemaître back in 1930's. The model, as predicted by Gamow in late 1940's [1], essentially states that our universe began at some finite past in a super-hot and super-dense state. It is the subsequent phase of cosmic expansion which led to adiabatic cooling and consequently at different phases of evolution galaxies, clusters and other cosmic objects were formed. In the early universe, soon after the Big-Bang radiation and matter are considered to be the primary components of a cosmic soup in thermal equilibrium. The photons, decoupled from matter due to the expansion of the universe, are found to follow Planckian distribution. Apart from understanding the cosmic expansion and creation of matter, the Big-Bang theory predicted the existence of a relic radiation which is now known as Cosmic Microwave Background Radiation (in short, CMBR). In 1965, Penzius and Wilson discovered [2] the existence of CMBR while scanning the whole sky for the microwave band of the electromagnetic spectra. The experiments, led by Smoot and Mather, with Cosmic Background Explorer (in short, COBE) satellite in 1990's observed the Planckian nature of the radiation for the first time [3, 4]. The new era of high precision measurements in cosmology started with the COBE experiments. Not only that COBE experiments confirmed the Planckian nature of the CMBR, it also revealed the fact that the radiation is, as expected, completely isotropic on large scale with tiny fluctuation embedded in it. The quantum fluctuations that might have developed during the evolution of the early universe played an important role in the formation of large scale structure of the universe. The detection of cosmic microwave background radiation as relic of the primordial super-hot phase is considered to be one of the observational evidences in favour of the Big-Bang model. However, the assumption that the universe, on large scale, is homogeneous and isotropic raises some obvious questions. There are some very well posed questions confronting observations in Big-Bang cosmology namely, *Horizon problem, Flatness problem, Singularity problem, Small scale inhomogeneity problem etc.* which have no answer in the framework of standard perfect fluid model (for detail see ref. [5, 2, 6, 7]). In order to sovle these problems, a phase of accelerated expansion, i.e. a huge expansion in a very brief period of time, is required in the early universe, which is termed as *inflation.*

In 1981, Guth [8] and Sato [9, 10] independently advocated the idea of inflation in cosmology. The model is known as *old inflation*. The old inflationary scenario is based on the temperature dependent first-order phase transition mechanism. In this scenario the universe, trapped in the false vacuum phase, transits to a true vacuum phase leading to exponential or quasi-exponential expansion. A small causally coherent region grows enormously to encompass the whole universe. Although the model can generate inflation it was found to have several shortcomings. Guth's original idea was that the bubbles of the inflationary universe would have marge giving birth to a mostly homogeneous universe. But if the inflation lasted long enough to solve initial condition problems bubble collisions would have become extremely rare for the background spacetime was expanding too fast [11].

Subsequently, a revised version, known as *new inflation*, was proposed by Linde [12] and Albrecht and Steinhardt [13] independently using slow-roll mechanism to obtain inflationary scenario with second-order phase transition. Although the new inflationary model solved the problems of the *old inflationary* models it was plagued with a fine tuning problem in its initial conditions. In 1983, Linde proposed a new variant of the slow-roll inflation [14] which permitted an inflationary universe for a scalar field (ϕ) in self-interacting potential. In this model the evolution of the universe was described by a scalar field where the field assumed a value other than zero from a chaotic initial distribution of field values, satisfying a limit $\phi > 3M_p$ for sufficient inflation. Earlier models of inflation needed a universe in thermal equilibrium as their starting phase. Later the chaotic model was implemented in a massive scalar field potential [15]. The work of Paul *et al.* [16] on chaotic inflation shows that it can be implemented even in a more general way namely, in anisotropic universe successfully. Further it is found that chaotic model can be implemented in the context of brane world [17, 18] also. The *chaotic inflation* model as mentioned above does not require such initial conditions apriori. Moreover, because in this model inflation could begin in a regime close to Planck density, the fine tuning problem of initial condition does not arise. During the last thirty years a number of inflationary models appeared in the literature in the context of different field theories of particle physics (*e. g.* ref. [6, 7]). Apart from more conventional frameworks inflationary models are also explored in the framework of superstring or supergravity theories (for review see ref. [19]). After thirty years of inflation, although there exists no single satisfactory model of inflation, the excitement in obtaining inflationary models of the early universe still remains. The inflationary universe scenario not only saves Big-Bang model from some of the fundamental problems, it is capable of solving outstanding problems of particle physics also. In addition, it is found that inflation is useful for generating quantum fluctuations in an otherwise homogeneous and isotropic universe as seeds for large scale structure formation [20, 21]. As discussed earlier such tiny fluctuations are detected in CMBR spectra by COBE. Together with modern quantum field theory from particle physics, cosmological models could help us to understand how quantum fluctuation evolved to become something of cosmic scale which can be detected by cosmological observational experiments like COBE. Several other more recent cosmological probes, such as BOOMERANG, MAXIMA, WMAP and now PLANCK satellite are sent to the space to measure CMBR and scanning the whole sky effectively. Experiments like the two degree Field (2dF) galaxy survey and the Sloan Digital Sky Survey (SDSS), among others, have confirmed some of the theoretical predictions of the standard Big-Bang cosmological models with great accuracy. These high precision probes have opened up the possibilities to test the particle physics theories applicable in the very early universe.

The standard model in cosmology is based on General theory of Relativity (GTR). The Einstein's field equation is:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \tag{1.1}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ represents Einstein's tensor and we consider unit where $c = 1$. $R_{\mu\nu}$ and $T_{\mu\nu}$ represent the Ricci tensor and the Energy-Momentum tensor respectively. G represents Newton's Gravitational constant. It is known from observations that our universe at large scale ($l > 100$ Mpc) is isotropic and homogeneous. Geometrically such a spacetime is described by Robertson Walker metric, which is given by:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right] \tag{1.2}$$

where $\kappa = 0, \pm 1$ characterises the curvature of the space at a fixed time. Using the metric (1.2) in Einstein's field eq. (1.1) one obtains from the dynamical equations in cosmology. Time-time component of the field equation is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \tag{1.3}$$

where over dot represents the derivative with respect to time. The conservation equation is

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0 \tag{1.4}$$

where ρ and p represent energy density and pressure respectively. It may be mentioned here that the space-space component is not required as eq. (1.3) and eq. (1.4) give rise to the space-space component. The Hubble parameter is defined as $H(t) = \frac{\dot{a}}{a}$. In terms of Hubble parameter the eq. (1.3) can be written as:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}. \tag{1.5}$$

Thus, for any value of Hubble parameter (H) there is a value of ρ which solves eq. (1.5) for zero spatial curvature (κ). The corresponding energy density is termed as critical density ($\rho_c = \frac{3H^2}{8\pi G}$). Evidently the two parameters that characterise the evolution of homogeneous and isotropic universe (i.e. κ and $a(t)$) are connected by the energy density ρ. Conveniently one can normalise the energy density with respect to the critical density by constructing a density parameter $\Omega = \frac{\rho}{\rho_c}$. If there is a number of matter field that contribute to the total energy density of the universe $\rho = \sum\limits_i \rho_i$ and the corresponding density parameter is $\Omega = \sum\limits_i \Omega_i$ where $\Omega_i = \frac{\rho_i}{\rho_c}$. Eq. (1.5) can be written in the form

$$\Omega_{total} - \frac{\kappa}{H^2 a^2} = 1. \tag{1.6}$$

Contribution of the matter (i.e. slowly moving particles which can fall into local gravitational potential well) can be measured from the observations on the dynamics of galaxies and clusters. This contribution from normal matter and dark matter as predicted by cosmological observations is found to be $\Omega_M = 0.3 \pm 0.1$. However, other cosmological observations on CMBR favour a universe which is nearly spatially flat. For a spatially flat universe one gets $\Omega_{total} = 1$ from eq. (1.6). Thus the component of dark energy contributing to the energy density of the universe is $\Omega_{dark} \approx 0.7$. This energy density must be smoothly distributed throughout space to avoid any influence on local motion of galaxies and clusters. Meanwhile recent observations of type Ia Supernovae predict that the present universe is passing through a phase of accelerated expansion [22]. It is straight forward to observe from the Friedmann equation (eq. (1.5)) that a phase of accelerated expansion can not be obtained with conventional form of perfect fluid. Equation (1.5) can be written as

$$\dot{a}^2 = \frac{8\pi G a^2}{3}\rho - \kappa. \tag{1.7}$$

Where, for matter domination the density of the universe follows $\rho_M \propto a^{-3}$ and for radiation it evolves as $\rho_R \propto a^{-4}$. In both the cases the first term in the right hand side is a decreasing function of time in an expanding universe, it never permits a universe with a phase of acceleration. It is known that such a phase of acceleration may be realised introducing a cosmological constant (Λ) or vacuum energy term which is a constant. A cosmological model with constant Λ permits a universe with $\rho_{vac} \approx 10^{-8} erg/cm^3$. Unfortunately any theoretical estimation of vacuum energy from fundamental physics greatly exceeds this value (in fact it is around 10^{55} orders of magnitude higher, for review see ref. [23]). Further, it remains a paradox why dark energy and matter in the universe are of the same order today ('Coincidence Problem'). There are two alternative ways to address these issues. (i) The dark energy density may not be strictly constant but a slowly varying fuction of time, mimicking a cosmological constant at present era. A number of literature came up following the above concept (for review on dark energy see e.g. [24, 25]). (ii) General theory of may be itself generalised to accommodate observed features of the universe. This is however not an easy task. There are many different proposals to be found in the contemporary literature. Some typical examples are DGP (Dvali- Gabadadze-

Porrati) gravity [26], Brane world gravity [27], Tensor Vector Scalar gravity [28] etc. In the present thesis modification of gravity achieved by straightforward modification of Einstein-Hilbert action is considered. The Einstein-Hilbert action, in presence of matter fields, is given by:

$$S_{EH} = \int \left[\frac{1}{2\kappa} R + \mathcal{L}_M \right] \sqrt{-g}\; d^4x \tag{1.8}$$

where $\kappa = 8\pi G c^{-4}$, G is Newton's Gravitational constant, R is the scalar Curvature and $\mathcal{L}_M$ is the Lagrangian density due to the matter fields of the theory. The modifications are made in the gravitational sector of the Einstein-Hilbert action by including a polynomial function of scalar curvature (R), for a more generalised theory of gravity. Within the modified gravity sector there have been proposals for inclusion of higher dimensions [26, 29, 30, 31, 32, 33]. It may be mentioned here that quadratic or higher order terms in the Riemann curvature tensor and its traces appear in the low energy limit of superstrings [34], as well as when the usual perturbation expansion is applied to GTR [35]. It is known further that with suitable counter terms, namely $C_{\mu\nu\rho\delta}C^{\mu\nu\rho\delta}$, R^2, Λ added to the Einstein action one obtains a perturbation theory which is well behaved, formally renormalisable and asymptotically free [36]. The higher derivative correction to the gravitational action is important since the renormalisation of higher loop contributions introduces terms to the effective action that are higher than quadratic in the Riemann curvature scalar [37]. The Einstein Hilbert action, therefore, can be modified by a Lagrangian which is a polynomial function of the scalar curvature (R) (e.g. $\mathcal{L} = \sum_{i=1}^{\infty} \lambda_i R^i$) [38, 39, 40, 41, 42] (for other ideas see ref. [43, 44]). Starobinsky [45] obtained inflation in a modified theory of gravity taking R^2 term in the Einstein-Hilbert action. However the efficacy of the inflationary universe scenario was known only after the seminal work of Guth [8]. It has been shown [46] in a quadratic Lagrangian gravity in 4-dimensions that it admits gravitational instantons which are singular and non-singular. The gravitational instantons are the Euclidean solutions of the relativistic field equations with finite action. It is known that gravitational instantons are important to understand the creation of the universe at the Planck era. Paul *et al.* [46] noted the existence of both singular Hawking-Turok instanton and non-singular de'Sitter instanton in a modified theory of gravity described by quadratic terms of the Ricci scalar in the Einstein's gravitational action. These are relevant for investigating early evolution of the universe. It is also known that the present accelerating phase may be realised by employing a modified theory gravity which includes $\frac{1}{R}$ term that is relevant at the present epoch [40]. Thereafter, non-linear Lagrangian (e.g. upto R^4 term in the action) is considered to study different issues in cosmology [47]. Such a nonlinear theory arises for (low energy) approximation to a more fundamental theory, which may bring out new possibilities for extracting the observed cosmic acceleration. Unfortunately a satisfactory theory with the modified theories of gravity is yet to come. In the present thesis modified gravitational theories will be considered to investigate evolution of both the early and late universe. Within the framework of modified gravity a combination of higher order terms namely, R^2, $R^{\mu\nu}R_{\mu\nu}$ and $R^{\mu\nu\gamma\delta}R_{\mu\nu\gamma\delta}$ is important also. These terms in definite combinations appeared in the literature as Gauss-Bonnet terms (for details see [48]). Gravitational theories with $f(G)$ as the Lagrangian for the Einstein-Hilbert action where $G \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\gamma\delta}R_{\mu\nu\gamma\delta}$ (Gauss-Bonnet term), are useful also for cosmological model building. Modification of Einstein-Hilbert action with GB terms may be useful for describing dark energy. It may be mentioned here that the Gauss-Bonnet term is a topological invariant in four dimensions. A modified theory of gravity including Gauss-Bonnet terms is motivated from effective low energy string theory action. The GB combination is generally taken in the above form as it leads to ghost free gravitation propagator near flat space [49]. Cosmological models with Gauss-Bonnet terms have been shown to pass solar system tests [50]. It may be pointed out here that for a dynamical contribution of the GB terms in the 4 dimensions one has to consider dilaton field coupled to it. In the framework of the Einstein-Hilbert action, one may conformally rescale a 4-dimensional metric which converts the dilaton into a self interacting scalar field, minimally coupled to gravity and the ground state becomes stable against its fluctuations. It has been shown [48] that a viable 4-dimensional cosmological model can be realised with an action in which R and the GB terms have opposite signs, contrary to what one obtains in the small slope expansion of the string theory. However, with appropriate compactification, one can recover at a large time usual Einstein equations. The calculations showed the rich structure of the theories with quadratic curvature terms which may be useful for model building.

Considerable work has been done during the last few decades to understand the quantum properties of gravity. The investigation seems to lead to a general belief that a consistent theory of quantum gravity cannot be obtained within the framework of point field theories. The advent of string theory has opened up new and interesting possibilities in this context. Superstring theory is considered to be a promising candidate for a consistent theory of quantum gravity. It needs dimensions more than the usual 4 dimensions for its consistency. Therefore, it is also interesting to look for cosmologies with dimensions more than usual 4 dimensions. Consequently higher dimensional theories are important to investigate cosmological issues. During inflation quantum fluctuations may produce primordial black holes (PBH) copiously. The mass of PBHs spans an extraordinarily large range. PBHs formed at Planck time ($10^{-43}s$) would be of Planck Mass ($10^{-5}g$) where as those formed at $1s$ would be as huge as $10^5 M_\odot$. PBHs of mass $M \approx 10^{15}g$ would not have evaporated by now. These are hot and might give signature for Hawking radiation. Some of the PBHs may evaporate out and contribute in the structure formation of the universe. Therefore, PBH are important objects in cosmology which may be useful as a candidate for dark energy leading to $\Omega \approx 1$ in 4 or higher dimensions.

1.2 Aim of the Work

The objective of the proposed research work is to study some theoretical and observational aspects of cosmological models. We intend to investigate modified gravity and dark energy models to accommodate both the early and late universe scenario. Apart from purely theoretical issues some relevant cosmological models are studied in the light of the recent observational data where we obtain constraints on the free parameters of the models. Specifically, the following problems are to be investigated:

I. Models of accelerating universe in $f(R)$ gravity theory.

II. Estimation of Primordial Black Hole (PBH) formation probability in higher dimensional $f(R)$ gravity.

III. Emergent Universe scenario in Gauss-Bonnet gravity with Dilaton.

IV. Observational Constraints on equation of state parameters for Emergent Universe.

V. Observational viability of different Emergent Universe models.

VI. Modified Generalized Chaplygin Gas and its observational constraints for cosmologies.

1.3 Summary of Work

Cahpter 1: Introduction

Brief review of the earlier work is discussed along with aims and objective of the thesis.

Chapter 2: Models of accelerating universe in $f(R)$ gravity theory.

A class of cosmological models are obtained in modified theories of gravity where the gravitational action is described by a Lagrangian density $f(R)$ which is a polynomial function of scalar curvature (R). The field equations obtained corresponding to the modified gravitational action are highly non linear and cannot be solved analytically. Consequently numerical techniques are adopted for solving them. We explore cosmological models which describe early inflationary phase and late time accelerating universe. The solution obtained here are new and interesting for cosmological model building. It is noted that the models permit late accelerating phase of the universe, duration of which is found to depend on the coupling parameters of the theory. The main objective of this work is to investigate if cosmological models permitted in $f(R)$ theories of gravity can accommodate features of early and late time universe fairly well without exotic matter in the theory.

Chapter 3: Estimation of Primordial Black Hole (PBH) formation probability in higher dimensional $f(R)$ gravity.

Primordial Black Holes (PBH) may be a useful candidate for dark matter. It is important to estimate the probability of creation of PBH pairs in the early universe. Bousso and Hawking(BH) [51] developed a method to estimate the probability of creation of universe with or without PBH pairs in presence of a cosmological constant. In this chapter using BH technique we obtain more general scenario in a higher dimensional universe in the framework of modified gravity described by nonlinear scalar curvature terms in Einstein-Hilbert action. We use semi-classical approach as suggested by BH in addition to the Hartle-Hawking 'No boundary' proposal [52] to investigate the creation probability of universe which may or may not contain PBH pair. Gravitational instantons are the Euclidean solution of the field equations. Using gravitational instantons obtained in two different topologies: namely, (a) S^{D-1} - topology and (b) $S^1 \times S^{D-2}$-topology we explore universe with or without a pair of PBH. The probability for quantum creation of an inflationary universe with a pair of PBH is then evaluated assuming a gravitational action which is described by a polynomial function of scalar curvature with or without a cosmological constant (Λ). The main objective of the work is to study if in a more generalised gravitational action, BH approach can be used to evaluate the probability of creation of universe with or without PBH pair.

Chapter 4: Emergent Universe scenario in Gauss-Bonnet gravity with Dilaton.

Gauss-Bonnet term in the Einstein-Hilbert action is a high energy modification of gravity which generally does not have any dynamical effect if we are confined in 4-dimensional theories. However, when GB term is coupled with a dilatonic field it might effect the dynamics of the evolution of the universe considerably. In this chapter we obtain cosmological solutions which admits Emergent Universe (EU) in the framework of Einstein Gauss-Bonnet gravity coupled with a dilaton field in four dimensions. The coupling parameter of the Gauss-Bonnet terms and the dilaton suitable for creation of EU are determined. Also the corresponding dilatonic potentials are studied. Interestingly, our study reveals that the Gauss- Bonnet (GB) terms coupled with a dilatonic field play a crucial role in describing the dynamics of the evolution of the early as well as the late universe. A very notable case is where the GB term dominates initially in the asymptotic past regime, decreases subsequently but dominates the cosmic dynamics again in late time. We also note that the Einstein static universe solution permitted here is unstable and an asymptotic EU might follow. The kind of EU considered here was first proposed by Mukherjee *et al.* [53] in the framework of GR but with exotic matter. Subsequently it was developed in many other framework. Our objective in this work is to investigate if the scenario which has many virtue on its own can be developed in a modified Einstein-Gauss-Bonnet gravity framework in presence of a dilatonic field.

Chapter 5: Observational Constraints on equation of state parameters for Emergent Universe.

Emergent Universe (EU) model proposed by Mukhrjee *et. al.* can successfully incorporate an early inflationary epoch as well as late time accelerating universe. However, the equation of state (EOS)($p = A\rho - B\rho^{1/2}$) required for realising the model is nonlinear and described by two parameters, namely A and B. The objective of this work [54] is to investigate the permitted values of the model parameters from the Observed Hubble Data ($H(z) - z$) data, measurement of a model independent BAO peak parameter (SDSS) and CMB shift parameter (WMAP7 data). It is found that although the value of the parameter A can be very close to zero, most of the observations favours a small and negative A. As a consequence, the effective Equation of State parameter for this class of Emergent Universe solutions remains negative always. We also compared the magnitude ($\mu(z)$) vs. redshift(z) curve obtained in the model with that obtained from the union compilation data [22]. According to the analysis the class of Emergent Universe solutions considered here cannot be ruled out by the present observations.

Chapter 6: Observational viability of different Emergent Universe models.

The motivation for studying EU scenario in flat four dimensional universe has already been stated. In the earlier chapters it is shown that recent observations put constraints on the model parameters of EU. However, strictly speaking, EU represents a class of solutions depending on the choice of a free parameter of the model (A). Since the model includes three independent free parameters we can in principle chose one for a certain model and still work with the other two. In this chapter different EU models are studied in the light of recent observational data. We note that some of the EU models permitted do not confront with observations. Thus from the analysis it is evident that the case $A = -\frac{1}{3}$ is ruled out with good confidence from the data available.

Chapter 7: Modified Generalized Chaplygin Gas and its observational constraints for cosmologies.

Modified Generalized Chaplygin Gas (MCG) as a prospective candidate for dark energy is taken up to construct cosmological models in this chapter. We determine the limiting values for the parameters of the model from recent observational data. The free parameters of the model are the ones coming from the equation of state of MCG ($p = B\rho - \frac{A}{\rho^{\alpha}}$), viz. B, A and α. The permitted values of these parameters are determined with the help of dimensionless age parameter ($H_o t_o$) or lookback time and ($H(z) - z$) data. More,specifically the objective of the work is to investigate the allowed ranges of values of B parameter in terms of α and A_s (A_s is defined in terms of the constants in the theory). Two popular models in cosmology, namely the Cold Dark Matter(CDM) model and Unified Dark Matter Energy model (UDME) are considered here.

Chapter 2

Models of accelerating universe in $f(R)$ gravity theory.

2.1 Introduction

The fact that we live in not only an expanding universe but also in one which is presently passing through an accelerated phase of expansion now seems undeniable. This was at first directly inferred from the study of light curve of hundreds of type Ia supernovae (SnIa) [22, 55] and was independently established from observations of cosmic microwave background (CMB) by the Wilkinson Microwave Anisotropy Probe (WMAP) [56, 57, 58, 59] and other astronomical observations on CMBR [60, 61] and large scale structures [62, 63, 64]. Since the advent of these precise measurements, cosmology has lost its earlier speculative status to become a true science where most of the theories can be verified by experiments. Current observations look particularly favourable towards what is known as Big-Bang cosmological model where universe starts in a super-hot and super-dense state followed by cosmic expansion leading to adiabatic cooling. However, this cosmic expansion is not the only signature of Big-Bang model since the model also predicts baryonic matter content in the universe and a relic radiation in microwave region from its hot phase. These signatures have been observed. In modern cosmology, a modification to the Big-Bang model has been proposed accommodating an inflationary phase at very early epoch that would render the geometry of the universe to essentially a flat one [8] and also give birth to tiny fluctuation to the temperature of CMB. Observationally both these signatures seem viable. As on today every cosmological model is based on the same theory of gravitation namely Einstein's General theory of Relativity (GTR) and according to this theory the average energy density of the universe determines the fate of the universe. A spatially flat universe corresponds to an equivalent matter density $\rho_c \sim 8 \times 10^{-27} kg.m^{-3}$ [65] known as critical density of the universe. An average density slightly more or less than this critical density would have made the universe either open or closed [5]. The cosmological observations when analyzed in the framework of Big-Bang model suggest that 4% matter in the universe constitutes baryonic matter. In order to describe the kinematics of the galaxies an additional matter component, commonly known as dark matter,is required to be introduced. However, the addition of this mysterious dark matter to the baryonic matter estimates about 27% of the total density of the universe. So, in order to construct a flat universe model accommodating an accelerating phase of expansion at present, it is necessary to take up further contribution for making $\Omega = 1$ known as dark energy. This is not a trivial task since in order to incorporate the cosmic acceleration within a GR based model a mysterious cosmic fluid with negative pressure must be evoked [66]. The simplest way to introduce such a candidate is to make provision for a new term called cosmological constant in the theory of gravitation [23, 67, 68, 69]. This is far from the remedy of the problem since the smallest possible estimate for the value of the cosmological constant is still about 55 orders of magnitude larger than the value required for a viable model [66]. This is one of the prime motivation of looking for an alternative theory [66, 39, 70, 71, 72, 73] to address the present observational issues. It, is worthwhile to mention here that none of these alternative ideas has been very successful and some of them have serious issues to be resolved. Consequently the idea that this cosmic speed up is not due to some extraordinary stuff like dark energy but can be due to a modification of gravitational sector has also been taken up [74, 75, 76]. The Einstein's field equation governing the dynamics of the universe in GTR can be obtained from Einstein-Hilbert action. The Einstein-Hilbert action for GR contains the Lagrangian density $\sqrt{-g}R$. Most natural choice is to add terms that are proportional to $\sqrt{-g}R^n$ where $n > 1$ would suggest a modification of the theory at an early epoch where curvature was high (e.g. Starobinsky inflation [45]). A low curvature modification of the theory taking into account terms that are relevant at late times are required to accommodate cosmic acceleration [41]. For this modification, terms with $n < 0$ are to be added to the action and the simplest of choices is the $\frac{1}{R}$-term [41, 40]. However, it was soon realized that a theory with $\frac{1}{R}$-term in the Einstein-Hilbert action inescapably leads to instabilities [42, 77, 78]. It was then suggested that further addition of a R^2- term [79, 80, 81, 82, 83] or even a $ln(R)$ term [42] to the action might lead to a consistent modified theory of gravity which not only would pass solar system tests but also free from instability problem. It thus makes sense to incorporate both the positive and negative powers of curvature scalar into the action in order to construct a model capable of accommodating both an early inflationary phase and a late time acceleration. A polynomial function of the form $f(R) = R + \alpha R^m + \beta \frac{1}{R^n}$ is useful. In the large curvature region, the term R^m dominates and permits power law inflation if $1 < m \leq 2$. Of late modified gravity with polynomial of R in the Einstein-Hilbert action, i.e., $f(R)$ -gravity has been thoroughly examined. It is now known that a large class of models including R^n-model do not permit a matter dominated phase. Capozziello *et al.* [82] criticized the claim made in [84, 85]. Tsujikawa [86] suggested some observational signature of $f(R)$ models which satisfy both cosmological and local gravity constraints fairly well. In some cases [87] does seem consistent with realistic cosmological model. However, till date there is no physical criteria known to select a particular kind of theory capable of matching the data at all scales and most of the $f(R)$ gravity models, including the one under consideration here, are toy models. The motivation of the present chapter is to investigate cosmological solutions considering the above functions in R in the Einstein-Hibert action. We explore the early inflationary phase as well as the present (late time) phase of acceleration within the $f(R)$-gravity. The field equations obtained here are highly non-linear and too complicated to obtain analytic solutions. To solve we adopt a numerical technique first adopted in [88] and more recently in [89]. The plan of the present work is as follows: in section 2.2, the relevant field equations in the modified theory of gravity will be set up, in section 2.3, cosmological evolutions are predicted in different models depending upon the coupling parameters of the gravitational action adopting numerical technique. Finally in section 2.4, we summarize and discuss the results obtained.

2.2 The $f(R)$ model of the universe

Let us consider a gravitational action containing a polynomial function $f(R)$ in scalar curvature (R) (in place of R in Einstein-Hilbert action (1.8)) which is given by:

$$I = -\int \left[\frac{1}{2}f(R) + L_m\right]\sqrt{-g}\; d^4x \tag{2.1}$$

where we chose natural unit i.e., $8\pi G = c = 1$, g is the determinant of the four dimensional metric tensor. Here $f(R)$ is a function of R and its higher power and L_m represents the matter Lagrangian. The above action (2.1) may be varied with respect to the metric which yields:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^c_{\mu\nu} + T^M_{\mu\nu} \tag{2.2}$$

where $T^M_{\mu\nu}$ represents the energy-momentum tensor scaled by a factor of $\frac{1}{f'(R)}$ and $T^c_{\mu\nu}$ denotes the contribution from the curvature to the effective stress-energy tensor. $T^c_{\mu\nu}$ is given by:

$$T^c_{\mu\nu} = \frac{1}{f'(R)}\left[\frac{1}{2}g_{\mu\nu}(f(R) - Rf^{'}(R)) + f^{'}(R)^{;\alpha\beta}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu}g_{\alpha\beta})\right], \tag{2.3}$$

where prime denotes the derivative with respect to R. During this work we would concentrate on the cosmological evolution driven by the geometry (i.e. the underlying theory of gravity) alone. Consequently, we set $L_m = 0$ which leads to $T^M_{\mu\nu} = 0$ in the subsequent sections. We consider a flat Robertson-Walker spacetime which is given by

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right] \tag{2.4}$$

where $a(t)$ is the scale factor of the universe. Using the metric (2.4) in the field eq. (2.2) (see also Ref. [24]) we get

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{f'}\left[\frac{1}{2}(f - Rf') - 3\frac{\dot{a}}{a}\dot{R}f''\right], \tag{2.5}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{f'}\left[2\frac{\dot{a}}{a}\dot{R}f'' + \ddot{R}f'' + \dot{R}^2 f''' - \frac{1}{2}(f - Rf')\right], \tag{2.6}$$

where an over dot indicates derivative with respect to the cosmic time t. The scalar curvature corresponding to the metric (2.4) is given by

$$R = -\,6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right). \tag{2.7}$$

The Ricci scalar R involves second order time derivative of the scale factor a. One might note that eq. (2.6) contains second order time derivative of curvature scalar ($\ddot{R}$). However, the curvature scalar itself contains double derivative of the scale factor with respect to time. Thus the field equations are in fact fourth order differential equations of the scale factor.

2.3 Numerical Analysis and Cosmological Solutions

From eqs. (2.5) and (2.6) we get:

$$\dot{H} = \frac{1}{2f'}\left[(H\dot{R} - \ddot{R})f'' - \dot{R}^2 f'''\right], \tag{2.8}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Here both R and H are functions of the scale factor (a) and its derivatives. Consequently, eq. (2.8) is a fourth order differential equation in a which is a highly non linear equation. For a given $f(R)$ theory the above differential equation can be solved. We adopt numerical technique to solve this equation as analytic solution in simple form can not be obtained. A number of $f(R)$ gravity models are available, out of which we consider the following:

I. $f(R) = R + \alpha R^2 - \frac{\mu^4}{R}$

II. $f(R) = R + b\, ln(R)$

III. $f(R) = R + m\, e^{[-nR]}$

2.3.1 Case I.

For a choice of $f(R) = R + \alpha R^2 - \frac{\mu^4}{R}$ the field equation (2.8) becomes

$$\dot{H} = \frac{1}{1 + 2\alpha R + \frac{\mu^4}{R^2}} \left[\frac{\mu^4}{R^2} \left(\frac{\ddot{R}}{R} - \frac{H\dot{R}}{R} - 3\frac{\dot{R}^2}{R^2} \right) + \alpha(H\dot{R} - \ddot{R}) \right]. \tag{2.9}$$

The above equation is, as noted earlier, a highly non-linear one. However, for different epochs of cosmic evolution it is possible to obtain asymptotic solutions.

(i) An exponential solution is permitted when $q = -1 : a(t) \sim e^{H_o t}$ in the early era.

(ii) A power law solution is permitted $a(t) \sim t^2$ which admits $q = -\frac{1}{2}$ at a late epoch. The deceleration parameter (q) is defined by

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1, \tag{2.10}$$

The deceleration parameter (q) is a function of H and its derivative and consequently q is a function of forth order time derivative of the scale factor. Thus, we can re-write the eq. (2.9) in form of a second order differential equation in q and H. Since

q contains terms with $\ddot{a}$, one can replace terms with fourth order derivative of the scale factor in eq. (2.9) by $\ddot{q}[H]$. Note tha the functions q and H are not independent. Using eq. (2.10) We get the following non-linear differential equation

$$q^{''} + \Pi(q,H)q^{'2} + \Theta(q,H)q' + \Psi(q,H) = 0 \tag{2.11}$$

where

$$\Pi(q,H) = -\frac{(2q+4)\mu^4 + 216\alpha(q-1)^4H^6}{(q^2-1)[\mu^4 - 216\alpha(q-1)^3H^6]},$$
$$\Theta(q,H) = -\frac{(4q+7)\mu^4 + 216\alpha(8q+5)(q-1)^3H^6}{(q+1)H[\mu^4 - 216\alpha(q-1)^3H^6]},$$
$$\Psi(q,H) = \frac{(q-1)[3\mu^4(2q+1) + 1296\alpha(q+1)(q-1)^3H^6 - 36(q-1)^2H^4]}{(q+1)H^2[\mu^4 - 216\alpha(q-1)^3H^6]},$$

where the prime indicates a differentiation with respect to H and the functions Π, Θ and Ψ depend on α and μ^4. The above equation, then, is a non-linear second order differential equation in q. Both q and H are time dependent and obtaining a known functional form as an analytic solution of the above equation is not possible. Therefore, we resort to numerical methods as suggested in Ref. [88]. Note that $\frac{1}{H}$ is a measure of the age of the universe with H being a monotonically decreasing function of time. One can use eq. (2.11) for a qualitative study of the evolution of the universe in terms of the deceleration parameter q. However, eq. (2.11) is a second order differential equation and for its numerical solution it is necessary to assume two initial conditions. We choose units so that H_o, the present value of H, is unity and pick up sets of values of q and q' for $H = 1$ (i.e. the present values). It is worthwhile to mention that this choice is not arbitrary but has to be consistent with observations such as *Ref.* [23]. We now plot q with H for different configuration of the system. Since the inverse of Hubble parameter (H) gives an estimate for the cosmic age, the region $H < 1$ describes the future evolution of the universe and $H > 1$ describes the past. The present universe is passing through a phase of accelerated expansion and to accommodate this fact we use a negative q at the present epoch i.e., $H = H_o = 1$ and also note that the universe has entered into this accelerated phase of expansion only in the recent past. The model would also predict the future course of the evolution of the universe in the above framework. We note the following points:

(i) In figure (2.1), q is plotted for different H with a given value of α and μ. The inverse of Hubble parameter $\frac{1}{H}$ being a measure of cosmic time, it is easily observed here that with an increase in H we are actually looking back in time. It is evident that the universe enters into the present accelerating phase (negative q) in the recent past. The rate of acceleration will increase further and subsequently it will attain a maximum at future epoch. Consequently there will be slowing down of the cosmic acceleration leading to an epoch when the universe expands without acceleration (which is transient) followed by another phase of accelerated expansion. For given values of $\mu^4 = 12$ and $\alpha = 2$, a sign flip for q at $H = 1.36$ is noted and it is also evident that in this case the universe never transits from accelerating phase to decelerating phase.

(ii) In figure (2.2), q vs H curve is plotted for a given set of values of α and μ but for different initial values of $q'[1]$ with $q[1] = -0.05$, when $\mu^4 = 12$ and $\alpha = 2$. It is observed that the universe entered into the accelerating phase in the recent past followed by another phase of deceleration. The duration of these phases are found to depend on the choice of $q'[1]$,and the duration increases with increasing initial values of $q'[1]$. We note the following critical points for transition from decelerating phase to

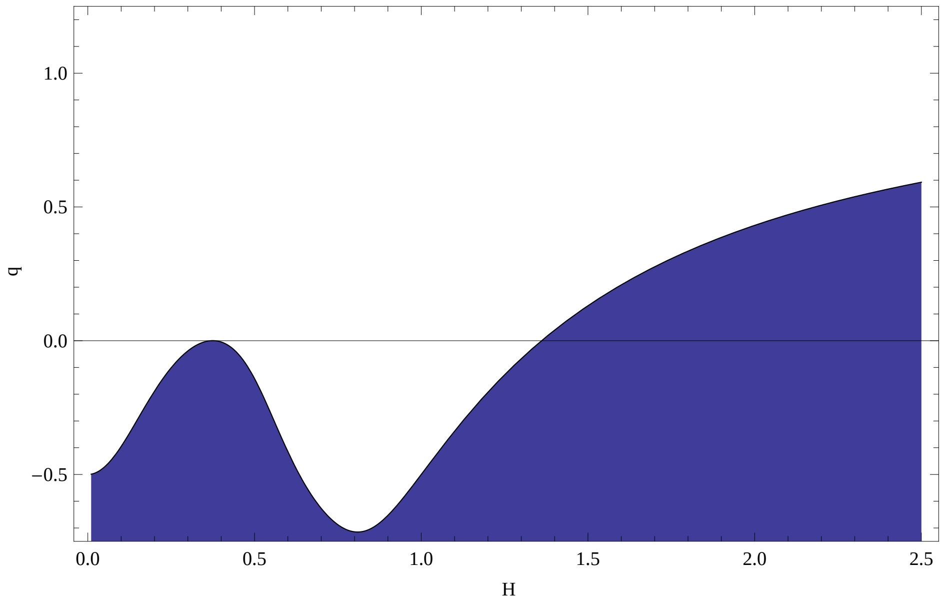


Figure 2.1: q with H plot for $\mu^4 = 12$ and $\alpha = 2$. Initial conditions are chosen as: q[1]= -0.5, q'[1]=1.655.

accelerating phase and thereafter accelerating to decelerating phase : (a) for $q'[1] = 1$: $H = 1.43$ and $H = 0.587$; (b) for $q'[1] = 1.2$: $H = 1.4$ and $H = 0.55$; (c) for $q'[1] = 1.5$: $H = 1.38$ and $H = 0.48$.

(iii) In figure (2.3), q vs. H is plotted for different values of α for $q[1] = -0.5$, $q'[1] = 1.2$ and $\mu^4 = 12$. It is observed that an increase in α reduces the duration of present accelerating phase, the universe will enter into decelerating phase with an increase in deceleration rate. The critical points are tabulated in table (2.1)

α	Critical Points $(q,\ H)$
0.815	(0,0.43)
2	(0,0.225),(0,0.547)
20	(0,0.072),(0,0.581)
200	(0,0.018),(0,0.581)

Table 2.1: Critical points in $q - H$ curve for different α

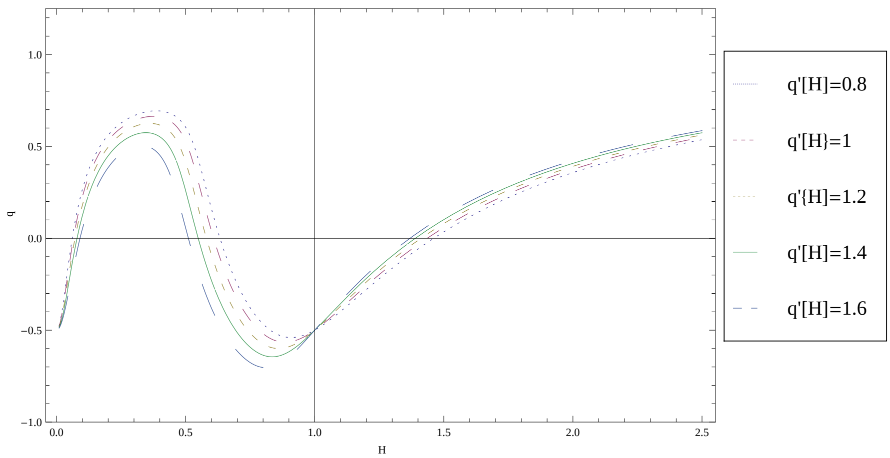


Figure 2.2: shows the plot of q with H for different initial conditions $q'[1] = 1$, $q'[1] = 1.2$, $q'[1]$= 1.5 with $q[1]$= -0.5.

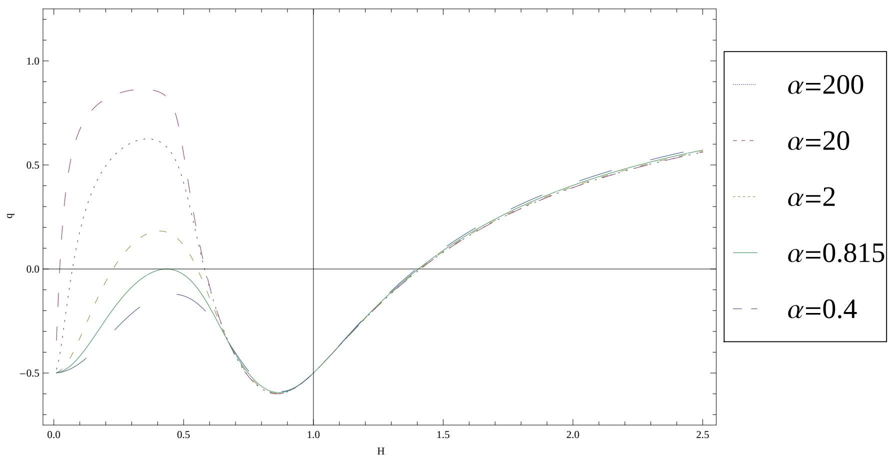


Figure 2.3: Plot of q with H for different value of α. Here we take $\mu^4 = 12$ and choose the initial conditions to be $q[1]$= -0.5, $q'[1]$=1.2.

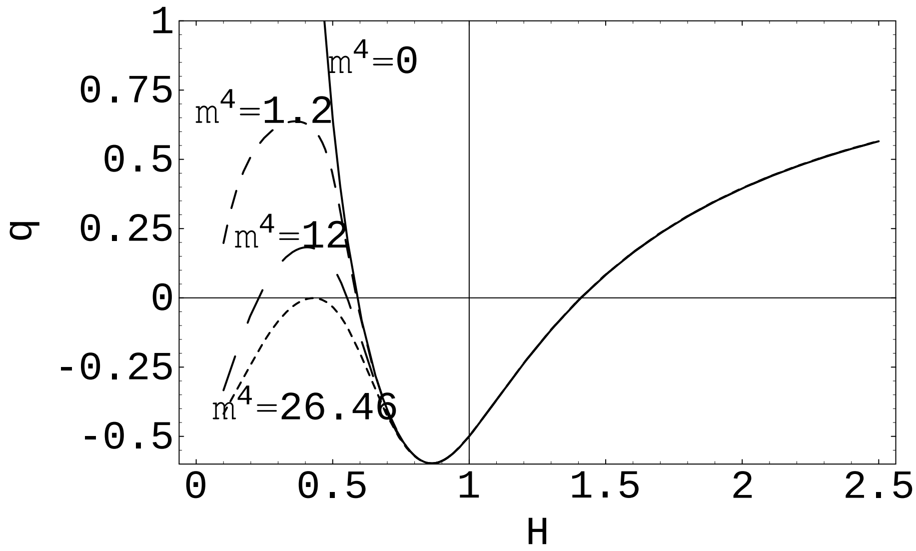


Figure 2.4: Plot of q with H for different values of μ^4 (shown by m^4) with $\alpha = 2$ and initial conditions: $q[1]$= -0.5, $q'[1]$=1.2.

(iv) In figure (2.4), q vrs. H is plotted for different values of coupling constant μ^4 with $\alpha = 2$. It is evident that in the absence of inverse Ricci scalar term in the action a universe is permitted which transits from a decelerating phase to an accelerating phase in the recent past ($H = 1.4$) allowing a further sign flip in q leading to a transition from accelerating to decelerating phase only. However, for $\mu^4 \neq 0$, an interesting evolutionary behaviour of a universe with three sign flips of q is found leading to a universe from accelerating to decelerating followed by a phase decelerating to accelerating and in future deceleration to acceleration. It is noted that as the values of μ^4 are increased, the duration of the present accelerating phase is found to increase for $\mu^4 > 26.46$ and the universe remains in the accelerating phase. However, the rate of acceleration changes with μ. The critical values are tabulated in Table. (2.2).

μ^4	Critical Points (q , H)
0.0	(0,0.587)
1.2	(0,0.587)
12	(0,0.227),(0,0.551)
26.46	(0,0.421)

Table 2.2: Critical points in $q - H$ curve for different μ

2.3.2 A Special Case

We consider here $\alpha = 0$ i.e., $f(R) = R - \frac{\mu^4}{R}$. Eq. (2.11) now takes the form

$$q'' - \frac{2q+4}{q^2-1}\, q'^2 - \frac{(4q+7)}{(q+1)H}\, q' - \frac{3(q-1)(2q+1)}{(q+1)H^2} + \frac{36(q-1)^3 H^2}{\mu^4(q+1)} = 0. \qquad (2.12)$$

This case was considered earlier by Das *et al.* [88]. However, the equation obtained in Ref. ([88]) is not correct. The plot of $q\ vrs.\ H$ is shown in (figure (2.5)). It is noted that a universe in the past might have started from a constant decelerating phase which transits to an accelerating phase thereafter. Once again, because of sign flip in q, it may transit to a decelerating phase. For $\mu^4 = 0.01$, it is noted that in future the universe might attain a constant decelerating phase. Thereafter, it re-enter into an accelerating phase. As μ^4 is increased the duration of the present accelerating phase is found to increase. However, there are epochs when it admits $q = 1$ before and after the present phase of acceleration. In all the cases the universe will end up with an acceleration having $q = -0.5$. We tabulated the critical values in Table (2.5).

2.3.3 Case II.

In this case we consider another form of higher derivative theory much discussed in the literature in recent times. The action contains a logarithmic function in the scalar

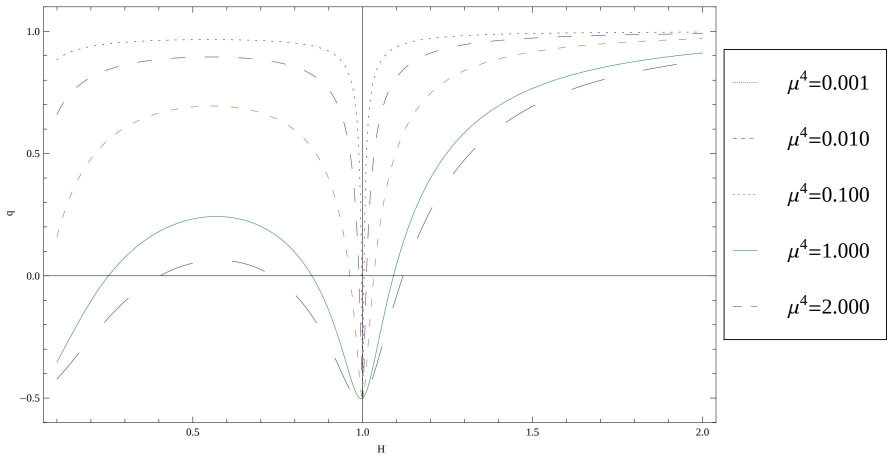


Figure 2.5: Plot of q versus H for $f(R) = R - \frac{\mu^4}{R}$ for different value of μ^4. Initial conditions:$q[1] = -0.5$, $q'[1] = 1.2$.

μ^4	Critical Points (q, H)
0.001	(0,0.996),(0,1.004)
0.010	(0,0.024),(0,0.987),(0,1.01)
0.100	(0,0.073),(0,0.958),(0,1.03)
1.000	(0,0.256),(0,0.849),(0,1.09)
2.000	(0,0.453),(0,0.261),(0,1.110)

Table 2.3: Critical points

curvature which is given by: $f(R) = R + b\, ln(R)$, may be relevant for a physically viable cosmological model. The relevant differential equation obtained from the field equation is given by

$$q'' - \frac{q+3}{q^2-1}\, q'^2 - \frac{3}{(q+1)H}\, q' + \frac{2(q-1)^2}{(q+1)}\left[\frac{6}{b} - \frac{1}{H^2}\right] = 0. \qquad (2.13)$$

We are interested to explore the evolution of universe for different values of the coupling parameter b in the action. For this purpose the curves drawn in figure (2.6). The universe enters an accelerating phase only in the recent past and seems to re-enter a decelerating phase once again in future. For $b < 0$ the duration of

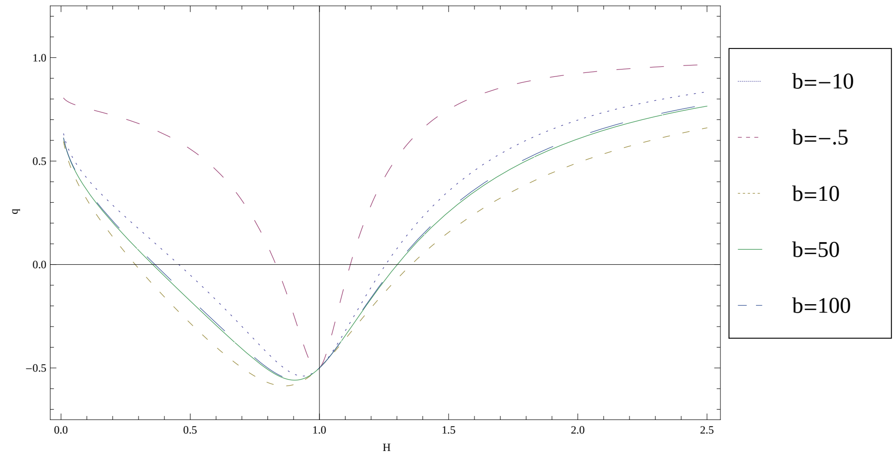


Figure 2.6: Plot of q versus H for $f(R) = R + b\ lnR$ for different value of b with $q[1] = -0.5$, $q'[1] = 1$.

accelerating phase is considerably shorter than that for $b > 0$. However, for b positive the duration of the accelerating phase will be longer if b is smaller. In the case of negative b, the universe is found to land up at a maximum possible acceleration at the present epoch, thereafter the rate of acceleration will decrease and consequently will transit to decelerating phase once again. For $b > 0$, the maximum rate of expansion is to be achieved in near future. The values for the critical points are listed in Table.(2.4)

b	Critical Points (q, H)
-0.2	(0,0.895),(0,1.08)
6.0	(0,0.241),(0,1.52)
100	(0,0.37),(0,1.3)

Table 2.4: Critical points for different b values

2.3.4 Case III.

In this case we consider gravitational action which includes an exponential function of scalar curvature : $f(R) = R + m\ e^{-nR}$. Consequently, from eq. (2.8), we obtain the following differential equation

$$q'' + \frac{1 - 6n(q+1)H^2}{q+1} q'^2 + \frac{8q + 5 - 24n(q^2-1)H^2}{(q+1)H} q' \qquad (2.14)$$
$$+ \frac{2(q-1)(3q+4)}{(q+1)H^2} - 24n(q-1)^2 - \frac{1}{3n^2 m(q+1)H^4}\left[e^{nR} - n\ m\right] = 0.$$

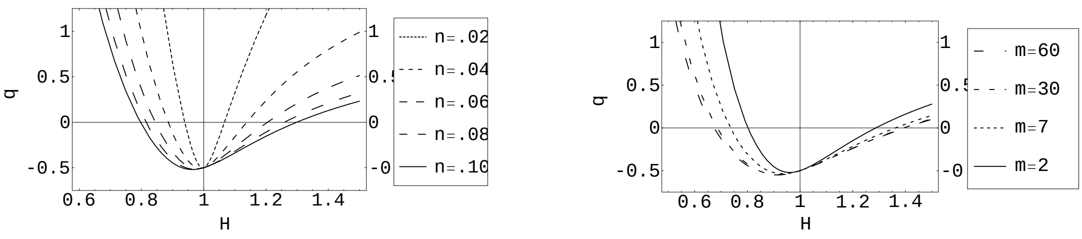
Solving it numerically some interesting studies can be made. We note

(i) The variation of q with H for different values of n is shown in the figure (2.7a). Evidently the time of transition of the universe from decelerating to accelerating phase depends on the parameter n. The time at which universe transits from decelerating to accelerating phase comes nearer to the present epoch for larger values of n . The duration of the present accelerating phase decreases with the decrease in n. The critical values are found in Table. (2.5).

n	Critical Points
0.01	(0,0.969),(0,1.03)
0.05	(0,0.847),(0,1.21)
0.1	(0,0.763),(0,1.34)

Table 2.5: Critical points for different 'n' values

(ii) We plot q vrs. H for different values of m taking $n = 0.1$ which is shown in fig. 10. The curves show that the universe at the present epoch entered from decelerating to acceleration and it will switch over to decelerating phase once again in future.



(a) (b)

Figure 2.7: q vs H plot $f(R) = R + me^{-nR}$ (a) variation in n (b) variation in m.

The smaller values of m leads to a shorter duration of the present accelerating phase. Table. (2.6) lists the critical values for the scenario.

m	Critical Points
2	(0,0.801),(0,1.29)
4	(0,0.764),(0,1.34)
40	(0,0.678),(0,1.39)

Table 2.6: Critical points for different 'm' values

Finally, since we have enough numerical values for $q[H]$ at different epochs (i.e. different H values), we might use those to obtain a closed analytic mathematical structure for q and H which can be determined using a polynomial function given by $q = \Sigma_0^n a_i H^i$. Indeed with initial value $q'[1] = 1.2$, the corresponding approximate analytic function may be expressed as

$$q[H] = 45.06381 - 515.24544H + 2478.58763H^2 - 6555.42602H^3 + 10556.26293H^4 \quad (2.15)$$
$$-10835.40545H^5 + 7153.27419H^6 - 2949.71637H^7 + 693.08477H^8 - 70.97952H^9.$$

The analytic function given in eq. (2.15) can be plotted along with the numerical solution obtained earlier. As can be seen from figure (2.8), two curves closely super-

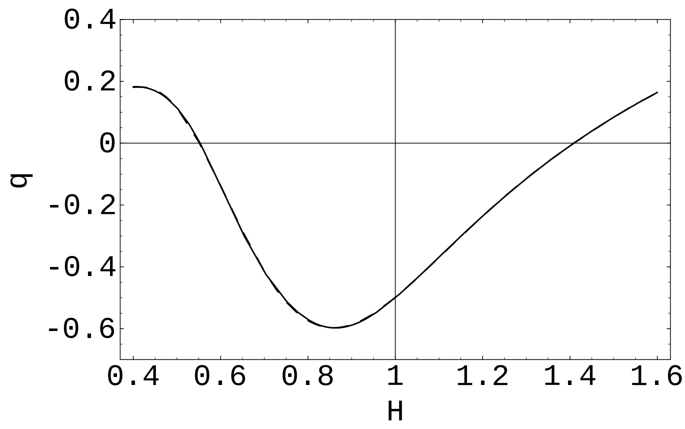


Figure 2.8: Polynomial fit for deceleration parameter: Dashed one is fitted curve

imposes. However, the comparison holds good only when H is reasonably close to one.

2.4 Discussion

Cosmological solutions in $f(R)$ theories of gravity are obtained here which predicts the past, present and future evolution. Different phases of expansion are explored for different choices of the modified theories of gravity described by the polynomial function $f(R)$. The field equation corresponding to the modified theory of gravity is highly non-linear. It is not suitable for obtaining simple analytic solutions in this framework, consequently we adopt numerical technique to investigate the nature of cosmological evolution. We stress in the fact that these are toy models of the universe and for a more realistic model one would have included matter components as well. A detailed study of the matter included cases is beyond the scope of the present work and will be considered elsewhere. From the present study it is found that a number of models can be constructed within $f(R)$ gravity which accommodate an accelerating phase at present epoch. The fact that the universe enters this phase of accelerated expansion only in recent past is also well described. Duration of the accelerating phase as well as future evolution are found to depend on various model parameters and it seems that there can be various phases of acceleration and deceleration in the course of evolution of the universe.

Chapter 3

Estimation of Primordial Black Hole (PBH) formation probability in higher dimensional $f(R)$ gravity.

3.1 Introduction

Recently cosmology is passing through a transformation from speculative science to an experimental one due to a number of precession measurements from various missions in cosmology and in astrophysics [22]. It is believed that the present universe emerged out from an inflationary phase of expansion in the early epoch of its evolution. Inflationary universe scenario, proposed by Guth [8] and elaborated by many others [12, 90, 91, 92], has slowly but surely made its way into the mainstream cosmology and now it is one of the most important and essential phases required to construct a model of the universe in the standard theory of cosmology. One of the main features of the inflation alongside is the fact that it is able to solve some of the conceptual issues not understood in the perfect fluid model of the universe. Inflation opens up new avenues in cosmology e.g., the perturbation that generated during inflation lead to a satisfactory explanation of the large scale structure formation [20, 21] in the universe. Inflation is helpful in many ways, one has the freedom to create the universe from a most generic situation as was shown by Linde [6]. At this point it is worth mentioning that the usual matter fields are not enough to correctly describe our universe as in one form or another, to realise an inflationary scenario one needs homogeneous scalar field, commonly called inflaton [5, 6] with suitable potential. It is also known that one or more scalar field with suitable potentials can produce sufficient inflation under certain conditions [5, 2, 6]. Alternatively one can realise inflation without inflaton field where inflation can be realised by adding higher order terms in scalar curvature (R) to Einstein-Hilbert action [38, 93]. Witten has shown that a modified theory of gravity is equivalent to a scalar field theory in Jordon frame. The construction of the theory is very simple and has its own advantages [38, 93, 94]. The higher order curvature terms lead to a modification of standard cosmology in the early epochs, i.e., high curvature region and admits a de Sitter phase in the early universe [93]. It is expected that there may be astrophysical objects which were formed during the creation of the universe from the primordial quantum fluctuations. These are known as Primordial Black Holes (in short, PBH). PBH are different from the astrophysical black holes which are formed out of the collapse of a very massive star. Primordial black holes of mass $\sim 10^{15}g$ may be created at the very beginning of the universe and these are hot. Observational detection of PBH are important as these are considered to be one of the relics of the very early universe [95]. Theoretically PBH are considered to be topological black holes and one can estimate their probability in the universe. Bousso and Hawking (hereafter BH)[51] first calculated the probability of quantum creation of universe with such a PBH pair in the presence of a cosmological constant. In the absence of a true quantum theory of gravity they took a semi classical approach and considered the possibility of creation of two types of Euclidean universe in 4 dimensions, one of which may accommodate a pair of PBH. Bousso and Hawking considered two different topologies : (i) a universe with space-like sections having S^3- topology and (ii) a universe having $S^1 \times S^2$- topology in space-like section. The first one describes an inflationary universe without PBH, while the later describes a Nariai universe [96] which may accommodate a PBH pair. Subsequently BH considered a massive scalar field instead of a cosmological constant to estimate the PBH pair creation employing slow roll in the potential (with mass term) in the early epoch. As the evolution of the universe can be traced back to the Planck time ($t \sim M_P^{-1}$) in the inflationary scenario, therefore, the evolution may be better understood in the framework of quantum gravity. But, as we know, a consistent quantum theory of gravity is yet to emerge and one has to consider higher dimensional theories which might be important at the epoch. Kaluza and Klein (in short, KK) [97, 98] independently initiated the formulation of a unified theory of gravity and electro-magnetic interaction by introducing an extra dimension. However, the extra dimension was taken compact to Planck scale so as to avoid any departure from general relativity at large scale. But the KK-approach did not work well subsequently because of uninteresting dimensional reduction technique employed. However, as far as cosmological models in higher dimensions are considered, of late there has been a paradigm shift. Advent of new theories like Superstring theory and M-theories rejuvenated the studies in higher dimensional framework and Generalized the approach considerably [99]. An interesting theoretical model known as Brane-world model proposed by Randall and Sundrum [100, 101] avoids dimensional reduction mechanism. In the Brane-world model the fields of the standard model are considered to be confined to (3+1) dimensional hyper-surface (referred to as 3-Brane) which is embedded in a higher dimensional space-time (bulk). The gravitational field may propagate through the bulk dimensions perpendicular to the brane. As shown by Randall-Sundrum [100, 101], the extra dimensions are no longer needed to be compact in these models. Recently another idea of a UV complete field theory of gravity has been put forward by Horava which is commonly known as Horava-Lifshitz gravity [102, 103]. At extremely high energy scale ($\sim 10^{19} GeV$), usual 4 dimensional Einstein field equation can no longer give correct predictions of the universe, therefore, it is essential for a modified field equation which reduces to the Einstein's field equations at low energy scale in 4-dimensions. Such a high enough energy scale might be available shortly after the hot Big-Bang. Thus our universe may be considered as an excellent laboratory for testing these new theories. However, at sufficiently low energy, Einstein's theory gives the correct predictions of the experiments with astonishing accuracy. So if there exists a more fundamental theory of gravity, preferably one which can be quantised, it has to reproduce the Einstein's theory at lower energy scales. In the usual 4-dimension the gravitational sector of the action may be modified by considering non-linear terms of scalar curvature (R). It is interesting to note that many of the theories of particle interactions, including String theory require space-time dimensions more than the usual four for their consistent formulation and this aspect is motivating enough to take up a higher dimensional scenario while studying different aspects of cosmology, specially in the early universe. In this chapter a modified theory of gravity with a polynomial of order 4 in scalar curvature is considered, taking into account a higher dimensional universe, to evaluate the probability of quantum creation of a universe with or without PBH pair. The Bousso-Hawking prescription will be employed here to estimate the probability of PBH pair using *Hartle-Hawking no boundary proposal* [52]. According to the no boundary proposal, the quantum state of the universe can be defined by path integrals over all Euclidean metrics $g_{\mu\nu}$ on compact manifolds M. From this it follows that the probability of finding a three metric h_{ij} on the spacelike hypersurface ∂M is given by a path integral over all $g_{\mu\nu}$ on M that agrees with h_{ij} on ∂M. Considering above geometries including matter fields, the wave function of the universe can be written as:

$$\Psi[h_{ij}, \Phi_{\partial M}] = \int D(g_{\mu\nu}, \Phi)\ exp[-I(g_{\mu\nu}, \Phi)], \tag{3.1}$$

where $(h_{ij}, \Phi_{\partial M})$ are the three-metric and matter fields on a spacelike boundary ∂M and the path integral is over all compact Euclidean metrics $g_{\mu\nu}$. We use a saddle point approximation of the path integral to estimate the wave function of the universe Ψ. To estimate the probability we follow BH. We consider two types of topologies for the spacelike sections that describe inflationary universe, namely (i) $R\times S^d$ -topology and (ii) $R \times S^1 \times S^d$ -topology. The former represents a universe without a pair of PBH and the later describes a universe which may accommodate a PBH pair. We obtain Euclidean solutions of the modified Einstein equation corresponding to each of these universes, which can be analytically continued to match a boundary ∂M of the corresponding topology. The Euclidean action (I) is evaluated for the above solutions. The wave function of the universe in the semiclassical approximation is given by

$$\Psi[h_{ij}, \Phi_{\partial M}] \approx \sum_n A_n e^{-I_n}. \tag{3.2}$$

In the above the sum is over the saddle points of the path integral, and I_n denotes the corresponding Euclidean action. We may assign a probability measure to each type of universe:

$$P[h_{ij}, \Phi_{\partial M}] \sim e^{\left(-2I^{Re}\right)}, \tag{3.3}$$

where the superscript Re stands for the real part of the action corresponding to the dominant saddle point, i.e., the classical solution satisfying the Hartle-Hawking (HH) boundary conditions [52].

It is generally established that a theory with higher order Lagrangian is conformally equivalent to Einstein gravity having a minimally coupled, self interacting scalar field in the a matter sector [93, 94]. However, the renormalisation of higher loop contributions introduces terms into the effective action that are higher than quadratic in R. Consequently it is important to study the effects of these terms in the quantum creation of a universe with a pair of PBH. Paul *et al.* [104] evaluated the probability of pair creation of PBH including R^2 term in 4 dimensional action. Subsequently considering an effective action described by a Lagrangian containing terms upto R^3 term was taken up to estimate the probability [105]. It is found that the probability of creation of a universe with a PBH pair is suppressed in the R^3-theory without a cosmological constant. The probability of BH pair creation rate [106] is then estimated in the case of a universe with higher dimensions. In 2005, Günther *et al.* [47] studied higher dimensional gravitational models with scalar curvature having non-linear terms of the types R^{-1} and R^4 to investigate the existence of at least one minimum of the effective potential for the volume moduli of the internal spaces with an emphasis to obtain stabilisation. It is desirable that the extra dimensional space components remains static or nearly static at least from the time of primordial nucleosynthesis. This is why the stabilisation of the internal space is of fundamental importance. In contrast to R^2-models, the R^4-model shows a richer stability region in parameter space which minimally depends on the total dimension of the bulk space. The modified gravitational action described by a Lagrangian polynomial in R and considering terms up to R^4 are taken up in this chapter to explore the PBH pair creation probability in the early universe.

3.2 Gravitational Instantons with or without PBH

Let us consider a higher dimensional Euclidean gravitational action which is given by

$$I_E = -\frac{1}{16\pi}\int d^D x\sqrt{g}f(R) - \frac{1}{8\pi}\int_{\partial M} d^{(D-1)}x\sqrt{h}Kf'(R), \tag{3.4}$$

where g is the D-dimensional Euclidean metric, R is the Ricciscalar, Λ is the cosmological constant, and $K = h^{ab}K_{ab}$ is the trace of the second fundamental form of the boundary ∂M in the metric. We consider a fourth order polynomial function in Ricci scalar $f(R) = \sum_i \lambda_i R^i$, in particular $\lambda_o = -2\Lambda, \lambda_1 = 1, \lambda_2 = \alpha, \lambda_3 = \beta, \lambda_4 = \gamma$, i.e., $f(R) = R + \alpha R^2 + \beta R^3 + \gamma R^4 - 2\Lambda$.

3.2.1 S^{D-1}- Topology, the de Sitter spacetimes

In this section, we study vacuum solutions of the field equation corresponding to the non-linear higher dimensional action (3.4). We look for a solution with space like section S^{D-1}. Let us choose the D-dimensional metric ansatz, which is given by

$$dS^2 = d\tau^2 + a^2(\tau)d\Omega_d^2, \tag{3.5}$$

where $d = D-1$, a is the scale factor of a D dimensional universe and $d\Omega_d^2$ is a line element of unit $(D-1)$-sphere. The scalar curvature is given by

$$R = -\left[2d\frac{\ddot{a}}{a} + d(d-1)(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2})\right], \tag{3.6}$$

where the overdots denote differentiation with respect to τ. We use the constraint through a Lagrangian multiplier $\tilde{\beta}$ and rewrite the action as

$$I_E = -V_o \int \left[f(R)a^d - \tilde{\beta}\left(R + 2d\frac{\ddot{a}}{a} + d(d-1)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right)\right)\right] d\tau - \frac{1}{8\pi}\int_{\delta M} d^{(D-1)}x\sqrt{h}f'(R), \tag{3.7}$$

where $V_o = \frac{1}{16\pi}\frac{2\pi^{(d+1)/2}}{\Gamma(d+1)/2}$. One can determine $\tilde{\beta}$ by varying the action with respect to R, which is

$$\tilde{\beta} = a^d f'(R). \tag{3.8}$$

Substituting the above constraint in the action (3.7) we get

$$I_E = -V_o \int_{\tau=0}^{\tau_{\delta M}} a^d \left[f(R) - f'(R)\left(R - d(d-1)\left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2}\right)\right) + 2d\dot{R}f''(R)\frac{\dot{a}}{a}\right] d\tau + \left[2dV_o\dot{a}a^{d-1}f'(R)\right]_{\tau=0}^{\tau_{\delta M}}. \tag{3.9}$$

Varying the above action with a and R respectively, we get:

$$f'(R)\left[2(d-1)\frac{\ddot{a}}{a} + (d-1)(d-2)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) + R\right] + f''(R)\left[(d-1)\frac{\dot{a}}{a}\dot{R} + 2\ddot{R}\right] + 2f'''(R)\dot{R}^2 - f(R) = 0, \tag{3.10}$$

$$f''(R)\left[2d\frac{\ddot{a}}{a}+d(d-1)\left(\frac{\dot{a}^2}{a^2}-\frac{1}{a^2}\right)+R\right]=0. \tag{3.11}$$

The above field equations admit an instanton solution given by:

$$a=\frac{1}{H_o}\,\sin\left(H_o\tau\right), \tag{3.12}$$

where H_o is determined from the constraint equation

$$d(d-1)H_o^2+\alpha d^2(d+1)(d-3)H_o^4+\beta d^3(d+1)^2(d-5)H_o^6+\gamma d^4(d+1)^3(d-7)H_o^8-2\Lambda=0. \tag{3.13}$$

Thus $H_o = f(d,\alpha,\beta,\gamma,\Lambda)$ determined by five parameters. We note that for (i) $D=4,\ H_o=f(d,\beta,\gamma,\Lambda)$; (ii) $D=6,\ H_o=f(d,\alpha,\gamma,\Lambda)$; and (iii) $D=8,\ H_o=f(d,\alpha,\beta,\Lambda)$. The above instanton solutions satisfy the HH boundary condition viz., $a(0)=0,\ \dot{a}(0)=0$. We can choose a path along the τ^{Re} axis to $\tau=\frac{\pi}{2H}$ and the solutions would describe half of the Euclidean de Sitter instanton in S^{D-1} -topology. Analytic continuation of the metic (3.5) to Lorentzian region $x_1\rightarrow\frac{\pi}{2}+i\sigma$ gives

$$ds^2=d\tau^2+a^2(\tau)\left[-d\sigma^2+cosh^2\sigma\ d\Omega_{d-2}^2\right], \tag{3.14}$$

which is nothing but a de Sitter like metric. However, if $\tau=it$ and $\sigma=\frac{i\pi}{2}+\chi$, the metric becomes

$$ds^2=-dt^2+S^2(t)\left[d\chi^2+sinh^2\chi\ d\Omega_{d-2}^2\right], \tag{3.15}$$

where $S(t)=-ia(it)$. The above metric describes an open universe. Thus, the creation of an open inflationary universe is realised in this case. It is worth pointing out here that a closed universe, created quantum mechanically, may be realised as an open inflationary universe under what is known as Wick rotation. Since it is not yet known whether our universe is exactly flat, it is interesting to study an open universe. The real part of the Euclidean action corresponding to the solution calculated by following the complex contour of τ suggested by BH is considered here which determines

$$I_E^{Re} = -\frac{V_o I_d}{H_o^{d+1}} \left[d(d+1)H_o^2 + \alpha d^2(d+1)^2 H_o^4 + \beta d^3(d+1)^3 H_o^6 + \gamma d^4(d+1)^4 H_o^8 - 2\Lambda \right], \tag{3.16}$$

where $I_d = \int_0^{\frac{\pi}{2H_o}} \sin^d y \, dy$, denoting $y = H_o\tau$. The value of the integral I_d depends on the dimensions of the universe. For odd number of dimensions (D),

$$I_d = \frac{d-1}{d}\frac{d-3}{d-2}........\frac{3}{4}\frac{1}{2}\frac{\pi}{2}, \quad for \;\; d \;\; even, \tag{3.17}$$

and for even number of dimensions (D),

$$I_d = \frac{d-1}{d}\frac{d-3}{d-2}.......\frac{4}{5}\frac{2}{3}1, \quad for \;\; d \;\; odd. \tag{3.18}$$

With the chosen path for τ, the solution describes half of the de Sitter instanton in a higher dimensional universe with S^d topology, joined to a real Lorentzian hyperboloid in ($R^1 \times S^d$) -topology. Note that it can be joined to any boundary satisfying the condition $a_{\partial M} > 0$. For $a_{\partial M} > H_o^{-1}$, the wave function oscillates and predicts a classical space-time.

3.2.2 S^1XS^{D-2}- Topology

In this section a vacuum solution of the field equation corresponding to the action (3.4) is considered in order to look for a universe with S^1XS^{D-2} -spacelike sections. The above topology accommodates a pair of PBH. The metric ansatz for $D = 1+1+d$ dimensions is given by:

$$ds^2 = d\tau^2 + a^2(\tau)dx^2 + b^2(\tau)d\Omega_d^2. \tag{3.19}$$

Here $a(\tau)$ is the scale factor of two sphere and $b(\tau)$ is the scale factor of the $^{D-2}$-sphere given by the metric

$$d\Omega_d^2 = dx_1^2 + sin^2 x_1 dx_2^2 + sin^2 x_1 sin^2 x_2 dx_3^2 + ... \ d \ space.$$

The scalar curvature is then given by:

$$R = -\left[2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 2d\frac{\dot{a}\dot{b}}{ab}\right]. \tag{3.20}$$

Using the constraint (3.13), the Eucledian action (3.4) may be rewritten as

$$\begin{aligned} I_E &= -V_o' \int \left[f(R)ab^d - \tilde{\beta}\left(R + 2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right) + 2d\frac{\dot{a}\dot{b}}{ab}\right)\right] d\tau - \\ & \frac{1}{8\pi}\int_{\delta M} d^{(D-1)}x\sqrt{h} f'(R), \end{aligned} \tag{3.21}$$

where $V_o' = \frac{1}{16\pi}\frac{2\pi^{(D+1)/2}}{\Gamma((D+1)/2)}$. The Lagrange's undetermined multiplier $\tilde{\beta}$ may now be calculated by varying the action and inserting the corresponding $\tilde{\beta}$ in the action. We finally obtain:

$$\begin{aligned} I_E = -V_o' \int_{\tau=0}^{\tau_{\delta M}} ab^d \left(f(R) - f'(R)\left(R - d(d-1)\frac{\dot{b}^2}{b^2} - 2d\frac{\dot{a}\dot{b}}{ab} - d(d-1)\frac{1}{b^2}\right)\right) d\tau - \\ V_o' \int_{\tau=0}^{\tau_{\delta M}} 2\dot{R} f''(R)\left(\frac{\dot{a}}{a} + d\frac{\dot{b}}{b}\right) d\tau + 2ab^d f'(R) V_o' \left[\frac{\dot{a}}{a} + d\frac{\dot{b}}{b}\right]_{\tau=0}^{\tau_{\delta M}} \end{aligned} \tag{3.22}$$

Varying the action with respect to a, b and R independently, we get the following field equations :

$$f''(R)\left[R + 2\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + 2d\frac{\dot{a}\dot{b}}{ab} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right] = 0, \tag{3.23}$$

$$\begin{aligned} f'(R)\left[R + 2d\frac{\ddot{b}}{b} + d(d-1)\left(\frac{\dot{b}^2}{b^2} - \frac{1}{b^2}\right)\right] + 2f''(R)\left(\ddot{R} + 2d\dot{R}\frac{\dot{b}}{b}\right) + \\ 2\dot{R}^2 f'''(R) - f(R) = 0, \end{aligned} \tag{3.24}$$

$$f'(R)\left[R+2(d-1)\frac{\ddot{b}}{b}+2(d-1)\frac{\dot{a}\dot{b}}{ab}+(d-1)(d-2)\left(\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}\right)+2\frac{\ddot{a}}{a}\right]+ \\ 2f'''(R)\dot{R}^2+2f''(R)\left[(d-1)\dot{R}\frac{\dot{b}}{b}+\ddot{R}+\dot{R}\frac{\dot{a}}{a}\right]-f(R)=0. \quad (3.25)$$

We consider the instanton solutions of the above field equations which is given by:

$$a=\frac{1}{H}\sin(H\tau),\ b=\sqrt{d-1}\ H^{-1},\ R=(2+d)H^2 \quad (3.26)$$

where H satisfies the following constraint equation

$$d\ H^2+\alpha(2+d)(d-2)H^4+\beta(2+d)^2(d-4)H^6+\gamma(2+d)^3(d-6)H^8-2\Lambda=0. \quad (3.27)$$

Evidently, H depends on the parameters $\alpha,\beta,\gamma,\Lambda$ i.e., $H=f(d,\alpha,\beta,\gamma,\Lambda)$ for a given extra space dimension (d). We note that for (i) $D=4,\ H=f(d,\beta,\gamma,\Lambda)$; (ii) $D=6,\ H=f(d,\alpha,\gamma,\Lambda)$; and (iii) $D=8,\ H=f(d,\alpha,\beta,\Lambda)$. The above solution satisfies the HH boundary conditions $a(0)=0,\ \dot{a}(0)=0,\ b(0)=0,\ \dot{b}(0)=0$. Analytic continuation of metric (3.19) to Lorentzian region can be obtained evoking Wick rotations, i.e., $\tau\rightarrow it$ and $x\rightarrow\frac{\pi}{2}+i\sigma$. We obtain:

$$ds^2=-dt^2+S(t)^2\ d\sigma^2+H^{-2}\ d\Omega_{d-2}^2, \quad (3.28)$$

where $S(t)=-ia(it)$. Clearly, in this case the analytic continuation of time and space is not an open universe and the space-time represents an anisotropic universe. The corresponding Lorentzian solution is given by

$a(\tau^{\mathrm{Im}})|_{\tau^{\mathrm{Re}}=\frac{\pi}{2H}}=H^{-1}coshH\tau^{\mathrm{Im}}$

$b(\tau^{\mathrm{Im}})|_{\tau^{\mathrm{Re}}=\frac{\pi}{2H}}=H^{-1}$

Its spacelike section represents $(D-2)$-spheres of radius a with holes of radius $b(=H_o^{-1})$ punched through two opposite poles. Physically this solution may be interpreted as $(d-1)$ spheres containing a PBH pair. PBHs accelerate away from each other as the universe expands. The real part of the action may be determined following the contour approach suggested by BH, which is

$$I^{Re}_{S^1\times S^d} = -\left[\frac{V'_o(d-1)^{d/2}}{H^{d+2}}\Big((2+d)H^2+\alpha(2+d)^2H^4+\beta(2+d)^3H^6+\gamma(2+d)^4H^8-2\Lambda\Big)\right], \tag{3.29}$$

where $d = D-2$. The solution (3.26) describes a universe with two PBH s at the poles of a $(D-2)$-sphere. It is of worth to note here that the non-zero contribution to the action comes from the surface terms only.

3.3 Evaluation of the probability for PBH

In the previous section, the action for an inflationary universe with or without a pair of PBH is computed. The result will be used here to estimate the probability for creation of a higher dimensional de Sitter universe in $f(R)$-theory. The probability for nucleation of a higher dimensional universe without PBH is given by:

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_oI_{D-1}}{H_o^D}\left[D(D-1)H_o^2+\alpha D^2(D-1)^2H_o^4+\beta D^3(D-1)^3H_o^6+\gamma D^4(D-1)^4H_o^8-2\Lambda\right]\right]}, \tag{3.30}$$

where H_o satisfies the constraint eq. (3.13). However, the probability of nucleation of an inflationary universe with a pair of black holes is obtained from action (3.22), which is

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V'_o(D-3)^{(D-2)/2}}{H^D}\left[DH^2+\alpha D^2H^4+\beta D^3H^6+\gamma D^4H^8-2\Lambda\right]\right]}, \tag{3.31}$$

where H satisfies constraint equation (3.27).

3.3.1 Special Cases

For $D \geq 4$ we note the following :

(i) $\alpha = \beta = \gamma = 0$, the probability estimates given in eqs. (3.30) and (3.31) reduce to

$$P_{S^{D-1}} \sim e^{\left[4V_o I_{D-1}(D-1)^{D/2}\left(\frac{D-2}{2\Lambda}\right)^{(D-2)/2}\right]}, \tag{3.32}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[4V_o'\left(\frac{(D-3)(D-2)}{2\Lambda}\right)^{(D-2)/2}\right]}. \tag{3.33}$$

We recover the probabilities obtained by Bousso and Hawking in Einstein theory when $D = 4$, which is

$$P_{S^3} \sim e^{3\pi/\Lambda}, \; P_{S^1\times S^2} \sim e^{2\pi/\Lambda}. \tag{3.34}$$

In the above de Sitter universe is more probable for a positive cosmological constant.

(ii) For $\alpha = \beta = 0, \; \gamma \neq 0$, the corresponding probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-6D(D-1)H_o^2+16\Lambda}{D-8}\right)\right]}, \tag{3.35}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-6DH^2+16\Lambda}{D-8}\right)\right]}, \tag{3.36}$$

where H_o and H are determined from eqs. (3.13) and (3.27) respectively for $D \neq 8$. In four dimensions the probabilities reduce to

$$P_{S^3} \sim e^{\left[\frac{\pi}{3H_o^4}(9H_o^2-2\Lambda)\right]}, \; P_{S^1\times S^2} \sim e^{\left[\frac{2\pi}{H^4}(3H^2-2\Lambda)\right]}. \tag{3.37}$$

For $\Lambda = 0$ and $D \neq 8$, we evaluate the probabilities which are

$$P_{S^{D-1}} \sim e^{\left[12V_o I_{D-1}(D(D-1))^{D/2}(8-D)^{(D-8)/6}\left(\frac{\gamma}{D-2}\right)^{(D-2)/6}\right]}, \tag{3.38}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[12V_o' D^{D/2}(8-D)^{(D-8)/6}(D-3)^{(D-2)/2}\left(\frac{\gamma}{D-2}\right)^{(D-2)/6}\right]}. \tag{3.39}$$

We note the following:

(i) a new gravitational instanton solution in higher dimensions is obtained for a R^4-theory with a negative coupling parameter (i.e., $\gamma < 0$) for $D > 8$, and that with a positive γ (i.e., $\gamma > 0$) for $D < 8$, (ii) de Sitter universe is more probable for $D \geq 3$. We also note that for $D = 8$, an instanton solution with $H_o^2 = \frac{\Lambda}{21}$ in S^7-topology and with $H^2 = \frac{\Lambda}{3}$ in $S^1 \times S^6$-topology are permitted. The probabilities of PBH for $D = 8$ dimensions are

$$P_{S^7} \sim e^{\left[\frac{1372\pi^3}{15\Lambda^3}\left(27+2048\gamma\Lambda^3\right)\right]}, P_{S^1\times S^6} \sim e^{\left[\frac{100\pi^3}{21\Lambda^3}\left(27+2048\gamma\Lambda^3\right)\right]}. \tag{3.40}$$

We further note that the probabilities are same if (i) $|\gamma| = \frac{27}{2048\Lambda^3}$ and $\Lambda > 0$ with negative γ and (ii) $\gamma = \frac{27}{2048\Lambda^3}$ and $\Lambda < 0$ with positive γ. An interesting gravitational instanton is permitted without a cosmological constant in $D = 8$.

(iii) For $\alpha \neq 0,\ \beta = \gamma = 0$, the probabilities are:

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-2D(D-1)H_o^2+8\Lambda}{D-4}\right)\right]}, \tag{3.41}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-2DH^2+8\Lambda}{D-4}\right)\right]}. \tag{3.42}$$

In this case PBH pair creation may be possible even without a cosmological constant. For $\Lambda = 0$, the probabilities are given by

$$P_{S^{D-1}} \sim e^{\left[\frac{4V_o I_{D-1}}{(D-2)^{(D-2)/2}}(D(D-1))^{D/2}(4-D)^{(D-4)/2}\right]}, \tag{3.43}$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[4V_o'\left(\frac{D-3}{D-2}\right)^{(D-2)/2}D^{D/2}(4-D)^{(D-4)/2}\right]}. \tag{3.44}$$

We note that the instanton solutions considered here for the two different topologies physically sensible when $\alpha < 0$ for $D > 4$ and $\alpha > 0$ for $D < 4$. We note that in higher dimensions with space-time dimensions $D \geq 3$ the probability of nucleation of a S^{D-1}-topology is found to be more than that of a universe with $S^1 \times S^{D-2}$ topology. In 4-dimensions the probabilities reduce to

$$P_{S^3} \sim e^{[24\pi\alpha + 3\pi/\Lambda]}, P_{S^1 \times S^2} \sim e^{[16\pi\alpha + 2\pi/\Lambda]}. \tag{3.45}$$

A theory with a positive cosmological constant and $\alpha > 0$, leads to a universe without PBH, which is more probable. Though a negative $\alpha < -\frac{1}{8\Lambda}$ leads to greater probability for a universe with PBH, the negative values of α lead to a classical instability in R^2-theory [11].

(iv) For $\alpha = \gamma = 0,\ \beta \neq 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left(\frac{-4D(D-1)H_o^2 + 12\Lambda}{D-6}\right)\right]}, \tag{3.46}$$

$$P_{S^1 \times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left(\frac{-4DH^2 + 12\Lambda}{D-6}\right)\right]}, \tag{3.47}$$

with $D \neq 6$. The probabilities in $D = 4$ dimensions may now be obtained as

$$P_{S^3} \sim e^{\left[\frac{\pi}{H_o^4}(H_o^2 - \Lambda)\right]}, P_{S^1 \times S^2} \sim e^{\left[\frac{2\pi}{H^4}(4H^2 - 3\Lambda)\right]}. \tag{3.48}$$

We note that PBH pair creation may be described even without a cosmological constant. The corresponding probabilities are

$$P_{S^{D-1}} \sim e^{\left[8V_o I_{D-1}\left(\frac{\beta}{D-2}\right)^{(D-2)/4}(D(D-1))^{D/2}(6-D)^{(D-6)/4}\right]}, \tag{3.49}$$

$$P_{S^1 \times S^{D-2}} \sim e^{\left[8V_o'(D-3)^{(D-2)/2}\left(\frac{\beta}{D-2}\right)^{(D-2)/4}D^{D/2}(6-D)^{(D-6)/4}\right]}. \tag{3.50}$$

We note that a universe with space-time dimensions (i) $D > 6$ admits an instanton solution with $\gamma < 0$ and (ii) $D < 6$, with a positive β. The probability measure for pair creation in inflationary universe is more comparable to that with a universe without primordial black holes.

Let us consider $D = 6$ dimensional universe. In this case $H_o^2 = \frac{\Lambda}{10}$ in S^5-topology and $H^2 = \frac{\Lambda}{2}$ in $S^1 \times S^4$-topology. The corresponding probabilities are

$$P_{S^5} \sim e^{\left[\frac{200\pi^2}{3\Lambda^2}\left(1+27\beta\Lambda^2\right)\right]}, P_{S^1\times S^4} \sim e^{\left[\frac{48\pi^2}{5\Lambda^2}\left(1+27\beta\Lambda^2\right)\right]}. \quad (3.51)$$

de Sitter universe without a pair of PBH is more probable.

(v) For $\Lambda = 0$, the probability measure in the two topologies under consideration are given by

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2 H_o^4+4\beta D^3(D-1)^3 H_o^6+6\gamma D^4(D-1)^4 H_o^8}{D-2}\right]\right]}, \quad (3.52)$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2 H^4+4\beta D^3 H^6+6\gamma D^4 H^8}{D-2}\right]\right]}. \quad (3.53)$$

In $D = 4$ dimensions the probabilities reduce to

$$P_{S^3} \sim e^{\left[\pi(24\alpha+576\beta H_o^2+10368\gamma H_o^4)\right]}, P_{S^1\times S^2} \sim e^{\left[\pi(16\alpha+128\beta H^2+768\gamma H^4)\right]}. \quad (3.54)$$

We now discuss few other special cases which are interesting and follow from the general result obtained above in higher dimensions.

(a) For $\gamma = 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2 H_o^4+4\beta D^3(D-1)^3 H_o^6}{D-2}\right]\right]}, \quad (3.55)$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2 H^4+4\beta D^3 H^6}{D-2}\right]\right]}, \quad (3.56)$$

one determines the instanton solution even with $\Lambda = 0$ it reduce to that obtained in *Ref.* [51] for $D = 4$

(b) For $\beta = 0$, and $\Lambda = 0$, the probabilities are

$$P_{S^{D-1}} \sim e^{\left[\frac{2V_o I_{D-1}}{H_o^D}\left[\frac{2\alpha D^2(D-1)^2 H_o^4+6\gamma D^4(D-1)^4 H_o^8}{D-2}\right]\right]}, \quad (3.57)$$

$$P_{S^1\times S^{D-2}} \sim e^{\left[\frac{2V_o'(D-3)^{(D-2)/2}}{H^D}\left[\frac{2\alpha D^2 H^4+6\gamma D^4 H^8}{D-2}\right]\right]}. \quad (3.58)$$

For $D = 4$

$$P_{S^3} \sim e^{\left[3\pi(8\alpha+3(128\gamma)^{1/3})\right]}, P_{S^1\times S^2} \sim e^{\left[2\pi(8\alpha+3(128\gamma)^{1/3})\right]}. \tag{3.59}$$

Here $H_o^2 = \sqrt[3]{\frac{1}{3456\gamma}}$ and $H^2 = \sqrt[3]{\frac{1}{128\gamma}}$, are new solutions. In this case a physically interesting instanton solution is obtained for a positive value of γ. For $\alpha < 0$ and if $|8\alpha| < 3(128\gamma)^{1/3}$, de Sitter universe without PBH is more probable. However interesting possibilities emerges when $|8\alpha| > 3(128\gamma)^{1/3}$.

3.4 Discussions

In this chapter, the probability of pair creation for primordial black holes in a non-linear theory of gravity is evaluated. We consider an action with a polynomial function of Ricci scalar R in the form $f(R) = \sum_i \lambda_i R^i$, to obtain gravitational instanton solutions in the multidimensional universes. The Euclidean action is then evaluated using instanton solutions in the two cases:(i) a universe with $R \times S^d$-topology and (ii) a universe with $R \times S^1 \times S^d$-topology in higher dimensions. The former admits an inflationary universe without PBH and later a pair of PBH. We present the corresponding estimation of the probability in a multidimensional universe and compare them to predict the probability of creation of universe. Higher order curvature terms are generally important in the very early universe and that is the epoch when PBH s might have been created. We determine that the probability measures in both the topologies in the framework of a non-linear theory of gravity with square, cubic and quadratic power of scalar curvature term (R) in the gravitational actions and estimate the coupling parameters for a physically realistic instanton solution. One interesting point is that in this $f(R)$ gravity model with square and higher order curvature terms one obtains gravitational instanton solution even without a cosmological constant. Note that in a $3+1$ dimensional universe with conventional GR this feature is absent. The de sitter universe without a pair of black holes is found more probable for $D \geq 3$. Also, the probability of a universe with topology $R \times S^3$- turns out to be much lower than a universe with topology $R \times S^1 \times S^2$ in R^2-theory unless $\Lambda < -\frac{1}{8\alpha}$ in 4-dimensions. The probability of creation of a universe with a pair of PBH is found to be suppressed if the R^3 term is included in the action under the constraints $|\alpha| < 2\sqrt{\beta}$ or $\alpha > -2\sqrt{\beta}$ with $\Lambda = 0$. Again the probability of creation of a universe with a pair of PBH is found to be strongly suppressed if one extends the polynomial function of $f(R)$ gravity model upto R^4-term with the constraints $|8\alpha| < 3(128\gamma)^{1/3}$ or $8\alpha > -3(128\gamma)^{1/3}$. One obtains Hawking-Turok [107, 108] type open inflationary universe in $R \times S^3$-topology, but in the other topology accommodating PBH does not permit an open universe. Instead, it gives rise to an anisotropic universe. We noted a dimensional dependence of the gravitational instanton and determined the different parameters to realise such instantons in the non-linear theory of gravity. A class of new gravitational instantons are presented here which are relevant for cosmological model building.

Chapter 4

Emergent Universe scenario in Gauss-Bonnet gravity with Dilaton

4.1 Introduction:

Modified theories of gravity is considered to be very much useful for describing various observational facts of the early as well as the late universe fairly well including an accelerated universe (see chapter (1,3)). In modern cosmology inflation is generally believed to be an essential ingredient to construct cosmological model. It is known that the early inflation can be realised consistently in a semi-classical theory of gravity [8, 12, 90, 91, 92]. It may be mentioned here that Starobinsky [45] obtained inflationary solution in a higher derivative theory of gravity long before the advent of inflationary scenario. The gravitational action in the Starobinsky model corresponds to a theory containing curvature squared terms in the the Einstein- Hilbert action. However, the efficacy of inflation is known only after the seminal work of Guth

[8] who used phase transition mechanism to obtain such scenario and proposed to resolve some of the outstanding problems of Big-Bang model. Although Guth's original model was soon found to be plagued with serious problems [12] the idea of inflation was happily accepted and within very short time Linde [12] and Steinhardt [13] came up with New Inflationary model. There has been a flurry of literature on inflation (for details see ref. [6, 7]) since then and many interesting theories came up. There are merits and demerits of all these models and none of them stands apart clearly today. Inflationary phase of the universe, even today, stores a lot of mysteries within. However, the late accelerating phase of universe is another big issue and carries overwhelming support from observational data. Sadly, a suitable explanation is yet to come out. To address the issue a number of proposals came up and the simplest one of them is most favoured by all observation, addition of a tiny cosmological constant [66]. Here, it should be noted that despite being absolute favourite from the point of view of observational cosmologies a cosmological constant as tiny as observations predict is simply an ad hoc addition to the EH action. Although, there has been a lot of effort to identify this term with vacuum energy density no fundamental physics can assign it such a small value [66]. In short both in early and late universe there is enough room for new theories to come up and explain the cosmological observations in a consistent way.

In higher derivative theories of gravity the gravitational Lagrangian density is generally considered as a polynomial function of scalar curvature R of the form $L - \sum_{i=o}^{n} \lambda_i R^i$. It is also known that quadratic terms in R in gravitational Lagrangian play a crucial role in the dynamics of the universe. A theory with R^2, $R_{ij}R^{ij}$, $R_{ijkl}R^{ijkl}$ and

Λ terms in the gravitational theory are also considered which is known as Generalized theory of gravity. It has been shown by Zweibach [49] long ago that the string corrections due to Einstein action up to first order in the slope parameter and fourth power of momenta should be proportional to Gauss-Bonnet (GB) terms which is a combination of the squared curvature terms namely, $GB = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. Subsequently it was realised that the field redefinition theorem of 't Hooft and Veltman [109, 110] may be applicable in this case. On the Einstein's shell ($R_{\mu\nu} = 0$), an action with curvature squared term of the form $R + aR_{\mu\nu}^2 + bR^2$ may be transformed into R itself (neglecting higher order terms) by field redefinition :

$$g'_{\mu\nu} = g_{\mu\nu} + aR_{\mu\nu} + g_{\mu\nu}\frac{a + 2b}{2 - D}R$$

where D represents the number of dimensions. Deser and coworkers [111, 112, 113, 114, 115, 116, 117, 118] have shown that on the linearised Einstein shell, the actions $R + \alpha'(GB)$ and $R + \alpha' R_{\mu\nu\gamma\delta}^2$ (here α' is the inverse of string tension) are not different and this result generalises to all higher-order ghost terms. GB terms arise naturally as the leading order of the α' expansion of heterotic superstring theory [119, 120, 121, 122]. It is known that GB terms in the higher dimensions lead to ghost free propagator. It may be mentioned here that in the ordinary 4 dimensions, GB terms do not contribute in the dynamics of evolution and it is just a Euler number. However, if GB terms are coupled with a dilaton field it may play an important role in the dynamics of the evolution via the dilaton field. In a string induced gravity [123], GB terms coupled with scalar field play an important role which avoids singularity in the theory. Later, in the braneworld scenario, it has been shown [124] that the naked singularities may not occur if dilaton with a Gauss-Bonnet term are considered.

The issues of fine tuning in a theory with a dilaton field and GB-terms interaction are also taken into account to study in details in [124, 125]. Recently, cosmological models with dark energy of the universe are probed in the Einstein-Hilbert action with GB terms and it is known that the theory accommodates the new form of energy [126, 50, 127, 128, 129, 130]. It has been shown [131] that scalar field entering the coupling with GB terms in the action plays an important role which may be used to explain different phases of expansion of the universe including the present accelerating phase. GB terms with dilaton is known to admit accelerating cosmologies too in higher dimensional theories [132]. The Gauss-Bonnet terms in the Einstein-Hilbert action are used to obtain new black holes solution [133] and Kaluza-Klein space-times [134, 135]. Paul and Mukherjee [48] earlier noted that a Gauss-Bonnet term in higher dimensions leads to a 4-dimensional universe at a later epoch with many good features when the sign of the coupling parameter is taken different from that one usually gets from the low energy limit of string theory. The model also gives a satisfactory explanation of the smallness of the effective 4-dimensional cosmological constant. Recently, Einstein-Hilbert action with a combination of higher order curvature terms e.g., GB terms including dilaton are employed to study the present acceleration of the universe [131, 133]. Therefore, the GB-theory has a rich structure that needs to be explored. In this chapter we explore what is known in literature as emergent universe scenario [136, 53] with GB terms coupled with a dilaton field in 4 dimensions. The emergent universe model is interesting as it avoids Big-Bang singularity.

4.1.1 Emergent Universe models in cosmology

In 1967, Harrison [137] obtained a cosmological solution with radiation in the presence of a cosmological constant in a closed model of the universe which asymptotically approaches to Einstein static universe but the scenario fails to exit from the inflationary phase. Recently, Ellis and Maartens [138] obtained similar cosmological solution considering a minimally coupled scalar field with a special choice of its potential where a graceful exit of the universe from its inflationary phase followed by reheating is possible. Subsequently it was shown by Ellis *et al.* [139] that the potential required to obtain such scenario may be obtained naturally by a conformal transformation of Einstein-Hilbert action with R^2-term for a proper choice of its coupling constant. The model incorporates an asymptotically Einstein static universe in the past and it evolves to an accelerating universe in the framework of a closed model of the universe subsequently. This model is usually known as emergent universe model. The salient features of an emergent universe scenario is that there is no time like singularity, it is ever existing and it approaches a static universe in the infinite past ($t \to -\infty$). It is interesting to construct an emergent universe model as it is capable of solving some conceptual issues of the standard Big-Bang model. The asymptotic Einstein static universe at some stage enters into the standard Big-Bang phase and might have features precisely known to us. Mukherjee *et al.* [136] obtained an emergent universe (EU) scenario in a flat universe in the modified Starobinsky model. Subsequently, they obtained EU scenario considering a polynomial form of equation of states admitting a mixture of three different types of cosmic fluid [53] including exotic matter. The possibilities of an emergent universe scenario have been studied recently in a number of theories like Brane world models [140], Brans-Dicke [141], because it permits a universe which is ever existing and large enough so that the spacetime may be treated as classical entities. In this chapter we present EU scenario considering a gravitational action with GB-terms coupled with a dilaton field.

4.2 Action and the Field Equations:

We consider a gravitational action given by:

$$I = -\int \left[\frac{R}{2\kappa^2} + f(\phi)(R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2) + L_\phi \right] \sqrt{-g}\, d^4x \tag{4.1}$$

where the Greek indices $\mu,\ \nu$ represents (0, 1, 2, 3), $f(\phi)$ represents the coupling factor of the Gauss-Bonnet (GB) term and GB is a combination of squared terms of Riemann tensor, Ricci tensor and Ricci scalar ($GB = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$), g represents the 4- dimensional metric, $8\pi G = \kappa^2$ and L_ϕ represents the Lagrangian for the dilaton field. The corresponding Lagrangian for the dilaton field is given by:

$$L_\phi = -\xi(\phi)\ \partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{4.2}$$

where $\xi(\phi)$ represents the coupling parameter for the field in the gravitational action and $V(\phi)$ represents potential of the dilaton field. Let us consider a flat, homogeneous and isotropic Robertson-Walker (RW) metric with scale factor $a(t)$, which is given by

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + sin^2\theta\ d\phi^2)\right]. \tag{4.3}$$

The action given in (4.1) with the RW metric (4.3) yields the following field equations

:

$$3\left(\frac{\dot{a}}{a}\right)^2 = \kappa^2\left[\xi(\phi)\dot{\phi}^2 + V(\phi) - 24f'(\phi)\dot{\phi}\frac{\dot{a}^3}{a^3}\right], \tag{4.4}$$

$$2\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2=-\kappa^2\left[\xi(\phi)\dot{\phi}^2-V(\phi)+16f'(\phi)\dot{\phi}\frac{\dot{a}\ddot{a}}{a^2}+8\left(f'(\phi)\ddot{\phi}+f''(\phi)\dot{\phi}^2\right)\frac{\dot{a}^2}{a^2}\right], \quad (4.5)$$

once again varying the action (4.1) with respect to the dilaton field ϕ, we get

$$\xi(\phi)\left[\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}+\frac{1}{2}\frac{\xi'}{\xi}\dot{\phi}^2+\frac{V'(\phi)}{2\xi}\right]=12f'(\phi)\frac{\dot{a}^2\ddot{a}}{a^3}, \quad (4.6)$$

where the over dot implies derivative w.r.t. time and prime ($'$) represents differentiation with respect to the field ϕ. In the above, out of the three eqs. (4.4)-(4.6), only two are independent, as the third can be derived from the first two. It is evident that there are altogether five unknowns namely, $a(t)$, ϕ, $V(\phi)$, $\xi(\phi)$ and $f(\phi)$ in the above field equations. In order to solve the equations additional assumptions are can be made. Let us first assume that

$$f'(\phi)\dot{\phi}=\eta, \quad (4.7)$$

where η is a constant. The above assumption leads to a relation $f(\phi)=\eta t(\phi)+\eta_o$, where η_o is a constant. The coupling parameter $f(\phi)$, therefore, grows with time. Consequently the effect of the GB terms becomes more and more important at late time, which may be useful for describing the present acceleration of the universe. Using the constraint given by eq. (4.7) in eqs. (4.4) and (4.5), one gets

$$3H^2=\kappa^2\left[\xi(\phi)\dot{\phi}^2+V(\phi)-24\eta H^3\right], \quad (4.8)$$

$$2\dot{H}+3H^2=-\kappa^2\left[\xi(\phi)\dot{\phi}^2-V(\phi)+16\eta H(\dot{H}+H^2)\right], \quad (4.9)$$

where $H=\frac{\dot{a}}{a}$ is the Hubble parameter. Now eliminating $V(\phi)$ from eqs. (4.8) and (4.9), we get

$$\dot{H}+\kappa^2\left[\xi(\phi)\dot{\phi}^2+8\eta H\dot{H}-4\eta H^3)\right]=0. \quad (4.10)$$

The dilaton potential can also be obtained from eqs. (4.8) and (4.9). By eliminating $\xi(\phi)\,\dot{\phi}^2$, the corresponding potential becomes a function of the Hubble parameter which is given by

$$V(H) = \frac{3}{\kappa^2}H^2 + 20\eta H^3 + \frac{1}{\kappa^2}(1 + 8\eta\kappa^2 H)\dot{H}, \tag{4.11}$$

Once the Hubble parameter (H) is known in terms of $\phi$the above potential may be expressed as a function of ϕ. We use eqs. (4.8) and (4.9) to determine the coupling parameters $\xi(\phi)$ and $f(\phi)$ in the theory for an EU cosmological solution following [131]. The set of eqs. (4.10) and (4.11) contain four unknowns, therefore, two more *ad hoc* assumptions are required to obtain a consistent cosmological solution. In the next section we begin with a known scale factor, which permits an emergent universe scenario.

4.3 Cosmological Solutions

Let us consider the evolution of the scale factor of the universe in the form

$$a(t) = a_o\left[A + e^{\alpha t}\right]^{\frac{1}{\beta}} \tag{4.12}$$

where a_o, α, β and A are positive constants. It gives an EU scenario as have been obtained in [137, 136]. The Hubble parameter corresponding to eq.(4.12) satisfies a first order differential equation given by

$$\dot{H} = \alpha H - \beta H^2. \tag{4.13}$$

The dilaton coupling and the dilaton potential can be determined eqs. (4.8)-(4.10). The field equations are highly non-linear and a general form of $\xi(\phi)$ and $V(\phi)$ can not be obtained in terms of the dilaton field. However, those parameters may be determined in terms of the Hubble parameter following [131], which are

$$\xi(H) = \frac{1}{\kappa^2 \dot{\phi}^2} \left[4\eta\kappa^2 (1 + 2\beta) H^3 + \left(\beta - 8\kappa^2 \alpha\eta \right) H^2 - \alpha H \right], \tag{4.14}$$

$$V(H) = \frac{3}{\kappa^2} H^2 + 20\eta H^3 + \frac{1}{\kappa^2} \left(1 + 8\kappa^2 \eta H \right) \left(\alpha H - \beta H^2 \right). \tag{4.15}$$

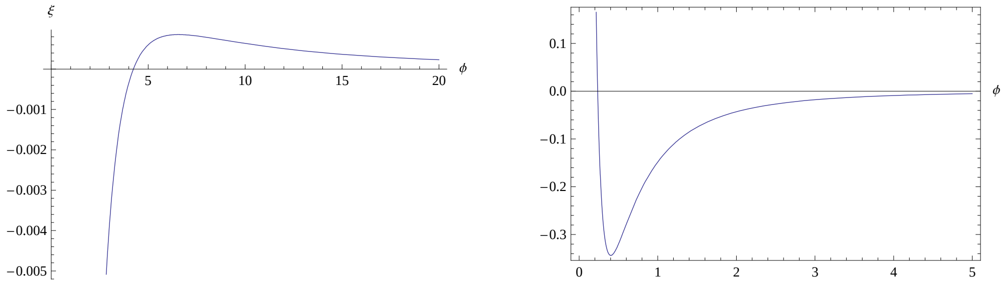
As a special case let us consider $\beta = 8\eta\kappa^2\alpha$ in the above. The corresponding dilaton field potential can be expressed in terms of the field. As the number of unknowns are one more than the number of relevant equations we can now make some choices. We look for emergent universe scenario for different behaviours of the dilaton field in the following.

4.3.1 Case I :

Consider an increasing dilaton field: $\phi = \phi_o e^{\alpha t}$, with the GB coupling terms $f(\phi) = \frac{\eta}{\alpha}\, ln\, \phi$. Dilaton coupling is given by:

$$\xi(\phi) = \frac{1}{2\kappa^2 \phi (1 + \beta\phi)^3} \left(\beta\phi^2 - 4\beta\phi - 2 \right) \tag{4.16}$$

We note the following: (i) $\xi(\phi) \to \infty$ when $\dot{a} \to 0$ i.e. $H \to 0$ and (ii) $\xi(\phi) \to 0$ at two points (i) for $H_1 = \frac{\alpha}{\beta}$ and (ii) for $H_2 = \frac{2\alpha^2}{\beta(1+2\beta)}$. We can plot the variation of the dilaton coupling (ξ) with ϕ as shown in the fig. (4.1a). It is evident that initially the dilaton coupling begins with negative value (phantom like property) but in due course it becomes positive and almost constant after attaining a peak. Thus in this model to begin with one can start with a field having negative kinetic energy, an interesting field which behaves like phantom [142, 143, 144, 145, 146, 147], now-a-days



(a) $\xi(\phi)$ Vs. ϕ curve for $\kappa = 1$

(b) $\xi(\phi)$ vs. ϕ curve for $\kappa = 1$ and $\beta = 2$

Figure 4.1: Case I (a) and Case II (b): Nature of $\xi(\phi)$ vs ϕ

it is considered as one of the candidate of dark matter and avoids the singularity. The potential is given by:

$$V(\phi) = \frac{\alpha^2}{\kappa^2(1+\beta\phi)^3}\left(\frac{11}{2}\beta\phi^3 + (2\beta+3)\phi^2 + \phi\right). \tag{4.17}$$

The Hubble parameter in this case is related to field as :

$$H = \frac{\alpha\phi}{1+\beta\phi}. \tag{4.18}$$

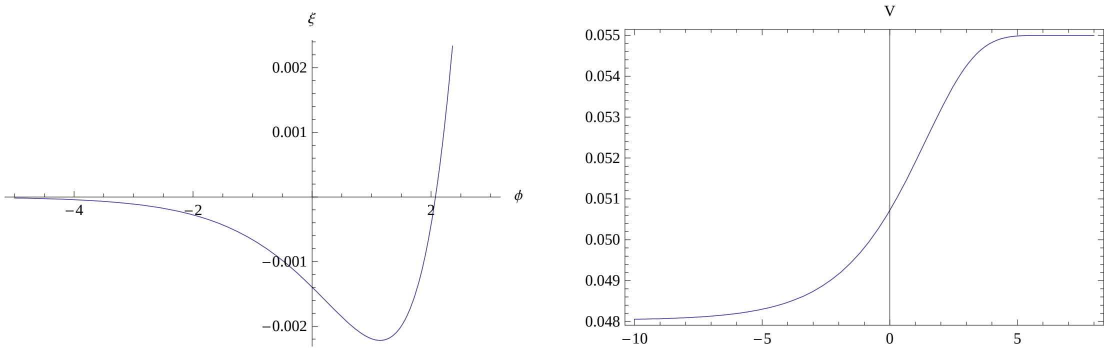
We determine $A = 1$, $\phi_o = \frac{1}{\beta}$ and the corresponding evolution of the scale factor becomes

$$a = a_o\left[1 + e^{\alpha t}\right]^{\frac{1}{\beta}}. \tag{4.19}$$

4.3.2 Case II :

For a decreasing dilaton field $\phi = \phi_o e^{-\alpha t}$, with the GB coupling term $f(\phi) = -\frac{\eta}{\alpha} ln\,\phi$, the dilaton coupling is given by:

$$\xi(\phi) = \frac{1}{2\kappa^2\phi^2\,(\beta+\phi)^3}\left(\beta - 4\beta\phi - 2\phi^2\right) \tag{4.20}$$



(a) $\xi(\phi)$ Vs. ϕ curve for $\beta = 5$ (b) $V(\phi)$ vs. ϕ curve for $\alpha = 0.5$ and $\beta = 5$

Figure 4.2: Case III: Nature of $\xi(\phi)$ and $V(\phi)$

In fig. (4.1b) we plot variation of dilaton coupling $\xi(\phi)$ with ϕ. It is evident that $\xi(\phi)$ begins with a negative value then attains a minimum and thereafter, it increases as ϕ decreases. The potential for the dilaton field is obtained as

$$V(\phi) = \frac{\alpha^2}{\kappa^2(\beta + \phi)^3)} \left(\frac{11}{2}\beta + (2\beta + 3)\phi + \phi^2 \right). \tag{4.21}$$

The Hubble parameter is given in terms of the dilaton as

$$H = \frac{\alpha}{\beta + \phi} \tag{4.22}$$

the corresponding scale factor is

$$a = a_o \left[\phi_o + \beta e^{\alpha t} \right]^{\frac{1}{\beta}}. \tag{4.23}$$

4.3.3 Case III :

Consider a slowly varying field $\phi = \frac{1}{\alpha} \ln t$, with $f(\phi) = \eta\, e^{\alpha\phi}$. A similar kind of field one expects in the string theory framework [148]. The dilaton coupling is given by:

$$\xi(\phi) = \frac{1}{\kappa^2} \left[\frac{\beta(1 + 2\beta)}{2\alpha} H^3 - \alpha H \right] \left(ln \frac{H}{\alpha - \beta H} \right)^2, \tag{4.24}$$

and the dilaton potential is:

$$V(H) = \frac{1}{\kappa^2}\left(\alpha H + 3H^2 + \frac{\beta}{\alpha}\left(\frac{5}{2} - \beta\right) H^3\right), \tag{4.25}$$

where the Hubble parameter is given by:

$$H = \frac{\alpha e^{\alpha e^{\alpha\phi}}}{1 + \beta e^{\alpha e^{\alpha\phi}}}. \tag{4.26}$$

Fig.(4.2a) shows variation of $\xi(\phi)$ with ϕ for $\beta = 5$. Here the dilaton coupling decreases as the dilaton increases to begin with from a negative value initially, attains a minimum, thereafter, it increases sharply. GB terms become important at late time in this case. The potential becomes flat as $\phi \to \infty$ which is shown in fig. 4.2b.

4.3.4 Case IV :

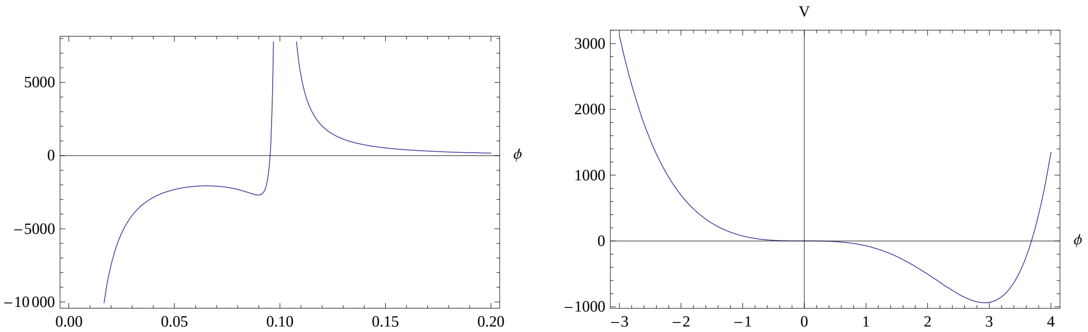
For a dilaton field that varies like the Hubble constant, i.e. $\phi = H$,, considering variation of the coupling of GB term as $f = \eta t$. We obtain :

$$\xi(\phi) = \frac{1}{\kappa^2\phi^2(\alpha - \beta\phi)^2}\left[\frac{\beta(1+2\beta)}{2\alpha}\phi^2 - \alpha\right] \tag{4.27}$$

with dilaton potential given by:

$$V(\phi) = \frac{1}{\kappa^2}\left(\alpha\phi + 3\phi^2 + \frac{\beta(5-2\beta)}{2\alpha}\phi^3\right) \tag{4.28}$$

The fig. (4.3a) shows the variation of $\xi(\phi)$ vs. ϕ, for $\beta = 5$. We note that the coupling parameter becomes undetermined when $\phi = \frac{\alpha}{\beta}$. In this case it is necessary to begin with an initial field which is greater than the above limiting value. In that case the coupling parameter always remains positive definite and it decreases from a large value to zero. Here the GB terms do not contribute at late time as the coupling



(a) $\xi(\phi)$ vs. ϕ curve for $\alpha = 0.5$ and $\beta = 5$ (b) $V(\phi)$ vs. ϕ curve for $\alpha = 0.5$ and $\beta = 5$

Figure 4.3: Case IV: Nature of $\xi(\phi)$ and $V(\phi)$

$\xi \to 0$. The corresponding dilaton potential required for the EU model is shown in fig.(4.3b). It has a minimum which is negative definite.The dilaton potential for $\beta = \frac{5}{2}$, becomes

$$V(\phi) = \frac{3}{\kappa^2}\left(\phi + \frac{\alpha}{6}\right)^2 - \frac{\alpha^2}{12\kappa^2}. \tag{4.29}$$

In this case the scale factor has a form:

$$a = a_o\left(1 + \frac{5}{2}\, e^{\alpha\, t}\right)^{\frac{2}{5}}. \tag{4.30}$$

4.3.5 Case V :

We now consider a special case $f(\phi) = \eta_o + \eta\phi^2$ to obtain EU scenario. In this case the dilaton field evolves as $\phi = \pm\sqrt{t}$ with the coupling parameter:

$$\xi(\phi) = \frac{4\phi^2}{\kappa^2}\left(-\alpha H + \frac{\beta}{2\alpha}(1 + 2\beta)H^3\right). \tag{4.31}$$

The dilaton field potential has a form:

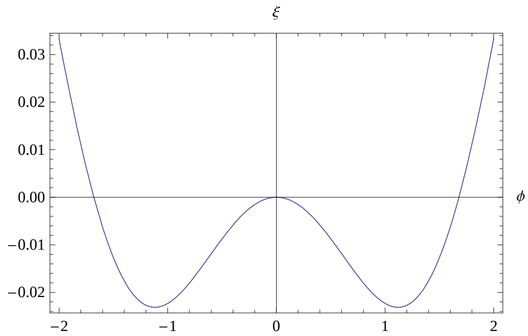
$$V(\phi) = \frac{1}{\kappa^2}\left(\alpha H + 3H^2 + \frac{\beta}{\alpha}\left(\frac{5}{2} - \beta\right)H^3\right) \tag{4.32}$$

where $H = \frac{\alpha e^{\alpha\phi^2}}{1+\beta e^{\alpha\phi^2}}$.

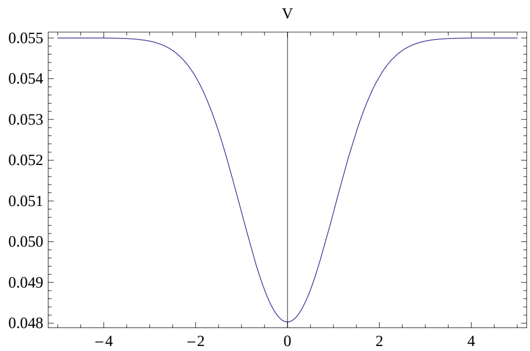
Fig. (4.4a) shows the variation of the dilaton coupling parameter with field, which is interesting. The variation of the potential is also shown in fig. (4.4b, which has two flat regions one in the early era and the other in the late era respectively. This is new and interesting as it permits both early inflation and late acceleration. In [136] it was shown that a particle creation may occur during a phase when the Hubble parameter varies slowly. In this case the GB coupling parameter $f(\phi) \to \pm\infty$ as $t \to \pm\infty$. The GB combination might have dominated in the early epoch of evolution which eventually decreases at later epoch (for $\eta_o = 0$) corresponding to a minimum of $f(\phi)$ (say at $t = 0$) thereafter it increases which is shown in fig. (4.4c). Thus GB terms play an important role in the early era which is subsequently important once again at late era contributing to the dark energy [41, 42, 81, 149, 150].

4.4 Stability of Einstein's Static Universe

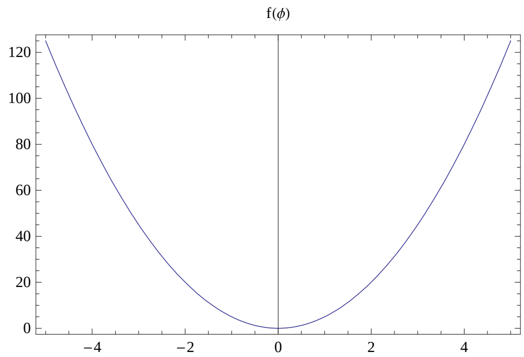
The EU scenario is characterised by the scale factor $a(t) = (a_0 + \beta e^{\alpha t})^{1/\beta}$ as given in eq. (4.12). It is worth pointing out that the universe (i) begins with singularity if $\beta < 0$, (ii) begins with singularity and asymptotically approaches Einstein static (ES) universe at late time if $\alpha < 0$ and $\beta \neq 0$, and (iii) spends infinite time near the Einstein static universe but pulls away and ends in an infinite inflating epoch if $\alpha > 0$ and $\beta > 0$. The first two cases lead to unstable solution [151]. Note that: (i) The existence of the ES universe in fourth order theories of gravity and stability of the ES has been studied in [152]. (ii) Stability of ES in $f(R)$ gravity has also been considered in [153]. (iii) It was also noted that ES universes are unstable in generic



(a) $\xi(\phi)$ vs. ϕ curve for $\beta = 5$



(b) $V(\phi)$ vs. ϕ curve for $\beta = 5$



(c) $f(\phi)$ vs. ϕ curve for $\eta_o = 0$ and $\eta = 5$

Figure 4.4: Case V: Nature of $\xi(\phi)$, $V(\phi)$ and $f(\phi)$

$f(R)$ models [154]. To analyse the stability of Einstein's Static (ES) universe (which happens to be an asymptotic past solution) [155] in the theory we start with a pair of differential equations in a and H which are given by

$$\dot{a} = aH. \tag{4.33}$$

$$\dot{H} = \alpha H - \beta H^2, \tag{4.34}$$

The above equations form an autonomous system which can be analysed by a standard technique. The Einstein static universe solution corresponds to the critical point of the system $(a_0, 0)$. The ES universe in this case is unstable for $\alpha > 0$. It is noted that the stability of the Einstein Static universe under inhomogeneous perturbations has recently been studied in [156, 157].

4.5 Distance modulus curve:

We now probe late universe in the Emergent Universe scenario taking into account the observational results available from SNIa data [22]. The distance modulus is $\mu = 5 \log d_L + 25$ where d_L is the luminosity distance (in the unit of mega parsecs), given by

$$d_L = r_1(1+z)a(t_0) \tag{4.35}$$

where r_1 is given by :

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2)}} = \int_{t_1}^{t_0} \frac{dt}{a(t)} \tag{4.36}$$

We consider the scale factor for a EU scenario as was given in eq. (4.12),

$$a(t) = a_0(\sigma + e^{\alpha t})^{\omega}, \tag{4.37}$$

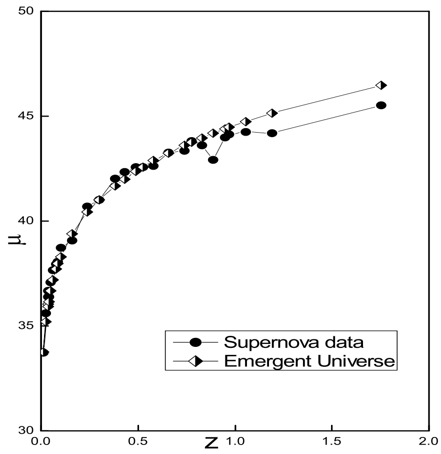


Figure 4.5: μ vs. z curve for Emergent universe and supernova data.

where a_0, σ and ω are constants. At late time the exponential term dominates and one can write

$$a(t) \sim a_0 e^{\alpha\omega t} \tag{4.38}$$

Since we are considering a flat universe, $k = 0$, eq. (4.36) yields

$$r_1 = \int_{t_1}^{t_0} \frac{dt}{a(t)} \tag{4.39}$$

Using the scale factor given by eq. (4.38) one obtains an expression for d_L :

$$d_L = \frac{z(1+z)}{H_0} \tag{4.40}$$

Note that the final expression for d_L does not depend on a_0, σ and ω. It is now possible to determine $\mu(z)$ numerically for an Emergent Universe scenario at different values of redshift parameters (z). The observed values of $\mu(z)$ at different z parameters [22, 55, 158, 159] along with that obtained from the present theory are given in Table (4.1).

z	*Supernova* μ	*EU* μ	z	*Supernova* μ	*EU* μ
0.038	36.67	36.0438240	0.430	42.33	42.0079418
0.014	33.73	33.7609468	0.490	42.58	42.3808310
0.026	35.62	35.1945228	0.526	42.56	42.5866208
0.036	36.39	35.9222306	0.581	42.63	42.8794593
0.040	36.38	36.1593860	0.657	43.27	43.2483587
0.050	37.08	36.6647158	0.740	43.35	43.6128241
0.063	37.67	37.1932883	0.778	43.81	43.7684760
0.079	37.94	37.7171620	0.828	43.61	43.9639519
0.088	38.07	37.9694771	0.886	42.91	44.1787963
0.101	38.73	38.2944627	0.949	43.99	44.3993095
0.160	39.08	39.4068091	0.970	44.13	44.4701090
0.240	40.68	40.4320839	1.056	44.25	44.7473544
0.300	41.01	41.0192423	1.190	44.19	45.1438747
0.380	42.02	41.6622327	1.755	45.53	46.4859129

Table 4.1: Data for Magnitude(μ) Redshift(z) plot from SNIa [22] and EU model

4.6 Discussion

In this chapter an EU scenario is constructed using a gravitational action with Gauss-Bonnet terms coupled with a dilaton field in the framework of a spatially flat 4 dimensional universe. It is observed that EU model evolved from a static universe in the infinite past. The Einstein static universe permitted here is unstable. We examine the evolution and different observational facts of the universe here. It is noted that the emergent universe scenario obtained in *Refs.* [136, 53] can be implemented in the modified theory of gravity with GB terms quite successfully with a suitable choice of coupled dilaton field and GB coupling parameters. The behaviour of the parameters are determined here and the results are classified into five different cases in the section (4.3). The result obtained here is new and interesting as it supports the view that the GB combination might have played a crucial role in driving both early inflation as well as late phase of acceleration. We note a case where the coupling parameter of the GB terms becomes dominant in the early era gradually decreases and thereafter increases in the late era. This result may be important to describe early inflation because of the dominance of the GB term in that epoch. The dominance of the GB term at late era is also important as the contribution of GB terms in the gravitational action is useful for accommodating dark energy. Another interesting solution is obtained with a new dilaton potential which is shown in fig. (4.4b). The late acceleration of the universe may be explained in this framework quite successfully as is shown in fig. (4.5) from the distance modulus curve. We also note cosmological solution where the dilaton field behaves like phantom [143, 144, 145, 146, 147] in the early era but it transits to non-minimally coupled scalar field in some later epoch in all the cases except in one case, where $\phi = H$ and $f(\phi)$ increases linearly with time. However it is evident from fig. (4.3b) that there is a regime where $V(\phi)$ remains negative. We compared our Emergent Universe model with recent SNeIa data [22] for late time evolution of the universe. Figure (4.5) shows a comparative study of supernovae magnitude ($\mu(z)$) vs. redshift (z) curve obtained from observation and that obtained theoretically from Emergent universe model. It is evident that observation fits well with the cosmological solution. Here we investigated an exponential evolution for the early era and an accelerated expansion for the late era. As pointed out in Ref. [136], there might exist an intermediate phase of particle creation which is required to be investigated in details.

Chapter 5

Observational Constraints on EOS parameters for Emergent Universe.

5.1 Introduction

Standard Big-Bang cosmology has been found to address a number of observational issues satisfactorily. Inspite of its overwhelming success people have often looked for an alternative cosmology, for example the steady state theory [160, 161] as the the Big-Bang model has fallen short in explaining quite a few issues. Not only that the model has failed in more than one occasion to provide us with explanation of observed phenomena in the universe, there are theoretical reasons also for which cosmologists have expressed their concerns over it. The main feature of the model, from where the name 'Big-Bang' appeared, is that it contains a timelike singularity in the past. Unless a consistent quantum theory of gravity comes to rescue our past is doomed in this singularity where laws of physics break down. In a way, then, Einstein's General Relativity (GR) predictions are no longer valied at some finite past. This is certainly a feature of much concern for the cosmologists and they sought to build cosmological models which does not have any timelike singularity in the finite past. In principle cosmologists looked for a viable model which permits an ever existing universe. Emergent Universe (EU) scenario is one of the alternative models talked about [137, 138]. A class of flat EU solution was proposed by Mukherjee *et al.* [53] which is interesting in this direction. Later the emergent universe scenario was implemented in the context of Brane World Gravity [140], Gauss Bonnet Gravity [162], Brans Dicke theory [141] etc. The model proposed by Mukherjee *et al.* [53] is however one of the simplest model and nicely fitted in the frame work of GTR with non linear EOS. The model was constructed on the following propositions:

1. The universe on large scale is completely homogeneous and isotropic.

2. As suggested by many experiments like WMAP the universe is spatially flat.

3. The universe is ever existing and remains large enough at any epoch so as to ignore quantum gravity effects.

4. The universe is accelerating as suggested by SNIa observation.

5. The universe may contain exotic kind of matter.

The last proposition makes it possible to consider a general kind of fluid equation such as the following:

$$p = A\rho - B\rho^{\frac{1}{2}} \tag{5.1}$$

where A and B are positive constants. This kind of equation of state (EOS) reminds us of a more general Chaplygin gas type one:

$$p = A\rho - B\rho^{\alpha} \tag{5.2}$$

The case considered by Mukherjee *et al.* [53] is a special case with $\alpha = 1/2$. These types of EOS were first introduced in cosmology to interpolate between two era like a matter dominated one and a de Sitter phase. In Chaplygin gas models, however, one would consider $\alpha < 0$. Fabris *et al.* [163] showed in 2007 that such interpolation is possible even with $\alpha > 0$ and those models can be considered as phenomenological realisation of a string specific configuration. The parameters involved in this model are to be fixed from observation and they play crucial role in the model itself. As shown by the authors [53] the parameter A decides the evolution of the fluid and it can mimic different composition of matter energy in universe for different choices of A. Other parameters play important role in determining different epochs of evolution of EU. In this work we are interested in constraining the parameters of EU model from observational data: (i) Observed Hubble Data (OHD) as quoted in [164]; (ii) Joint analysis of OHD with a certain measurement of BAO peak parameter as proposed in [165] where the authors consider a sample of Luminous Red Galaxy (LRG) measurement in Sloan Digital Sky Survey (SDSS); (iii) the joint analysis with OHD, BAO and CMB shift parameter as measured in WMAP3 results.

5.2 Field Equations

Hubble parameter (H) can be expreseed in terms of the redshift parameter (z) which is given by:

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{5.3}$$

Since components of matter (baryon) and Dark energy (exotic matter) are conserved separately, we can use energy conservation equation (1.3) together with EOS given by eq. (5.1) to determine the expression for the energy density:

$$\rho_{emu} = \left[\frac{B}{1+A} + \frac{1}{A+1}\frac{K}{a^{\frac{3(A+1)}{2}}}\right]^2, \tag{5.4}$$

where K is an integration constant that is required to be positive if we are to get an EU [53]. From eq. (5.4) it is evident that the energy density is composed of three different terms, where a constant term $((\frac{B}{1+A})^2)$ may be identified with a cosmological constant and the other two terms are identified with two kinds of fluids determined by the parameters A. For simplicity, eq. (5.4) can be rewritten as:

$$\rho_{emu} = \rho_{em_0}\left[A_s + \frac{1-A_s}{a^{3(A+1)/2}}\right]^2 \tag{5.5}$$

where $A_s = \frac{B}{1+A}\frac{1}{\rho_{em_0}^{\frac{1}{2}}}$ and $\frac{K}{A+1} = \rho_{em_0}^{\frac{1}{2}} - \frac{B}{A+1}$

. Using the Friedmann equation we express H in terms of redshift parameter (z) for the model, which is given by:

$$H(z) = H_0[\Omega_{b_0}(1+z)^3 + (1-\Omega_{b_0})[\frac{B+K(1+z)^{\frac{3(A+1)}{2}}}{B+K}]^2]^{\frac{1}{2}}, \tag{5.6}$$

with $\Omega = \Omega_{b_0} + \Omega_{em_0} = 1$, where Ω is composed of baryon and exotic fluids. Ω_{b_0} represents baryon energy density and Ω_{em_0} represents the exotic fluid density at the present epoch.

5.3 Method of model fitting and Contours :

5.3.1 Chi-square Minimisation Technique

Suppose there are N data points $(x_i, y_i), i = 1, 2, .., N$, to be fitted to a model that have M adjustable parameters $a_j, j = 1, 2, ..., M$. It is possible to predict a functional relationship between the measures of independent and dependent variables,

$$y(x) = y(x; a_1...a_M) \tag{5.7}$$

. For a particular set of data of x_i's and y_i's, we want to select fitted parameters that are "most likely". Suppose that each data point y_i has a measurement error that is independently random and distributed as a normal or Gaussian distribution around the true model $y(x)$. Also each data point (x_i, y_i) has its own known standard deviation σ_i. Then the probability of the data set is the product of the probabilities of each point,

$$P \propto \prod_{i=1}^{N} \left\{ exp\left[-\frac{1}{2}\left(\frac{y_i - y(x_i)}{\sigma_i}\right)^2 \right] \Delta y_i \right\}. \tag{5.8}$$

The factor Δy_i are the spread due to error occurring in each data point. To obtain the maximum probability data set it is necessary to maximise the above function. This is equivalent to maximising its logarithm or minimising the negative of its logarithm,i.e.,

$$\left[\sum_{i=1}^{N} \frac{[y_i - y(x_i)]^2}{2\sigma_i^2} \right] + constant\ terms \tag{5.9}$$

Subsequently one can minimise the chi-square function (χ^2) which is given by:

$$\chi^2 \equiv \sum_{i=1}^{N} \frac{[y_i - y(x_i)]^2}{2\sigma_i^2} \tag{5.10}$$

Redshift(z)	Hubble ($H(z)$)	Error (σ)	Redshift(z)	Hubble ($H(z)$)	Error (σ)
0.00	73	08.0	0.88	97	40.4
0.10	69	12.0	0.90	117	23.0
0.17	83	08.0	1.30	168	17.4
0.27	77	14.0	1.43	177	18.2
0.40	95	17.4	1.53	140	14.0
0.48	90	60.0	1.75	202	40.4

Table 5.1: Table for Observed Hubble Data (OHD) [164]

5.3.2 Constant Chi-square Boundaries as Confidence Limits

It is usual practice to summarise the probability distribution in the form of confidence limits rather than presenting all the details of the distribution. The total probability distribution is a function defined in the M dimensional parameter space spanned by all the parameters $\mathbf{a}$ $(a_1, a_2, ..., a_M)$. A confidence region is a region of that space containing a certain percentage of total probability distribution; e.g 99% confidence region would mean that there is a 99% chance that true parameter values fall within this region around the measured value. In general both the confidence level and the shape of the confidence region can be chosen by the experimentalist. Certain percentages are, however, customary in scientific usage: 68.3 % (generally the lowest confidence worth quoting), 95.4 % etc. Also when the method used to estimate the parameters is chi-square minimisation there is a natural choice for the shape of the confidence region (ellipsoids). For the observed data set the value of χ^2 is minimum for certain parameter values. If the vector of the parameter values is perturbed slightly from the best fit value then χ^2 increases. The region within which χ^2 does not increase more than a given amount $\Delta\chi^2$ defines some confidence region in the parameter space around the best fit values.

5.4 Observed Hubble Data (OHD) as a constraining tool

The EU considered here is implemented in a flat universe. Consequently, we consider baryonic matter and the exotic matter in a flat Friedmann universe to constrain the parameters. The Hubble parameter given by eq. (5.6) is a function of a number of variables, consequently we can re-write eq. (5.6) as :

$$H^2(H_0, A, B, K, z) = H_0^2 E^2(A, B, K, z), \tag{5.11}$$

where

$$E(A, B, K, z) = [\Omega_{b_0}(1+z)^3 + (1-\Omega_{b_0})[\frac{B + K(1+z)^{\frac{3(A+1)}{2}}}{B+K}]^2]^{\frac{1}{2}}. \tag{5.12}$$

For a given value of K, the best fit values for the unknown parameters of the model, namely A and B are determined by minimising a χ^2_{H-z} function which is given below.

$$\chi^2_{H-z}(H_0, A, B, K, z) = \sum \frac{(H(H_0, A, B, K, z) - H_{obs}(z))^2}{\sigma_z^2} \tag{5.13}$$

where $H_{obs}(z)$ is the observed Hubble parameter at redshift z and σ_z is the error associated with that particular observation (table.5.1). Since we are interested in determining the model parameters, H_0 is not important for our analysis. So we marginalise over H_0 to get the probability distribution function in terms of A, B, K only, which is given by

$$L(A, B, K) = \int dH_0 P(H_0) \exp\left(\frac{-\chi^2_{H-z}(H_0, A, B, K, z)}{2}\right), \tag{5.14}$$

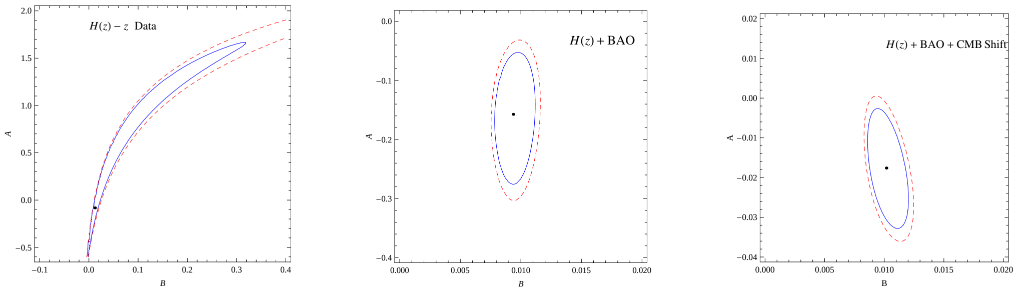
where $P(H_0)$ is the prior distribution function for the present Hubble constant. Here we consider Gaussian priors, $H_0 = 72 \pm 8$. One can minimise χ^2 by maximising the function $L(A, B, K)$. First of all we fix K at the best fitted value and contours in the two dimensional plane A-B are obtained. We are interested to obtain the range of parameters A and B respectively for a given K permitted by the EOS given by eq. (5.1). In fig. (5.1a) we draw 99% and 95% contours on A-B plane. It is evident that within 99% confidence one obtains $-0.5949 \leq A \leq 1.663$ and $-0.0022 \leq B \leq 0.3189$. Of course theoretically we must have $B \geq 0$ to obtain an EU solution for the model we are considering here. Thus some of the models permitted in the theory are ruled out by observations.

5.5 Joint analysis of OHD with BAO peak parameter

In this section we consider a different constraining tool where OHD and BAO peak parameter data [165] are analysed jointly.

5.5.1 BAO peak parameter $\mathcal{A}$

The angular scale of CMB acoustic peaks constraint the angular diameter distance to $z = 1089$ which is measured with very high accuracy provided that $\Omega_m h^2$ and $\Omega_b h^2$ values are known. For a flat cosmological model with cosmological constant as dark energy candidate, the distance depends on only one parameter (either Ω_m or Ω_Λ) and subsequently the distance measurement puts a constraint on Ω_m, Ω_Λ and H_0 with high precision. For a more general dark energy model (with varying equation of state parameter, $\omega(z)$) the parameter space is found larger and a degeneracy may be found



(a) Constraints:OHD

(b) Constraints:OHD + SDSS LRG

(c) constraints:OHD + SDSS LRG + WMAP 3

Figure 5.1: Constraints on Emergent Universe model from different observations: 99% (Dashed) and 95% (Solid) confidence contours are shown

for CMB. However Eisenstein *et al.* noted in [165] that one can still add high quality constraints from the measurements of acoustic scales at $z = 0.35$ if it is used with some other data, e.g. large scale structure at another redshift, supernovae distance measurement or a Hubble constant measurement. A new parameter namely, $\mathcal{A}$ is defined which for a flat universe with constant ω is given by (in low redshift region):

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{\int_0^{z_1} \frac{dz}{E(z)}}{z_1} \right)^{2/3} \tag{5.15}$$

where $\Omega_m = \Omega_b + (1 - \Omega_b)(1 - B/(K + B))^2$ and $z_1 = 0.35$ and $\mathcal{A} = 0.469 \pm 0.017$. Even if ω is not constant it gives a reasonable approximation for such small interval.

5.5.2 Joint Analysis with BAO parameter

We define $\chi^2_{BAO} = \frac{(\mathcal{A}-0.469)^2}{(0.017)^2}$, and for joint analysis we consider $\chi^2_{joint} = \chi^2_{H-z} + \chi^2_{BAO}$. The above joint analysis scheme with BAO sets new constraints on A and B (fig. (5.1b)), which up to 95% confidence level is: $-0.3053 \leq A \leq -0.0306$ and

$0.0077 \leq B \leq 0.0116$. Up to 99% confidence level the ranges found are: $-0.2757 \leq A \leq -0.0500$ and $0.0078 \leq B \leq 0.0114$.

5.6 Joint analysis with $H(z)$-z , BAO peak parameter and CMB shift parameter ($\mathcal{R}$)

Cosmic Microwave Background provides us with yet another tool to test different cosmological models. In this section we briefly discuss this option.

5.6.1 CMB Shift parameter

To tighten up the constraints on the Dark Energy models it is a common approach to use additional information about the distance scale from CMB observation in the form of so-called CMB Shift parameter [166]. The use of CMB Shift parameter as a probe of dark energy is based on the observation that different dark energy models will have almost identical CMB power spectra, provided the following conditions are satisfied:

1. $\omega_c = \Omega_c h^2$ and $\omega_b = \Omega_b h^2$ are equal,
2. primordial fluctuation spectrum is unchanged,
3. and the shift parameter ($\mathcal{R}$) is constant,

where $\mathcal{R}$ is given by:

$$\mathcal{R} = \frac{\omega_m^{1/2}}{\omega_k^{1/2}} sinn_k(\omega_k^{1/2} y) \tag{5.16}$$

with $sinn_k(x) = \{sin(x), x, sinh(x)\}$ for $k = +1, 0, -1$ respectively and

$$y = \int_{a_r}^{1} \frac{da}{\sqrt{\omega_m a + \omega_k a^2 + \omega_\Lambda a^4 + \omega_Q a^{1-3w}}}. \tag{5.17}$$

To define Shift parameter, the universe is assumed to be filled with matter (ω_m, ω_b), curvature (ω_k), cosmological constant (ω_Λ) and some form of dark energy (ω_Q) with constant equation of state parameter w. The integration is carried out from the time of recombination, a_r, until today, $a = 1$. In a spatially flat universe the Shift parameter is given by:

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_{ls}} \frac{dz'}{H(z')/H_0} \tag{5.18}$$

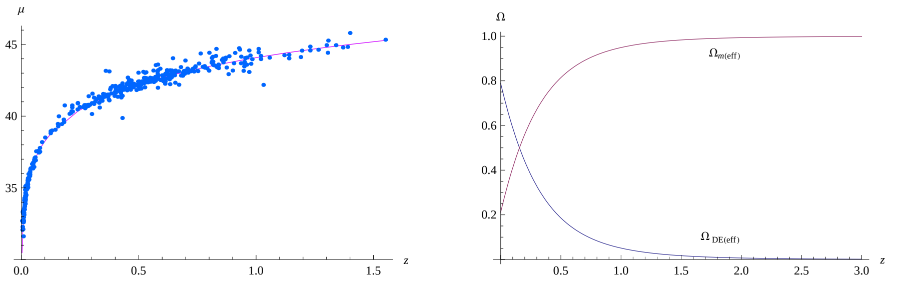
where z_{ls} is the z at the surface of last scattering. In this form shift parameter has been used in many works [166].

5.6.2 Constraints from Joint Analysis with CMB Shift Parameter

The WMAP3 data [166] gives us $\mathcal{R} = 1.70 \pm 0.03$. Thus we consider $\chi^2_{CMB} = \frac{(\mathcal{R}-1.70)^2}{(0.03)^2}$, with $\chi^2_{Tot} = \chi^2_{H-z} + \chi^2_{BAO} + \chi^2_{CMB}$ which imposes additional constraints on the model parameters. The statistical analysis with χ^2_{Tot} further tightens up the bounds on A and B. In fig.(5.1c) 95% and 99% contours are plotted on A-B plane. We determine constraints from this analysis: within 95% confidence level $-0.0360 \leq A \leq 0.0005$ and $0.0083 \leq B \leq 0.0125$. However, within 99% confidence limit we get $-0.0328 \leq A \leq -0.0024$ and $0.0086 \leq B \leq 0.0120$. The best fit value obtained here is given by $A = -0.0176$ and $B = 0.0102$. Finally we draw a magnitude ($\mu(z)$) vs. redshift (z) curve for our model with the best fit values of A, B and K and also show the same curve drawn from union compilation data for SNeIa [22] in fig. (5.1c).

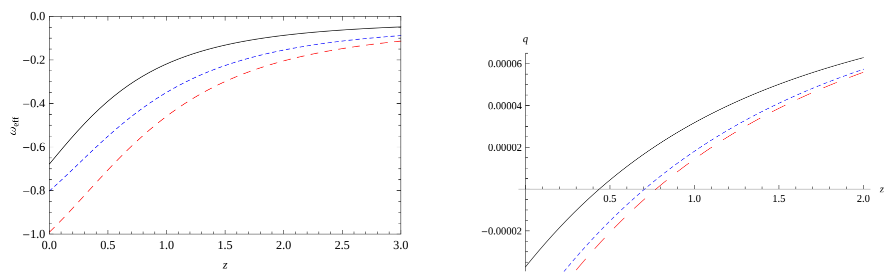
5.7 Discussions

A class of EU solutions are possible theoretically with a non-linear equation of state. It is shown that a non-linear EOS in turn represents a composition of different kinds of fluids [53] depending on the choice of the model parameter A. In this work we determine allowed ranges for the model parameters (particularly those involved with the EOS i.e., A and B). The EOS required for a flat emergent universe is considered here to obtain the constraints on the parameters using data available from cosmological observations. From the analysis it is noted that the EOS permitting the desired class of EU solutions might contain exotic matter ($A < 0$, $B > 0$). This is certainly not ruled out by the contemporary theory itself. It is shown that $H(z)$-z data puts a bound on the model parameters which is further studied in the light of the other observational data such as the measurement of BAO peak parameter value and CMB shift parameter. Most importantly, we see here that the later observations do not permit a positive value for the parameter A. Only small negative values seem to be allowed. Positive A values are permitted when we consider $H(z)$-z data only but the best fit value is found to be negative. However the possibility that $A \approx 0$ can not be ruled out with sufficient confidence since our analysis permits values of A which are even very close to zero ($A = 0$) and the model may be realised in the presence of dust and dark energy. Later in next chapter we present a detail analysis of EU for different choices of A. We also study the evolution of various cosmological parameters of the model. We plot density parameter for effective dark energy and effective matter content of the universe with the redshift in fig. (5.2b). We note that almost 80% of the present matter-energy content is effective dark energy and baryonic and nonbaryonic matter constitute the remaining part. The effective equation of state (ω_{eff}) for EU remains negative always which we plot in fig. (5.3a). The solid line corresponds to the curve drawn using best fitted values. Dash and dotted curves are drawn with typical model parameters values within 95% and 99% confidence levels respectively. The transition of the universe from a deceleration phase to an accelerating phase in recent past is depicted from the curve of deceleration parameter against redshift plotted in fig. (5.3b). The solid curve describes the one drawn with best fitted values and dotted and dash curves represent curves drawn with values within 99% and 95% confidence level respectively. It is found that certain class of EU solutions considered here is not ruled out by the observations. However, this class of EU solutions admits different composition of matter-energies in the universe and the nature of composition depends on the value of parameter A in particular. The observations do in fact severely constrain the nature of allowed composition of matter-energy by constraining the range of the values the parameters which may be important for a physically viable model.



(a) (b)

Figure 5.2: (a) $\mu(z)$ $vs.$ z curve comparison with SNIa data (b) Density parameters for EU



(a) (b)

Figure 5.3: (a)Effective EOS parameter for EU (ω_{eff})(b)Deceleration parameter (q) is plotted with redshift. Plot with the best fitted values of model parameters $k = 0.0101$, $A = -0.0176$ and $B = 0.0102$ (Solid) within 99% confidence (Dashing) and within 95% (Dotted).

Chapter 6

Observational viability of different Emergent Universe models

6.1 Introduction

In the previous chapter (chapter (5)) a class of Emergent Universe (henceforth, EU) model proposed by Mukherjee *et al.* [53] was considered to determine the constraints of the model parameters from observational considerations. In general the analysis of the EU model is based on three parameters, A and B from EOS and K, which is an integration constant obtained on integrating conservation equation. However, it was shown that for different values of the parameter A corresponds to different matter-energy density of EU model. The expression for ρ obtained is given in eq. (5.4) :

$$\rho_{emu} = \left(\frac{B}{1+A}\right)^2 + 2\frac{BK}{(1+A)^2}a^{-\frac{3(A+1)}{2}} + \frac{K^2}{(1+A)^2}a^{-3(A+1)}. \quad (6.1)$$

Evidently ρ has three different components evolving differently for different choices of A. The first term in equation (6.1) is a constant which may be identified with a cosmological constant. Mukherjee *et al.* quoted a few possible A values and discussed a composition of cosmic soup from which EU can be obtained. The values of A and corresponding composition are tabulated in (table. 6.1). Some of the candidates in

A Value	Composition
1	Dark energy, dust and stiff matter
$-1/3$	Dark energy, domain walls and cosmic strings
0	Dark energy, Exotic Matter and Dust
$1/3$	Dark energy, cosmic strings and radiation

Table 6.1: Different EU Models from ([53])

table (6.1) appear to be very much realistic. In this chapter different models of EU, suggested in [53] will be analysed using observational results. For a specific choice of composition of matter for EU model A is fixed. Earlier for a given K, contours for A and B are obtained which gives a limiting value for them. In this chapter, the range of values of K and B for a given A from the two dimensional contours drawn using different observational data will be calculated. A two dimensional plot between the density parameters of the models will be drawn to estimate their effective values. As far as observational constraints on the model parameters are concerned the method used has already been described in chapter (5). The observational data for (i) Observed Hubble Data (OHD) and (ii) measurement of BAO peak parameter from the study of Luminous Red Galaxies (LRG) in Sloan Digital Sky Survey (SDSS) will be considered here. Additionally we adopt the technique proposed by Vishwakarma and Narlikar [167] to compare the goodness of fit in each model.

6.2 Testing Credibility of EU Model Through Goodness of Fit

It has been seen in chapter (5) that observational data puts an effective constraint on model parameters through statistical hypothesis testing (in our case Maximum Likelihood). However, as argued by Vishwakarma and Narlikar [167] that constraints can also be obtained for model parameters irrespective of the quality of fit. Obviously, estimated parameters and their uncertainties are rendered useless if the fit itself is poor. In general a good fit is obtained if the minimised χ^2 function roughly equals the number of data points or more precisely the degrees of Freedom (Dof) which is nothing but $Number\ of\ data\ points - Number\ of\ fitted\ parameters$. There is, however, a more quantitative way to asses the goodness of fit in terms of χ^2- probability. If a fitted model provides the value of χ^2 as x in n DoF, this probability is given by:

$$P\left(x,n\right) = \frac{1}{\Gamma(n/2)} \int_{\frac{x}{2}}^{\infty} e^{-u} u^{n/2-1} du. \tag{6.2}$$

Qualitatively, P represents the probability of finding a worse fit to the data. But the probability, P, holds strictly only when the measurement errors are normally distributed, which in general not correct. But largely the effect of non-Gaussianity in errors is to create an abundance of outer points which thereby reduces the probability P. Usually, for this reason, models with $P > 0.001$ are considered acceptable. Keeping this in mind we try to find the χ^2- probabilities for different EU models. The analysis is presented in table (6.2).

It is evident that the the model with $A = -1/3$ can not be fitted sufficiently well.

Model	$P(OHD)$	$P(BAO)$	$P(CMB)$
$A = 1$	0.5721	0.562	0.0.520
$A = 1/3$	0.689	0.733	0.004
$A = -1/3$	0.715	0.004	$0 << .001$

Table 6.2: Goodness of fit for EU models

Therefore we look for other values of A discussed above.

6.3 Field Equations

Friedmann equation in a flat universe becomes:

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho}{3}} \tag{6.3}$$

where H is the Hubble parameter and a is the scale factor of the Universe.The usual conservation equation holds:

$$\frac{d\rho}{dt} + 3H\left(p + \rho\right) = 0 \tag{6.4}$$

Using the EOS given by eq. (5.1) in eq.(6.3) and eq. (6.4) one obtains:

$$\rho\left(z\right) = \left(\frac{B}{A+1}\right)^2 + \frac{2BK}{\left(A+1\right)^2}\left(1+z\right)^{\frac{3(A+1)}{2}} + \left(\frac{K}{A+1}\right)^2\left(1+z\right)^{3(A+1)} \tag{6.5}$$

where 'z' represents the cosmological redshift. The first term in the right hand side of eq.(6.5) is a constant which can be interpreted as cosmological constant and describing dark energy. Eq. (6.5) can be written as:

$$\rho\left(z\right) = \rho_1 + \rho_2\left(1+z\right)^{\frac{3(A+1)}{2}} + \rho_3\left(1+z\right)^{3(A+1)} \tag{6.6}$$

where $\rho_1 = \left(\frac{B}{A+1}\right)^2$, $\rho_2 = \frac{2BK}{(A+1)^2}$ and $\rho_3 = \left(\frac{K}{A+1}\right)^2$ represents densities at the present epoch. The Friedmann equation (eq. 6.3) can now be written in terms of redshift and density parameter as follows:

$$H^2(z) = H_0^2\left(\Omega_1 + \Omega_2(1+z)^{\frac{3(A+1)}{2}} + \Omega_3(1+z)^{3(A+1)}\right) \tag{6.7}$$

where we define density parameter: $\Omega = \frac{8\pi G\rho}{3H_0^2} = \Omega(A, B, K)$. For a given $A = A_0$ (say) we note that the nature of evolution for the variable parts of the matter energy density may now be established. Hence, choice of a suitable value for A leads to a known composition of fluids. For example, [168] considered the case $A = 0$ with dark energy, dark matter and dust in the Universe. Fixing A one can re-write eq. (6.7) as:

$$H^2(H_0, B, K, z) = H_0^2 E^2(B, K, z) \tag{6.8}$$

where,

$$E^2(B, K, z) = \Omega_\Lambda + \Omega_2(1+z)^{\frac{3(A+1)}{2}} + \Omega_3(1+z)^{3(A+1)}. \tag{6.9}$$

Here we have replaced the constant part of the DP (Ω_1) by a new notation Ω_Λ.

6.4 Analysis with observational data

6.4.1 Observed Hubble Data (OHD)

Using observed value of Hubble parameter at different redshifts (twelve data points listed in Observed Hubble Data by [164]) we analyse the model in this section. For the analysis we first define a chi square function as follows:

$$\chi^2_{OHD} = \sum \frac{\left(H_{Theory}(H_0, B, K, z) - H_{Obs}\right)^2}{2\sigma^2} \tag{6.10}$$

Model	B	K	χ^2_{min} (d.o.f)
$A = 0$	0.261	0.474	0.718
$A = 1$	1.931	0.166	0.818
$A = 1/3$	1.5600	0.470	0.737

Table 6.3: Findings: OHD

where H_{Theory} and H_{Obs} are theoretical and observational values of Hubble parameter at different redshifts respectively and σ is the corresponding error. Here, H_0 is a nuisance parameter and can be safely marginalised. We consider $H_0 = 72 \pm 8$ and a fixed prior distribution. A reduced chi square function can be defined as follows:

$$\chi^2_{red} = -2ln \int \left[e^{-\frac{\chi^2_{OHD}}{2}} P\left(H_0\right) \right] dH_0 \tag{6.11}$$

where $P\left(H_0\right)$ is the prior distribution. The regions of 68.3%, 95.5% and 99.8% confidence are shown in fig.(6.1a), in fig. (6.1b) and in fig. (6.1c) for different $A = 0$, $A = 1$ and $A = 1/3$ respectively. The best fit values are tabulated in table (6.3).

6.4.2 Joint analysis with BAO peak parameter

In the previous sections the standard values for H_0 are used. In this section we consider analysis which is independent of the measurement of H_0 and does not consider any particular dark energy model. We use here a method proposed by [165]. A model independent BAO (Baryon Acoustic Oscillation) peak parameter is defined for low redshift (z_1) measurements in a flat universe as:

$$\mathcal{A} = \frac{\Omega_m}{E\left(z_1\right)} \frac{\int_0^{z_1} \frac{dz}{E(z)}}{z_1} \tag{6.12}$$

Model	B	K	χ^2_{min} (d.o.f)
$A = 0$	0.260	0.475	0.818
$A = 1$	1.905	0.168	0.875
$A = 1/3$	1.646	0.451	0.707

Table 6.4: Findings: OHD+SDSS(BAO)

where Ω_m is the matter density parameter for the Universe. Now the chi square function can be defined as follows:

$$\chi^2_{BAO} = \frac{(\mathcal{A} - 0.469)^2}{2\,(0.017)^2} \tag{6.13}$$

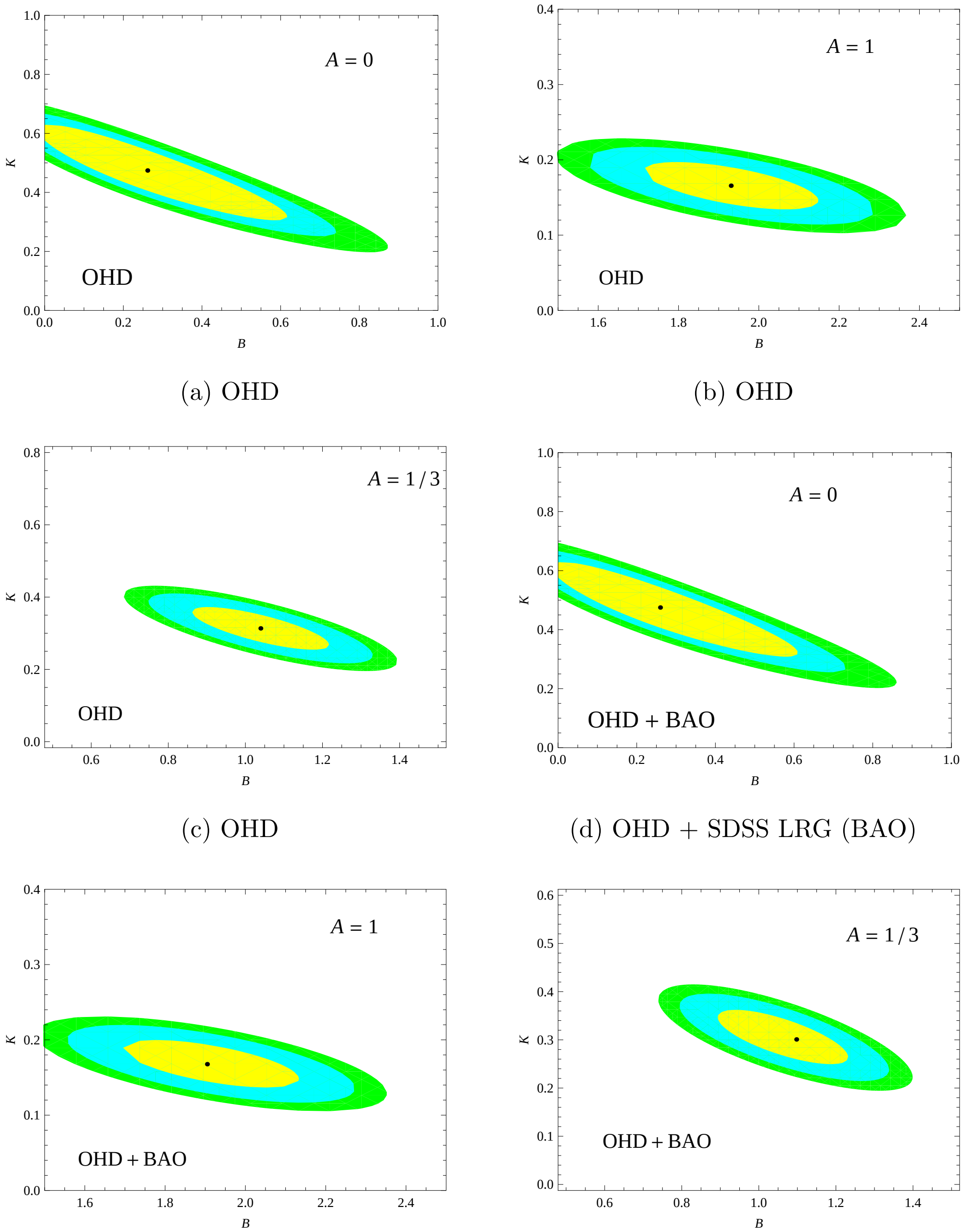
where we have used the measured value for $\mathcal{A}$ ($0.469 \pm .0.017$) as was obtained by [165] from the SDSS data for LRG (Luminous Red Galaxies) survey. Now we can define a total chi square function for our joint analysis as:

$$\chi^2_{tot} = \chi^2_{red} + \chi^2_{BAO} \tag{6.14}$$

The 68.3%, 95.5% and 99.8% regions obtained from this joint analysis are given in fig.(6.1d) for $A = 0$, in fig.(6.1e) for $A = 1$ and in fig. (6.1f) for $A = 1/3$. The Best fit values are displayed in table (6.4).

6.5 Density Parameters in defferent EU models

The best fit values for B and K corresponding to different models evoked by different choices of A are studied in the previous section. Here, we plot contours corresponding to $\Omega_1 - \Omega_2$ plane. The 68.3% (solid), 95.5% (dashed) and 99.8% (dotted) contours are shown in fig.(6.2a-6.2c). EU model with $A = 1$ permits a composition of dark energy (Ω_Λ), dust (Ω_1) and stiff matter (Ω_2) [53]. For $A = 1/3$, Ω_1 represents

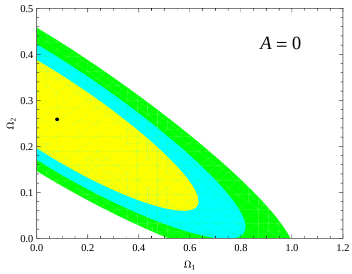


(a) OHD (b) OHD

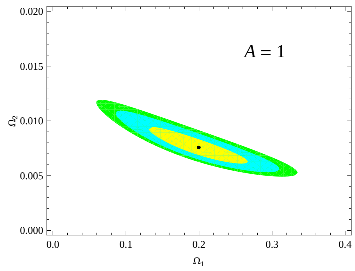
(c) OHD (d) OHD + SDSS LRG (BAO)

(e) OHD + SDSS LRG (BAO) (f) OHD + SDSS LRG (BAO)

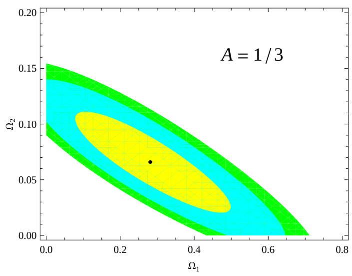
Figure 6.1: Constraints on model with different A value: 68.3% (innermost region), 95.5% (middle) and 99.8% (outermost region) regions are shown



(a)



(b)



(c)

Figure 6.2: Contours on $\Omega_2 - \Omega_3$ plane for (a) $A = 0$ (b) $A = 1$ and $A = 1/3$: 68.3%, 95.5% and 99.8% confidence Regions are shown.

Model	Ω_1	Ω_2	Ω_Λ
$A = 0$	0.079	0.259	0.662
$A = 1$	0.200	0.008	0.792
$A = 1/3$	0.281	0.066	0.653

Table 6.5: Findings: Analysis of Density Parameters

DP for cosmic strings and Ω_2 represents DP for radiation. The best fit values for Ω_1 and Ω_2 for different models are obtained which in turn determines the best fit values for Ω_Λ in the corresponding model as:

$$\Omega_\Lambda = 1 - \Omega_1 - \Omega_2 \tag{6.15}$$

We have shown the best fit values for the parameters of EU in table(6.5). However with WMAP 7 data the case with $A = -1/3$ gives a very poor fit (P value is much lower than acceptable) so the corresponding cosmological model is ruled out by observations..

6.6 Discussion

In this chapter we obtain observational constraints on the model parameters for different EU models in respect of different A values (5). The model parameters of the EU are constrained using the observed Hubble data (OHD) as well as using a joint analysis with the measurement of a BAO peak parameter from SDSS results on LRG study [165]. BAO peak parameter is so defined (for details see [165]) that it is independent of dark energy model. It is observed that the case $A = -1/3$ cannot be fitted well with WMAP 7 data. The present day value of the density parameters also become unrealistic in this case. From the analysis it came out that the case $A = -1/3$, which corresponds to evolution of the cosmic fluid which mimics as a composition of dark energy (cosmomological constant), domain walls and cosmic strings is not a viable model . As the evaluated value of the present day density parameter in the model with $A = -1/3$ is not realistic, the model with $A = -1/3$ may be ruled out. In the other three cases, namely for $A = 0$, $A = 1$ and $A = 1/3$, we obtain cosmological models with physically realistic density parameter. The best fit values for the model parameters B and K are determined. It is found that the model admits dark energy density close to that predicted by observations in ΛCDM cosmology with sufficient confidence. One model with $A = 0$ is particularly interesting because the fluid here behaves as a composition of dark energy, dust and exotic matter which is a reasonable option as far as composition of cosmic fluid is concerned. The analysis we adopted here involves kinematics only. It would be interesting to analyse and determine the model constraints using the dynamical aspects like structure formation etc. A more stringent constraint on the EU may be obtained for a viable candidate for cosmology. Also it will be useful to crosscheck whether $A = -1/3$ model can be rejected with enough confidence. All these issues may be interesting to explore in future with precision observations.

Chapter 7

Modified Generalized Chaplygin Gas and its observational constraints for cosmologies

7.1 Introduction

It is now generally believed from the recent cosmological observations like Type Ia Supernovae (SNIa)[22, 55], WMAP [56, 57, 58, 59] that the present universe is passing through a phase of cosmic acceleration. The origin of such expansion is not yet understood properly. When observational results are analysed in the framework of Big-Bang model it came out that more than 70% matter of our universe is composed of something having a negative pressure ! Commonly this type of matter is known as dark energy. More surprising is the fact that our known form of baryonic matter constitutes only about 4% of total matter energy density of universe. There is around 26% of Cold Dark Matter to account for the rest. In the literature a lot of choices have been discussed as prospective candidate for dark energy which in turn drives the present phase of acceleration. The simplest of choices is the cosmological constant. To name a few of other choices, there are model with: (i) homogeneous scalar field ϕ, whose effective potential $V(\phi)$ leads to an accelerated phase at a later stage of the universe [169, 170], (ii) a X-matter component, which is characterised by an equation of state $p = \omega\rho$, where, $-1 \leq \omega < 0$ [171], (iii) effects from extra dimensions [172, 173], (iv) an exotic fluid [53] including modification of the gravitational sector (Chapter 3).

In this chapter we consider CDM (Cold Dark Matter) and UDME (Unified Dark Matter Energy) models of the universe which use Modified Generalized Chaplygin Gas (MGCG) as the dark component of the cosmic fluid. In the case of CDM, MGCG is present along with radiation and baryonic matter. However in UDME, MGCG represents both dark energy and dark matter as a whole. Recently cosmologists have developed interest in Chaplygin Gas (henceforth, CG) as a plausible dark energy candidate. CG was first introduced in 1904 but in a very different field namely in aerodynamics. In fact CG emerges in string theory from the dynamics of a Generalized d-brane in a higher dimensional spacetime. It can be described by a complex scalar field which is obtained from a Generalized Born-Infeld action. The equation of state is given by

$$p = -\frac{A}{\rho} \tag{7.1}$$

where A is a positive constant, p and ρ are pressure and density respectively. Later a modified form of the equation of state was introduced [174, 175] of the form

$$p = -\frac{A}{\rho^{\alpha}} \tag{7.2}$$

with $0 < \alpha \leq 1$ which is known as Generalized Chaplygin Gas (GCG). It has two free parameter A (positive), α. Recently another modification of CG has come up [176]. The Modified Generalized Chaplygin gas (MGCG) is more general and contains three free parameters. The idea is to interpolate states of standard fluids at high pressures and at high energy densities to a constant negative pressure at low energy densities [177]. In addition it covers whole aspects of GCG. This model accommodates consistent (i) Gravitational lensing test [178, 179], (ii) Gamma-ray bursts [180]. The equation of state for this Modified Chaplygin Gas is given by

$$p = B\rho - \frac{A}{\rho^{\alpha}} \tag{7.3}$$

where A, B, α are arbitrary constants with $0 \leq \alpha \leq 1$. In what follows we show how the model parameters can be constrained from cosmological observations such as (i) Measurement of the dimensionless age parameter of the universe and (ii) Measurement of the Hubble Constant (H). There are some theoretical limits on both A and α from the stability criteria of the Chaplygin gas model [174]. In this work we look for the observational constraints on the parameter B for different values of A and α and see if a realistic model is permitted. We obtain observational bounds on B and describe how the viable range for B varies for CDM and UDME models.

As there are three free parameters, unlike GCG, we look for a suitable range of B parameter for MCG for a viable cosmological model accommodating the observational evidences. The parameters are determined by (i) Considering a dimensionless age parameter $H_o t_o$ and (ii) $H(z) - z$ Data analysis [181].

We investigate both CDM and UDME (Unified Dark Matter Energy) models in the following sections. UDME model refers to the model in which the Modified Chaplygin gas(MCG) represents dark matter and dark energy as a whole, where the

z Data	H(z)	σ
0.09	69	$\pm$ 12.0
0.17	83	$\pm$ 8.3
0.27	70	$\pm$ 14.0
0.40	87	$\pm$ 17.4
0.88	117	$\pm$ 23.4
1.30	168	$\pm$ 13.4
1.43	177	$\pm$ 14.2
1.53	140	$\pm$ 14.0
1.75	202	$\pm$ 40.4

Table 7.1: OHD Data from Wu and Yu work [181]

total energy density comprises of radiation, baryon and MCG energy density. In the case of Cold dark matter (CDM) model the constituents of our universe are radiation, CDM and MCG.

7.2 Field equations of the model

Using EOS (7.3) in the conservation equation (1.3) and on integration we get:

$$\rho = \left[\frac{A}{1+B} + \frac{C}{a^{3n}}\right]^{\frac{1}{1+\alpha}} \tag{7.4}$$

where C is an arbitrary constant and we denote $(1+B)(1+\alpha) = n$. The above equation (eq. (7.4)) can be written as:

$$\rho = \rho_o\left[A_S + \frac{1-A_S}{a^{3n}}\right]^{\frac{1}{1+\alpha}} \tag{7.5}$$

where z is redshift parameter, $A_S = \frac{A}{1+B}\frac{1}{\rho_o^{\alpha+1}}$, $\frac{a}{a_0} = \frac{1}{1+z}$ and we chose $a_0 = 1$ for convenience. Note that MCG reduces to GCG model when we set $B = 0$ in the above equation. The Friedmann equations can be written as:

$$H(z) = H_0[\Omega_{r0}(1+z)^4 + \Omega_{j0}(1+z)^3 +$$

$$(1-\Omega_{r0}-\Omega_{j0})[(A_s+(1-A_s)(1+z)^{3n})^{\frac{1}{1+\alpha}}]]^{\frac{1}{2}}. \tag{7.6}$$

The above equation can be written in terms of $a(t)$:

$$H(a) = H_0[\frac{\Omega_{r0}}{a^4}+\frac{\Omega_{j0}}{a^3} \\ +(1-\Omega_{r0}-\Omega_{j0})(A_s+\frac{1-A_s}{a^{3n}})^{\frac{1}{1+\alpha}}]^{\frac{1}{2}} \tag{7.7}$$

where $j=m$ for CDM model and $j=b$ for UDME model. The deceleration parameter ($q_0=-(\frac{a\ddot{a}}{\dot{a}^2})_{t_0}$) at the present time can be written as

$$q_0=\frac{3}{2}\left[\frac{\Omega_{j0}+\frac{4}{3}\Omega_{r0}+(1+B)(1-\Omega_{j0}-\Omega_{r0})(1-A_S)}{\Omega_{j0}+\Omega_{Cg_0}+\Omega_{r0}}\right]-1 \tag{7.8}$$

For a flat universe we have $\Omega_{j0}+\Omega_{Cg_0}+\Omega_{r0}=1$ which will be used to measure the parameters in the next section. In the above Ω_{Cg_0} represents the present day Modified Chaplygin gas energy density, Ω_{j0} is the present energy density of either cold dark matter (in CDM model) or of the baryonic energy density (in UDME model) and Ω_{r0} represents the present radiation energy density of our universe.

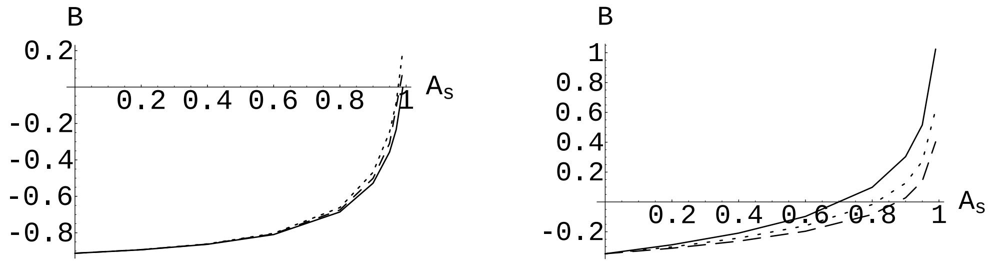
7.3 Constraints from the age of the universe

The age parameter for the universe is written as [182]:

$$t_0=\int_0^1\left[\frac{da}{aH(a)}\right] \tag{7.9}$$

where $\frac{a}{a_0}=\frac{1}{1+z}$ and $H(a)$ is given by eq. (7.7). MCG model predicts an age of the universe:

$$t_0=\frac{1}{H_0}\int_0^1\left[\frac{da}{af\left(a,\Omega_{j0},\Omega_{r0},A_S,B,\alpha\right)}\right] \tag{7.10}$$



(a) For CDM model:Dotted line for $\alpha = 0.0$,Dashed line for $\alpha = 0.2$,Thin line for $\alpha = 0.39$

(b) For UDME model:Thin line for $\alpha = 0.0$,Dotted line for $\alpha = 0.5$,Dashed line for $\alpha = 1.0$

Figure 7.1: Constraints on MCG model parameters in CDM and UDME model from the estimated age of the universe

with

$$f(a, \Omega_{j0}, \Omega_{r0}, A_S, B, \alpha) = \frac{H(a)}{H_0}. \tag{7.11}$$

We consider here from experimental facts the value $H_o t_o = 0.95$. Although it has some error limit in both sides, we take this value as standard. From the constancy of this parameter, we derive constraints on the parameters of the theory. For a given value of alpha we plot variation of A_s with B. We note the following :

Fig. (7.1a) shows variation of B with A_s for $\alpha = 0, 0.20, 0.39$ by dotted, dashed and thin lines respectively in CDM model. The value of A_S approaches 1 (0.97 to 1) for $0 \leq \alpha \leq 0.39$ the B parameter picks up positive values with a maximum 0.20. In fig. (7.1b) variation of B with A_s is shown for different α α=0,0.5 and 1 with thin, dotted and dashed lines respectively in UDME. In this case as the value of A_S is increased from 0.7 to 1 it is evident that the B parameter picks up positive value up to a maximum 1.02 for $0 \leq \alpha \leq 1$ in UDME model.

Look back time or the age of the universe can provide us with useful information about the model parameters of MCG both in CDM and UDME scenario. The variation of B along with A_s has been shown in fig. (7.1a) and (7.1b) for CDM and UDME models respectively. We note that in CDM model B lies between 0 to 0.20, where as in UDME model B lies between 0 to 1.02. Moreover, in CDM model B is positive when $0 \leq \alpha \leq 0.39$ and A_s between 0.97 to 1. In UDME we note that B is positive for $0 \leq \alpha \leq 1$ and A_s between 0.7 to 1.

7.4 Constraints $H(z) - z$ data

For a flat universe containing only radiation, cold matter (or baryon) and the MCG, the Friedmann equation can be expressed as

$$H^2(H_0, A_s, B, \alpha, z) = H_0^2 f^2(A_s, B, \alpha, z) \tag{7.12}$$

where,

$$f(A_s, B, \alpha, z) = [\Omega_{r0}(1+z)^4 + \Omega_{j0}(1+z)^3 +$$
$$(1 - \Omega_{r0} - \Omega_{j0})(A_s + (1 - A_s)(1+z)^{3n})^{\frac{1}{1+\alpha}}]^{\frac{1}{2}} \tag{7.13}$$

with $j = m$ for CDM model and $j = b$ for UDME model. The best fit values for model parameters A_s, B, α and H_0 can be determined by minimising as follows

$$\chi^2(H_0, A_s, B, \alpha, z) = \sum \frac{(H(H_0, A_s, B, \alpha, z) - H_{obs}(z))^2}{\sigma_z^2}. \tag{7.14}$$

We are interested in determining the model parameters here and H_0 is not important. We can marginalise over H_0 to evaluate the probability distribution function

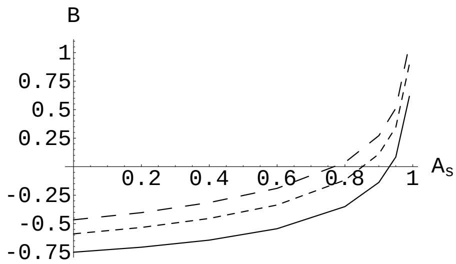
[182] for A_s, B, α as

$$L(A_s, B, \alpha) = \int \left[dH_0 P(H_0) \exp(\frac{-\chi^2(H_0, A_s, B, \alpha, z)}{2}) \right] \tag{7.15}$$

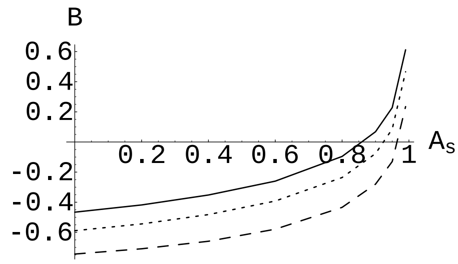
where $P(H_0)$ is the prior distribution function for the present Hubble constant. We consider Gaussian priors $H_0 = 72 \pm 8$. For -1σ level calculation the limit of integration will be from 64 to 72 and for 1σ level calculation the limit of integration becomes 72 to 80. Minimising χ^2 determines the maximum $L(A_s, B, \alpha)$ value. We determine the maximum value of the function $L(A_s, B, \alpha)$ by plotting the function with any of its parameter keeping other two fixed. As a result we get the maximum values of the parameters A_s, B and α. Consequently a relation between B and A_s for various α can be established. Here we use the 9 data points quoted in [181] from Observed Hubble Data (OHD).

In CDM model, variation of B with A_s for $\alpha = 0.01,\ 0.5,\ 0.99$ at -1σ, -2σ and -3σ level respectively are shown in figs. (7.2a)-(7.2f). We note that as the value of A_S tends to 1. We see that the B parameter picks up positive values (i) up to 1.07 (fig.7.2a), (ii) upto 0.62 (fig.7.2b), (iii) upto 0.36 (fig.7.2c) in accordance with the $H(z) - z$ data. Thus as α increases, B decreases. In UDME model variation of B with A_s for $\alpha = 0,\ 0.5,\ 1$ at $\pm 1\sigma$, $\pm 2\sigma$ and $\pm 3\sigma$ level respectively are shown in figs. (7.2d)-(7.2f). We note that as the value of A_S tends to 1 and the B parameter picks up positive values (i) upto 1.35 (fig.7.2d), (ii) upto 0.84 (fig.7.2e), (iii) upto 0.58 (fig.7.2f) in accordance with the $H(z) - z$ data. Thus as α increases, B decreases but compared to CDM model B parameter values in UDME is more for a given α and A_s.

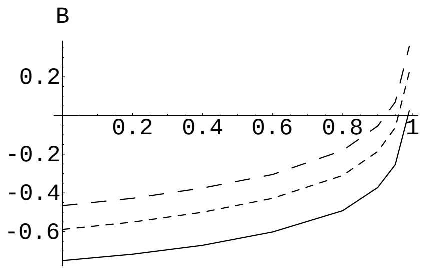
Thus in CDM the range for B is 0 to 1.07 and in Unified Dark Energy model the



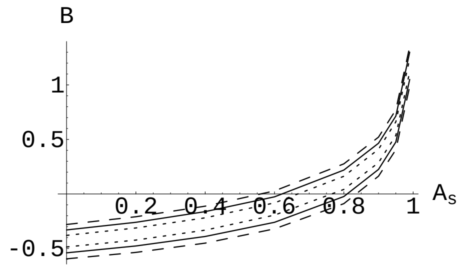
(a) CDM, $\alpha = 0.01$



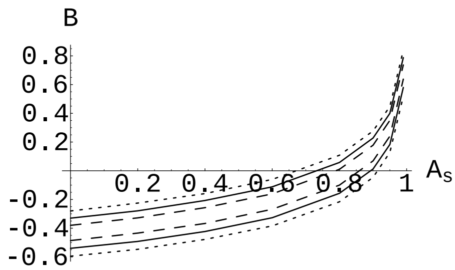
(b) CDM, $\alpha = 0.5$



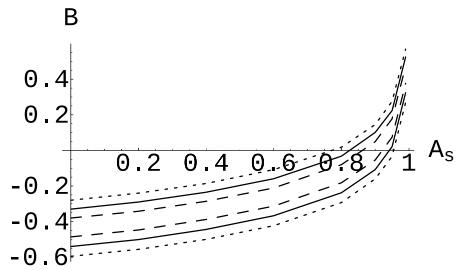
(c) CDM, $\alpha = 0.99$



(d) UDME, $\alpha = 0$



(e) UDME, $\alpha = 0.5$



(f) UDME, $\alpha = 1.0$

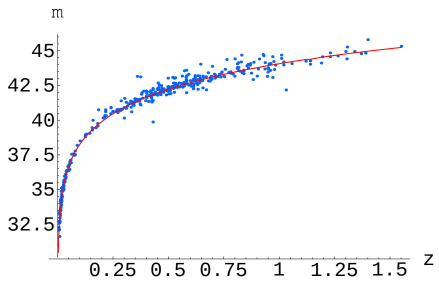
Figure 7.2: Constraints on MCG model parameters in CDM and UDME model from $H(z)$ vs. z data: 1σ, 2σ and 3σ levels are shown.

range is 0 to 1.35 upto 99.7 percent confidence level.

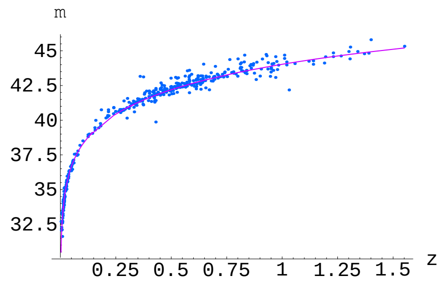
In CDM B is positive only when A_s is within 0.76 to 1 for α 0 to 1 and in UDME B is positive (so, permissible) only when A_s is within 0.57 to 1 for α lying between 0 to 1.

7.5 Viability of the model from union compilation of supernova magnitudes and redshift data

We have already found the best-fit values of the parameters of MCG in our models from H(z)-z data part. For CDM model the best-fit values of the parameters are $A_s = .99$, $B = .01$, $\alpha = .01$ and for UDME model it is $A_s = .8$,$B = .06$, $\alpha = .11$. In order to check its validity we used these best-fit values to find supernovae magnitudes at different redshift for the two models under our consideration. From there we have drawn supernovae magnitudes vs redshift curve using those best-fit values of the models. We compared these supernovae magnitude (μ) vs. redshift (z) curve of those two models to the original curve of union compilation data [183] (between those two parameters). Fig.(7.3a) shows a plot of $\mu(z)$ vs. z obtained from the CDM model (the continuous line) along with that obtained from union compilation data (the dots). The same curve is drawn for the UDME model in Fig. (7.3b) (continuous line for UDME model and dots for union compilation data). As one can see from these two curves, both the CDM and UDME models are in excellent agreement with union compilation data.



(a) CDM model



(b) UDME model

Figure 7.3: $\mu(z)$ vs. Z curves for CDM (a) and UDME (b) model and union compilation data (in the figure m stands for $\mu(z)$)

7.6 Discussion

Cosmological models with Modified Chaplygin Gas, presented here, contain three different parameters: A_s, B and α. We determine permissible range of values of B parameter from the age constancy. In section (7.3) we plot B *vs.* A_s for different values of α in figs. (7.1a) and (7.1b). The figures are plotted for both positive and negative values of B. In the case of CDM we note that B can pick up positive values upto 0.2 for $.97 \leq A_s < 1$, $0 \leq \alpha \leq 0.39$. However, for UDME model we note that B can pick up positive values upto 1.02 for $0.7 \leq A_s < 1$, $0 \leq \alpha \leq 1$.

In section (7.4), chi-square minimisation technique is used to constrain B from the Hubble parameter vs. redshift data. Here, we restrict to positive values of B to obtain a viable cosmology with MCG. The constraints on B are : (i) $0 \leq B \leq 1.07$ for $0.76 \leq A_s < 1$, and $0 \leq \alpha \leq 1$ in CDM and (ii) $0 \leq B \leq 1.35$ for $0.56 \leq A_s < 1$, $0 \leq \alpha \leq 1$ for UDME. For UDME model the range of values of B is found to be more than that of CDM. If the age constant parameter is decreased then we note that the values of B permitted by CDM and UDME model are in agreement with that found by chi-square minimization of the observed $H(z) vs z$ data. Consequently the limiting value of the age of our universe is pushed to lower values ($t < 13.6$ *Billion years*).

The best-fit values of the parameters obtained here for CDM and UDME models are in agreement with union compilation data. We note that the best-fit values of our models are $A_s = 0.99$,$B = 0.01$,$\alpha = 0.01$ for CDM and $A_s = 0.8$,$B = 0.06$,$\alpha = 0.11$ for UDME model.

Chapter 8

Concluding Remarks and Future Plan

8.1 Concluding Remarks

Since the discovery of cosmic acceleration from Supernova observations hundreds of supernovae have been observed only to arrive at the same conclusion. These observations are over broad range of redshifts so as to reduce the systematic errors and statistically strengthen the idea that we live in an accelerating universe. Recent studies, solely based on SN-Hubble observations and independent of GR, strongly favour the accelerating universe (at $7 - 9\sigma$ level) as well [22]. However, the physical origin of such a phenomenon is still a mystery tied together with many other important problems in cosmology. Apart from these problems in late universe there are problems in the description of early universe as well. Although the inflationary universe has become a part of the standard cosmological models till today we do not have a standard theory for the inflation itself. Moreover the physics of early universe is very speculative in nature as we are yet to find a theory of quantum gravity. In

this thesis results of investigation on some of these issues, particularly problems of early universe and late universe are presented. The accelerating universe problem is generally addressed from two alternative view points : (i) considering some modified theory of gravity and (ii) evoking the concept of dark energy in the framework of general theory of relativity. Regarding early universe and its problem we ventured to test some models of cosmology, namely a class of Emergent Universe (EU) models, which have the prospect of solving many of the long standing problems arising from the standard Big-Bang models. It is worth mentioning here that these models are by no means complete but deserves some exploration.
In this thesis I have explored some aspects of both the alternatives mentioned above.

In chapter (2), cosmological models with $f(R)$ gravity are explored. The models studied here are toy models of the universe. We found that a late accelerating universe can be successfully incorporated within the framework of $f(R)$ gravity. In fact, we have found that there can be a number of models within $f(R)$ gravity framework which accommodate a late time accelerating phase of the universe successfully. The duration of the accelerating phases in different models depend on the various model parameters.
In chapter (3), the creation of a universe with or without Primordial Black Hole pairs is studied. Primordial Black Holes are important cosmological objects for many reasons and can serve as relics of early universe when they were formed. We have estimated the probability of creation of universe with or without Primordial Black Hole Pairs in a higher dimensional universe within the framework of $f(R)$ gravity.

In chapter (4), Emergent Universe scenario in Gauss-Bonnet gravity coupled with a dilaton field is explored. It is found that Emergent Universe model can be successfully implemented in this framework. An interesting solution is noted where Gauss-Bonnet term dominates the dynamics of the universe in the past and then becomes negligible. However, the terms again dominates the late time evolution of universe and gives rise to an accelerating phase. GB term is very important and it has a rich structure in the theory.

In chapter (5, 6), a class of Emergent Universe model in the light of recent cosmological observations are investigated. It is found that a number of models belonging to the class can not be ruled out from present observational data. We also obtained constraints on model parameters imposed by various observations.

Finally in chapter (7), the observational constraints imposed on parameters of Modified Chaplygin Gas models are determined for viable cosmology.

A scientific view of cosmology has matured during 20^{th} century. Over the last decade the subject was brought under sharp focus both theoretically and experimentally. We are now in an era of precession cosmology. However, theoretical riddles are yet to be settled. There are conceptual problems in our understanding of both early and late universe. The whole evolutionary scenario of the universe right from the beginning to the present is not yet understood in a single theoretical framework. I hope that the high precision experiments in future, particularly PLANCK mission which can probe very near to the origin of the universe will enrich us with knowledge on cosmic evolution. The advances of High-Energy physics would be immensely helpful for cosmologists also. The ongoing Large Hadron Collider (LHC) experiments at CERN, Switzerland is expected to give hints on some of the phases of matter we need in building cosmological models including existence of higher dimensions. The next generation experiments in cosmology will be aimed at more challenging problems such as constraining equation of state parameter (ω), nutrino masses etc. The progress is likely to be slower since it would be difficult to control the systematic errors to the required level of precession which is already very high. However, all these, let us hope, will illuminate to understand some of the conceptual and technical issues in cosmology and open up new horizons in our understanding of the universe.

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